MTH 451 Quiz 14

Evan Fox (efox20@uri.edu)

March 24, 2022

Question 1.

(a) Find $P(X \geq k)$.

Let $X \sim \text{Geom}(p)$. Then the PMF is $f(x) = p(1-p)^x$ for $x = 0, 1, 2, 3, ...$ Then we have the CMF given by $F(X) = \sum_0^x p(1-p)^i$. It is clear that

$$P(X \geq k) = 1 - F(x) = 1 - \sum_{i=0}^{k-1} p(1-p)^i = \sum_{i=k}^{\infty} p(1-p)^i =$$

Now since our index starts at k each summand will contain a copy of $(1-p)^k$ which we can factor out giving,

$$= (1-p)^k \sum_{i=k}^{\infty} p(1-p)^{i-k} = (1-p)^k \sum_{i=0}^{\infty} p(1-p)^i = (1-p)^k$$

Where the last equality follows since $\sum_{i=0}^{\infty} p(1-p)^i$ is just a sum of the PDF over the support. So we have

$$P(X \geq k) = (1-p)^k$$

(b) *Proof.* We can prove this with a string of equalitys(recall def of conditional probability), Observe

$$P(X \geq m+n | X \geq n) = \frac{P(X \geq m+n \& X \geq n)}{P(X \geq n)} = \frac{P(X \geq m+n)}{P(X \geq n)}$$

where the third equality follows since if the random variable X is greater than $m+n$ it is necessarily greater than n. Now using the formula derived in part (a) we get

$$\frac{P(X \geq m+n)}{P(X \geq n)} = \frac{(1-p)^{m+n}}{(1-p)^n} = (1-p)^m = P(X \geq m)$$

and this completes the proof. $\square$

Question 2.

The probility of 3 coins being heads is $.5^3 = 1/8$. Now define a random variable $X \sim \text{Geom}(1/8)$ as the number of attempts until all three are heads. We then define the random variable $Y = 70 - 10(X - 1)$ as the amount of winnings. We want to find the expectation of Y.

$$E(Y) = E(70 - 10(X - 1)) = 60 - 10E(X)$$

by linearity of expectation. Then from class we have $E(X) = (1 - p)/p = \frac{7/8}{1/8} = 7$. so

$$E(Y) = 60 - 10(7) = -10$$

Since the expectation is negitive you should not take the bet.