

## A practical model for predicting soil water deficit in New Zealand pastures

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of potential evapotranspiration, *PET*, and total readily available water (*RAW*) per day. *RAW* is defined as all of the water in the rapidly recharged surface zone plus a proportion of the water in the remainder of the soil profile. By validation against 11 historical data sets, the model is shown to give accurate predictions of soil water deficit across a range of New Zealand flat-land pastoral soils. The model parameters can be easily estimated from commonly available soil properties (soil order classification, and available water holding capacity) without the need for additional site-specific calibration. This model provides an easily used, practical decision tool for the management of drought, allowing early prediction of decline in pasture growth and estimates of required irrigation.

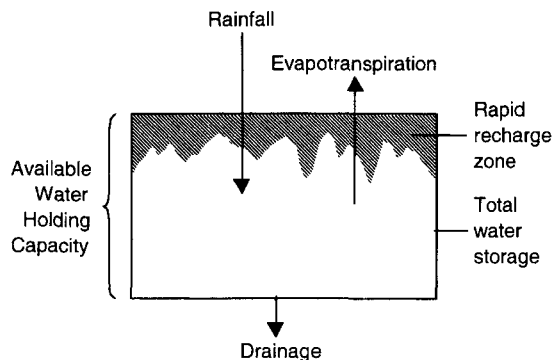
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**Abstract** Soil water is the single most important resource for pasture and crop production in New Zealand farms. Because soil water is difficult to measure, however, the ability to predict soil water status from daily weather data is valuable, and has application for on-farm irrigation, stocking, and supplementation decisions. In this paper a practical water balance model is presented. The model uses daily rainfall and potential evapotranspiration (*PET*) estimates to predict changes in the water content in two overlapping soil zones: a rapidly recharged (and depleted) zone of unspecified depth, and the total plant rooting zone. The use of two zones improves predictions of actual evapotranspiration and plant stress compared with models that use only one zone. An important factor determining the success of soil water models is the ability to predict actual evapotranspiration, *AET*. In this model actual evapotranspiration, *AET*, is calculated as the lesser

## INTRODUCTION

As well as providing an essential component of pasture growth models (e.g., McCall 1984; Thornley 1998; Woodward 1999), the ability to predict soil water status may be used to schedule irrigation or forecast the onset of drought, allowing de-stocking, feed rationing, or early purchase of supplementary feeds. A variety of soil water models exist in the literature, ranging from simple water balances to detailed mechanistic models based on Richards' equation (Feddes et al. 1988; Leenhardt et al. 1995; de Jong & Bootsma 1996; Akinremi & McGinn 1996). For practical application the former are often adequate (Rickert 1984; de Jong & Bootsma 1996), although multi-layer water balance models are preferred over those predicting water in a single layer only (Calder et al. 1983). Most of these models have been designed only for deep soils at flat sites.

The simple two-layer water balance model published by Scotter et al. (1979a) represented an



**Fig. 1** Structure of Scotter et al.'s (1979a) simple water balance model.

important advance in modelling soil water deficit for flat-land pastoral soils in New Zealand. By including a zone which is depleted and recharged preferentially, the Scotter et al. (1979a) model offers an ingenious solution to the problem encountered by single-layer water balance models (e.g., Baars et al. 1977; McAneney et al. 1982), namely that full-profile models under-predict pasture transpiration and growth following short-lived re-wetting of the topsoil during a prolonged dry period. Furthermore, because the two zones in the Scotter et al. (1979a) model overlap, water movement between, or water extraction from, the two zones does not need to be modelled explicitly. The structure of the Scotter et al. (1979a) model is shown in Fig. 1.

A major component of all soil water models is the prediction of evapotranspiration (Leenhardt et al. 1995). Whereas a number of proven models exist for (weather limited) potential evapotranspiration, *PET* (Priestley & Taylor 1972; Coulter 1973; Smith et al. 1996), models of actual (soil water and/or vegetation limited) evapotranspiration, *AET*, have generally been empirical, and only locally applicable (Baier & Robertson 1966; Calder et al. 1983; de Jong & Bootsma 1996). *AET* models for use in pasture are less problematic in this regard, as the "crop" canopy is usually closed, meaning that evaporation from the bare soil can be neglected (McAneney et al. 1982), and a mature root network may be assumed (cf. Baier & Robertson 1966). Furthermore, studies on the effects of grazing intensity or pasture cover on soil water status have yielded equivocal results (Barker et al. 1985; Barker & Chu 1985; Harris et al. 1999) and for this reason are often also neglected (Barker & Chu 1985; Leenhardt et al. 1995).

Scotter et al. (1979a) used the Priestley & Taylor (1972) *PET* model, and proposed a linear relationship between soil water deficit and *AET* on both theoretical and observational grounds. A similar approach has been used by other authors (Baier & Robertson 1966; McAneney et al. 1982; Barker et al. 1985), and has been shown by Calder et al. (1983) to work well in comparison with other *AET* models. However, like other published *AET* models, the Scotter et al. (1979a) *AET* model requires soil-specific empirical constants, which can only be obtained through detailed trial work at the site in question, or potentially from published values, which have not been forthcoming. This has probably limited the uptake and utility of Scotter et al.'s (1979a) model, despite it having been validated for an irrigated and un-irrigated Tokomaru silt loam soil at Palmerston North (Scotter et al. 1979a; Barker et al. 1985). One goal of the work described in this paper, therefore, was to modify Scotter et al.'s (1979a) model so that a commonly measured soil property (available water holding capacity) could be used in place of empirical constants in estimating *AET*.

A second goal was to rewrite Scotter et al.'s (1979a) model in differential equation notation to make it more compatible with the many recent pasture growth models which use this formalism (e.g., Johnson & Thornley 1983; Woodward 1997, 1999; Thornley 1998). This has several advantages. Firstly, the model can be written compactly, and the formalism enforces a certain degree of internal consistency which is not possible in computer code descriptions of models. Secondly, the model definition is independent of the simulation timestep (or method of integration, Press et al. 1989). This allows different timesteps or integration methods to be used on the same model, if desired.

The main goal of this work, however, was to provide a simple and reliable water balance model for use in decision-support tools for New Zealand farms.

### Model development and testing

The main processes resulting in water loss from soil are drainage, evaporation from the soil surface, and transpiration by plants. In most soils, rainfall and irrigation applied in excess of water storage capacity ("field capacity") drains rapidly, and soil moisture can usually be assumed to be near field capacity after 3 days (Scotter 1977). In poorly drained soils, however, excess water may form a "perched" water table. In the soil water balance model of Scotter et

al. (1979a), drainage of water above field capacity was assumed to be instantaneous, and this assumption was found to work well for the data sets studied here also.

Soil water deficit ( $W$ ) is defined as the difference between current soil water content and field capacity, in millimetres. This and other symbols used in this paper are listed in Table 1. A number of data sets, from flat-land sites representing a range of New Zealand climatic regions, soil types, and irrigation regimes, were collected in order to study the pattern of soil water depletion and recharge, and to construct and validate a model to predict soil water deficit under pasture. These data sets are listed in Table 2.

In each case daily weather data from the experimental site (or nearby) were used to calculate recharging of soil moisture due to rainfall or irrigation and losses of soil water due to evapotranspiration. For each data set the root mean squared error of prediction ( $RMSEP$ , Wallach & Goffinet 1989) was calculated to assess the model goodness-of-fit.

## Potential evapotranspiration

Evaporation and transpiration are usually considered together as evapotranspiration. Unless soils are particularly dry (as typically occurs in later summer), evapotranspiration is limited by weather and pasture cover only. Potential (weather limited) evapotranspiration ( $PET$ ) can be estimated from daily climatic data such as temperature, radiation, or sunshine hours, using any one of a number of models (Smith et al. 1996). Scotter et al. (1979a) used the model of Priestley & Taylor (1972), which requires as inputs only mean daily temperature and net radiation receipt. The Priestley-Taylor model (Priestley & Taylor 1972) has been shown to work well in humid regions provided the effect of ventilation is small relative to that of radiation (Tanner & Ritchie 1974; Smith et al. 1996). Analysis of weather data across a range of sites, however, shows that this requirement is often not met in New Zealand, where the relative contribution of the ventilation effect is frequently large enough to require a Priestley-Taylor factor considerably greater than the 1.26 commonly used (see

