Resonance-Based Spectral Operator for Prime Detection Using Zeta Zeros

Evgeny Stupakov
Independent Researcher
Email: evgeny.stupakov96@gmail.com

2026

Abstract

A spectral resonance operator for detecting prime numbers is constructed using the non-trivial zeros of the Riemann zeta function and the von Mangoldt function. The operator acts as a finite spectral projection that produces constructive interference at prime arguments. The construction interprets zeta zeros as spectral frequencies and primes as resonance locations. A shifted spectral line $\sigma = 0.002551280340399997$ is used for stable numerical evaluation. Numerical validation confirms reliable detection of primes up to $N = 2^{61} - 1$ using 500 zeta zeros. This establishes a direct connection between analytic number theory, spectral operator theory, and computational prime detection.

1 Introduction

The distribution of prime numbers is fundamentally connected to the analytic structure of the Riemann zeta function $\zeta(s)$. The explicit formula expresses prime structure in terms of oscillatory contributions from non-trivial zeros.

Let

$$\rho = \beta + it_\rho$$

denote non-trivial zeros of $\zeta(s)$.

The Hilbert–Pólya conjecture suggests that these zeros correspond to eigenvalues of a self-adjoint operator.

This work constructs an explicit spectral resonance operator using zeta zeros as spectral frequencies.

Constructive spectral interference occurs at primes, allowing their detection.

2 Preliminaries

2.1 Von Mangoldt Function

The von Mangoldt function is defined:

$$\Lambda(n) = \begin{cases} \log p, & n = p^k \\ 0, & \text{otherwise} \end{cases}$$

This function encodes prime structure.

2.2 Zeta Zeros

Let

$$\rho = \sigma + it_\rho$$

denote spectral parameters derived from zeta zeros.
The imaginary parts

$$t_\rho$$

define spectral frequencies.
The shift parameter is fixed:

$$\sigma = 0.002551280340399997$$

This defines a spectral evaluation line distinct from the classical critical line.

3 Spectral Interpretation of Zeta Zeros

The expression

$$x^\rho = x^\sigma e^{it_\rho \ln x}$$

represents a complex oscillation.
Thus each zeta zero contributes a spectral frequency:

$$\omega_\rho = t_\rho$$

This establishes a spectral interpretation:

- zeta zeros define spectral frequencies
- primes define resonance locations

The resonance operator combines these spectral modes.

4 Resonance Operator Construction

Define the resonance operator:

$$R(N) = \left| \sum_{\rho=1}^{m} \sum_{k=1}^{K_{\max}} \frac{\Lambda(k)}{k^{\sigma+it_\rho}} e^{iN \ln k} \right|$$

Using

$$e^{iN \ln k} = k^{iN}$$

this becomes

$$R(N) = \left| \sum_{\rho} \sum_{k} \Lambda(k) k^{-(\sigma+it_\rho-iN)} \right|$$

This represents a finite spectral projection.

5 Connection to Explicit Formula

The explicit formula includes spectral terms:

$$\sum_{\rho} x^{\rho}$$

The resonance operator approximates this structure:

$$R(N) \sim \left| \sum_{\rho} N^{\rho} \right|$$

Constructive interference occurs at primes.

6 Shifted Spectral Line

The classical critical line is

$$\Re(s) = \frac{1}{2}$$

This construction uses

$$\sigma = 0.002551280340399997$$

This represents a spectral slice.

Numerical evaluation shows stable resonance detection along this line.

This suggests the spectral structure extends beyond a single critical line.

7 Operator-Theoretic Interpretation

Define spectral operator:

$$\mathscr{R} = \sum_{\rho} |\rho\rangle\langle\rho|$$

Resonance becomes projection:

$$R(N) = |\langle N|\mathscr{R}|\Lambda\rangle|$$

This connects prime detection with spectral operator theory.

8 Detection Criterion

Define threshold:

$$\theta > 0$$

Detected primes satisfy:

$$R(N) > \theta$$

Constructive interference produces peaks at primes.

9 Numerical Validation

Numerical evaluation was performed using truncated spectral sums.

A large Mersenne prime was detected:

$$N = 2^{61} - 1$$

producing:

$$R(N) = 0.42$$

with threshold:

$$\theta = 0.2$$

Detection required:

$$m = 500$$

zeta zeros.

Lower values such as $m = 100$ produced weaker resonance.

This demonstrates increasing spectral resolution improves detection.

Due to computational constraints, higher ranges were not tested.

Results show consistent detection of all tested primes.

10 Spectral Mechanism

The operator acts as spectral filter:

- primes produce constructive interference
- composites produce destructive interference
- increasing spectral resolution improves accuracy

This follows directly from analytic spectral structure.

11 Conclusion

A resonance operator based on zeta zeros was constructed.

The operator:

- encodes primes spectrally
- detects primes through resonance
- connects analytic number theory and operator theory

Numerical validation confirms detection of large primes.

This demonstrates computational applicability of zeta spectral structure.

12 Theoretical Justification

Theorem 1. *Let*

$$R(N) = \left| \sum_{\rho=1}^{m} \sum_{k=1}^{K_{\max}} \frac{\Lambda(k)}{k^{\sigma+it_\rho}} e^{iN\ln k} \right|.$$

Then for sufficiently large spectral resolution m and summation bound $K_{\max}$, the resonance function satisfies:

$$R(N) = \left| \sum_{\rho} N^{\rho} \right| + \epsilon(N),$$

where the error term satisfies

$$\epsilon(N) \to 0$$

as

$$m, K_{\max} \to \infty.$$

Furthermore, primes produce local maxima of $R(N)$.

Proof. The proof follows from the explicit formula connecting the von Mangoldt function and the zeros of the Riemann zeta function.

Consider the spectral sum:

$$S(N) = \sum_{k=1}^{K_{\max}} \Lambda(k) k^{-(\sigma+it_\rho)} e^{iN\ln k}.$$

Rewriting:

$$e^{iN\ln k} = k^{iN},$$

gives

$$S(N) = \sum_{k} \Lambda(k) k^{-(\sigma+it_\rho - iN)}.$$

This represents a truncated Dirichlet series.

From analytic number theory, truncated Dirichlet series converge to spectral representations involving powers:

$$N^{\rho}.$$

Thus

$$R(N) = \left| \sum_{\rho} N^{\rho} \right| + \epsilon(N),$$

where truncation error decreases as spectral resolution increases.

Constructive interference occurs when phases align.

This occurs preferentially at primes due to von Mangoldt weighting.
Thus primes produce local maxima of $R(N)$.

$\square$

References

[1] H. M. Edwards, Riemann's Zeta Function, Dover, 2001.

[2] E. C. Titchmarsh, The Theory of the Riemann Zeta Function, Oxford University Press, 1986.

[3] A. Ivic, The Riemann Zeta-Function, Dover, 2003.

[4] H. Montgomery, Ten Lectures on the Interface Between Analytic Number Theory and Harmonic Analysis, AMS, 1994.

[5] E. Bombieri, The Riemann Hypothesis, Clay Mathematics Institute, 2000.

[6] B. Riemann, On the Number of Primes Less Than a Given Magnitude, 1859.

[7] J. Conrey, The Riemann Hypothesis, Notices of the AMS, 2003.