Model Development Using Machine Learning Models

September 18, 2023

Outline

- Tree Ensembles (review)
- Artificial Neural Networks
- Hyper-Parameter tuning

Last Time: Generalized Linear Model Framework

In the GLM framework:

- A distribution for the response Y is appropriately chosen
- A function of the conditional mean of the response is modeled as a linear combination of the predictors:

$$- g(E[Y|X]) = g(\mu) = X\beta = \beta_0 + \beta_1 X_1 + \dots + \beta_p X_p$$

- where $g(\cdot)$ is called the link function, and relates the linear combination of predictors to the mean of the distribution of y
- Nonlinearity and interactions need to be included with appropriately selected variable transformations

Generalized Additive Model (GAM):

$$E(Y|X)$$
 or $logodds(p) = f(X) = \alpha + f_1(X_1) + f_2(X_2) + ... + f_p(X_p)$

 $f_i(X_i)$: unspecified smooth non-parametric functions

Tree Ensemble Review

Decision Trees

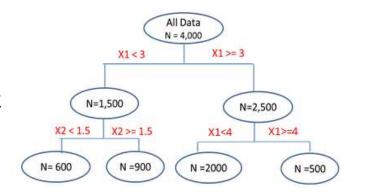
• Decision trees partition the feature space into a set of rectangles and fit a simple model (e.g., constant) in each one.

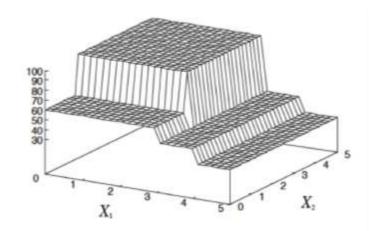


- Fast, intuitive
- Able to handle both numeric and categorical data
- Robust to outliers in predictors
- Model interaction and nonlinearity automatically (little data transformation)

• Disadvantage:

- High bias for shallow trees, for example trying to model linear relationships
- Unstable, high variance for deep trees. Small change in data can result in a completely different tree



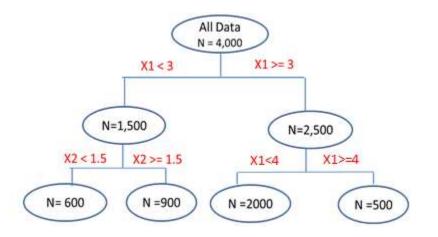


CART

- CART (classification and regression tree) by Breiman et. al.(1984).
 - Starts from the root node with all data
 - Splits into child nodes based on a variable → child node as homogeneous as possible.
 - Loss functions for measuring homogeneity:
 - MSE(continuous)
 - Gini-Index/cross-entropy/logloss (binary/multi-class)
 - Pruning: grow deep tree → prune to minimize cost complexity
 - Results can be viewed as a simple piecewise constant regression model

-
$$\hat{Y} = c_1 \times I(X_1 < 3, X_2 < 1.5) + c_2 \times I(X_1 < 3, X_2 \ge 1.5) + c_3 \times I(3 \le X_1 < 4) + c_4 \times I(X_1 \ge 4)$$

- Model based trees fit a simple model at each node instead of constant
- Algorithms with more than two splits (CHAID by Kass 1980)
- M5(Quinlan (1992)), LOTUS(Chan and Loh(2004)), SLIM(Hu et. Al. 2020)
- Implementations
 - Scikit-learn: DecisionTreeRegressor and DecisionTreeClassifier
 - R: rpart package
 - Spark: mllib library



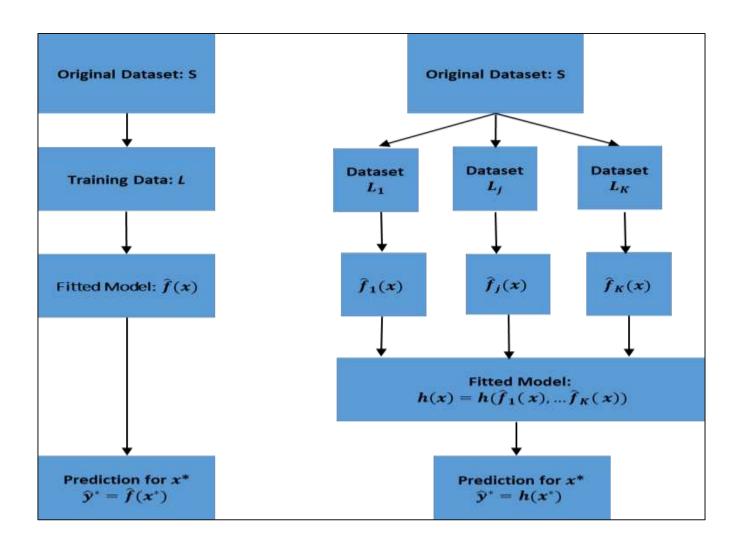
Performance comparison

• Using the UCI <u>Bike Share data</u> on rental counts.

