Low-overhead error detection with spacetime codes

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Trading off samples vs. qubits

Thousands of gates

Millions of gates

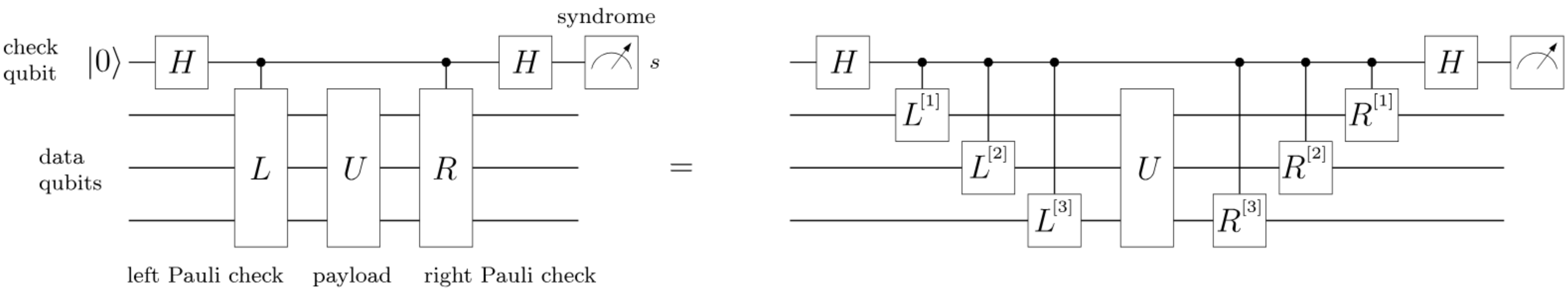
PEC, ZNE, TEM, Circuit Cutting (expectation values only)

Error detection uses extra qubits to reduce sampling cost. It is also **single-shot**.

Fault-tolerance (single-shot)

But it is critical that the detection circuit is efficient, otherwise it introduces more errors than it removes.

Refresher on coherent Pauli checks[1-3]



Entangle extra "check" qubits with a Clifford payload.

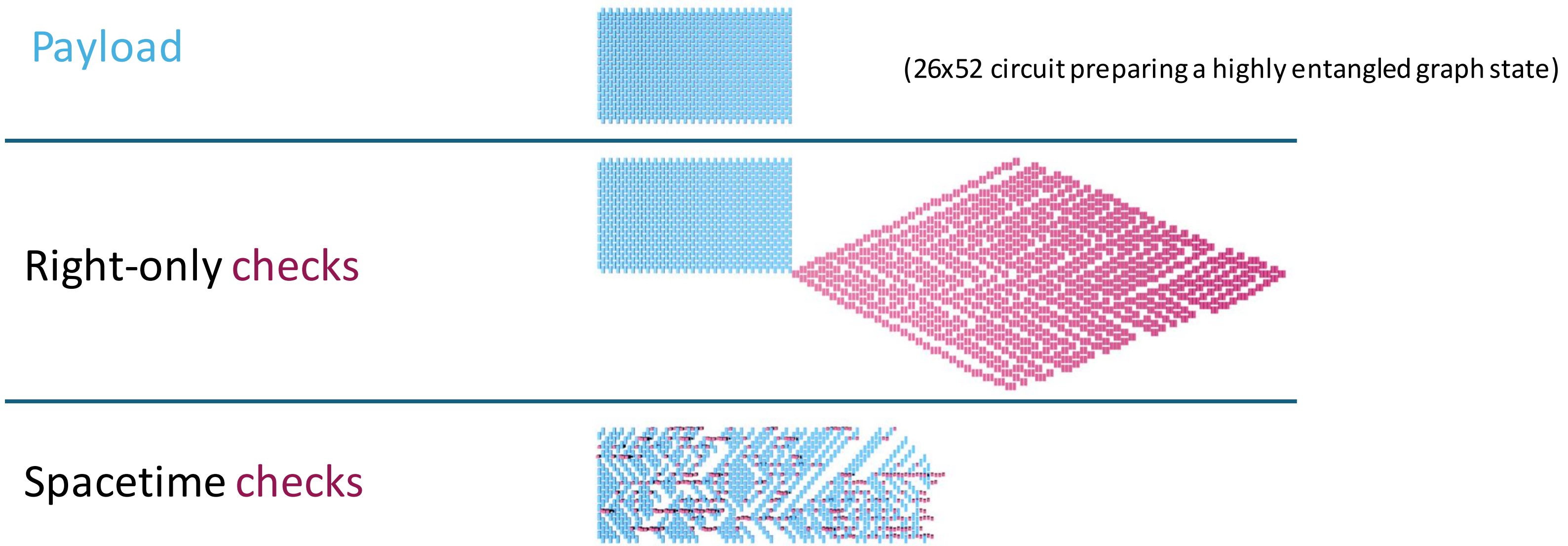
Measuring the check qubit reveals whether the Clifford payload correctly maps the **L**eft Pauli to the **R**ight Pauli.

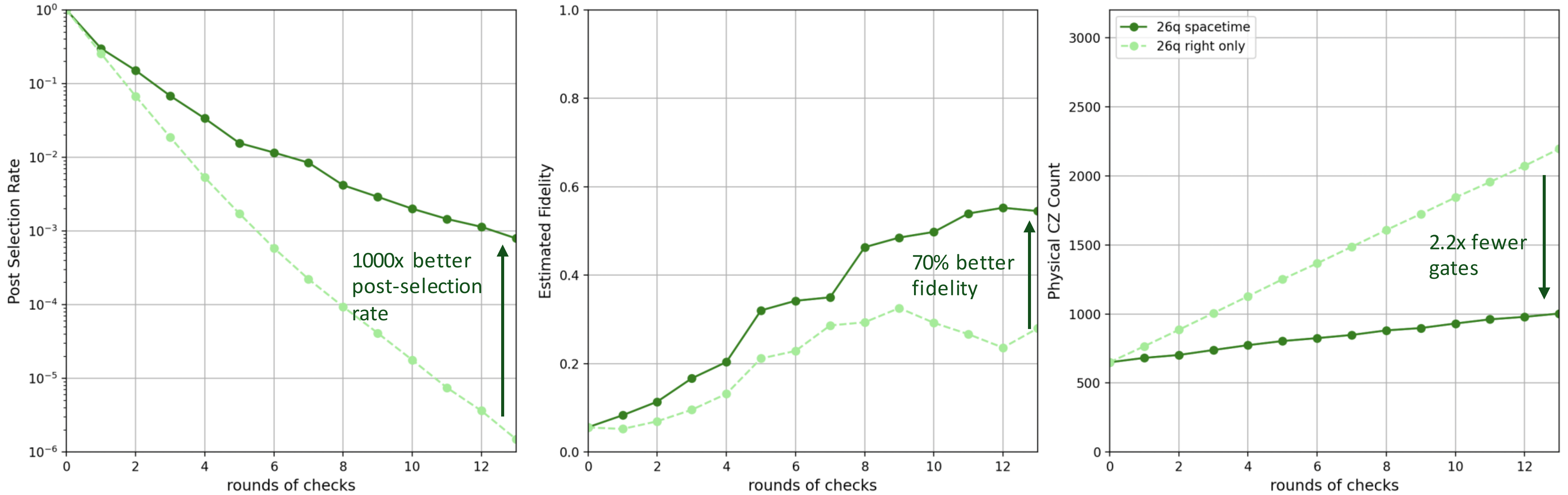
More checks detect more errors. But check overhead can tank post-selection rate and introduce many new errors.

We use spacetime codes[4-6] to probe for errors more efficiently.

