Leafs

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Introduction

The intent of this paper is to develop a method for classifying leaves as either Cherry or Pear, based on their measured length and width. This method was developed for Dr. Steven Vamosi, a botanist from the University of Calgary.

The classification method used was Linear discriminant analysis, developed by R.A Fischer. To . . . . To take training samples from a sampled population take measurements, from these measurements classification rules are created. This will then be tested against the classifying sample to see if there are any miss classifications.

Cherry and Pear leaves are both leaves from fruit trees. Cherry trees belong to the genus Prunus and Pear trees belong to the genus Pyrus [2],[3]. A common feature amongst the leaves is that they both have a midrib, which is the central vein of the leaf which extends along the leaf's center line.

Data

Measurement Process

The first step taken in the measurement of the leaves was to give each leaf an identification number based on the species. The method used to measure the dimensions was to create a box with the minimum length and width in which the entire leaf would be encompassed in the box.

To begin creating the sides of the box, a ruler was aligned parallel to the midrib, which is the central vein in the leaf and moved towards the left and the right of the picture until only one point on the leaf remained [1]. From the single point on the side of the leaf, a line was drawn parallel to the midrib of the leaf.

Next, the base and point of the leaf were measured, a ruler was placed perpendicular to the midrib and the ruler was moved towards to tip of the leaf until a single point remained, a line was draw perpendicular to the midrib at this point. At the base of the leaves the length of the leaf was set as the point where the leaf ends and the stem begins, at this point a line was drawn perpendicular to the midrib.

After all the boxes were created, the width (lines parallel to midrib) and the length (lines perpendicular to midrib) were measured and the results were recorded in a spread sheet.

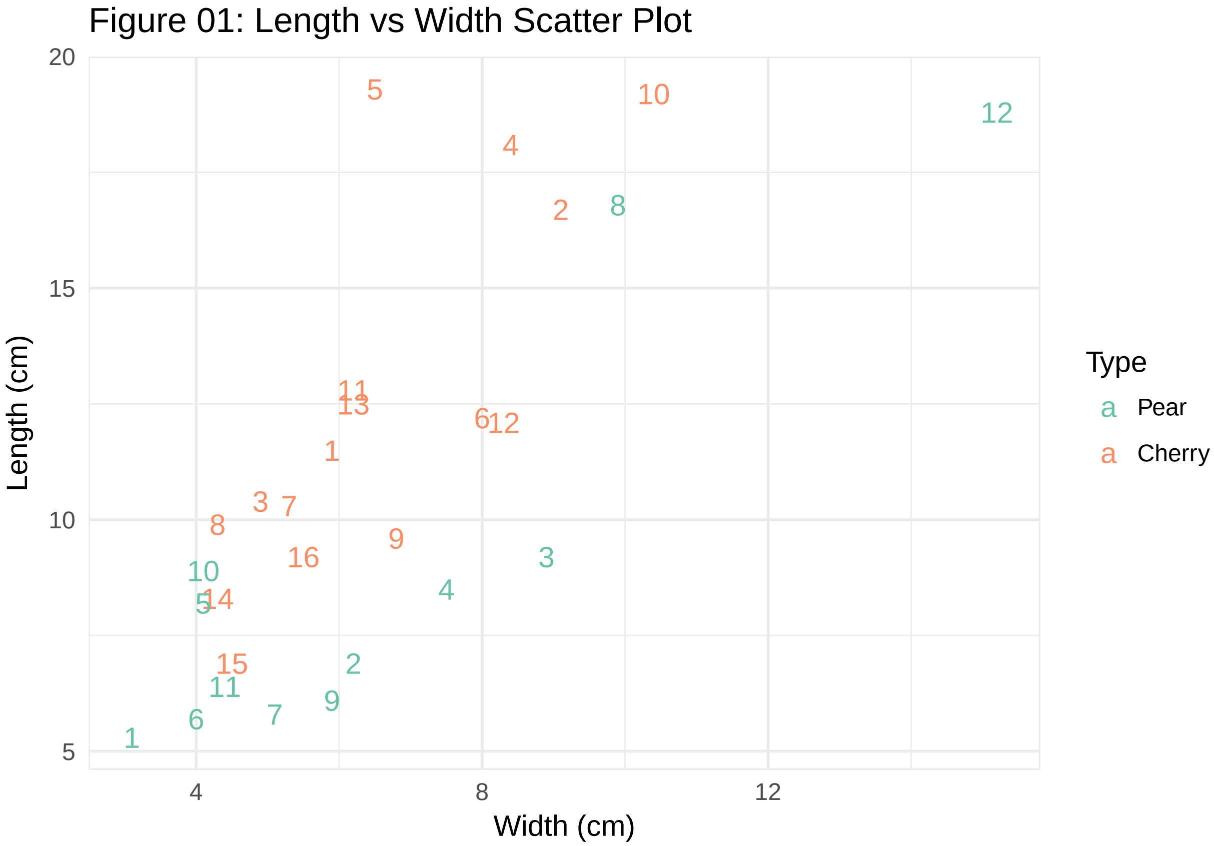
Data Creation

Table 1: Data Summary

Type	Length	Width
Pear :12	Min. : 5.300	Min. : 3.100
Cherry:16	1st Qu.: 7.875	1st Qu.: 4.475

Type	Length	Width
NA	Median : 9.750	Median : 6.050
NA	Mean :10.914	Mean : 6.536
NA	3rd Qu.:12.575	3rd Qu.: 8.075
NA	Max. :19.300	Max. :15.200

In this original data set there are a few issues that need to be acknowledged. The first issues that occurred during the data measurements was the result of the leaves that were distributed as the training sample were images, in which the images were not to scale. This resulted in a few outlines, which much larger lengths and widths compared to the other leaves in the set. These outlines included Pear#12, Cherry#10 and Cherry#5. However, based on the nature of this project in just observing the ratio between the length and width, this should not be affected by the size of the image, unless the image was stretched in either direction.



In Figure 01 there is a distinct separation in the data of the cherry and the pear. As mentioned above, the outlines are Cherry#10, Cherry#5 and Pear#12 these outlines appear to follow a similar grouping and therefore they were kept in the data set. The raw data is located in Appendix A.

