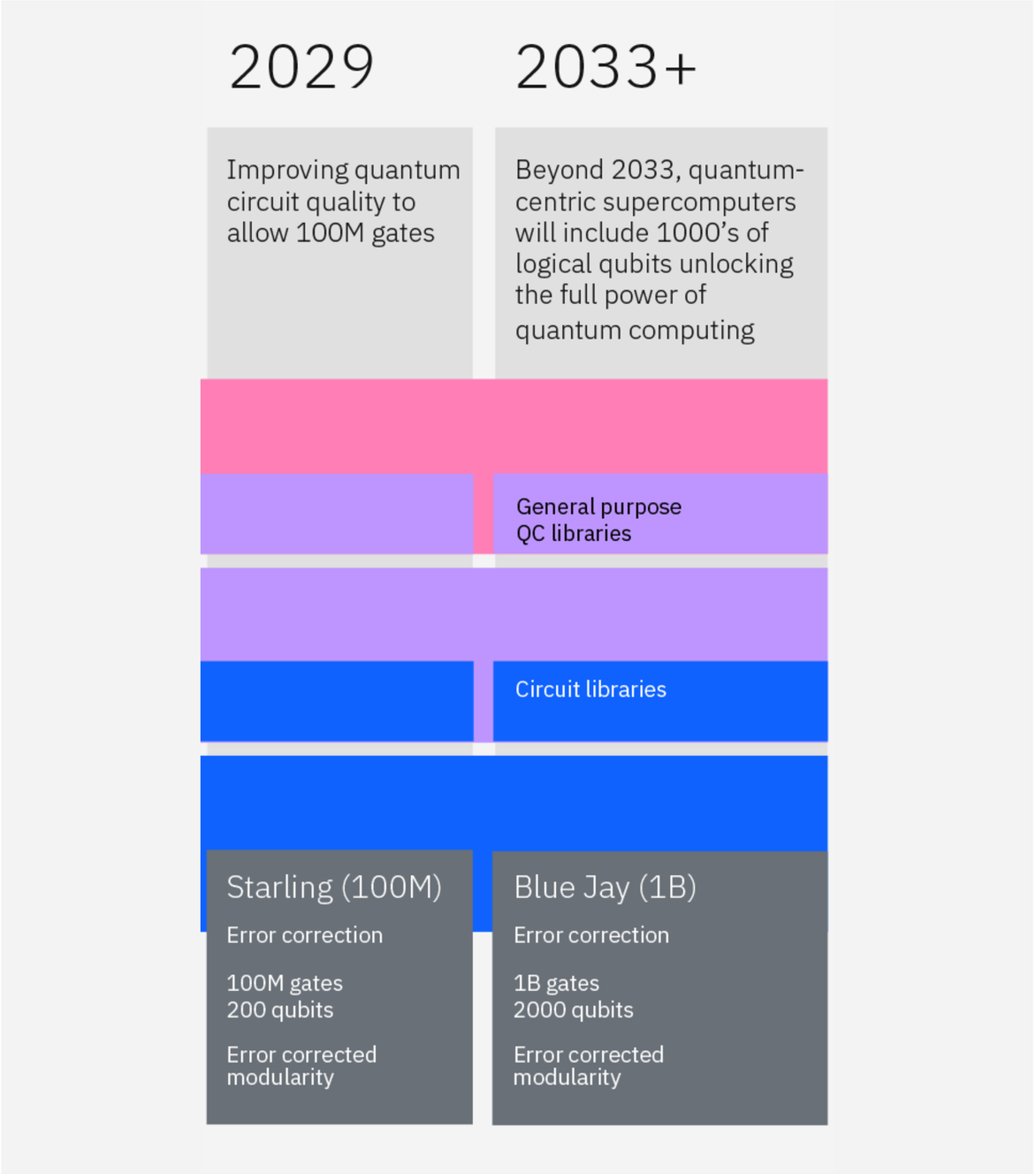
Towards Fault Tolerant Quantum Computing with IBM qLDPC Codes

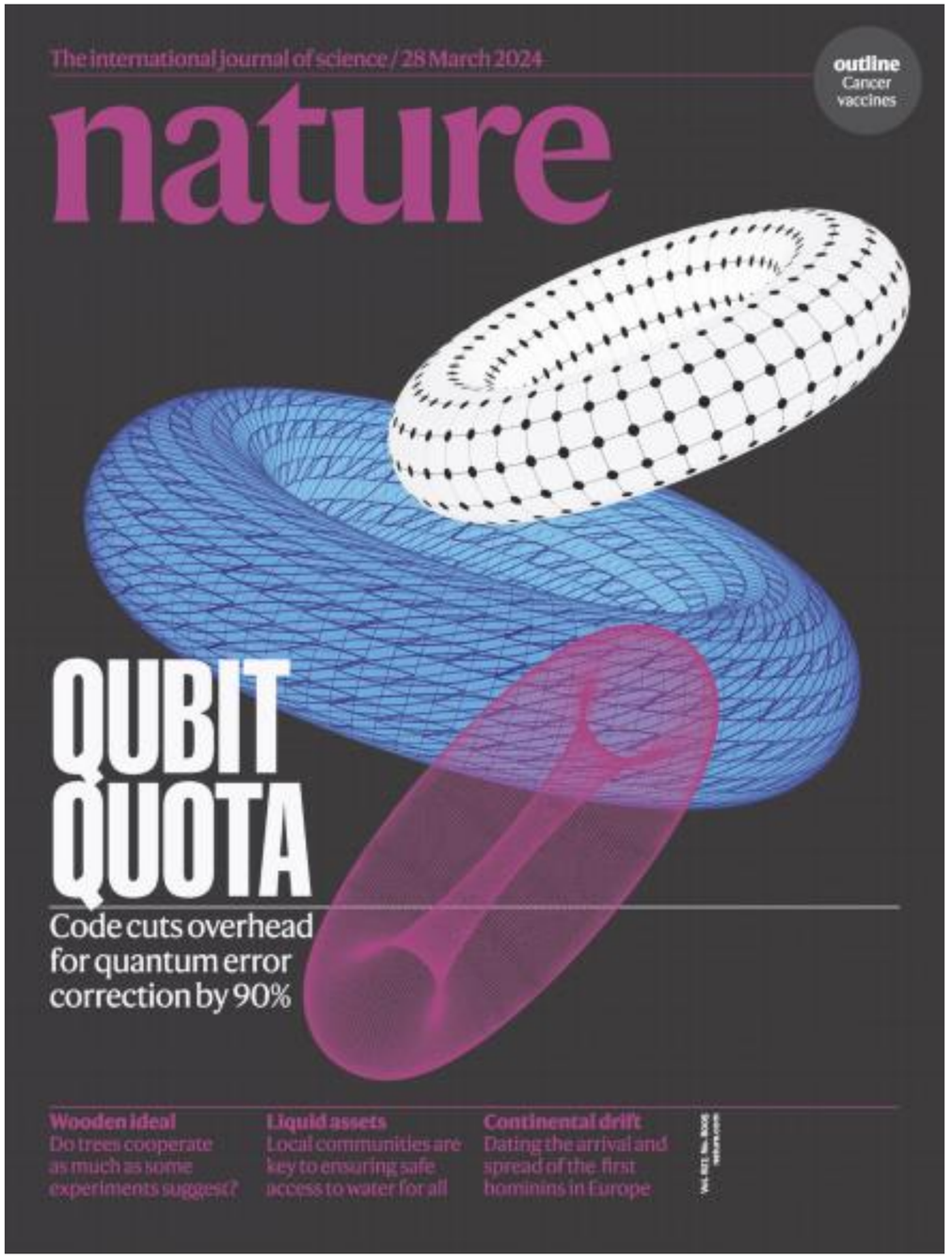
IBM **Quantum**

IBM

IBM’s Roadmap: FTQC by 2029



The Gross Code and the Bivariate Bicycle Family



Criteria for a **fault tolerant computer architecture**

Fault tolerant

Logical errors are suppressed enough for meaningful algorithms to succeed.

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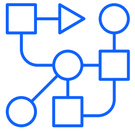
Universal

A universal set of quantum instructions can be applied to the logical qubits.

Modular

The hardware can be distributed across a set of connected, replaceable, modules.



Bivariate bicycle codes

The code has CSS check matrices

X-checks: $H_X = [\ A\ |\ B\ ]$ Z-checks: $H_Z = [\ B^T\ |\ A^T\ ]$

where A, B commute with each other.

Construct A, B from cyclic shifts $\mathrm{S_l}$ for integers a_i, b_i that can be varied.

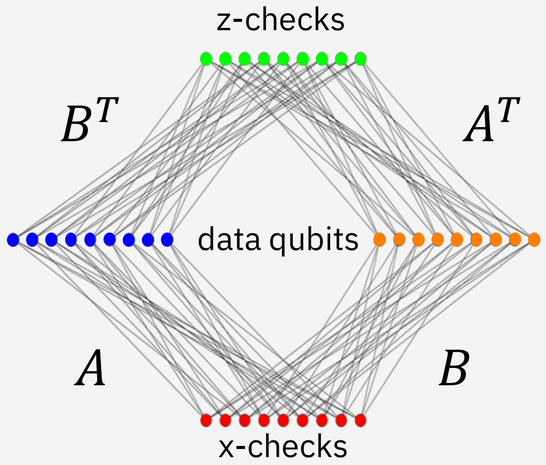
$x = S_l \otimes I_m$ $y = I_l \otimes S_m$

$A = x^{a_1} + y^{a_2} + y^{a_3}$ $B = y^{b_1} + x^{b_2} + x^{b_3}$

→ Each check acts on 6 qubits.
→ Each qubit is in 6 checks.