1. Van den Berg et al. Single-shot error mitigation by coherent Pauli checks, PRR 2023
2. Debroy, Brown, Extended flag gadgets for low-overhead circuit verification, PRA 2020
3. Gonzales et al. Quantum error mitigation by Pauli check sandwiching, Scientific Reports 2023
4. Bacon, Flammia, Harrow, Shi, Sparse quantum codes from quantum circuits, IEEE Info. Theory 2014
5. Gottesman, Opportunities and challenges in fault-tolerant quantum computation, arxiv:2210.15844
6. Delfosse, Paetznick, Spacetime codes of Clifford circuits, arxiv:2304.05943

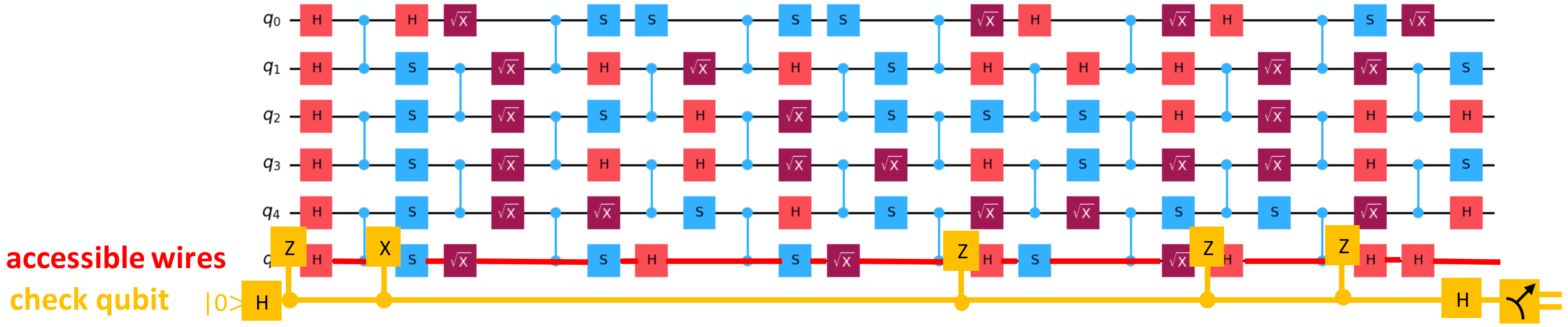
Spacetime checks vs. one-sided coherent Pauli checks

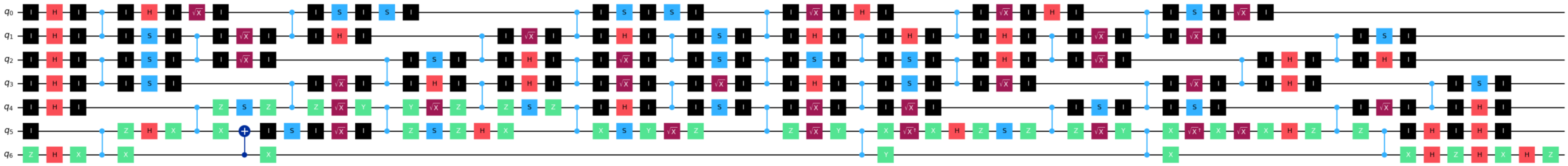




How spacetime checks work

1) Start with a payload.
2) Consider an adjacent **check qubit** with some **accessible wires**.
3) Find **valid low-weight** Pauli supported on those wires.
4) Implement the Pauli check.
5) Compute **coverage** of check by computing its “cumulant”.
6) Score and repeat.

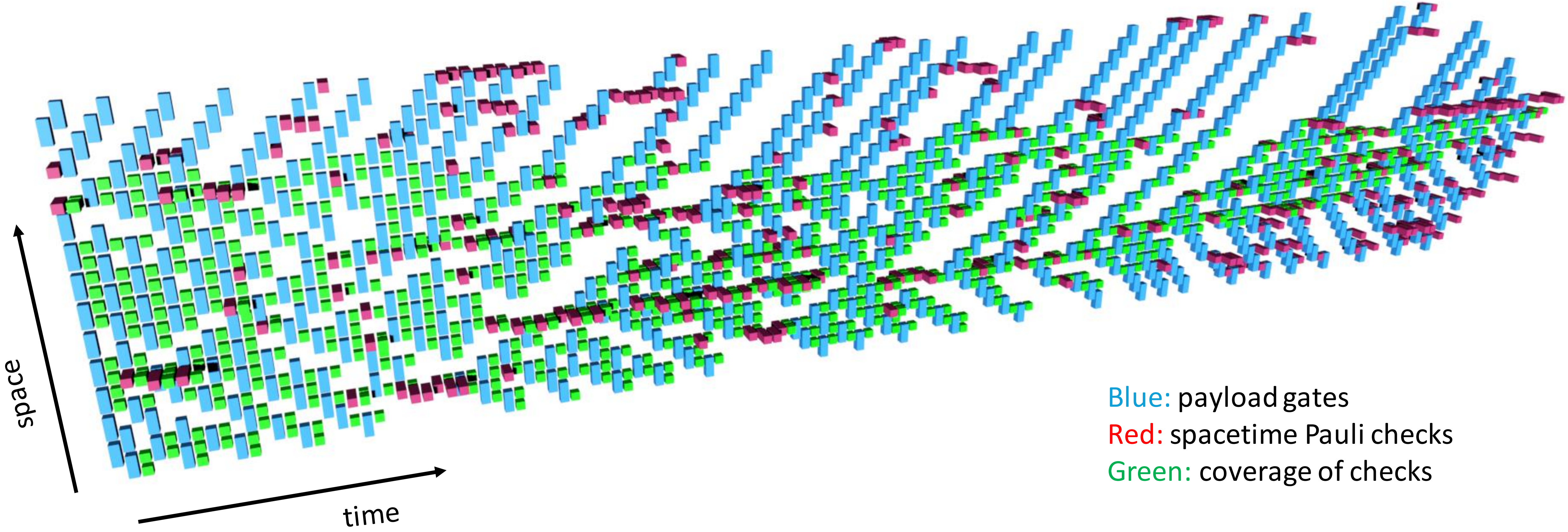




2/3 of errors originating in green boxes will be caught - errors in black boxes will go through

Visualizing spacetime checks and cumulants

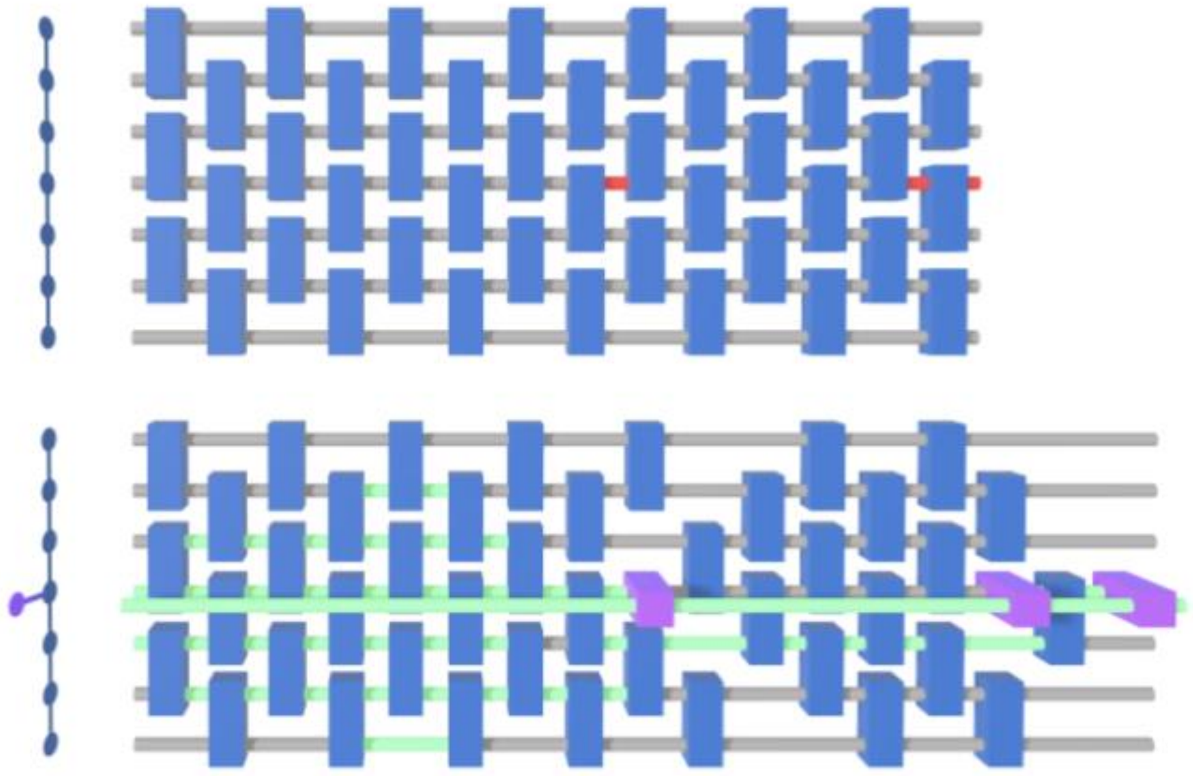
Given a Clifford circuit, we can "sprinkle" it with Pauli operators that stabilize the circuit in both space and time. These checks can be made hardware efficient by solving a decoding problem to find low-weight checks.



