

# Rechnernetze – Computer Networks

## Lecture 4: Direct Link Networks

### Error Correction: Automatic Repeat Request

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## Channel capacity

- ▶ AWGN channel:  $C = B \log_2(1 + S/N)$

## Forward error correction (FEC)

- ▶ triple modular redundancy
- ▶ linear block codes

## Error detection: frame check sequence (FCS)

- ▶ Internet checksum
- ▶ cyclic redundancy check (CRC)

## Today

- ▶ FEC can correct some but not all bit errors
- ▶ FCS can detect erroneous frames but cannot correct these
- ▶ Solutions
  - ▶ FEC at frame level: redundant repair frames
  - ▶ automatic repeat request (ARQ): retransmit erroneous frames



Redundant repair frames

Automatic repeat request

- Stop-and-wait

- Pipelining

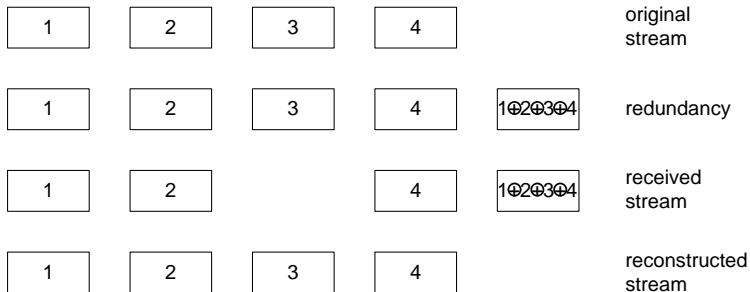
- Sliding window protocols

  - Go-back-n

  - Selective repeat

Data link layer protocols

- HDLC



- ▶ exclusive or (XOR) operation:  $a \oplus b = 1$  iff  $a \neq b$
- ▶ thus  $a \oplus a = 0$  and  $a \oplus 0 = a$  for any  $a = 0, 1$
- ▶ the XOR operation is commutative and associative
- ▶ hence  $f_1 \oplus f_2 \oplus f_3 \oplus f_4 \oplus f_1 \oplus f_2 \oplus f_4 = f_3$
- ▶ example: redundancy recovers one lost frame out of five



## ARQ error control

- ▶ concerned with retransmission of lost or erroneous frames
- ▶ uses feedback from receiver to sender
- ▶ acknowledgements to confirm the correct receipt of frames
- ▶ timeouts to trigger retransmissions
- ▶ ideally, ARQ retransmits frames only if needed  
(as opposed to the constant overhead of FEC frames)

Frames may, however, also be lost despite error-free transmission

- ▶ if the receiver cannot process data at the speed of the sender
- ▶ flow control to adapt the sender to the receiver's ability
- ▶ uses feedback from receiver to sender

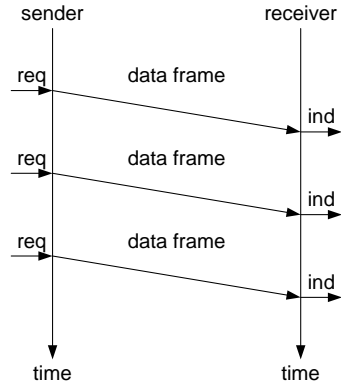
Often, the mechanisms for error and flow control are interlinked.

## Assumptions

- ▶ error-free channel
- ▶ infinitely large buffer
- ▶ infinitely fast receiver

## But

- ▶ finite receiver buffer
- ▶ finite processing speed
- ▶ ...



