

# Does Size Matter? (Estimation of Banana Weight with a Regression Modeling Approach)

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## Summary

This study developed linear regression models to effectively predict the weight of bananas based on external measurements. In order to develop the different models the techniques used were, determining significance of variables, elimination of variables and elimination of outliers. The models were then compared by ANOVA. This study concludes that the weight of the banana is dependent on the length and the radius of the banana.

## Introduction

The weight of a banana is difficult to predict since it depends on density and volume, which are not easily measurable. In this study it was assumed that the density of the bananas remained consistent throughout the entire banana and that volume was approximately

$$V = \pi r^2 * l$$

, where  $r$  was the radius of the body and  $l$  was the curved length.

This report states the data collection methods, the different methods and techniques used for creating regression models, the analysis of the regression models and the recommendations for the best method to predict banana weight. For the purpose of this study, it is assumed  $\alpha = 0.05$  for all tests done.

## Data Collection

First a small sample set of 24 bananas were purchased from the Real Canadian Superstore. The weight, length, diameter and circumference were then calculated using a scale and a ruler. This data was recorded using an Excel spreadsheet for further analysis.

Table 1: Summary Statistics for Simulated R-Squared

Min.	1st Qu.	Median	Mean	3rd Qu.	Max.
0.0004513	0.4005	0.6352	0.5719	0.7617	0.9812

In order to determine the minimum sample size needed, random sample sizes of 10 were generated using radius and length as the predictors. The correlation of the random sample sizes were calculated and a matrix of the correlations were generated. The value of the squared population multiple correlation coefficients with two predictor variables was then calculated and determined to be approximately 0.5719. From this the minimum sample size required was then determined from the table from Gregory T. Knochinski's Sample Size When Using Multiple Linear Regression for Prediction, the minimum sample size was determined to be between 15 and 35, therefore the minimum number of bananas required was finalized at 24 bananas.

# Analysis

## Preliminary Analysis

To begin the analysis the summary statistics of all predictor variables were calculated and recorded in Table 2. The summary statistics were used to highlight the data to obtain a general understanding of the spread for each predictor variable.

Next graphs of the sampling distributions of the predictor variables were used to visualize the trends of the data in Figure 1, and were examined for skewness and normality. Length appears to be highly skewed to the left, weight appears to be more normally distributed circumference and radius are very similarly distributed with potential bimodal distribution.

Table 2: Banana Summary Statistics

Statistic	Weight	Radius	Length	Circumference
Min.	157.0000	18.30000	128.0000	115.0000
1st Qu.	179.5000	18.90000	243.2500	119.0000
Median	193.5000	19.30000	256.5000	121.0000
Mean	193.7917	19.50417	250.2083	122.5417
3rd Qu.	203.2500	20.20000	268.5000	127.0000
Max	231.0000	20.50000	283.0000	129.0000

