Т-4

$\xi \sim R[\theta, 2\theta]$

а) <u>Метод моментов:</u>

$$\mathcal{L}_1 = M[\xi] = \int_{-\infty}^{+\infty} x\, p(x)\, dx = \int_{\theta}^{2\theta} x \frac{1}{\theta} dx = \frac{3}{2}\theta$$

$$\tilde{\mathcal{L}}_1 = \overline{x} \Rightarrow \mathcal{L}_1 = \tilde{\mathcal{L}}_1 \Leftrightarrow \frac{3}{2}\theta = \overline{x} \Rightarrow \tilde{\theta} = \frac{2}{3}\overline{x}$$

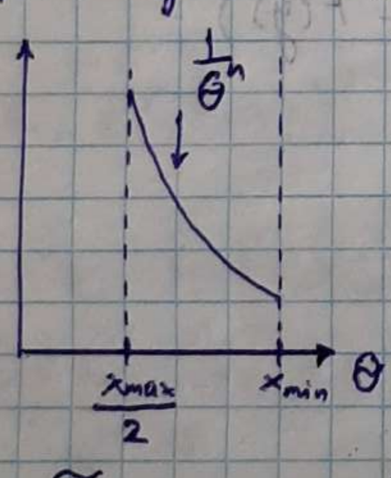
Оценка: $\tilde{\theta} = \frac{2}{3}\overline{x}$

<u>Метод максимального правдоподобия:</u>

$$L(\theta) = \prod_{i=1}^{n} p(x_i, \theta) = \frac{1}{\theta^n} \text{ при } \theta \le x_i \le 2\theta \ \forall i = \overline{1,n}$$

$$\theta \le x_i \le 2\theta \Rightarrow \begin{cases} \theta \le x_{min} \\ 2\theta \ge x_{max} \end{cases} \Leftrightarrow \frac{x_{max}}{2} \le \theta \le x_{min}$$

При условии, что $x_{max} \le 2x_{min}$



Из графика очевидно, что

$$\max L(\theta) \Rightarrow \tilde{\theta} = \frac{x_{max}}{2}$$

Оценка: $\tilde{\theta} = \frac{x_{max}}{2}$

б) $\tilde{\theta}_1 = \frac{2}{3}\overline{x}, \quad \tilde{\theta}_2 = \frac{x_{max}}{2}, \quad \tilde{\theta}_3 = \frac{1}{5}(x_{min} + 2x_{max})$

<u>Первая оценка:</u>

$M[\xi] = \frac{3}{2}\theta$ Несмещённость:

$M[\tilde{\theta}_1] = M[\frac{2}{3}\bar{x}] = \frac{2}{3} M[\frac{1}{n}\sum_{i=1}^{n} x_i] = \frac{2}{3}\frac{1}{n}\sum_{i=1}^{n} M[\xi] =$

$= \frac{2}{3} \cdot \frac{3}{2}\theta = \theta \Leftrightarrow M[\tilde{\theta}_1] = \theta \Rightarrow \tilde{\theta}_1$ не смещена

Состоятельность:

По достаточному условию:

$D[\tilde{\theta}_1] = M[\tilde{\theta}_1^2] - M^2[\tilde{\theta}_1] =$

$M \cancel{D}[\xi^2] = \int_{-\infty}^{+\infty} \frac{x^2}{\theta} dx = \frac{x^3}{3\theta}\Big|_{\theta}^{2\theta} = \frac{7}{3}\theta^2$

$M[\xi^2] - M^2[\xi] = \frac{7}{3}\theta^2 - \frac{9}{4}\theta^2 = \frac{1}{12}\theta^2 = D[\xi]$

$D[\tilde{\theta}_1] = D[\frac{3}{2}\bar{x}] = \frac{9}{4}\frac{1}{n^2}\sum_{i=1}^{n} D[\xi] =$

$= \frac{9}{4n} \cdot \frac{1}{12}\theta^2 \xrightarrow[n\to\infty]{} 0 \Rightarrow \tilde{\theta}_1$ – состоятельная.