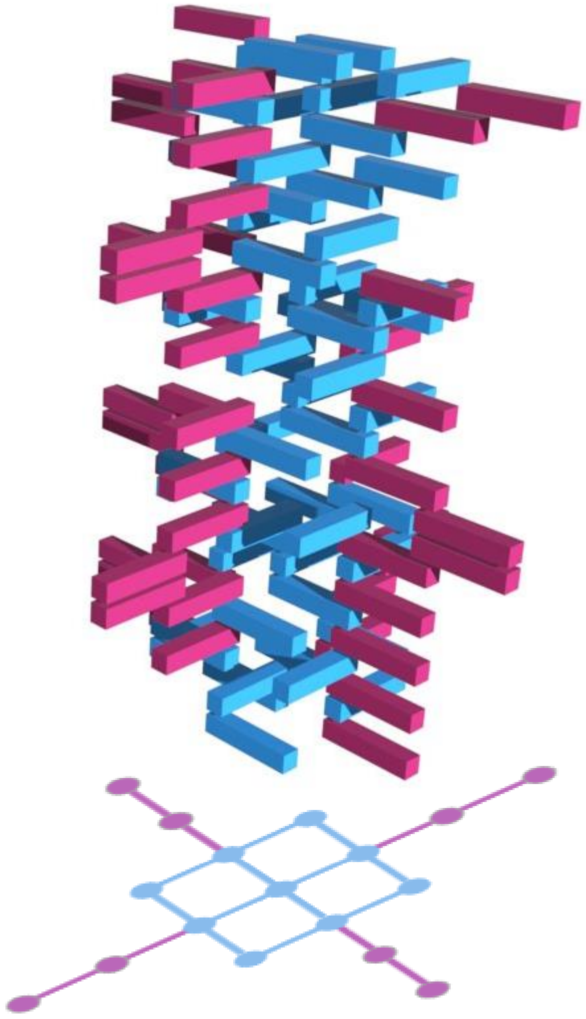
Blue: payload gates
Red: spacetime Pauli checks
Green: coverage of checks

In a nutshell

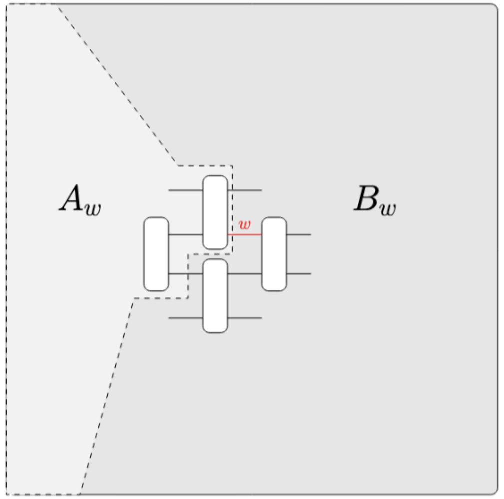
Keep checks spatially local and cheap, by distributing in time.



Works for any connectivity.



More formally



$$B(P, w) = A_w^{\dagger} P_w A_w$$

Define *B(P, w) = Pauli operator P on wire w and pulled to the beginning.*

A check $\{(P_1, w_1), \ldots, (P_k, w_k)\}$ is **valid** iff $\prod_i B(P_i, w_i) = I$

If the input state is stabilized by some input group S: $\prod_i B(P_i, w_i) \in S$

We search for checks using a Boolean encoding of Pauli operators:

$$\prod_{w \in L} B(P_w, w) = I \Leftrightarrow xB = 0$$

We can restrict our search on some fixed set of wires L
and use a *decoding algorithm* ***to find a low weight x***
(low weight => low gate overhead)

$$B = \begin{bmatrix} 00000100000000 \\ 00000100000010 \\ 00000000000010 \\ 00011100000100 \\ 00011000000110 \\ 00000100000010 \\ 01100100010100 \\ 01100000010110 \\ 00000100000010 \\ 00011000000110 \\ 01111000010000 \\ 01100000010110 \\ 00011000001110 \\ 01111000011000 \\ 01100000010110 \\ 00011100001100 \\ 01100100010100 \\ 01111000011000 \\ 00011101100100 \\ 00000001101000 \\ 00011100001100 \\ 00011001101100 \\ 00011000000100 \\ 00000001101000 \end{bmatrix}$$

Use Monte Carlo sampling to approximate the logical error after a check.

$$LER(C_1, \ldots, C_l) = \sum_{k=1}^{\infty} \sum_{\substack{E=(P_1,w_1)\ldots(P_k,w_k) \\ \text{not detected} \\ \text{logical error}}} \mathbb{P}(E)$$

$$= \sum_{k=1}^{\infty} \sum_{\substack{E=(P_1,w_1)\ldots(P_k,w_k) \\ \text{not detected} \\ \text{logical error}}} \prod_{i=1}^{k} \mathbb{P}((P_i, w_i))$$

What is this useful for?

Error detection methods based on Pauli checks are suitable for Clifford or Clifford-dominated circuits.

A few use cases:

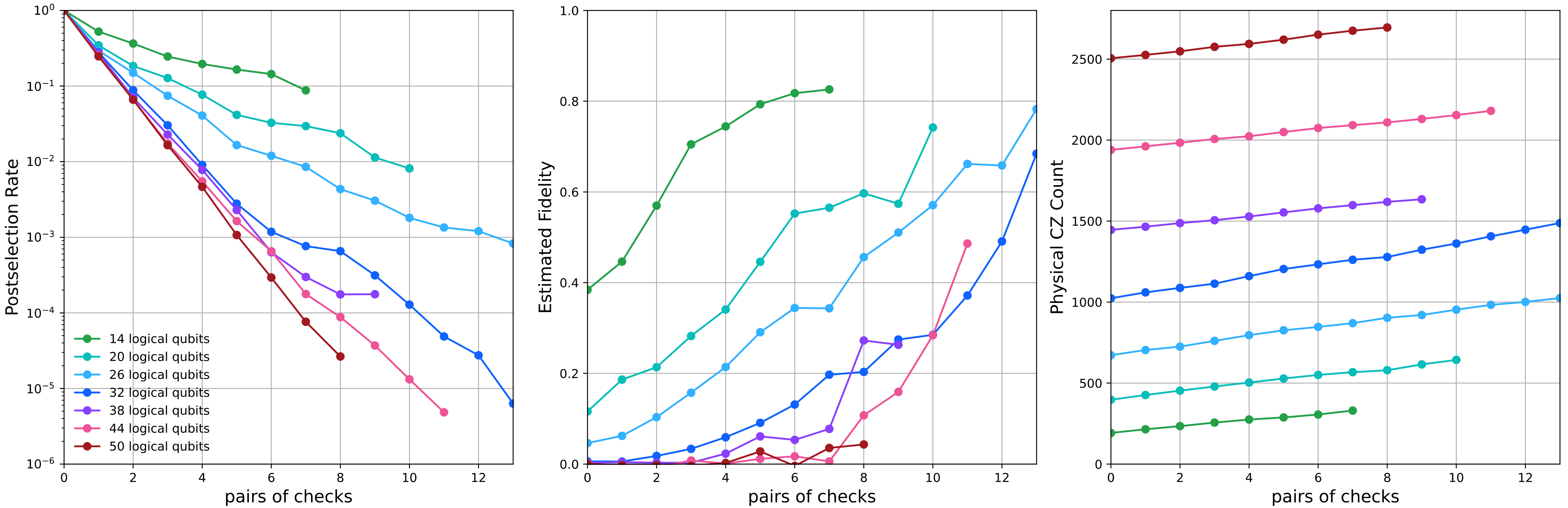
- Any circuit can be written as Clifford applied to magic state inputs[1]