## Assumptions

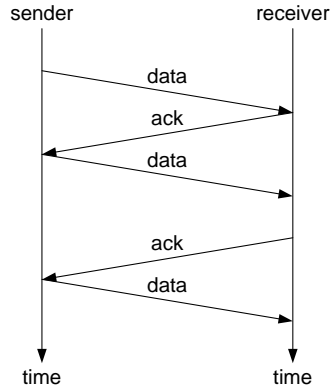
- ▶ error-free channel
- ▶ finite buffer
- ▶ slow receiver
- ▶ (half-) duplex

## Solution

- ▶ feedback, here acknowledgements
- ▶ stop-and-wait flow control

## But, additionally

- ▶ error-prone channel



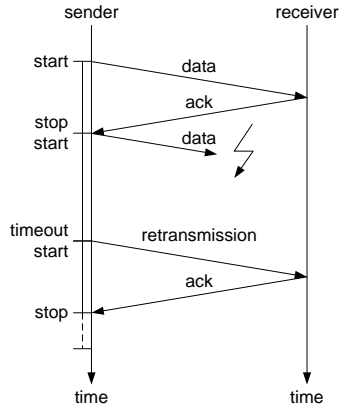
Assumptions: error-prone (half-) duplex channel

Solution

- ▶ positive acknowledgement with retransmit (PAR)
- ▶ also called automatic repeat request (ARQ)
- ▶ timeout triggers retransmissions
  - ▶ timeout too long  $\Rightarrow$  unnecessary waiting
  - ▶ timeout too short  $\Rightarrow$  needless retransmissions

But, additionally

- ▶ acks may be lost





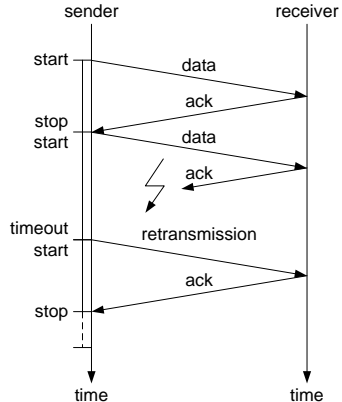
Assumptions: error-prone (half-) duplex channel

## Problem

- ▶ loss of acknowledgements causes duplicated frames
- ▶ receiver accepts duplicates as if they were new data

## Solution

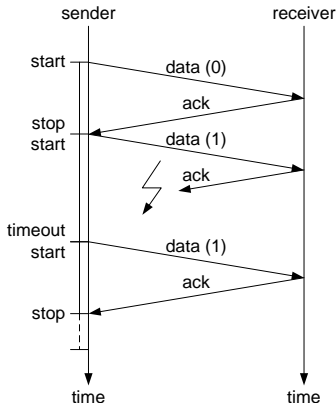
- ▶ need to mark retransmissions
- ▶ numbering of frames



Assumptions: error-prone (half-) duplex channel

## Solution

- ▶ sequence numbers
- ▶ each frame is assigned a consecutive SeqNo
- ▶ retransmissions have to use the same SeqNo as the original transmission
- ▶ receiver discards duplicates
- ▶ range  $[0, \dots k - 1] \bmod k$  where  $k = 2^n$
- ▶ stop-and-wait  $[0, 1] \bmod 2$



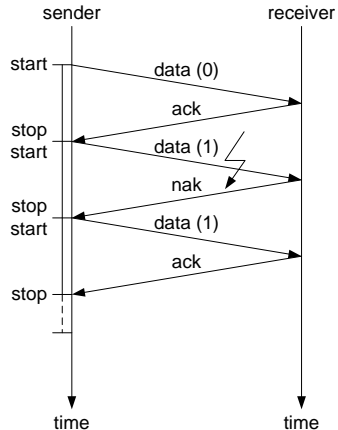
So far no differentiation  
between

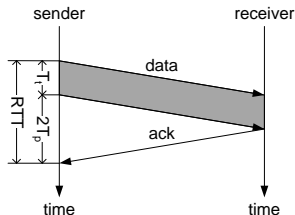
- ▶ faulty frame
- ▶ missing frame

need to wait for timeout in any  
case.