Figure 01: Sample Distributions of Banana Data

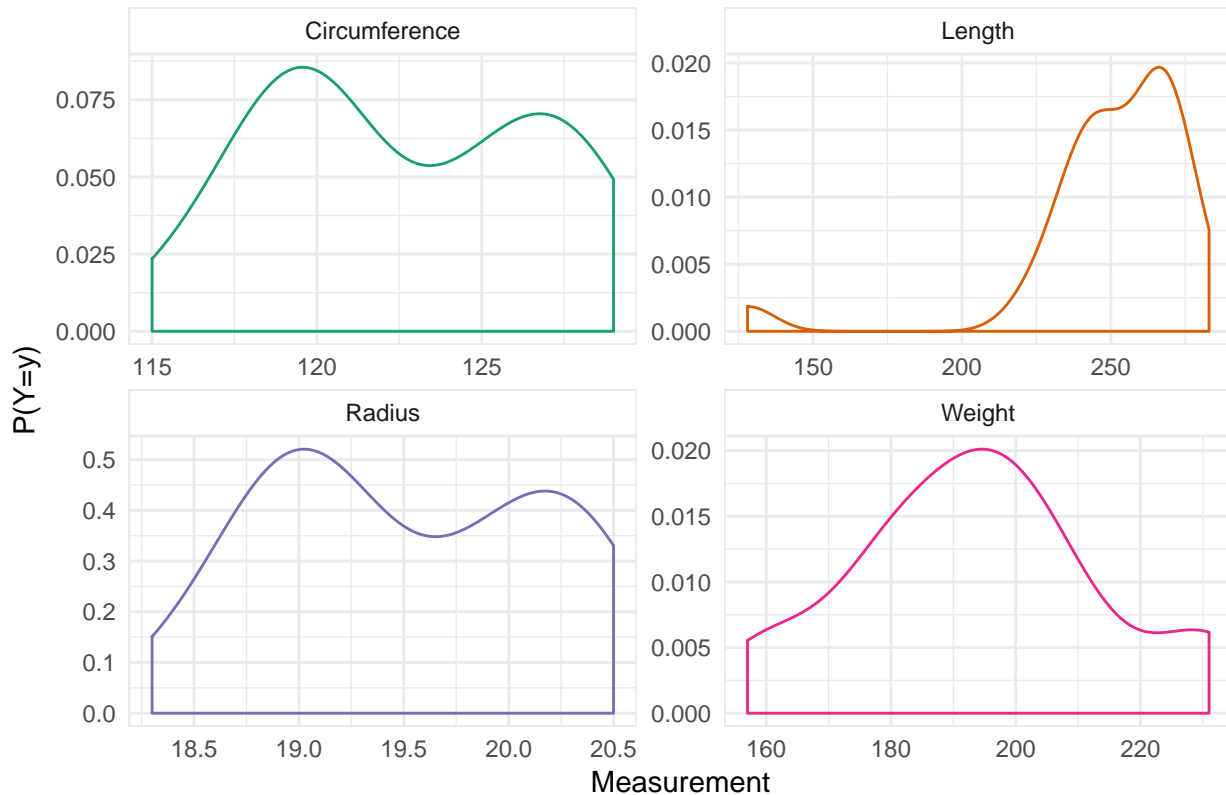
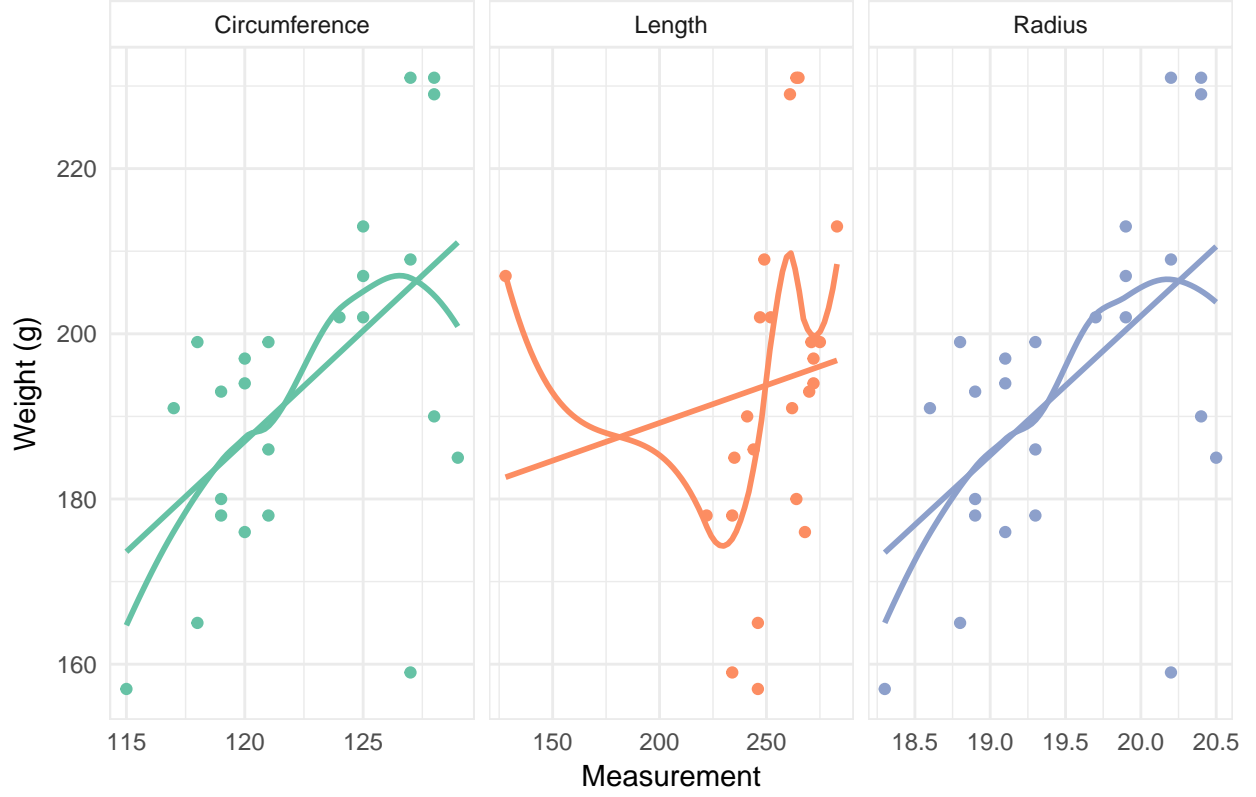


