Does Size Matter? (Estimation of Banana Weight with a Regression Modeling Approach)

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Summary

Introduction

The purpose of this study was to determine the most effective regression model to predict the weight of a banana using external measurements. This study also demonstrated multiple techniques for developing regression models. These models were then examined to demonstrate their effectiveness at creating regression models.

Data Collection

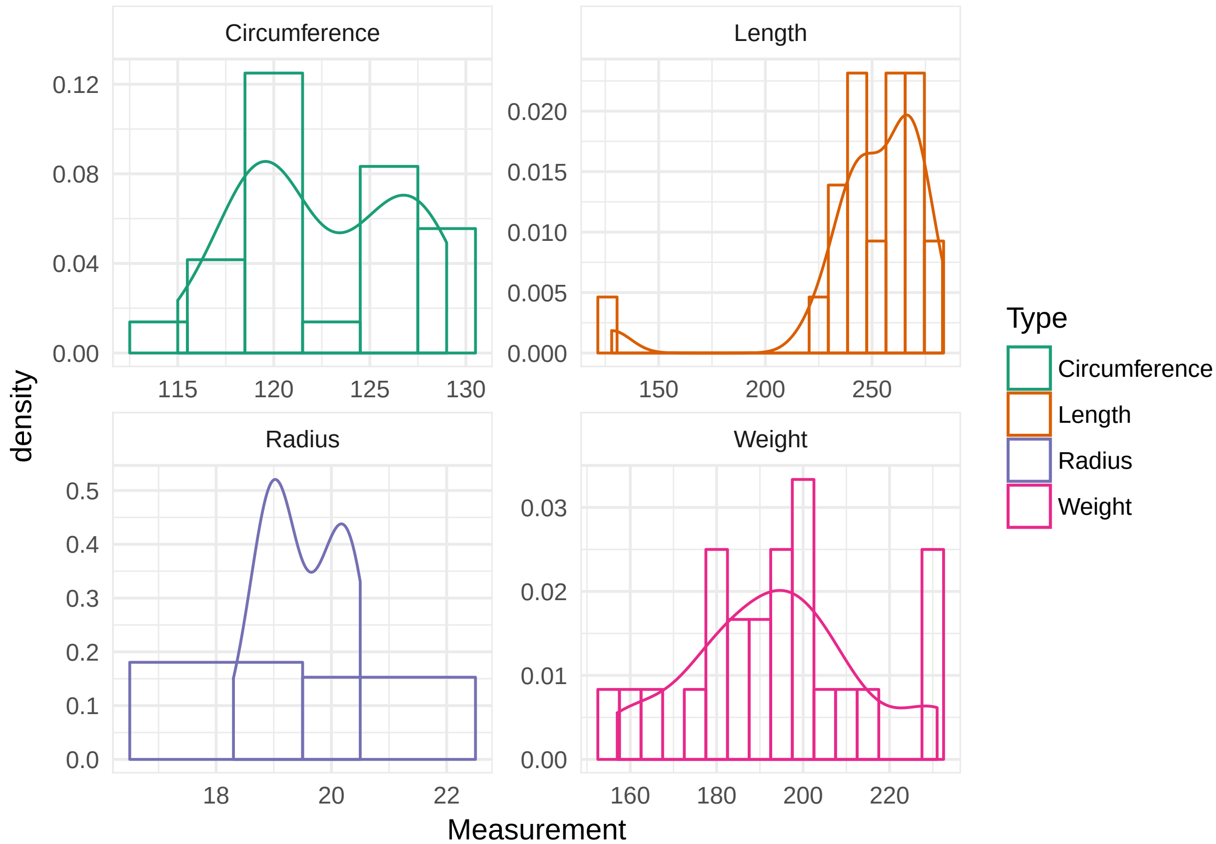
First a small sample set bananas were purchased from the Real Canadian Superstore. The weight, length, diameter and circumference were then calculated using a scale and a ruler.