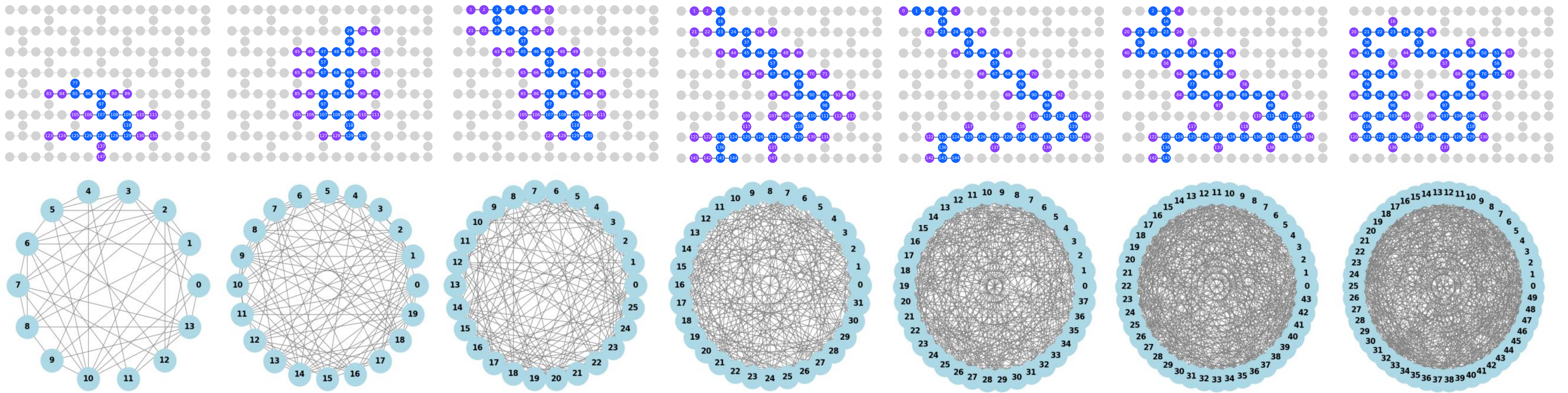
- Cliffords are key to many quantum advantage proposals:
  - Conjugated Clifford Circuits sampling[2]
  - Extended Clifford circuit sampling[3]
  - Graph states
    - Classically hard to sample in non-Clifford basis[4]
    - Critical resource for measurement-based quantum computing[5]

We focus on preparing & verifying highly-entangled graph states.

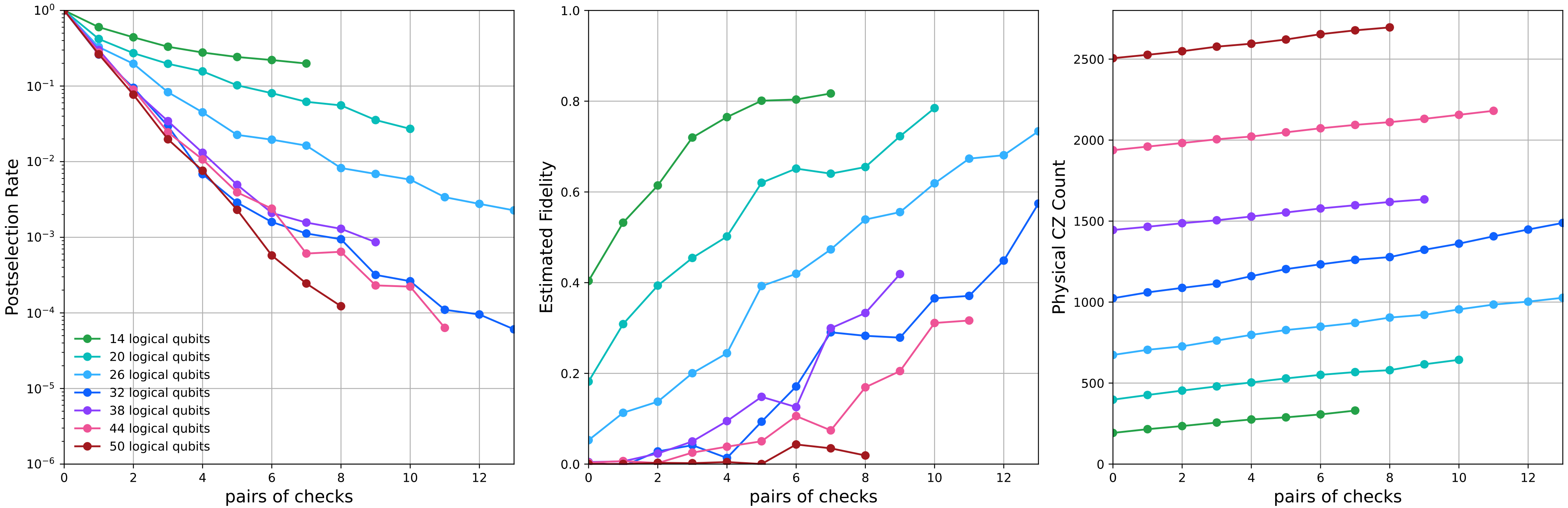
1. Gottesman and Chuang, Quantum teleportation is a universal computational primitive, Nature 1999
2. Bouland, Fitzsimons, Koh, Complexity classification of conjugated Clifford circuits, arXiv:1709.01805
3. Jozsa, Van den Nest, Classical simulation complexity of extended Clifford circuits, arXiv:1305.6190
4. Ghosh et al. Complexity phase transitions generated by entanglement, PRL 2023
5. Van den Nest et al. Universal Resources for Measurement-Based Quantum Computation, PRL 2006

