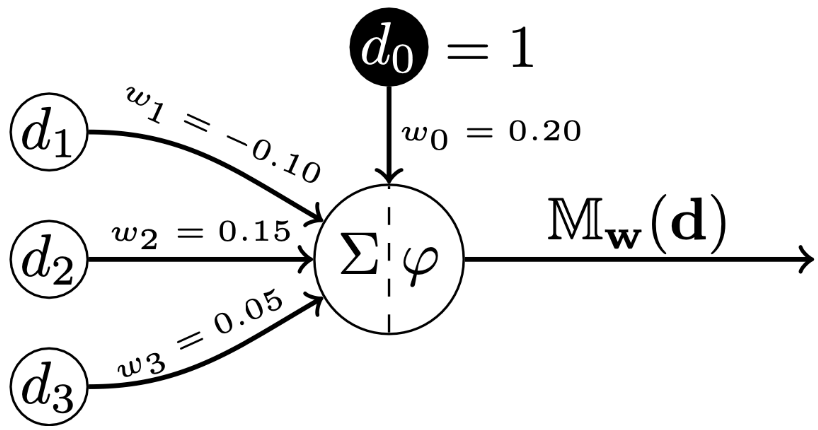
DSBA 6156
Assignment #3
Module 8

1. The following image shows an artificial neuron that takes 3 inputs: (4 points)



   a. Calculate the weighted sum for this neuron for the input vector: d = [0.2, 0.5, 0.7]
   b. What would be the output from this neuron if the activation function φ is a threshold activation with $\theta = 1$?

$$\mathbb{M}_{\mathbf{w}}(\mathbf{d}) = \begin{cases} 1 & \text{if } z \geq \theta \\ 0 & otherwise \end{cases}$$

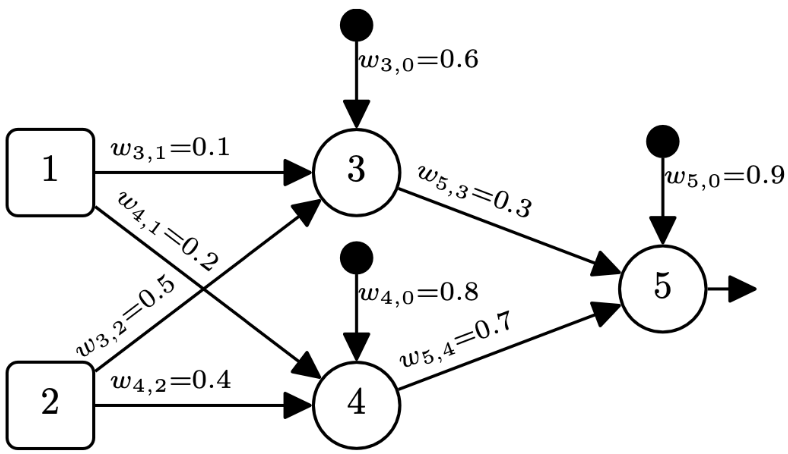
   c. What would be the output from this neuron if the activation function φ is the logistic function?

$$logistic(z) = \frac{1}{1 + e^{-z}}$$

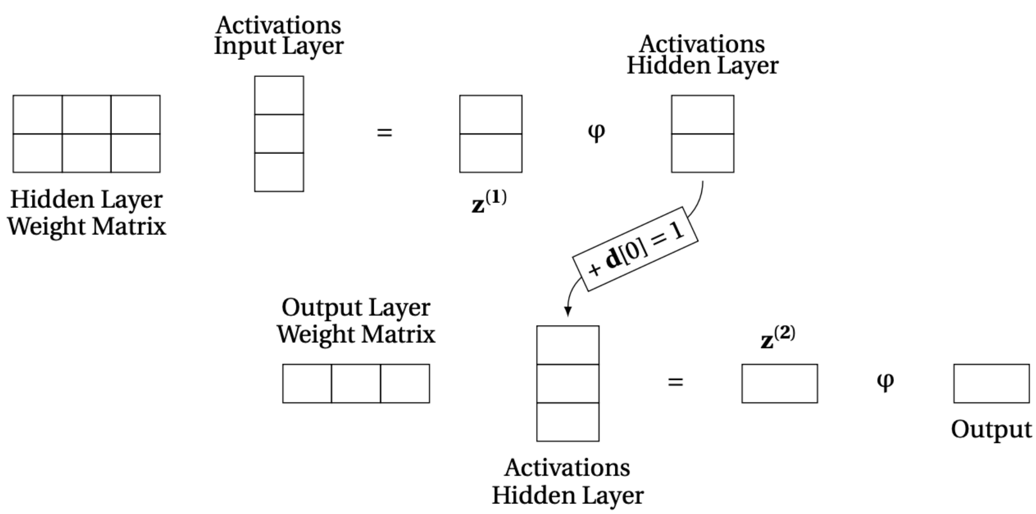
   d. What would be the output from this neuron if the activation function φ is the rectified linear function?