| | | Model | test_MSE | test_MAE | test_R2 | train_MSE | train_MAE | train_R2 | Time |
|--|---|-------|----------|----------|---------|-----------|-----------|----------|------|
| | 2 | Tree | 0.0052 | 0.0436 | 0.8511 | 0.0040 | 0.0383 | 0.8868 | 0.0 |
| | 1 | GAM | 0.0114 | 0.0791 | 0.6762 | 0.0110 | 0.0778 | 0.6868 | 0.2 |
| | 0 | GLM | 0.0221 | 0.1103 | 0.3727 | 0.0223 | 0.1122 | 0.3637 | 0.0 |

Able to capture **interactions** in the data and hence higher predictive performance

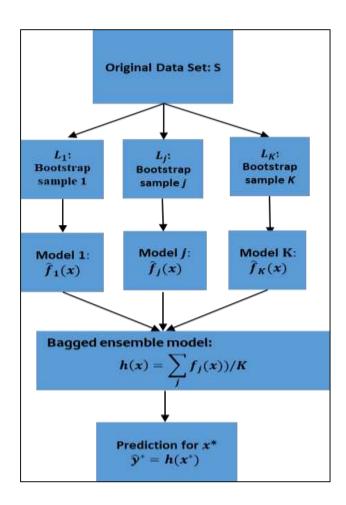
Ensemble algorithms



- Improve performance by combining the outputs of several individual learners.
- Examples:
 - Bagging
 - Boosting
 - Model Averaging
 - Majority Voting
 - Ensemble Stacking
- Mostly bagging and boosting algorithms use tree based learners.

Bagging

- Bagging: bootstrap aggregating(Breiman in 1994)
 - "improvements for unstable procedures" (Breiman 1996, example: deep decision tree
- Algorithmic framework:
 - bootstrap sample at each iteration i, i = 1, 2, ..., n.
 - Fit base learner to bootstrap sample $\rightarrow \hat{f}_i(x)$
 - Combine base model predictions :
 - Averaging (Regression): $\hat{f}(x) = \frac{1}{n} \sum_{i} \hat{f}_{i}(x)$
 - Majority voting (Classification): $\hat{f}(x) = \arg \max_{k} \frac{1}{n} \sum_{i} I(\hat{f}_{i}(x) == k)$
- Loss functions:
 - Tree based learners grow trees using same loss functions as in CART
 - Loss functions for measuring homogeneity in leaves:
 - MSE(continuous)
 - Gini-Index/cross-entropy/logloss (binary/multi-class)
- Combining base model predictions reduces variance, making model stable
 - More base learners → better prediction, higher computational complexity
 - Base learners: low bias high variance



Why bagging works?

- Continuous response:
 - Assuming independence of base learners, averaging reduces variance

$$- Var\left(\frac{1}{n}\sum_{i}\hat{f}_{i}(x)\right) = \frac{Var(\hat{f}_{i}(x))}{n}$$

- Binary response:
 - Assume Bayesian optimal decision at x is 1
 - $-P(\hat{f}_i(x)=1)=e>0.5$
 - $\sum_{i} I(\hat{f}_{i}(x) = 1) \sim Binomial(n, e)$ [i.i.d base learners]

$$-P(\hat{f}(x)=1) = P(\frac{1}{n}\sum_{i}I(\hat{f}_{i}(x)=1) > 0.5) \to 1 \text{ as } n \to \infty$$

- In reality, base learners are not independent due to overlap in bootstrap samples
 - Variance $\rightarrow c \neq 0$ as *n* increases.
 - Correlation limits the reduction of variance. How to further de-correlate the base models?

Random Forests

- Random Forests (Breiman 2001)
 - Bagging and random feature subsampling
- Deep Trees
 - Low bias, high variance
 - Reduce variance through bagging
 - A variant uses sample without replacement
- De-correlate trees
 - use random subset of features in each split instead of entire feature set
 - Tries to achieve maximum variance reduction
- Typically over-fits the data
- Typical Hyper-Parameters (HP) to be tuned (sklearn)
 - N_estimators : number of forests
 - Max_depth: maximum depth of the trees
 - Min_samples_leaf (MSL): the minimum samples required at each leaf
 - Max_features: number of features to consider at each split

Boosting

- Boosting is a different type of ensemble algorithm, based on removing bias of a simple learner.
- Given a simple learner, can you improve it to be a strong learner? (Kearns and Valiant 1988)
- Schapire (1989): Yes → by a technique called "boosting"
- Freund and Schapire (1995): AdaBoost for classification

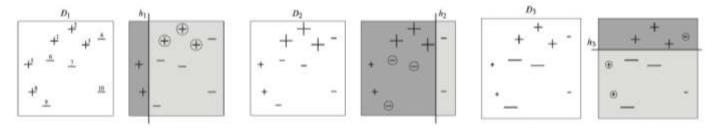
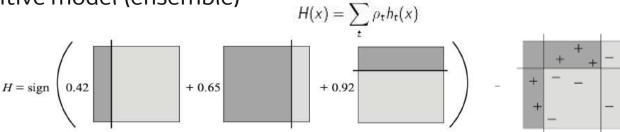


