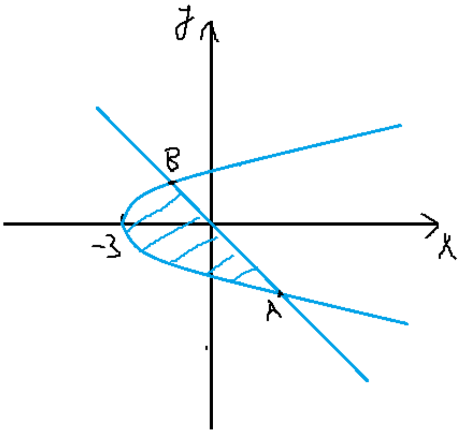
17 вариант

1. $D: y = -x,\ \ y^2 = x + 3$

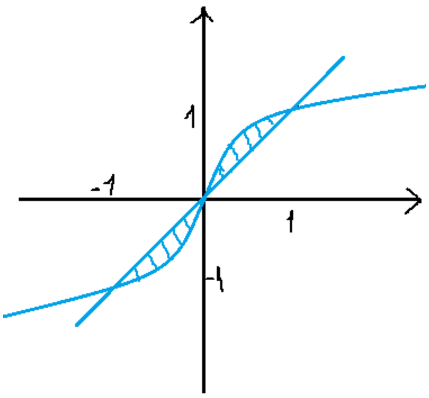


Найдем точки пересечения графиков: $y^2 - 3 = -y \Rightarrow y^2 + y - 3 = 0 \quad D = 13$

$$y_A = \frac{-1-\sqrt{13}}{2} \quad x_A = \frac{1+\sqrt{13}}{2}; \quad y_B = \frac{-1+\sqrt{13}}{2} \quad x_B = \frac{1-\sqrt{13}}{2}$$

$$\iint\limits_D f(x,y)dxdy = \int\limits_{\frac{-1-\sqrt{13}}{2}}^{\frac{-1+\sqrt{13}}{2}} dy \int\limits_{y^2-3}^{-y} f(x,y)dx = \int\limits_{-3}^{\frac{1-\sqrt{13}}{2}} dx \int\limits_{-\sqrt{x+3}}^{\sqrt{x+3}} f(x,y)dy + \int\limits_{\frac{1-\sqrt{13}}{2}}^{\frac{1+\sqrt{13}}{2}} dx \int\limits_{-\sqrt{x+3}}^{-x} f(x,y)dy$$

2. $D: y^3 = x, y = x \quad \iint_D y(1-x)dxdy = \int_{-1}^{0} dx \int_{\sqrt[3]{x}}^{x} y(1-x)dy + \int_0^1 dx \int_x^{\sqrt[3]{x}} y(1-x)dy$



1) $\int_{-1}^{0} dx \int_{\sqrt[3]{x}}^{x} y(1-x)dy = \frac{1}{2}\int_{-1}^{0}(1-x)dx\, y^2|_{\sqrt[3]{x}}^{x} = \frac{1}{2}\int_{-1}^{0}(1-x)\left(x^2 - x^{\frac{2}{3}}\right)dx =$

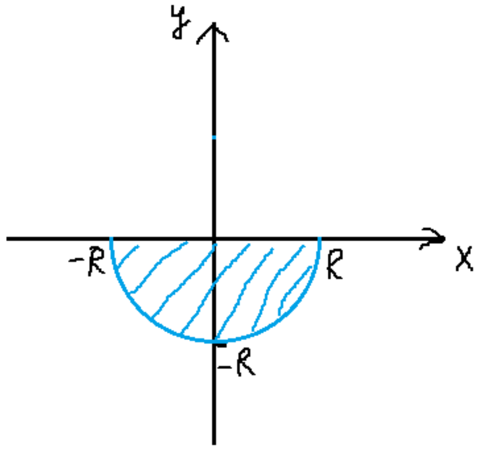
$$= \frac{1}{2}\int\limits_{-1}^{0}\left(x^2 - x^{\frac{2}{3}} - x^3 + x^{\frac{5}{3}}\right)dx = \frac{1}{2}\left(\frac{x^3}{3} - \frac{3x^{\frac{5}{3}}}{5} - \frac{x^4}{4} + \frac{3x^{\frac{8}{3}}}{8}\right)\Big|_{-1}^{0} = \frac{1}{2}\left(\frac{1}{3} - \frac{3}{5} + \frac{1}{4} - \frac{3}{8}\right) = -\frac{47}{240}$$