Two-block codes: A. Kovalev, L. Pryadko, 1212.6703, (2012); H. Lin, L. Pryadko, 2306.16400, (2023)
Lifted product codes: P. Panteleev and G. Kalachev, arXiv:1904.02703, 2012.04068, 2111.03654
BB codes: S. Bravyi et al, Nature 627 (2024); arXiv:2308.07915

Tanner graph



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X-checks: $H_X = [\ A\ |\ B\ ]$ Z-checks: $H_Z = [\ B^T\ |\ A^T\ ]$

where A, B commute with each other.

Constructing A, B from cyclic shifts S_l gives us, for example, the [[144,12,12]] gross code.

$x = S_{12} \otimes I_6$ $y = I_6 \otimes S_{12}$

$A = x^3 + y^2 + y^1$ $B = y^3 + x^2 + x^1$

→ Each check acts on 6 qubits.
→ Each qubit is in 6 checks.
→ The connectivity graph has thickness 2 and a toric layout.

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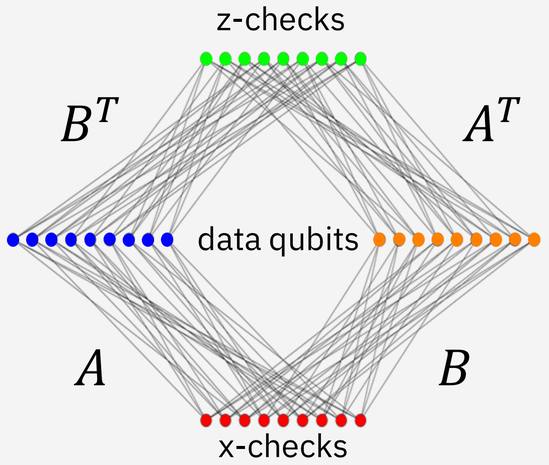
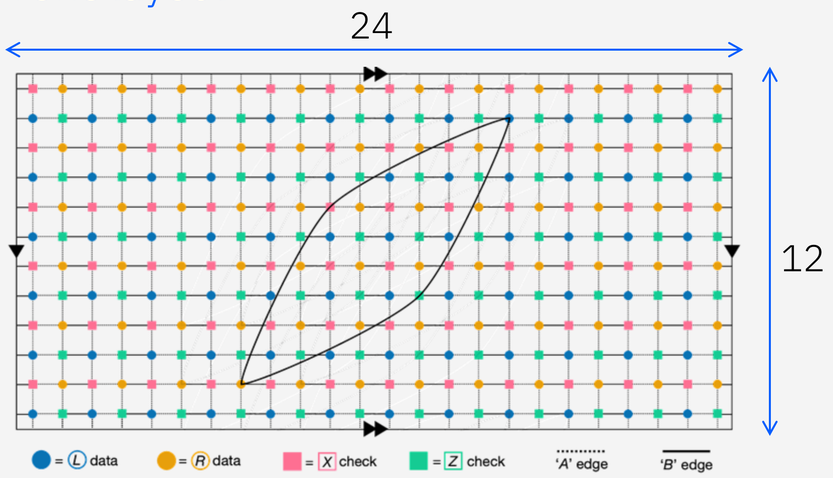


Toric layout



Examples of bivariate bicycle codes

Table of bivariate bicycle codes (not exhaustive). These have check weight 6 and toric layouts.*

$[[n, k, d]]$	Net Encoding Rate r	ℓ, m	A	B	
$[[72, 12, 6]]$	$1/12$	$6, 6$	$x^3 + y + y^2$	$y^3 + x + x^2$	
$[[90, 8, 10]]$	$1/23$	$15, 3$	$x^9 + y + y^2$	$1 + x^2 + x^7$	
$[[108, 8, 10]]$	$1/27$	$9, 6$	$x^3 + y + y^2$	$y^3 + x + x^2$	
$[[144, 12, 12]]$	$1/24$	$12, 6$	$x^3 + y + y^2$	$y^3 + x + x^2$	**gross code**
$[[288, 12, 18]]$	$1/48$	$12, 12$	$x^3 + y^2 + y^7$	$y^3 + x + x^2$	**two-gross code**
$[[360, 12, \leq 24]]$	$1/60$	$30, 6$	$x^9 + y + y^2$	$y^3 + x^{25} + x^{26}$	
$[[756, 16, \leq 34]]$	$1/95$	$21, 18$	$x^3 + y^{10} + y^{17}$	$y^5 + x^3 + x^{19}$	