Figure: AdaBoost. Source: Figure 1.1 of [Schapire and Freund, 2012]

- "Base learner": simple rectangular classification regions at each stage
- Reweighting at each stage more weight to data that are misclassified
- Fit an additive model (ensemble)



Source: Figure 1.2 of [Schapire and Freund, 2012]

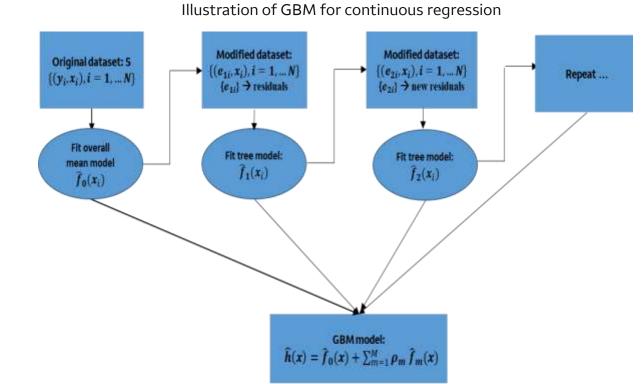
Gradient Boosting

- Breiman (1998+): Boosting → optimization algorithm
- Friedman (2000+): Extended concept to gradient boosting (gradient descent)
- Loss function to minimize: L(y, f).
 - squared error loss $(y f)^2$ for regression
 - deviance $yf \log(1 + e^f)$ for binary classification (f is the logodds) (deviance = logloss)
 - Other loss functions: absolute error loss, partial likelihood, etc
- Find the prediction function f(x) that minimize the total loss $\sum_{i=1}^{N} L(y_i, f(x_i))$.
 - f(x) is optimized in an additive, stage-wise way: $f(x) = T_0(x) + \sum_{m=1}^{M} \eta_m T_m(x)$, where $T_0(x)$ is baseline (e.g., overall mean in regression).
 - In each stage m, update f(x) in the direction $T_m(x)$ where the total loss decreases, for a step size/learn rate of η_m .
 - Each base learner $T_m(x_i)$ is fit to the negative gradient (gradient descent) from the previous iteration

$$- T_m(x_i) = -\frac{\partial L(x_i, y_i)}{\partial \hat{f}_{m-1}(x_i)}$$

Gradient Boosting

- Stochastic gradient boosting (Friedman 1999): fit each tree with a subsample instead of the entire data. This can be more robust and less overfitting.
- Hyper-Parameters (HP):
 - number of trees,
 - learn rate, tree
 - depth, ...
- Implementation:
 - Scikit learn: GradientBoostingClassifier and GradientBoostingRegressor
 - R: gbm package
 - Spark: mllib library
 - H2o: h2o.gbm
- Other popular variations
 - XGBoost
 - LightGBM
 - CatBoost



XGBoost

- XGBoost (Extreme Gradient Boosting) (Chen and Guestrin 2016) a variant of GBM that uses second order Hessian
- XGBoost adds a **penalty** term to the loss function

$$\sum_{i=1}^{N} L(y_i, f(x_i)) + \sum_{m=1}^{M} \Omega(T_m(x))$$

to control overfitting.

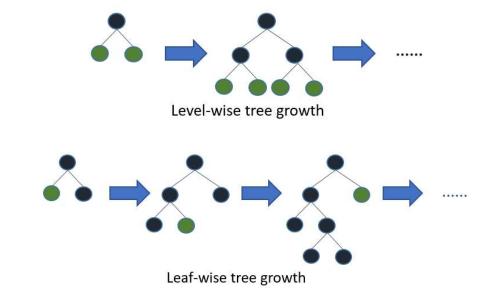
- $-\Omega(T)=\gamma |T|+rac{1}{2}\lambda\sum_j w_j^2$, where |T| is the number of terminal nodes in the tree and w_j is the fit in terminal node j.
- The parameter γ is a pruning parameter: any split with the improvement below γ is pruned.
- The parameter λ acts like a shrinkage parameter: the prediction in each tree node is shrunk $w_j = -\frac{G_j}{H_j + \lambda}$, where $G_j = \sum_{i \in I_j} g_i$, $H_j = \sum_{i \in I_j} h_i$ are the total gradients and total **Hessian** in Node j.
- An L1 penalty ($\alpha \sum_i |w_i|$) can be added as well.
- The trees are built **depth-wise**.
- XGBoost has the following advantages:
 - Parallelized. So it is scalable.
 - Penalized, to reduce overfitting issue.
- It was originally implemented in C++ but it is available in Java, Python, R through APIs. Has Scikit-learn wrapper.
- Hyper-Parameters(HP) include: max_depth, learning_rate (lr_rate), N_estimators (# trees), L1 (alpha), L2 (lambda), min_child_weight, ...

