MTH 316 Homework 1

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Question 1.

$$\mathbb{Z}^2/\langle a, b\rangle \sim (a, b)\mathbb{Z}$$

where (a, b) is the GCD and $(a, b)\mathbb{Z}$ denotes the coset

Proof. Let $(a, b) = g$ and fix $s, t \in \mathbb{Z}$ satisfying $as+bt = g$. Define $\phi : \mathbb{Z}^2 \to \mathbb{Z}$ by $\phi(x, y) = bx - ay$. it is clear ϕ is a homomorphism with $\ker \phi = \langle a, b\rangle$. Then $\phi(t, -s) = bt + as = g$ so $g \in \operatorname{im} \phi \leq \mathbb{Z}$. But every subgroup of $\mathbb{Z}$ is of the form $n\mathbb{Z}$ (since it is cyclic group every subgroup must be cyclic + every non id element has inf order) so $\operatorname{im} \phi = g\mathbb{Z}$ □