

Дискретная математика. Задание 4

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Исходный граф:

	e1	e2	e3	e4	e5	e6	e7	e8	e9	e10	e11	e12
e1	0	4		2	1				2			2
e2	4	0		3	1		3		4	5		2
e3			0			3		2	5	5		
e4	2	3		0	1		2	4	4		5	1
e5	1	1		1	0	1			3		1	
e6			3		1	0		4	4	3		2
e7		3		2			0	3		4	1	4
e8			2	4		4	3	0	2	4	4	3
e9	2	4	5	4	3	4		2	0			1
e10		5	5			3	4	4		0	1	
e11				5	1		1	4		1	0	2
e12	2	2		1		2	4	3	1		2	0

1. Нахождение Гамильтонова цикла

$S = \{x_1\}$

Включим x_2 , $S = \{x_1, x_2\}$

Включим x_4 , $S = \{x_1, x_2, x_4\}$

Включим x_5 , $S = \{x_1, x_2, x_4, x_5\}$

Включим x_6 , $S = \{x_1, x_2, x_4, x_5, x_6\}$

Включим x_6 , $S = \{x_1, x_2, x_4, x_5, x_6\}$

Включим x_3 , $S = \{x_1, x_2, x_4, x_5, x_6, x_3\}$

Включим x_8 , $S = \{x_1, x_2, x_4, x_5, x_6, x_3, x_8\}$

Включим x_7 , $S = \{x_1, x_2, x_4, x_5, x_6, x_3, x_8, x_7\}$

Включим x_{10} , $S = \{x_1, x_2, x_4, x_5, x_6, x_3, x_8, x_7, x_{10}\}$

Включим x_{11} , $S = \{x_1, x_2, x_4, x_5, x_6, x_3, x_8, x_7, x_{10}, x_{11}\}$

Включим x_{12} , $S = \{x_1, x_2, x_4, x_5, x_6, x_3, x_8, x_7, x_{10}, x_{11}, x_{12}\}$

Включим x_9 $S = \{x_1, x_2, x_4, x_5, x_6, x_3, x_8, x_7, x_{10}, x_{11}, x_{12}, x_9\}$

Гамильтонов цикл: $S = \{x_1, x_2, x_4, x_5, x_6, x_3, x_8, x_7, x_{10}, x_{11}, x_{12}, x_9\}$

2. Матрица смежности с перенумерованными вершинами

	e1	e2	e4	e5	e6	e3	e8	e7	e10	e11	e12	e9
e1	0	4	2	1							2	2
e2	4	0	3	1				3	5		2	4
e4	2	3	0	1			4	2		5	1	4
e5	1	1	1	0	1					1		3
e6				1	0	3	4		3		2	4
e3					3	0	2		5			5
e8			4		4	2	0	3	4	4	3	2
e7		3	2				3	0	4	1	4	
e10		5			3	5	4	4	0	1		
e11			5	1			4	1	1	0	2	
e12	2	2	1		2		3	4		2	0	1
e9	2	4	4	3	4	5	2				1	0

3. Построение графа пересечений G'

Определим p_{212} , для чего в матрице R выделим подматрицу R_{212} .

Ребро (x_2x_{12}) пересекается с $(x_1x_3), (x_1x_4), (x_1x_{11})$

Определим p_{211} , для чего в матрице R выделим подматрицу R_{211} .

Ребро (x_2x_{11}) пересекается с $(x_1x_3), (x_1x_4)$

Определим p_{29} , для чего в матрице R выделим подматрицу R_{29} .

Ребро (x_2x_9) пересекается с $(x_1x_3), (x_1x_4)$

Определим p_{28} , для чего в матрице R выделим подматрицу R_{28} .

Ребро (x_2x_8) пересекается с $(x_1x_3), (x_1x_4)$

Определим p_{24} , для чего в матрице R выделим подматрицу R_{24} .

Ребро (x_2x_4) пересекается с (x_1x_3)

Определим p_{312} , для чего в матрице R выделим подматрицу R_{312} .
 Ребро (x_3x_{12}) пересекается с $(x_1x_4), (x_1x_{11}), (x_2x_4), (x_2x_8), (x_2x_9), (x_2x_{11})$
 Определим p_{311} , для чего в матрице R выделим подматрицу R_{311} .
 Ребро (x_3x_{11}) пересекается с $(x_1x_4), (x_2x_4), (x_2x_8), (x_2x_9)$
 Определим p_{310} , для чего в матрице R выделим подматрицу R_{310} .
 Ребро (x_3x_{10}) пересекается с $(x_1x_4), (x_2x_4), (x_2x_8), (x_2x_9)$
 Определим p_{38} , для чего в матрице R выделим подматрицу R_{38} .
 Ребро (x_3x_8) пересекается с $(x_1x_4), (x_2x_4)$
 Определим p_{37} , для чего в матрице R выделим подматрицу R_{37} .
 Ребро (x_3x_7) пересекается с $(x_1x_4), (x_2x_4)$
 Определим p_{412} , для чего в матрице R выделим подматрицу R_{412} .
 Ребро (x_4x_{12}) пересекается с $(x_1x_{11}), (x_2x_8), (x_2x_9), (x_2x_{11}), (x_3x_7), (x_3x_8), (x_3x_{10}), (x_3x_{11})$
 Определим p_{410} , для чего в матрице R выделим подматрицу R_{410} .
 Ребро (x_4x_{10}) пересекается с $(x_2x_8), (x_2x_9), (x_3x_7), (x_3x_8)$
 15 пересечений графа найдено, закончим поиск.

	p_{13}	p_{212}	p_{14}	p_{111}	p_{211}	p_{29}	p_{28}	p_{24}	p_{312}	p_{311}	p_{310}	p_{38}	p_{37}	p_{412}	p_{410}
p_{13}	1	1	0	0	1	1	1	1	0	0	0	0	0	0	0
p_{212}	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0
p_{14}	0	1	1	0	1	1	1	0	1	1	1	1	1	0	0
p_{111}	0	1	0	1	0	0	0	0	1	0	0	0	0	1	0
p_{211}	1	0	1	0	1	0	0	0	1	0	0	0	0	1	0
p_{29}	1	0	1	0	0	1	0	0	1	1	1	0	0	1	1
p_{28}	1	0	1	0	0	0	1	0	1	1	1	0	0	1	1
p_{24}	1	0	0	0	0	0	0	1	1	1	1	1	1	0	0
p_{312}	0	0	1	1	1	1	1	1	1	0	0	0	0	0	0
p_{311}	0	0	1	0	0	1	1	1	0	1	0	0	0	1	0
p_{310}	0	0	1	0	0	1	1	1	0	0	1	0	0	1	0
p_{38}	0	0	1	0	0	0	0	1	0	0	0	1	0	1	1
p_{37}	0	0	1	0	0	0	0	1	0	0	0	0	1	1	1
p_{412}	0	0	0	1	1	1	1	0	0	1	1	1	1	1	0
p_{410}	0	0	0	0	0	1	1	0	0	0	0	1	1	0	1