Min.	1st Qu.	Median	Mean	3rd Qu.	Max.
0.1885	0.6148	0.713	0.7039	0.8061	0.9782

In order to determine the minimum sample size needed, random sample sizes of 10 were generated using radius and length as the predictors. The correlation of the random sample sizes were calculated and a matrix of the correlations were generated. The value of the squared population multiple correlation coefficients with two predictor variables was then calculated and determined to be approximately 0.7039. From this the minimum sample size required was then determined from the table from Gregory T. Knofcznski's Sample Size When Using Multiple Linear Regression for Prediction, the minimum sample size was determined to be between 15 and 35, therefore the minimum number of bananas required was finalized at 24 bananas.

Analysis

Statistic	Weight	Radius	Length	Circumference
Min.	157.0000	18.30000	128.0000	115.0000
1st Qu.	179.5000	18.90000	243.2500	119.0000
Median	193.5000	19.30000	256.5000	121.0000
Mean	193.7917	19.50417	250.2083	122.5417
3rd Qu.	203.2500	20.20000	268.5000	127.0000
Max	231.0000	20.50000	283.0000	129.0000



To begin analysis, a model using all predictor variables was created. In this case the density of the banana is assumed to be a constant. In the following models all measured bananas were considered.

Let:

$$W = \text{Weight (g)},\ L = \text{Length (mm)},\ R = \text{Radius (mm)}$$

Then:

$$\log(W) = \beta_0 + \beta_1 \log(L) + \beta_2 \log(R) + \beta_3 \log(C) \implies W = e^{\beta_0} \times L^{\beta_1} \times R^{\beta_2} \times C^{\beta_3}$$

	Estimate	Std. Error	t value	Pr(>|t|)
(Intercept)	10.06	26.32	0.3822	0.7063
Length_log	0.123	0.1275	0.9652	0.346
Radius_log	7.526	14.09	0.5341	0.5992
Circumference_log	-5.788	14.16	-0.4088	0.687

Table 4: Fitting linear model: Weight_log ~ Length_log + Radius_log + Circumference_log

Observations	Residual Std. Error	R^2	Adjusted R^2
24	0.09248	0.3318	0.2316

In the second model the predictor variable, circumference, was removed. This is because $C = 2\pi R$.

$$\log(W) = \beta_0 + \beta_1 \log(L) + \beta_2 \log(R)$$

	Estimate	Std. Error	t value	Pr(>|t|)
(Intercept)	-0.6702	1.913	-0.3503	0.7296
Length_log	0.1223	0.1249	0.9787	0.3389
Radius_log	1.77	0.5596	3.163	0.004684

Table 6: Fitting linear model: Weight_log ~ Length_log + Radius_log

Observations	Residual Std. Error	R^2	Adjusted R^2
24	0.09062	0.3262	0.2621

The third model considered the predictor, length.

$$\log(W) = \beta_0 + \beta_1 \log(L)$$

	Estimate	Std. Error	t value	Pr(>|t|)
(Intercept)	4.99	0.8037	6.209	3e-06
Length_log	0.04917	0.1457	0.3374	0.739

Table 8: Fitting linear model: Weight_log ~ Length_log

Observations	Residual Std. Error	R^2	Adjusted R^2
24	0.1076	0.005146	-0.04007

The fourth model considered only one predictor, radius.

$$\log(W) = \beta_0 + \beta_2 \log(R)$$

	Estimate	Std. Error	t value	Pr(>|t|)
(Intercept)	0.3046	1.632	0.1867	0.8536
Radius_log	1.669	0.5494	3.038	0.006043

