实验 3 序贯最小优化算法(SMO)

一、实验目的

理解 SMO 算法的工作原理,编程实现该算法并用于非线性分类问题。

二、实验内容

基于 SMO 算法,自定义一个实现核 SVM 分类器的类,其中核函数分别取 RBF 核和多项式核。利用所设计的 SMO 类,在乳腺癌数据训练一个 SVM 分类器,测试其分类精度,并与 SKlearn 中的 SVM 进行比较。

三、实验环境

硬件: CPU i5-8300H, 内存 8G, 硬盘 SAMSUNG 512G 固态

软件: win10 家庭版、python3.7、Visual Studio Code

数据: 乳腺癌数据集

四、实验原理

1. 序贯最小优化算法(Sequential Minimal Optimization,SMO)基本思想

SMO 算法用于求解凸二次规划问题。其主要思想是选取两个变量(lagranger 乘子)作为待更新的变量,固定其他变量,针对这两个变量变量构造一个二次规划问题,用解析方法求解该子问题,如此反复直到所有的解都满足该最优化问题的 KKT 条件。

2. 算法描述

表 1 算法描述

输入: 训练数据集 T = { $(x_1,y_1),...,(x_N,y_N)$ } 测试数据 $x^* = (x^{*(1)},...,x^{*(M)})^T$

过程:

(1) 初始化参数:

$$W_0 = 0, k = 0, \alpha^{(0)} = (0,...,0)$$

计算数据的核函数矩阵 $K_{ii} = K(x_{ii}, x_{i})$ 和 $E^{(0)}$,其中 i = 1,...,N, j = 1,...,N

(2) 选取优化变量 $\alpha_m^{(k)}$, $\alpha_n^{(k)}$:

选取第一个变量 $\alpha_m^{(k)}$ 的过程为外层循环。交替地在"整个样本集上"和"非界样本子集上多次遍历"选取。

选取第二个变量 $\alpha_n^{(k)}$ 的过程为内层循环。选择这一变量的原则是使优化过程中步长最大,加速收敛。即选取 $|E_m-E_n|$ 最大的 $\alpha_n^{(k)}$ 。但是这样一来,同一个变量 $\alpha_n^{(k)}$ 很可能被多次选择,因此,在部分迭代过程中也通过随机选取来获得第二个变量 $\alpha_n^{(k)}$ 。

(3) 求解两个变量的最优化问题,求得最优解 $\alpha_m^{(k+1)}$, $\alpha_n^{(k+1)}$,更新参数 $\alpha^{(k)}$ 为 $\alpha^{(k+1)}$,并利用更新后的参数计算 $W_0^{(k+1)}$ 和 $E^{(k+1)}$,更新参数的计算公式如下:

$$\alpha_n^{(k+1)} = \max\left(\min\left(\alpha_n^{(k)} + \frac{y_n(E_m^{(k)} - E_n^{(k)})}{K_{mm} + K_{nn} - 2K_{mn}}\right), L\right)$$

$$\alpha_m^{(k+1)} = \alpha_m^{(k)} + y_m y_n (\alpha_n^{(k)} - \alpha_n^{(k+1)})$$

$$W_{0(1)}^{(k+1)} = -E_m^{(k)} + W_0^{(k)} + \left(\alpha_m^{(k)} - \alpha_m^{(k+1)}\right) y_m K_{mm} + (\alpha_n^{(k)} - \alpha_n^{(k+1)}) y_n K_{nm}$$

$$W_{0(2)}^{(k+1)} = -E_n^{(k)} + W_0^{(k)} + \left(\alpha_m^{(k)} - \alpha_m^{(k+1)}\right) y_m K_{mn} + (\alpha_n^{(k)} - \alpha_n^{(k+1)}) y_n K_{nn}$$

$$W_0^{(k+1)} = \frac{W_{0(1)}^{(k+1)} + W_{0(2)}^{(k+1)}}{2}$$

$$E_j^{(k+1)} = g(y_j) - y_j = \left(\sum_{i=1}^N \alpha_i^{(k+1)} y_i K_{ij} + W_0^{(k+1)}\right) - y_j$$