Figure 02: Weight vs. Predictors



## Initial Regression Models

To begin analysis, a model using all predictor variables was created. In this case the density of the banana is assumed to be a constant. In the following models all measured bananas were considered.

Let:

$$W = \text{Weight (g)}, L = \text{Length (mm)}, R = \text{Radius (mm)}, C = \text{Circumference (mm)}$$

Then:

$$\ln(W) = \beta_0 + \beta_1 \ln(L) + \beta_2 \ln(R) + \beta_3 \ln(C) \implies W = e^{\beta_0} \times L^{\beta_1} \times R^{\beta_2} \times C^{\beta_3} \quad (1)$$

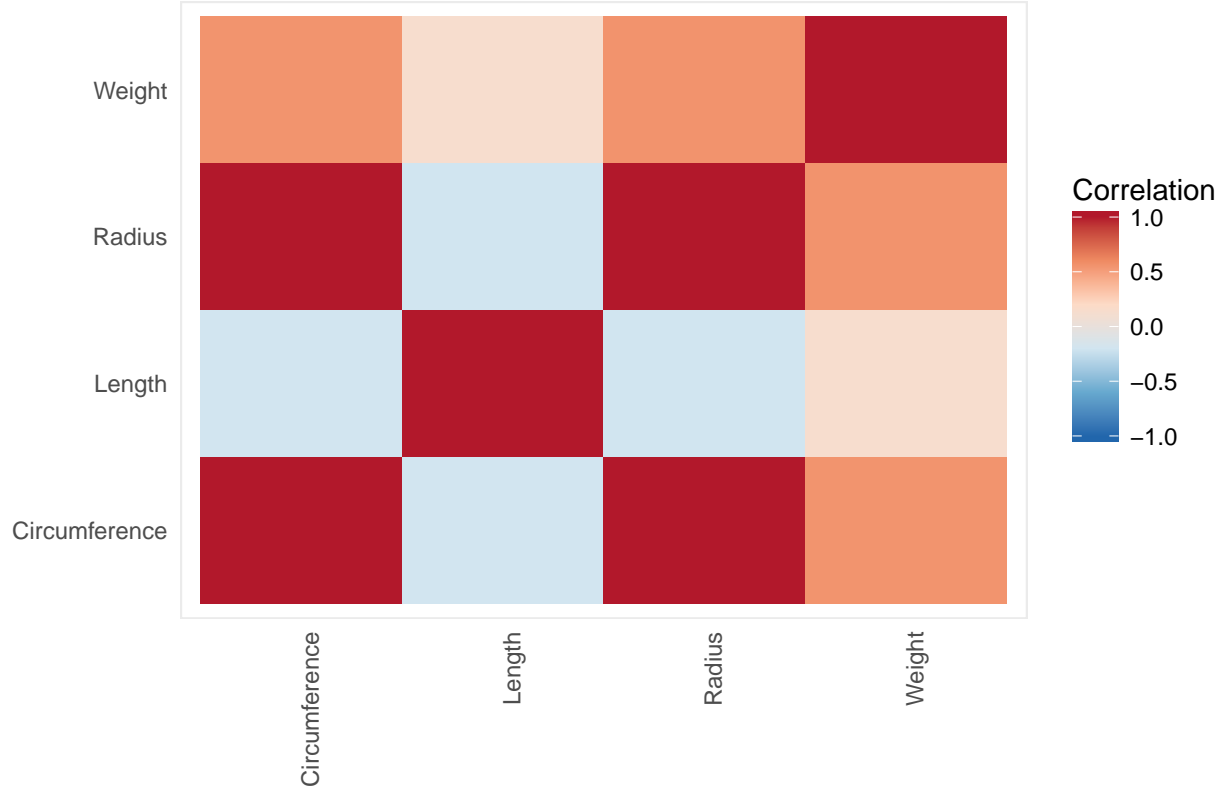
	Estimate	Std. Error	t value	Pr(> t )
<b>(Intercept)</b>	10.06	26.32	0.3822	0.7063
<b>Length_log</b>	0.123	0.1275	0.9652	0.346
<b>Radius_log</b>	7.526	14.09	0.5341	0.5992
<b>Circumference_log</b>	-5.788	14.16	-0.4088	0.687

