MTH 316 Homework 2

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Question 1.

Prove all subgroups of a cyclic group are cyclic

Proof. Let G be a group such that $< g >= G$ and let $H \leq G$. If $g \in H$ then it is clear $H = G$ and H is cyclic. So Assume $g \notin H$. Then we write the elements of H

$$H = \{e, g^{\alpha_1}, g^{\alpha_2}, g^{\alpha_3}, ...\}$$

To prove H is cyclic we show there exists $i \in \mathbb{N}$ such that for all $j \in \mathbb{N}$ $\alpha_i | \alpha_j$. Let S be the set of powers of g in H that is, $S = \{\alpha \in \mathbb{N} | g^\alpha \in H\}$. Then S is obviously non-empty so fix α as the least element of S. Then assume there exists $\beta \in S$ such that α does not devide β, then using the divison algorithm we may write

$$\beta = \alpha q + r$$

For $0 < r < \alpha$ But this implies $g^\beta = g^{\alpha q} g^r$ so

$$g^{-\alpha q} g^\beta = g^r$$

so $r \in S$ but $r < \alpha$, a contradiction. Thus all positive exponents of g in H are multiples of α_i, then since $g^{-x} \in H$ iff $g^x \in H$ it follows that $< \alpha >= H$.

a $\neq$ b

$\square$