<u>Вторая оценка:</u>

$F(x) = \int_{\theta}^{x} \frac{1}{\theta} dt = \frac{t}{\theta}\Big|_{\theta}^{x} = \frac{x}{\theta} - 1$

$F_{max}(x) = (F(x))^n = (\frac{x}{\theta} - 1)^n, \quad p(x) = \frac{n}{\theta}(\frac{x}{\theta} - 1)^{n-1}$

$M[\xi_{max}] = \int_{\theta}^{2\theta} \frac{nx}{\theta}(\frac{x}{\theta} - 1)^{n-1} dx = x(\frac{x}{\theta} - 1)^n\Big|_{\theta}^{2\theta} -$

$- \int_{\theta}^{2\theta} (\frac{x}{\theta} - 1)^n dx = 2\theta - \frac{\theta}{n+1}(\frac{x}{\theta} - 1)^{n+1}\Big|_{\theta}^{2\theta} =$

$$= 2\theta - \frac{\theta}{n+1} = \frac{2n+1}{n+1}\theta$$

Несмещённость:

$$M[\tilde{\theta}_2] = M[\frac{x_{max}}{2}] = \frac{1}{2} M[x_{max}] = \frac{1}{2} \cdot \frac{2n+1}{n+1}\theta \Rightarrow$$

$\Rightarrow \tilde{\theta}_2$ - смещена

Коррекция: $\tilde{\theta}_2' = \frac{2n+2}{2n+1}\tilde{\theta}_2$

Состоятельность:

$$D[\theta x_{max}] = M[x_{max}^2] - M^2[x_{max}]$$

$$M[\xi_{max}^2] = \int_{\theta}^{2\theta} \frac{x^2 n}{\theta}\left(\frac{x}{\theta}-1\right)^{n-1} dx =$$

$$= x^2\left(\frac{x}{\theta}-1\right)^n \Big|_{\theta}^{2\theta} - 2\int_{\theta}^{2\theta} x\left(\frac{x}{\theta}-1\right)^n dx =$$

$$= 4\theta^2 - 2\frac{\theta}{n+1} x\left(\frac{x}{\theta}-1\right)^{n+1}\Big|_{\theta}^{2\theta} + \int_{\theta}^{2\theta}\frac{2\theta}{n+1}\left(\frac{x}{\theta}-1\right)^{n+1} dx$$

$$= 4\theta^2 - \frac{4\theta^2}{n+1} + \frac{2\theta^2}{(n+1)(n+2)}\left(\frac{x}{\theta}-1\right)^{n+2}\Big|_{\theta}^{2\theta} =$$

$$= 4\theta^2\frac{n}{n+1} + 2\theta^2\frac{1}{(n+1)(n+2)} = 2\theta^2\left(\frac{2n^2+4n+1}{(n+1)(n+2)}\right)$$

$$= \frac{4n^2+8n+2}{(n+1)(n+2)}\theta^2$$

$$D[x_{max}] = M[x_{max}^2] - M^2[x_{max}] =$$

$$= \frac{4n^2+8n+2}{(n+1)(n+2)}\theta^2 - \frac{4n^2+4n+1}{(n+1)^2}\theta^2 =$$

$$\theta^2\left(\frac{4n^3+8n^2+2n+4n^2+8n+2-4n^3-4n^2-n-8n^2-8n-2}{(n+1)^2(n+2)}\right)=$$

$$=\theta^2\frac{n}{(n+1)^2(n+2)}$$

Достаточное условие:

$$D[\tilde{\theta}_2'] = D\left[\frac{2n+2}{2n+1}\frac{x_{max}}{2}\right] = \frac{(n+1)^2}{(2n+1)^2}D[x_{max}] =$$

$$=\frac{(n+1)^2}{(2n+1)^2}\frac{\theta^2 n}{(n+1)^2(n+2)} = \frac{\theta^2 n}{(n+2)(2n+1)^2}\xrightarrow[n\to\infty]{} 0 \Rightarrow$$

$\Rightarrow \tilde{\theta}_2'$ - состоятельна

Третья оценка:

max: $p(x) = \frac{n}{\theta}\left(\frac{x}{\theta}-1\right)^{n-1}$

min: $F_{min}(x) = 1-\left(1-\frac{\theta x}{\theta}+1\right)^n = 1-\left(2-\frac{x}{\theta}\right)^n$

$$p_{min}(x) = \frac{n}{\theta}\left(2-\frac{x}{\theta}\right)^{n-1}$$

$$M[x_{min}] = \int_\theta^{2\theta}\frac{nx}{\theta}\left(2-\frac{x}{\theta}\right)^{n-1}dx =$$

$$= -x\left(2-\frac{x}{\theta}\right)^n\Big|_\theta^{2\theta} + \int_\theta^{2\theta}\left(2-\frac{x}{\theta}\right)^n dx =$$

$$=\theta - \frac{\theta}{n+1}\left(2-\frac{x}{\theta}\right)^{n+1}\Big|_\theta^{2\theta} = \theta + \frac{\theta}{n+1} = \frac{n+2}{n+1}\theta$$