Classification Procedure (LDA)

Training Data

Table 2: LDA Prior Probabilities

Type	Probability
Pear	0.4286
Cherry	0.5714

Table 3: LDA Group Means

	Length	Width
Pear	8.8833	6.5333
Cherry	12.4375	6.5375

Table 4: LDA Coefficients of Linear Discriminants

Dimension	Coefficient
Length	0.4194
Width	-0.5311

Tables 2-4 represent the output of a linear discriminant analysis (LDA) done on the raw data. Table 2 represents the prior probabilities of falling in a particular type. The prior probability of being a Pear leaf is 0.4286. The prior probability of being a Cherry leaf is 0.5714.

NOTE INCLUDE LDA COEFFICIENT MEANINGS

Table 5: LDA Misclassification Results

Predicted	Actual	Length	Width	Cherry Probability	Pear Probability	Correct Prediction
Pear	Cherry	9.6	6.8	0.3582	0.6418	FALSE
Pear	Cherry	12.1	8.3	0.4481	0.5519	FALSE
Pear	Cherry	6.9	4.5	0.3892	0.6108	FALSE
Cherry	Pear	8.2	4.1	0.6631	0.3369	FALSE
Cherry	Pear	16.8	9.9	0.8116	0.1884	FALSE
Cherry	Pear	8.9	4.1	0.7528	0.2472	FALSE

Table 6: LDA Confusion Matrix

	Pear	Cherry
Pear	9	3
Cherry	3	13

Table 7: LDA Confusion Matrix Stats

	x
Sensitivity	0.7500000
Specificity	0.8125000
Pos Pred Value	0.7500000
Neg Pred Value	0.8125000

	x
Precision	0.7500000
Recall	0.7500000
F1	0.7500000
Prevalence	0.4285714
Detection Rate	0.3214286
Detection Prevalence	0.4285714
Balanced Accuracy	0.7812500

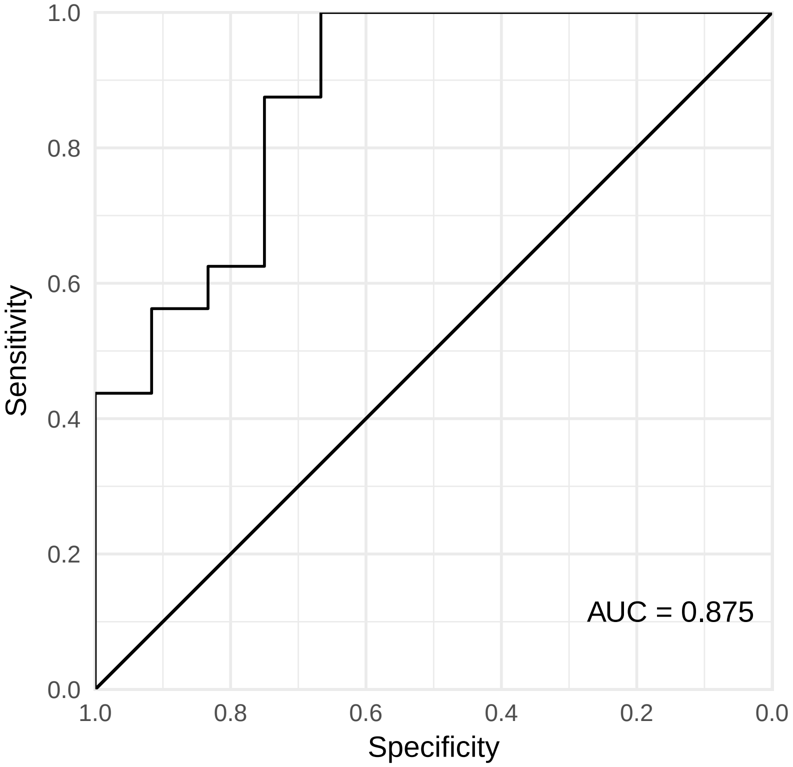
Tables 5-6 are the results from the LDA, in this model six leaves were classifieds, which included three pear and three cherry. Upon examining where these leaves are situated in the scatter plot these leaves are along the boundary lines.

In the confusion matrix, out of the 12 pear leaves 9 were classified correctly and 3 were classifieds and out of the 16 cherry leaves 13 were classified correctly and 3 were misclassified.

The sensitivity represents the proportion of predicted pear leaves that were actually pear leaves, which was 0.75. The specificity represents the proportion of predicted cherry leaves that were actually cherry leaves, which was 0.8125.

Figure 02: ROC Curve

Based on the LDA Model



The Receiver Operating Characteristic (ROC) Curve represents the matched pairs of Specificity and Sensitivity at different threshold levels. What this means is that for a given data point we can assign a leaf type based on the estimated probability. We choose a threshold for this assignment, for example anything with a probability of being a cherry leaf of ≥ 0.60 we would assign a predicted type of cherry, with a threshold of 60%.

It is important to have an Area Under the Curve (AUC) that approaches one, as this ensures that the ROC curve approaces 1 for both specificity and sensitivity. For the LDA, we have a AUC of 0.875, which is pretty good.