Table 4: Fitting linear model:  $\text{Weight\_log} \sim \text{Length\_log} + \text{Radius\_log} + \text{Circumference\_log}$

Observations	Residual Std. Error	$R^2$	Adjusted $R^2$
24	0.09248	0.3318	0.2316

None of the predictor variables were found to be statistically significant. This is due to the high degree of collinearity exhibited between Radius and Circumference. As such, in the second model, the predictor variable, circumference, was removed. This is because  $C = 2\pi R$ , leading to collinearity. This can also be seen by examining the correlation plot produced by the variables:

**Figure 03: Correlation Plot**



The second model considered only the predictor variables length and radius:

$$\ln(W) = \beta_0 + \beta_1 \ln(L) + \beta_2 \ln(R) \quad (2)$$

	Estimate	Std. Error	t value	Pr(> t )
<b>(Intercept)</b>	-0.6702	1.913	-0.3503	0.7296
<b>Length_log</b>	0.1223	0.1249	0.9787	0.3389
<b>Radius_log</b>	1.77	0.5596	3.163	0.004684

Table 6: Fitting linear model:  $\text{Weight\_log} \sim \text{Length\_log} + \text{Radius\_log}$

Observations	Residual Std. Error	$R^2$	Adjusted $R^2$
24	0.09062	0.3262	0.2621

The third model considered the predictor, length.

$$\ln(W) = \beta_0 + \beta_1 \ln(L) \quad (3)$$

	Estimate	Std. Error	t value	Pr(> t )
<b>(Intercept)</b>	4.99	0.8037	6.209	3e-06
<b>Length_log</b>	0.04917	0.1457	0.3374	0.739

Table 8: Fitting linear model: Weight\_log ~ Length\_log

Observations	Residual Std. Error	$R^2$	Adjusted $R^2$
24	0.1076	0.005146	-0.04007

The fourth model considered only one predictor, radius.

$$\ln(W) = \beta_0 + \beta_2 \ln(R) \quad (4)$$

	Estimate	Std. Error	t value	Pr(> t )
<b>(Intercept)</b>	0.3046	1.632	0.1867	0.8536
<b>Radius_log</b>	1.669	0.5494	3.038	0.006043

Table 10: Fitting linear model: Weight\_log ~ Radius\_log

Observations	Residual Std. Error	$R^2$	Adjusted $R^2$
24	0.09054	0.2955	0.2635

ANOVAs were then done to determine if a statistically significant difference in levels of explained variation existed between the 4 models. 1st, 01 and 02 were compared, testing the effect of Circumference:

Table 11: Analysis of Variance Table: Model 01 vs. Model 02

Res.Df	RSS	Df	Sum of Sq	F	Pr(>F)
21	0.1725	NA	NA	NA	NA
20	0.171	1	0.001429	0.1671	0.687

Based on the data, we failed to reject the null hypothesis of equal explained variance between the two models. As such, we believe the model 02 is more appropriate, as it is smaller and more parsimonious.