Table 10: Fitting linear model: Weight_log ~ Radius_log

Observations	Residual Std. Error	R^2	Adjusted R^2
24	0.09054	0.2955	0.2635

Table 11: Analysis of Variance Table: Model 1 vs. Model 2

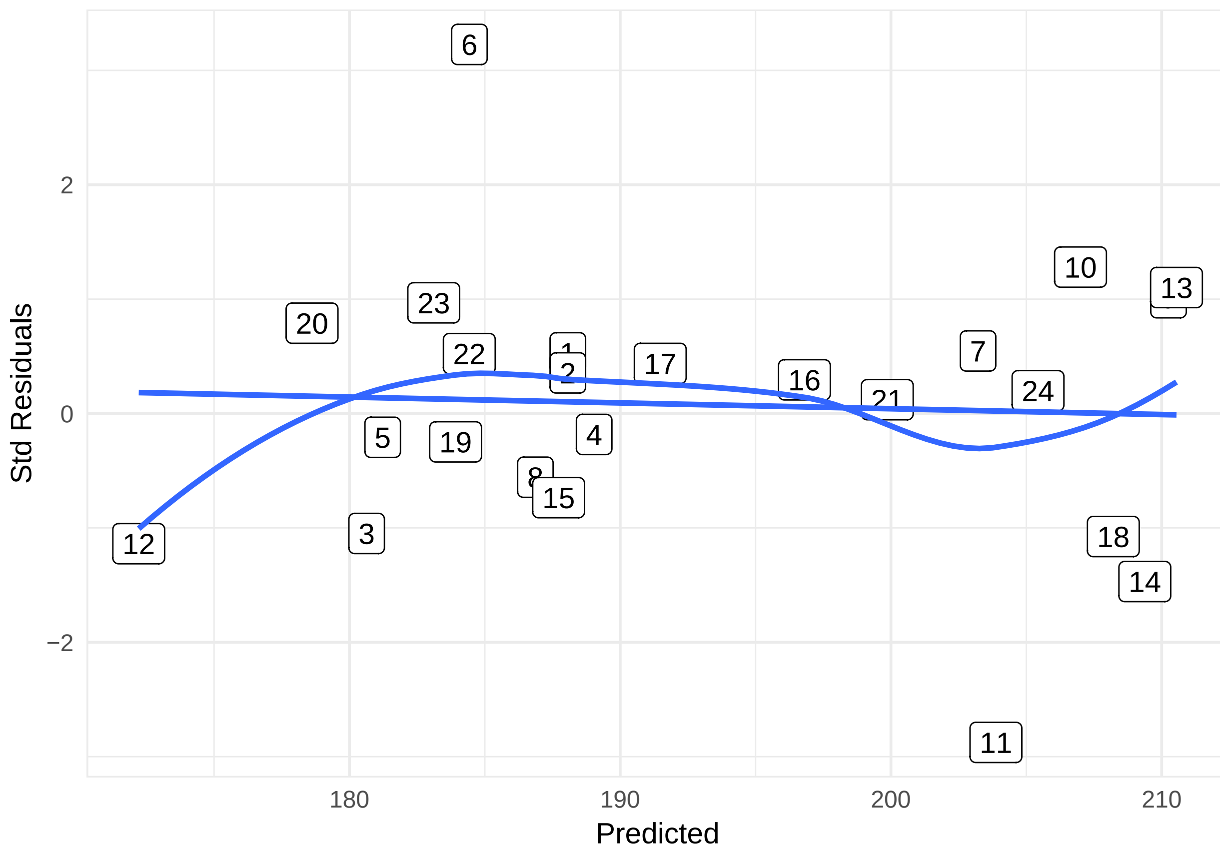
Res.Df	RSS	Df	Sum of Sq	F	Pr(>F)
21	0.1725	NA	NA	NA	NA
20	0.171	1	0.001429	0.1671	0.687

Table 12: Analysis of Variance Table: Model 1 vs. Model 3

Res.Df	RSS	Df	Sum of Sq	F	Pr(>F)
22	0.2547	NA	NA	NA	NA
20	0.171	2	0.08362	4.889	0.01868