Active error control

- ▶ positive acks
- ▶ negative naks





- ▶ transmission time  $T_t = \text{frame length } l / \text{capacity } C$
- ▶ propagation delay  $T_p = \text{distance} / \text{speed of light}$
- ▶ channel utilization  $T_t / (T_t + 2T_p)$
- ▶ e.g.  $T_p = 50 \text{ ms}$  trans Atlantic,  $l = 1500 \text{ Byte}$ ,  $C = 2 \text{ Mb/s}$  gives  $T_t = 6 \text{ ms}$  and a utilization of 5 %
- ▶ e.g.  $T_p = 0.1 \text{ ms}$ ,  $l = 1500 \text{ Byte}$ ,  $C = 54 \text{ Mb/s}$  gives  $T_t = 0.22 \text{ ms}$ , and a utilization of roughly 50 %



effective data rate  $R = \frac{\text{number of bits accepted by receiver}}{\text{total time for delivery}}$

- ▶ frame length  $l$
- ▶ header/trailer length  $h$
- ▶ payload  $l - h$
- ▶ capacity  $C$
- ▶ transmission time  $T_t = l/C$
- ▶ propagation delay  $T_p$

$$R = \frac{l - h}{\frac{l}{C} + 2T_p}$$

$$\left( \text{formally w. ergodicity: } R = \lim_{j \rightarrow \infty} \frac{j(l - h)}{\sum_{i=1}^j t_i} = \lim_{j \rightarrow \infty} \frac{l - h}{\frac{1}{j} \sum_{i=1}^j t_i} = \frac{l - h}{\mathbb{E}[t]} \right)$$



Consider transmission errors: Denote  $N$  the average number of transmission attempts until a frame is accepted (timeout =  $T_t + 2T_p$ ).

$$R = \frac{l - h}{N \left( \frac{l}{C} + 2T_p \right)}$$

To derive  $N$

- ▶ let  $P$  be the probability that the transmission of a frame is erroneous
- ▶ assume  $k$  frames shall be transmitted
- ▶ assume that errors in different frames are independent
- ▶ of the  $k$  frames transmitted  $(1 - P)k$  are on average received correctly, requiring  $Pk$  retransmissions



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- ▶ of the  $Pk$  retransmissions  $(1 - P)Pk$  are on average received correctly, requiring another  $P^2k$  retransmissions
- ▶ continuing  $(1 + P + P^2 + \dots)k$  transmissions are needed



The sum  $k(1 + P + P^2 + \dots) = k \sum_{i=0}^{\infty} P^i$  is a geometric sum that can be easily solved (the trick is to solve  $(1 - P) \sum_{i=0}^j P^i$  first).





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We can also find the solution from the following argument

- ▶ from  $n$  transmissions  $(1 - P)n$  frames are received correctly
- ▶ hence, if we want  $k$  frames to be received correctly we equate

$$k = (1 - P)n$$

and solve for the number of transmissions required yielding

$$n = k/(1 - P)$$



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Normalizing  $n$  by  $k$  the average number of transmissions required until one frame is received correctly is

$$N = \frac{n}{k} = \frac{1}{1 - P}$$

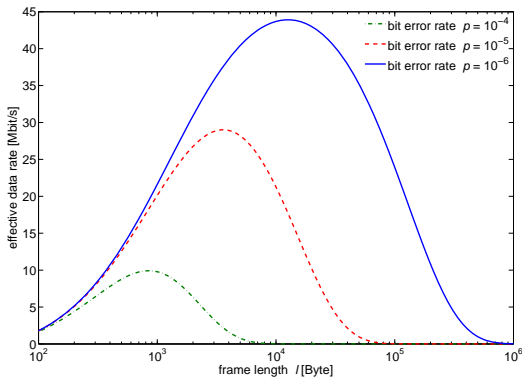


Computation of the frame error probability  $P$

- ▶ assume bit errors occur with probability  $p$
- ▶ hence a bit is transmitted correctly with probability  $1 - p$
- ▶ we assume that bit errors are statistically independent
- ▶ the probability that an entire frame of  $l$  bits is transmitted correctly becomes  $(1 - p)^l$
- ▶ the frame error probability follows as  $P = 1 - (1 - p)^l$

Putting all pieces together the effective data rate becomes

$$R = \frac{(l - h)(1 - p)^l}{\frac{l}{C} + 2T_p}$$



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►  $C = 54 \text{ Mbit/s}$

►  $T_p = 0.1 \text{ ms}$

►  $h = 52 \text{ Byte}$



How to improve the performance of stop-and-wait ARQ?

