# Flow modelling and Cellular Automata

Shaun Schreiber (16715128)

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# Outline

# Lattice gas cellular automata

## Difference

What is the difference between lattice gas cellular automata (LGCA) and CA?

## Basic structure

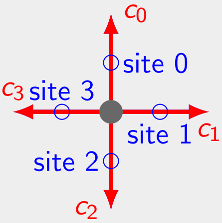



Figure: Here is an example of a single cell with 4 velocity vectors and 4 empty sites.

# LGCA in flow modelling

## Lattices Models

- HPP
- FHP
- LBM

# HPP

## History

- Developed in 1973.
- By Hardy, de Pazzis and Pomeau.
- Fail Navier-Stokes.

## One cell.

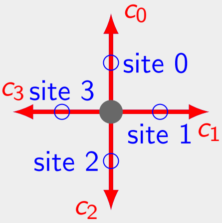
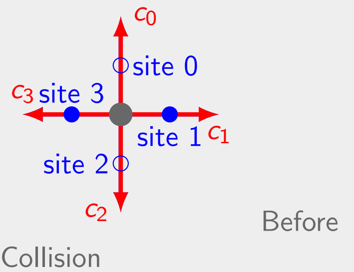


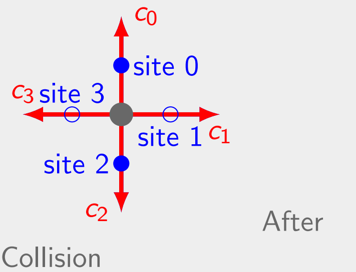

Figure: Here is an example of a single cell with 4 velocity vectors and 4 empty sites.

# HPP Collision Phase

## Before

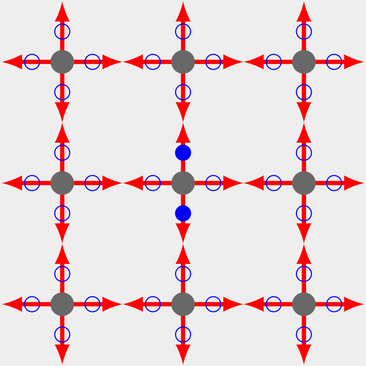



## After



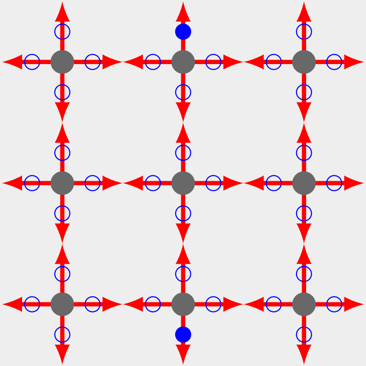
# HPP Propagation Phase

## Before



After Collision and before propagation

## After



After propagation

# HPP Mass and Momentum densities

The lgca can be fully describe by the boolean fields $n_i(t, \mathbf{r}_j)$, where $i \in \{0, 1, 2, 3\}$, $\mathbf{r}$ is the position of the cell and t is the discrete time. $n_i(t, \mathbf{r}_j)$ means the boolean value of the ith site at position $\mathbf{r}$ at time t. Before calculating the mass and momentum densities we require the mean occupation numbers. The mean occupation number for site $i$ at position $\mathbf{x}$, is the average of $n_i(t, \mathbf{r}_j)$ where $\mathbf{r}_j$ are the the positions of the neighbours of $\mathbf{x}$

$$N_i(t, \mathbf{x}) = \frac{\sum_{j=0}^{3} n_i(t, \mathbf{r}_j)}{4}.$$

The mass density for time t and position $\mathbf{x}$ is defined as follow:

$$\rho(t, \mathbf{x}) = \sum_{i=0}^{3} N_i(t, \mathbf{x}).$$

# HPP Mass and Momentum densities continued

The momentum densities for time t and position $\mathbf{x}$ is defined as follow:

$$\mathbf{j}(t, \mathbf{x}) = \rho \mathbf{u} = \sum_{i=0}^{3} \mathbf{c}_i N_i(t, \mathbf{x}).$$

Given $\rho$ and $\mathbf{j}$ the mass and momentum densities we can calculate $N_i$ assuming a linear relationship

$$N_i = \xi \rho + \eta \mathbf{c}_i \mathbf{j}.$$

We get: $\xi = \frac{1}{4}$; and $\eta = \frac{1}{2}$ thus

$$N_i = \frac{1}{4} \rho + \frac{1}{2} \mathbf{c}_i \mathbf{j}.$$

# FHP

## History

- Developed in 1986.
- By Frisch, Hasslacher and Pomeau.
- Navier-Stokes success.
- pauli exclusion principle.
- FHP-I, FHP-II and FHP-III.