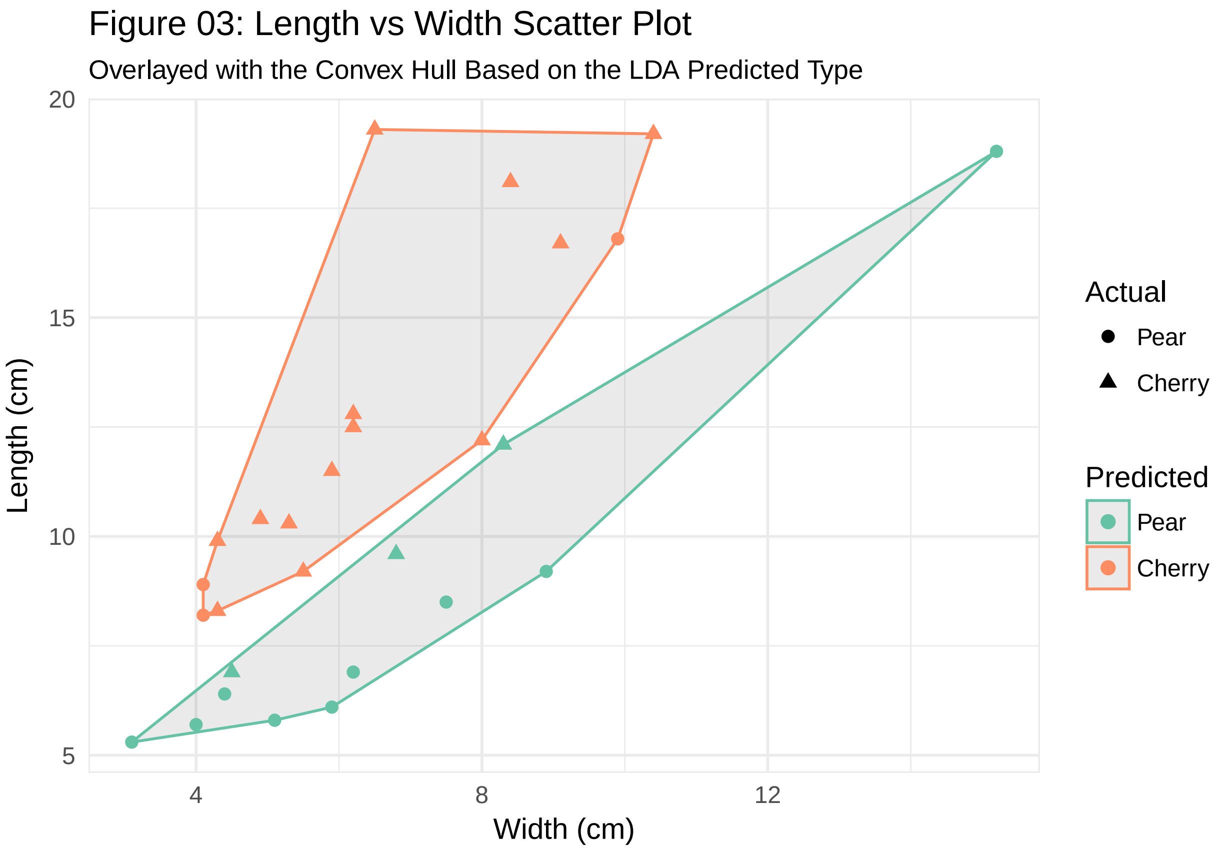
New Data

Table 8: LDA New Data Predictions

Predicted	Cherry	Pear	Number	Length	Width
Cherry	0.8003229	0.1996771	1	8.2	3.2
Pear	0.2772400	0.7227600	2	5.2	3.8
Cherry	0.5942515	0.4057485	3	7.6	4.0

Table 8 represents the predicted lead type based on data that was not originally included in the data set.

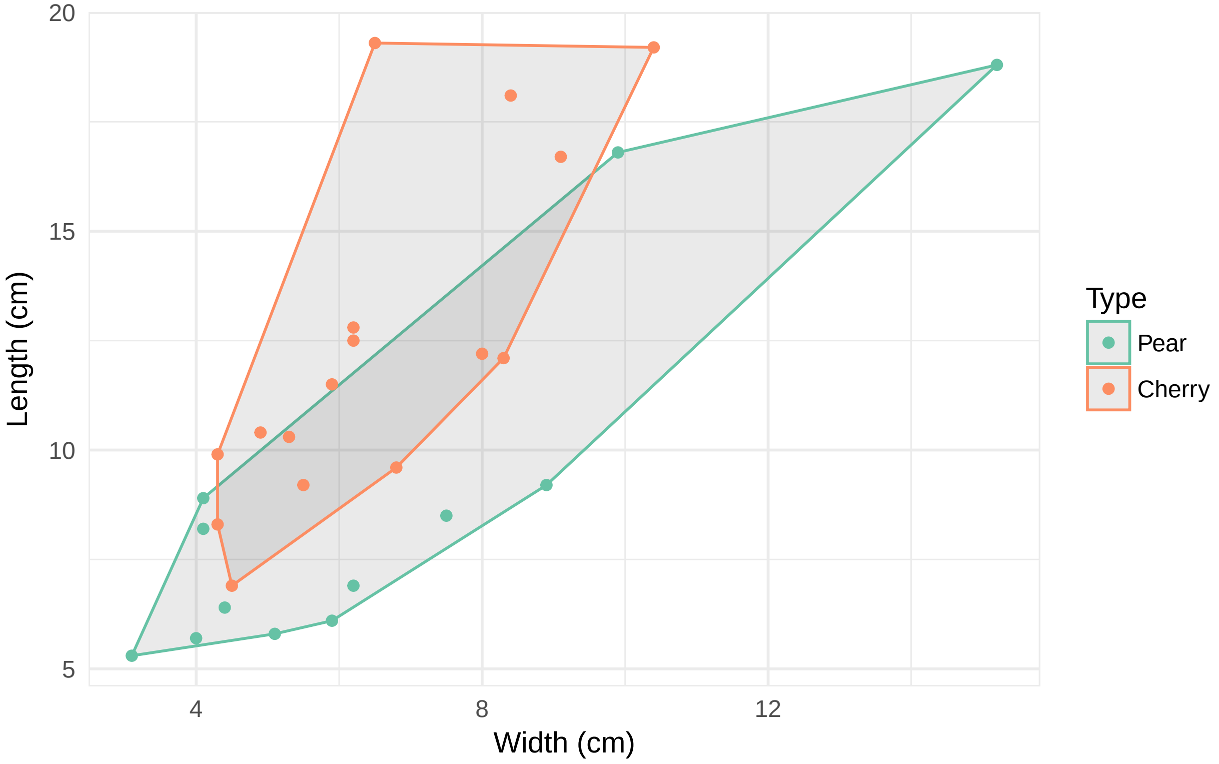
Observation Space



The convex hull in Figure 03 represents the region that captures all the points of a given leaf type, and is convex in nature. The convexity ensures that any linear combination of points in the set is still in the set. We can evaluate this on the predicted types given by the LDA to see the separating hyper plane between the two convex sets, as this gives a good approximation of the line used to differentiate between the types by the LDA. As well, by looking at the combination of shape and colour, we can see which points were misclassified as per Table 3.