4. Построение семейства ψG

$$\begin{aligned}
M_{13} &= r_1 \vee r_3 = 11101111111100 \\
M_{134} &= M_{13} \vee r_4 = 11111111111110 \\
M_{13415} &= M_{134} \vee r_{15} = 11111111111111 \\
M_{1314} &= M_{13} \vee r_{14} = 11111111111110 \\
M_{131415} &= M_{1314} \vee r_{15} = 11111111111111 \\
M_{1315} &= M_{13} \vee r_{15} = 11101111111101 \\
M_{14} &= r_1 \vee r_4 = 110111111000010 \\
M_{1410} &= M_{14} \vee r_{10} = 11111111100010 \\
M_{141011} &= M_{1410} \vee r_{11} = 11111111110010 \\
M_{14101112} &= M_{141011} \vee r_{12} = 11111111111011 \\
M_{1410111213} &= M_{14101112} \vee r_{13} = 11111111111111 \\
M_{14101113} &= M_{141011} \vee r_{13} = 11111111111011 \\
M_{14101115} &= M_{141011} \vee r_{15} = 11111111111111 \\
M_{141012} &= M_{1410} \vee r_{12} = 11111111110101 \\
M_{141013} &= M_{1410} \vee r_{13} = 11111111110011 \\
M_{141015} &= M_{1410} \vee r_{15} = 11111111110111 \\
M_{1411} &= M_{14} \vee r_{11} = 111111111010010 \\
M_{1412} &= M_{14} \vee r_{12} = 111111111001011 \\
M_{1413} &= M_{14} \vee r_{13} = 111111111000111 \\
M_{1415} &= M_{14} \vee r_{15} = 110111111001111 \\
M_{19} &= r_1 \vee r_9 = 111111111000000 \\
M_{1910} &= M_{19} \vee r_{10} = 111111111100010 \\
M_{191011} &= M_{1910} \vee r_{11} = 111111111110010 \\
M_{19101112} &= M_{191011} \vee r_{12} = 111111111111011 \\
M_{1910111213} &= M_{19101112} \vee r_{13} = 11111111111111 \\
M_{19101113} &= M_{191011} \vee r_{13} = 111111111110111 \\
M_{19101115} &= M_{191011} \vee r_{15} = 11111111111111 \\
M_{191012} &= M_{1910} \vee r_{12} = 111111111101011 \\
M_{191013} &= M_{1910} \vee r_{13} = 111111111100111 \\
M_{191015} &= M_{1910} \vee r_{15} = 111111111101111 \\
M_{1911} &= M_{19} \vee r_{11} = 111111111010010 \\
M_{1912} &= M_{19} \vee r_{12} = 111111111001011 \\
M_{1913} &= M_{19} \vee r_{13} = 111111111000111 \\
M_{1914} &= M_{19} \vee r_{14} = 11111111111110 \\
M_{191415} &= M_{1914} \vee r_{15} = 11111111111111 \\
M_{1915} &= M_{19} \vee r_{15} = 111111111001101 \\
M_{110} &= r_1 \vee r_{10} = 111011110100010 \\
M_{111} &= r_1 \vee r_{11} = 111011110010010 \\
M_{112} &= r_1 \vee r_{12} = 111011110001011 \\
M_{113} &= r_1 \vee r_{13} = 111011110000111 \\
M_{114} &= r_1 \vee r_{14} = 110111110111110 \\
M_{115} &= r_1 \vee r_{15} = 110011110001101 \\
M_{25} &= r_2 \vee r_5 = 111110001000010 \\
M_{256} &= M_{25} \vee r_6 = 11111001110011 \\
M_{2567} &= M_{256} \vee r_7 = 11111101110011 \\
M_{25678} &= M_{2567} \vee r_8 = 11111111111111 \\
M_{256712} &= M_{2567} \vee r_{12} = 11111111111011 \\
M_{25671213} &= M_{256712} \vee r_{13} = 11111111111111 \\
M_{256713} &= M_{2567} \vee r_{13} = 11111111111011 \\
M_{2568} &= M_{256} \vee r_8 = 11111011111111 \\
M_{25612} &= M_{256} \vee r_{12} = 11111011111011 \\
M_{25613} &= M_{256} \vee r_{13} = 11111011111011 \\
M_{257} &= M_{25} \vee r_7 = 111110101110011 \\
M_{258} &= M_{25} \vee r_8 = 111110011111110 \\
M_{25815} &= M_{258} \vee r_{15} = 11111111111111 \\
M_{2510} &= M_{25} \vee r_{10} = 11111111100010 \\
M_{251011} &= M_{2510} \vee r_{11} = 111111111110010 \\
M_{25101112} &= M_{251011} \vee r_{12} = 111111111111011 \\
M_{2510111213} &= M_{25101112} \vee r_{13} = 11111111111111 \\
M_{25101113} &= M_{251011} \vee r_{13} = 111111111110111 \\
M_{25101115} &= M_{251011} \vee r_{15} = 11111111111111 \\
M_{251012} &= M_{2510} \vee r_{12} = 111111111101011 \\
M_{251013} &= M_{2510} \vee r_{13} = 111111111100111 \\
M_{251015} &= M_{2510} \vee r_{15} = 111111111101111 \\
M_{2511} &= M_{25} \vee r_{11} = 111111111010010 \\
M_{2512} &= M_{25} \vee r_{12} = 111110011001011 \\
M_{2513} &= M_{25} \vee r_{13} = 111110011000111 \\
M_{2515} &= M_{25} \vee r_{15} = 11111101001111 \\
M_{26} &= r_2 \vee r_6 = 111101001110011 \\
M_{27} &= r_2 \vee r_7 = 111100101110011 \\
M_{28} &= r_2 \vee r_8 = 111100011111100 \\
M_{2814} &= M_{28} \vee r_{14} = 111111111111110 \\
M_{281415} &= M_{2814} \vee r_{15} = 111111111111111 \\
M_{2815} &= M_{28} \vee r_{15} = 111101111111101 \\
M_{29} &= r_2 \vee r_9 = 111111111000000 \\
M_{2910} &= M_{29} \vee r_{10} = 111111111100010 \\
M_{291011} &= M_{2910} \vee r_{11} = 111111111110010 \\
M_{29101112} &= M_{291011} \vee r_{12} = 111111111111011 \\
M_{2910111213} &= M_{29101112} \vee r_{13} = 111111111111111 \\
M_{29101113} &= M_{291011} \vee r_{13} = 111111111110111 \\
M_{29101115} &= M_{291011} \vee r_{15} = 111111111111111 \\
M_{291012} &= M_{2910} \vee r_{12} = 111111111101011 \\
M_{291013} &= M_{2910} \vee r_{13} = 111111111100111 \\
M_{291015} &= M_{2910} \vee r_{15} = 111111111101111
\end{aligned}$$

$$\begin{aligned}
M_{2911} &= M_{29} \vee r_{11} = 111111111010010 \\
M_{2912} &= M_{29} \vee r_{12} = 111111111001011 \\
M_{2913} &= M_{29} \vee r_{13} = 111111111000111 \\
M_{2914} &= M_{29} \vee r_{14} = 111111111111110 \\
M_{291415} &= M_{2914} \vee r_{15} = 111111111111111 \\
M_{2915} &= M_{29} \vee r_{15} = 111111111001101 \\
M_{210} &= r_2 \vee r_{10} = 111101110100010 \\
M_{211} &= r_2 \vee r_{11} = 111101110010010 \\
M_{212} &= r_2 \vee r_{12} = 111100010001011 \\
M_{213} &= r_2 \vee r_{13} = 111100010000111 \\
M_{214} &= r_2 \vee r_{14} = 111111100111110 \\
M_{215} &= r_2 \vee r_{15} = 111101100001101
\end{aligned}$$

Из матрицы $R(G')$ видно, что строки с номерами $j > 4$ не смогут закрыть ноль в позиции 2.