S. Bravyi et al, Nature 627 (2024); arXiv:2308.07915

gross: n, a dozen dozen

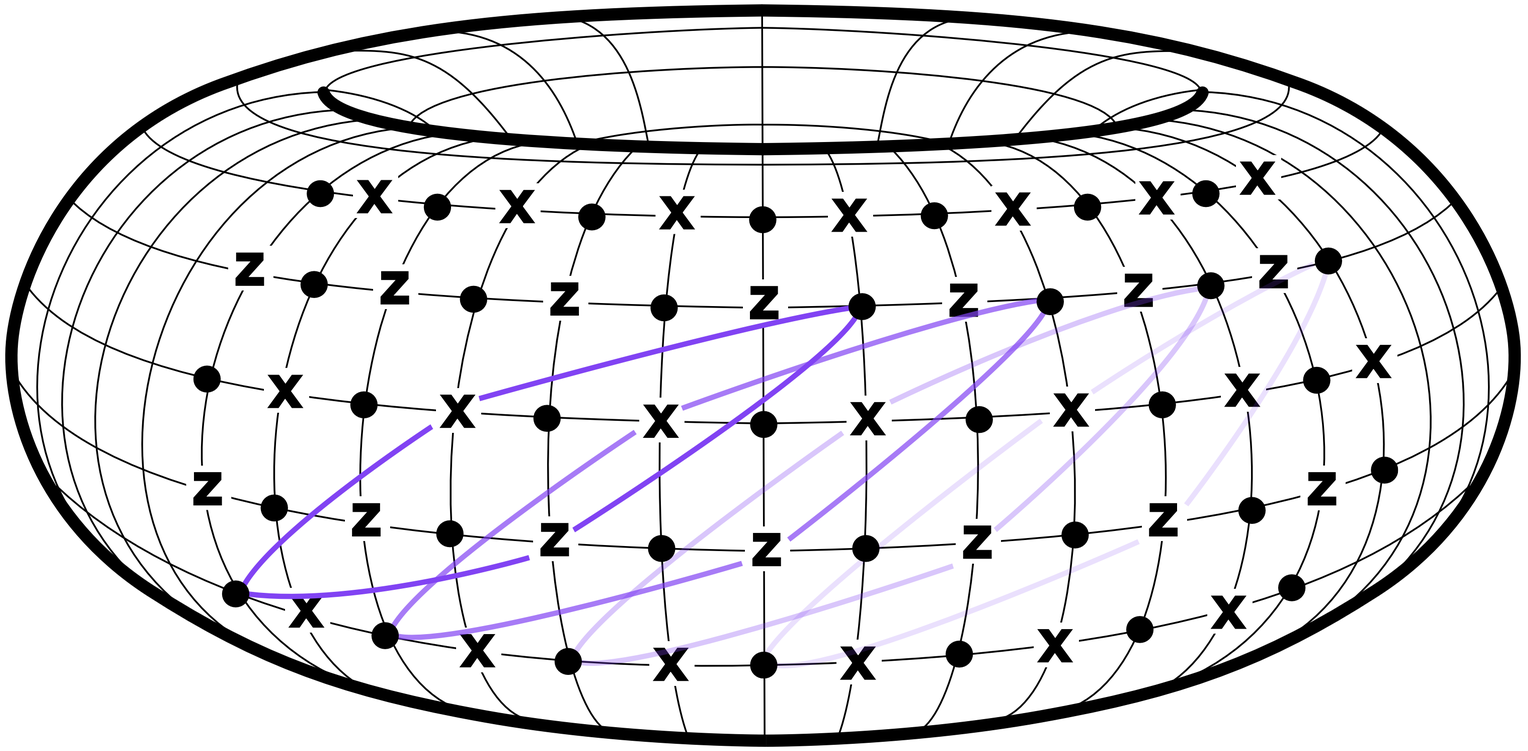
Number of code qubits

$$n = 2lm$$

Net encoding rate

$$r = \frac{k}{2n}$$

Building Bivariate Bicycle Codes