2) $\int_0^1 dx \int_x^{\sqrt[3]{x}} y(1-x)dy = \frac{1}{2}\int_0^1(1-x)\left(x^{\frac{2}{3}} - x^2\right)dx = \frac{1}{2}\int_0^1\left(x^{\frac{2}{3}} - x^2 - x^{\frac{5}{3}} + x^3\right)dx =$

$$= \frac{1}{2}\left(\frac{3x^{\frac{5}{3}}}{5} - \frac{x^3}{3} - \frac{3x^{\frac{8}{3}}}{8} + \frac{x^4}{4}\right)\Big|_0^1 = \frac{1}{2}\left(\frac{3}{5} - \frac{1}{3} + \frac{1}{4} - \frac{3}{8}\right) = \frac{17}{240}$$

$$\iint\limits_D y(1-x)dxdy = -\frac{47}{240} + \frac{17}{240} = -\frac{30}{240} = -\frac{1}{8}$$

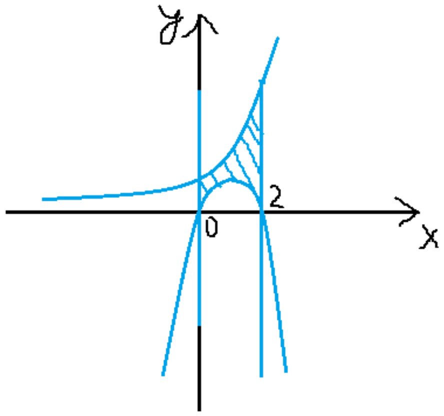
3. $\int_{-R}^{R} dx \int_{-\sqrt{R^2-x^2}}^{0} \frac{dy}{\sqrt{x^2+y^2}\sin^2\sqrt{x^2+y^2}} = \begin{vmatrix} x = \rho\cos\phi \\ x = \rho\sin\phi \\ dx, dy = d\phi, \rho d\rho \end{vmatrix} =$



$$= \int_{-\pi}^{0} d\phi \int_{0}^{R} \frac{\rho d\rho}{\sqrt{\rho^2\cos^2\phi + \rho^2\sin^2\phi}\,\sin^2\sqrt{\rho^2\cos^2\phi + \rho^2\sin^2\phi}} = \int_{-\pi}^{0} d\phi \int_{0}^{R} \frac{d\rho}{\sin^2\rho} = \int_{-\pi}^{0} -\cot\rho\,\Big|_0^R d\phi$$

$\cot 0 \ \nexists$ => интеграл расходится

4. $D: y = 2^x,\ y = 2x - x^2,\ x = 2,\ x = 0$



$$S = \iint\limits_D dxdy = \int_0^2 dx \int_{2x-x^2}^{2^x} dy = \int_0^2 (2^x - 2x + x^2)dx = \left(\frac{2^x}{\ln 2} - x^2 + \frac{x^3}{3}\right)\Big|_0^2 = \frac{4}{\ln 2} - 4 + \frac{8}{3} - \frac{1}{\ln 2} =$$

$$= \frac{3}{\ln 2} - \frac{4}{3}$$

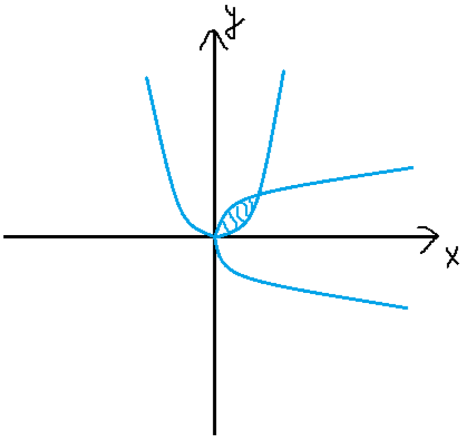
5. $\rho^2 = a^2(1+\sin^2\phi)\ \ =>\ \ \rho = a\sqrt{1+\sin^2\phi}$

$\begin{cases} 1+\sin^2\phi \geq 0 \\ -\pi \leq \phi \leq \pi \end{cases}$ => верно для $\forall\phi$

$$S = \int_{-\pi}^{\pi} d\phi \int_{0}^{a\sqrt{1+\sin^2\phi}} \rho d\rho = \frac{a^2}{2}\int_{-\pi}^{\pi}(1+\sin^2\phi)d\phi = \frac{a^2}{2}\int_{-\pi}^{\pi}\left(1+\frac{1-\cos 2\phi}{2}\right)d\phi =$$

$$= \frac{a^2}{4}\int_{-\pi}^{\pi}(3-\cos 2\phi)d\phi = \frac{a^2}{4}\left(3\phi - \frac{\sin 2\phi}{2}\right)\Big|_{-\pi}^{\pi} = \frac{a^2}{4}(3\pi+3\pi) = \frac{3\pi a^2}{2}$$

