MTH 451 Worksheet 3

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Question 1.

First I calculate $C(7,x) = \binom{7}{x}$ for $x \in \{1,2,3,4,5,6,7\}$.

$$C(7,0) = 1$$

$$C(7,1) = 7$$

$$C(7,2) = 21$$

$$C(7,3) = 35$$

$$C(7,4) = 35$$

$$C(7,5) = 21$$

$$C(7,6) = 7$$

$$C(7,7) = 1$$

In order for f to be a PMF each value of the function must be between 0 and 1 and the sum over all values of f must equal 1. We note that $\sum_{i=0}^{7} C(7,i) = 128$, so setting $c = 1/128$ will have the desired effect.

Question 2.

For f to be a PMF it must have all outputs between 0 and 1 and it must sum to 1. Let $c = 3$, then it is clear that $f(x) < 1$ for all $x \in \mathbb{N}$. Let $S = \sum_{i=1}^{\infty} 3(1/4)^i$. We must show that $S = 1$, note that S is a geometric series with common ratio 1/4 then since $1/4 < 1$ the series will converge to 3r/1 - r = (3/4)/(1 - 1/4) = 1. Thus for $C = 3$, f is a probability mass function as desired.

Question 3.

(a) We need $\int_{-\infty}^{\infty} f(x)\,\mathrm{d}x = 1$. Note that there is only a contribution on the interval from 0 to 4. Then

$$\begin{aligned}\int_0^4 \frac{c}{\sqrt{x}}\,\mathrm{d}x &= c\int_0^4 x^{-1/2}\,\mathrm{d}x \\ &= \tfrac{c}{2}x^{1/2}\Big|_0^4 = \frac{c}{2}2 = c\end{aligned}$$

Thus we select $c = 1$ so that the intergral equals 1.

(b) We want to find $P(X > 1) = \int_1^4 f(x)\,\mathrm{d}x = \int_1^4 \frac{1}{\sqrt{x}}\,\mathrm{d}x$ which is $\frac{1}{2}x^{1/2}\Big|_1^4$ so we get $2 - \frac{1}{2} = \frac{3}{2}$.

Question 4.

(a) We have $E(x) = \int_{-\infty}^{\infty} x f(X)\,\mathrm{d}x$, then plugging in $f(x)$ and noting there is only a non zero contribution on the interval $(0, 2)$ we get

$$\tfrac{1}{2}\int_0^2 x^2\,\mathrm{d}x = \tfrac{1}{2}(\tfrac{1}{3}x^3\big|_0^2) = \frac{4}{3}$$

(b) To find $E(x^2)$ we use the law of the unconscious statistican, and get

$$\tfrac{1}{2}\int_0^2 x^3\,\mathrm{d}x = \tfrac{1}{2}(\tfrac{1}{4}x^4\big|_0^2) = 2$$

(c) We use the identity $V(X) = E(x^2) - E(x)^2$ to find the variance so

$$V(X) = 2 - (\frac{4}{3})^2 = \frac{2}{9}$$

Question 5.

(a) $P(X \leq 0)$ is by definition equivalent to $F(X)$ evaluated at zero, so $P(X \leq 0) = 0.61$

(b) We want $P(X = 1)$, since for $x \in [0, 3)$ we have $F(x) = 0.61$ we can see that $P(X = 1) = 0$, since 1 cannot be in the support of X. For instance if $P(X = 1) = c$ and $c \neq 0$ then we should have $P(X \leq 0) < P(\leq 1)$ but this is not the case.

(c) We note $P(-1 < X < 4) = P(-2 \leq X \leq 3) = F(-2) - F(3) = 0.52$

(d) To find $P(X > 2)$ we observe that $P(X \leq 2) + P(X > 2) = 1$ so $P(X > 2) = 1 - F(2) = 0.39$.