MTH 435: Analysis and Topology

Evan Fox

September 7, 2022

Contents

Chapter 1

Introduction

1.1 Algebraic and Order properties

The Real numbers, denoted $\mathbb{R}$ form a field, that is $\mathbb{R}$ is an Abelian group under addition and multiplication that has distinct identites and satisfies the distributive property.

Aside from the algebraic structure we also have a total order on $\mathbb{R}$ that turns it into an orderd field.

1. Tricotomy: For $x, y \in \mathbb{R}$ exactly one of $a = y$, $x < y$, or $x > y$ holds
2. For $x, y, z \in \mathbb{R}$ if $x < y$ then $x + z < y + z$
3. For $x, y \in \mathbb{R}_{>0}$ we have $xy > 0$.
4. Transitivity: For $x, y, z \in \mathbb{R}$ if $x < y$ and $y < z$ then $x < z$.

These are the properties of an ordered field and from them we can prove other familliar facts about real numbers, for example

Lemma 1.1.1
Let $x, y \in \mathbb{R}$ and $x < y$ then if $z > 0$ we have

$$xz < yz$$

if $z < 0$ we have

$$xz > yz$$

Proof. $x < y$ implies $0 < y - x$ by subtracting x from both sides. Then we use the 3rd axiom to multipliy by z and get

$$0 < (y - x)z \implies xz < yz$$

after an application of the distrubutive property and adding xz to both sides. $\square$