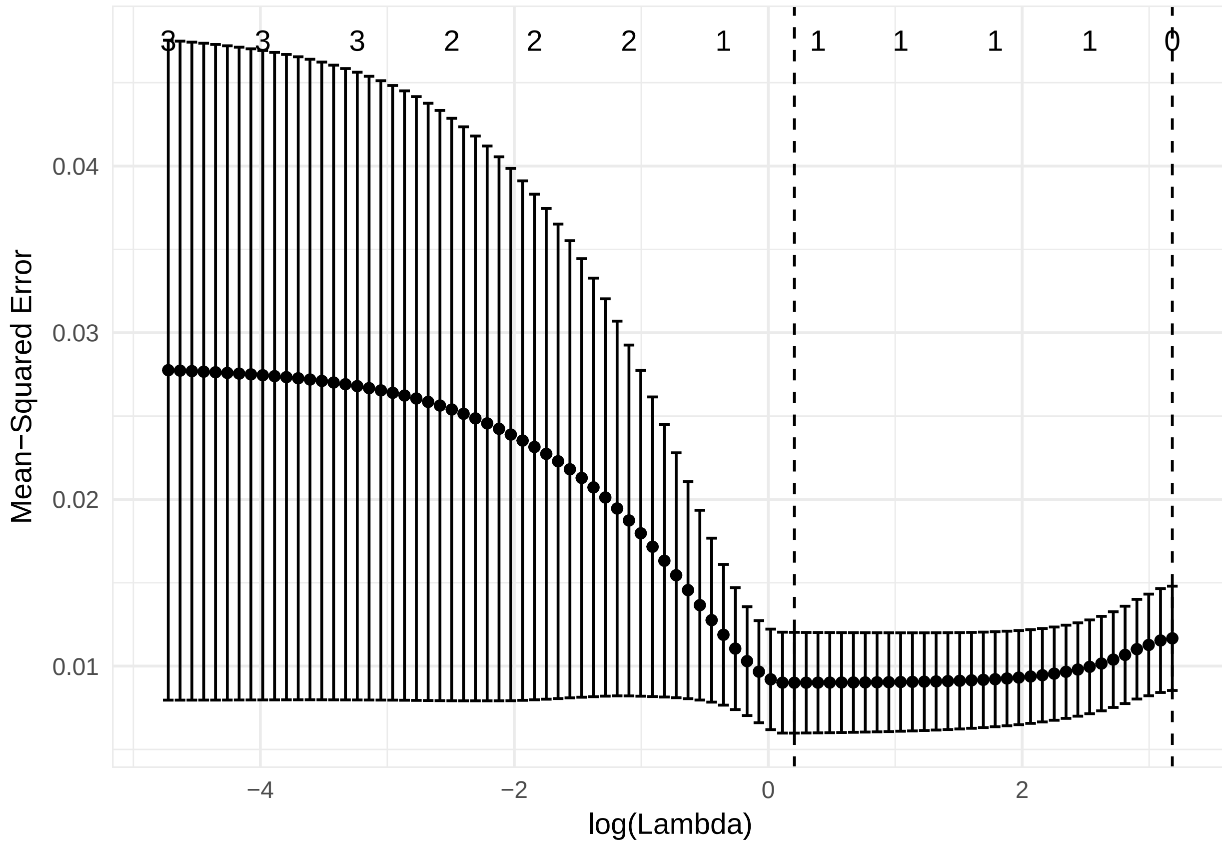
Table 13: Analysis of Variance Table: Model 2 vs. Model 3

Res.Df	RSS	Df	Sum of Sq	F	Pr(>F)
22	0.2547	NA	NA	NA	NA
21	0.1725	1	0.08219	10.01	0.004684




```
## Warning: Option grouped=FALSE enforced in cv.glmnet, since < 3 observations
## per fold

## Warning: Option grouped=FALSE enforced in cv.glmnet, since < 3 observations
## per fold
```



	1
(Intercept)	0.55713
Length	0.00000
Radius	0.00000
Circumference	0.00000
Length_log	0.00000
Radius_log	1.58392
Circumference_log	0.00000

	1
(Intercept)	5.261471
Length	0.000000
Radius	0.000000
Circumference	0.000000
Length_log	0.000000
Radius_log	0.000000
Circumference_log	0.000000

Removal of Outliers

The second part of the analysis included removing any potential outliers from the analysis. In this case, one outlier was removed.