LightGBM

- Introduces two new concepts (<u>Ke et al, 2017</u>)
 - Gradient One-Sided sampling(GOSS)
 - At each iteration, keep all observations with large gradients and random subset on instances with smaller gradients.
 - Exclusive feature bundling (EFB)
 - Bundles up mutually exclusive features.
 - Helpful in feature reduction for high dimensional data with large number of 0-1 encoded columns.
- Builds trees **leaf-wise**
- Hyper-Parameters:
 - Lr_rate
 - Max_depth
 - Num_leaves
 - Min_data_in_leaf
- Developed by Microsoft. Built in C++, has python and R interface.

XGBoost vs LightGBM

- Numerous blogs on comparison
- Similar performance
- LightGBM is faster
- XGB by default grows tree level-wise(split node closer to root)
- LightGBM grows trees leaf-wise
- Difference in the algorithm may lead to different feature importance and other diagnostics.



Artificial Neural Networks

Neural Networks: history and inspiration from neuroscience

Wave 1: "Cybernetics": [1940s-1960s]

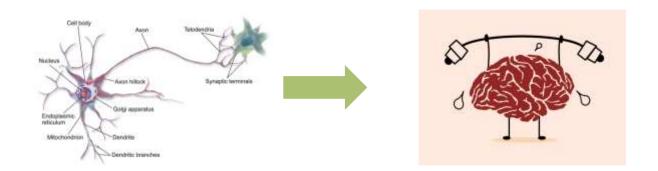
- Model of biological function of brain
- 1958 Rosenblatt: Developed "perceptron", a training algorithm
- Effectively ended by Perceptrons (Minsky and Papert), who showed difficulties with current approaches.

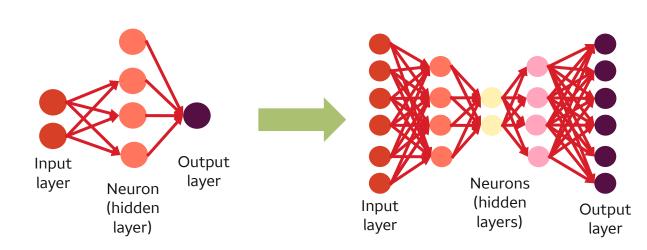
Wave 2: "Connectionism": [1980s-1990s]

- From the study of how the brain can form connections.
- Central Idea: A network of many simple units can learn complex patterns.
- The backpropagation algorithm provided an essential advance in training neural networks.

Third Wave [2006-present]: "Deep Learning"

- Ability to train neural networks with more layers.
- Able to outperform other ML systems:
 - Object recognition in pictures
 - Natural Language Processing



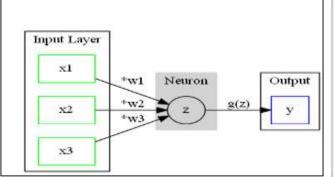


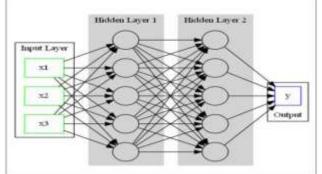
Feedforward Neural Networks (FFNN):

- Activation function: $g(w^Tx)$
 - Sigmoidal, Hyperbolic Tan, ReLU
 - Connection to additive index models:

$$f(x) = g(w_1x_1 + ... + w_Px_P)$$

- FFNN architecture
 - Nodes (Neurons)
 - Input, Output, and Hidden Layers
 - All nodes connected with others in next layer
- Hyper-Parameters(HP)
 - Learning rate
 - Number of layers
 - Neurons in each layer
 - L1, L2, dropout
 - batch-size
 - activation function...
- Implementation
 - Scikit-learn MLP
 - Keras
 - PyTorch





Single Layer

- "Universal Approximation Theorem":
 Wide hidden layer +squashing activation function
 can approximate any well behaved function arbitrarily well
- Possible over-fitting

Multi-Layer

- Increased representation power
- Less prone to over-fitting
- Increase in computation cost and lack of interpretability

Fitting a Neural Network to Data: Learning the Weights

- The weights (and bias) of each neuron are the unknown parameters in an ANN that need to be learned from data.
- To do so, we define an appropriate cost function that:
 - Represents an average of the cost of individual observations in the training set.
 - Should be a function of the outputs from the ANN and the response y.
 - Examples:
 - For continuous responses, we use squared error loss:

$$\frac{1}{n}\sum_{i}(y_{i}-\hat{y}_{i})^{2}.$$

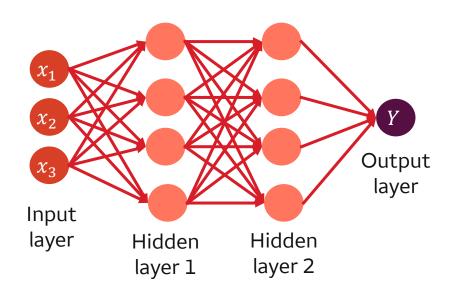
- For binary response, we use cross entropy, or log loss: $-\frac{1}{n}\sum_{i}[y_{i}\log\hat{y}_{i}+(1-y_{i})\log(1-\hat{y}_{i})]$
- For multinomial responses, we use a generalization of cross-entropy, with *j* indexing category:

$$-\frac{1}{n} \sum_{i} \sum_{j} \left[y_i^{(j)} \log \hat{y}_i^{(j)} \right]$$

- Choose the weights and biases that minimize the cost function.
 - In principle, this can be achieved using calculus.
 - However, many of these solutions are not easily implemented in ANNs.