Несмещённость:

$$M[\tilde{\theta}_3] = M\left[\frac{1}{5}x_{min} + \frac{2}{5}x_{max}\right] =$$

$= \frac{1}{5} M[x_{min}] + \frac{2}{5} M[x_{max}] =$

$= \frac{1}{5} \frac{n+2}{n+1}\theta + \frac{2}{5} \frac{2n+1}{n+1}\theta =$

$= \frac{n+2 + 4n+2}{5(n+1)}\theta = \frac{5n+4}{5(n+1)}\theta \Rightarrow$ смещена

Коррекция: $\tilde{\theta}_3' = \frac{5(n+1)}{5n+4}\tilde{\theta}_3$

Состоятельность:

$D[\tilde{\theta}_3] = D[\frac{1}{5}x_{min} + \frac{2}{5}x_{max}] =$

$= \frac{1}{25}D[x_{min}] + \frac{4}{25}D[x_{max}] + \frac{4}{25}cov(x_{max}, x_{min})$

$D[x_{max}] = \frac{n\theta^2}{(n+1)^2(n+2)}$

$D[x_{min}] = M[x_{min}^2] - M^2[x_{min}]$

$M[x_{min}^2] = \int_\theta^{2\theta} \frac{nx^2}{\theta}(2-\frac{x}{\theta})^{n-1}dx =$

$= -x^2(2-\frac{x}{\theta})^n \Big|_\theta^{2\theta} + 2\int_\theta^{2\theta} x(2-\frac{x}{\theta})^n dx =$

$= \theta^2 - \frac{2\theta}{n+1}x(2-\frac{x}{\theta})^{n+1}\Big|_\theta^{2\theta} + \frac{2\theta}{n+1}\int_\theta^{2\theta}(2-\frac{x}{\theta})^{n+2}dx =$

$= \theta^2 + \frac{2\theta^2}{n+1} - \frac{2\theta^2}{(n+1)(n+2)}(2-\frac{x}{\theta})^{n+2}\Big|_\theta^{2\theta} =$

$$= \theta^2 + \frac{2\theta^2}{n+1} + \frac{2\theta^2}{(n+1)(n+2)} =$$

$$= \theta^2\left(\frac{n^2+3n+2 + 2n+4 + 2}{(n+1)(n+2)}\right) = \frac{n^2+5n+8}{(n+1)(n+2)}\theta^2$$

$$D[x_{min}] = \frac{n^2+5n+8}{(n+1)(n+2)}\theta^2 - \frac{n^2+4n+4}{(n+1)^2}\theta^2 =$$

$$= \frac{\cancel{n^3}+\cancel{5n^2}+\cancel{8n}+\cancel{n^2}+5n+\cancel{8}-\cancel{n^3}-\cancel{4n^2}-4n-\cancel{2n^2}-\cancel{8n}-\cancel{8}}{(n+1)^2(n+2)}\theta^2 =$$

$$= \frac{n}{(n+1)^2(n+2)}\theta^2$$

$$\mathrm{cov}(x_{max}, x_{min}] = M[x_{min}x_{max}] - M[x_{max}]M[x_{min}]$$

$$M[x_{min}x_{max}] = \iint\limits_{-\infty}^{+\infty} yz\,\frac{n(n-1)}{\theta^2}\left(\frac{z}{\theta}-\frac{y}{\theta}\right)^{n-2} dy\,dz =$$

$$= \int\limits_{\theta}^{2\theta} \frac{n(n-1)}{\theta^2} z\,dz \int\limits_{\theta}^{z} y\left(\frac{z}{\theta}-\frac{y}{\theta}\right)^{n-2} dy$$

Внутренний интеграл:

$$\int\limits_{\theta}^{z} y\left(\frac{z}{\theta}-\frac{y}{\theta}\right)^{n-2} dy = -\frac{\theta}{n-1}\,y\left(\frac{z}{\theta}-\frac{y}{\theta}\right)^{n-1}\Bigg|_{\theta}^{z} +$$

$$+ \frac{\theta}{n-1}\int\limits_{\theta}^{z}\left(\frac{z}{\theta}-\frac{y}{\theta}\right)^{n-1} dy = \frac{\theta^2}{n-1}\left(\frac{z}{\theta}-1\right)^{n-1}$$

$$- \frac{\theta^2}{n(n-1)}\left(\frac{z}{\theta}-\frac{y}{\theta}\right)^{n}\Bigg|_{\theta}^{z} = \frac{\theta^2}{n-1}\left(\frac{z}{\theta}-1\right)^{n-1} + \frac{\theta^2}{n(n-1)}\left(\frac{z}{\theta}-1\right)^{n}$$