Cross Validation

```
## Analysis of Variance Table
##
## Response: Weight_log
##            Df Sum Sq Mean Sq F value Pr(>F)
## Length_log  1 0.0013  0.0013    0.16 0.6928
## Radius_log  1 0.0822  0.0822   10.01 0.0047 **
## Residuals  21 0.1725  0.0082
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
##
## fold 1
## Observations in test set: 8
##               [,1]   [,2]   [,3]    [,4]   [,5]    [,6]   [,7]   [,8]
## Predicted   5.2368 5.2368  5.196  5.1998 5.3143  5.2304  5.344  5.215
## cvpred      5.2428 5.2428  5.207  5.2131 5.3184  5.2464  5.358  5.222
## Weight_log  5.2832 5.2679  5.106  5.1818 5.3613  5.1818  5.220  5.193
## CV residual 0.0404 0.0251 -0.101 -0.0314 0.0429 -0.0646 -0.137 -0.029
##
## Sum of squares = 0.04    Mean square = 0    n = 8
##
## fold 2
## Observations in test set: 8
##                 [,1]  [,2]  [,3]    [,4]   [,5]    [,6]   [,7]   [,8]
## Predicted    5.24195 5.348 5.333  5.2350 5.2548  5.3386 5.1853 5.3251
## cvpred       5.23389 5.324 5.310  5.2248 5.2417  5.3184 5.1822 5.3056
## Weight_log   5.22575 5.434 5.442  5.1705 5.2933  5.2470 5.2523 5.3423
## CV residual -0.00814 0.109 0.132 -0.0543 0.0516 -0.0714 0.0701 0.0367
##
## Sum of squares = 0.05    Mean square = 0.01    n = 8
##
## fold 3
## Observations in test set: 8
##             [,1]   [,2]    [,3]   [,4]   [,5]   [,6]   [,7]   [,8]
## Predicted   5.22  5.318  5.1488 5.3497 5.2822 5.2976 5.2173 5.2101
## cvpred      4.78  5.267  5.1137 5.3771 5.2744 5.2788 5.2464 5.2500
## Weight_log  5.33  5.069  5.0562 5.4424 5.3083 5.3083 5.2627 5.2933
## CV residual 0.55 -0.198 -0.0574 0.0654 0.0339 0.0295 0.0163 0.0433
##
## Sum of squares = 0.35    Mean square = 0.04    n = 8
##
## Overall (Sum over all 8 folds)
##     ms
## 0.0183
```

```
## # A tibble: 24 x 13
##       ID Weight Radius Length Circumference Weight_log Radius_log
##    <int>  <int>  <dbl>  <int>         <int>      <dbl>      <dbl>
##  1     1    197   19.1    272           120       5.28       2.95
##  2     2    194   19.1    272           120       5.27       2.95
##  3     3    165   18.8    246           118       5.11       2.93
##  4     4    186   19.3    244           121       5.23       2.96
```

```
##  5     5    178   18.9    234            119       5.18       2.94
##  6     6    207   19.9    128            125       5.33       2.99
##  7     7    213   19.9    283            125       5.36       2.99
##  8     8    178   19.3    222            121       5.18       2.96
##  9     9    229   20.4    261            128       5.43       3.02
## 10    10    231   20.2    265            127       5.44       3.01
## # ... with 14 more rows, and 6 more variables: Length_log <dbl>,
## #   Circumference_log <dbl>, Predicted <dbl>, cvpred <dbl>, `CV
## #   Residual` <dbl>, Residual <dbl>
```

