MTH 316 Homework 5

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Question 1.

Let $|G| = pq$ for primes p, q. Show that all proper subgroups of G are cyclic. Must G be cyclic?

Proof. Let $H \leq G$ be a proper subgroup so $|H| \neq |G|$. Then by Lagrange's theorem we have $|H| \mid |G|$, then the possible orders for H are $1, p$, and q. Note the trivial subgroup is generated by the identity. Then if H has prime order p each non-identity element must also have order p since the order of any element in a group must devide the order of the group and the only divisor of p greater than one is p itself, and so, it follows every element must generate H. The same argument clearly works if H has order q since all we needed was that p was prime.

G need not be cyclic: consider S_3.

□

Question 2.

Let G be an abelian group of odd order. Show that the product of all elements in G is equal to the identity.

Proof. Note that since G has odd order, no element can have order two since the order of an element must devide the order of the group. So then every element in G has a distinct inverse. Then since G is abelian we may rearange the product how we like and get

$$\Pi G = (g_1 \times g_1^{-1}) \times (g_2 \times g_2^{-1}) \cdots = e \times e \times \cdots = e$$

This completes the proof. □