MTH 451 Quiz 1

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February 24, 2022

Question 1.

We have that $\sigma_{XY} = E(XY) - E(X)E(Y)$ and $E(X) = \sum_{x \in X} x f_X(x)$. We calculate the marginals for X and Y first.

$$f_X(0) = \frac{6}{23}, f_X(1) = \frac{10}{23}, f_X(2) = \frac{5}{23}, f_X(3) = \frac{2}{23}$$

and

$$f_Y(0) = \frac{10}{23}, f_Y(1) = \frac{10}{23}, f_Y(2) = \frac{3}{23},$$

so then

$$E(X) = 0 * \frac{6}{23} + 1 * \frac{10}{23} + 2 * \frac{5}{23} + 3 * \frac{2}{23} = \frac{26}{23}$$

and

$$E(Y) = 0 * \frac{10}{23} + 1 * \frac{10}{23} + 2 * \frac{3}{23} = \frac{16}{23}$$

Then we have $E(XY) = \sum\sum_{(x,y)\in S} xyf(x,y)$. Note there is no positive contribution from any point containg a zero and hence we only need to consider $\{(1,1),(1,2),(2,1)\}$. Then

$$E(XY) = 1 * 1 * \frac{5}{23} + 1 * 2 * \frac{2}{23} + 1 * 2 * \frac{1}{23} = \frac{11}{23}$$

Then $\sigma_{XY} = \frac{11}{23} - \frac{26}{23}(\frac{16}{23}) = -17.6$.

Question 2.

Since we have that independant random variables have a covarience of 0, X and Y cannot be independant since they have a non zero covarience.

Question 3.

$E(X|Y=1) = \sum_{x \in X} x f_{X|Y}(x|1)$. We first find $f_{X|Y}(x|1)$.

$$f_{X|Y}(x|1) = \frac{f(x,1)}{f_Y(1)} = \frac{f(x,1)}{\frac{10}{23}} = \frac{23}{10} f(x,1)$$

then

$$E(x|Y=1) = \tfrac{23}{10}[0 * f(0,1) + 1 * f(1,1) + 2 * f(2,1)] = \frac{9}{10}$$

Question 4.

We have $V(X|Y=1) = E(x^2|Y=1) - E(x|Y=1)^2$. We have the first moment from the previous calculation, now we compute the second moment.

$$E(x^2|Y=1) = \sum_{x \in X} x^2 f_{X|Y}(x,1) = I$$

We already have $f_{X|Y}(x,1) = \frac{23}{10} f(x,1)$. So

$$I = \tfrac{23}{10}[0^2 * f(0,1) + 1^2 * f(1,1) + 2^2 * f(2,1)] = \frac{13}{10}$$

Then using the short-cut formula for conditional varience we get

$$V(x|Y=1) = \frac{13}{10} - \frac{9^2}{10} = \frac{13}{10} - \frac{81}{10} = \frac{-68}{10}$$