MTH 451 Worksheet 8

Evan Fox (efox20@uri.edu)

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Question 1.

Let X be a random variable that counts the number of sixes. Then X is a binomial random variable. $X \sim binom(20, 1/6)$. We want the probability that X is a most 2. The PDF of X is given by

$$P(X = x) = f(x) = \binom{20}{x}(\tfrac{1}{6})^x(1 - \tfrac{1}{6})^{20-x}$$

Now we must use the CDF to calculate

$$P(X \leq 2) = \sum_{x=0}^{2} f(x) = (1 - \tfrac{1}{6})^{20} + 20(\tfrac{1}{6})^1(1 - \tfrac{1}{6})^{19}$$
$$+\binom{20}{2}(\tfrac{1}{6})^2(1 - \tfrac{1}{6})^{18}$$

The R command that we would use is pbinom(2, size=20, prob=(1/6)). When we put this into R we get 0.3287.

Question 2.

Test Size: The test size α is given by the probability we reject the null hypothesis given it is true. So assume $p = .02$ and we want to find the probability that at most one subject contracts shingles within five years. The probability that 0 people contract shingles out of 300 is $(1-.02)^{300} = 0.00233$; then the probability that one person contracts shingles is $\binom{300}{1}(0.02)(1-.02)^{299} = 0.0142$. Now to find the test size we add the previous two calculations and get

$$\alpha = .0166$$

Power: The power is very similar to the test size. We want to find the probability that we reject the null hypothesis given it is false. So let $p = .01$ then we want the probability that at most one subject contracts shingles within 5 years. The calculations are very similar

$$\binom{\beta = (1 - 0.01)^{300} + (300}{1)(.01)(1 - .01)^{300} = 0.1961}$$

Let X be a random variable that counts the number of patients that contract shingles within 5 years of getting the vaccine. Then it is clear that X will have a binomial distribution where the chance of success is $p = 0.02$ and the number of trials is $n = 300$. Thus $X \sim binom(300, .02)$.

$$P(X = x) = \binom{300}{x}(.02)^x(1 - .02)^{300-x}$$

I am not sure which probability you want the R command for, but to calculate the probability that at most 1 gets shingles I would use pbinom(1, size=300, prob = 0.02); this gives $\alpha = .01661$ as expected.

Question 3.

Let X be a random variable that counts the number of licensed drivers who get into an accident. We have $X \sim binom(150, .04)$. Let $\lambda = 150 * .04 = 6$. Then by the possion approximation distribution $X \sim pois(6)$

$$P(X \leq 3) \approx \sum_{0}^{3} \frac{e^{-6}6^x}{x!}$$

The R command will be ppois(3, 6) = 0.1512

Question 4.

Let X be a random variable that counts the number of hurricanes that hit the US. Then $X \sim pois(1.8)$. To find the p - value we want the probability that X is greater than or equal to 7 given that H_0 is true.

$$P(x \geq 7) = 1 - P(x \leq 6) = 1 - F(6)$$

where $F(x)$ is the CDF, i.e.

$$F(x) = \sum_{0}^{6} f(x) = \sum_{0}^{6} \frac{e^{-1.8}(1.8)^x}{x!}$$

So we have

$$P(x \geq 7) = 1 - \sum_{x=0}^{6} \frac{e^{-1.8}(1.8)^x}{x!}$$

The R command will be (1 - ppois(6, 1.8)) = 0.0026

Question 5.

Let X be a random variable that counts the number of bee stings per student. Then $X \sim pois(0.8)$

(a) To find the test size α we want to find the probability that 5 randomly selected campers have each had at least one bee sting given that $X \sim pois(0.8)$. Since X counts the number of stings per kid we are looking for

$$P(X \geq 1) = 1 - P(x = 0) = 1 - \frac{e^{-.8}}{1}$$

The R command gives 1 - ppois(0, .8) = 0.5506

(b) To find the power assume $X \sim pois(1.2)$ and we want to find the probability we reject the null hypothesis under this distribution.

$$P(X \geq 1) = 1 - P(x = 0) = 1 - \frac{e^{-1.2}}{1}$$

The R command gives 1 - ppois(0, 1.2) = 0.6988