Understanding Analysis Chapter 1.2

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Question 1.

Give a definition for greatest lower bound and prove a lemma analagous to 1.3.8

Definition 1. *Let* $A \subseteq \mathbb{R}$ *and let* $l \in \mathbb{R}$. *We say that* $l = \inf(A)$ *if and only if*

1. *l is a lower bound for A (i.e. $l \leq a$ for all $a \in A$).*
2. *For an arbitrary lower bound L, we have that $L \leq l$.*

Lemma 1.1. *Assume that* $l \in \mathbb{R}$ *is a lower bound for a set* $A \subseteq \mathbb{R}$. *Then,* $l = \inf(A)$ *if and only if, for all choices* $\varepsilon > 0$*, we have that* $l + \varepsilon > a$ *for some* $a \in A$.

Proof. Assume $l = \inf(A)$. Then note for all $\varepsilon \geq 0$ that $l < l + \varepsilon$. Then since l is the greatest lower bound for A by definiton, we have that $l + \varepsilon$ is not a lower bound. But then there must exist $a \in A$ such that $l + \varepsilon > a$.

To prove the Other direction assume we have $l \in \mathbb{R}$ such that l is a lower bound for A with the property that for all $\varepsilon > 0$ we have $l + \varepsilon > a$ for some $a \in A$. For the sake of contradiction assume that we have $L \in \mathbb{R}$ such that $L > l$ and that $L = \inf(A)$. Then we note that by choosing $\varepsilon = -l + L$ we get that $L > a$ for some $a \in A$. Hence L is not a lower bound for A contradicting our assumtion. Hence $l = \inf(A)$. $\square$