$$rectifier(z) = max(0, z)$$

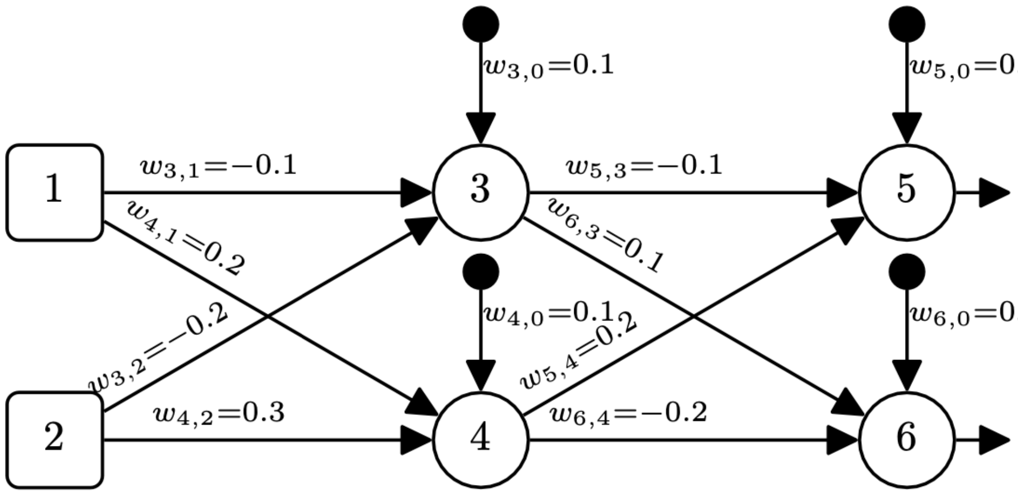
2. The following image shows an artificial neural network with two sensing neurons (Neurons 1 and 2) and 3 processing neurons (Neurons 3, 4, and 5)



a. Assuming that the processing neurons in this network use a logistic activation function, what would be the output of Neuron 5 if the network received the input vector: Neuron 1 = 0.7 and Neuron 2 = 0.3? (2 points)
b. Assuming that the processing neurons in this network use a ReLU activation function, what would be the output of Neuron 5 if the network received the input vector: Neuron 1 = 0.7 and Neuron 2 = 0.3? (1 point)
c. The following image provides a template diagram for the sequence of matrix operations that our neural network would use to process the input vector Neuron 1 = 0.7 and Neuron 2 = 0.3. Assuming that the processing neurons in the network use a ReLU activation function, fill in the diagram (Excel) with the appropriate weights, bias terms, weighted sum values, and activations. (4 points)



3. The following image illustrates the topology of a feedforward neural network that has two sensing neurons (Neurons 1 and 2), two hidden processing neurons (Neurons 3, and 4), and two processing output neurons (Neurons 5 and 6).



a. Assuming that the processing neurons use a rectifier activation functions, that the input to the network is Neuron 1 = 0.3 and Neuron 2 = 0.6 and that the desired output for this input is Neuron 5 = 0.7 and Neuron 6 = 0.4:
   i. Calculate the output generated in response to this input. (2 points)
   ii. Calculate the sum of squared errors for this network in this example. (1 point)
   iii. Calculate the δ values for each of the processing neurons in the network (i.e., $\delta 6,\ \delta 5,\ \delta 4,\ \delta 3$). (2 points)
   iv. Using the δ values you calculated above, calculate the sensitivity of the error of the network to changes in each of the weights of the network i.e.

$\partial\mathcal{E}/\partial w_{6,4},\ \partial\mathcal{E}/\partial w_{6,3}, \partial\mathcal{E}/\partial w_{6,0}, \partial\mathcal{E}/\partial w_{5,4}, \partial\mathcal{E}/\partial w_{5,3}, \partial\mathcal{E}/\partial w_{5,0}, \partial\mathcal{E}/\partial w_{4,2}, \partial\mathcal{E}/\partial w_{4,1}, \partial\mathcal{E}/\partial w_{4,0}, \partial\mathcal{E}/\partial w_{3,2}, \partial\mathcal{E}/\partial w_{3,1}, \partial\mathcal{E}/\partial w_{3,0}$. (3 points)

   v. Assuming a learning rate of $\alpha = 0.1$, calculate the updated values for each of the weights in the network $(w_{6,4}, w_{6,3}, w_{6,0}, w_{5,4}, w_{5,3}, w_{5,0}, w_{4,2}, w_{4,1}, w_{4,0}, w_{3,2}, w_{3,1}, w_{3,0})$ after the processing of this single training example.(3 points)
   vi. Calculate the reduction in the error of the network for this example using the new weights, compared with using the original weights.(3 points)