Figure 04: Length vs Width Scatter Plot

Overlayed with the Convex Hull of that Type



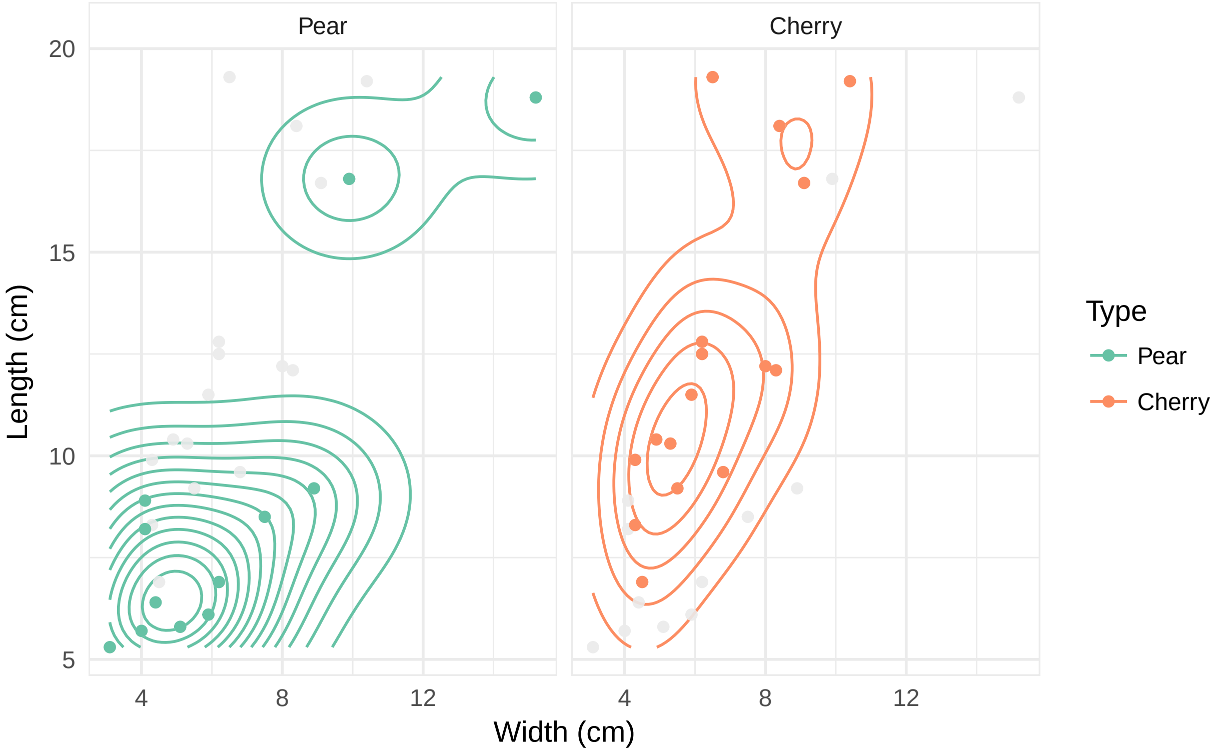
This is the convex hall of the raw data, and as you can see there is an overlap which indicates that there is no strict separation in the raw data and therefore it was necessary to conduct the LDA.

Probability Distributions

Contour

Figure 05: Length vs Width Scatter Plot

Overlayed with the Contour Plot



Contour plots show the clustering of data for pear and cherry trees. Each contour line represents the same density anywhere along that line. As the contour line density increases so does the steepness of the graph and the probability that a given leaf will have those characteristics.

In Figure 05, there are two disctrint shapes for the contour plot of pear and cherry leaves, this is due to the different covariance matrices of the leaves as seen in Table 10 and 11. The pear leaves tended to have a more similar length and width whereas, the cherry leaves tended to have a longer length and a skinnier width.

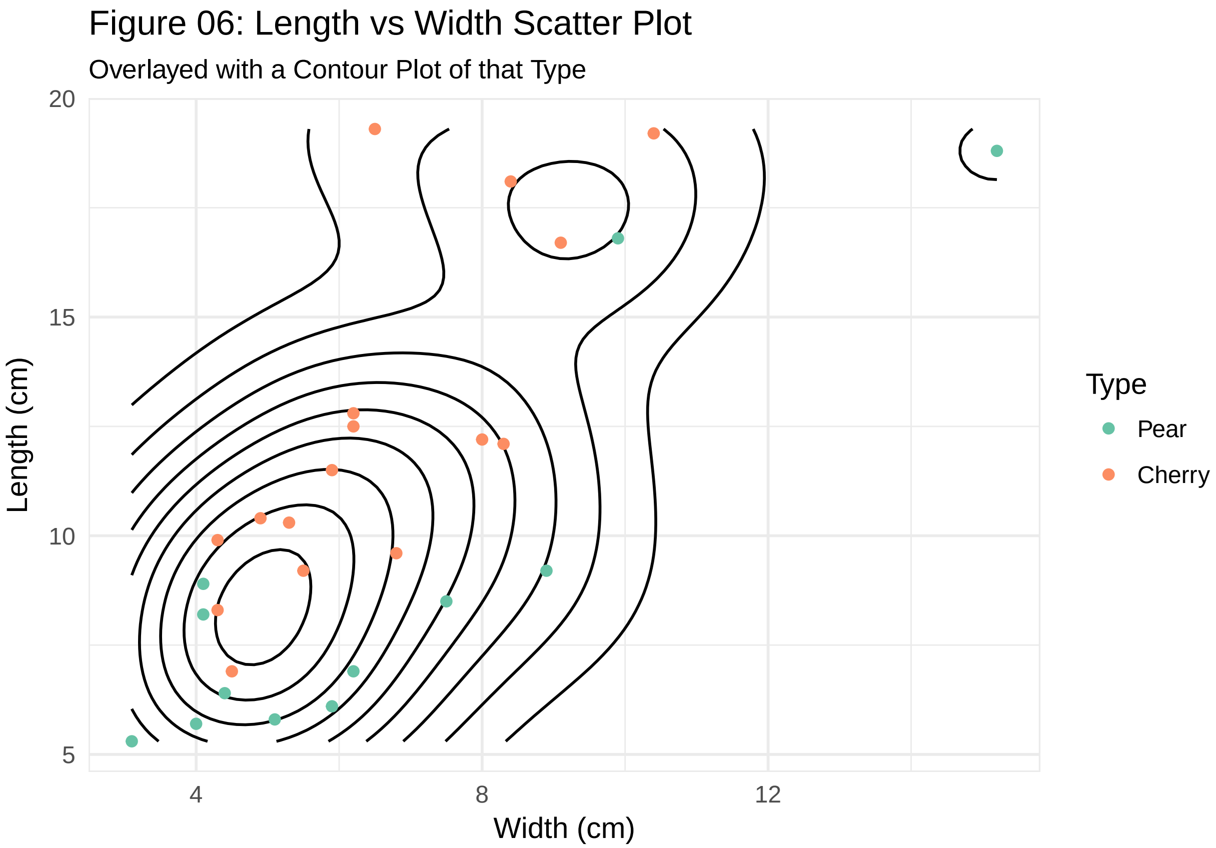


Figure 06: Length vs Width Scatter Plot

Overlayed with a Contour Plot of that Type

Figure 6 is the combined contour plot of the raw data. In this contour plot there is potential bimodality, as seen with the two peaks in the contour plot. However, these peaks fall along the diagonal and not along the vertical or horizontal axises, which indcates that the bimodality is shared between the length and the width.