Optimization: Back Propagation Algorithm

- Gradient Descent, can be challenging in NNs, due to computation of gradient.
- Preparation: Input the data x, and initialize all weights in the network.
- In practice, optimization of ANNs has become a research area:
 - many other algorithms and research (Adam, SGD, etc)
 - mini-batch training



Feedforward

- Feed the data through the network
- Compute the output of each node based on the current weights

Update

- Update the weights using gradient descent
- Return to step 1 "Feedforward"

Gradient

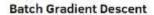
 Compute the gradient of the cost function with respect to the last hidden layer

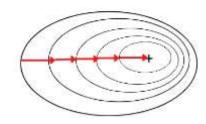
Backward propagation

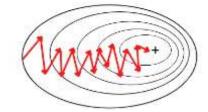
- Work backwards through the network
- Compute the gradient of the cost function as they depend on the weights in the lower layers

Different forms of gradient descent

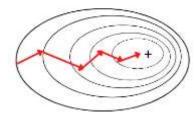
- Batch gradient descent
 - computes gradients of all samples at each iteration
- Stochastic gradient descent
 - computes gradient of single random sample at each iteration
- Mini batch gradient descent
 - partitions data and computes gradient of one partition at each iter stochastic Gradient Descent
- Variations include
 - Adaptive learning rates
 - Adding momentum
- Different variations of gradient descent
 - Adagrad
 - RMSProp
 - Adam







Mini-Batch Gradient Descent



Using ANNs in Practice

General Usage

- Very flexible. Choices for:
 - Structure: Number of Hidden Layers,
 Number of nodes on Each Hidden
 Layer, Activation Functions for each
 Hidden Layer
 - Regularization Strategy/ Parameters
 - Additional Features: Skip connections, Batch Normalization, Dropout, Constraints
 - Training/Optimization Algorithms
- Can make grid search difficult
- Note: Much of the literature available gives advice in the context of unstructured data (images/text/speech).

Such advice may not always be useful in banking problems.

Training effectively

- Training can be challenging; saddle points and local minima can result in a sub-optimal model.
- Tips:
 - Standardize or Normalize data (X) before training. Avoids vanishing gradient problem.
 - Min/Max scaling
 - Gaussian standardization (perform better with large outliers)
 - Batch Normalize hidden layers.
 - optimization routine with learning rate decay (e.g. Adam).
 - batch size used in training smaller batches can be slow and volatile, but help escape local minima/saddle points.
 - Use early stopping to determine number of training epochs.

Overfitting

- ANNs are flexible models, with a (potentially) large number of parameters, therefore overfitting is a concern.
- Strategies to avoid overfitting in ANNs include:
 - Multiple narrow layers vs. Single wide layer
 - Weight Regularization: Penalizing large weights in the cost function.
 - Dropout: Randomly set a fraction of neurons' activations to 0 during training. This forces redundancy of neurons, and reduces neuron specialization.
 - Early Stopping via a validation set.

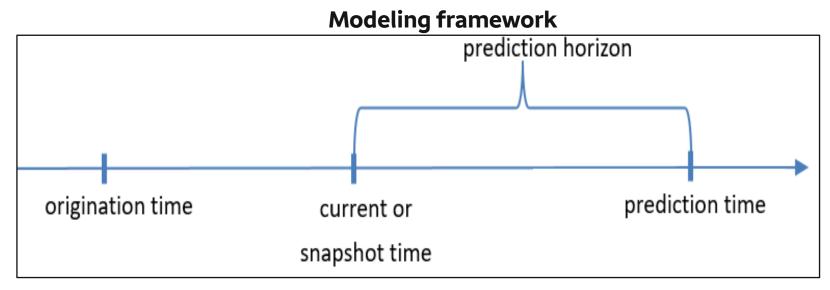
Comparison of ML Algorithms

Below is comparison among some most popular machine learning algorithms.