Семейство максимальных внутренне устойчивых множеств ψG построено. Это:

$$\begin{aligned}
\psi_1 &= \{u_1 3, u_1 4, u_1 11, u_4 10\} \\
\psi_2 &= \{u_1 3, u_1 4, u_4 12, u_4 10\} \\
\psi_3 &= \{u_1 3, u_1 11, u_3 11, u_3 10, u_3 8, u_3 7\} \\
\psi_4 &= \{u_1 3, u_1 11, u_3 11, u_3 10, u_4 10\} \\
\psi_5 &= \{u_1 3, u_3 12, u_3 11, u_3 10, u_3 8, u_3 7\} \\
\psi_6 &= \{u_1 3, u_3 12, u_3 11, u_3 10, u_4 10\} \\
\psi_7 &= \{u_1 3, u_3 12, u_4 12, u_4 10\} \\
\psi_8 &= \{u_2 12, u_2 11, u_2 9, u_2 8, u_2 4\} \\
\psi_9 &= \{u_2 12, u_2 11, u_2 9, u_2 8, u_3 8, u_3 7\} \\
\psi_{10} &= \{u_2 12, u_2 11, u_2 4, u_4 10\} \\
\psi_{11} &= \{u_2 12, u_2 11, u_3 11, u_3 10, u_3 8, u_3 7\} \\
\psi_{12} &= \{u_2 12, u_2 11, u_3 11, u_3 10, u_4 10\} \\
\psi_{13} &= \{u_2 12, u_2 4, u_4 12, u_4 10\} \\
\psi_{14} &= \{u_2 12, u_3 12, u_3 11, u_3 10, u_3 8, u_3 7\} \\
\psi_{15} &= \{u_2 12, u_3 12, u_3 11, u_3 10, u_4 10\} \\
\psi_{16} &= \{u_2 12, u_3 12, u_4 12, u_4 10\} \\
\psi_{17} &= \{u_1 4, u_1 11, u_2 4, u_4 10\} \\
\psi_{18} &= \{u_1 4, u_2 4, u_4 12, u_4 10\} \\
\psi_{19} &= \{u_1 11, u_2 11, u_2 9, u_2 8, u_2 4\} \\
\psi_{20} &= \{u_1 11, u_2 11, u_2 9, u_2 8, u_3 8, u_3 7\} \\
\psi_{21} &= \{u_1 11, u_2 11, u_2 4, u_4 10\} \\
\psi_{22} &= \{u_1 11, u_2 11, u_3 11, u_3 10, u_3 8, u_3 7\} \\
\psi_{23} &= \{u_1 11, u_2 11, u_3 11, u_3 10, u_4 10\}
\end{aligned}$$

5. Выделение из G' максимального двудольного подграфа H'

Для каждой пары множеств вычислим значение критерия $\alpha\psi\beta = |\psi\gamma| + |\psi\beta| - |\psi\gamma \cap \psi\beta|$:

$$\alpha_{12} = |\psi_1| + |\psi_2| - |\psi_1 \cap \psi_2| = 4 + 4 - 3 = 5$$

$$\alpha_{13} = |\psi_1| + |\psi_3| - |\psi_1 \cap \psi_3| = 4 + 6 - 2 = 8$$

$\alpha_{14}=|\psi_1|+|\psi_4|-|\psi_1\cap\psi_4|=4+5-3=6$
 $\alpha_{15}=|\psi_1|+|\psi_5|-|\psi_1\cap\psi_5|=4+6-1=9$
 $\alpha_{16}=|\psi_1|+|\psi_6|-|\psi_1\cap\psi_6|=4+5-2=7$
 $\alpha_{17}=|\psi_1|+|\psi_7|-|\psi_1\cap\psi_7|=4+4-2=6$
 $\alpha_{18}=|\psi_1|+|\psi_8|-|\psi_1\cap\psi_8|=4+5-0=9$
 $\alpha_{19}=|\psi_1|+|\psi_9|-|\psi_1\cap\psi_9|=4+6-0=10$
 $\alpha_{110}=|\psi_1|+|\psi_{10}|-|\psi_1\cap\psi_{10}|=4+4-1=7$
 $\alpha_{111}=|\psi_1|+|\psi_{11}|-|\psi_1\cap\psi_{11}|=4+6-0=10$
 $\alpha_{112}=|\psi_1|+|\psi_{12}|-|\psi_1\cap\psi_{12}|=4+5-1=8$
 $\alpha_{113}=|\psi_1|+|\psi_{13}|-|\psi_1\cap\psi_{13}|=4+4-1=7$
 $\alpha_{114}=|\psi_1|+|\psi_{14}|-|\psi_1\cap\psi_{14}|=4+6-0=10$
 $\alpha_{115}=|\psi_1|+|\psi_{15}|-|\psi_1\cap\psi_{15}|=4+5-1=8$
 $\alpha_{116}=|\psi_1|+|\psi_{16}|-|\psi_1\cap\psi_{16}|=4+4-1=7$
 $\alpha_{117}=|\psi_1|+|\psi_{17}|-|\psi_1\cap\psi_{17}|=4+4-3=5$
 $\alpha_{118}=|\psi_1|+|\psi_{18}|-|\psi_1\cap\psi_{18}|=4+4-2=6$
 $\alpha_{119}=|\psi_1|+|\psi_{19}|-|\psi_1\cap\psi_{19}|=4+5-1=8$
 $\alpha_{120}=|\psi_1|+|\psi_{20}|-|\psi_1\cap\psi_{20}|=4+6-1=9$
 $\alpha_{121}=|\psi_1|+|\psi_{21}|-|\psi_1\cap\psi_{21}|=4+4-2=6$
 $\alpha_{122}=|\psi_1|+|\psi_{22}|-|\psi_1\cap\psi_{22}|=4+6-1=9$
 $\alpha_{123}=|\psi_1|+|\psi_{23}|-|\psi_1\cap\psi_{23}|=4+5-2=7$
 $\alpha_{23}=|\psi_2|+|\psi_3|-|\psi_2\cap\psi_3|=4+6-1=9$
 $\alpha_{24}=|\psi_2|+|\psi_4|-|\psi_2\cap\psi_4|=4+5-2=7$
 $\alpha_{25}=|\psi_2|+|\psi_5|-|\psi_2\cap\psi_5|=4+6-1=9$
 $\alpha_{26}=|\psi_2|+|\psi_6|-|\psi_2\cap\psi_6|=4+5-2=7$
 $\alpha_{27}=|\psi_2|+|\psi_7|-|\psi_2\cap\psi_7|=4+4-3=5$
 $\alpha_{28}=|\psi_2|+|\psi_8|-|\psi_2\cap\psi_8|=4+5-0=9$
 $\alpha_{29}=|\psi_2|+|\psi_9|-|\psi_2\cap\psi_9|=4+6-0=10$
 $\alpha_{210}=|\psi_2|+|\psi_{10}|-|\psi_2\cap\psi_{10}|=4+4-1=7$
 $\alpha_{211}=|\psi_2|+|\psi_{11}|-|\psi_2\cap\psi_{11}|=4+6-0=10$
 $\alpha_{212}=|\psi_2|+|\psi_{12}|-|\psi_2\cap\psi_{12}|=4+5-1=8$
 $\alpha_{213}=|\psi_2|+|\psi_{13}|-|\psi_2\cap\psi_{13}|=4+4-2=6$
 $\alpha_{214}=|\psi_2|+|\psi_{14}|-|\psi_2\cap\psi_{14}|=4+6-0=10$
 $\alpha_{215}=|\psi_2|+|\psi_{15}|-|\psi_2\cap\psi_{15}|=4+5-1=8$
 $\alpha_{216}=|\psi_2|+|\psi_{16}|-|\psi_2\cap\psi_{16}|=4+4-2=6$
 $\alpha_{217}=|\psi_2|+|\psi_{17}|-|\psi_2\cap\psi_{17}|=4+4-2=6$
 $\alpha_{218}=|\psi_2|+|\psi_{18}|-|\psi_2\cap\psi_{18}|=4+4-3=5$
 $\alpha_{219}=|\psi_2|+|\psi_{19}|-|\psi_2\cap\psi_{19}|=4+5-0=9$
 $\alpha_{220}=|\psi_2|+|\psi_{20}|-|\psi_2\cap\psi_{20}|=4+6-0=10$
 $\alpha_{221}=|\psi_2|+|\psi_{21}|-|\psi_2\cap\psi_{21}|=4+4-1=7$
 $\alpha_{222}=|\psi_2|+|\psi_{22}|-|\psi_2\cap\psi_{22}|=4+6-0=10$
 $\alpha_{223}=|\psi_2|+|\psi_{23}|-|\psi_2\cap\psi_{23}|=4+5-1=8$
 $\alpha_{34}=|\psi_3|+|\psi_4|-|\psi_3\cap\psi_4|=6+5-4=7$
 $\alpha_{35}=|\psi_3|+|\psi_5|-|\psi_3\cap\psi_5|=6+6-5=7$
 $\alpha_{36}=|\psi_3|+|\psi_6|-|\psi_3\cap\psi_6|=6+5-3=8$