- (4) 重复步骤(2)(3)直到精度 ϵ 范围内满足停机条件。取 $\hat{\alpha} = \alpha^{(k+1)}$
- (5) 根据 $\hat{\alpha}$ 计算 $f(x^*)$

$$W_0 = W_0^{(k+1)}$$

$$f(x^*) = \sum_{i=1}^{N} \widehat{\alpha}_i y_i K(x_i, x^*) + W_0$$

(6) 输出数据所属类别v*

$$y^* = \begin{cases} 1, & f(x^*) \ge 0 \\ -1, & f(x^*) < 0 \end{cases}$$

输出: 测试数据 $x^* = (x^{*(1)},...,x^{*(M)})^T$ 所属类别 y^*

3. 类设计

表 2 SVM_Model 类的方法

方法	描述
init	初始化模型参数
svm_fit	训练 svm 模型
predict	对输入数据进行预测
update_am_an	更新 smo 算法的两个优化变量
update_b	更新参数 b
SMO	smo 算法集成
smo_iter	smo 算法单次迭代
smo_update	smo 更新参数
find_an	选择 SMO 算法的第二个优化变量
KKT_check	检查 KKT 条件
g_x	计算 g(x)
set_kernel	设置核
K_xy	计算核函数
validate	验证预测的精度
cross_validate	对模型进行交叉验证

五、实验步骤

实验数据简介:

实验所用到的乳腺癌数据集总共 569 个样本数据,每个样本的特征维度为 30 维。

1、 载入乳腺癌数据

```
# 导入数据集
breast_cancer = load_breast_cancer()
dataset = breast_cancer['data']
feature_names = breast_cancer['feature_names']
target = breast_cancer['target']
target_names = breast_cancer['target_names']
```

2、 建立 SVM 的模型,并初始化

```
# 模型初始化

svm_model = SVM_Model()

# SVM模型初始化

svm_model.init(C=10,kernel="rbf",max_iter=100)
```

3、 对乳腺癌数据集进行标准化,以便能够正确的进行训练和分类

```
# 对数据进行标准化
X,Y = svm_model.normalize(dataset,target)
```

4、划分训练集和测试集

```
# 划分训练集和测试集
x_train,x_test, y_train, y_test = train_test_split(X,Y,test_size = 0.15,random_state = 1)
```

5、 训练模型——迭代

① 将训练数据载入模型,同时初始化模型参数,计算核函数映射矩阵等

② SMO 算法外层循环,交替遍历所有样本和非界样本,选取第一个优化变量

③ 内层循环,根据优化原则选取第二个优化变量,如果优化后参数无明显变化,则第二个优化变量采用随机选取

```
# 内层循环,选取第二个优化点
am = self.A[m]
n,an = self.find_an(m)
if(n ==-1):
    return
# smo迭代更新参数
am_new,an_new = self.smo_update(m,n)

# 如果参数不变,则随机选不为m的点
if(an_new == an) and (am_new == am):
    # 随机选取不为m的点
    n = m
    while n == m:
        n = randint(1,self.n_data-1)
# smo更新参数
am_new,an_new = self.smo_update(m,n)
return
```

④ 根据选取的优化变量更新模型参数

```
# SMO 单次更新

def smo_update(self,m,n):

smo_update_start = time.time()

# 更新am,an

am,an = self.update_am_an(m,n)

# 更新b

self.update_b(m,n,am,an)

# 更新E

for i in range(self.n_data):

    self.E[i] = self.g_x(i) - self.Y[i]

self.A[m] = am

self.A[n] = an

smo_update_end = time.time()

self.smo_update_count += 1

#print("SMO update cost = {:.4f} s".format(smo_update_end - smo_update_start))

return am,an
```

⑤ 当满足 KKT 条件,模型参数迭代过程中无明显变化或超出最大迭代次数时退出

```
# 满足KKT条件退出
if(KKT_correct_cnt >= self.n_data):
    break
dec_A = np.sum(A_Record-self.A)

# 参数无明显变化退出
if (dec_A == 0):
    break
```

6、对输入数据进行预测

```
# svm预测

def predict(self,x_predict):
    y_predict = list()
    for xi in x_predict:
        yi = 0
        ay = np.multiply(self.A,self.Y)
        for i in range(self.n_data):
            yi += ay[i]*self.K_xy(self.X[i],xi)
        yi += self.b
        if yi >= 0:
            yi =1
        else:
            yi =-1
        y_predict.append(yi)
    return y_predict
```