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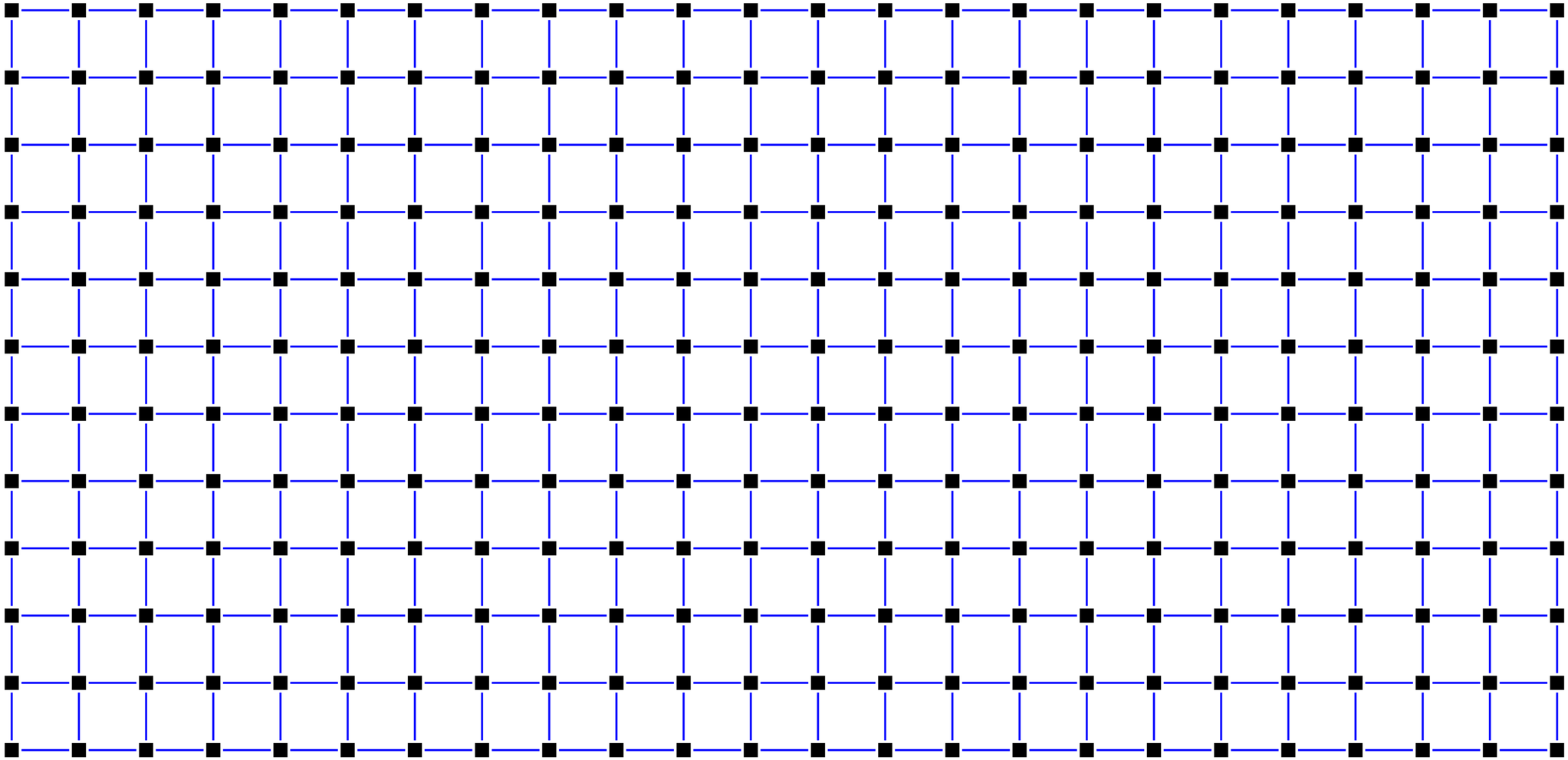

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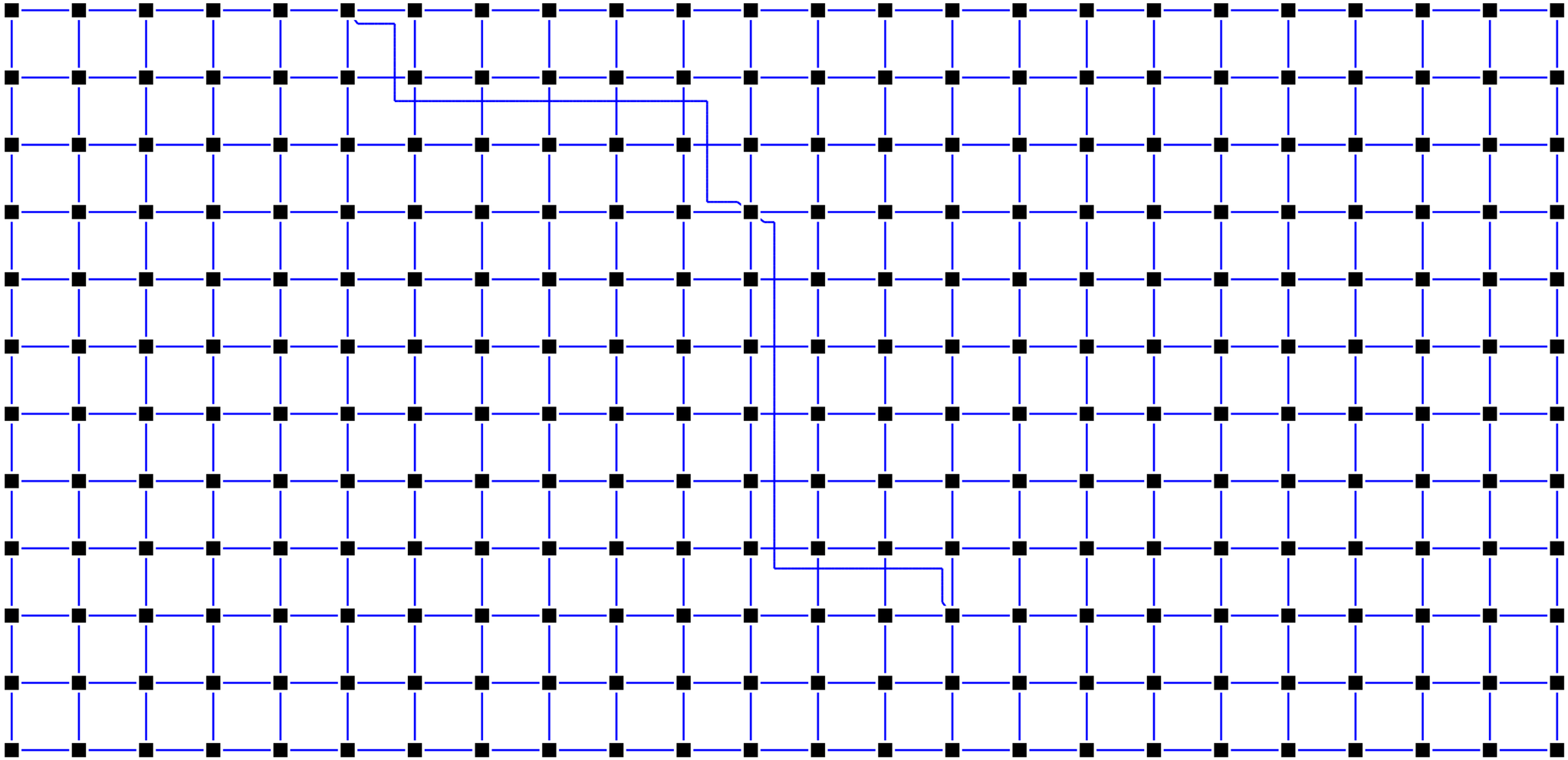
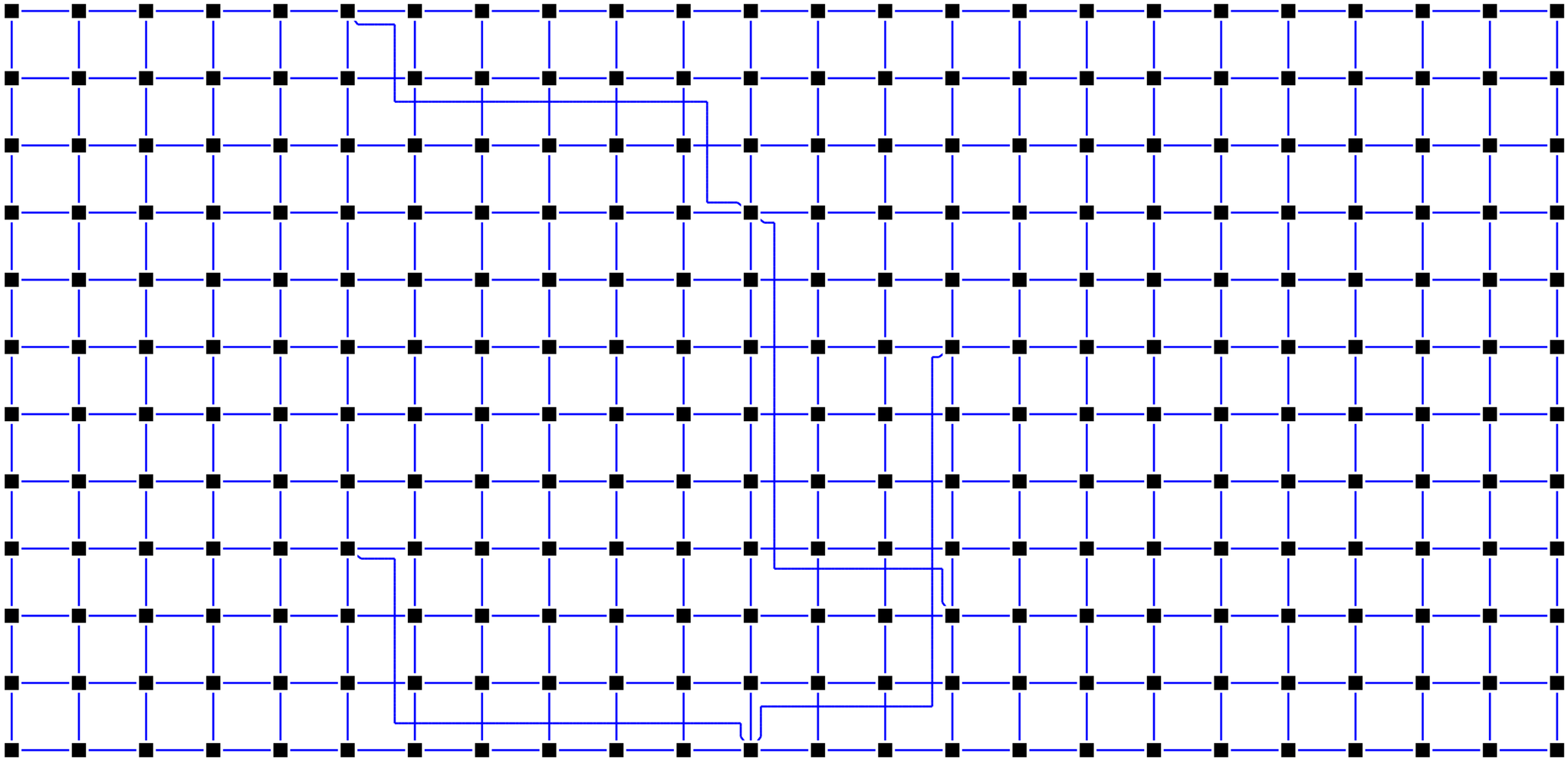
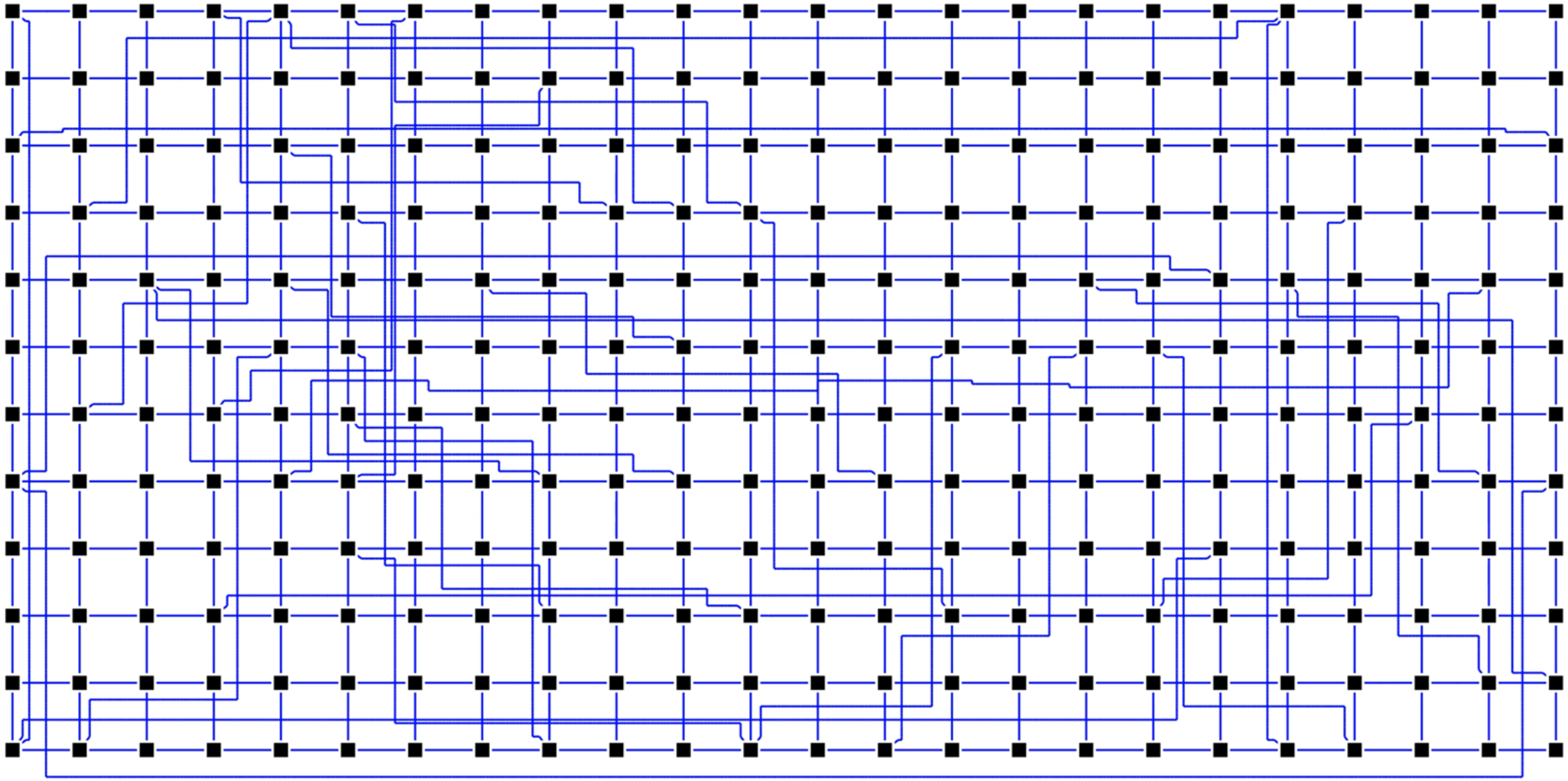