$$\begin{aligned}
\alpha_{37} &= |\psi_3| + |\psi_7| - |\psi_3 \cap \psi_7| = 6 + 4 - 1 = 9 \\
\alpha_{38} &= |\psi_3| + |\psi_8| - |\psi_3 \cap \psi_8| = 6 + 5 - 0 = 11 \\
\alpha_{39} &= |\psi_3| + |\psi_9| - |\psi_3 \cap \psi_9| = 6 + 6 - 2 = 10 \\
\alpha_{310} &= |\psi_3| + |\psi_{10}| - |\psi_3 \cap \psi_{10}| = 6 + 4 - 0 = 10 \\
\alpha_{311} &= |\psi_3| + |\psi_{11}| - |\psi_3 \cap \psi_{11}| = 6 + 6 - 4 = 8 \\
\alpha_{312} &= |\psi_3| + |\psi_{12}| - |\psi_3 \cap \psi_{12}| = 6 + 5 - 2 = 9 \\
\alpha_{313} &= |\psi_3| + |\psi_{13}| - |\psi_3 \cap \psi_{13}| = 6 + 4 - 0 = 10 \\
\alpha_{314} &= |\psi_3| + |\psi_{14}| - |\psi_3 \cap \psi_{14}| = 6 + 6 - 4 = 8 \\
\alpha_{315} &= |\psi_3| + |\psi_{15}| - |\psi_3 \cap \psi_{15}| = 6 + 5 - 2 = 9 \\
\alpha_{316} &= |\psi_3| + |\psi_{16}| - |\psi_3 \cap \psi_{16}| = 6 + 4 - 0 = 10 \\
\alpha_{317} &= |\psi_3| + |\psi_{17}| - |\psi_3 \cap \psi_{17}| = 6 + 4 - 1 = 9 \\
\alpha_{318} &= |\psi_3| + |\psi_{18}| - |\psi_3 \cap \psi_{18}| = 6 + 4 - 0 = 10 \\
\alpha_{319} &= |\psi_3| + |\psi_{19}| - |\psi_3 \cap \psi_{19}| = 6 + 5 - 1 = 10 \\
\alpha_{320} &= |\psi_3| + |\psi_{20}| - |\psi_3 \cap \psi_{20}| = 6 + 6 - 3 = 9 \\
\alpha_{321} &= |\psi_3| + |\psi_{21}| - |\psi_3 \cap \psi_{21}| = 6 + 4 - 1 = 9 \\
\alpha_{322} &= |\psi_3| + |\psi_{22}| - |\psi_3 \cap \psi_{22}| = 6 + 6 - 5 = 7 \\
\alpha_{323} &= |\psi_3| + |\psi_{23}| - |\psi_3 \cap \psi_{23}| = 6 + 5 - 3 = 8 \\
\alpha_{45} &= |\psi_4| + |\psi_5| - |\psi_4 \cap \psi_5| = 5 + 6 - 3 = 8 \\
\alpha_{46} &= |\psi_4| + |\psi_6| - |\psi_4 \cap \psi_6| = 5 + 5 - 4 = 6 \\
\alpha_{47} &= |\psi_4| + |\psi_7| - |\psi_4 \cap \psi_7| = 5 + 4 - 2 = 7 \\
\alpha_{48} &= |\psi_4| + |\psi_8| - |\psi_4 \cap \psi_8| = 5 + 5 - 0 = 10 \\
\alpha_{49} &= |\psi_4| + |\psi_9| - |\psi_4 \cap \psi_9| = 5 + 6 - 0 = 11 \\
\alpha_{410} &= |\psi_4| + |\psi_{10}| - |\psi_4 \cap \psi_{10}| = 5 + 4 - 1 = 8 \\
\alpha_{411} &= |\psi_4| + |\psi_{11}| - |\psi_4 \cap \psi_{11}| = 5 + 6 - 2 = 9 \\
\alpha_{412} &= |\psi_4| + |\psi_{12}| - |\psi_4 \cap \psi_{12}| = 5 + 5 - 3 = 7 \\
\alpha_{413} &= |\psi_4| + |\psi_{13}| - |\psi_4 \cap \psi_{13}| = 5 + 4 - 1 = 8 \\
\alpha_{414} &= |\psi_4| + |\psi_{14}| - |\psi_4 \cap \psi_{14}| = 5 + 6 - 2 = 9 \\
\alpha_{415} &= |\psi_4| + |\psi_{15}| - |\psi_4 \cap \psi_{15}| = 5 + 5 - 3 = 7 \\
\alpha_{416} &= |\psi_4| + |\psi_{16}| - |\psi_4 \cap \psi_{16}| = 5 + 4 - 1 = 8 \\
\alpha_{417} &= |\psi_4| + |\psi_{17}| - |\psi_4 \cap \psi_{17}| = 5 + 4 - 2 = 7 \\
\alpha_{418} &= |\psi_4| + |\psi_{18}| - |\psi_4 \cap \psi_{18}| = 5 + 4 - 1 = 8 \\
\alpha_{419} &= |\psi_4| + |\psi_{19}| - |\psi_4 \cap \psi_{19}| = 5 + 5 - 1 = 9 \\
\alpha_{420} &= |\psi_4| + |\psi_{20}| - |\psi_4 \cap \psi_{20}| = 5 + 6 - 1 = 10 \\
\alpha_{421} &= |\psi_4| + |\psi_{21}| - |\psi_4 \cap \psi_{21}| = 5 + 4 - 2 = 7 \\
\alpha_{422} &= |\psi_4| + |\psi_{22}| - |\psi_4 \cap \psi_{22}| = 5 + 6 - 3 = 8 \\
\alpha_{423} &= |\psi_4| + |\psi_{23}| - |\psi_4 \cap \psi_{23}| = 5 + 5 - 4 = 6 \\
\alpha_{56} &= |\psi_5| + |\psi_6| - |\psi_5 \cap \psi_6| = 6 + 5 - 4 = 7 \\
\alpha_{57} &= |\psi_5| + |\psi_7| - |\psi_5 \cap \psi_7| = 6 + 4 - 2 = 8 \\
\alpha_{58} &= |\psi_5| + |\psi_8| - |\psi_5 \cap \psi_8| = 6 + 5 - 0 = 11 \\
\alpha_{59} &= |\psi_5| + |\psi_9| - |\psi_5 \cap \psi_9| = 6 + 6 - 2 = 10 \\
\alpha_{510} &= |\psi_5| + |\psi_{10}| - |\psi_5 \cap \psi_{10}| = 6 + 4 - 0 = 10 \\
\alpha_{511} &= |\psi_5| + |\psi_{11}| - |\psi_5 \cap \psi_{11}| = 6 + 6 - 4 = 8 \\
\alpha_{512} &= |\psi_5| + |\psi_{12}| - |\psi_5 \cap \psi_{12}| = 6 + 5 - 2 = 9 \\
\alpha_{513} &= |\psi_5| + |\psi_{13}| - |\psi_5 \cap \psi_{13}| = 6 + 4 - 0 = 10
\end{aligned}$$

