MTH 316 Homework 1

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Question 1.

Let G be a group such that for all $g \in G$, $g^2 = e$. Prove G is Abelian

Proof. Let $g \in G$. Then $g^2 = e$ so $g = g^{-1}$; it follows that every element of G is it's own inverse. So for $a, b \in G$ we have by closure that $ab \in G$. Thus

$$ab = (ab)^{-1} = b^{-1}a^{-1}$$

by the socks-shoes property. But then since $g = g^{-1}$ for all $g \in G$

$$ab = b^{-1}a^{-1} = ba$$

thus G is Abelian. $\square$

Question 2.

Let G be Abelian and define

$$H = \{g \in G | g^4 = e\}$$

Prove $H \leq G$.

Proof. Note $e \in G$ and $e^4 = e$ so $e \in H$. Then using the two step subgroup test we show $H \leq G$. So let $a, b \in H$, then $a^4 = e$ and $b^4 = e$. Since G is Abelian we have

$$(ab)^4 = a^4b^4 = ee = e$$

so $ab \in H$. Now we must show $b^{-1} \in H$ whenever $b \in H$. Note

$$b^4b^{-4} = b^{4-4} = b^0 = e$$

so

$$e = b^4b^{-4} = eb^{-4} = b^{-4} = (b^{-1})^4$$

Hence $b^{-1} \in H$. So by the two step subgroup test $H \leq G$. $\square$