7、 对模型输入进行验证

```
# 验证数据

def validate(self,y_predict,y_test):
    accuracy = np.sum(y_predict==y_test)/len(y_test)
    return accuracy
```

8、与 sklearn 的 svm 模型进行比对

```
print("-----
                    .....")
print("My SVM Model:")
# 使用My SVM Model
smo_time_start = time.time()
svm_model.svm_fit(x_train,y_train)
y_predict = svm_model.predict(x_test)
accuracy = svm_model.validate(y_predict,y_test)
smo_time_end = time.time()
print("Time Cost = {:.3f} s".format(smo_time_end - smo_time_start))
print("My SVM accuracy : {:.4f}".format(accuracy))
smo_time_start = time.time()
svm = SVC(kernel='rbf',C=1.0,random_state= 0)
svm.fit(x_train,y_train)
y_result = svm.predict(x_test) # 使用模型预测值
accuracy = svm_model.validate(y_result,y_test)
smo_time_end = time.time()
print("Time Cost = {:.3f} s".format(smo_time_end - smo_time_start))
print("Sklearn SVM accuracy : {:.4f}".format(accuracy))
```

9、对模型进行交叉验证,评价模型指标

```
# 交叉验证
svm_model.cross_validate(X,Y,cv=7)
```

五、实验结果

① 使用 RBF 核(gamma = 1/30) 交叉验证

```
=====Cross validation Begin=======
Iter 1:
SVM Time Cost = 3.666 s
accuracy: 0.9643
Iter 2:
SVM Time Cost = 3.702 s
accuracy : 0.9464
SVM Time Cost = 2.882 s
accuracy: 0.8929
SVM Time Cost = 3.023 s
accuracy : 0.9286
Iter 5:
SVM Time Cost = 2.788 s
accuracy : 0.8750
Iter 6:
SVM Time Cost = 3.062 s
accuracy : 0.9464
SVM Time Cost = 3.164 s
accuracy: 0.9286
Iter 8:
SVM Time Cost = 3.459 s
accuracy : 0.9286
Iter 9:
SVM Time Cost = 2.942 s
accuracy: 0.8036
Iter 10:
SVM Time Cost = 3.461 s
accuracy: 0.9821
           -----Cross validation End-----
Average accuracy : 0.9196
```

模型比较

② 使用多项式核(degree=3, coef0=0)

交叉验证

```
-----Cross validation Begin-----
Iter 1:
SVM Time Cost = 2.464 s
accuracy : 0.9821
Iter 2:
SVM Time Cost = 2.219 s
accuracy: 0.9821
Iter 3:
SVM Time Cost = 2.258 s
accuracy : 0.8393
Iter 4:

SVM Time Cost = 2.635 s

accuracy : 1.0000

Iter 5:
SVM Time Cost = 2.007 s
accuracy: 0.9107
Iter 6:
SVM Time Cost = 2.307 \text{ s}
accuracy: 0.9286
Iter 7:
SVM Time Cost = 2.301 s
accuracy : 0.8214
Iter 8:
SVM Time Cost = 2.587 s
accuracy : 0.8571
Iter 9:
SVM Time Cost = 2.340 s
accuracy : 0.9107
Iter 10:
SVM Time Cost = 2.552 \text{ s}
accuracy : 0.9286
             ====Cross validation End======
Average accuracy : 0.9161
```