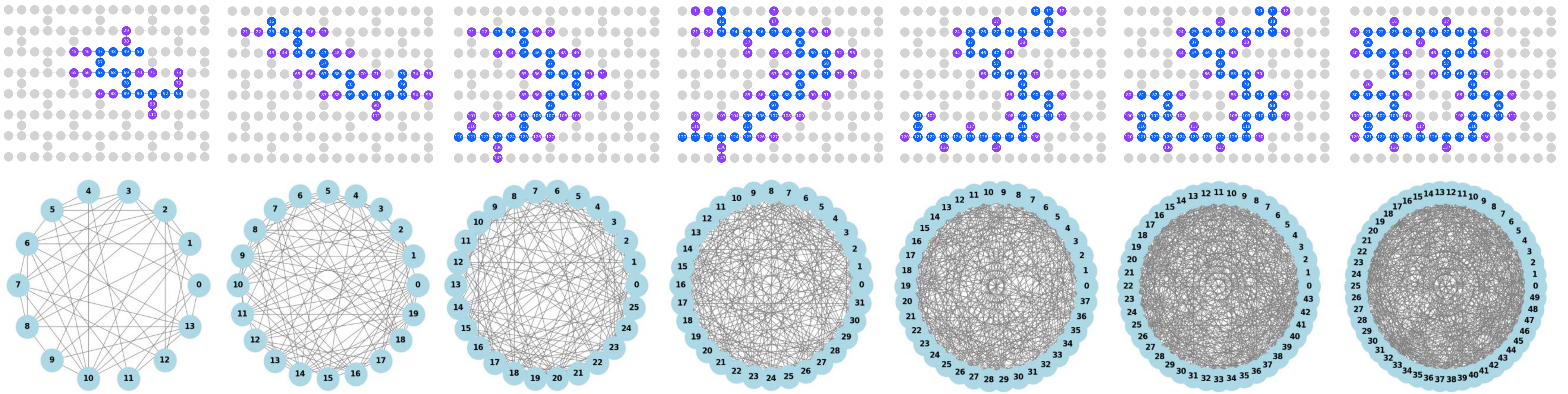
Experiments:
Fez





Experiments: Kingston

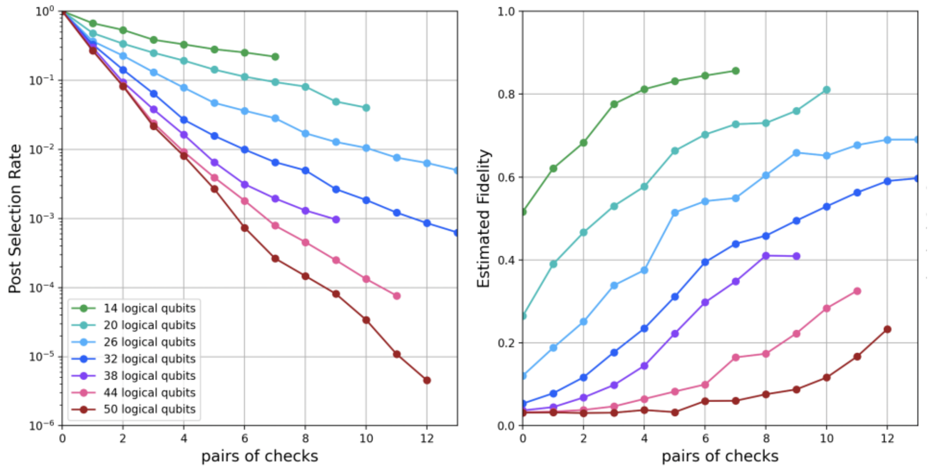




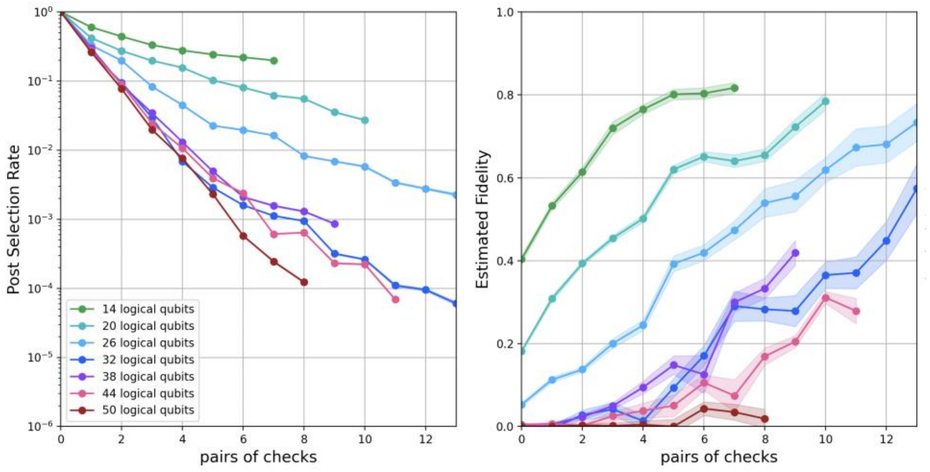
Basically agrees with simulations under a simplified noise model

> Uniform Pauli noise **p** after each two-qubit gate + Pauli noise for long delays **1 - e(-t/T)**

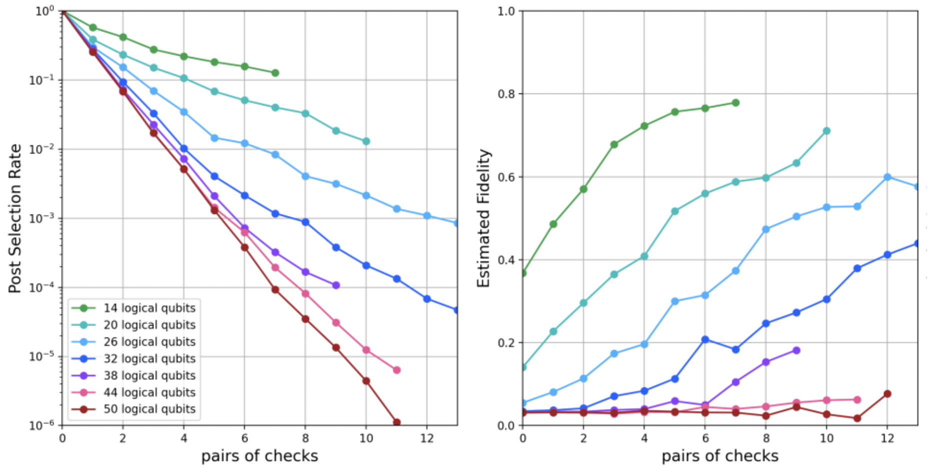
p=0.003, T=100us



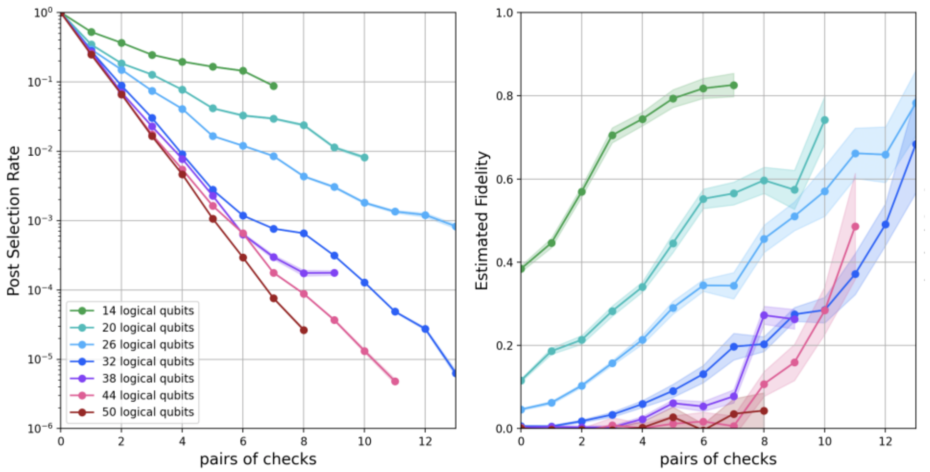
Kingston



p=0.005, T=100us



Fez



Takeaway: relatively small sample overhead and relatively mild circuit overhead can boost the signal substantially.

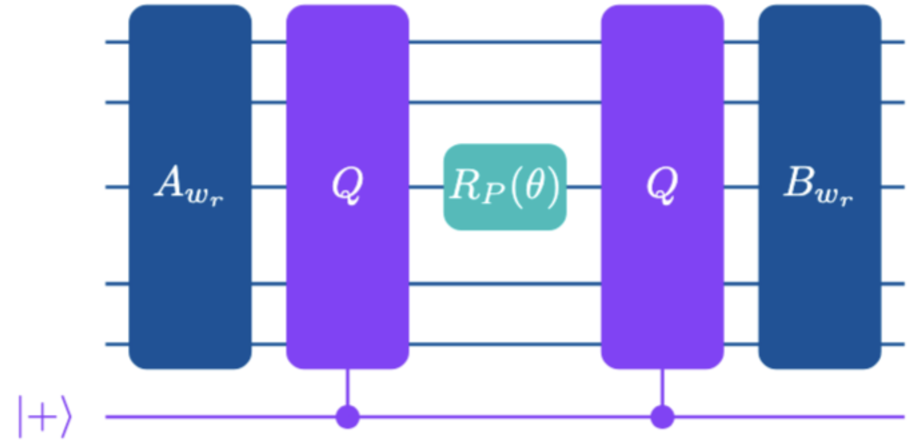
Bare circuit				Sampling overhead		This work			
Qubits	Gates	Rank width	LF	PS	PEC	Qubit overhead	Gate overhead	Sampling overhead	Fidelity gain
14	182	[4,5]	0.960	2.9	6.9×10^1	14	148	5.0	2×
20	380	[5,6]	0.937	1.4×10^1	3.3×10^4	20	263	3.7×10^1	4×
26	650	[7,9]	0.908	1.6×10^2	5.8×10^8	26	376	4.4×10^2	14×
32	992	[9,11]	0.869	8.2×10^3	4.4×10^{15}	26	496	1.7×10^4	131×
38	1406	[10,13]	0.884	1.2×10^4	1.9×10^{16}	18	228	1.2×10^3	92×
44	1892	[9,15]	0.875	1.3×10^5	2.8×10^{18}	22	288	1.5×10^4	104×
50	2450	[8,17]	0.870	1.2×10^6	1.9×10^{24}	16	245	8.2×10^3	69×