- ▶ if the frame error rate is high  
⇒ reduce the frame size
- ▶ if the waiting time is large compared to the transmission time  
⇒ increase the frame size

Cannot do both at the same time!



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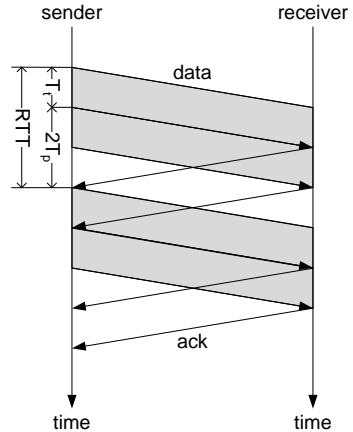
Solution: Pipelining

- ▶ several frames may be in flight simultaneously
- ▶ at any time at most  $n$  of the frames transmitted may be unacknowledged (can also be used for flow control)
- ▶ stop-and-wait is the special case  $n = 1$
- ▶ ideally, there is a continuous flow of frames and acknowledgements such that no waiting occurs at all
- ▶ each frame can be small to account for high bit error rates



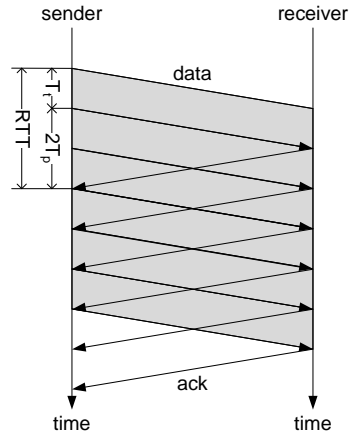
The sender can transmit  $n$  frames at line speed before waiting for acks. Consider the figure, which  $n$  achieves full utilization?

- $n = 2$ : utilization  $2/3$   
still limited by  $T_p$



The sender can transmit  $n$  frames at line speed before waiting for acks. Consider the figure, which  $n$  achieves full utilization?

- ▶  $n = 2$ : utilization  $2/3$   
still limited by  $T_p$
- ▶  $n = 3$ : utilization 1  
 $n l/C$  matches  $RTT$
- ▶  $n > 3$ : utilization 1  
limited by the capacity  $C$







A continuous flow of data (full utilization) is achieved if

$$n l \geq C \cdot RTT$$

The term  $C \cdot RTT$  is called the bandwidth delay product.

- ▶  $C$  and  $RTT$  are external parameters of the physical layer connection (from point of view of the data link protocol)
  - ▶ the  $RTT$  consists (mainly) of the transmission time  $T_t$  and two propagation delays  $T_p$
  - ▶  $T_p$  is distance divided by propagation speed
  - ▶ propagation occurs with speed of light  $v_l$  where depending on the medium  $v_l \approx 200\,000 - 300\,000$  km/s
- ▶  $n$  and  $l$  are internal parameters of the data link protocol that can be adapted to optimize the utilization



The term  $C \cdot T_p$  denotes the number of bits that are in flight simultaneously during a (serial) transmission at full utilization.

To see this:

- ▶ the duration of a bit sent at rate  $C$  is  $T_b = 1/C$
- ▶ the bit propagates at speed of light  $v_l$ , i.e. it is spread over (occupies) a section of  $s_b = v_l T_b = v_l / C$ 
  - ▶ on a 10 Mb/s Ethernet bits are 20 meters long



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  - ▶ on a 10 Mb/s Ethernet bits are 20 meters long
- ▶ it follows that a cable of length  $s_c$  has room for  $s_c / s_b$  bits
- ▶ the propagation delay is  $T_p = s_c / v_l$  respectively  $s_c = v_l T_p$
- ▶ there can be  $C \cdot T_p$  bits in flight on the cable
  - ▶ on a 10 meter cable used at 100 Mb/s is room for 5 bits
  - ▶ on a 10 000 km trans Atlantic fibre used at 10 Gb/s is room for 500 000 000 bits = 62.5 MByte