The following test compares models 02 and 03, testing the effect of Radius:

Table 12: Analysis of Variance Table: Model 02 vs. Model 03

Res.Df	RSS	Df	Sum of Sq	F	Pr(>F)
22	0.2547	NA	NA	NA	NA
21	0.1725	1	0.08219	10.01	0.004684

Based on the data, we rejected the null hypothesis of equal explained variance between the two models. As such, we believe the model 02 is more appropriate, as it does a better job at explaining the underlying

variance in the data.

The following test compares models 02 and 03, testing the effect of Length:

Table 13: Analysis of Variance Table: Model 02 vs. Model 04

Res.Df	RSS	Df	Sum of Sq	F	Pr(>F)
22	0.1803	NA	NA	NA	NA
21	0.1725	1	0.007867	0.9578	0.3389

Based on the data, we failed to reject the null hypothesis of equal explained variance between the two models. As such, we believe the model 04 is more appropriate, as it is smaller and more parsimonious.

From this, we found the most appropriate model was model 04, as it is the smallest model that explains an adequate amount of variance. This model being:

$$\ln(W) = 0.3046 + 1.6690 \ln(R) \implies W = e^{0.3046} R^{1.6690}$$

This is an unexpected result, as it doesn't include the Length predictor, implying that the weight of a banana is purely a function of its radius. Due to this, further analysis was required.

## Removal of Outliers

Due to the surprising results obtained above, the choice to check for potential outliers was made. For this, model 02 was chosen, as we wished to examine the potential presence of outlier with respect to both Length and Radius, which model 04 would fail to accomplish.