| | Preprocess | Robustness to outliers in input space | Computational scalability (large N) | Predictive power | Smoothness of response surface | Feature engineering | Hyper parameter tuning |
|-------------------|--|--|--|---|---|---|------------------------------|
| GBM | No, some require dummy coding | Yes, tree based methods are robust to outliers | Depend on software implementation. XG B, LightGBM and H2O GBM are scalable | Good, often outperforms random forest and neural network in prediction | The response surface for tree based methods are often jumpy and not smooth, especially for small data | Good for manually created features; may not be good for raw features, e.g., transaction data, image data. | Relatively easy |
| Random Forest | No, some require dummy coding | Yes, tree based methods are robust to outliers | Depend on software implement ation. Scikit learn random forest, H2O random forest are scalable | Good | The response surface for tree based methods are often jumpy and not smooth, especially for small data | Good for manually created features; may not be good for raw features, e.g., transaction data, image data. | Relatively easy |
| Neural Network | dummy coding and standardizatio n | No | Large neural network with large data requires GPU. Computation with simple network or small data is scalable on CPU. | Good. Best for image, speech, | Usually smooth | Powerful in feature engineering with a variety of deep neural network structures. | Complicated |

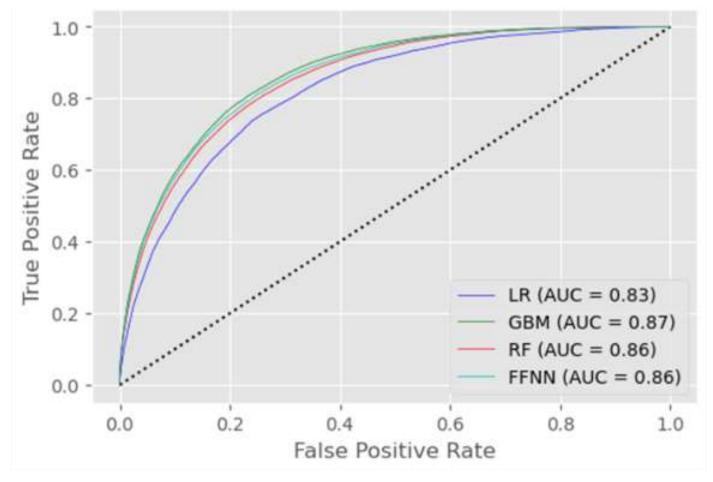
Application to Home Mortgage: Modeling "In-Trouble" Loans

- One portfolio: ~ 5 million observations
- Response: binary = loan is "in trouble" (multiple failures and connections to competing risks)
- 20+ predictors: credit history, type of loan, loan amount, loan age, loan-to-value ratios, interest rates at origination and current, loan payments up-to-date, etc. (origination and over time)



Loan origination, current (snapshot) and prediction times

Comparison of Predictive Performance: ROC and AUC on Test Data



- ML with 22 predictors
- LR model: eight carefully selected variables
 - snapshot fico (credit history);
 - Itv (loan-to-value ratio);
 - ind_financial-crisis;
 - o pred_unemp_rate;
 - two delinquency status variables;
 - horizon

How typical is this "lift" in our applications?

Findings from internal study -XGB and FFNN are competitive and exhibit better model performance across a variety of functional forms than RF

https://arxiv.org/ftp/arxiv/papers/2204/2204.12868.pdf

Neural Network – other architectures

Complex Architectures

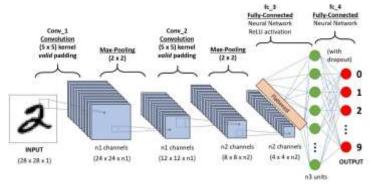
Convolutional NN

Used in images, text, time series

- Key Features:
 - Convolutional layers, where inputs are convolved with their neighbors.
 - Each output is a weighted average of the inputs:

$$\sum_{i,j} a_{i,j} \, x_{i,j}$$

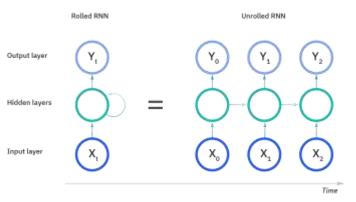
- The weights remain constant as the convolution is applied to successive windows of data.
- Other tricks, like Pooling



A CNN sequence to classify handwritten digits

Recurrent NNs

- Useful in studying sequences, such as in natural language context.
- Defined by "recurrent" connections, where the output of a downstream unit serves as input to an upstream neuron.
- Several variations; "Long Short-Term Memory" (LTSM) was popular; now focus on Attention Networks.

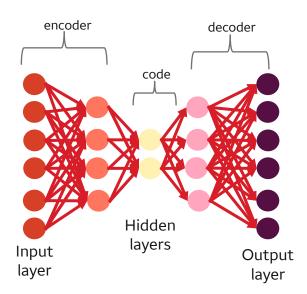


https://www.ibm.com/cloud/learn/recurrent-neural-networks

Complex Architectures 2

Auto-Encoders

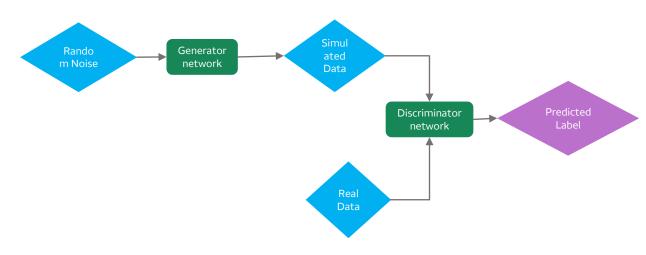
- Dimension Reduction Network
- Predicts input from input.
- Bottleneck layer engineers lower-dimensional features.
- Different types
 - De-noising AE
 - Sparse AE
 - Variational AE ...



