

MTH 316 Homework 7

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April 20, 2022

Question 1.

Since the cosets of a group form a partition, if H has index 2, $H \cup aH = G$ and $H \cup Ha = G$ are the only cosets. then $aH = G - H = Ha$ so H is normal. Note that exactly half of the permutations in S_n are even and thus A_n has index 2. Then A_n is normal.

Question 2.

After listing all 8 cosets we can check their order. Note $3^8 = 1 \pmod{32}$ and so it generates the subgroup and implies $U(32)/\{1, 17\}$ is isomorphic to \mathbb{Z}_8 .