Redundant repair frames

Automatic repeat request

- Stop-and-wait

- Pipelining

- Sliding window protocols

  - Go-back-n

  - Selective repeat

Data link layer protocols

- HDLC

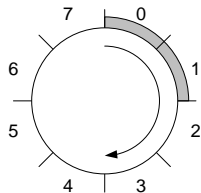


## Sender

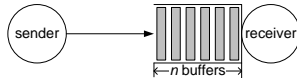
- ▶ frames are numbered in ascending order
- ▶ sequence numbers SeqNo  $\in [0, \dots, k-1]$
- ▶ SeqNo is incremented modulo  $k$

## Receiver

- ▶ frames received in sequence are acknowledged and passed to layer 3
- ▶ acknowledgements contain an AckNo  $\in [0, \dots, k-1]$
- ▶ cumulative acknowledgements acknowledge all frames up to AckNo



With pipelining, how to ensure that the receiver is not flooded?

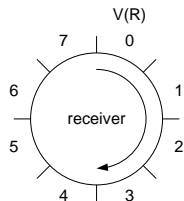
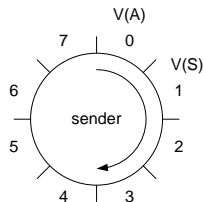


Solution: Sliding window protocol

- ▶ receive window: number of receiver buffers  $n$ , advertised to sender
- ▶ send window: number of frames transmitted but not yet acknowledged

Condition: send window  $\leq$  receive window

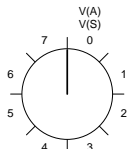
- ▶ sender: send window  $V(S) - V(A)$ 
  - ▶  $V(A)$ : acknowledge state variable
    - ▶ smallest not acknowledged SeqNo
    - ▶ increment on receipt of an ack
  - ▶  $V(S)$ : send state variable
    - ▶ next SeqNo to be sent
    - ▶ increment when sending a frame
    - ▶ largest SeqNo allowed:  $V(A) + n - 1$
- ▶ receiver: receive window  $n$  buffers
  - ▶  $V(R)$ : receive state variable
    - ▶ next SeqNo to be received in sequence
    - ▶ increment on receipt of a frame
    - ▶ largest SeqNo allowed:  $V(R) + n - 1$



Window size  $n = 3$

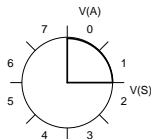
1.  $V(S) = V(A)$

- ▶ no frames stored, i.e. all transmitted frames have been ack'ed
- ▶ sender may send up to three new frames



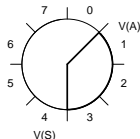
2.  $V(S) = V(A) + 2$

- ▶ two frames stored, i.e. two transmitted frames have not been ack'ed
- ▶ sender may send one new frame



3.  $V(S) = V(A) + 3$

- ▶ three frames stored
- ▶ sender is not permitted to send further frames





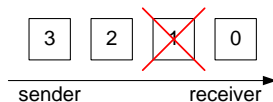


## Expected order

- ▶ window size  $n = 1$  (stop-and-wait)
  - ▶ transmission/reception generally in sequence
- ▶ window size  $n > 1$ , two options
  - ▶ go-back-n
    - ▶ reception only in sequence
    - ▶ out-of sequence frames are discarded
  - ▶ selective repeat
    - ▶ no requirement for in-sequence reception
    - ▶ only limited by the receive window

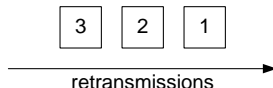
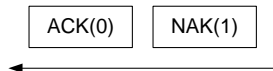
## Receiver

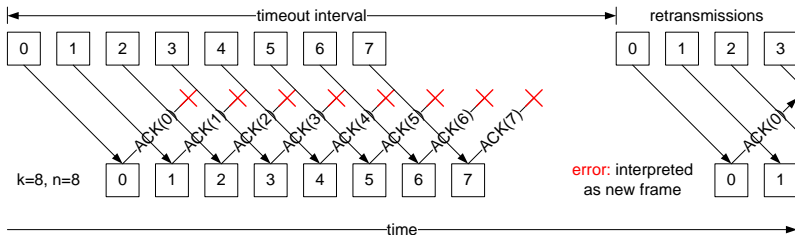
- ▶ accepts frames only in order of increasing SeqNo
- ▶ if a frame is erroneous or missing all subsequent frames are discarded