## History

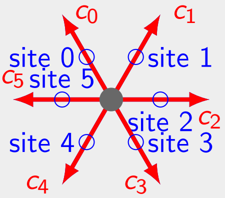
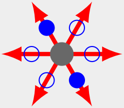



Figure: Here is an example of a single hexagonal cell with 6 velocity vectors and 6 empty sites.

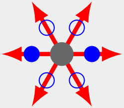
# FHP Collision Phase1

## Before collision.



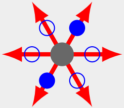
Before collision.

## First possible outcome.



First possible outcome.

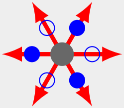
## Second possible outcome.



Second possible outcome.

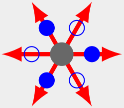
# FHP Collision Phase2

**Before collision.**



Before collision.

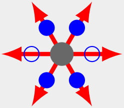
**After collision.**



After collision.

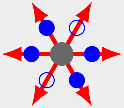
# FHP Collision Phase3

## Before collision.



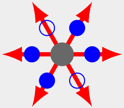
Before collision.

## First possible outcome.



First possible outcome.

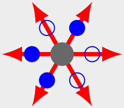
## Second possible outcome.



Second possible outcome.

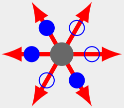
# FHP Collision Phase4

**Before collision.**



Before collision.

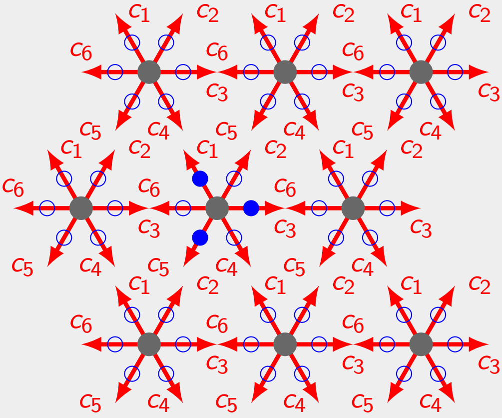
**After collision.**



After collision.

# FHP Propagation Phase

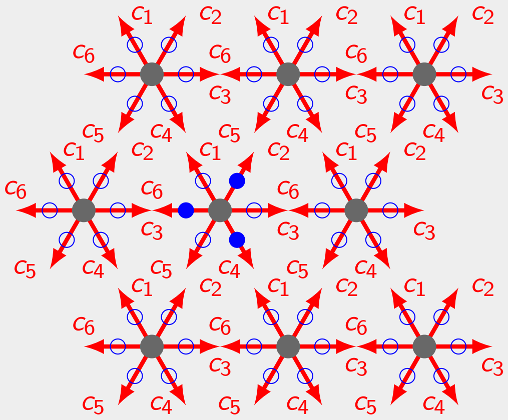
## History




Before collision phase

# FHP Propagation Phase

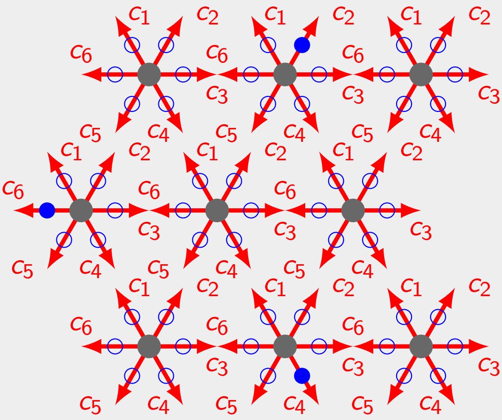
## After collision phase and before propagation phase.




After collision phase and before propagation

# FHP Propagation Phase

## After propagation phase.




After propagation phase.

# Applications

## Applications

- Simulating rain in games.
- Simulating rivers in games.
- 2D fluid simulations.
- Network congestion simulation.

# Bibliography

Wolf-Gladrow, D. A. 2000. Lattice-gas cellular automata and lattice Boltzmann models. New York: Springer.