Flow modelling and Cellular Automata

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Outline

Lattice gas cellular automata

Difference

What is the difference between lattice gas cellular automata (LGCA) and CA?

Basic structure

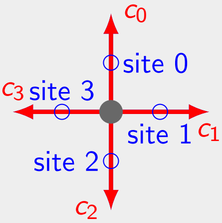


Figure: Here is an example of a single cell with 4 velocity vectors and 4 empty sites.

LGCA in flow modelling

Lattices Models

- HPP
- FHP
- LBM

Why?

Why was LGCA developed?

HPP

History

- Developed in 1973.
- By Hardy, de Pazzis and Pomeau.
- Fail Navier-Stokes.

One cell.

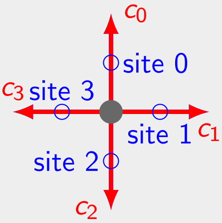
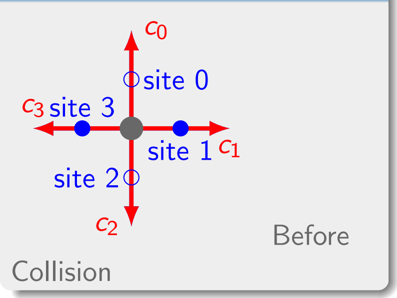


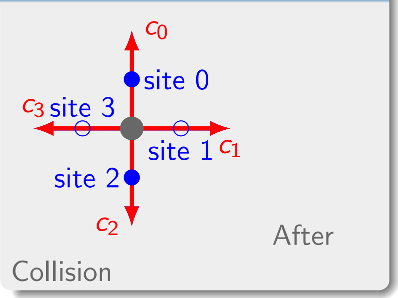
Figure: Here is an example of a single cell with 4 velocity vectors and 4 empty sites.

HPP Collision Phase

Before

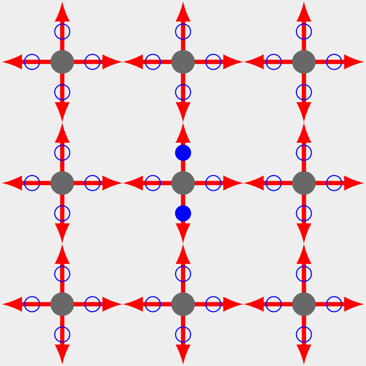


After



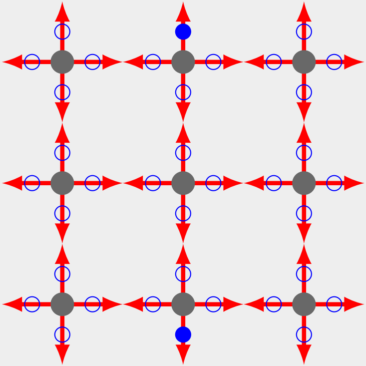
HPP Propagation Phase

Before



After Collision and before propagation

After



After propagation

HPP Mass and Momentum densities

The lgca can be fully describe by the boolean fields $n_i(t, \mathbf{r}_j)$, where $i \in \{0, 1, 2, 3\}$, $\mathbf{r}$ is the position of the cell and t is the discrete time. $n_i(t, \mathbf{r}_j)$ means the boolean value of the ith site at position $\mathbf{r}$ at time t. Before calculating the mass and momentum densities we require the mean occupation numbers. The mean occupation number for site i at position $\mathbf{x}$, is the average of $n_i(t, \mathbf{r}_j)$ where $\mathbf{r}_j$ are the the positions of the neighbours of $\mathbf{x}$

$$N_i(t, \mathbf{x}) = \frac{\sum_{j=0}^{3} n_i(t, \mathbf{r}_j)}{4}.$$

The mass density for time t and position $\mathbf{x}$ is defined as follow:

$$\rho(t, \mathbf{x}) = \sum_{i=0}^{3} N_i(t, \mathbf{x}).$$

HPP Mass and Momentum densities continued

The momentum densities for time t and position $\mathbf{x}$ is defined as follow:

$$\mathbf{j}(t, \mathbf{x}) = \rho \mathbf{u} = \sum_{i=0}^{3} \mathbf{c}_i N_i(t, \mathbf{x}).$$

Given ρ and $\mathbf{j}$ the mass and momentum densities we can calculate N_i assuming a linear relationship

$$N_i = \xi \rho + \eta \mathbf{c}_i \mathbf{j}.$$

We get: $\xi = \frac{1}{4}$; and $\eta = \frac{1}{2}$ thus

$$N_i = \frac{1}{4}\rho + \frac{1}{2}\mathbf{c}_i \mathbf{j}.$$

FHP

History

- Developed in 1986.
- By Frisch, Hasslacher and Pomeau.
- Navier-Stokes success.
- pauli exclusion principle.
- FHP-I, FHP-II and FHP-III.