Figure 04: Standardized Residuals vs. Predicted for Model 02

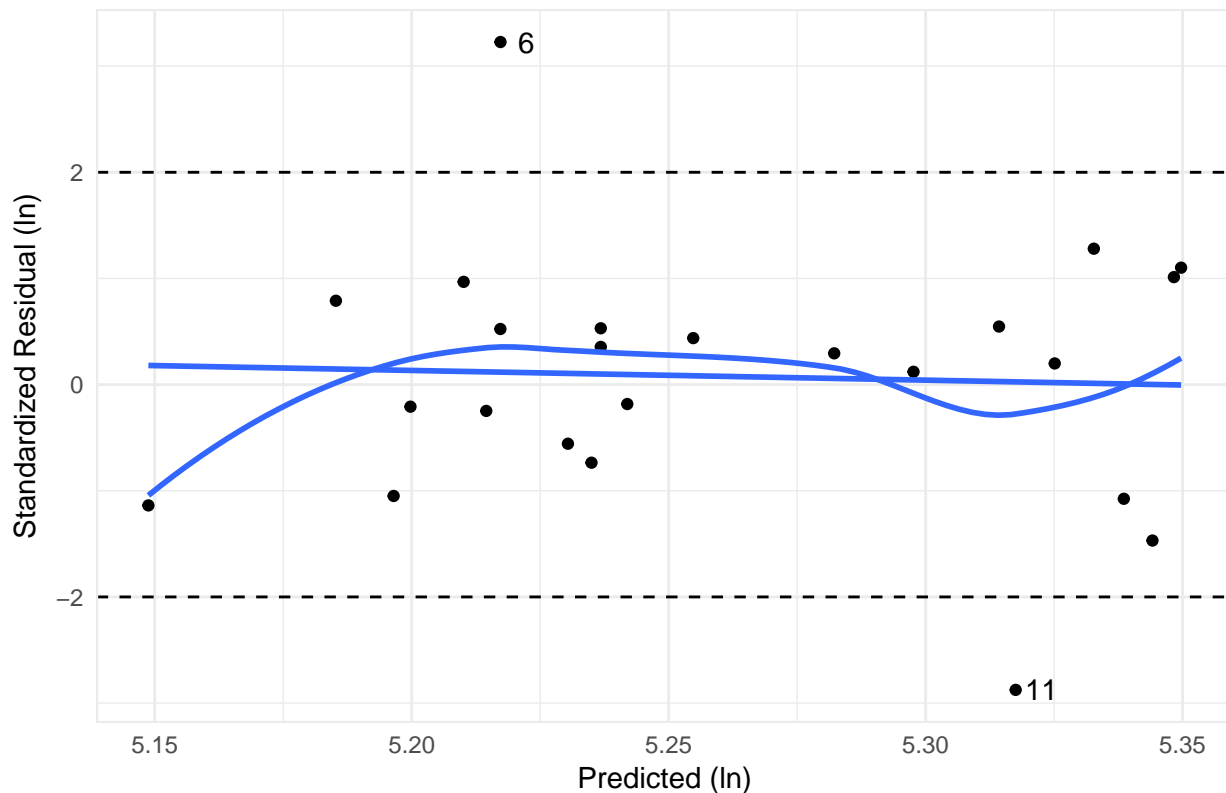


Figure 05: Standardized Residuals vs. Leverage for Model 02

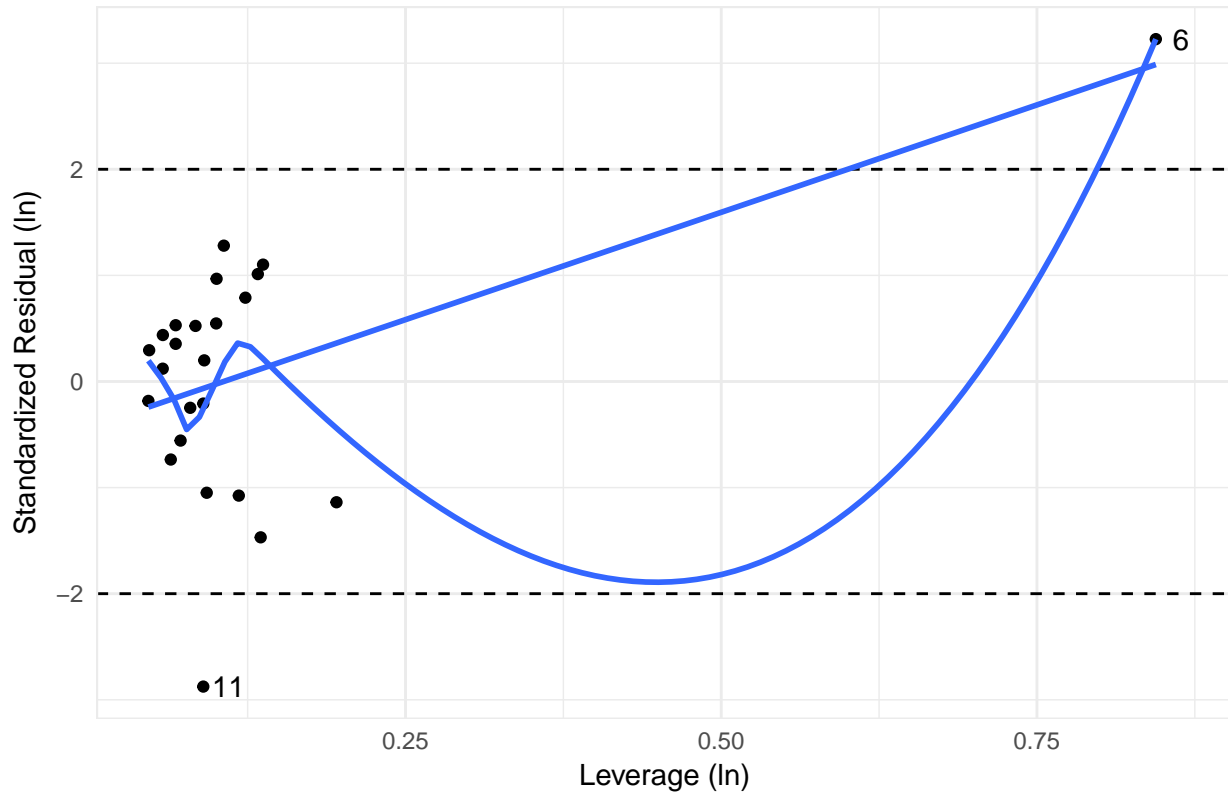


Table 14: Entries with a  $|\text{Standardized Residual}| > 2$

ID	Weight	Radius	Length	Circumference	Std Residuals	Leverage
6	207	19.9	128	125	3.226762	0.8441502
11	159	20.2	234	127	-2.875567	0.0898602

By looking at values with  $|\text{Std. Residual}| > 2$ , we can identify potential outliers. Then, by examining the leverage of those outliers we can make a determination of whether or not they are severely distorting the regression line from the true slope. Although both 6 and 11 have a large standardized residual, only 6 has a large leverage associated with it, and as such is excluded from the dataset.

## Cross Validation

```
## Analysis of Variance Table
##
## Response: Weight_log
##           Df Sum Sq Mean Sq F value Pr(>F)
## Length_log  1  0.0643   0.0643    14.8  0.00101 **
## Radius_log   1  0.0994   0.0994    22.9  0.00011 ***
## Residuals   20  0.0870   0.0043
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
##
```

```