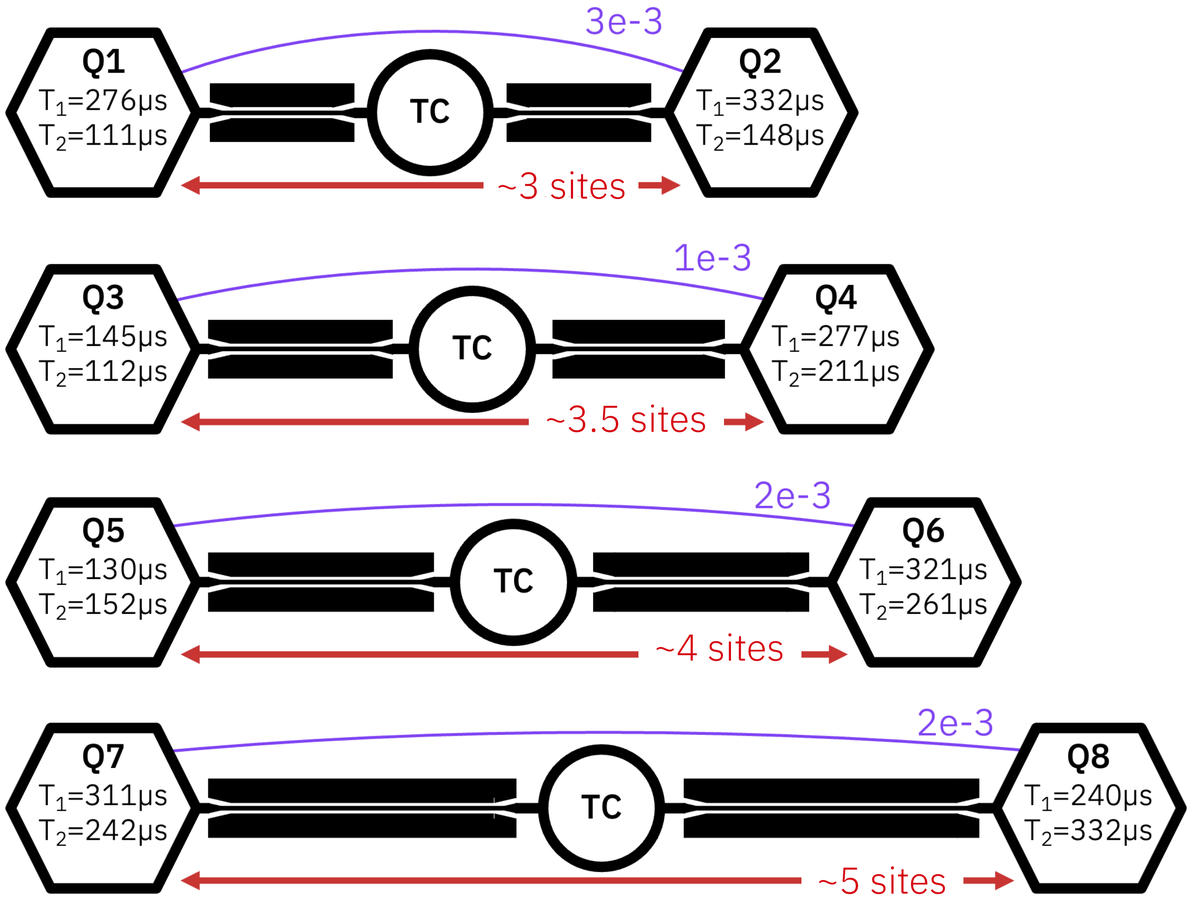
Building Bivariate Bicycle Codes

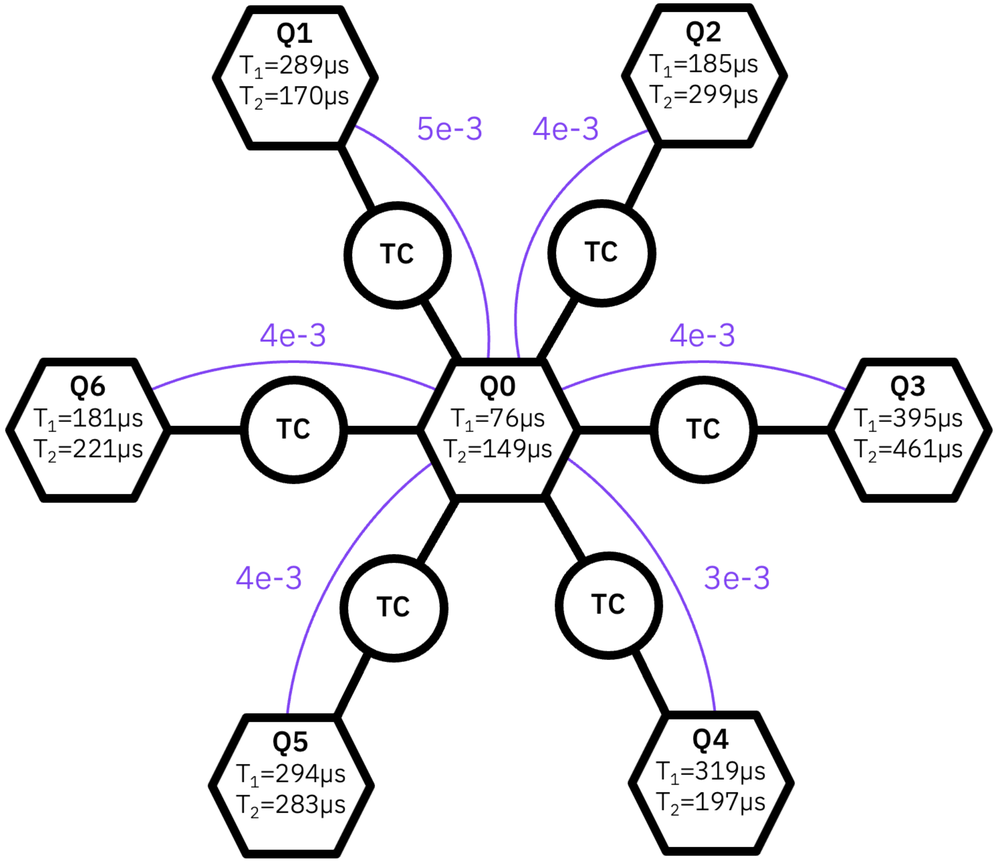


Building Bivariate Bicycle Codes

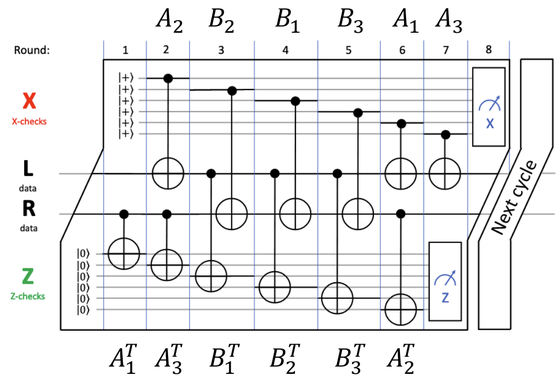


Extending coupler length and increasing connectivity to degree-6: benchmarking test devices





Syndrome measurement circuits



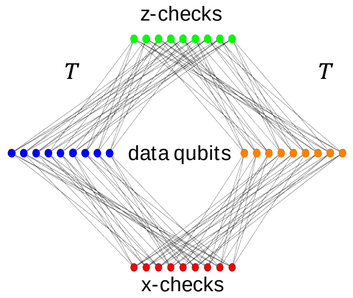
$$A = A_1 + A_2 + A_3$$

$$B = B_1 + B_2 + B_3$$

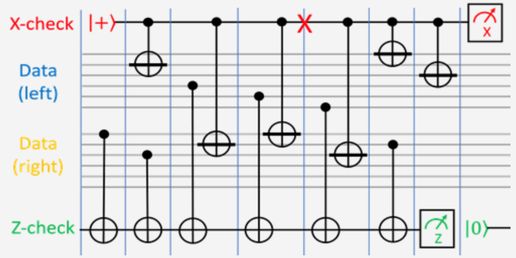
→ Same circuit design works for all bivariate bicycle codes in ref [1].

→ One measurement qubit per check (*i.e.*, “bare”).

→ All CNOT gates have edges in the connectivity graph.



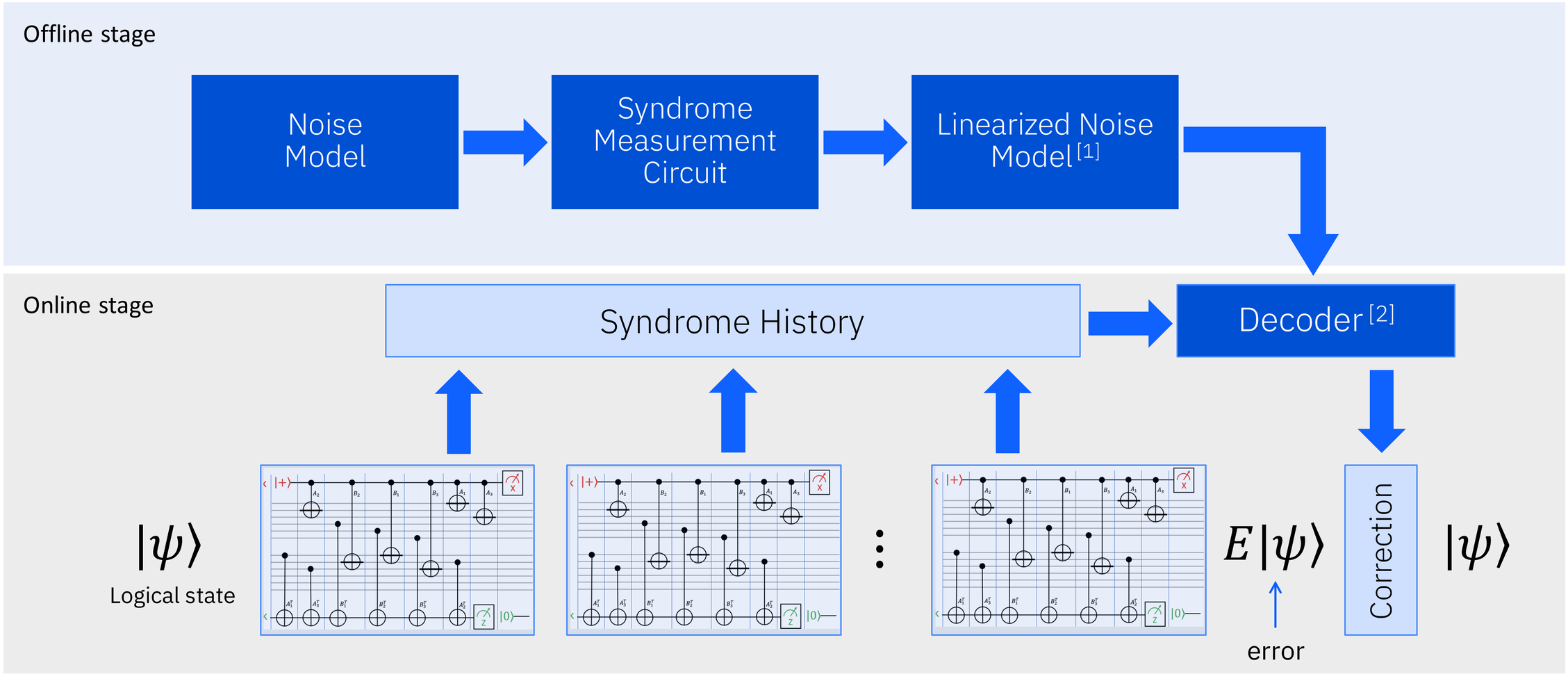
Hook errors can effectively reduce the code distance d.



→ *Circuit-level distance* d_{circ} is the smallest number of faulty operations in the circuit that can generate a logical error without triggering any syndromes.