模型比对

附件

实验代码: SMO 算法

```
from sklearn.svm import SVC
from sklearn.preprocessing import StandardScaler
from sklearn.datasets import load_breast_cancer
from sklearn.model_selection import train_test_split
import numpy as np
import matplotlib.pyplot as plt
import math
from random import randint
import time
# SVM 模型,使用 SMO 算法进行优化求解
class SVM Model:
   def SVM_Model():
       self.init()
   def init(self,C = np.inf,e p = 1e-6,KKT e=0.1,kernel = "linear",max ite
r = 100,gamma = "auto",coef0=0,degree=3):
       # 软间隔容忍因子
       self.C = C
       # 精度范围
       self.e_p = e_p
       # KKT 条件容忍度
       self.KKT e = KKT e
       # 设置映射核函数
       self.kernel = dict()
       self.max_iter = max_iter
       # 设置核函数
       self.set_kernel(kernel,gamma=gamma,coef0=coef0,degree=degree)
   # svm 数据标准化
   def normalize(self,dataset,target):
       Y = np.array(target)
       for i in range(len(Y)):
```

```
if Y[i] != 1:
           Y[i] = -1
   X = np.array(dataset)
   X -= np.mean(X,0)
   X \neq np.std(X,0)
   return X,Y
def svm_fit(self,dataset,target):
   # 数据
   self.data_shape = dataset.shape
   self.n_data = self.data_shape[0]
   self.n_feature = self.data_shape[1]
   self.X = np.array(dataset) # 训练数据
   self.Y = np.array(target) # 标签
   # 分类超平面参数
   self.b = 0
    self.K = np.zeros([self.n_data,self.n_data])
    for i in range(self.n_data):
        for j in range(self.n_data):
           self.K[i][j] = self.K_xy(self.X[i],self.X[j])
   # 约束最优化参数
   self.A = np.zeros(self.n_data)
   self.E = np.zeros(self.n_data)
    for i in range(self.n_data):
        self.E[i] = self.g_x(i) - self.Y[i]
    # SMO 算法选取两个 a 更新
    self.SMO(self.max_iter)
# svm 预测
def predict(self,x_predict):
   y_predict =list()
   for xi in x_predict:
       yi = 0
        ay = np.multiply(self.A, self.Y)
       for i in range(self.n_data):
```

```
yi += ay[i]*self.K_xy(self.X[i],xi)
           yi += self.b
           if yi >= 0:
               yi =1
               yi = -1
           y_predict.append(yi)
       return y_predict
   # SMO 更新参数 a1,a2
   def update_am_an(self,m,n):
       am = self.A[m]
       an = self.A[n]
       E1 = self.E[m]
       E2 = self.E[n]
       K11 = self.K[m,m]
       K22 = self.K[n,n]
       K12 = self.K[m,n]
       u = K11+K22 - 2*K12
       u = max(self.e_p,u)
       an += self.Y[n]*(E1-E2)/u
       L = 0
       H = self.C
       if self.Y[m] == self.Y[n]:
           L = max(0, self.A[m] + self.A[n] - self.C) # max(0,a2+a1-C)
           H = min(self.C, self.A[m] + self.A[n]) # min(C,a2+a1)
           L = max(0, self.A[n] - self.A[m]) # max(0,a2-a1)
           H = min(self.C, self.A[n] - self.A[m] + self.C) # min(C,a2-a1+
       # 控制 an 范围
       if(H<L):
           print(" ======== H({}) < L({}) ========".format(H,</pre>
L))
           print("m = {} n = {} a1 = {} a2 = {} equal_flag = {}".format(
m,n,am,an,self.Y[m] == self.Y[n]))
           an = 0
       else:
           an = max(min(an,H),L)
       # 计算 am
```

```
am += self.Y[m]*self.Y[n]*(self.A[n]-an)
   # 控制 am 范围
   am = max(0,am)
   return am, an
# SMO 更新 b
def update_b(self,m,n,a1,a2):
   E1 = self.E[m]
   E2 = self.E[n]
   K11 = self.K[m,m]
   K12 = self.K[m,n]
   K21 = self.K[n,m]
   K22 = self.K[n,n]
   y1 = self.Y[m]
   y2 = self.Y[n]
   # 计算 b1
   b1 = -E1 - y1*K11*(a1 - self.A[m]) - y2*K21*(a2 - self.A[n]) + self
   # 计算 b2
   b2 = -E2 - y1*K12*(a1 - self.A[m]) - y2*K22*(a2 - self.A[n]) + self.
   self.b = (b1+b2)/2
# SMO 算法求解约束最优化问题
def SMO(self,max_iter):
   # SMO 算法迭代
   iter count = 0
   Out_Loop_flag = 0 #外层循环的 flag
   # print('||========SMO START========||')
   while iter_count<max_iter:</pre>
       self.smo_update_count = 0
       # print('\n----Iter {}----'.format(iter_count))