Внешний интеграл

$$\int_{\theta}^{2\theta} nz\left(\frac{z}{\theta}-1\right)^{n-1}dz + \int_{\theta}^{2\theta} z\left(\frac{z}{\theta}-1\right)^{n}dz = J_1 + J_2$$

$$J_1: \int_{\theta}^{2\theta} nz\left(\frac{z}{\theta}-1\right)^{n-1}dz = \theta z\left(\frac{z}{\theta}-1\right)^{n}\Big|_{\theta}^{2\theta} -$$

$$- \theta\int_{\theta}^{2\theta}\left(\frac{z}{\theta}-1\right)^{n}dz = 2\theta^2 - \theta^2\frac{1}{n+1}\left(\frac{z}{\theta}-1\right)^{n+1}\Big|_{\theta}^{2\theta} =$$

$$= 2\theta^2 - \theta^2\frac{1}{n+1} = \frac{2n+1}{n+1}\theta^2$$

$$J_2: \int_{\theta}^{2\theta} z\left(\frac{z}{\theta}-1\right)^{n}dz = \frac{\theta}{n+1} z\left(\frac{z}{\theta}-1\right)^{n+1}\Big|_{\theta}^{2\theta} -$$

$$- \frac{\theta}{n+1}\int_{\theta}^{2\theta}\left(\frac{z}{\theta}-1\right)^{n+1}dz = \frac{2\theta^2}{n+1} - \frac{\theta^2}{(n+1)(n+2)}\left(\frac{z}{\theta}-1\right)^{n+2}\Big|_{\theta}^{2\theta} =$$

$$= \frac{2\theta^2}{n+1} - \frac{\theta^2}{(n+1)(n+2)} = \frac{2n+4-1}{(n+1)(n+2)} = \frac{2n+3}{(n+1)(n+2)}\theta^2$$

$$J_1 + J_2 = \frac{2n+1}{n+1}\theta^2 + \frac{2n+3}{(n+1)(n+2)}\theta^2 =$$

$$= \frac{2n^2+4n+n+2+2n+3}{(n+1)(n+2)}\theta^2 = \frac{2n^2+7n+5}{(n+1)(n+2)}\theta^2$$

$$M[x_{max}x_{min}] = \frac{2n^2+7n+5}{(n+1)(n+2)}\theta^2$$

$$M[x_{max}]\,M[x_{min}] = \frac{2n+1}{n+1}\cdot\frac{n+2}{n+1}\theta^2 =$$

$$= \frac{2n^2+5n+2}{(n+1)^2}\theta^2$$

$$\operatorname{cov}(X_{min}, X_{max}) = \frac{2n^2+7n+5}{(n+1)(n+2)}\theta^2 - \frac{2n^2+5n+2}{(n+1)(n+1)}\theta^2 =$$

$$= \frac{\cancel{2n^3}+\cancel{7n^2}+\cancel{5n}+\cancel{2n^2}+\cancel{7n}+5-\cancel{2n^3}-\cancel{5n^2}-\cancel{2n}-\cancel{4n^2}-\cancel{10n}-4}{(n+1)^2(n+2)}\theta^2 =$$

$$= \frac{\theta^2}{(n+1)^2(n+2)}$$

$$D[\tilde{\theta}_3] = \frac{1}{25}D[X_{min}] + \frac{4}{25}D[X_{max}] + \frac{4}{25}\operatorname{cov}(X_{max}, X_{min})$$

$$= \frac{1}{25}\left(\frac{n}{(n+1)^2(n+2)}\theta^2 + 4\frac{\theta^2 n}{(n+1)^2(n+2)} + 4\frac{\theta^2}{(n+1)^2(n+2)}\right) =$$

$$= \frac{1}{25}\left(\frac{5\theta^2 n + 4\theta^2}{(n+1)^2(n+2)}\right) = \frac{1}{25}\frac{5n+4}{(n+1)^2(n+2)}\theta^2$$

$$D[\tilde{\theta}_3'] = D\left[\frac{5(n+1)}{5n+4}\tilde{\theta}_3\right] =$$

$$= \frac{25(n+1)^2}{(5n+4)^2}D[\tilde{\theta}_3] = \frac{\theta^2}{(5n+4)(n+2)}$$