$[[n, k, d]]$	Net Encoding Rate r	Circuit-level distance d_{circ}
$[[72, 12, 6]]$	$1/12$	≤ 6
$[[90, 8, 10]]$	$1/23$	≤ 8
$[[108, 8, 10]]$	$1/27$	≤ 8
$[[144, 12, 12]]$	$1/24$	≤ 10
$[[288, 12, 18]]$	$1/48$	≤ 18

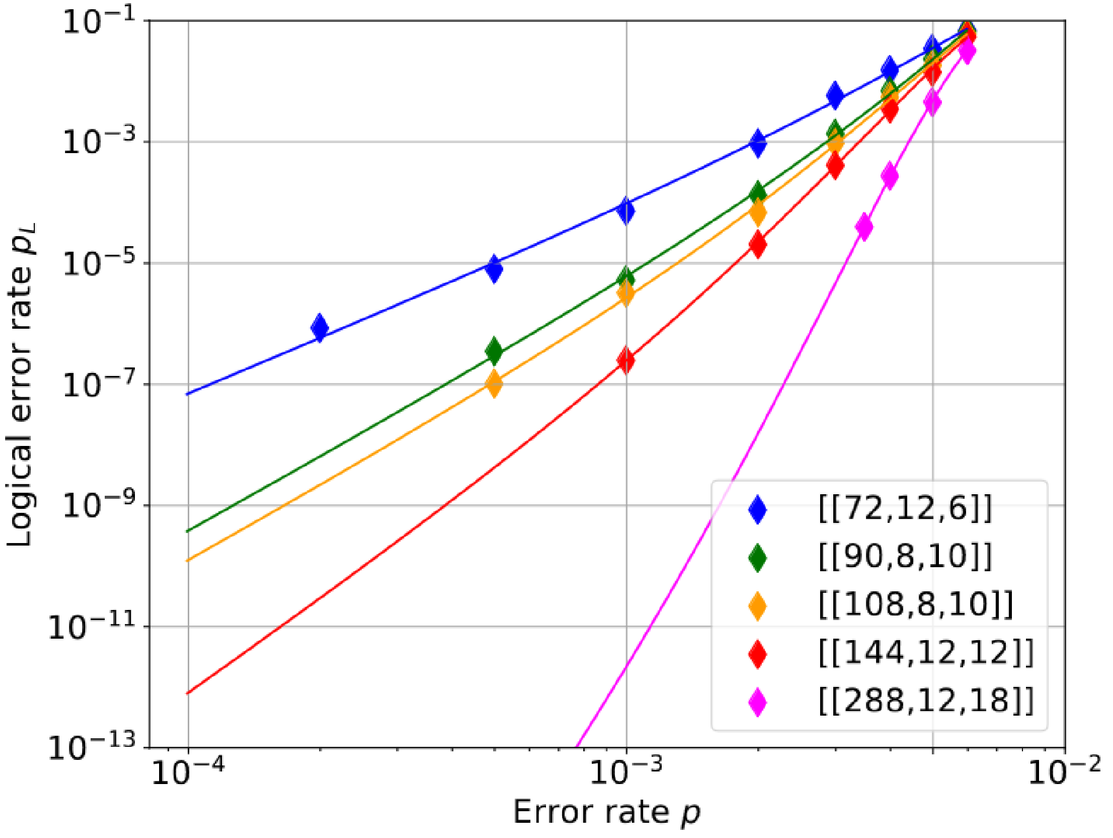
Syndrome processing for circuit-level noise

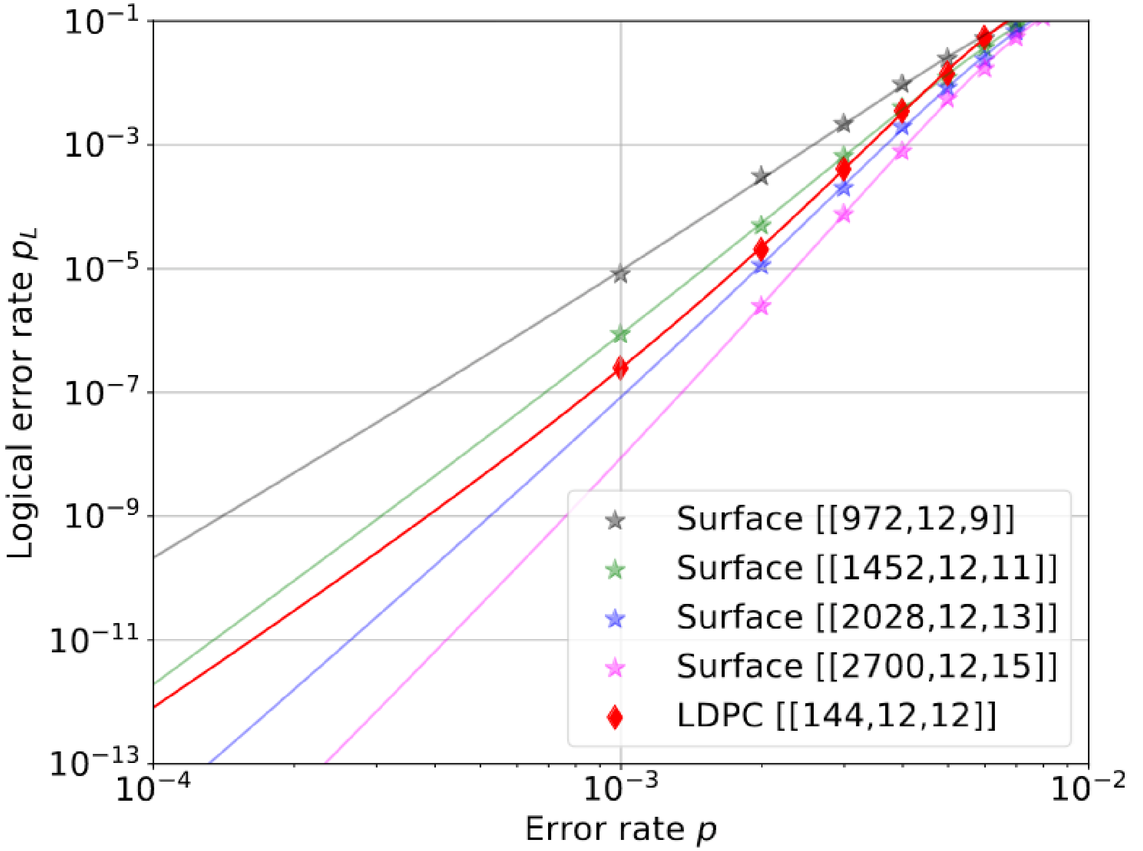


[1] For example, independent X and Z errors on an extended graph, like the surface code "decoding graph"
[2] e.g., BP-OSD* §, GDG †, BP-LSD ◊, DTD‡, ...

* P. Panteleev, G. Kalachev, Quantum 5 (2021)
§ J. Roffe, D. White, S. Burton, E. Campbell, Phys. Rev. Research 2, 043423 (2020)
† A. Gong, S. Cammerer, J. Renes, arXiv:2403.18901 (2024)
◊ T. Hillmann et al, arXiv:2406.18655 (2024)
‡ K. Ott, B. Hetenyi, M. Beverland, arXiv:2502.16408 (2025)

Quantum memory simulation with circuit noise





S. Bravyi et al, Nature 627 (2024); arXiv:2308.07915

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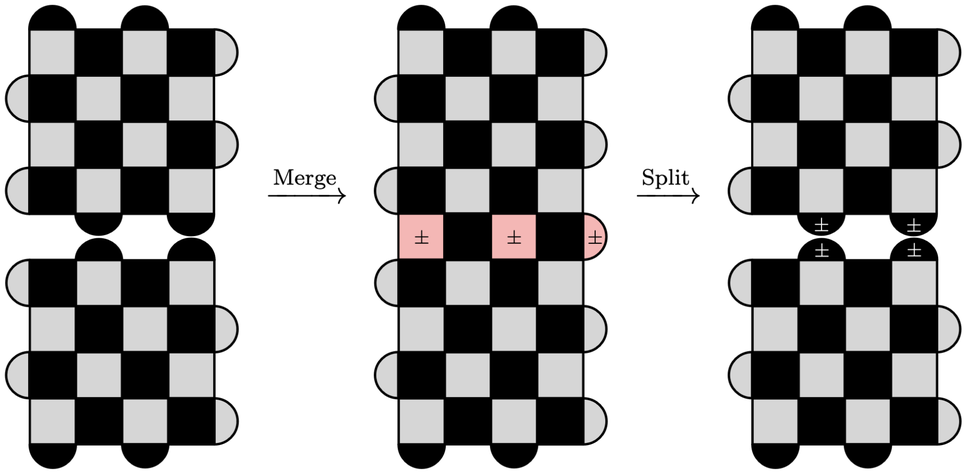
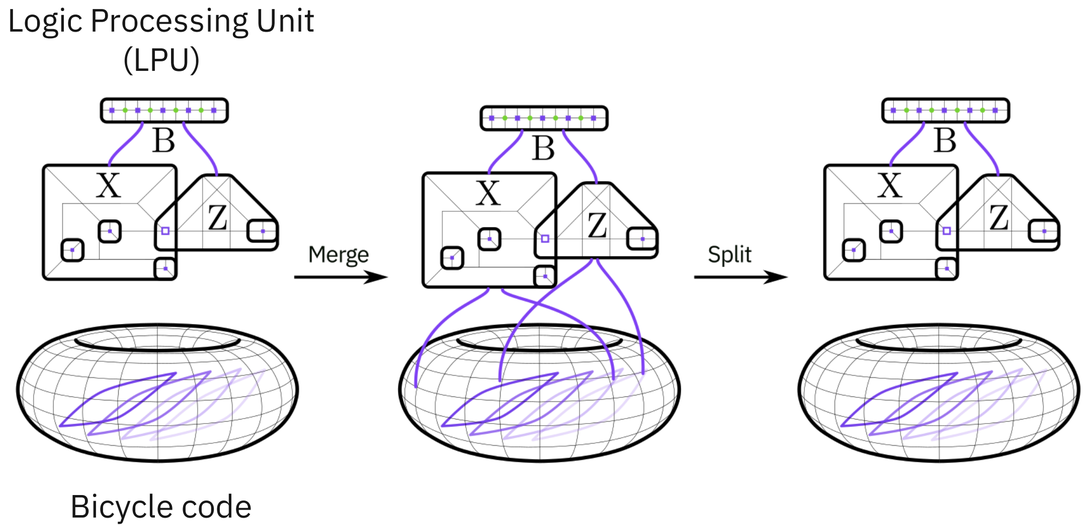
Surface code lattice surgery



