**Given**

Let S be the implicit surface defined by 

**Question**

Let us prove that the normal at point  is proportional to 

**Definitions**

Since  it means that

At point  the gradient  is defined to be 

**Prove**

In order to prove that, we would like to show that both the gradient  and the normal  at point  are perpendicular to the isosurface 

Regarding the normal, by its definition it is perpendicular

Regarding the gradient, let us prove that

Any curve in space can be written as  for a parameter , and if we require this curve to be on the isosurface, we have that 

Differentiating this with respect to  and using the chain rule, we have that:



Where  Is the tangent vector of the surface at , for any curve  on the surface that passes through .

So the gradient must be in the direction of the normal to the surface