Radix Complements

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Radix complements are used to simplify subtraction operations in positional number systems. Instead of directly performing subtraction, we can **transform the problem into an addition operation using the complement of a number**. This is particularly useful for computers because addition circuits are simpler than subtraction circuits.

Types of Complements

- **Diminished Radix Complement**: Also referred to as the $(r - 1)$'s complement. It is the complement relative to one less than the base.
- **Radix Complement**: Also referred to as the r's complement. It is the complement relative to the full range of the base.

FAQ: Is this related to the concept of 1's complement and 2's complement that we learned in CCICOMP?

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Yes! When the value of the **radix *r* is substituted in the name for binary numbers** (meaning *r* is 2), the two types are referred to as the 1's complement and 2's complement for binary numbers. In the same sense, you can infer that the complements of decimal numbers are the 9's complement and 10's complement.

Diminished Radix Complement

Given a number N in base-r having n digits, the diminished radix complement is defined as $(r^n - 1) - N$.

- For decimal numbers, $r = 10$ and $r - 1 = 9$. Therefore, the 9's complement of N is $(10^n - 1) - N$.
  - 10^n represents a number that consists of a single 1 followed by n amount of 0s.
  - Essentially, the 9's complement is obtained by subtracting each digit from 9.

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Example 1: Let N be 245 in base-10.

N = 245
r = 10
n = 3
```

$$r^n - 1 - N = (10^3)_{10} - 1 - 245 = 999 - 245 = 754$$

```
Put simply:

   9 9 9   (subtract r − 1 in all digits)
-  2 4 5
--------
   7 5 4   (notice that 245 + 754 = 999)
```

- For binary numbers, $r = 2$ and $r - 1 = 1$. Therefore, the 1's complement of N is $(2^n - 1) - N$.
    - The 1's complement is obtained by subtracting each digit from 1.
    - Notice how this essentially means that the 1's complement is obtained by simply switching 1s to 0s and 0s to 1s.

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Example 1: Let N be 010 1101 in base-2.

N = 010 1101
r = 2
n = 7
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$$r^n - 1 - N = (2^7)_2 - 1 - 010\,1101 = 111\,1111 - 010\,1101 = 101\,0010$$

```
Put simply:

   1 1 1 1 1 1 1   (subtract r − 1 in all digits)
-  0 1 0 1 1 0 1
----------------
   1 0 1 0 0 1 0   (notice that 010 1101 + 101 0010 = 111 1111)
```

- In a similar manner, the diminished radix complement of octal or hexadecimal numbers is obtained by subtracting each digit from 7 or F (decimal 15), respectively.

Radix Complement

The r's complement of an n-digit number N in base-r is defined as $r^n - N$ for $N \neq 0$ and as 0 for $N = 0$. It is obtained by adding 1 to the diminished radix complement, since $r^n - N = [(r^n - 1) - N] + 1$. Note that the formula can also be seen as leaving all least significant 0s unchanged, subtracting the first nonzero least significant digit from r, and subtracting all higher significant digits from $r - 1$.

- For decimal numbers, which means $r = 10$:

```
Example 1: Let N be 245 in base-10.

   9 9 10   (subtract r in the least significant non-0 digit)
-  2 4  5   (r − 1 for the rest)
---------
   7 5  5   (notice that 245 + 755 = 1000)
```

- Similarly, the 2's complement can be formed by leaving all least significant 0's and the first 1 unchanged and replacing 1's with 0's and 0's with 1's in all other higher significant digits.

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Example 1: Let N be 100 1100 in base-2.

(least significant non-0 digit)
           |
           v
   1 0 0 1 1 0 0
----------------
   0 1 1 0 1 0 0   (notice that 100 1100 + 011 0100 = 1000 0000)
```

FAQ: Should I add 1 to the diminished radix complement or calculate it the standard way?

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While adding 1 may be the faster way, it is also easier to make mistakes especially **if you forget to keep track of the base** (e.g., $(7)_8 + (1)_8 = (10)_8$ which is equal to but not the same as $(8)_{10}$). Either way, both methods are still viable as long as it is done carefully.