Worksheet 1.1: Lectures 2 and 3
Growth of Functions and Run-Time Estimation

Question 1: For each pair of functions below, first determine which **function grows asymptotically faster**, and then express the relation between the two functions using **all** the asymptotic notations (**θ, O, Ω, o** and **ω)** that apply. Only give the relations in which **f(n)** appears on the left-hand side. **Justify your answer**, and use the limit definition if necessary.

(1) $f(n) = n^2 + (\log n)^3$ $g(n) = n^2 \log n$ No justification is required

(2) $f(n) = 4^n$ $g(n) = 3^n + n$ Justify your answer using the limit definition. Show your work.

Question 2: If we know that the running time **T(n)** of some algorithm satisfies the relations:
$T(n) = o(n^{2.5} \log n)$ and **$T(n) = \Omega(n(\log n)^3)$,**
which of the following functions can **T(n)** possibly be? Circle **all** that apply.

$n^{2.5} \log n$ $n(\log n)^3$ $n(\log n)^5$ $n^{1.01}$ $n^{2.6}$

Question 3: Consider two algorithms for solving a certain problem: Algorithm X with an asymptotic complexity of **θ**$(n^2 \log n)$ and Algorithm Y with an asymptotic complexity of **θ**(n^3). Algorithm X is run on a machine that can execute 10^7 operation per second. Compute the speed of the machine that we need to run Algorithm Y on in order to get the same execution time as Algorithm X for an input of size **one million**. Assume base 2 for the log. Compute the best possible approximation without using a calculator. **Show your work.**