**Table 1** Table of symbols used in the main body of the paper.

| Symbol      | Value/Units                  | Description   |
|-------------|------------------------------|---|
| $\alpha$    | $\text{d mm}^{-1}$           | "readily available" water coefficient                               |
| $\beta$     |                              | ratio of $AWHC$ to $AWHC_{76}$                                      |
| $\rho$      | $\text{d}^{-1}$              | proportion of $RAW$ able to be extracted in one day                 |
| $\theta_s$  | $\text{cm}^3 \text{cm}^{-3}$ | volumetric soil water content near the soil surface                 |
| $\psi_s$    | kPa                          | soil water potential near the soil surface                          |
| $A$         | kPa                          | empirical constant in the $\psi_s$ conversion                       |
| $AET$       | $\text{mm d}^{-1}$           | actual evapotranspiration   |
| $AWHC$      | mm                           | available water holding capacity of soil to rooting depth           |
| $AWHC_s$    | mm                           | available water holding capacity of soil in the rapid recharge zone |
| $AWHC_{76}$ | mm                           | measured available water holding capacity of soil to 76 cm depth    |
| $B$         | 2–18                         | empirical constant in the $\psi_s$ conversion                       |
| $BD$        | $\text{g cm}^{-3}$           | bulk density of soil  |
| $d$         | cm                           | maximum root depth of pasture                                       |
| $FC$        | $\text{cm}^3 \text{cm}^{-3}$ | volumetric field capacity near the soil surface                     |
| $GSWC$      | $\text{g g}^{-1}$            | gravimetric soil water content                                      |
| $GSWC_s$    | $\text{g g}^{-1}$            | gravimetric soil water content near the soil surface                |
| $LAI$       | $\text{m}^2 \text{m}^{-2}$   | leaf area index of pasture  |
| $PET$       | $\text{mm d}^{-1}$           | potential evapotranspiration  |
| $PWP$       | $\text{cm}^3 \text{cm}^{-3}$ | volumetric permanent wilting point near the soil surface            |
| $r$         | $\text{d}^{-1}$              | relative rate of drainage   |
| $rain$      | $\text{mm d}^{-1}$           | daily rainfall  |
| $RAW$       | mm                           | readily available water   |
| $RAW_s$     | mm                           | readily available water in the rapid recharge zone                  |
| $SWD$       | mm                           | soil water deficit  |
| $W$         | mm                           | soil water deficit (usually negative)                               |
| $W_s$       | mm                           | soil water deficit in the rapid recharge zone                       |
| $W_z$       | mm                           | soil water deficit in the top $z$ cm of soil                        |
| $z$         | cm                           | depth to which soil water measurements are taken                    |

**Table 2** List of data sets used in testing the water balance model.  $AWHC_{76}$ , measured available water holding capacity between  $-0.02$  MPa and  $-1.5$  MPa, to 76 cm depth;  $\beta$ , best fit multiplier for  $AWHC_{76}$ ;  $RMSEP$ , root mean squared error of prediction of the model (in the same units as the data measured); N/A, not available.

| Fig. | Data set             | Soil type               | Soil order | Data measured           | Location                               | $AWHC_{76}$     | Fitted $\beta$ | $RMSEP$           |
|------|----------------------|-------------------------|------------|-------------------------|--|-----------------|----------------|-------------------|
| 2    | Parfitt et al. 1985a | Judgeford silt loam     | Brown      | 170 cm SWD              | Taita                                  | 92              | 2.4            | 12.6              |
| 3    | Watt 1977            | Waikiwi silt loam       | Brown      | 90 cm SWD               | Invercargill                           | 128             | N/A            | 11.6 <sup>d</sup> |
| 4    | Watt 1977            | Otokia silt loam        | Pallic     | 200 cm SWD              | Dunedin Airport                        | 96              | 2.3            | 15.2 <sup>e</sup> |
| 5    | Scotter et al. 1979a | Tokomaru silt loam      | Pallic     | 170 cm SWD              | Massey University                      | 91              | 2.5            | 14.1              |
| 6    | Barker et al. 1985   | Tokomaru silt loam      | Pallic     | 100 cm SWD <sup>a</sup> | Massey University                      | 96 <sub>f</sub> | 2.0            | 6.5               |
| 7    | Korte & Chu 1983     | Ohakea silt loam        | Pallic     | 30 cm SWD <sup>b</sup>  | Massey University                      | 92 <sup>f</sup> | 2.4            | 5.7               |
| 8    | Fraser et al. 1999   | Lismore stony silt loam | Brown      | 30 cm GSWC              | Winchmore                              | 61 <sup>g</sup> | 1.2            | 1.4               |
| 9    | Parfitt et al. 1985b | Stratford silt loam     | Allophanic | 180 cm SWD              | Taranaki Agricultural Research Station | 112             | 1.7            | 11.5              |
| 10   | McAneney & Judd 1983 | Horotiu sandy loam      | Allophanic | 140 cm SWD <sup>c</sup> | Hamilton Airport                       | 101             | 2.0            | 4.7               |
| 11   | Baars et al. 1977    | Horotiu sandy loam      | Allophanic | 30 cm SWD               | Rukuhia                                | 101             | 1.9            | 8.1               |
| 12   | Baars et al. 1977    | Te Kowhai clay loam     | Gley       | 30 cm SWD               | Rukuhia                                | 92              | 1.5            | 9.8               |
| 13   | Harris et al. 1999   | Te Kowhai silt loam     | Gley       | 7.5 cm GSWC             | Ruakura                                | 176             | 1.1            | 6.8               |

<sup>a</sup> These data were reduced by 25 mm to account for initial pore content above field capacity on Day 0, and then increased by 13% to account for extraction below 100 cm depth.

<sup>b</sup> These data were increased by 130% to account for extraction below 30 cm depth.

<sup>c</sup> The 1980-81 data were increased by 26 mm to correct for the calculated soil water deficit on 9 October 1980. McAneney & Judd (1983) assumed that the soil was at field capacity on this day.

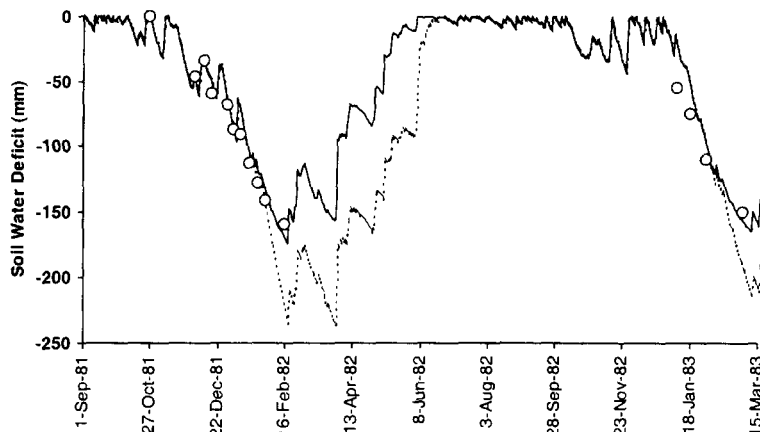
<sup>d</sup> The best fit was obtained by reducing PET to 82% of the FAO Penman-Monteith value. This may have been due to low LAI at this site.

<sup>e</sup> The best fit was obtained by reducing PET to 69% of the FAO Penman-Monteith value. This may have been due to low LAI at this site.

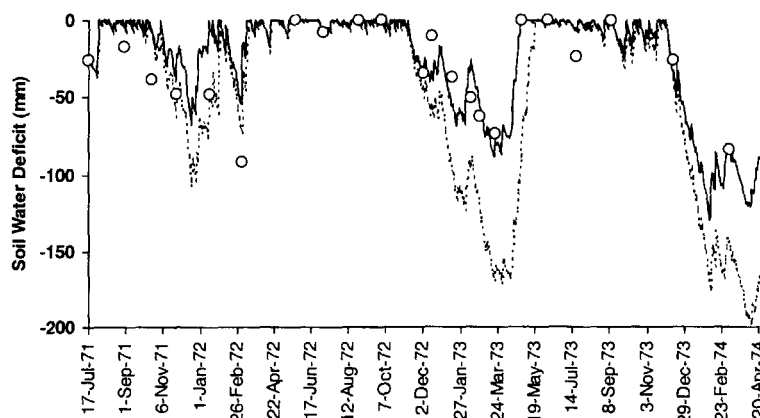
<sup>f</sup> Data from Paraha silt loam, a similar soil, were used.

<sup>g</sup> A. C. Bywater, E. S. Burt, R. F. Zyskowski unpubl. data. This soil is shallow and stony, so fitted parameters may not be reliable.

**Fig. 2** Fit of the water balance model (solid line) to soil water data (*SWD* to 170 cm depth, circles) from Taita (Parfitt et al. 1985a). The dotted line shows the prediction if *PET* is used instead of *AET*.



**Fig. 3** Fit of the water balance model (solid line) to soil water data (*SWD* to 90 cm depth, circles) from Invercargill (Watt 1977). The dotted line shows the prediction if *PET* is used instead of *AET*. Evapotranspiration was probably limited by pasture LAI at this site: the best fit (solid line) was obtained when *PET* was reduced to 82% of its expected value.



