

Homework

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Question 2.1

(a)

Parse tree

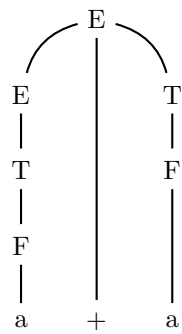
$E \rightarrow T \rightarrow F \rightarrow a$

Derivation

$E \rightarrow T \rightarrow F \rightarrow a$

(b)

Parse tree

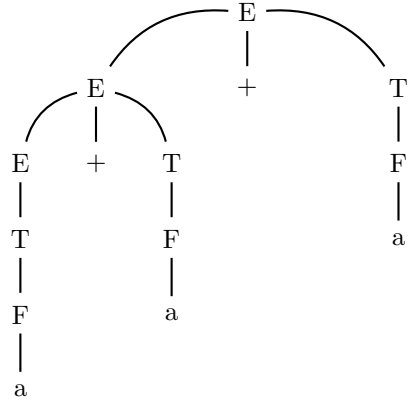


Derivation

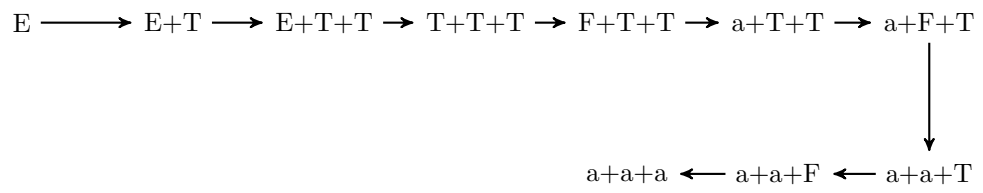
$E \rightarrow E+T \rightarrow T+T \rightarrow F+T \rightarrow a+T \rightarrow a+F \rightarrow a+a$

(c)

Parse tree

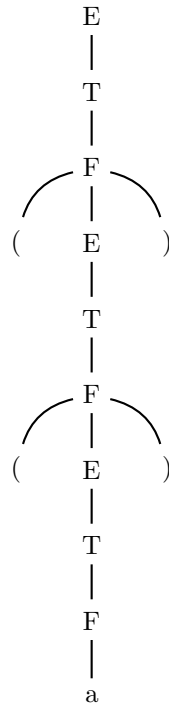


Derivation

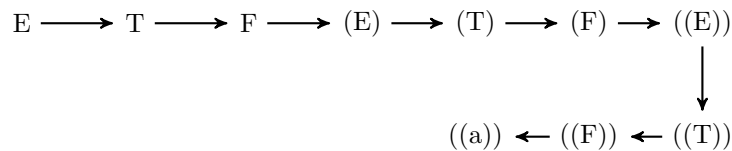


(d)

Parse tree



Derivation



Question 2.2

(a)

Language $A = \{a^m b^n c^n \mid m, n \geq 0\}$. Language $B = \{a^n b^n c^m \mid m, n \geq 0\}$. The language that is produced by taking the intersection of language A and B is $\{a^n b^n c^n \mid n \geq 0\}$. Call this new language C. To prove that CFL are not closed under intersection we'll assume that C is a CFL then prove by contradiction that C is not and thus proving that CFL in general are not closed under intersection.

Proof Let $C = \{a^n b^n c^n \mid n \geq 0\}$ and let the pumping length be p given by the pumping lemma. Let s be the string $a^p b^p c^p$. Divide s up into 5 section $uvxyz$.

If v and y consisted of the same alphabet element and $s uv^i xy^i z$ is pumped with $i = 0$ then the string in the form uxz is formed. This string does not contain the same amount of a 's, b 's and c 's which is a contradiction.

If v and y consisted of different alphabet letters and s is pumped with $i = 0$. Then number of a 's, b 's and c 's will not be the same because only two will change and one will remain constant. This causes a contradiction.

If v and y consisted of multiple different alphabet letters and s is pumped with $i = 2$ then the string in the form $uvvxyyx$ is formed. Since v and/or y consists of multiple different different alphabet letters this new string contradicts the original description.

This thus proves that CFL are not closed under intersection.

(b)

In 2.2.a we proved that CFL are not closed under intersection. Now we will prove that CFL are not closed under complementation by using DeMorgan's Law ($\overline{A \cup B} = \overline{A} \cap \overline{B}$). DeMorgan's law can be rewritten into the follow form $\overline{A \cup B} = A \cap B$. Lets assume that A and B are the same as defined in 2.2.a. The right hand side is the intersection of A and B . Since CFL is not closed under intersection the right hand side doesn't hold for CFL thus the left hand side also doesn't hold for CFL. Thus CFL are not closed under complementation.

Question 2.4

(a)

$$\begin{aligned} A &= 0A|1B \\ B &= 0B|1C \\ C &= 0C|1D \\ D &= 0D|1D|\epsilon \end{aligned}$$

(b)

$$\begin{aligned} A &= 0B0|1B1|1|0 \\ B &= 0B|1B|\epsilon \end{aligned}$$

(c)

$$\begin{aligned} A &= 0B|1B \\ B &= 0A|1A|\epsilon \end{aligned}$$

(b)

$$\begin{aligned} A &= BAB|0 \\ B &= 0|1 \end{aligned}$$

(e)

$$A = 1A1|0A0|\epsilon$$

(f)

$$A = A$$

Question 2.12

Question 2.14

Step 1

$$\begin{aligned} S_0 &= A \\ A &= BAB|B|\epsilon \\ B &= 00|\epsilon \end{aligned}$$

Step 2

$$\begin{aligned} S_0 &= A \\ A &= BAB|AB|BA|B|\epsilon \\ B &= 00 \end{aligned}$$

Step 3

$$\begin{aligned} S_0 &= BAB|AB|BA|B|\epsilon \\ A &= BAB|AB|BA|B|\epsilon \\ B &= 00 \end{aligned}$$

Step 4

$$\begin{aligned} S_0 &= BAB|AB|BA|00|\epsilon \\ A &= BAB|AB|BA|00|\epsilon \\ B &= 00 \end{aligned}$$

Step 5

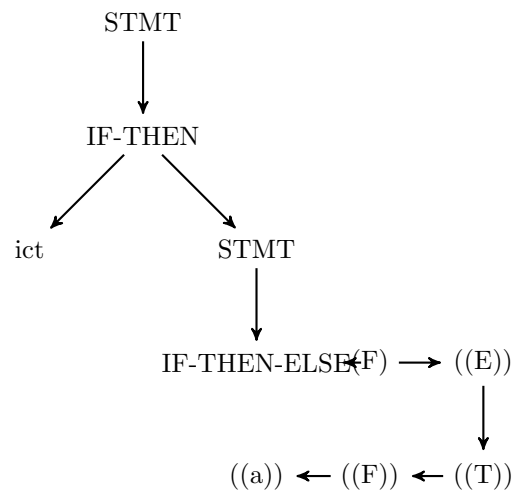
$$\begin{aligned} S_0 &= BB_1|AB|BA|00|\epsilon \\ A &= BB_1|AB|BA|00|\epsilon \\ B &= 00 \\ B_1 &= AB \end{aligned}$$

Question 2.27

(a)

For a grammar to be ambiguous it needs to have atleast two left most derivations for a string w. The given grammer has atleast two left most derivations for the string “if condition then if condition then a:=1 else a:=1”. Note `itc = if condition then`.

Derivation 1



Derivation 2