MTH 435

Homework I

(1) Show that there doesn't exist a rational number s such that $s^2 = 6$.

(2) If $a,\, b \in \mathbb{R}$, show that $|a+b| = |a| + |b|$ if and only if $ab \geq 0$.

(3) Find all $x \in \mathbb{R}$ that satisfy the inequality

$$4 < |x+2| + |x-1| < 5 \ .$$

(4) Show that if $a, b \in \mathbb{R}$ then
- $\max\{a,b\} = \frac{1}{2}(a+b+|a-b|)$ and $\min\{a,b\} = \frac{1}{2}(a+b-|a-b|)$
- $\min\{a,b,c\} = \min\{\min\{a,b\},c\}$

(5) Let $S_4 = \{1-(-1)^n/n : n \in \mathbb{N}\}$. Find $\inf S_4$ and $\sup S_4$

(6) Let A and B bounded nonempty subsets of $\mathbb{R}$, and let $A+B = \{a+b : a \in A,\, b \in B\}$. Prove that $\sup(A+B) = \sup A + \sup B$ and $\inf(A+B) = \inf A + \inf B$.

(7) Let X and Y be nonempty sets and let $h : X \times Y \to \mathbb{R}$ have bounded range in $\mathbb{R}$. Let $f : X \to \mathbb{R}$ and $g : Y \to \mathbb{R}$ be defined by

$$f(x) := \sup\{h(x,y) : y \in Y\}, \quad g(y) := \inf\{h(x,y) : x \in X\}$$

Prove that

$$\sup\{g(y) : y \in Y\} \leq \inf\{f(x) : x \in X\}$$

(8) If $u > 0$ is any real number and $x < y$, show that there exists a rational number r such that $x < ru < y$. (Hence the set $\{ru : r \in \mathbb{Q}\}$ is dense in $\mathbb{R}$.)