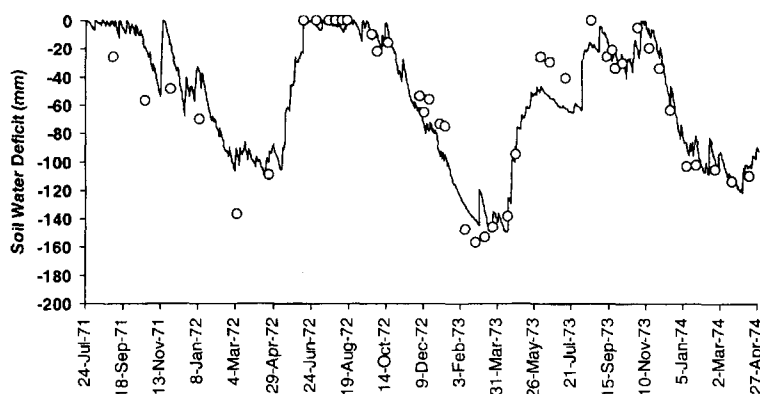
Appendix). Models that take ventilation effects into account explicitly are therefore preferred. The FAO version of the Penman-Monteith *PET* model was used in this paper, because it allows for the effects of humidity and wind, and has been validated globally (Smith et al. 1996). This model is described in detail in the Appendix.

The dotted line in Fig. 2 shows predictions of soil water where water loss is assumed to be equal to *PET* against soil water data measured by Parfitt et al. (1985a) at Taita, southern North Island (New Zealand Soil Bureau). Except where soil water deficit fell below about 140 mm in late summer, the predictions are good. The methods used to calculate water loss during dry periods (the solid line in Fig. 2) will be discussed below.

### Leaf Area Index effects

Studies of pasture management effects on evapotranspiration have often yielded equivocal results

(Barker et al. 1985; Barker & Chu 1985; Harris et al. 1999) and are therefore often neglected in modelling (Barker & Chu 1985; Leenhardt et al. 1995). If pasture Leaf Area Index (LAI) is low, or there are patches of bare ground, pasture plants may not be able to extract the potential amount of evapotranspiration even when the soil is wet. This may have been the case for the data measured by Watt (1977) at Invercargill, southern South Island (Woodlands Research Station) shown in Fig. 3. Even though the soil water deficit remained at less than 100 mm at this site, water extraction was much less than *PET* (represented by the dotted line in Fig. 3). The most likely explanation for this is that the pasture, which was continuously grazed by sheep, had very low LAI and so a low potential for transpiration (J. P. C. Watt pers. comm.). The solid line in Fig. 3 shows the best-fit prediction, which occurred when actual evapotranspiration (*AET*) was only 82% of potential (after the first year, during



**Fig. 4** Fit of the water balance model (solid line) to soil water data (*SWD* to 200 cm depth, circles) from Dunedin airport (Watt 1977). Evapotranspiration was probably limited by pasture LAI at this site: the best fit was obtained when *PET* was reduced to 69% of its expected value.

which no limitation was observed). Unfortunately, the data sets in Table 2 were not accompanied by sufficient pasture data to examine water extraction of low LAI pastures in detail. Rosenberg et al. (1983) gave empirical relationships for the decline in evapotranspiration at low LAI which may be used to estimate these effects if required. Comparison of our data with soil water predictions based on *PET* alone, suggested that evapotranspiration was restricted by pasture LAI only at the Invercargill and Dunedin Airport sites (Fig. 3, 4).

### Soil moisture effects

Even with non-limiting pasture cover, actual evapotranspiration (*AET*) is limited when the pasture roots are unable to access sufficient water. This can occur due to shallow soil, low soil water content, or limited pasture root depth. With the exception of the data from Winchmore, central South Island (Fraser et al. 1999), which came from a flat site with shallow, stony soil, the data collected for this study (Table 2) all come from flat sites with deep, predominantly well-drained soils. The vegetation at all sites was established ryegrass-white clover pasture, so that maximum potential rooting depth is likely to have been realised. This makes these data unsuitable for formulation of a model to extend to shallow soils such as those found in New Zealand hill country, or poorly drained clays. A simple water balance model for hill-country soils has previously been published by Bircham & Gillingham (1986).

For pasture with mature root systems on deep soils, water extraction in late summer may be limited by low soil water content. A number of studies have shown that during soil drying, *AET* continues at potential (i.e.,  $AET = PET$ ) until some

critical soil water deficit is reached, after which the ratio of *AET* to *PET* drops off linearly (Priestley & Taylor 1972; Watt 1977; Scotter et al. 1979a; McAneney & Judd 1983; Barker et al. 1985; Parfitt et al. 1985a, 1985b). This can be modelled using the equation,

$$AET = \min(PET, \rho RAW) \quad (1)$$

where readily available water, *RAW*, is calculated as (recalling that *W* is negative)

$$RAW = \alpha PET(AWHC + W)$$

and where  $\rho$  is the proportion of *RAW* able to be extracted by the plants in one day,  $\alpha PET$  is the availability of the soil water, and *AWHC* is the available water holding capacity of the soil. Note that in this paper, the acronym *AWHC* refers to the model parameter which describes the maximum amount of water that can be extracted from the soil, not the laboratory measurement of the same name. This model parameter is discussed in detail below.

The same formulation has been independently proposed by other researchers (M.-J. Cros, M. Duru, F. Garcia, & R. Martin-Clouaire unpubl. data). The key to Equation 1 is the definition of readily available water, *RAW*, as water that is readily extracted, so that  $\rho = 1 \text{ d}^{-1}$ . This definition of *RAW* is unique to this model, and means that not all water is readily available and that the availability of water may vary depending on its depth in the soil or its matric potential.

The parameter  $\alpha$  can be estimated by examining the ratio of actual ET (calculated as the difference between subsequent soil measurements plus any rainfall for the period) to calculated *PET* when the soils are particularly dry, following the method used by McAneney & Judd (1983) and Parfitt et al. (1985a, 1985b). Six of our data sets (Massey

University (3 sets), Hamilton Airport, Taranaki Agricultural Research Station, and Taita) were suitable for estimating  $\alpha$  in this way. Based on analysis of these data sets, it was found that the value of  $\alpha$  was similar for all of these soils, and the pooled best fit value for  $\alpha$  was found to be 0.0073. This implies that soils lose a further 137 mm of water ( $= 1/\alpha$ ) from the time that *AET* first begins to be soil-moisture restricted to the time that *ET* ceases altogether (when  $W = -AWHC$ ). Therefore, pastures growing in soils with low *AWHC* experience water stress at a smaller soil water deficit than those growing in soils with high *AWHC*. Subsequent data fitting indicated that  $\alpha = 0.0073$  worked well across all data sets in Table 2.

The associated fitted values of *AWHC* ranged from 191 to 247 mm and reflect the maximum amount of water that could be extracted from these soils in practice. These fitted *AWHC* values are much higher than those obtained from laboratory measurements, indicating that the potential for water extraction *in situ*, which is the parameter used by the model, is not well described by using standard matric potentials in the laboratory.

Prediction of *AET* immediately following rainfall is more complicated because in this case the soil zone containing the majority of roots tends to be far wetter than the overall profile (Webb 1989; Barker & Dymock 1997). Any single soil layer water balance model will always underestimate evapotranspiration under such circumstances. An elegant and straightforward way around this problem was that proposed by Scotter et al. (1979a) to include an additional variable  $W_s$  to represent water in the rewetted (surface) zone (Fig. 1). This is the soil zone that is readily depleted or recharged, but need not correspond to a fixed depth of soil, as rewetting tends to follow the preferential flow patterns of the soil (Webb 1989). However, since grass and clover roots are concentrated near the surface, the majority of roots would be expected to lie in this layer (Barker & Dymock 1997).

On this basis, we propose that *RAW* be calculated as the sum of available water in the surface zone ( $RAW_s$ ) and a proportion of the available water in the remainder of the profile, i.e.,

$$\begin{aligned} AET_s &= \min(PET, \rho RAW_s) \\ AET &= \min(PET, \rho RAW) \end{aligned} \quad (2)$$

where

$$\begin{aligned} RAW_s &= AWHC_s + W_s \\ RAW &= RAW_s + \alpha PET[(AWHC + W) - RAW_s] \end{aligned}$$

Water in the  $W_s$  layer is all considered to be readily available (Scotter et al. 1979a), whereas only a proportion  $\alpha PET$  of available water in the remaining soil profile is considered readily available. This approach is easily extendable to models with more than two soil layers.