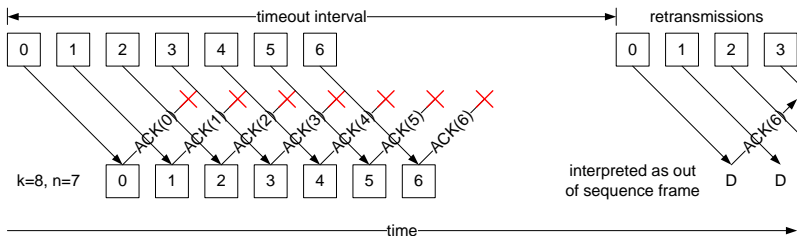
## Sender

- ▶ transmits frames in sequence
- ▶ if a frame with a certain SeqNo is retransmitted all subsequent frames are retransmitted, too

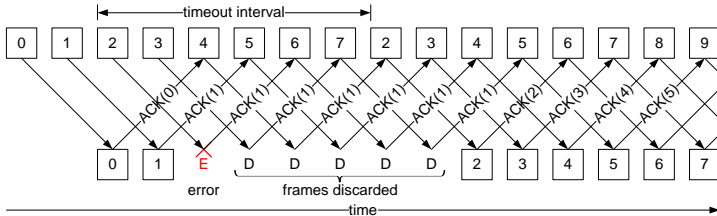


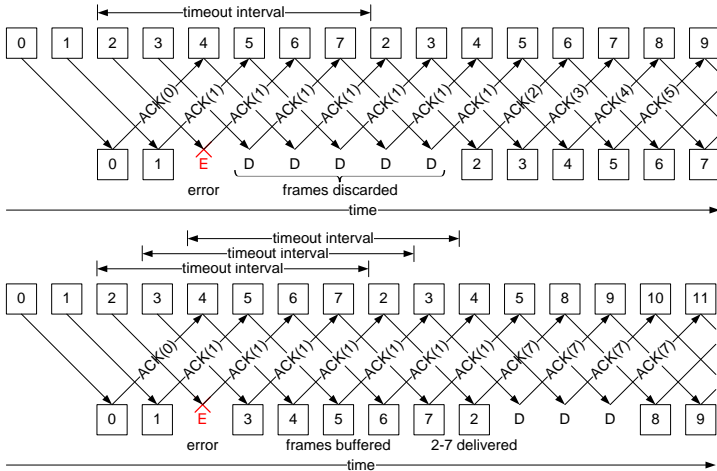


- ▶ number of distinct sequence numbers  $k$
- ▶ sequence numbers  $[0, \dots, k-1]$  modulo  $k$
- ▶ window size  $n$
- ▶ go-back-n: frames are only accepted in sequence



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- ▶ window size  $n$
- ▶ go-back- $n$ : frames are only accepted in sequence
- ▶ need  $n \leq k-1$  to distinguish retransmissions





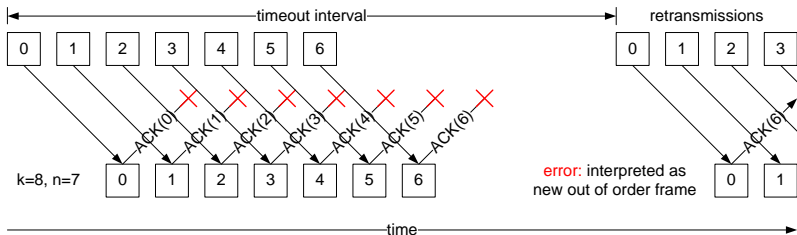


## Receiver

- ▶ stores all correctly received frames
  - ▶ that fall into the receive window
  - ▶ regardless whether in sequence or not
- ▶ hands over data to the network layer only in sequence
- ▶ may involve extensive buffering of received frames while waiting for missing frames