6. $y = x^2, x = y^2, z = 3x+2y+6, z = 0$



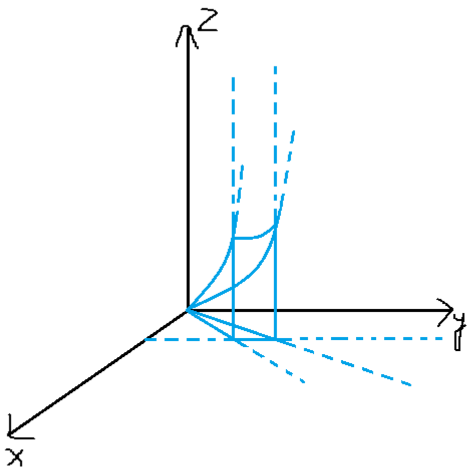
$$V = \int_0^1 dx \int_{x^2}^{\sqrt{x}} (3x+2y+6)dy = \int_0^1 dx(3xy+y^2+6y)\Big|_{x^2}^{\sqrt{x}} =$$

$$= \int_0^1 \left(3x^{\frac{3}{2}} + x + 6x^{\frac{1}{2}} - 3x^3 - x^4 - 6x^2\right)dx = \left(\frac{6x^{\frac{5}{2}}}{2} + \frac{x^2}{2} + \frac{12x^{\frac{3}{2}}}{3} - \frac{3x^4}{4} - \frac{x^5}{5} - 2x^3\right)\Big|_0^1 =$$

$$= \frac{6}{5} + \frac{1}{2} + 4 - \frac{3}{4} - \frac{1}{5} - 2 = \frac{74}{20} - \frac{15}{20} - \frac{4}{20} = \frac{55}{20} = \frac{11}{4}$$

7. $V: x = 1, y = 2x, y = 3x, z \geq 0, z = 2x^2 + y^2$

$$\iiint_V f(x,y,z)dxdydz = \int_0^1 dx \int_{2x}^{3x} dy \int_0^{2x^2+y^2} f(x,y,z)dz$$



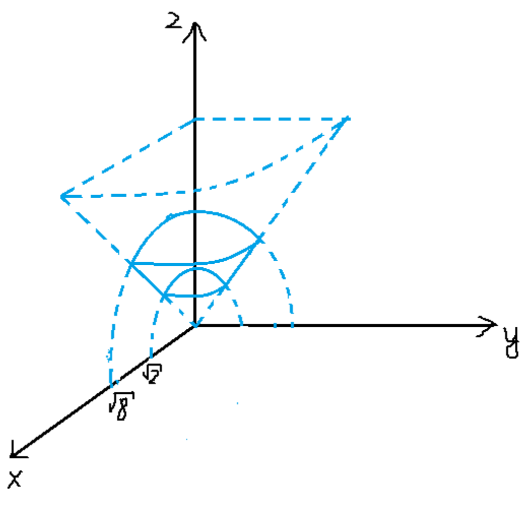
8. $V: 0 \le x \le 1, -2 \le y \le 1,\ 0 \le z \le 1 \quad \iiint_V (2x^2 + y - z^3)dxdydz =$

$$= \int_0^1 dx \int_{-2}^1 dy \int_0^1 (2x^2 + y - z^3)dz = \int_0^1 dx \int_{-2}^1 dy \left(2x^2 z + yz - \frac{z^4}{4}\right)\Big|_0^1 = \int_0^1 dx \int_{-2}^1 \left(2x^2 + y - \frac{1}{4}\right) dy =$$

$$= \int_0^1 dx \left(2x^2 y + \frac{y^2}{2} - \frac{y}{4}\right)\Big|_{-2}^1 = \int_0^1 \left(2x^2 + \frac{1}{2} - \frac{1}{4} + 4x^2 - 2 - \frac{1}{2}\right) dx = \int_0^1 \left(6x^2 - \frac{9}{4}\right) dx =$$

$$= \left(2x^3 - \frac{9}{4}x\right)\Big|_0^1 = 2 - \frac{9}{4} = -0.25$$

9. $\iiint_V xydxdydz = (*);\quad V: 2 \le x^2 + y^2 + z^2 \le 8,\ z^2 = x^2 + y^2,\ x \ge 0,\ y \ge 0,\ z \ge 0$



$x = r\sin\theta\cos\phi\,, y = r\sin\theta\sin\phi\,, z = r\cos\theta\,, xy = r^2\sin^2\theta\sin\phi\cos\phi,$