Equation (2) differs from Scotter et al.'s (1979a) model by allowing the capacity of the rapid zone  $AWHC_s$  to vary from 25 mm of water. Estimates for  $AWHC_s$  are discussed in the following section. As before, the parameter  $\alpha$  is the increase in *AET*/*PET* per mm of water in the soil (while the surface zone remains dry). It may be that this parameter can be related to the proportion of roots in the lower soil layer (Baier & Robertson 1966; Calder et al. 1983). If so, pasture species effects on  $\alpha$  may be able to be identified. It was not possible to evaluate this possibility using the data sets in this paper, as pasture species were not reported.

Equation 2 is a simple, robust, flexible, and easily calibrated model for *AET*, and captures some of the same logic implied by Scotter et al.'s (1979a) original *AET* model.

#### Available water holding capacity

The *AET* model (Equation 2) requires two soil specific parameters, *AWHC* and  $AWHC_s$ . Analytically, available water holding capacity is usually defined as the water held between field capacity (typically  $-0.01$  MPa or  $-0.02$  MPa) and permanent wilting point (typically  $-1.5$  MPa) to a given soil depth (Scotter 1977; Rickert 1984). Using this definition, available water holding capacity may be measured in the laboratory, and has been tabulated for many New Zealand soils (Gradwell 1968, 1971, 1974; Griffiths 1985).

While available water holding capacity is a commonly measured property of New Zealand soils, its use still presents some problems. These arise because available water holding capacity is sometimes measured between arbitrary water tensions and to arbitrary soil depths (Scotter 1977; Rickert 1984). For ease of use in our model, we wish to adopt a reference definition of available water holding capacity that gives a useful description of the soil properties with regards to soil water use by pasture, allows the use of published values, and can be scaled to provide estimates of the model parameter *AWHC*. It should be noted that the model parameter *AWHC* represents the potential water extraction by the plants in the model, and is not equal to the available water holding capacity of the soil as measured in laboratory analysis.

The most comprehensive set of published available water holding capacity values are those of Gradwell (1968, 1971, 1974). A selection of Gradwell's data is given in Table 3. Available water holding capacity in these studies was measured between  $-0.02$  MPa and  $-1.5$  MPa tension and to a depth of 76 cm. Given the large amount of published data, it seems reasonable to use Gradwell's (1968) definition (which we denote  $AWHC_{76}$ ) as the reference available water holding capacity for use in estimating the parameter  $AWHC$  in our model. A scaling factor  $\beta$  is used to estimate the model  $AWHC$  from  $AWHC_{76}$ , i.e.,

$$AWHC = \beta AWHC_{76} \quad (3)$$

Analysis of the data sets used in this study indicates that the model parameter  $AWHC$  is typically twice as large as available water holding capacity values measured in the laboratory ( $AWHC_{76}$ ). This suggests that plants are able to extract a greater proportion of the soil water than is usually assumed in laboratory measurements, and that a significant amount of water is extracted from matric potentials below  $-15$  MPa in the field.

Available water holding capacity data from several other studies are also given in Table 3. In these studies the tension used to define field capacity ranged from  $-0.0049$  to  $-0.01$  MPa. Equation 11 (below) was used to estimate (by

interpolation) the volumetric soil water content ( $\theta_s$ ) at  $-0.02$  MPa and, thus,  $AWHC_{76}$ . The differences between the  $AWHC_{76}$  values measured by Gradwell (1974) and those measured by other researchers for the same soils (Tokomaru silt loam, Judgeford silt loam) are within the expected magnitude of variation between sites.

In Equation 2,  $AWHC_s$  represents the water holding capacity of the rapidly recharged surface zone. This parameter becomes important during the rewetting of the soil (especially when dry summer soils are recharged to field capacity, usually during April and May), as it controls the value of  $AET$  immediately following rainfall when  $W$  is low (dry). Because the data sets used in this study (Table 2) contained relatively few data during this period (see, for example, Fig. 2) it was not possible to obtain reliable estimates for  $AWHC_s$ . A value of 25 mm was therefore retained (Scotter et al. 1979a).

### Water balance equations

The water balance model can now be written as

$$\begin{aligned} W(t+1) &= \min(0, W(t) + \text{rain}(t) - AET(t)) \\ W_s(t+1) &= \min(0, W_s(t) + \text{rain}(t) - AET_s(t)) \end{aligned} \quad (4)$$

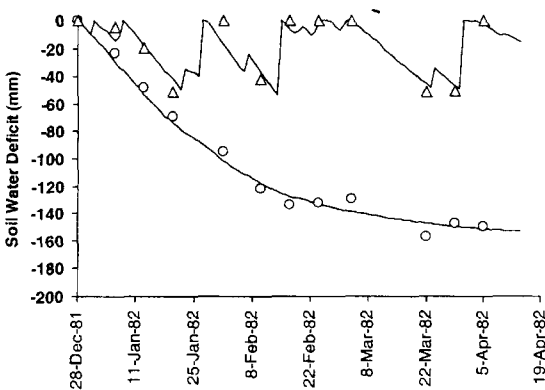
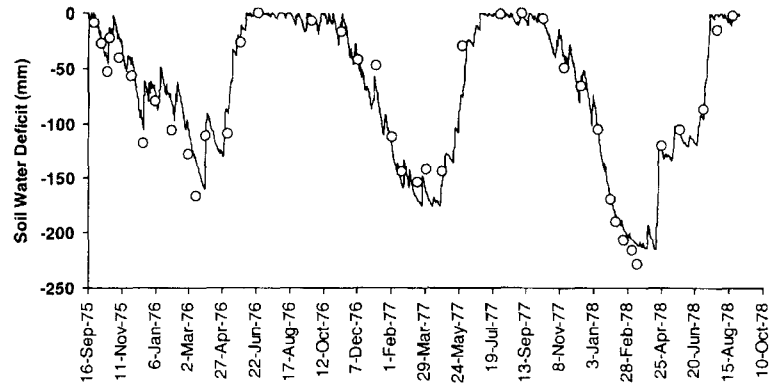
where  $t$  is time (in days). The model (Equation 4) was encoded in an Excel spreadsheet for evaluation. The water balance model can also be written in differential equation notation, as

**Table 3** Laboratory measurements of available water holding capacity (mm water held between  $-0.02$  MPa and  $-1.5$  MPa tension to 76 cm soil depth) for a range of soils representing the main flat-land soil orders in New Zealand.

| Soil type                 | Soil order | Source                      | $AWHC_{76}$ |
|---------------------------|------------|-----------------------------|-------------|
| Tokomaru silt loam        | Pallic     | Scotter et al. 1979b        | 91          |
| Tokomaru silt loam        | Pallic     | Barker et al. 1985          | 96          |
| Tokomaru silt loam        | Pallic     | Gradwell 1974               | 114         |
| Paraha silt loam          | Pallic     | H. Wilde pers. comm.        | 92          |
| Otokia silt loam          | Pallic     | Gradwell 1974               | 96          |
| Waikiwi silt loam         | Brown      | Gradwell 1974               | 128         |
| Mangapiri heavy silt loam | Brown      | Gradwell 1974               | 93          |
| Mokotua peaty silt loam   | Brown      | Gradwell 1974               | 153         |
| Taita clay loam           | Brown      | Gradwell 1968               | 72          |
| Judgeford silt loam       | Brown      | Parfitt et al. 1985a        | 92          |
| Judgeford silt loam       | Brown      | Gradwell 1974               | 114         |
| Ohaupo silt loam          | Allophanic | Gradwell 1968               | 118         |
| Horotiu sandy loam        | Allophanic | Gradwell 1968               | 101         |
| Stratford silt loam       | Allophanic | Parfitt et al. 1985b        | 112         |
| Tirau silt loam           | Allophanic | Gradwell 1968               | 131         |
| Hamilton clay loam        | Granular   | Gradwell 1968               | 88          |
| Naikē clay                | Granular   | Gradwell 1968               | 70          |
| Te Kowhai silt loam       | Gley       | Gradwell 1968               | 176         |
| Te Kowhai clay loam       | Gley       | P. L. Singleton pers. comm. | 92          |
| Himatangi sand            | Recent     | Swain & Scotter 1988        | 36          |



**Fig. 5** Fit of the water balance model (solid line) to the soil water data (*SWD* to 170 cm depth, circles) of Scotter et al. (1979a) from Massey University.