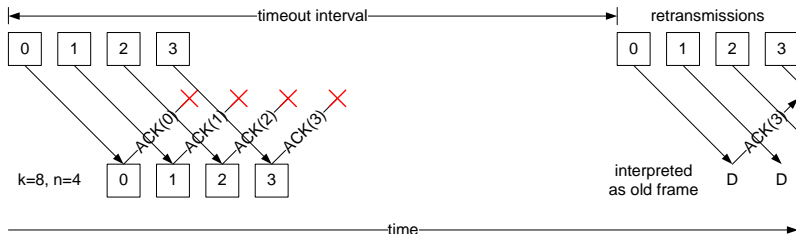
## Sender

- ▶ seeks to retransmit only erroneous frames  
(as opposed to all frames starting at the erroneous one)
- ▶ notification of missing frames
  - ▶ timeout, one timer per frame
  - ▶ optional stalled positive acknowledgements
  - ▶ optional negative acknowledgements



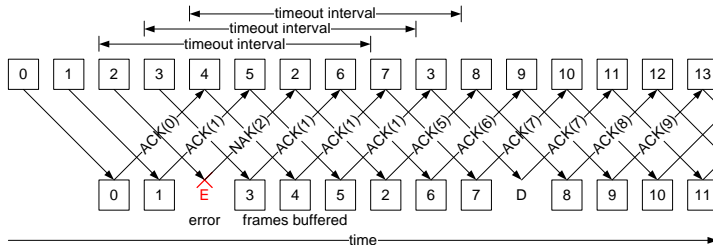
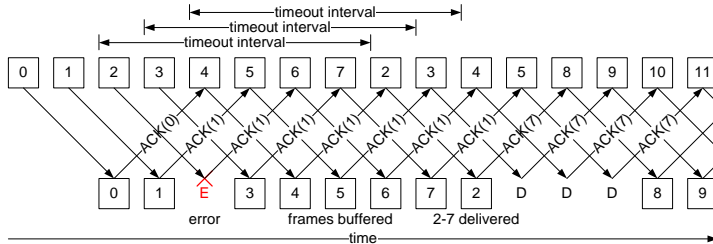
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- ▶ out of sequence frames are buffered
- ▶ need  $n \leq k/2$  to distinguish retransmissions

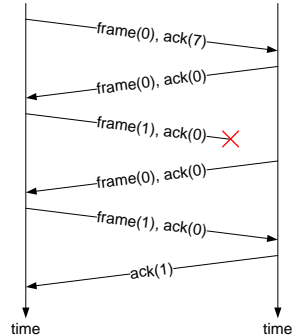
# Selective repeat with negative acknowledgements



# Duplex: piggyback acknowledgements



- ▶ adds acks to data frames
- ▶ frames have
  - ▶ SeqNo
  - ▶ AckNo
- ▶ need to determine when to send standalone acks
  - ▶ if a frame is received start a timer
  - ▶ if there is data for the reverse direction add the piggyback ack
  - ▶ otherwise send an ack once the timer expires
  - ▶ ack timer must be smaller than the retransmit timer





Redundant repair frames

Automatic repeat request

- Stop-and-wait

- Pipelining

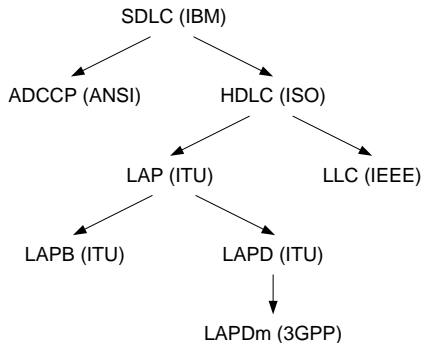
- Sliding window protocols

  - Go-back-n

  - Selective repeat

Data link layer protocols

- HDLC



Synchronous Data Link Control  
Advanced Data Communication  
Control Procedure  
High-Level Data Link Control  
Link Access Procedure  
Logical Link Control  
Link Access Procedure Balanced  
Link Access Procedure D  
signalling channel (ISDN)  
Link Access Procedure D modified  
signalling channel (GSM)