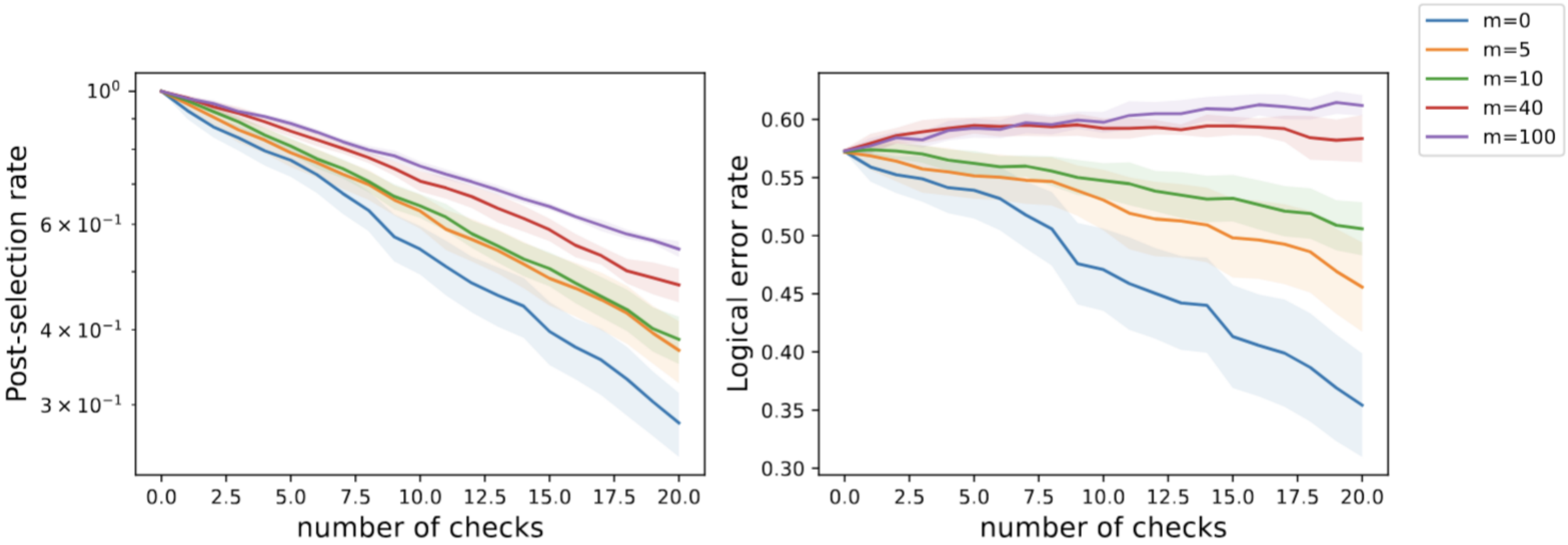
Table 1: Summary of experiments performed on *ibm_kingston*. We characterized layer fidelity (LF) for one layer of each circuit, which (due to the brickwork nature of the circuit) lets us infer the sampling overhead needed to remove all errors using either idealized postselection (PS) or probabilistic error cancellation (PEC). "Gates" here means two-qubit gates, which is controlled-Z in our case. Sampling overhead for error detection is the inverse of postselection rate.

Beyond Clifford circuits

Any non-Clifford rotation Rp partitions the circuit. A check is still valid if, pulled to that partition point, commutes with Rp.

We can still efficiently find checks by enforcing the new constraints, but the space of valid checks shrinks exponentially with increasing non-Clifford rotations.





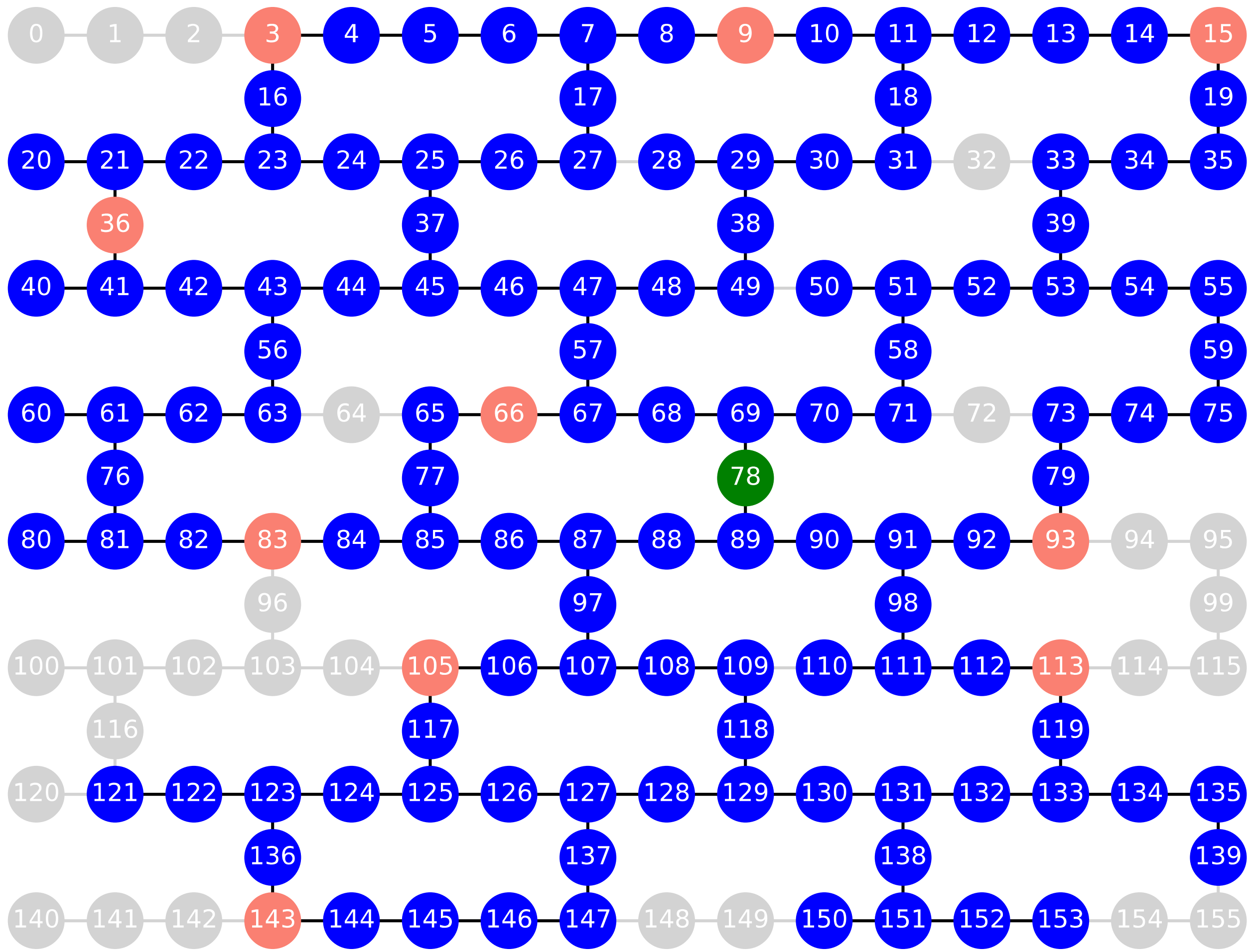
Application: Preparation of GHZ states

GHZ states are a common benchmark for quality of hardware and software.

It can be prepared in short depth by breadth-first search from a root of a tree.

The Pauli checks are particularly simple: the state is permutation-invariant and it has low-weight ZZ stabilizer. Measuring ZZ of any pair of qubits yields a valid check.[1, 2]

But not all checks are the same in terms of coverage. We use the tools we developed to optimize the coverage of these checks.