**Fig. 6** Fit of the water balance model (solid line) to the soil water data (*SWD* to 100 cm depth) of Barker et al. (1985) from Massey University.  $\circ$ , dry treatment;  $\Delta$ , irrigated treatment.

$$\frac{dW}{dt} = \text{rain} - AET - r \max(0, W)$$

$$\frac{dW_s}{dt} = \text{rain} - AET_s - r \max(0, W_s) \quad (5)$$

where  $r$  is the instantaneous rate of drainage ( $\text{d}^{-1}$ ). While the Scotter et al. (1979a) model and Equation 4 work only on daily time steps, writing the model in this form allows it to be used with sub-daily steps, as required by some pasture growth models (e.g., Thornley 1998; Woodward 1999). This formulation also extends Equation 4 to situations where  $W > 0$  (above field capacity), and drainage occurs.

### Model validation

In order to assess the applicability and reliability of the model, and to examine how the  $\beta$  multiplier (Equation 3) varies among soil types, the model

was fitted to each data set, using  $\alpha = 0.0073$  and adjusting  $\beta$ , in order to minimise *RMSEP* (Table 2; Fig. 2–13). Good fits were obtained to all data sets, and the best fit values of  $\beta$  appeared to be consistent within soil orders (Table 4). Some variation is to be expected, given that published values of  $AWHC_{76}$  are variable (Table 3), as are measurements of soil water deficit in the field. This result suggests that soil order alone may be sufficient information to use this model at a particular site.

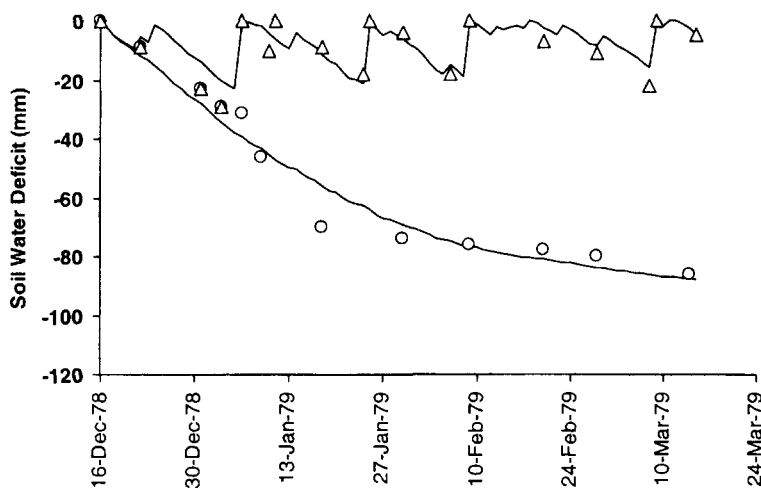
### Conversion of units

Soil water measurements in some of the data sets fitted were reported in units other than whole-profile soil water deficit ( $W$ ). In order to make comparisons with these data sets, conversions were required. For example, the data of Harris et al. (1999) were reported as gravimetric soil water content  $GSWC_s$  (i.e., % by weight) in the top 7.5 cm of soil, while the data of Fraser et al. (1999) were reported as  $GSWC_s$  in the top 30 cm (Fig. 8, 13). Gravimetric soil water content can be calculated from volumetric soil water content  $\theta_s$  by dividing by soil bulk density,  $BD$ :

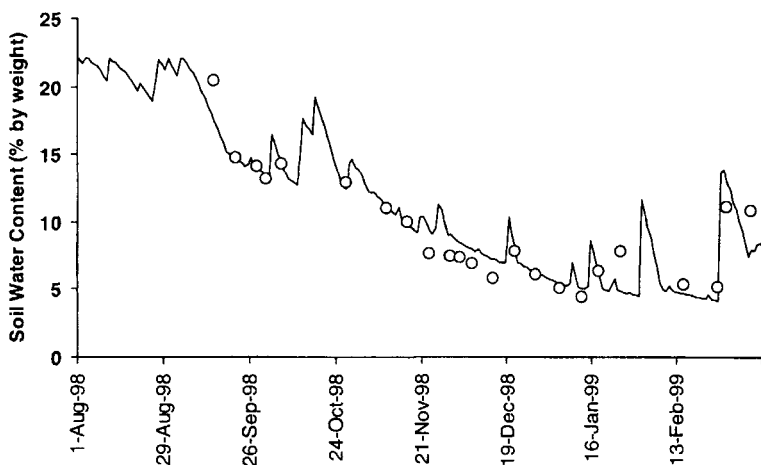
$$GSWC_s = \frac{\theta_s}{BD} \quad (6)$$

**Table 4** Model  $\beta$  parameter for the some of the main flat-land soil orders in New Zealand.  $\beta$ , multiplier to estimate full profile  $AWHC$  values (required for the model) from laboratory  $AWHC_{76}$  values;  $n$ , number of data sets used to estimate  $\beta$  (not all data sets were used).

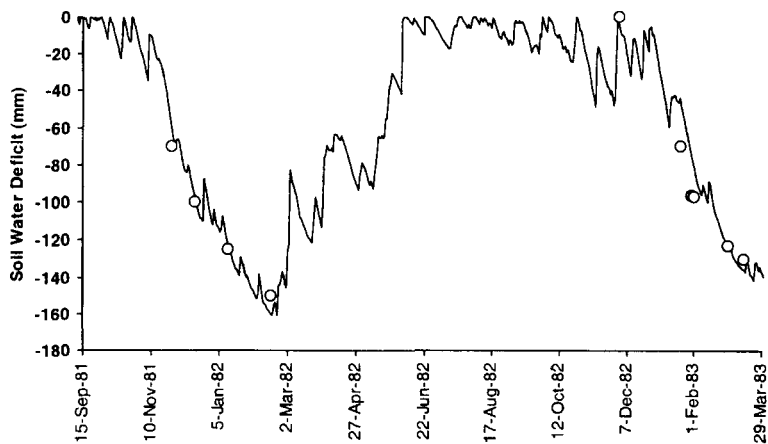
| Soil order       | $\beta$         |
|------------------|-----------------|
| Pallic soils     | 2.4 ( $n = 3$ ) |
| Brown soils      | 2.4 ( $n = 1$ ) |
| Allophanic soils | 1.9 ( $n = 3$ ) |
| Gley soils       | 1.3 ( $n = 2$ ) |



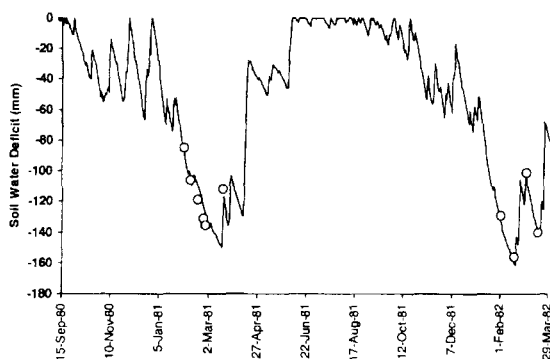
**Fig. 7** Fit of the water balance model (solid line) to the soil water data (*SWD* to 30 cm depth) of Korte & Chu (1983) from Massey University. ○, dry treatment; △, irrigated treatment.



**Fig. 8** Fit of the water balance model (solid line) to the soil water data (*GSWC* to 30 cm depth, circles) of Fraser et al. (1999) from Winchmore.



**Fig. 9** Fit of the water balance model (solid line) to soil water data (*SWD* to 180 cm depth, circles) from Taranaki Agricultural Research Station (Parfitt et al. 1985b).



**Fig. 10** Fit of the water balance model (solid line) to the soil water data (*SWD* to 140 cm depth, circles) of McAneney & Judd (1983) from near Hamilton airport.

While the root mass density under mature pasture typically declines exponentially with depth (Scotter 1977; Rickert 1984; Barker & Dymock 1997), water extraction typically decreases linearly

with depth (Fig. 14) (Scotter et al. 1979b; Barker et al. 1985; Prasad 1988; Durand et al. 1997). This implies that volumetric soil water content near the soil surface  $\theta_s$  can be estimated from,

