MTH 451 Quiz 1

Evan Fox (efox20@uri.edu)

February 17, 2022

Question 1.

For f to be a PDF it must be non-negitive and the intergral must equal 1. The fact that f is non-negitive is clear so we must choose c so that

$$\iint_I c(x^2 - xy)\,\mathrm{d}x\,\mathrm{d}y = 1$$

so we compute

$$c\int_0^1 \int_{-x}^{x} x^2 - xy\,\mathrm{d}y\,\mathrm{d}x = c\int_0^1 (x^2y - \tfrac{1}{2}xy^2\big|_{-x}^{x}\,\mathrm{d}x$$
$$c\int_0^1 (x^3 - \tfrac{1}{2}x^3 + x^3 + \tfrac{1}{2}x^3\,\mathrm{d}x = c\int_0^1 2x^3\,\mathrm{d}x$$
$$2c(\tfrac{1}{4}x^4\big|_0^1) = \tfrac{1}{2}c = 1$$
$$c = 2$$

so $c = 2$

Question 2.

To find the joint PDF we find the second order mixed partial derivative of F. for $x, y > 0$

$$\frac{\partial^2 F}{\partial x \partial y} = \frac{\partial}{\partial x}(1 - e^{-x^2})\frac{\partial}{\partial y}(1 - e^{-y^2})$$

so

$$\frac{\partial^2 F}{\partial x \partial y} = 2xe^{-x^2}(2ye^{-y^2})$$

Then the joint PDF is $f(x, y) = 2xe^{-x^2}(2ye^{-y^2})I\{x > 0, y > 0\}$.

Question 3.

We just use the definiton of the marginal distribution of X discussed in class. That is,

$$f_X(x) = \int_{-\infty}^{\infty} f(x, y)\,\mathrm{d}y$$

so plugging in given values gives

$$\begin{aligned} f_X(x) &= \int_0^2 \tfrac{1}{4}(2x + y)\,\mathrm{d}y \\ &= \tfrac{1}{4}(4x + 2) = x + \tfrac{1}{2} \end{aligned}$$

and so

$$f_X(x) = \tfrac{1}{4}(4x + 2) = x + \tfrac{1}{2}$$