Green: GHZ root, **Blue**: GHZ qubits, **Pink**: checks

1. Mooney et al. Journal of physics communications 2021
2. Liao et al, Achieving computational gains with quantum error correction primitives.., arXiv 2411.14638

Recipe:

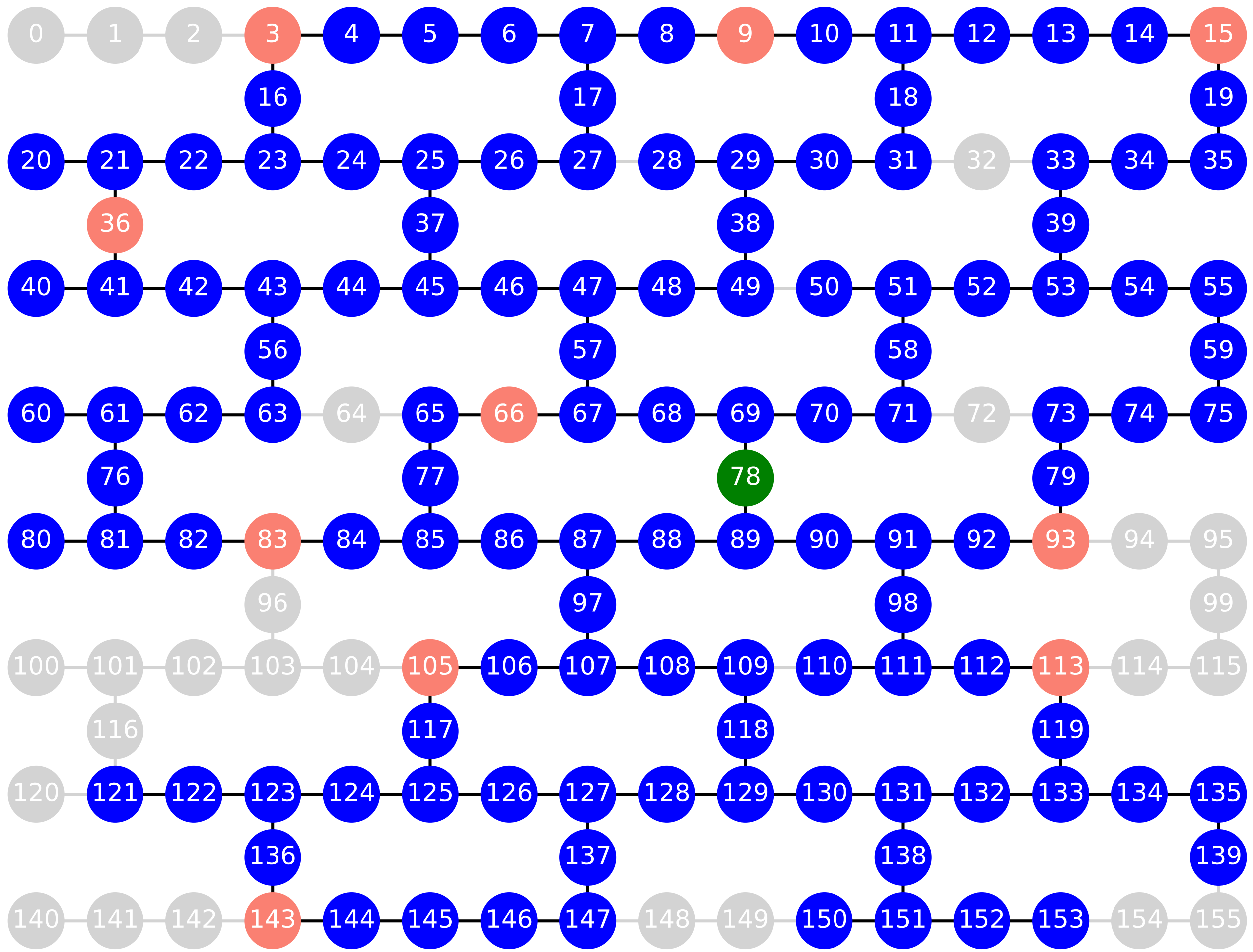
1- Start by removing bad nodes and edges from the graph (errors above a given threshold).

2- For each remaining qubit, build a BFS tree starting at that qubit as root. Choose the root that yields the shortest depth GHZ.

3- Starting at the chosen root, randomly block a small number of remaining nodes and build a BFS tree again.

4- Now find some valid checks and compute their coverage.

5- Repeat this randomization until finding the best coverage.

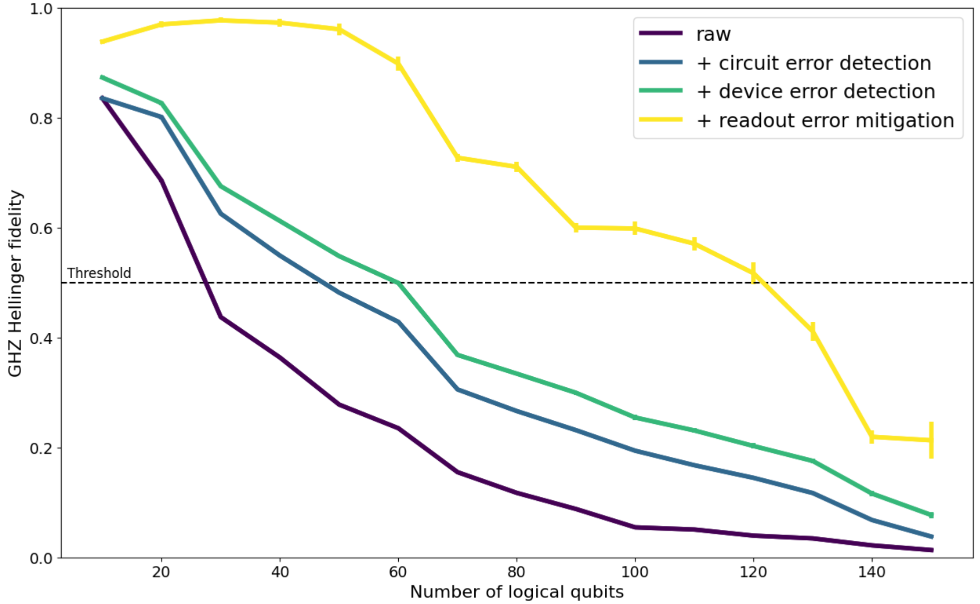


How to verify the fidelity of GHZ state

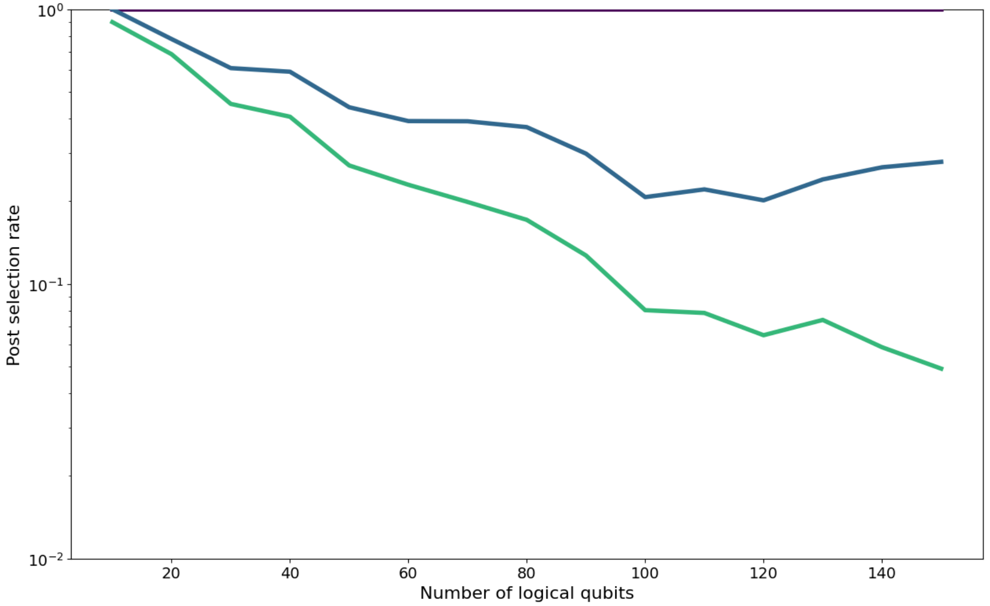
We can do direct fidelity estimation as before, however the literature on GHZ typically looks at oscillations by varying rotation angles. Can we relate these 2?

Ongoing study (with Alireza Seif, Ken Wei, Simon Martiel) --- but the answer seems to be yes.