$$\begin{aligned}
\alpha_{514} &= |\psi_5| + |\psi_{14}| - |\psi_5 \cap \psi_{14}| = 6 + 6 - 5 = 7 \\
\alpha_{515} &= |\psi_5| + |\psi_{15}| - |\psi_5 \cap \psi_{15}| = 6 + 5 - 3 = 8 \\
\alpha_{516} &= |\psi_5| + |\psi_{16}| - |\psi_5 \cap \psi_{16}| = 6 + 4 - 1 = 9 \\
\alpha_{517} &= |\psi_5| + |\psi_{17}| - |\psi_5 \cap \psi_{17}| = 6 + 4 - 0 = 10 \\
\alpha_{518} &= |\psi_5| + |\psi_{18}| - |\psi_5 \cap \psi_{18}| = 6 + 4 - 0 = 10 \\
\alpha_{519} &= |\psi_5| + |\psi_{19}| - |\psi_5 \cap \psi_{19}| = 6 + 5 - 0 = 11 \\
\alpha_{520} &= |\psi_5| + |\psi_{20}| - |\psi_5 \cap \psi_{20}| = 6 + 6 - 2 = 10 \\
\alpha_{521} &= |\psi_5| + |\psi_{21}| - |\psi_5 \cap \psi_{21}| = 6 + 4 - 0 = 10 \\
\alpha_{522} &= |\psi_5| + |\psi_{22}| - |\psi_5 \cap \psi_{22}| = 6 + 6 - 4 = 8 \\
\alpha_{523} &= |\psi_5| + |\psi_{23}| - |\psi_5 \cap \psi_{23}| = 6 + 5 - 2 = 9 \\
\alpha_{67} &= |\psi_6| + |\psi_7| - |\psi_6 \cap \psi_7| = 5 + 4 - 3 = 6 \\
\alpha_{68} &= |\psi_6| + |\psi_8| - |\psi_6 \cap \psi_8| = 5 + 5 - 0 = 10 \\
\alpha_{69} &= |\psi_6| + |\psi_9| - |\psi_6 \cap \psi_9| = 5 + 6 - 0 = 11 \\
\alpha_{610} &= |\psi_6| + |\psi_{10}| - |\psi_6 \cap \psi_{10}| = 5 + 4 - 1 = 8 \\
\alpha_{611} &= |\psi_6| + |\psi_{11}| - |\psi_6 \cap \psi_{11}| = 5 + 6 - 2 = 9 \\
\alpha_{612} &= |\psi_6| + |\psi_{12}| - |\psi_6 \cap \psi_{12}| = 5 + 5 - 3 = 7 \\
\alpha_{613} &= |\psi_6| + |\psi_{13}| - |\psi_6 \cap \psi_{13}| = 5 + 4 - 1 = 8 \\
\alpha_{614} &= |\psi_6| + |\psi_{14}| - |\psi_6 \cap \psi_{14}| = 5 + 6 - 3 = 8 \\
\alpha_{615} &= |\psi_6| + |\psi_{15}| - |\psi_6 \cap \psi_{15}| = 5 + 5 - 4 = 6 \\
\alpha_{616} &= |\psi_6| + |\psi_{16}| - |\psi_6 \cap \psi_{16}| = 5 + 4 - 2 = 7 \\
\alpha_{617} &= |\psi_6| + |\psi_{17}| - |\psi_6 \cap \psi_{17}| = 5 + 4 - 1 = 8 \\
\alpha_{618} &= |\psi_6| + |\psi_{18}| - |\psi_6 \cap \psi_{18}| = 5 + 4 - 1 = 8 \\
\alpha_{619} &= |\psi_6| + |\psi_{19}| - |\psi_6 \cap \psi_{19}| = 5 + 5 - 0 = 10 \\
\alpha_{620} &= |\psi_6| + |\psi_{20}| - |\psi_6 \cap \psi_{20}| = 5 + 6 - 0 = 11 \\
\alpha_{621} &= |\psi_6| + |\psi_{21}| - |\psi_6 \cap \psi_{21}| = 5 + 4 - 1 = 8 \\
\alpha_{622} &= |\psi_6| + |\psi_{22}| - |\psi_6 \cap \psi_{22}| = 5 + 6 - 2 = 9 \\
\alpha_{623} &= |\psi_6| + |\psi_{23}| - |\psi_6 \cap \psi_{23}| = 5 + 5 - 3 = 7 \\
\alpha_{78} &= |\psi_7| + |\psi_8| - |\psi_7 \cap \psi_8| = 4 + 5 - 0 = 9 \\
\alpha_{79} &= |\psi_7| + |\psi_9| - |\psi_7 \cap \psi_9| = 4 + 6 - 0 = 10 \\
\alpha_{710} &= |\psi_7| + |\psi_{10}| - |\psi_7 \cap \psi_{10}| = 4 + 4 - 1 = 7 \\
\alpha_{711} &= |\psi_7| + |\psi_{11}| - |\psi_7 \cap \psi_{11}| = 4 + 6 - 0 = 10 \\
\alpha_{712} &= |\psi_7| + |\psi_{12}| - |\psi_7 \cap \psi_{12}| = 4 + 5 - 1 = 8 \\
\alpha_{713} &= |\psi_7| + |\psi_{13}| - |\psi_7 \cap \psi_{13}| = 4 + 4 - 2 = 6 \\
\alpha_{714} &= |\psi_7| + |\psi_{14}| - |\psi_7 \cap \psi_{14}| = 4 + 6 - 1 = 9 \\
\alpha_{715} &= |\psi_7| + |\psi_{15}| - |\psi_7 \cap \psi_{15}| = 4 + 5 - 2 = 7 \\
\alpha_{716} &= |\psi_7| + |\psi_{16}| - |\psi_7 \cap \psi_{16}| = 4 + 4 - 3 = 5 \\
\alpha_{717} &= |\psi_7| + |\psi_{17}| - |\psi_7 \cap \psi_{17}| = 4 + 4 - 1 = 7 \\
\alpha_{718} &= |\psi_7| + |\psi_{18}| - |\psi_7 \cap \psi_{18}| = 4 + 4 - 2 = 6 \\
\alpha_{719} &= |\psi_7| + |\psi_{19}| - |\psi_7 \cap \psi_{19}| = 4 + 5 - 0 = 9 \\
\alpha_{720} &= |\psi_7| + |\psi_{20}| - |\psi_7 \cap \psi_{20}| = 4 + 6 - 0 = 10 \\
\alpha_{721} &= |\psi_7| + |\psi_{21}| - |\psi_7 \cap \psi_{21}| = 4 + 4 - 1 = 7 \\
\alpha_{722} &= |\psi_7| + |\psi_{22}| - |\psi_7 \cap \psi_{22}| = 4 + 6 - 0 = 10 \\
\alpha_{723} &= |\psi_7| + |\psi_{23}| - |\psi_7 \cap \psi_{23}| = 4 + 5 - 1 = 8 \\
\alpha_{89} &= |\psi_8| + |\psi_9| - |\psi_8 \cap \psi_9| = 5 + 6 - 4 = 7
\end{aligned}$$

