MTH 451 Quiz 1

Evan Fox (efox20@uri.edu)

January 27, 2022

Question 1.

(a) These values are not permissible because $P(D) = -0.2$ violates axiom 1 which requires that all probabliltys are non-negative.

(b) We first see that since all values are non-negative, axiom 1 is not violated. Similarly axiom 2 is not violated since the sum of all events equals 1. There is no contradiction with the axioms and so this assignment is allowable.

(c) Note by axiom two we must have $P(S) = 1$, then since A, B, C, and D are disjoint, we may apply axiom 3 to get

$$P(S) = P(A \cup B \cup C \cup D) = +P(A) + P(B) + P(C) + P(D) = \frac{18}{19} \neq 1$$

Hence by axioms 2 and 3, this probability distribution is impossible.

Question 2.

(a) We use $P(A \cup B) = P(A) + P(B) - P(A \cap B)$.

Proof. Let $A, B \subseteq S$ and let $C = A \cap B$. Then $P(A \cup B) = P(A - C \cup B) = P(A - C) + P(B)$. Now consider

$$P(A^c \cup C) + P(B^c) = 1 - P(A - C) + 1 - P(B)$$

$$P(A^c) + P(C) + P(B^c) = 1 - P(A - C) + 1 - P(B)$$

$$P(A - C) + P(B) = 1 - P(A^c) + 1 - P(B^c) - P(C)$$

$$P(A - C) + P(B) = P(A) + P(B) - P(C)$$

As desired. $\square$

Since all of these values are given we simply plug in to get

$$P(A \cup B) = 0.3 + 0.5 - 0.25 = .55$$

(b) We use $P(B/A) = P(B) - P(A \cap B)$

Proof. By definition of set difference $P(B/A) = P(B/(A \cap B))$. Now let $C = A \cap B$, then observe $B^c \cap C = \varnothing$ since $C \subseteq B$, so our use of axiom 3 in the next step is permissable. We compute

$$P(B/C) = 1 - P(B^c \cup C) = 1 - (P(B^c) + P(C))$$

$$1 - ((1 - P(B)) + P(C)) = P(B) - P(C)$$

as desired. □

Thus $P(B/C) = P(B/A) = P(B) - P(A \cap B) = 0.5 - 0.25 = 0.25$.