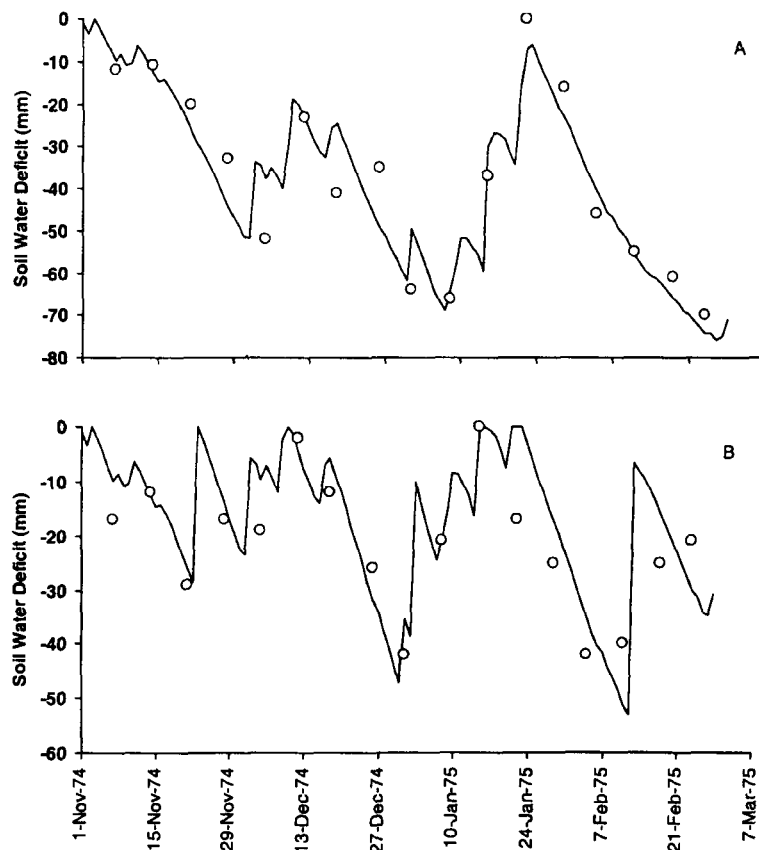
$$-W = \frac{1}{2} d(FC - \theta_s) \quad (7)$$

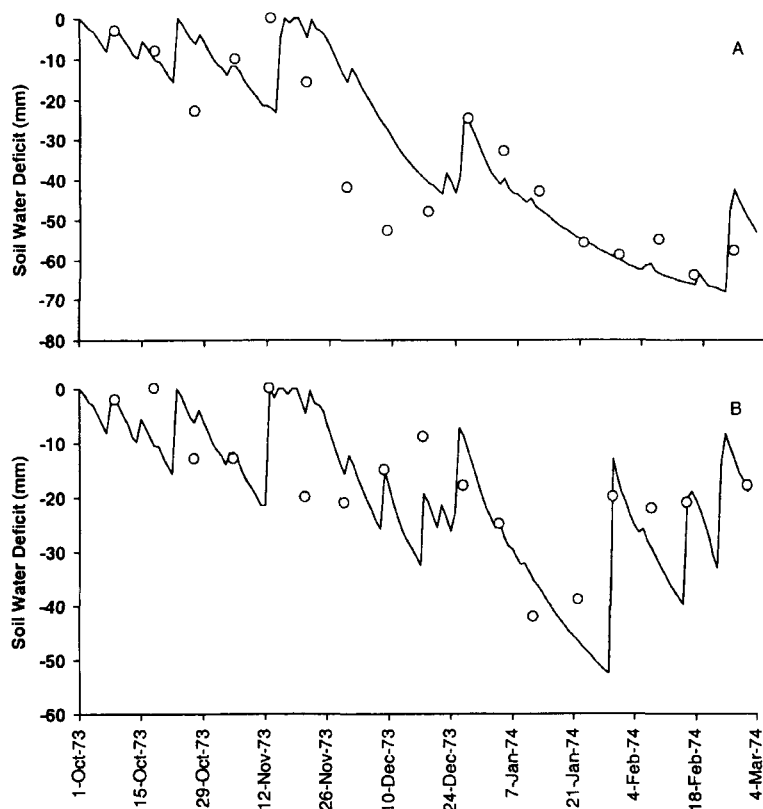
where  $d$  is the maximum rooting (or extraction) depth, and  $FC$  is the (volumetric) field capacity near the surface. Rooting depth is usually greater than 1 m for pasture species (Hebblethwaite & McGowan 1977; Hayman & Stocker 1982; McAneney & Judd 1983; Parfitt et al. 1985a; Barker et al. 1985; Parry et al. 1992) and can be estimated from,

$$AWHC = \frac{1}{2} d(FC - PWP) \quad (8)$$

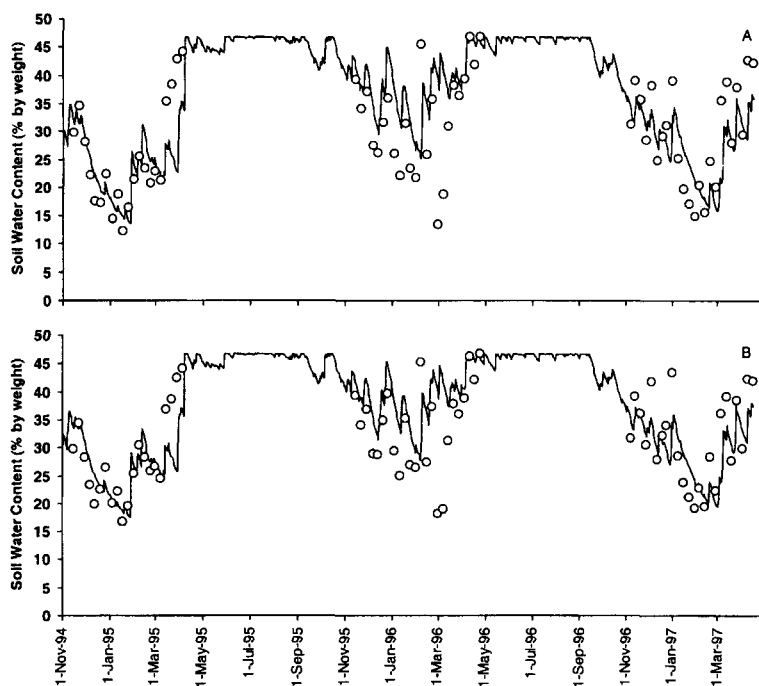
where  $PWP$  is the (volumetric) permanent wilting point near the surface. Note that permanent wilting point potential assumed by most laboratory analyses ( $-15$  MPa) underestimates the amount of water that plants can extract under field conditions, and hence

**Fig. 11** Fit of the water balance model (solid line) to data (*SWD* to 30 cm depth, circles) from Horotiu sandy loam at Rukuhia (Baars et al. 1977). **A**, non-irrigated; **B**, irrigated.

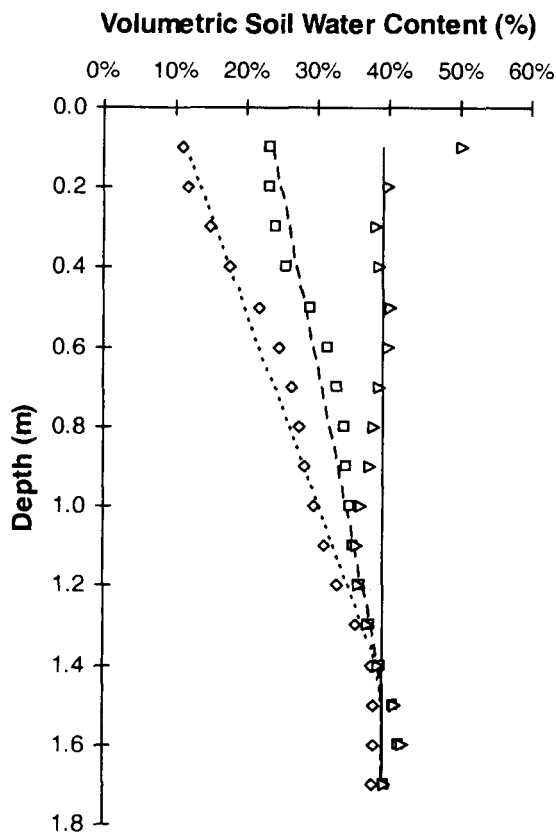




**Fig. 12** Fit of the water balance model (solid line) to data (*SWD* to 30 cm depth, circles) from Te Kowhai clay loam at Rukuhia (Baars et al. 1977). **A**, non-irrigated; **B**, irrigated.



**Fig. 13** Fit of the water balance model (solid line) to the soil water data (*GSD* to 7.5 cm depth, circles) of Harris et al. (1999) from Ruakura. **A**, grazed treatment; **B**, deferred treatment.



**Fig. 14** Pattern of water extraction with depth at  $\Delta$ , field capacity;  $\square$ , 127 mm;  $\diamond$ , 226 mm deficit (Scotter et al. 1979b). Rooting depth was estimated to be 1.44 m. Note the approximately linear pattern of water extraction with depth.

overestimates the minimum soil water content (or *PWP*). Combining Equations 7 and 8 gives (remembering that  $W$  is negative),

$$\theta_s = FC + \frac{W}{AWHC}(FC - PWP) \quad (9)$$

Some studies measured only soil water deficit in the top  $z$  cm of soil (e.g., Baars et al. 1977; Korte

& Chu 1983; Barker et al. 1985), where  $z$  is the depth to which the measurements were taken. Model predictions for these scenarios ( $W_z$ ) can be calculated by assuming that a fixed proportion of soil water extraction is taken from below the measured zone. Assuming a linear decline in soil water content with depth, then,

$$W_z = W \left( 1 - \left( \frac{d-z}{d} \right)^2 \right) \quad (10)$$

where the bracketed term is a constant.