$\alpha_{810}=|\psi_8|+|\psi_{10}|-|\psi_8\cap\psi_{10}|=5+4-3=6$
 $\alpha_{811}=|\psi_8|+|\psi_{11}|-|\psi_8\cap\psi_{11}|=5+6-2=9$
 $\alpha_{812}=|\psi_8|+|\psi_{12}|-|\psi_8\cap\psi_{12}|=5+5-2=8$
 $\alpha_{813}=|\psi_8|+|\psi_{13}|-|\psi_8\cap\psi_{13}|=5+4-2=7$
 $\alpha_{814}=|\psi_8|+|\psi_{14}|-|\psi_8\cap\psi_{14}|=5+6-1=10$
 $\alpha_{815}=|\psi_8|+|\psi_{15}|-|\psi_8\cap\psi_{15}|=5+5-1=9$
 $\alpha_{816}=|\psi_8|+|\psi_{16}|-|\psi_8\cap\psi_{16}|=5+4-1=8$
 $\alpha_{817}=|\psi_8|+|\psi_{17}|-|\psi_8\cap\psi_{17}|=5+4-1=8$
 $\alpha_{818}=|\psi_8|+|\psi_{18}|-|\psi_8\cap\psi_{18}|=5+4-1=8$
 $\alpha_{819}=|\psi_8|+|\psi_{19}|-|\psi_8\cap\psi_{19}|=5+5-4=6$
 $\alpha_{820}=|\psi_8|+|\psi_{20}|-|\psi_8\cap\psi_{20}|=5+6-3=8$
 $\alpha_{821}=|\psi_8|+|\psi_{21}|-|\psi_8\cap\psi_{21}|=5+4-2=7$
 $\alpha_{822}=|\psi_8|+|\psi_{22}|-|\psi_8\cap\psi_{22}|=5+6-1=10$
 $\alpha_{823}=|\psi_8|+|\psi_{23}|-|\psi_8\cap\psi_{23}|=5+5-1=9$
 $\alpha_{910}=|\psi_9|+|\psi_{10}|-|\psi_9\cap\psi_{10}|=6+4-2=8$
 $\alpha_{911}=|\psi_9|+|\psi_{11}|-|\psi_9\cap\psi_{11}|=6+6-4=8$
 $\alpha_{912}=|\psi_9|+|\psi_{12}|-|\psi_9\cap\psi_{12}|=6+5-2=9$
 $\alpha_{913}=|\psi_9|+|\psi_{13}|-|\psi_9\cap\psi_{13}|=6+4-1=9$
 $\alpha_{914}=|\psi_9|+|\psi_{14}|-|\psi_9\cap\psi_{14}|=6+6-3=9$
 $\alpha_{915}=|\psi_9|+|\psi_{15}|-|\psi_9\cap\psi_{15}|=6+5-1=10$
 $\alpha_{916}=|\psi_9|+|\psi_{16}|-|\psi_9\cap\psi_{16}|=6+4-1=9$
 $\alpha_{917}=|\psi_9|+|\psi_{17}|-|\psi_9\cap\psi_{17}|=6+4-0=10$
 $\alpha_{918}=|\psi_9|+|\psi_{18}|-|\psi_9\cap\psi_{18}|=6+4-0=10$
 $\alpha_{919}=|\psi_9|+|\psi_{19}|-|\psi_9\cap\psi_{19}|=6+5-3=8$
 $\alpha_{920}=|\psi_9|+|\psi_{20}|-|\psi_9\cap\psi_{20}|=6+6-5=7$
 $\alpha_{921}=|\psi_9|+|\psi_{21}|-|\psi_9\cap\psi_{21}|=6+4-1=9$
 $\alpha_{922}=|\psi_9|+|\psi_{22}|-|\psi_9\cap\psi_{22}|=6+6-3=9$
 $\alpha_{923}=|\psi_9|+|\psi_{23}|-|\psi_9\cap\psi_{23}|=6+5-1=10$
 $\alpha_{1011}=|\psi_{10}|+|\psi_{11}|-|\psi_{10}\cap\psi_{11}|=4+6-2=8$
 $\alpha_{1012}=|\psi_{10}|+|\psi_{12}|-|\psi_{10}\cap\psi_{12}|=4+5-3=6$
 $\alpha_{1013}=|\psi_{10}|+|\psi_{13}|-|\psi_{10}\cap\psi_{13}|=4+4-3=5$
 $\alpha_{1014}=|\psi_{10}|+|\psi_{14}|-|\psi_{10}\cap\psi_{14}|=4+6-1=9$
 $\alpha_{1015}=|\psi_{10}|+|\psi_{15}|-|\psi_{10}\cap\psi_{15}|=4+5-2=7$
 $\alpha_{1016}=|\psi_{10}|+|\psi_{16}|-|\psi_{10}\cap\psi_{16}|=4+4-2=6$
 $\alpha_{1017}=|\psi_{10}|+|\psi_{17}|-|\psi_{10}\cap\psi_{17}|=4+4-2=6$
 $\alpha_{1018}=|\psi_{10}|+|\psi_{18}|-|\psi_{10}\cap\psi_{18}|=4+4-2=6$
 $\alpha_{1019}=|\psi_{10}|+|\psi_{19}|-|\psi_{10}\cap\psi_{19}|=4+5-2=7$
 $\alpha_{1020}=|\psi_{10}|+|\psi_{20}|-|\psi_{10}\cap\psi_{20}|=4+6-1=9$
 $\alpha_{1021}=|\psi_{10}|+|\psi_{21}|-|\psi_{10}\cap\psi_{21}|=4+4-3=5$
 $\alpha_{1022}=|\psi_{10}|+|\psi_{22}|-|\psi_{10}\cap\psi_{22}|=4+6-1=9$
 $\alpha_{1023}=|\psi_{10}|+|\psi_{23}|-|\psi_{10}\cap\psi_{23}|=4+5-2=7$
 $\alpha_{1112}=|\psi_{11}|+|\psi_{12}|-|\psi_{11}\cap\psi_{12}|=6+5-4=7$
 $\alpha_{1113}=|\psi_{11}|+|\psi_{13}|-|\psi_{11}\cap\psi_{13}|=6+4-1=9$
 $\alpha_{1114}=|\psi_{11}|+|\psi_{14}|-|\psi_{11}\cap\psi_{14}|=6+6-5=7$

