

Resonance-Based Detection of Prime Numbers Using Riemann Zeta Zeros

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Abstract

Method to detect prime numbers using a *resonance operator* constructed from the first non-trivial zeros of the Riemann zeta function. This approach leverages the analytic properties of $\zeta(s)$ and the von Mangoldt function $\Lambda(n)$ to create a spectral filter that highlights primes in a given range.

1 Preliminaries

Let $\Lambda(n)$ denote the von Mangoldt function:

$$\Lambda(n) = \begin{cases} \log p, & \text{if } n = p^k \text{ for some prime } p \text{ and integer } k \geq 1, \\ 0, & \text{otherwise.} \end{cases}$$

Let $\sigma \in \mathbb{R}$ denote a small positive shift along the critical line (e.g., $\sigma = 0.00255128$). Let $\{t_\rho\}_{\rho=1}^m$ denote the first m non-trivial zeros of the Riemann zeta function:

$$\zeta(\sigma + it_\rho) = 0.$$

2 Resonance Operator

We define the *resonance function* $R(N)$ for integers $N \geq 1$ as:

$$R(N) = \left| \sum_{\rho=1}^m \sum_{k=1}^{K_{\max}} \frac{\Lambda(k)}{k^{\sigma+it_\rho}} e^{iN \ln k} \right|. \quad (1)$$

Here:

- The inner sum runs over integers $k = 1, \dots, K_{\max}$,
- Each term is weighted by $\Lambda(k)$ to emphasize prime powers,
- The factor $k^{-(\sigma+it_\rho)}$ introduces spectral modulation based on Riemann zeros,
- $e^{iN \ln k}$ creates constructive interference at prime N ,
- The absolute value captures the amplitude of the resonance.

For sufficiently large K_{\max} , this sum can be approximated by

$$R(N) \sim \left| \sum_{\rho=1}^m N^\rho \right|, \quad (2)$$

which clearly shows the connection between resonance peaks and the distribution of primes. This approximation highlights that the resonance operator effectively encodes the same information as the explicit formula for primes in terms of Riemann zeros.

3 Prime Detection Criterion

To detect primes, we introduce a threshold $\theta > 0$:

$$\text{Resonant numbers} = \{N : R(N) > \theta\}.$$

The set of correctly detected primes is

$$\text{Correct primes} = \{N \in \text{Resonant numbers} : N \text{ is prime}\},$$

and false positives correspond to resonant numbers that are not prime.

4 Interpretation and Implementation

The resonance operator acts as a spectral filter:

- Constructive interference occurs for prime N , producing peaks in $R(N)$.
- Destructive interference reduces the signal for composite numbers.
- Only the first m zeros are required for practical ranges (e.g., $m = 10$).
- Threshold θ can be empirically chosen to separate primes from non-primes.

The function can be evaluated numerically over a range $[N_{\min}, N_{\max}]$ to avoid memory issues. This method provides a bridge between analytic number theory and computational prime search.

5 Summary

This resonance-based approach demonstrates a practical application of Riemann zeta zeros in detecting prime numbers. By leveraging the spectral properties of $\zeta(s)$ and the von Mangoldt function, it:

- Produces a spectral signature for primes,
- Offers flexibility to scan large numerical ranges efficiently,
- Connects analytic number theory with computational prime detection.