Finally, soil water potential  $\psi_s$  is commonly used in detailed plant growth models (e.g., Thornley 1998). Thornley & Johnson (1990) gave,

$$\psi_s = -A(\theta_s)^B \text{ kPa} \quad (11)$$

where  $A$  and  $B$  are soil dependent parameters.  $B$  has values between about 2 (for sands) and 18 (for clays), and Gregson et al. (1987) suggested that  $A$  could be estimated as,

$$A = 0.375(0.557)^B \text{ kPa} \quad (12)$$

Better estimates of  $A$  and  $B$  are given in Table 5 for several of the soils used in this study.

## CONCLUSION

The two-zone water balance model suggested by Scotter et al. (1979a) and modified here provides a simple means to predict patterns of water recharge and depletion in some flat-land pastoral soils. The excellent results obtained by comparing the model's predictions with a range of data sets from pastoral soils from Waikato to Southland show that the model gives accurate predictions of soil water status and can be expected to be reliable also for other soil types and climatic regions. Climatic inputs required are daily rainfall and potential evapotranspiration (*PET*). The latter can be estimated from daily mean temperature and net radiation receipt (Priestley & Taylor 1972; Scotter et al. 1979a). Soil parameters required are available water

**Table 5** Parameters for calculating soil water tension in the surface zone ( $\psi_s$ , Equation 11) for several soil types used in this study.

| Data set             | Soil type           | $A$                   | $B$  |
|----------------------|---------------------|-----------------------|------|
| Scotter et al. 1979a | Tokomaru silt loam  | $3.78 \times 10^{-3}$ | 7.76 |
| Barker et al. 1985   | Tokomaru silt loam  | $1.53 \times 10^{-3}$ | 7.75 |
| Parfitt et al. 1985a | Judgeford silt loam | $2.42 \times 10^{-3}$ | 8.28 |
| Parfitt et al. 1985b | Stratford silt loam | $6.60 \times 10^{-3}$ | 8.40 |
| Swain & Scotter 1988 | Himatangi sand      | $4.58 \times 10^{-3}$ | 3.82 |

holding capacity to 76 cm ( $AWHC_{76}$ ), which has been tabulated for many New Zealand soils (Gradwell 1968, 1971, 1974) and soil orders (pallic, brown, allophanic, gley, etc.). This is multiplied by a soil order specific parameter  $\beta$  (Table 4) to estimate the available water holding capacity of the soil in the field,  $AWHC$ , which is typically twice  $AWHC_{76}$  as measured in the laboratory. This indicates that laboratory measurements, which typically assume that water extraction ceases at  $-15$  MPa matric potential, underestimate the amount of water extracted by plants in the field. The model can be applied without further site-specific parameterisation at many sites.

This type of model is suitable for use in irrigation scheduling (de Jong & Bootsma 1996), early drought warning, and in pasture growth models for management decisions on-farm (McCall 1984; Woodward 1999). However, it is probably not suitable for use in pesticide fate, nutrient transport, soil erosion, or salinisation studies, etc., for which a more detailed mechanistic model would be required, which can describe the transport of water, solutes, and sediment (de Jong & Bootsma 1996).

The model takes no account of lateral surface and subsurface flow, which may be important on sloping sites (Tian et al. 1998), nor of runoff from, or preferential flow through, non-saturated soils resulting in a loss of effectiveness of applied water. Capillary rise of groundwater is also not considered. Further modification would also be necessary to model soil water balance on hill-country soil (Bircham & Gillingham 1986). These include adjusting  $PET$  predictions for slope, altitude, and aspect, and including an estimate of surface runoff. Hill-country soils are also typically shallow, so the use of  $AWHC_{76}$  and  $\beta$  may also need to be modified for these soils.

The rapidly recharged zone (which should not be considered a physical layer) is a key feature in the success of this model, allowing rapid response of  $AET$  (and plant growth) to rain events during prolonged dry periods. The use of the status of both zones in the  $AET$  function (Equation 2), as well as available water holding capacity and the parameter  $\alpha$ , allows a simple but biologically reasonable plant response to be calculated using these parameters. Further work is required to determine the dependence of  $\alpha$  on pasture and soil types. If  $\alpha$  can be successfully related to pasture species, this may provide a means to study the relative response of alternative pasture species to drought conditions

(Parry et al. 1992), as plant stress is closely related to  $AET$  (McAneney et al. 1982; McAneney & Judd 1983; Parfitt et al. 1985a, 1985b; Barker & Chu 1985; Rosenthal et al. 1987; Barker et al. 1989; Sadras & Milroy 1996). This would then allow a practical model of plant growth response to water stress to be formulated.

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## Appendix

### Calculating potential evapotranspiration

A number of methods exist for estimating daily potential evapotranspiration. Scotter et al. (1979a) used the Priestley-Taylor method (Priestley & Taylor 1972), which requires as inputs mean daily air temperature,  $T$  ( $^{\circ}\text{C}$ ), and daily net radiation receipt,  $R_n$  ( $\text{MJ m}^{-2} \text{d}^{-1}$ ). In this method  $PET$  is calculated as,

$$PET = 1.26 \frac{s}{s + \gamma} \left( \frac{R_n - G}{L} \right) \quad (\text{A1})$$

where,

$$s = 0.61 \exp \left( \frac{17.3T}{237.3 + T} \right) \frac{17.3 \times 237.3}{(237.3 + T)} \text{ kPa } ^{\circ}\text{C}^{-1} \quad (\text{A2})$$

is the slope of the saturation vapour pressure curve,  $\gamma = 0.067 \text{ kPa } ^{\circ}\text{C}^{-1}$  is the psychrometric constant (Monteith 1998), and  $L = 2.45 \text{ (MJ kg}^{-1}\text{)}$  is the latent heat of vaporisation of water (Scotter et al. 1979a). While the daily soil heat flux  $G$  ( $\text{MJ m}^{-2} \text{d}^{-1}$ ) is often assumed to be negligible, it can be easily estimated (FAO 1990) as,

$$G = 0.38 (T - T_{d-1}) \quad (\text{A3})$$

where  $T_{d-1}$  is the mean air temperature on the previous day. Methods for estimation of daily net radiation receipt  $R_n$  are discussed below.

The Priestley-Taylor model has been shown to work well in humid regions provided the effect of aerodynamics is small relative to that of radiation

(Tanner & Ritchie 1974; Smith et al. 1996). Analysis of weather data across a range of sites, however, showed that this requirement is often not met in New Zealand, where the relative contribution of the aerodynamic effect is frequently large enough to require a Priestley-Taylor factor considerably greater than the 1.26 commonly used (Fig. A1). Models that take aerodynamic effects into account explicitly are therefore preferred. The FAO version of the Penman-Monteith  $PET$  model, for example, allows for the effects of humidity and wind and has been validated globally (Smith et al. 1996). This model gives  $PET$  for a well-watered 12-cm-high grass sward (Smith et al. 1996) as,

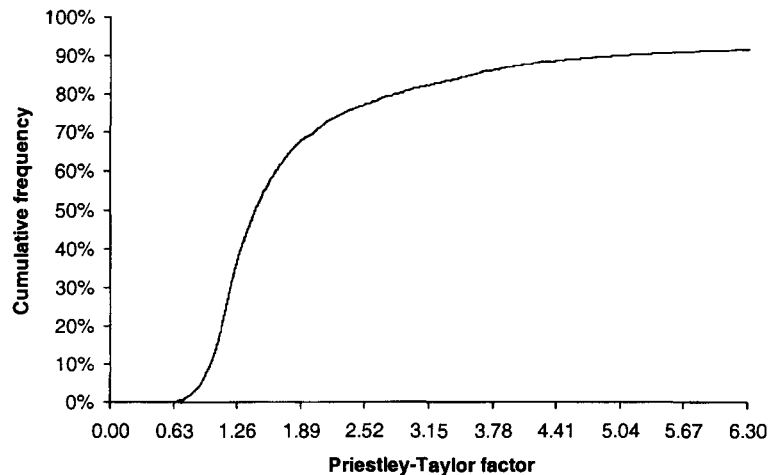
$$PET = \frac{s \frac{R_n - G}{L} + \gamma \frac{900}{T + 273} U_2 (e_a - e_d)}{s + \gamma (1 + 0.34 U_2)} \quad (\text{A4})$$

where  $U_2$  is the average daily wind speed measured at 2 m above the ground ( $\text{m s}^{-1}$ ), and  $e_a - e_d$  is the vapour pressure deficit (kPa). Saturation ( $e_a$ , kPa) and dewpoint ( $e_d$ , kPa) vapour pressures can be estimated from relative humidity (FAO 1990), or from daily maximum and minimum temperature (FAO 1990; Smith et al. 1996),

$$e_a = \frac{1}{2} \left( 0.61 \exp \left( \frac{17.3 T_{\max}}{T_{\max} + 237.3} \right) + 0.61 \exp \left( \frac{17.3 T_{\min}}{T_{\min} + 237.3} \right) \right)$$

$$e_d = 0.61 \exp \left( \frac{17.3 T_{\min}}{T_{\min} + 237.3} \right) \quad (\text{A5})$$

**Fig. A1** Cumulative frequency plot of the Priestley-Taylor factor (usually given a value of 1.26) required to equate Priestley-Taylor  $PET$  predictions with those calculated using the FAO Penman-Monteith formula, for 2495 days of data (approximately one year from each of Invercargill Airport, Dunedin Airport, Taita, Massey University, Normanby, Hamilton Airport, and Ruakura), each of which included windrun.