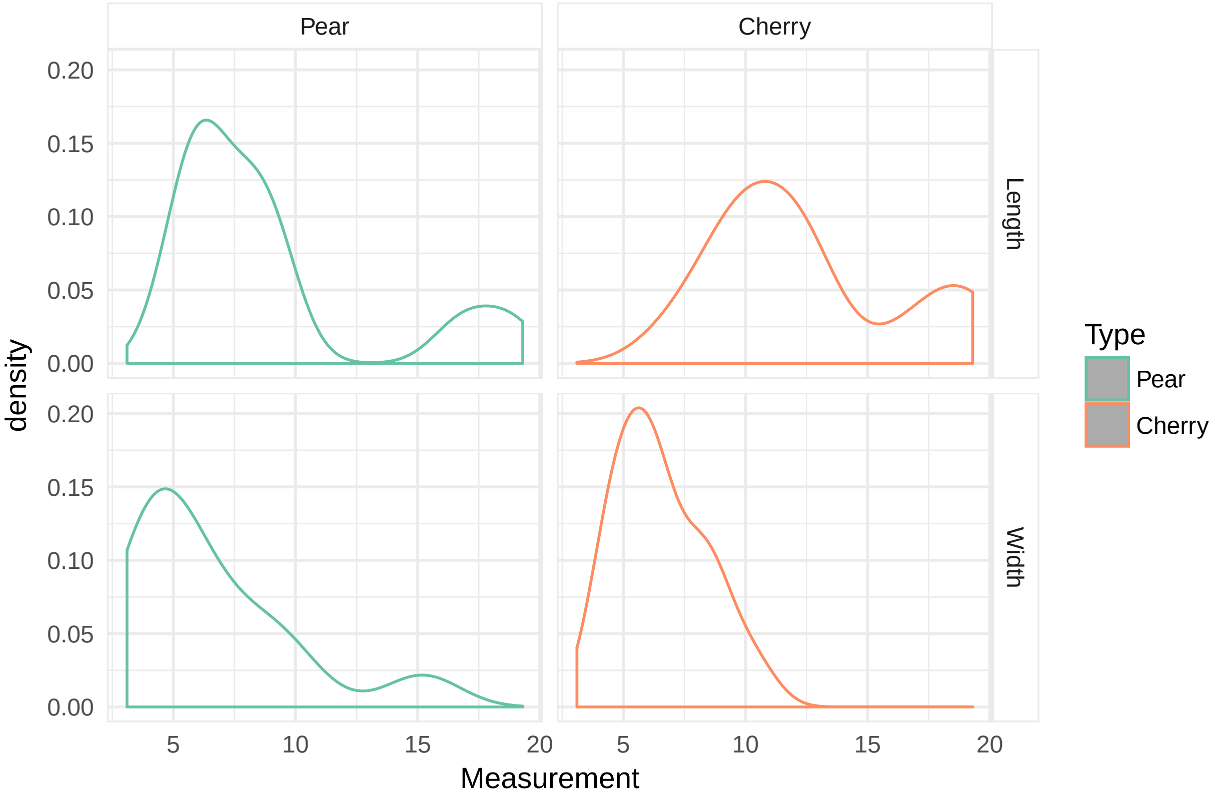
```
## Warning: Computation failed in `stat_bin()`:
## is.numeric(width) is not TRUE

## Warning: Computation failed in `stat_bin()`:
## is.numeric(width) is not TRUE

## Warning: Computation failed in `stat_bin()`:
## is.numeric(width) is not TRUE

## Warning: Computation failed in `stat_bin()`:
## is.numeric(width) is not TRUE
```

Figure 07: Density Plot by Type



```
## Warning: Computation failed in `stat_bin()`:
## is.numeric(width) is not TRUE

## Warning: Computation failed in `stat_bin()`:
## is.numeric(width) is not TRUE
```