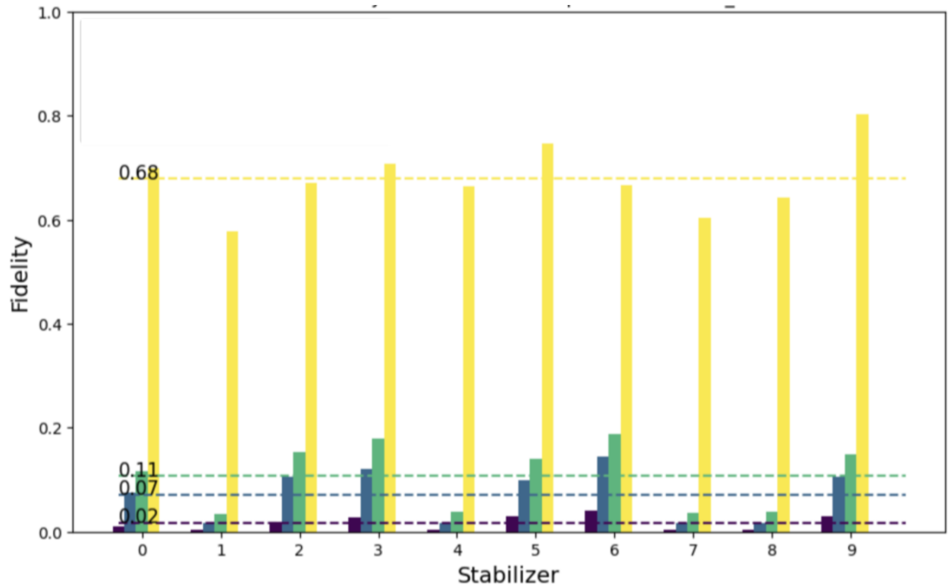
GHZ states beyond 100 qubits (postselection using circuit info and device info)



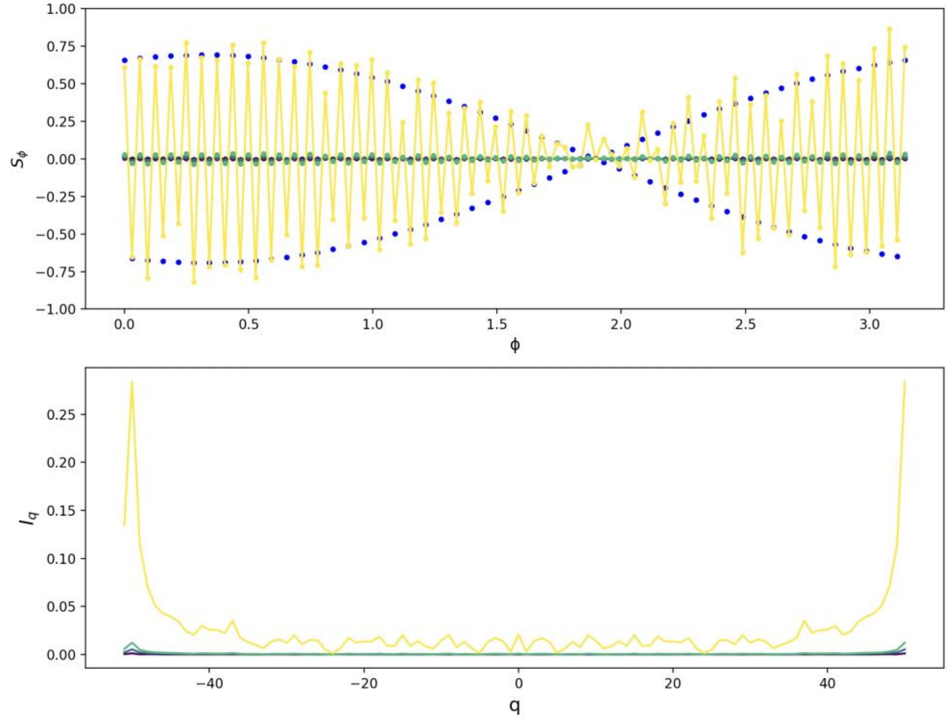
Hellinger fidelity



Post selection rate



Direct fidelity estimation



Parity oscillations

Conclusions

Error detection in Clifford circuits inspired by spacetime codes.

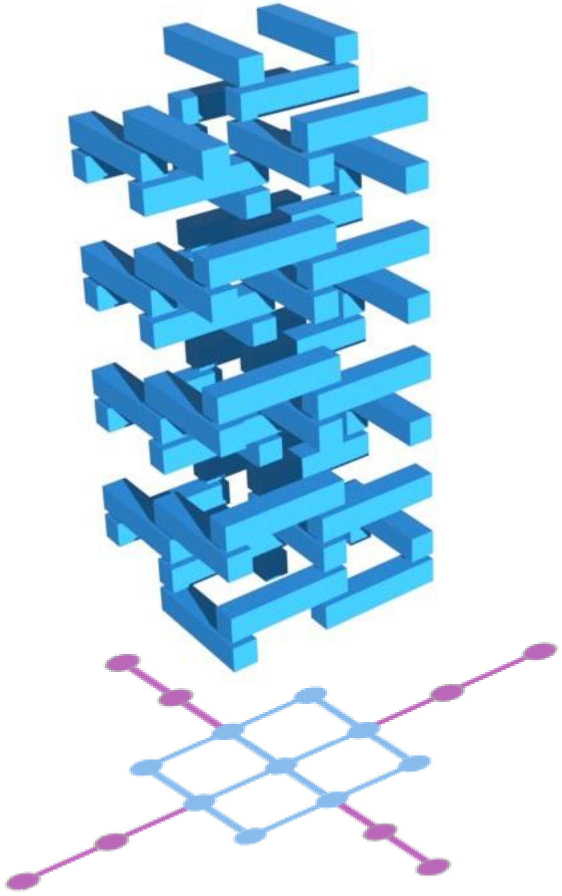
- We develop tools that allow us to find valid checks and score them.
- Works for any qubit connectivity graph.
- Simulations match experiments.
- Unlocks large experiments: highly entangled graph states with 2.5k gates; or 100+ qubit GHZ states.

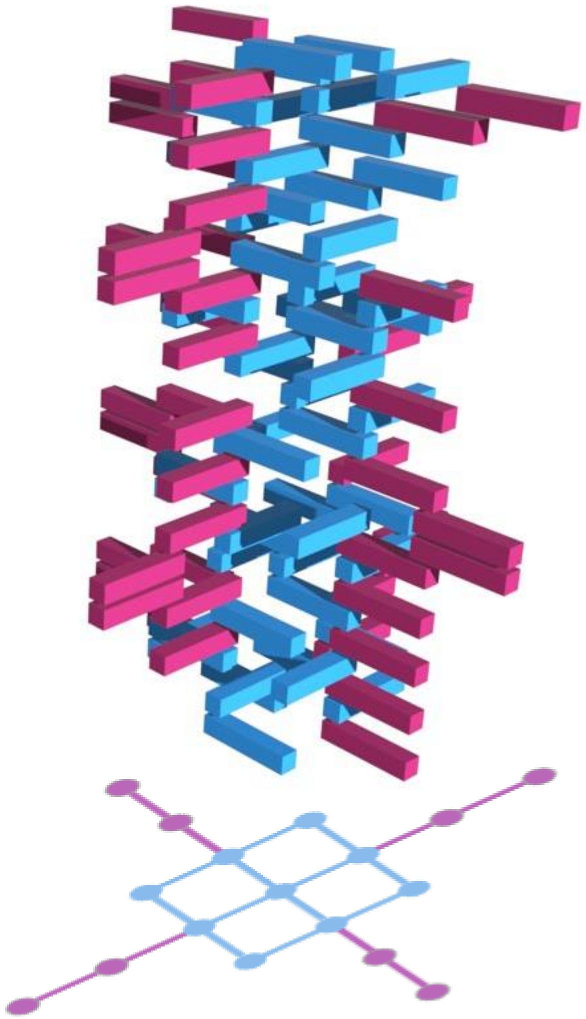
Open questions/future work

- Can we pick checks such that we correct errors?
- Is there something to exploit in more structured circuits to design better checks?

Further details

Checks are spatially local but distributed in time





Bare circuit vs. Left-Right checks vs. Spacetime checks