History

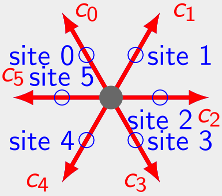
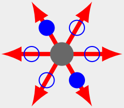


Figure: Here is an example of a single hexagonal cell with 6 velocity vectors and 6 empty sites.

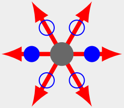
FHP Collision Phase1

Before collision.



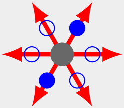
Before collision.

First possible outcome.



First possible outcome.

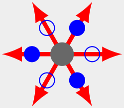
Second possible outcome.



Second possible outcome.

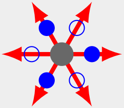
FHP Collision Phase2

Before collision.



Before collision.

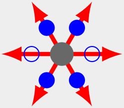
After collision.



After collision.

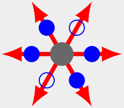
FHP Collision Phase3

Before collision.



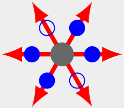
Before collision.

First possible outcome.



First possible outcome.

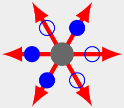
Second possible outcome.



Second possible outcome.

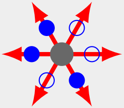
FHP Collision Phase4

Before collision.



Before collision.

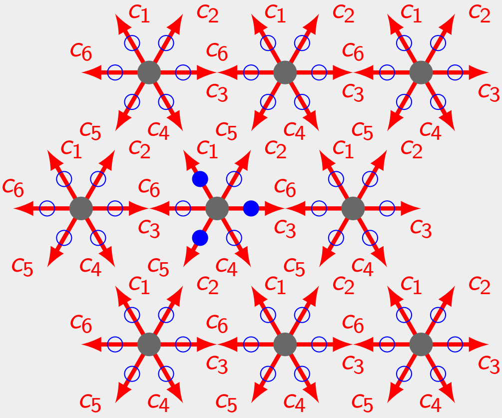
After collision.



After collision.

FHP Propagation Phase

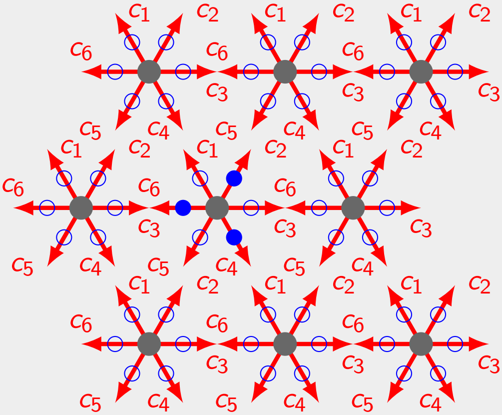
History



Before collision phase

FHP Propagation Phase

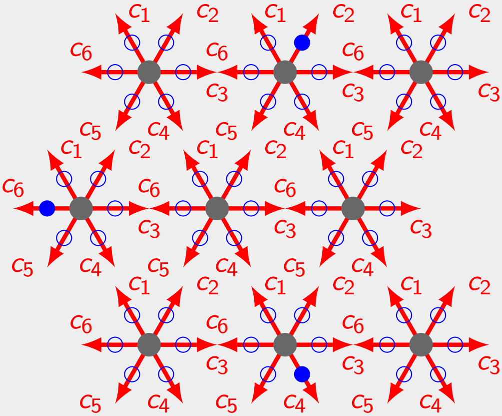
After collision phase and before propagation phase.



After collision phase and before propagation

FHP Propagation Phase

After propagation phase.



After propagation phase.

Applications

Applications

- Simulating rain in games.
- Simulating rivers in games.
- 2D fluid simulations.
- Network congestion simulation.

Bibliography

Wolf-Gladrow, D. A. 2000. Lattice-gas cellular automata and lattice Boltzmann models. New York: Springer.