$\alpha_{1115}=|\psi_{11}|+|\psi_{15}|-|\psi_{11}\cap\psi_{15}|=6+5-3=8$
 $\alpha_{1116}=|\psi_{11}|+|\psi_{16}|-|\psi_{11}\cap\psi_{16}|=6+4-1=9$
 $\alpha_{1117}=|\psi_{11}|+|\psi_{17}|-|\psi_{11}\cap\psi_{17}|=6+4-0=10$
 $\alpha_{1118}=|\psi_{11}|+|\psi_{18}|-|\psi_{11}\cap\psi_{18}|=6+4-0=10$
 $\alpha_{1119}=|\psi_{11}|+|\psi_{19}|-|\psi_{11}\cap\psi_{19}|=6+5-1=10$
 $\alpha_{1120}=|\psi_{11}|+|\psi_{20}|-|\psi_{11}\cap\psi_{20}|=6+6-3=9$
 $\alpha_{1121}=|\psi_{11}|+|\psi_{21}|-|\psi_{11}\cap\psi_{21}|=6+4-1=9$
 $\alpha_{1122}=|\psi_{11}|+|\psi_{22}|-|\psi_{11}\cap\psi_{22}|=6+6-5=7$
 $\alpha_{1123}=|\psi_{11}|+|\psi_{23}|-|\psi_{11}\cap\psi_{23}|=6+5-3=8$
 $\alpha_{1213}=|\psi_{12}|+|\psi_{13}|-|\psi_{12}\cap\psi_{13}|=5+4-2=7$
 $\alpha_{1214}=|\psi_{12}|+|\psi_{14}|-|\psi_{12}\cap\psi_{14}|=5+6-3=8$
 $\alpha_{1215}=|\psi_{12}|+|\psi_{15}|-|\psi_{12}\cap\psi_{15}|=5+5-4=6$
 $\alpha_{1216}=|\psi_{12}|+|\psi_{16}|-|\psi_{12}\cap\psi_{16}|=5+4-2=7$
 $\alpha_{1217}=|\psi_{12}|+|\psi_{17}|-|\psi_{12}\cap\psi_{17}|=5+4-1=8$
 $\alpha_{1218}=|\psi_{12}|+|\psi_{18}|-|\psi_{12}\cap\psi_{18}|=5+4-1=8$
 $\alpha_{1219}=|\psi_{12}|+|\psi_{19}|-|\psi_{12}\cap\psi_{19}|=5+5-1=9$
 $\alpha_{1220}=|\psi_{12}|+|\psi_{20}|-|\psi_{12}\cap\psi_{20}|=5+6-1=10$
 $\alpha_{1221}=|\psi_{12}|+|\psi_{21}|-|\psi_{12}\cap\psi_{21}|=5+4-2=7$
 $\alpha_{1222}=|\psi_{12}|+|\psi_{22}|-|\psi_{12}\cap\psi_{22}|=5+6-3=8$
 $\alpha_{1223}=|\psi_{12}|+|\psi_{23}|-|\psi_{12}\cap\psi_{23}|=5+5-4=6$
 $\alpha_{1314}=|\psi_{13}|+|\psi_{14}|-|\psi_{13}\cap\psi_{14}|=4+6-1=9$
 $\alpha_{1315}=|\psi_{13}|+|\psi_{15}|-|\psi_{13}\cap\psi_{15}|=4+5-2=7$
 $\alpha_{1316}=|\psi_{13}|+|\psi_{16}|-|\psi_{13}\cap\psi_{16}|=4+4-3=5$
 $\alpha_{1317}=|\psi_{13}|+|\psi_{17}|-|\psi_{13}\cap\psi_{17}|=4+4-2=6$
 $\alpha_{1318}=|\psi_{13}|+|\psi_{18}|-|\psi_{13}\cap\psi_{18}|=4+4-3=5$
 $\alpha_{1319}=|\psi_{13}|+|\psi_{19}|-|\psi_{13}\cap\psi_{19}|=4+5-1=8$
 $\alpha_{1320}=|\psi_{13}|+|\psi_{20}|-|\psi_{13}\cap\psi_{20}|=4+6-0=10$
 $\alpha_{1321}=|\psi_{13}|+|\psi_{21}|-|\psi_{13}\cap\psi_{21}|=4+4-2=6$
 $\alpha_{1322}=|\psi_{13}|+|\psi_{22}|-|\psi_{13}\cap\psi_{22}|=4+6-0=10$
 $\alpha_{1323}=|\psi_{13}|+|\psi_{23}|-|\psi_{13}\cap\psi_{23}|=4+5-1=8$
 $\alpha_{1415}=|\psi_{14}|+|\psi_{15}|-|\psi_{14}\cap\psi_{15}|=6+5-4=7$
 $\alpha_{1416}=|\psi_{14}|+|\psi_{16}|-|\psi_{14}\cap\psi_{16}|=6+4-2=8$
 $\alpha_{1417}=|\psi_{14}|+|\psi_{17}|-|\psi_{14}\cap\psi_{17}|=6+4-0=10$
 $\alpha_{1418}=|\psi_{14}|+|\psi_{18}|-|\psi_{14}\cap\psi_{18}|=6+4-0=10$
 $\alpha_{1419}=|\psi_{14}|+|\psi_{19}|-|\psi_{14}\cap\psi_{19}|=6+5-0=11$
 $\alpha_{1420}=|\psi_{14}|+|\psi_{20}|-|\psi_{14}\cap\psi_{20}|=6+6-2=10$
 $\alpha_{1421}=|\psi_{14}|+|\psi_{21}|-|\psi_{14}\cap\psi_{21}|=6+4-0=10$
 $\alpha_{1422}=|\psi_{14}|+|\psi_{22}|-|\psi_{14}\cap\psi_{22}|=6+6-4=8$
 $\alpha_{1423}=|\psi_{14}|+|\psi_{23}|-|\psi_{14}\cap\psi_{23}|=6+5-2=9$
 $\alpha_{1516}=|\psi_{15}|+|\psi_{16}|-|\psi_{15}\cap\psi_{16}|=5+4-3=6$
 $\alpha_{1517}=|\psi_{15}|+|\psi_{17}|-|\psi_{15}\cap\psi_{17}|=5+4-1=8$
 $\alpha_{1518}=|\psi_{15}|+|\psi_{18}|-|\psi_{15}\cap\psi_{18}|=5+4-1=8$
 $\alpha_{1519}=|\psi_{15}|+|\psi_{19}|-|\psi_{15}\cap\psi_{19}|=5+5-0=10$
 $\alpha_{1520}=|\psi_{15}|+|\psi_{20}|-|\psi_{15}\cap\psi_{20}|=5+6-0=11$