(figure from ‡)

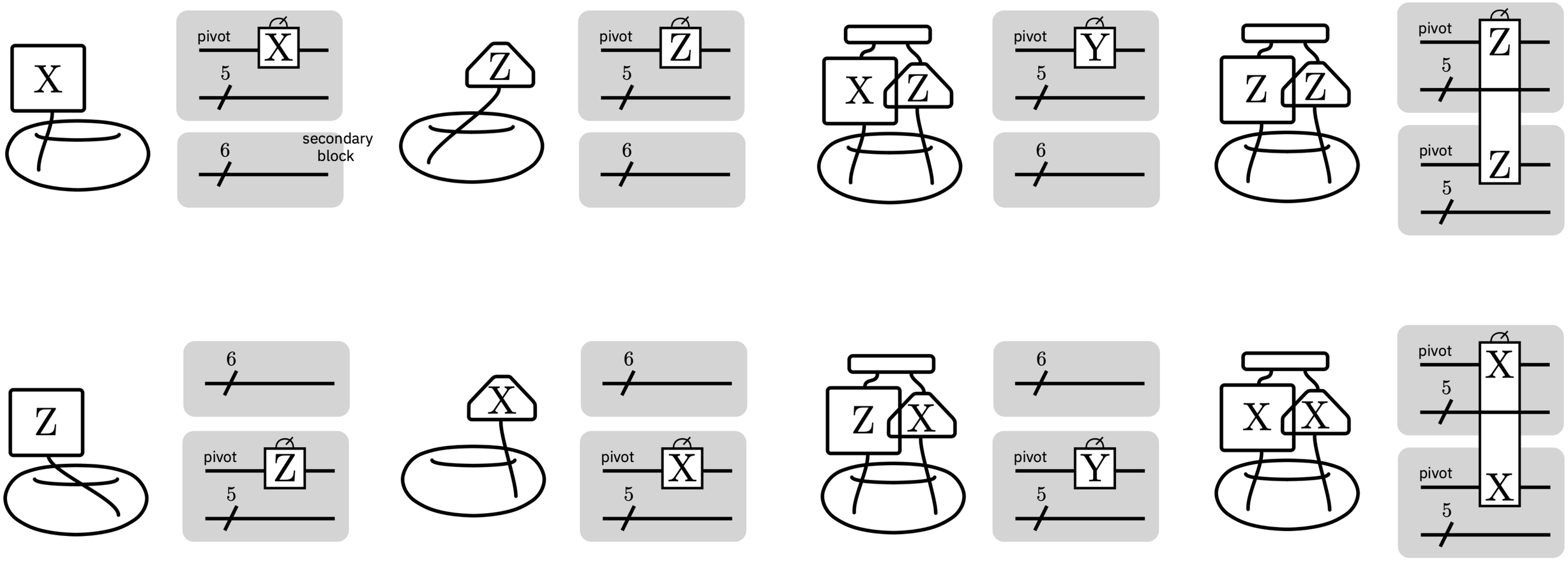
D. Horsman, A. Fowler, S. Devitt, R. Van Meter, NJP 14 123011 (2012)
‡ C. Vuillot, L. Lao, B. Criger, C. Almudever, K. Bertels, B. Terhal, NJP 21 033028 (2019)

Generalized surgery



L. Cohen, I. Kim, S. Bartlett, B. Brown, Sci. Adv. 8, eabn1717 (2022)
M. Hastings, arXiv:2102.10030 (2021); arXiv:1611.03790 (2016)
E. Sabo, L. Gunderman, B. Ide, M. Vastmer, G. Dauphinais, arXiv:2402.05228 (2024)
A. Cross, Z. He, P. Rall, T. Yoder, arXiv:2407.18393 (2024)
D. Williamson, T. Yoder, arXiv:2410.02213, (2024)
E. Swaroop, T. Jochym-O'Connor, T. Yoder, arXiv:2410.03628, (2024)
Ide *et. al.*, arXiv:2410.02753 (2024)

LPU Measurement Library

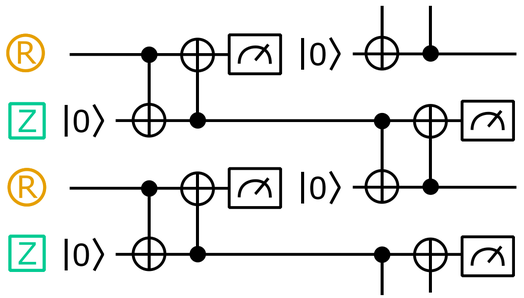


Automorphisms: Move the LPU to a different Spot

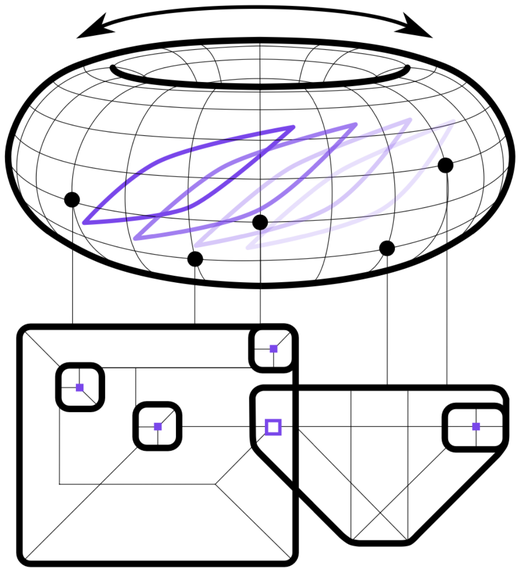
Modulo the stabilizer, the [[144,12,12]] gross code has 36 permutation automorphisms.

One can implement 36 x 8 = 288 *native measurements*.

These 288 measurements include all single qubit Pauli measurements.



Shallow fault-tolerant circuit



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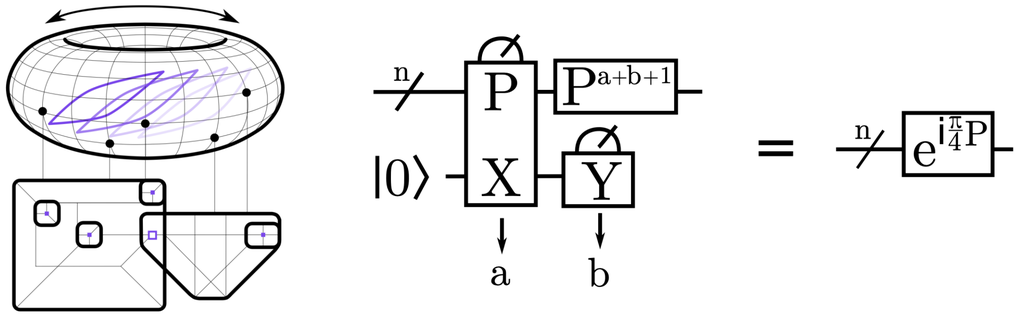
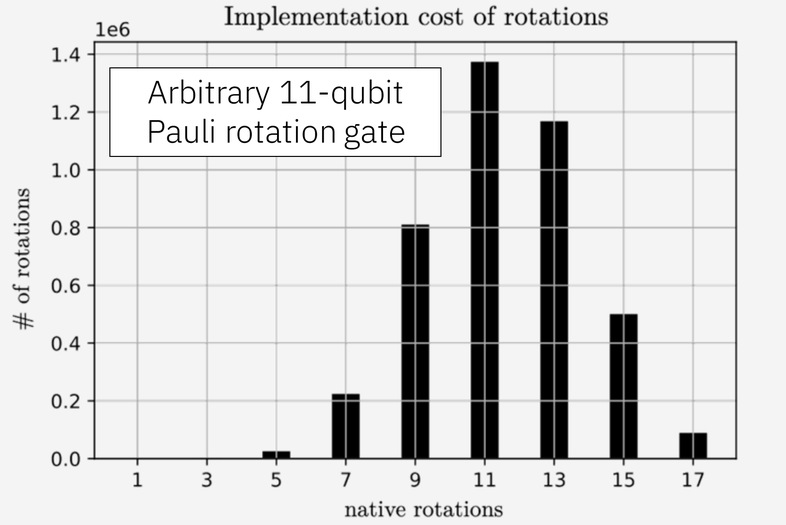
Logical Clifford operations on a [[144,12,12]] code block

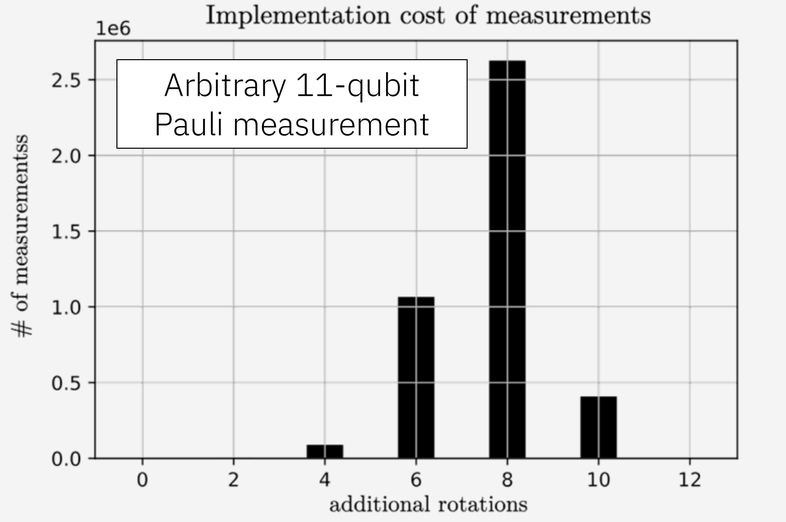
→ Together with automorphisms, a gauge-fixed ancilla system suffices to implement the full Clifford group on 11 of 12 logical qubits (the last logical qubit is an ancilla for measurement-based gates).