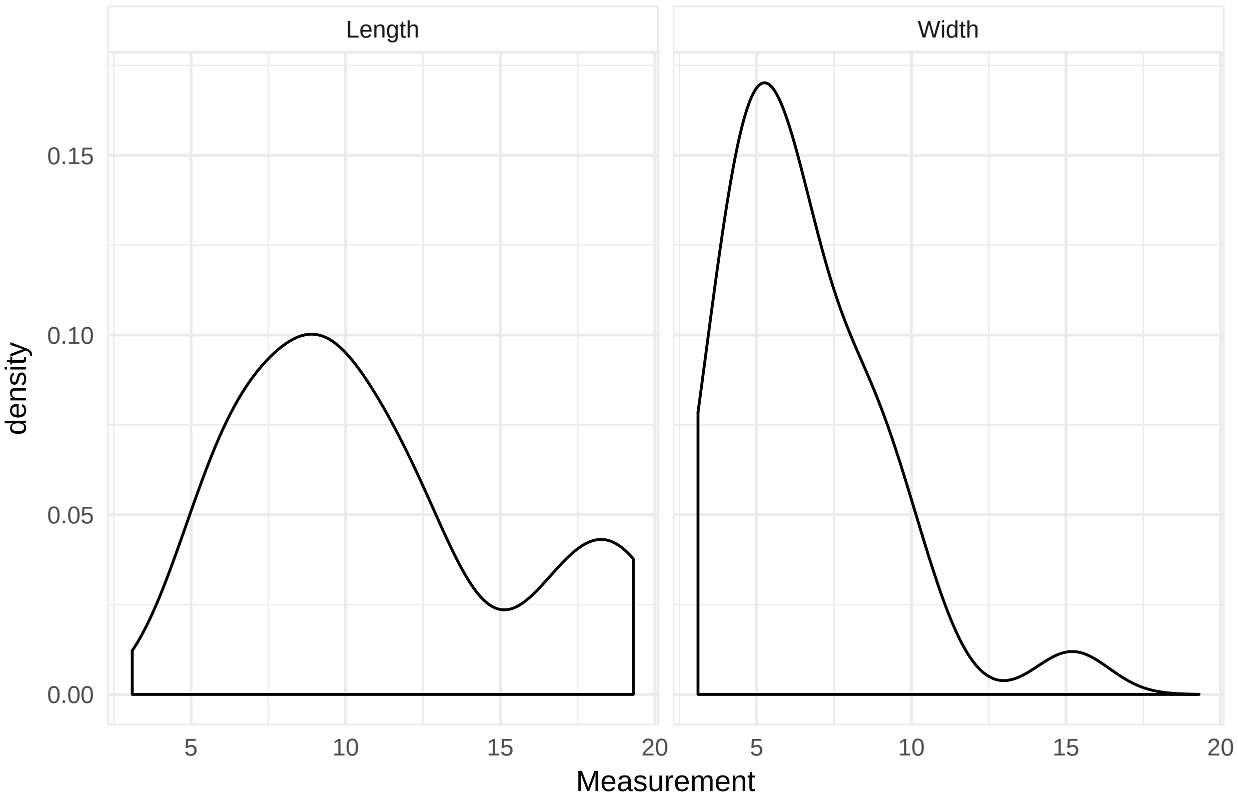


Figure 08: Density Plot

Figure 8 shows the bimodality of the data for both the length and the width. The bimodaility of the data is more prominent in the length than in the width, which becomes especially apparent when you split it up by type.

Covariance Matrix

Table 9: Shared Covariance Matrix

	Length	Width
Length	19.422011	8.476508
Width	8.476508	6.685344

Table 10: Cherry Covariance Matrix

	Length	Width
Length	15.047833	5.443833
Width	5.443833	3.357167

Table 11: Pear Covariance Matrix

	Length	Width
Length	19.27788	13.37333

	Length	Width
Width	13.37333	11.83152

```
## [1] 14.67862
```

Classification Procedure (QDA)

The difference between LDA and Quadratic Discriminat Analysis (QDA) is that QDA doesn't rely on the assumption that both classes of data share a covariance matrix, which is a crucial assumption in LDA. This allows us to perform analysis on data where this assumption may not hold in exchange for an increased variance. As well, it doesn't require the classification rule to be linear, but instead can be a quadratic function.

Training Data

Table 12: QDA Prior Probabilities

Type	Probability
Pear	0.4286
Cherry	0.5714

Table 13: QDA Group Means

	Length	Width
Pear	8.8833	6.5333
Cherry	12.4375	6.5375

The prior probability of being a Pear leaf is 0.4286. The prior probability of being a Cherry leaf is 0.5714.

Table 14: QDA Misclassification Results

Predicted	Actual	Length	Width	Cherry Probability	Pear Probability	Correct Prediction
Pear	Cherry	9.6	6.8	0.4686	0.5314	FALSE
Pear	Cherry	12.1	8.3	0.4363	0.5637	FALSE
Pear	Cherry	6.9	4.5	0.4710	0.5290	FALSE
Cherry	Pear	8.2	4.1	0.6431	0.3569	FALSE
Cherry	Pear	16.8	9.9	0.8095	0.1905	FALSE
Cherry	Pear	8.9	4.1	0.7287	0.2713	FALSE

Table 15: QDA Confusion Matrix

	Pear	Cherry
Pear	9	3
Cherry	3	13

Table 16: QDA Confusion Matrix Stats

	x
Sensitivity	0.7500000
Specificity	0.8125000
Pos Pred Value	0.7500000
Neg Pred Value	0.8125000
Precision	0.7500000
Recall	0.7500000
F1	0.7500000
Prevalence	0.4285714
Detection Rate	0.3214286
Detection Prevalence	0.4285714
Balanced Accuracy	0.7812500

Tables 12-14 are the results from the QDA, in this model six leaves were classifieds, which included three pear and three cherry.

In the confusion matrix, out of the 12 pear leaves 9 were classified correctly and 3 were classifieds and out of the 16 cherry leaves 13 were classified correctly and 3 were misclassified.

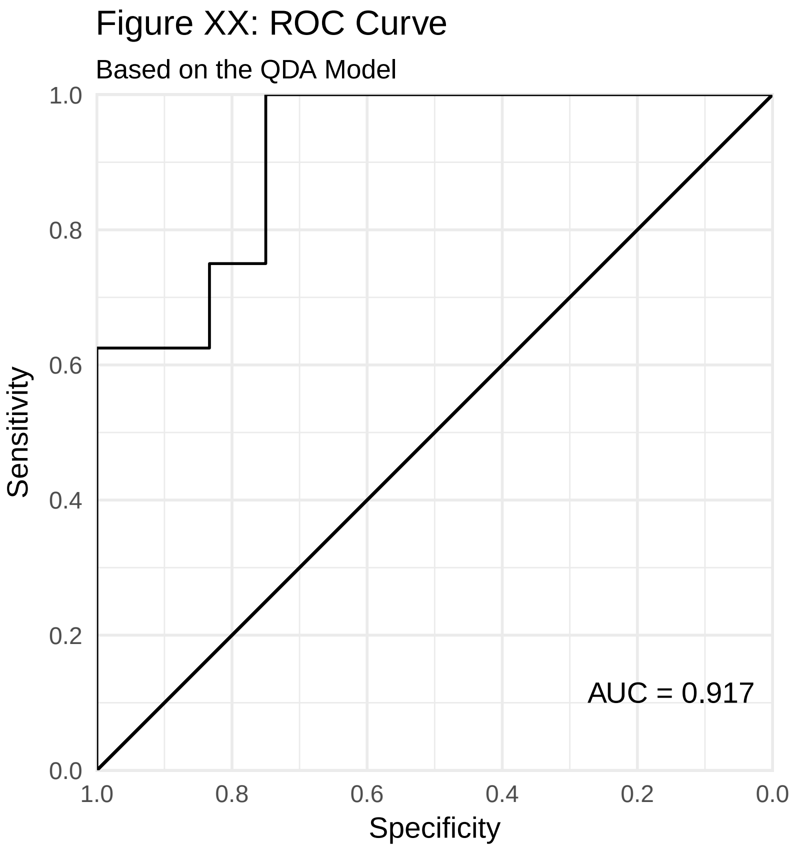
The sensitivity represents the proportion of predicted pear leaves that were actually pear leaves, which was 0.75. The specificity represents the proportion of predicted cherry leaves that were actually cherry leaves, which was 0.8125.