$\alpha_{1521}=|\psi_{15}|+|\psi_{21}|-|\psi_{15}\cap\psi_{21}|=5+4-1=8$
 $\alpha_{1522}=|\psi_{15}|+|\psi_{22}|-|\psi_{15}\cap\psi_{22}|=5+6-2=9$
 $\alpha_{1523}=|\psi_{15}|+|\psi_{23}|-|\psi_{15}\cap\psi_{23}|=5+5-3=7$
 $\alpha_{1617}=|\psi_{16}|+|\psi_{17}|-|\psi_{16}\cap\psi_{17}|=4+4-1=7$
 $\alpha_{1618}=|\psi_{16}|+|\psi_{18}|-|\psi_{16}\cap\psi_{18}|=4+4-2=6$
 $\alpha_{1619}=|\psi_{16}|+|\psi_{19}|-|\psi_{16}\cap\psi_{19}|=4+5-0=9$
 $\alpha_{1620}=|\psi_{16}|+|\psi_{20}|-|\psi_{16}\cap\psi_{20}|=4+6-0=10$
 $\alpha_{1621}=|\psi_{16}|+|\psi_{21}|-|\psi_{16}\cap\psi_{21}|=4+4-1=7$
 $\alpha_{1622}=|\psi_{16}|+|\psi_{22}|-|\psi_{16}\cap\psi_{22}|=4+6-0=10$
 $\alpha_{1623}=|\psi_{16}|+|\psi_{23}|-|\psi_{16}\cap\psi_{23}|=4+5-1=8$
 $\alpha_{1718}=|\psi_{17}|+|\psi_{18}|-|\psi_{17}\cap\psi_{18}|=4+4-3=5$
 $\alpha_{1719}=|\psi_{17}|+|\psi_{19}|-|\psi_{17}\cap\psi_{19}|=4+5-2=7$
 $\alpha_{1720}=|\psi_{17}|+|\psi_{20}|-|\psi_{17}\cap\psi_{20}|=4+6-1=9$
 $\alpha_{1721}=|\psi_{17}|+|\psi_{21}|-|\psi_{17}\cap\psi_{21}|=4+4-3=5$
 $\alpha_{1722}=|\psi_{17}|+|\psi_{22}|-|\psi_{17}\cap\psi_{22}|=4+6-1=9$
 $\alpha_{1723}=|\psi_{17}|+|\psi_{23}|-|\psi_{17}\cap\psi_{23}|=4+5-2=7$
 $\alpha_{1819}=|\psi_{18}|+|\psi_{19}|-|\psi_{18}\cap\psi_{19}|=4+5-1=8$
 $\alpha_{1820}=|\psi_{18}|+|\psi_{20}|-|\psi_{18}\cap\psi_{20}|=4+6-0=10$
 $\alpha_{1821}=|\psi_{18}|+|\psi_{21}|-|\psi_{18}\cap\psi_{21}|=4+4-2=6$
 $\alpha_{1822}=|\psi_{18}|+|\psi_{22}|-|\psi_{18}\cap\psi_{22}|=4+6-0=10$
 $\alpha_{1823}=|\psi_{18}|+|\psi_{23}|-|\psi_{18}\cap\psi_{23}|=4+5-1=8$
 $\alpha_{1920}=|\psi_{19}|+|\psi_{20}|-|\psi_{19}\cap\psi_{20}|=5+6-4=7$
 $\alpha_{1921}=|\psi_{19}|+|\psi_{21}|-|\psi_{19}\cap\psi_{21}|=5+4-3=6$
 $\alpha_{1922}=|\psi_{19}|+|\psi_{22}|-|\psi_{19}\cap\psi_{22}|=5+6-2=9$
 $\alpha_{1923}=|\psi_{19}|+|\psi_{23}|-|\psi_{19}\cap\psi_{23}|=5+5-2=8$
 $\alpha_{2021}=|\psi_{20}|+|\psi_{21}|-|\psi_{20}\cap\psi_{21}|=6+4-2=8$
 $\alpha_{2022}=|\psi_{20}|+|\psi_{22}|-|\psi_{20}\cap\psi_{22}|=6+6-4=8$
 $\alpha_{2023}=|\psi_{20}|+|\psi_{23}|-|\psi_{20}\cap\psi_{23}|=6+5-2=9$
 $\alpha_{2122}=|\psi_{21}|+|\psi_{22}|-|\psi_{21}\cap\psi_{22}|=4+6-2=8$
 $\alpha_{2123}=|\psi_{21}|+|\psi_{23}|-|\psi_{21}\cap\psi_{23}|=4+5-3=6$
 $\alpha_{2223}=|\psi_{22}|+|\psi_{23}|-|\psi_{22}\cap\psi_{23}|=6+5-4=7$

- 5 8 6 9 7 6 9 10 7 10 8 7 10 8 7 5 6 8 9 6 9 7
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 - - - 7 7 8 9 11 10 10 8 9 10 8 9 10 9 10 10 9 9 7 8
 - - - - 8 6 7 10 11 8 9 7 8 9 7 8 7 8 9 10 7 8 6
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 - - - - - - - - - - - - - - - - - - 8 6
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