       iter_time_start = time.time()
       if(iter_count % 5) == 0:
           Out_Loop_flag = 0 # 遍历所有样本
           Out_Loop_flag = 1 # 遍历非界样本
       iter_count += 1
```

```
# 外层循环,找到最违反 KKT 条件的点 m
m = 0
am = 0
exceed_error = 0
error yg = 0
A Record = self.A.copy()
# 外层循环,选取所有违反 KKT 条件的参数,将其作为第一个优化参数
for i in range(self.n_data):
    if Out_Loop_flag:
       ai = self.A[i]
       if not self.KKT_check(i,ai):
           if (np.abs(ai)<self.e_p):</pre>
               self.smo_iter(i)
    else:
       ai = self.A[i]
       if not self.KKT_check(i,ai):
           if (np.abs(ai)<self.e_p):</pre>
               self.smo_iter(i)
# 计算是否满足 KKT 条件
KKT_correct_cnt = 0
for i in range(self.n_data):
   ai = self.A[i]
   if self.KKT_check(i,ai):
       KKT_correct_cnt +=1
iter_time_end = time.time()
```

```
time start))
          # print("Update Count = {:d} ".format(self.smo_update_count))
          #满足KKT条件退出
          if(KKT_correct_cnt >= self.n_data):
              break
          dec_A = np.sum(A_Record-self.A)
          # 参数无明显变化退出
          if (dec_A == 0):
              break
  # SMO 单次迭代
  def smo_iter(self,m):
      # 内层循环,选取第二个优化点
      am = self.A[m]
      n,an = self.find_an(m)
      if(n ==-1):
          return
      # smo 迭代更新参数
      am_new,an_new = self.smo_update(m,n)
      # 如果参数不变,则随机选不为 m 的点
      if(an_new == an) and (am_new == am):
          # 随机选取不为 m 的点
          n = m
          while n == m:
              n = randint(1,self.n_data-1)
          # smo 更新参数
          am_new,an_new = self.smo_update(m,n)
          return
  # SMO 单次更新
  def smo_update(self,m,n):
      smo_update_start = time.time()
      # 更新 am, an
      am,an = self.update_am_an(m,n)
```

```
# 更新 b
       self.update_b(m,n,am,an)
       # 更新 E
       for i in range(self.n_data):
           self.E[i] = self.g_x(i) - self.Y[i]
       self.A[m] = am
       self.A[n] = an
       smo_update_end = time.time()
       self.smo_update_count += 1
       #print("SMO update cost = {:.4f} s".format(smo_update_end - smo_upd
ate_start))
       return am, an
   # SMO 第二参数选取
   def find_an(self,m):
       E1 = self.E[m]
       E2 = 0
       e f = 1
       an = 0
       if E1 < 0:
          # 取最大的 E2
           e_f = 1
       else:
           # 取最小的 E2
           e_f = -1
       for i in range(self.n_data):
           Ei = self.E[i]
           if (i!=m) and (Ei*e_f >= E2*e_f):
               E2 = Ei
               an = self.A[n]
       return n,an
   #核对是否满足KKT条件
   def KKT_check(self,i,ai):
       KKT_flag = False
       g = self.g_x(i)
       yg = self.Y[i]*g
       if (np.abs(ai)<self.e_p):</pre>
           KKT_flag = (yg >= 1-self.KKT_e)
```

```
elif (ai>=self.e_p) and (ai <=self.C-self.e_p):</pre>
        KKT_flag = (np.abs(yg-self.C) <self.e_p)</pre>
    elif np.abs(ai-self.C)<self.e_p:</pre>
        KKT_flag = (yg <= 1 + self.KKT_e)</pre>
    return KKT_flag
# 计算 g(x)
def g_x(self,i):
    \# g = w*fi(x) + b = sumj(aj*yj*Kji) + b
    g = np.dot(np.multiply(self.A,self.Y),self.K[:,i]) + self.b
    return g
# degree 当为 poly 核时多项式的最高次数
# 设置映射核函数
def set_kernel(self,kernel = 'None',gamma = "auto",coef0=0,degree=3):
    if kernel == 'rbf':
        self.kernel['name'] = 'rbf'
        self.kernel['gamma'] = gamma
    elif kernel == 'linear':
        self.kernel['name'] = 'linear'
    elif kernel == 'poly':
        self.kernel['name'] = 'poly'
        self.kernel['degree'] = degree
        self.kernel['coef0'] = coef0
    else:
        self.kernel['name'] = 'None'
# 计算核函数映射后的 K(x,y)
def K_xy(self,x,y):
    K_xy = 0
    if self.kernel['name'] == 'rbf':
        gamma = 0
        if self.kernel['gamma'] == 'auto':
            gamma = 1/self.n_feature
        else:
            gamma = self.kernel['gamma']
        K_xy = np.exp(-gamma*np.linalg.norm(x-y)**2)
    elif self.kernel['name'] == 'linear':
        K_xy = np.dot(x,y)
    elif self.kernel['name'] == 'poly':
        degree = self.kernel['degree']
        coef0 = self.kernel['coef0']
```

```
K_xy = (np.dot(x,y) + coef0)**degree
   else:
       K_xy = np.dot(x,y)
    return K_xy
# 验证数据
def validate(self,y_predict,y_test):
   accuracy = np.sum(y_predict==y_test)/len(y_test)
   return accuracy
# 交叉验证数据
def cross_validate(self,X,Y,cv = 10):
   n_data = X.shape[0]
   n_feature = X.shape[1]
   # 合并数据和标签
   DataSet = np.array(np.c_[X,Y.T])
   # 打乱数据集
   np.random.shuffle(DataSet)
   # print(DataSet.shape)
   folds = list()
   # 分割数据集
   for k in range(cv):
       fold_size = np.int(n_data/cv)
       l_index = max(k*fold_size,0)
       h_index = min((k+1)*fold_size,n_data)
       fold_k =DataSet[l_index:h_index]
       folds.append(fold_k)
   accuracy = 0
   print("=========Cross validation Begin========")
    # 交叉验证
    for i in range(cv):
       # 划分数据集和测试集
       x_train = np.array([])
       y_train = np.array([])
       x_test = np.array([])
       y_test = np.array([])
       for k in range(cv):
           X_k = folds[k][:,0:n_feature]
          Y_k = folds[k][:,n_feature]
```

```
if k != i:
                   if np.any(x_train):
                       x_train = np.r_[x_train,X_k]
                       y_train = np.r_[y_train,Y_k]
                   else:
                       x_{train} = X_k
                       y_train = Y_k
               else:
                   x_{test} = X_k
                   y_{test} = Y_k
           smo_time_start = time.time()
           self.svm_fit(x_train,y_train)
           y_predict = self.predict(x_test)
           accuracy_k = self.validate(y_predict,y_test)
           accuracy += accuracy_k
           smo_time_end = time.time()
           print("Iter {}:".format(i+1))
           print("SVM Time Cost = {:.3f} s".format(smo_time_end - smo_time
_start))
           print("accuracy : {:.4f}".format(accuracy_k))
       accuracy /= cv
       print("=======Cross validation End=======")
       print("Average accuracy : {:.4f}".format(accuracy))
def main():
   breast_cancer = load_breast_cancer()
   dataset = breast_cancer['data']
   feature_names = breast_cancer['feature_names']
   target = breast_cancer['target']
   target_names = breast_cancer['target_names']
   svm model = SVM Model()
```

```
# SVM 模型初始化
   svm model.init(C=10,kernel="poly",max iter=100)
   # 对数据进行标准化
   X,Y = svm_model.normalize(dataset, target)
   # 交叉验证
   svm_model.cross_validate(X,Y,cv=10)
   print("=========Model comparison========")
   # 划分训练集和测试集
   x_train,x_test, y_train, y_test = train_test_split(X,Y,test_size = 0.15
random state = 1)
   print("train size: {:d} test size: {:d}".format(len(x_train),len(y_tes
t)))
   print("----")
   print("My SVM Model:")
   smo_time_start = time.time()
   svm_model.svm_fit(x_train,y_train)
   y_predict = svm_model.predict(x_test)
   accuracy = svm_model.validate(y_predict,y_test)
   smo_time_end = time.time()
   print("Time Cost = {:.3f} s".format(smo_time_end - smo_time_start))
   print("My SVM accuracy : {:.4f}".format(accuracy))
   print("----")
   # 使用 Sklearn SVM
   smo time start = time.time()
   svm = SVC(kernel='poly',C=1.0,random_state= 0,gamma='auto')
   svm.fit(x_train,y_train)
   y_result = svm.predict(x_test) # 使用模型预测值
   accuracy = svm_model.validate(y_result,y_test)
   smo_time_end = time.time()
   print("Time Cost = {:.3f} s".format(smo_time_end - smo_time_start))
   print("Sklearn SVM accuracy : {:.4f}".format(accuracy))
   print("----")
if __name__ == '__main__':
  main()
```