New Data

Table 17: QDA New Data Predictions

Predicted	Cherry	Pear	Number	Length	Width
Cherry	0.6362745	0.3637255	1	8.2	3.2
Pear	0.3382849	0.6617151	2	5.2	3.8
Cherry	0.5712093	0.4287907	3	7.6	4.0

Table 17 represents the predicted lead type based on data that was not originally included in the data set.

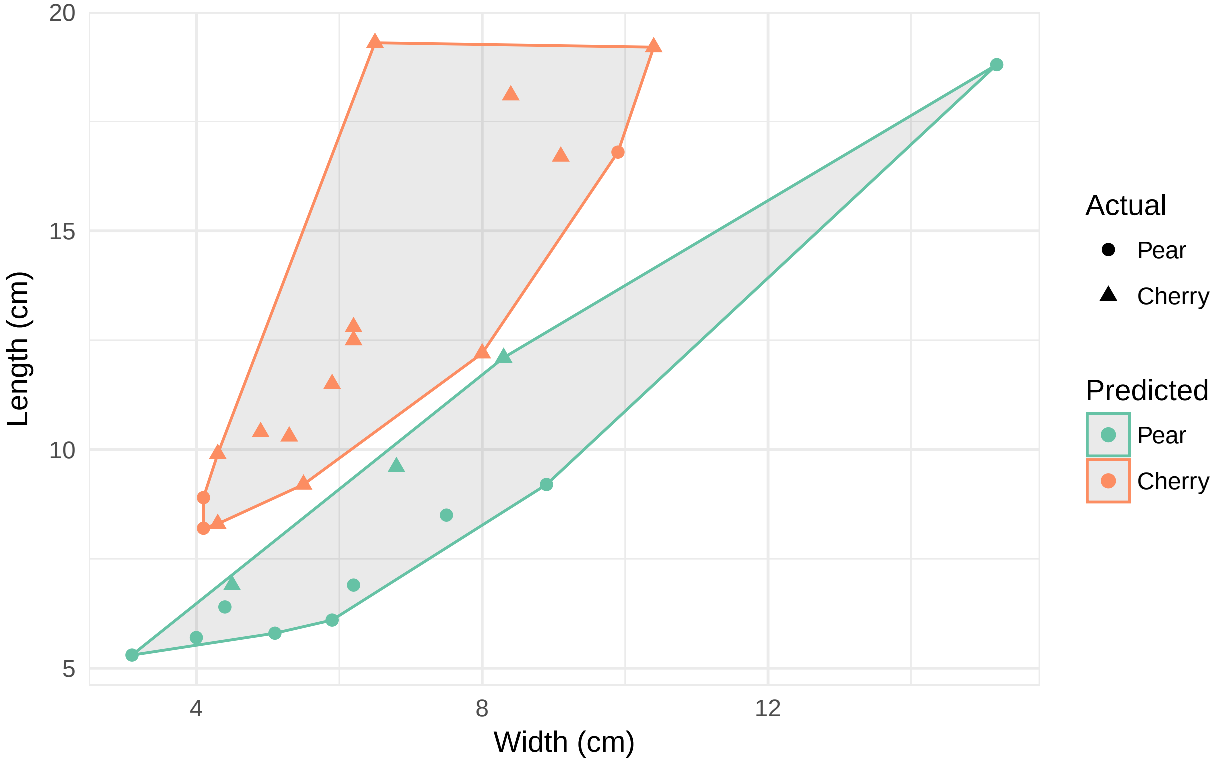


For the QDA, we have a AUC of 0.9166667, which is better than our LDA model.

Observation Space

Figure XX: Length vs Width Scatter Plot

Overlayed with the Convex Hull Based on the QDA Predicted Type



Classification Procedure (GLM)

Training Data

	Estimate	Std. Error	z value	Pr(>|z|)
(Intercept)	-1.74	1.519	-1.145	0.2522
Length	0.7764	0.2875	2.7	0.006931
Width	-0.9338	0.3815	-2.448	0.01438

(Dispersion parameter for binomial family taken to be 1)

Table 19: Logit Misclassification Results

Null deviance:	38.24 on 27 degrees of freedom
Residual deviance:	24.26 on 25 degrees of freedom

Table 20: Logit Confusion Matrix

Predicted	Actual	Length	Width	Cherry Probability	Pear Probability	Correct Prediction
Pear	Cherry	9.6	6.8	0.3460	0.6540	FALSE
Pear	Cherry	12.1	8.3	0.4759	0.5241	FALSE
Pear	Cherry	6.9	4.5	0.3578	0.6422	FALSE
Cherry	Pear	8.2	4.1	0.6895	0.3105	FALSE
Cherry	Pear	16.8	9.9	0.8868	0.1132	FALSE
Cherry	Pear	8.9	4.1	0.7927	0.2073	FALSE

