MTH 451 Worksheet 6

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Question 1.

We have

$$M_X(t) = \sum_{x=1}^{\infty} 2(\frac{1}{3}^x e^{tx}) = 2\sum_{x=1}^{\infty} \left(\frac{e^t}{3}^x\right) = \frac{-2e^t}{e^t - 3}$$

This is vaild as long as $t \neq ln(3)$.

To find the mean we compute

$$E(X = M_X'(0) = \left(\frac{-2e^t}{e^t - 3}\right)' = \frac{6e^t}{(e^t - 3)^2}\Big|_{t=0} = \frac{6}{4}$$

Then to find the second moment we take the second derivitive of the MGF

$$E(X^2) = M_X^{(2)}(0) = \left(\frac{6e^t}{(e^t - 3)^2}\right)'\Big|_{t=0} = 3$$

Question 2.

(a) To find c we sum over the natural numbers union 0 and choose c such that the summation is equal to 1.

$$\sum_{x=0}^{\infty} \frac{c}{x!} = c \sum_{x=0}^{\infty} \frac{1}{x!} = c(1 + 1 + \frac{1}{2!} + ...) = ce = 1$$

hence $c = e^{-1}$.

(b)

$$M_X(t) = e^{-1} \sum e^{xt} \frac{1}{x!} = e^{-1} \times (1 + e^t + \frac{e^{2t}}{2!} + \frac{e^{3t}}{3!} + ...)$$

which gives

$$e^{-1} e^{e^t} = e^{e^t - 1}$$

Question 3.

We use the given formulas.

$$E(X) = [ln(e^{4(e^t-1)})]'\big|_{t=0} = 4e^0 = 4$$

Then

$$E(X^2) = [ln(e^{4(e^t-1)})]''\big|_{t=0} = 4e^0 = 4$$

Then using the fact that varience is given by second moment subtract the square of the first moment we get

$$V(X) = 4 - 4^2 = -12$$

We have that

$$M_Z(t) = E(e^{tZ}) = E(e^{t(\frac{x-3}{4})}) = e^{-3/4}E(e^{\frac{1}{4}tX}) = e^{-3/4}M_X(\tfrac{1}{4}t)$$

so then

$$M_Z(t) = e^{-3/4}((e^{3/4t+2t^2})) = (e^{3/4t+2t^2-\frac{3}{4}})$$

Then

$$E(Z) = \left(e^{3/4(t-1)+2t^2})\right)'\Big|_{t=0} = (3/4+4t)e^{3/4(t-1)+2t^2} = 3/4e^{-3/4}$$

and

$$E(Z^2) = \left((3/4+4t)e^{3/4(t-1)+2t^2})\right)'\Big|_{t=0} =$$

$$= (3/4)e^{3/4(t-1)+2t^2} + (3/4+4t)^2e^{3/4(t-1)+2t^2} = (3/4)e^{-3/4} + (3/4)^2e^{-3/4}$$

Hence

$$v(Z) = (3/4)e^{-3/4} + (3/4)^2e^{-3/4} - (3/4e^{-3/4})^2$$