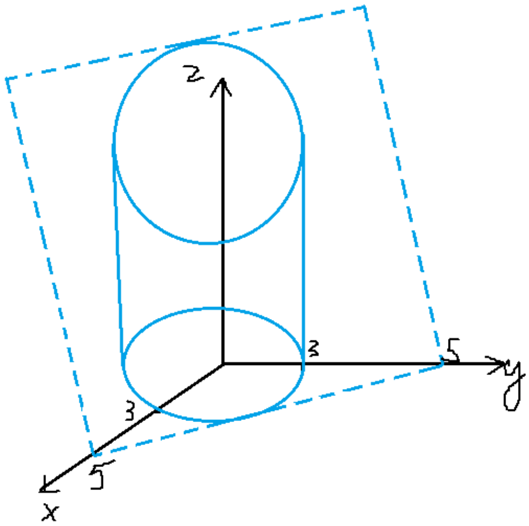
$dxdydz = r^2\sin\theta\, d\phi d\theta dr$

$$(*) = \iiint_V r^4 \sin^3\theta \sin\phi\cos\phi\, d\phi d\theta dr = \int_0^{\frac{\pi}{2}} \sin\phi\cos\phi\, d\phi \int_0^{\frac{\pi}{4}} \sin^3\theta\, d\theta \int_{\sqrt{2}}^{2\sqrt{2}} r^4 dr =$$

$$=\frac{124\sqrt{2}}{5}\int_0^{\frac{\pi}{2}}\sin\phi\cos\phi\,d\phi\int_0^{\frac{\pi}{4}}\sin^3\theta\,d\theta=\frac{124\sqrt{2}}{5}\int_0^{\frac{\pi}{2}}\sin\phi\cos\phi\left(\frac{\cos^3\theta}{3}-\cos\theta\right)\Big|_0^{\frac{\pi}{4}}d\phi=$$

$$=\frac{62(4\sqrt{2}-5)}{15}\int_0^{\frac{\pi}{2}}\sin\phi\cos\phi\,d\phi=\frac{62(4\sqrt{2}-5)}{15}*\frac{\sin^2\phi}{2}\Big|_0^{\frac{\pi}{2}}=\frac{31(4\sqrt{2}-5)}{15}$$

10. $V: z\geq 0,\ x^2+y^2=9,\ z=5-x-y$



$x=r\cos\phi\,, y=r\sin\phi\,, z=z, dxdy=rdrd\phi$

$$V=\int_0^{2\pi}d\phi\int_0^3 rdr\int_0^{5-r(\sin\phi+\cos\phi)}dz=\int_0^{2\pi}d\phi\int_0^3 r(5-r(\sin\phi+\cos\phi))dr=$$

$$=\int_0^{2\pi}d\phi\int_0^3(5r-r^2\sin\phi-r^2\cos\phi)dr=\int_0^{2\pi}\left(\frac{5r^2}{2}-\frac{r^3\sin\phi}{3}-\frac{r^3\cos\phi}{3}\right)\Big|_0^3 d\phi=$$

$$=\int_0^{2\pi}\left(\frac{45}{2}-9\sin\phi-9\cos\phi\right)d\phi=\left(\frac{45\phi}{2}+9\cos\phi-9\sin\phi\right)\Big|_0^{2\pi}=45\pi+9-0-9=45\pi$$

11. $V: z=3(x^2+y^2),\ x^2+y^2=9,\ z=0$ Т.к. тело однородное, плотность можно не учитывать, при вычислениях она сократится

$$m=\iiint_V dxdydz=\int_0^{2\pi}d\phi\int_0^3 rdr\int_0^{3r^2}dz=3\int_0^{2\pi}d\phi\int_0^3 r^3dr=\frac{243}{4}\int_0^{2\pi}d\phi=\frac{243\pi}{2}$$

$$S_{xy}=\int_0^{2\pi}d\phi\int_0^3 rdr\int_0^{3r^2}zdz=\frac{9}{2}\int_0^{2\pi}d\phi\int_0^3 r^5dr=\frac{6561}{12}\int_0^{2\pi}d\phi=\frac{6561\pi}{6}$$

$$S_{yz} = \int_0^{2\pi} d\phi \int_0^3 r dr \int_0^{3r^2} r\cos\phi\, dz = 3\int_0^{2\pi} \cos\phi\, d\phi \int_0^3 r^4 dr = \frac{729}{5}\int_0^{2\pi} \cos\phi\, d\phi = \frac{729}{5}\sin\phi \,\Big|_0^{2\pi} = 0$$

$$S_{xz} = \int_0^{2\pi} d\phi \int_0^3 r dr \int_0^{3r^2} r\sin\phi\, dz = 3\int_0^{2\pi} \sin\phi\, d\phi \int_0^3 r^4 dr = \frac{729}{5}\int_0^{2\pi} \sin\phi\, d\phi = -\frac{729}{5}\cos\phi \,\Big|_0^{2\pi} = 0$$

$x_c = \dfrac{S_{yz}}{m} = 0;\ \ y_c = \dfrac{S_{xz}}{m} = 0;\ \ z_c = \dfrac{S_{xy}}{m} = \dfrac{6561\pi}{6} * \dfrac{2}{243\pi} = 9$ Ответ: (0,0,9)