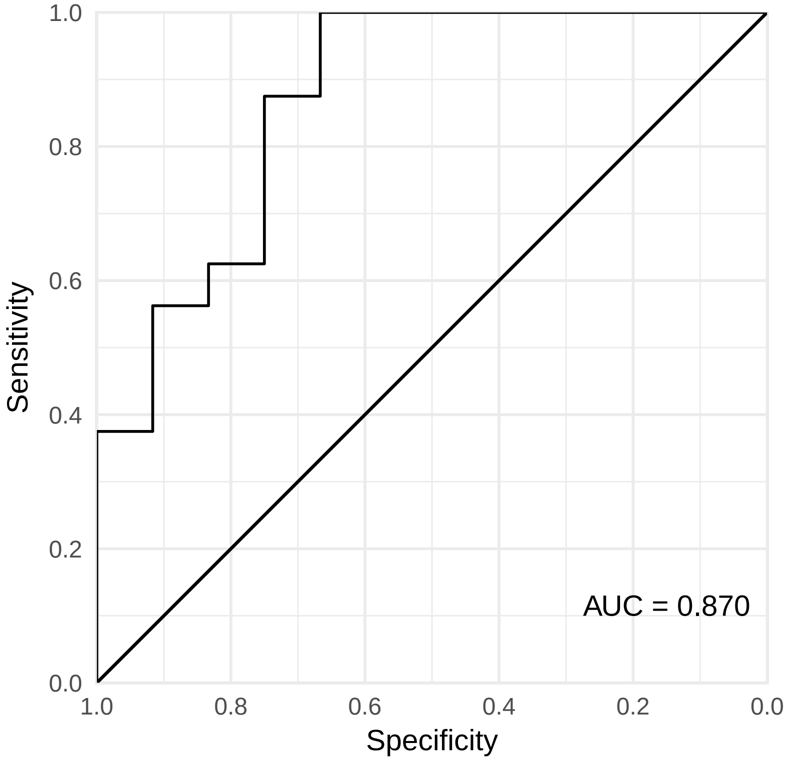
Table 21: Logit Confusion Matrix Stats

	Pear	Cherry
Pear	9	3
Cherry	3	13

	x
Sensitivity	0.7500000
Specificity	0.8125000
Pos Pred Value	0.7500000
Neg Pred Value	0.8125000
Precision	0.7500000
Recall	0.7500000
F1	0.7500000
Prevalence	0.4285714
Detection Rate	0.3214286
Detection Prevalence	0.4285714
Balanced Accuracy	0.7812500

Figure XX: ROC Curve

Based on the Logit Model



New Data

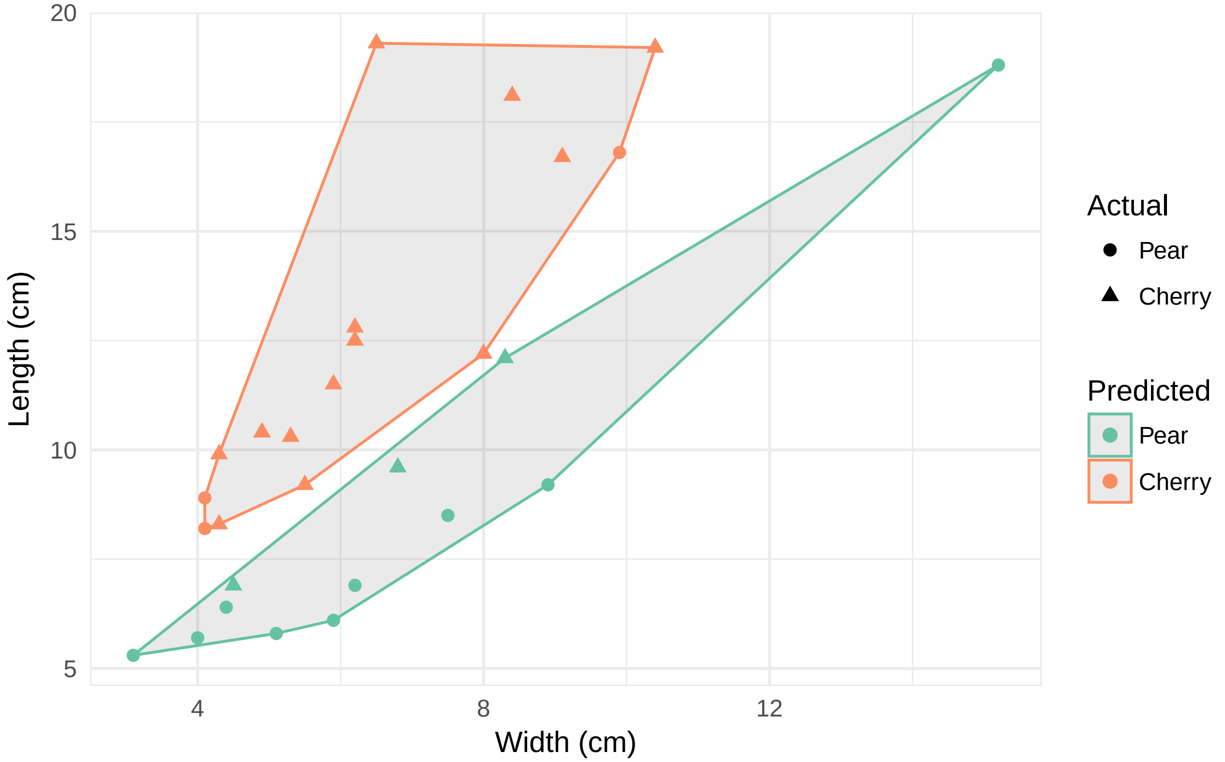
Table 23: Logit New Data Predictions

Predicted	Cherry Probability	Pear Probability	Number	Length	Width
Cherry	0.8373176	0.1626824	1	8.2	3.2
Pear	0.2225230	0.7774770	2	5.2	3.8
Cherry	0.6047999	0.3952001	3	7.6	4.0

Observation Space

Figure 09: Length vs Width Scatter Plot

Overlayed with the Convex Hull Based on the Logit Predicted Type



Conclusion

Appendix

Appendix A

Table 24: Data

Number By Type	Type	Length	Width
1	Cherry	11.5	5.9
2	Cherry	16.7	9.1
3	Cherry	10.4	4.9
4	Cherry	18.1	8.4
5	Cherry	19.3	6.5
6	Cherry	12.2	8.0
7	Cherry	10.3	5.3
8	Cherry	9.9	4.3
9	Cherry	9.6	6.8
10	Cherry	19.2	10.4
11	Cherry	12.8	6.2
12	Cherry	12.1	8.3

Number By Type	Type	Length	Width
13	Cherry	12.5	6.2
14	Cherry	8.3	4.3
15	Cherry	6.9	4.5
16	Cherry	9.2	5.5
1	Pear	5.3	3.1
2	Pear	6.9	6.2
3	Pear	9.2	8.9
4	Pear	8.5	7.5
5	Pear	8.2	4.1
6	Pear	5.7	4.0
7	Pear	5.8	5.1
8	Pear	16.8	9.9
9	Pear	6.1	5.9
10	Pear	8.9	4.1
11	Pear	6.4	4.4
12	Pear	18.8	15.2

References

[1] The Parts of a Leaf. (17, October 30). Retrieved March 20, 18, from http://www.robinsonlibrary.com/science/botany/anatomy/leafparts.htm

[2] Britannica, T. E. (2016, November 11). Cherry. Retrieved March 20, 2018, from https://www.britannica.com/plant/cherry

[3] Britannica, T. E. (2015, May 13). Pear. Retrieved March 20, 2018, from https://www.britannica.com/plant/pear