MTH 451 Worksheet 3

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Question 1.

First I calculate $C(7, x) = \binom{7}{x}$ for $x \in \{1, 2, 3, 4, 5, 6, 7\}$.

$$C(7, 0) = 1$$

$$C(7, 1) = 7$$

$$C(7, 2) = 21$$

$$C(7, 3) = 35$$

$$C(7, 4) = 35$$

$$C(7, 5) = 21$$

$$C(7, 6) = 7$$

$$C(7, 7) = 1$$

In order for f to be a PMF each value of the function must be between 0 and 1 and the sum over all values of f must equal 1. We note that $\sum_{i=0}^{7} C(7, i) = 128$, so setting $c = 1/128$ will have the desired effect.

Question 2.

For f to be a PMF it must have all outputs between 0 and 1 and it must sum to 1. Let $c = 3$, then it is clear that $f(x) < 1$ for all $x \in \mathbb{N}$. Let $S = \sum_{i=1}^{\infty} 3(1/4)^i$. We must show that $S = 1$, note that S is a geometric series with common ratio 1/4 then since $1/4 < 1$ the series will converge to 3r/1 - r = (3/4)/(1 - 1/4) = 1. Thus for $C = 3$, f is a probability mass function as desired.

Question 3.

(a) We need $\int_{-\infty}^{\infty} f(x)\,\mathrm{d}x = 1$. Note that there is only a contribution on the interval from 0 to 4. Then

$$\begin{aligned}\int_0^4 \frac{c}{\sqrt{x}}\,\mathrm{d}x &= c\int_0^4 x^{-1/2}\,\mathrm{d}x \\ &= \tfrac{c}{2}x^{1/2}\Big|_0^4 = \frac{c}{2}2 = c\end{aligned}$$

Thus we select $c = 1$ so that the intergral equals 1.

(b) We want to find $P(X > 1) = \int_1^4 f(x)\,\mathrm{d}x = \int_1^4 \frac{1}{\sqrt{x}}\,\mathrm{d}x$ which is $\frac{1}{2}x^{1/2}\Big|_1^4$ so we get $2 - \frac{1}{2} = \frac{3}{2}$.