”The nice thing about standards is that you have so many to choose from“ [Tanenbaum]. The HDLC protocol family consists of

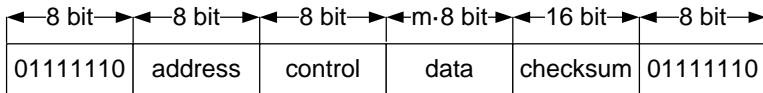
- ▶ bit oriented protocols
- ▶ with confusing little differences

## HDLC protocol

- ▶ full duplex
- ▶ bit-oriented with bit stuffing (after five 1s a 0 is stuffed)

## Frame format

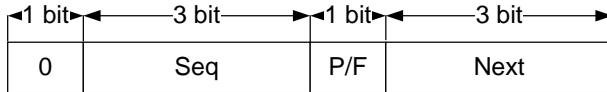
- ▶ flag: 01111110
- ▶ address: used if not point-to-point
- ▶ control: sequence number, acknowledgement
- ▶ data: arbitrary number of payload bytes
- ▶ checksum: cyclic redundancy check



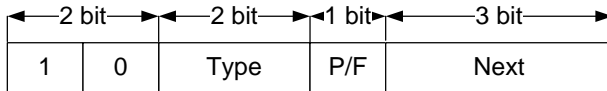


The control field specifies three different types of frames

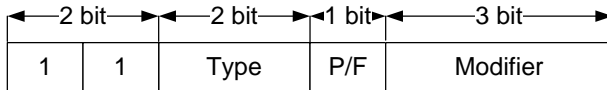
► information frames

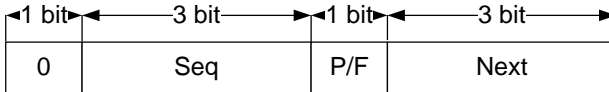


► supervisory frames



► unnumbered frames

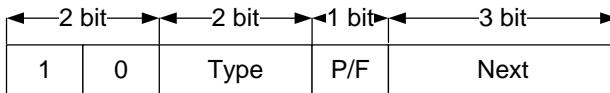




Sliding window with 3 bit sequence number  
(extended version with 7 bit)

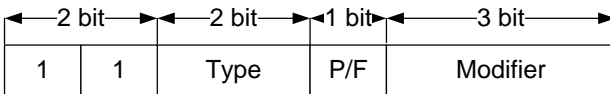
- ▶ Seq: sequence number of this frame
- ▶ Next: piggyback acknowledgement
  - ▶ specifies the next frame expected
  - ▶ last frame received in sequence plus one
- ▶ P/F: Poll/Final
  - ▶ master requests data transmission from a slave by setting P
  - ▶ the slave sends data with P and the final data frame with F





## Type

- ▶ 00: RECEIVE READY
  - ▶ acknowledgement (not piggyback)
  - ▶ Next: next frame expected
- ▶ 01: REJECT
  - ▶ negative acknowledgement, go-back-n
  - ▶ Next: first frame to be retransmitted
- ▶ 10: RECEIVE NOT READY
  - ▶ stop command to make communication peer pause
  - ▶ start again after RECEIVE READY, REJECT, ...
- ▶ 11: SELECTIVE REJECT
  - ▶ negative acknowledgement, selective repeat
  - ▶ Next: frame to be retransmitted



## Type, modifier

- ▶ SNRM: Set Normal Response Mode
  - ▶ reports availability, initializes unbalanced master/slave mode
- ▶ SABM: Set Asynchronous Balanced Mode
  - ▶ reports availability, initializes balanced mode, i.e. equal communication peers
- ▶ DISC: Disconnect
  - ▶ reports unavailability
- ▶ UA: Unnumbered Acknowledgement
  - ▶ acknowledgement for unnumbered frames
  - ▶ only one unnumbered frame may be outstanding at any time
- ▶ FRMR: Frame Reject
  - ▶ report frames with incorrect semantics