If average daily wind speed data are not available, Smith et al. (1996) suggest using  $2 \text{ m s}^{-1}$  as a typical value for  $U_2$ , or  $3 \text{ m s}^{-1}$  at windy sites. The FAO Penman-Monteith *PET* model was used in the simulations presented in this paper.

An alternative approach to these, which may be adequate for many applications, is to use a simple empirical function relating *PET* to time of year. The following relation was derived by fitting a cosine curve to predictions made using Equation A4 for eight years of weather data (one each from Invercargill Airport, Dunedin Airport, Winchmore, Taita, Massey University, Normanby, Hamilton Airport, and Ruakura).

$$PET \approx 2.43 + 1.56 \cos\left(2\pi \frac{d}{365}\right) \quad (R^2 = 0.62) \quad (\text{A6})$$

where  $d$  is the day of the year. This gives the mean annual trend of *PET* for these sites. No significant differences were found between the eight sites.

### Estimation of net radiation

Daily net radiation,  $R_n$  ( $\text{MJ m}^{-2} \text{ d}^{-1}$ ), is an important weather variable required by many *PET* models but not commonly available in the field. Net radiation is the difference between net incoming shortwave and net outgoing longwave radiation (Rosenberg et al. 1983; FAO 1990), and may be estimated as,

$$R_n = (1 - a)rad - R_{n,l} \quad (\text{A7})$$

where  $a = 0.23$  is the canopy relectivity (albedo) of grass, and  $rad$  ( $\text{MJ m}^{-2} \text{ d}^{-1}$ ) is the solar radiation receipt (e.g., measured with a pyranometer).  $R_{n,l}$  ( $\text{MJ m}^{-2} \text{ d}^{-1}$ ) is the net outgoing longwave radiation, which can be estimated (FAO 1990) as,

$$R_{n,l} = f(\epsilon_{vs} - \epsilon_n) \frac{\sigma}{2} \left[ (T_{\min} + 273)^4 + (T_{\max} + 273)^4 \right] \quad (\text{A8})$$

where  $f$  is an adjustment for cloud cover,  $\epsilon_{vs}$  and  $\epsilon_n$  are the vegetation-soil and atmosphere emissivities, respectively, and  $\sigma$  is the Stefan-Boltzmann constant ( $4.90 \times 10^{-9} \text{ MJ m}^{-2} \text{ K}^{-4} \text{ d}^{-1}$ ). The cloud cover constant  $f$  can be estimated (FAO 1990) as,

$$f = 1.0 - 0.9c \quad (\text{A9})$$

where  $c$  is the average daily cloud cover. Assuming clouds are randomly distributed throughout the day, cloud cover  $c$  is estimated as,

$$c = 1 - \frac{\text{sun}}{h} \quad (\text{A10})$$

where *sun* is sunshine hours per day. Day length  $h$  is calculated by,

$$h = \frac{24}{\pi} \cos^{-1}(-\tan \lambda \tan \delta) \quad (\text{A11})$$

where  $\lambda$  is the latitude (in radians from the equator, negative for the Southern Hemisphere) and,

$$\delta = -0.4084 \cos\left(2\pi \frac{d+10}{365}\right) \quad (\text{A12})$$

is the solar declination angle (in radians) on the current day of the year.

The net emissivity in Equation A8 can be estimated from dewpoint vapour pressure (Equation A5) (FAO 1990) as,

$$\epsilon_{vs} - \epsilon_n = 0.34 - 0.14 \sqrt{e_d} \quad (\text{A13})$$

The above calculations require solar radiation and/or sunshine data. The next section discusses methods for estimating these if measured values are not available.

### Estimation of solar radiation

Solar radiation  $rad$  ( $\text{MJ m}^{-2} \text{ d}^{-1}$ ) may be estimated from calibrations against pan evaporation (Scotter et al. 1979a),  $pan$  ( $\text{mm d}^{-1}$ ), or temperature (Smith et al. 1996). The following relations are based on calibrations from a range of sites in New Zealand (same sites as listed above),

$$rad \approx 3.36pan + 4.64 \quad (R^2 = 0.70) \quad (\text{A14})$$

$$rad \approx 2.60h \sin \phi - 8.9 \quad (R^2 = 0.59)$$

where  $\phi$  is the solar elevation at solar noon, which depends on latitude and solar elevation,

$$\sin \phi = \sin \lambda \sin \delta + \cos \lambda \cos \delta \quad (\text{A15})$$

If sunshine data are available, however, solar radiation  $rad$  can be more accurately estimated using the method described in Johnson et al. (1995), as follows. In clear skies, the mean daily irradiances due to total and direct beam light (Johnson et al. 1995) are, respectively,

$$J_{0,p} = 1367 \frac{p}{\pi} \sin \phi (1 + \tau^{1/\sin \phi})$$

$$J_{0,s} = 1367 \frac{2p}{\pi} \sin \phi (\tau^{1/\sin \phi}) \quad (\text{A16})$$

where  $1367 \text{ J m}^{-2} \text{ s}^{-1}$  is the solar constant and  $p$  is the relevant fraction of radiation in full spectrum sunlight (here taken to be 1). The parameter  $\tau$  is the clear sky transmissivity at the site, which represents

the degree of absorption and scattering of solar radiation as it passes through the atmosphere. Analysis of climate data (from Invercargill, Dunedin, Winchmore, Palmerston North, and Hamilton) suggests that,

$$\tau = 0.64 + 0.12 \cos\left(2\pi \frac{d-174}{365}\right) \quad (\text{A17})$$

gives a good representation of the atmospheric transmissivity in New Zealand.

Daily light receipt  $J_0$  can now be seen to satisfy the equation,

$$h J_0 = sun J_{0,s} + h J_{0,d} \quad (\text{A18})$$

where  $J_{0,d}$  is the mean daily irradiance due to diffuse light (from both cloud and blue sky scattering).  $J_{0,d}$  can be written as,

$$J_{0,d} = J_{0,p}(f_{blue}(1-c) + f_{cloud}c) \quad (\text{A19})$$

where the relative intensities of radiation from cloud and blue sky are, respectively,

$$\begin{aligned} f_{blue} &= \frac{1 - \tau^{1/\sin\phi}}{1 + \tau^{1/\sin\phi}} \\ f_{cloud} &= 1.11 f_{blue} \end{aligned} \quad (\text{A20})$$

The factor 1.11 in Equation A20 is an empirical parameter that works well across the New Zealand sites listed above.

Daily solar radiation  $rad$  can now be estimated as,

$$rad \approx \frac{3600h J_0}{10^6 p} \quad (\text{A21})$$

This method gives an estimate of  $rad$  from sunshine hours data with an  $R^2$  of 0.91 and an expected error of  $2.4 \text{ MJ m}^{-2} \text{ d}^{-1}$  across the sites mentioned above, and can be expected to work well at other New Zealand locations also. Aside from the improved accuracy of prediction, an additional benefit of calculating daily direct and diffuse radiation explicitly is that daily photosynthesis can then also be calculated (Johnson et al. 1995).

### Estimation of sunshine hours

The procedure above may also be reversed to calculate sunshine hours from daily solar radiation data, by inverting Equation A18,

$$sun = \max\left(0, h \frac{J_0 - f_{cloud}J_{0,p}}{J_{0,s} - f_{cloud}J_{0,p} + f_{blue}J_{0,p}}\right) \quad (\text{A22})$$

This method gives a prediction for  $sun$  with an expected error of 1.6 hours  $\text{d}^{-1}$  ( $R^2 = 0.82$ ) across the sites mentioned above.