GAN: Generative Adversarial Networks

Unsupervised technique

- Pair of ANNs, trained with simultaneous backpropagation
- A Generator Network, which produces candidate data examples
- A Discriminator Network, which learns to distinguish the generated data from the real data. (Classification)
- Simultaneous training improves the performance of both networks.



Advanced Architectures and Ongoing Research

Attention Networks

- Added mechanisms to model dependencies across data regardless of positions.
- Originally proposed as a component of RNN structures; now used independently.
- Reference "Attention Is All You Need" https://arxiv.org/pdf/1706.03762.pdf
- <u>Temporal Fusion Transformers</u>
 - Use a combination of RNN and Attention structures to predict multi-horizon time series
 - Provides some measure of interpretability into temporal patterns
 - Reference: "Temporal Fusion Transformers for Interpretable Multi-horizon Time Series Forecasting" https://arxiv.org/pdf/1912.09363.pdf
- Other exciting work, both in the bank and in the broader research community

Pros and Cons of using Neural Network Algorithms

Pros

- Flexibility: ANNs can be use in a variety of machine learning tasks:
 - Regression
 - Classification
 - Dimension Reduction/ Feature Engineering
 - Text Processing
- Feature Engineering: ANNs show potential to extract meaningful features from raw data.
- Exploiting Large Data: ANNs have been observed to continue receiving performance gains from larger training data sizes when other algorithms reach a plateau.
- Flexibility in Inputs: ANNs are capable using a variety of data types, as well as combining differing data types.

Cons

- The "black box" problem: Often difficult/impossible to explain why the network makes the predictions it does.
- Computational Cost: Training ANNs can have a high computational cost, and may not be efficient when another tool (e.g. Gradient Boosting) will perform well.

Software

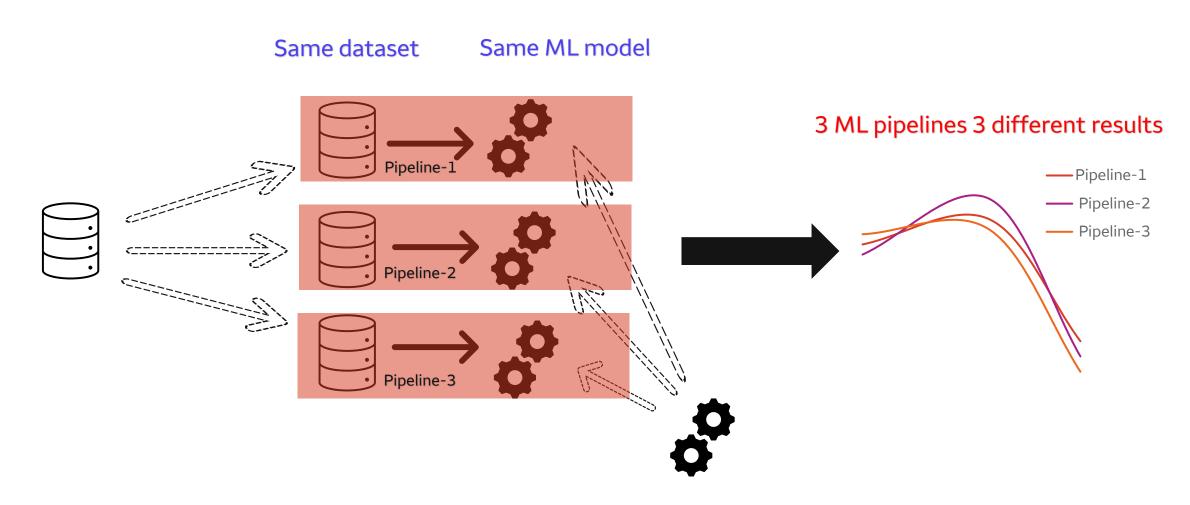
- Specialized software packages exploit computational graphs, symbolic differentiation, back propagation and mini-batch training.
- Popular Packages include:
 - TensorFlow / Keras (Google)
 - PyTorch/Torch
 - Caffe/Caffe2
 - CNTK (Cognitive Toolkit) (Microsoft)
 - Theano
- Wrappers exist to provide abstraction layers (Keras)
- Many general-purpose scientific packages have more limited implementations (R, MatLAB, scikit-learn)
- Many offer distributed or flexible computing
- NVidia providing growing support for GPU computation