## fold 1
## Observations in test set: 7
##      [,1]    [,2]    [,3]    [,4]    [,5]    [,6]    [,7]
## Predicted  5.2865  5.2865  5.111  5.098  5.3693  5.235  5.268
## cvpred     5.2964  5.2964  5.059  5.026  5.3739  5.232  5.242
## Weight_log  5.2832  5.2679  5.182  5.182  5.4424  5.193  5.308
## CV residual -0.0132 -0.0285  0.123  0.155  0.0685 -0.039  0.066
##
## Sum of squares = 0.05    Mean square = 0.01    n = 7
##
## fold 2
## Observations in test set: 8
##      [,1]    [,2]    [,3]    [,4]    [,5]    [,6]    [,7]    [,8]
## Predicted  5.19520  5.24  5.0995  5.275  5.30308  5.291  5.2668  5.3053
## cvpred     5.23493  5.34  5.0927  5.381  5.28996  5.381  5.2254  5.3711
## Weight_log  5.22575  5.07  5.0562  5.220  5.29330  5.247  5.2933  5.3423
## CV residual -0.00918 -0.27 -0.0364 -0.161  0.00334 -0.134  0.0679 -0.0287
##
## Sum of squares = 0.12    Mean square = 0.02    n = 8
##
## fold 3
## Observations in test set: 8
##      [,1]    [,2]    [,3]    [,4]    [,5]    [,6]    [,7]    [,8]
## Predicted  5.152  5.4075  5.3729  5.3847  5.271  5.2685  5.1961  5.25829
## cvpred     5.163  5.3975  5.3550  5.3667  5.276  5.2629  5.2099  5.26643
## Weight_log  5.106  5.3613  5.4337  5.4424  5.170  5.3083  5.2523  5.26269
## CV residual -0.057 -0.0363  0.0787  0.0757 -0.105  0.0453  0.0423 -0.00374
##
## Sum of squares = 0.03    Mean square = 0    n = 8
##
## Overall (Sum over all 8 folds)
##      ms
## 0.00894

```

Table 15: CV Model (continued below)