→ The 95 native rotations are generated from the subset of native measurements that are supported on pivot and the rest of the system.





$$e^{i\frac{\pi}{4}P} = e^{-i\frac{\pi}{4}Q_1}...e^{-i\frac{\pi}{4}Q_{n-1}}e^{i\frac{\pi}{4}Q_n}e^{i\frac{\pi}{4}Q_{n-1}}...e^{i\frac{\pi}{4}Q_1}$$



$$\mathcal{M}_P = e^{-i\frac{\pi}{4}Q_1}...e^{-i\frac{\pi}{4}Q_{n-1}}\mathcal{M}_{Q_n}e^{i\frac{\pi}{4}Q_{n-1}}...e^{i\frac{\pi}{4}Q_1}$$

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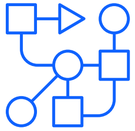
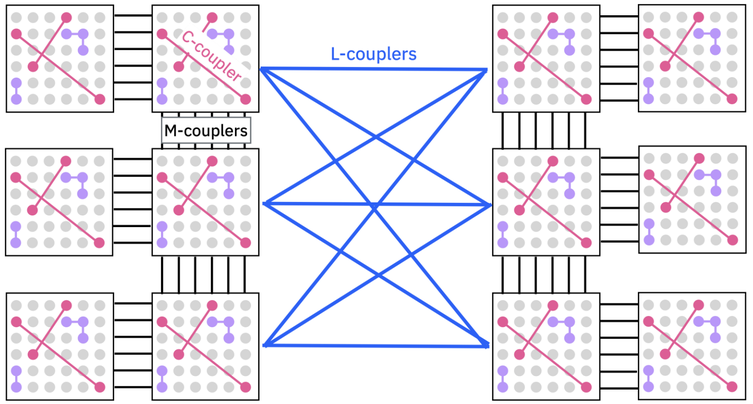
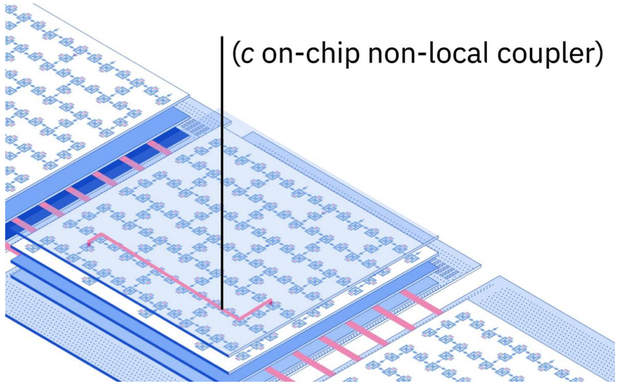
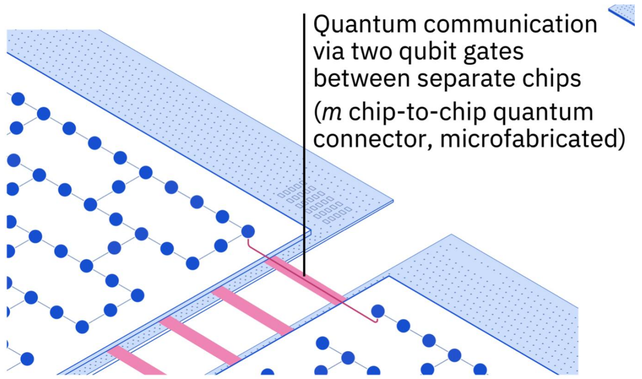
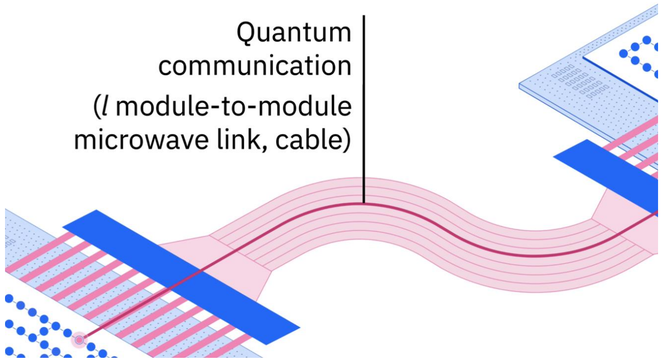


Modular quantum computing architecture



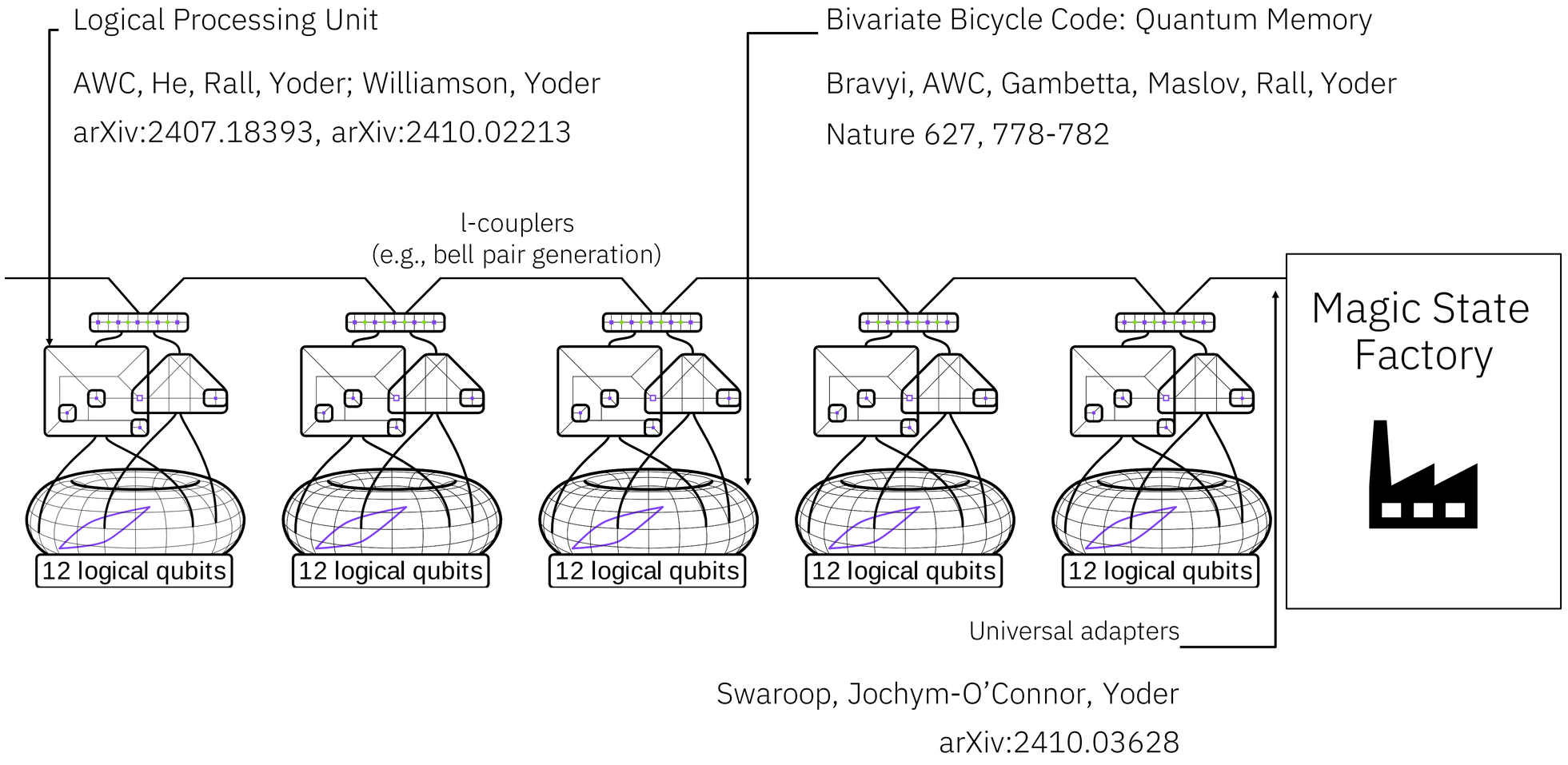






Coupler figures and naming convention: S. Bravyi, O. Dial, J. Gambetta, D. Gil, Z. Nazario, Journal of Applied Physics 132, 160902 (2022)

Fault-tolerant modular architecture



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