MTH 451 Quiz 8

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Question 1.

In order to be independant we must have that for all x, y in the support, the value of the joint density function must equal the product of the marginal distributions. Observe $f(2, 1) = 0$ but $f_X(2)f_Y(1) = \frac{1}{4} * \frac{1}{4} \neq 0$, hence we have a counter example.

Question 2.

(a) We calculate the marginal distributions and check to see if their product is equalivalent to the density function.

$$f_X(x) = \frac{1}{96}\int_o^\infty e^{-(\frac{x}{8}+\frac{y}{12})}\,\mathrm{d}y = \frac{1}{96}[-12e^{-(\frac{x}{8}+\frac{y}{12})}\big|_0^\infty]$$

$$\frac{1}{96}12e^{-\frac{x}{6}}$$

Now we do the same for Y

$$f_Y(y) = \frac{1}{96}\int_0^\infty e^{-(\frac{x}{8}+\frac{y}{12})}\,\mathrm{d}x = \frac{1}{96}[-8e^{-(\frac{x}{8}+\frac{y}{12})}\big|_0^\infty]$$

$$\frac{1}{96}8e^{-\frac{y}{12}}$$

Then when we multiply these together we get

$$f_X(x) \times f_Y(y) = \frac{1}{96}e^{-(\frac{x}{8}+\frac{y}{12})}$$

So yes they are independant.

(b) we have that $\sigma_{XY} = E(XY) - E(X)E(Y)$ But since X, Y are independant we can split $E(XY) = E(X)E(Y)$. Hence $\sigma_{XY} = 0$.

Question 3.

Since we are assume X and Y to be independant we can get the joint probablity function by multiplying the marginal distributions. We find $P(X+Y=3) = f(2,1) + f(1,2) = f_X(2)f_Y(1) + f_X(1)f_Y(2) = \frac{1}{6} + \frac{2}{6} = \frac{1}{2}$.