MAE

```
## # A tibble: 1 x 2
##      MAE   MPAE
##    <dbl>  <dbl>
## 1  13.7 0.0722
```

Recommendations

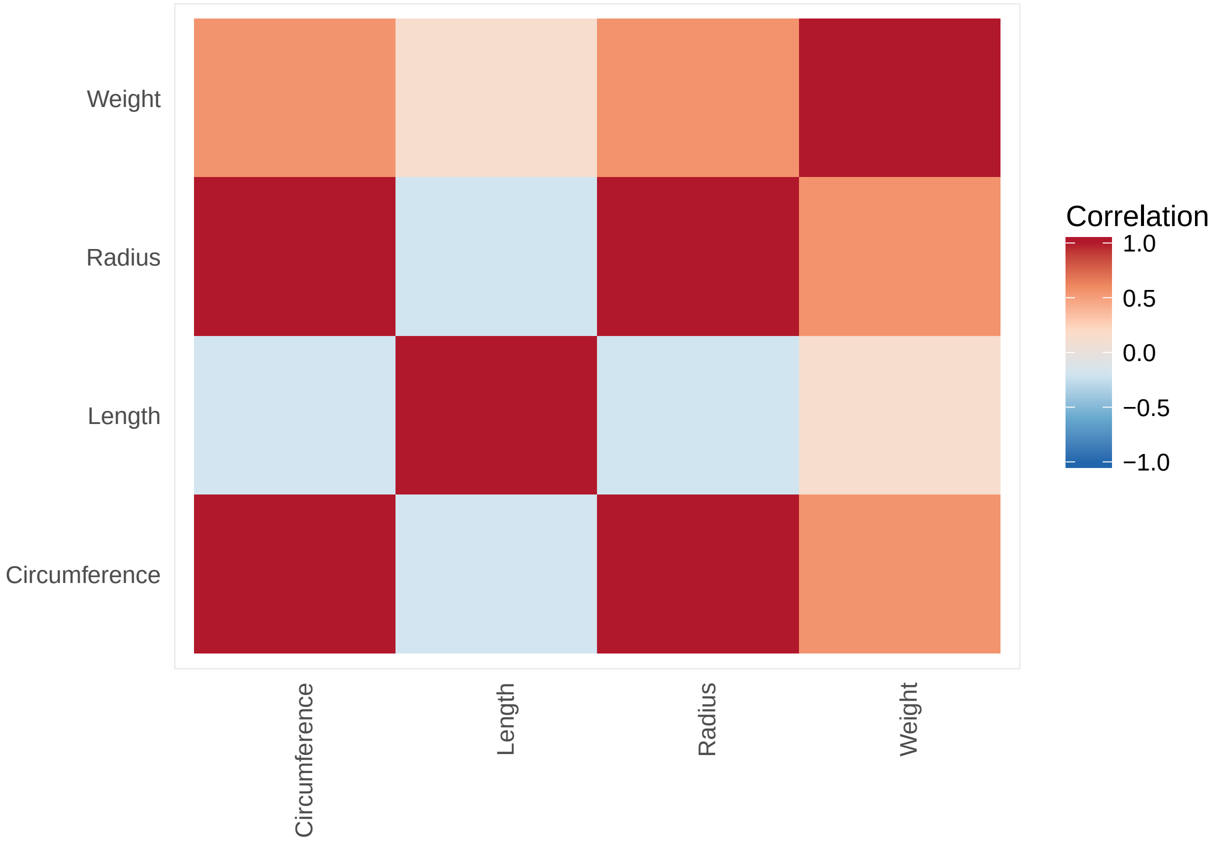
Using the first set of data before the outlier was removed, it can be determiend that the best way to predict the weight of a banana is by measuring the radius of the banana. The model that is then used for banana weight prediction is the following:

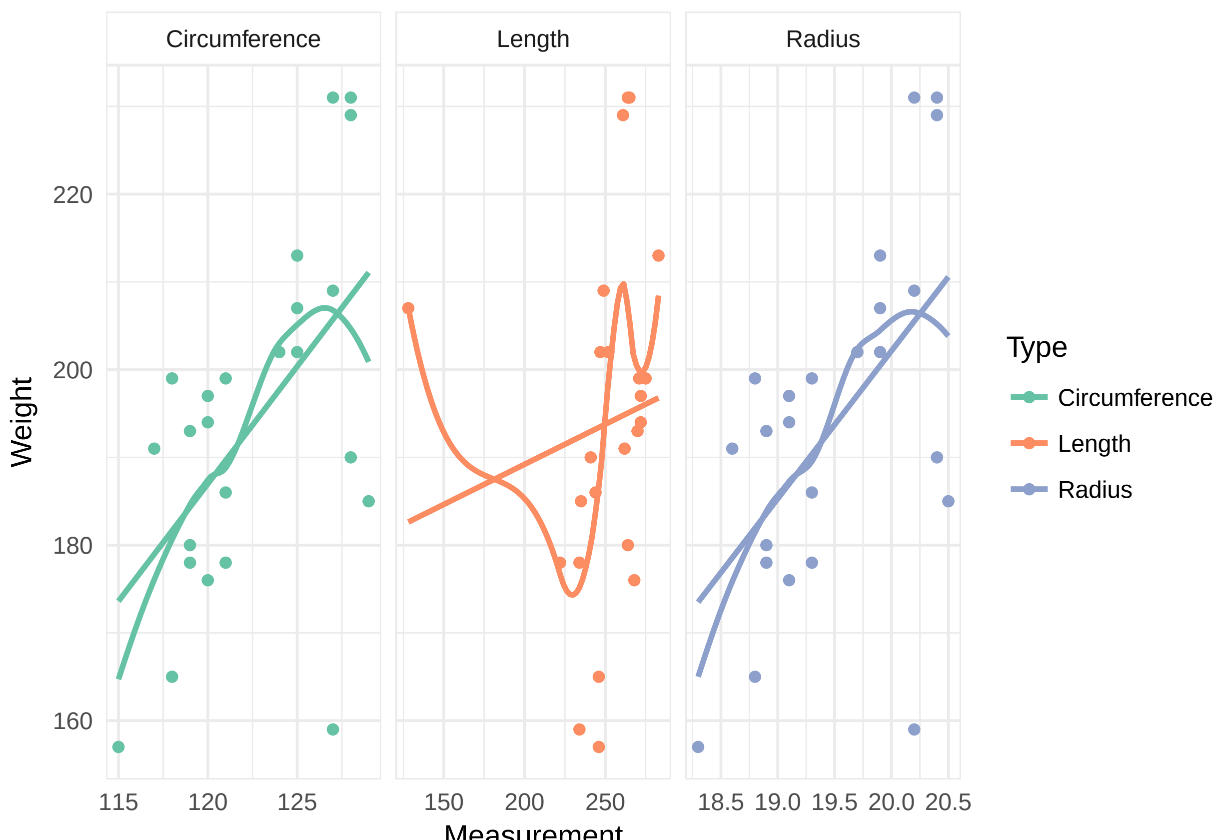
$$\log(W) = \beta_0 + \beta_1 \log(R)$$

$$\log(W) = 0.3046 + 1.669 \log(R)$$

After the removal of the outlier, the model that was determined to be the best predictor for banana weight was the following:

Appendix


Weight
Radius
Length
Circumference
Circumference
Length
Radius
Weight
Correlation
1.0
0.5
0.0
−0.5
−1.0


Circumference
Length
Radius
Weight
220
200
180
160
115
120
125
150
200
250
18.5
19.0
19.5
20.0
20.5
Measurement
Type
Circumference
Length
Radius