Additional Considerations: Monotonicity and Reproducibility

Monotone constraints on ML algorithms

- Monotonicity is an important model requirement in some applications like credit scoring. Customer with better credit, longer history, less inquires should be more likely to be approved.
- A variable x_j is said to be monotone increasing, if $f(x_j = a, x_{-j}) \le f(x_j = b, x_{-j})$ for any $a \le b$ and x_{-j} . Similarly, monotone decreasing can be defined.
- Different monotone ML algorithms are available:
 - Soft monotone ML algorithms. These algorithms add a penalty on the loss function, eg,
 - KiGB (knowledge-intensive gradient boosting) algorithm: it penalizes the difference between average of all leaves in left vs right subtree. This method is flawed since mono on average is a weaker requirement.
 - \circ **Certified monotonic neural networks** (Liu et al. 2020) calculates the gradient of a nnet and adds a penalty if the gradient violates the monotonicity. If violations are found, the penalty parameter λ is increased and the model is retrained again until no violations can be found. But in practice, it is NP hard to find all violations.
 - Hard monotone ML algorithms: these algorithms are 100% monotone by design
 - o **XGBoost** imposes split-wise and branch-wise limit to achieve monotonicity. It works well for shallow trees
 - o **Isotonic classification tree RF** (Bonakdarpour et al. 2018): fit a conventional RF, then use isotonic regression to update leaf values so monotonicity is satisfied.
 - o **Tensorflow Lattice** (Gupta et al. 2016): use Lattice with monotonicity constraints.

A Note on Reproducibility in Machine learning



Many Machine learning models (Xgboost, Random forest, DNN, etc) have dependence on parameters that are randomized

Randomization exists at different levels in ML

| Dataset | Sources of randomness | | |
|--|---|--|--|
| | -Train/Test/Validation split -Cross Validation split | | |
| Split is randomly chosen and depends on a random seed. Different training splits will generate a different model and hence difference in results. | | | |

| Model | Sources of randomness controlled by model random seed | | |
|-------|---|--|--|
| RF | BootstrappingRow subsamplingColumn subsampling | | |
| GBM | Row subsampling Column subsampling Selection of validation set while early stopping | | |
| XGB | Row subsampling Column subsampling Selection of validation set while early stopping | | |

Summary

- We talked about the different machine learning models, focusing on supervised machine learning methods like random forest, GBM and neural network.
- Random forests and GBM are tree based ensemble methods, designed to improve the performance of a single tree using a collective set of trees. Neural network is a biologically inspired method designed to mimic the function of brain.
- In structured data XGB and FFNN are shown to have competitive performance and both outperform RF.
- Machine learning algorithm can achieve better performance than traditional statistical regression, without manual variable selection or transformation.
- Hyper-Parameter tuning is an essential component of training Machine Learning models
- Reproducibility is important to replicate results as claimed and hence, it is important to understand the sources of randomness in machine learning modeling.

References

- Breiman, L. (1996). Bagging predictors. Machine Learning, 24 (2): 123–140.
- Quinlan, J. R. (1986). Induction of decision trees. Machine Learning, 1(1):81-106.
- Breiman, L.; Friedman, J. H.; Olshen, R. A.; Stone, C. J. (1984). Classification and regression trees. Monterey, CA: Wadsworth & Brooks/Cole Advanced Books & Software.
- Friedman, J. H (2001). Greedy Function Approximation: A Gradient Boosting Machine. The Annals of Statistics, 29 (5): 1189-1232.
- Friedman, J. H. (1999). Stochastic gradient boosting. Stanford University.
- Hastie, T.; Tibshirani, R.; Friedman, J. H. (2001). The elements of statistical learning: Data mining, inference, and prediction. New York: Springer Verlag.
- Breiman, Leo (2001). Random Forests. Machine Learning, 45 (1)
- Deep Learning, by I. Goodfellow, Y. Bengio, and A. Courville, available: http://www.deeplearningbook.org/
- Explained: Neural networks, by L. Hardesty, available: http://news.mit.edu/2017/explained-neural-networks-deep-learning-0414
- M Nielsen (2017), Neural Network and Deep Learning online book: http://neuralnetworksanddeeplearning.com/index.html
- Goldstein, A., Kapelner, A., Bleich, J., & Pitkin, E. (2013). Peeking Inside the Black Box: Visualizing Statistical Learning with Plots of Individual Conditional Expectation. *eprint arXiv:1309.6392*.
- Ribeiro, M. T., Singh, S., & Guestrin, C. (2016). Why should I trust you?: Explaining the predictions of any classifier. *Proceedings of the 22nd ACM SIGKDD International Conference on Knowledge Discovery and Data Mining*, (pp. 1135-1144)
- Hu, L et. al (2022). Surrogate locally-interpretable models with supervised machine learning algorithms, Journal of Indian Statistical Association.
- Chen, J. et al (2020). Adaptive Explainable Neural Networks, arXiv:2004.02352
- Vaughan, J. et al (2018). Explainable Neural Networks based on Additive Index Models. The RMA Journal