ID	Weight	Radius	Length	Circumference	Weight_log	Radius_log
1	197	19.1	272	120	5.283	2.95
2	194	19.1	272	120	5.268	2.95
3	165	18.8	246	118	5.106	2.934
4	186	19.3	244	121	5.226	2.96
5	178	18.9	234	119	5.182	2.939
7	213	19.9	283	125	5.361	2.991
8	178	19.3	222	121	5.182	2.96
9	229	20.4	261	128	5.434	3.016
10	231	20.2	265	127	5.442	3.006
11	159	20.2	234	127	5.069	3.006
12	157	18.3	246	115	5.056	2.907
13	231	20.4	264	128	5.442	3.016
14	185	20.5	235	129	5.22	3.02
15	176	19.1	268	120	5.17	2.95
16	202	19.7	252	124	5.308	2.981
17	199	19.3	271	121	5.293	2.96
18	190	20.4	241	128	5.247	3.016



ID	Weight	Radius	Length	Circumference	Weight_log	Radius_log
19	180	18.9	264	119	5.193	2.939
20	191	18.6	262	117	5.252	2.923
21	202	19.9	247	125	5.308	2.991
22	193	18.9	270	119	5.263	2.939
23	199	18.8	275	118	5.293	2.934
24	209	20.2	249	127	5.342	3.006

Length_log	Circumference_log	Predicted	cvpred
5.606	4.787	5.286	5.296
5.606	4.787	5.286	5.296
5.505	4.771	5.152	5.163
5.497	4.796	5.195	5.235
5.455	4.779	5.111	5.059
5.645	4.828	5.408	5.398
5.403	4.796	5.098	5.026
5.565	4.852	5.373	5.355
5.58	4.844	5.369	5.374
5.455	4.844	5.241	5.338
5.505	4.745	5.099	5.093
5.576	4.852	5.385	5.367
5.46	4.86	5.275	5.381
5.591	4.787	5.271	5.276
5.529	4.82	5.269	5.263
5.602	4.796	5.303	5.29
5.485	4.852	5.291	5.381
5.576	4.779	5.235	5.232
5.568	4.762	5.196	5.21
5.509	4.828	5.268	5.242
5.598	4.779	5.258	5.266
5.617	4.771	5.267	5.225
5.517	4.844	5.305	5.371

**MAE**

Table 17: Calculated Error Terms for Log CV Model

MSE	MAE	MPAE
0.009	0.072	0.014

Table 18: Calculated Error Terms for CV Model

MSE	MAE	MPAE
317	13.7	0.073

## Recommendations

Using the first set of data before the outlier was removed, it can be determined that the best way to predict the weight of a banana is by measuring the radius of the banana. The model that is then used for banana weight prediction is the following:

$$\ln(W) = \beta_0 + \beta_1 \ln(R)$$
$$\ln(W) = 0.3046 + 1.669 \ln(R)$$

After the removal of the outlier, the model that was determined to be the best predictor for banana weight was the following:

## Appendix

### Appendix A: Code

```
library(pander, warn.conflicts = FALSE, quietly = TRUE)
library(MAAS, warn.conflicts = FALSE, quietly = TRUE)
library(DAAG, warn.conflicts = FALSE, quietly = TRUE)
library(tidyverse, warn.conflicts = FALSE, quietly = TRUE)
library(magrittr, warn.conflicts = FALSE, quietly = TRUE)
library(ggfortify, warn.conflicts = FALSE, quietly = TRUE)
library(knitr, warn.conflicts = FALSE, quietly = TRUE)

set.seed(5609)

theme_minimal2 <- theme_minimal() %>% theme_set()
theme_minimal2 <-
  theme_update(
    panel.border = element_rect(
      linetype = "solid"
      ,colour = "grey92"
      ,fill = NA
    )
    ,strip.background = element_rect(
      linetype = "solid"
      ,colour = "grey92"
      ,fill = NA
    )
  )

banana_data <-
  "mybanana.txt" %>%
  read_tsv()

banana_data <-
  banana_data %>%
  mutate_at(
    .vars = vars(Weight:Circumference)
    ,.funs = funs(log = log)
  )
```

```
banana_tidy <-  
  banana_data %>%  
  select(  
    -c(  
      Weight_log  
      ,Radius_log  
      ,Length_log  
      ,Circumference_log  
    )  
  ) %>%  
  gather(  
    key = "Type"  
    ,value = "Measurement"  
    ,-ID  
  )
```