Состоятельность:

$$D[\tilde{\theta}_3'] = \frac{\theta^2}{(5n+4)(n+2)} \xrightarrow[n\to\infty]{} 0 \Rightarrow$$ состоятельна

по достаточному условию.

c) $D[\theta_1] = \frac{3}{16n}\theta^2$; $D[\tilde{\theta}_2'] = \frac{\theta^2 n}{(n+2)(2n+1)^2}$

$$D[\tilde{\theta}_3'] = \frac{\theta^2}{(5n+4)(n+2)}$$

$$D[\tilde{\theta}_1] \geq D[\tilde{\theta}_2'] \geq D[\tilde{\theta}_3']$$

$\tilde{\Theta}_3'$ – наиболее эффективная

Аналогично и для асимптотической оценки.

$\tilde{\Theta}_3' \xrightarrow[n\to\infty]{} \theta$ с большей скоростью.

e) Доверительный интервал для метода моментов:

$\overline{x} = \frac{3}{2}\tilde{\theta}$; $\quad M[\xi] = \frac{3}{2}\theta$

ЦПТ: $\dfrac{\overline{x} - M[\xi]}{\sqrt{D[\xi]}}\sqrt{n} \sim N(0,1)$

$$q_{\frac{1-\beta}{2}} < \frac{\overline{x} - M[\xi]}{\sqrt{D[\xi]}}\sqrt{n} < q_{\frac{1+\beta}{2}}$$

$$q_{\frac{1-\beta}{2}} < \frac{3}{2}\,\frac{\tilde{\theta} - \theta}{s}\sqrt{n} < q_{\frac{1+\beta}{2}}$$

$$\tilde{\theta}_1 - \frac{2}{3}\frac{s}{\sqrt{n}}\,q_{\frac{1+\beta}{2}} < \theta < \tilde{\theta}_1 - \frac{2}{3}\frac{s}{\sqrt{n}}\,q_{\frac{1-\beta}{2}}$$

$s^2 = \frac{1}{n-1}\sum_{i=1}^{n}(x_i - \overline{x})^2$, β – доверительная вероятность

q – квантиль.

f) Доверительный асимптотический интервал для $n=100$, $\Theta=10$, $\beta=0{,}95$

$9{,}69 < \Theta < 10.49$

g) Численный доверительный интервал для $n=100$, $N=1000$, $\Theta=10$, $\beta=0{,}95$

$9{,}67 < \Theta < 10.49$

h) в условиях данной выборки разброс значений Θ меньше при асимптотическом подходе.

Т-5

$$\xi \sim p(x) = \begin{cases} \frac{\theta-1}{x^{\theta}}, & x \geq 1 \\ 0, & x < 1 \end{cases}, \quad \theta > 1$$

а) Получение оценки методом максимального правдоподобия:

$$L(\theta) = \prod_{i=1}^{n} p(x_i, \theta) = \prod_{i=1}^{n} \frac{\theta-1}{x_i^{\theta}}$$

$$L(\theta) \to \max, \Leftrightarrow \ln L(\theta) \to \max$$

$$\ln L(\theta) = \sum_{i=1}^{n} [\ln(\theta-1) - \theta \ln x_i] =$$

$$= n \ln(\theta-1) - \theta \sum_{i=1}^{n} \ln x_i = n \ln(\theta-1) - \theta C$$

$$\frac{\partial \ln L(\theta)}{\partial \theta} = \frac{n}{\theta-1} - C = 0 \Rightarrow n - \theta C + C = 0$$

$$\theta = \frac{n}{C} + 1 = \frac{n}{\sum_{i=1}^{n} \ln x_i} + 1$$

Проверка на максимум:

$$\frac{\partial^2 \ln L(\theta)}{\partial \theta^2} = -\frac{n}{(\theta-1)^2} < 0 \Rightarrow \theta = \frac{n}{\sum_{i=1}^{n} \ln x_i} + 1 - \max$$

Оценка: $\tilde{\theta} = \frac{n}{\sum_{i=1}^{n} \ln x_i} + 1$

в) Построение доверительного интервала для медианы:

$med(\vec{x}_n) \Rightarrow F(med) = \frac{1}{2}$

$$F(x) = \int_1^x \frac{\theta-1}{t^\theta} dt = -\frac{1}{t^{\theta-1}}\Big|_1^x = 1 - \frac{1}{x^{\theta-1}} = \frac{1}{2}$$

$$x = 2^{\frac{1}{\theta-1}}$$

$\tilde{\theta}$ — состоятельна, т.к. модель регулярна, p — дважды непрерывно дифференцируема при $x \geq 1$, $\theta > 1$, Θ — открытое множество, $\Rightarrow$ по теореме о наследовании

$x = 2^{\frac{1}{\theta-1}} = 2^{\frac{\sum \ln x_i}{n}}$ — состоятельная оценка методом максимального правдоподобия медианы.

$$\tau(\theta) = 2^{\frac{1}{\theta-1}}, \quad \tilde{g}(\vec{x}_n) = 2^{\frac{\sum_{i=1}^n \ln x_i}{n}}$$

$$\tau'(\theta) = -\ln 2 \cdot 2^{\frac{1}{\theta-1}} \frac{1}{(\theta-1)^2}$$

$$I(\theta) = M\left[\left(\frac{\partial \ln p(x,\theta)}{\partial \theta}\right)^2\right] = -M\left[\frac{\partial^2 \ln p(x,\theta)}{\partial^2 \theta}\right]$$

$$M\left[\frac{\partial^2 \ln p(x,\theta)}{\partial^2\theta}\right] = \int_1^{+\infty} \frac{1}{1-\theta}\,\frac{1}{x^{\theta}}\,dx =$$

$$= \frac{1}{(1-\theta)^2}\,\frac{1}{x^{\theta-1}}\Big|_1^{+\infty} = -\frac{1}{(\theta-1)^2}$$

$$I(\theta) = \frac{1}{(\theta-1)^2}$$

$$\sqrt{n I(\theta)}\;\frac{\tilde{g}(\vec{x}_n) - \tau(\theta)}{\tau'(\theta)} \rightsquigarrow N(0,1)$$

$$\tilde{g}(\vec{x}_n) = \tilde{m}, \quad \tau(\theta) = m$$

$$\sqrt{\frac{n}{(\theta-1)^2}}\;\frac{\tilde{m} - m}{(-\ln 2)\cdot 2^{\frac{1}{\theta-1}}\,\frac{1}{(\theta-1)^2}} \rightsquigarrow N(0,1)$$

$$\sqrt{n}\,(\theta-1)\;\frac{m-\tilde{m}}{\ln 2\cdot 2^{\frac{1}{\theta-1}}} \rightsquigarrow N(0,1)$$

$$\sqrt{n}\,(\tilde{\theta}-1)\;\frac{m-\tilde{m}}{\ln 2\cdot \tilde{m}} \rightsquigarrow N(0,1)$$

$$q_{\frac{1-\beta}{2}} < \frac{\sqrt{n}\,(\tilde{\theta}-1)}{\ln 2\cdot \text{[redacted]}}\left(\frac{m}{\tilde{m}} - 1\right) < q_{\frac{1+\beta}{2}}$$

$$\tilde{m} + \frac{q_{\frac{1-\beta}{2}}\ln 2\,\tilde{m}}{\sqrt{n}\,(\tilde{\theta}-1)} < m < \tilde{m} + \frac{q_{\frac{1+\beta}{2}}\ln 2\,\tilde{m}}{\sqrt{n}\,(\tilde{\theta}-1)}$$

d) Построение асимптотического доверительного интервала:

$$(\tilde{\theta} - \theta)\sqrt{n I_1(\theta)} \sim N(0,1)$$

$$\frac{q_{\frac{1-\beta}{2}}}{\sqrt{n I_1(\theta)}} < \tilde{\theta} - \theta < \frac{q_{\frac{1+\beta}{2}}}{\sqrt{n I_1(\theta)}}$$

$$\tilde{\theta} - \frac{q_{\frac{1+\beta}{2}}}{\sqrt{n}}(\tilde{\theta}-1) < \theta < \tilde{\theta} - \frac{q_{\frac{1-\beta}{2}}}{\sqrt{n}}(\tilde{\theta}-1)$$

e) Доверительный асимптотический интервал при $n = 100$, $\theta = 10$, $\beta = 0,95$.

θ: $8,87 < \theta < 12,71$

med: $1,06 < med < 1,09$

med для данной выборки $\overline{x_n}$: $med = 1,07$

f) Численный доверительный интервал при $\theta = 10$, $n = 100$, $N = 1000$, $\beta = 0,95$

$9.06 < \theta < 13.33$

g) Разброс (доверительный интервал) для оценки методом моментов уже => ОМП точнее для данной выборки.