

# EVOLUTIONARY DYNAMICS OF MULTILINGUAL COMPETITION UNDER INTERVENTION

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An evolutionary dynamics model is investigated for the evolution of multilingual populations. The model consists of two different parts, formulated as two different evolutionary games. The first part accounts for the selection of languages based on the competition for popularity and social or economic advantages. The second part relates to the circumstance when the selection is altered, for better or worse, by forces other than competition such as public policies, education, or family influences. By combining competition with intervention, the model shows how a multilingual population may evolve under these two different sources of influences, and the languages may co-exist in evolutionarily stable multilingual forms with appropriate interventional measures. This is in contrast with the predictions from previous studies that the co-existence of languages is unstable in general, and one language will eventually dominate while all others become extinct.

## 1. Introduction

While many languages are in danger of extinction, multilingualism is being adopted as a common practice for communication among different language groups, and is playing a unique role in preserving language and cultural diversities (Gabszewicz et al., 2011; Grosjean, 2012). Languages compete and spread among their speakers, as genes are inherited and passed down to biological generations, where some are selected while others become extinct (Pinker & Bloom, 1990). Genes may be carried over in mixed forms. So are languages by multilingual speakers. In this paper, a mixed population of multilingual speakers and bilingual speakers in particular is considered, with multilingual defined broadly as zero, limited, or full uses of multiple languages or dialects, and an evolutionary dynamics model for its evolution is proposed, similar to that for genetic evolution (Burger, 2000).

Unlike genetic evolution though, the uses of languages are not only dependent of competition, but also subject to various societal interventions, common in social or cultural evolution. The proposed model consists of two different parts accordingly, formulated as two different evolutionary games, respectively. The first part accounts for the selection of languages based on the competition for popularity and social or economic advantages. The second part relates to the circumstance

when the selection of languages is altered, for better or worse, by forces other than competition such as public policies, education, or family influences.

Much work has been done on modeling language competition, although not specifically for the evolution of multilingualism. A well known model was proposed by Abrams & Strogatz, 2003 for the study of language death. The model was later extended to more general and complex cases by several other groups (Mira & Paredes, 2005; Patriaca & Heinsalu, 2009; Vazquez et al., 2010; Fujie et al., 2013). The models along this line focus mainly on language competition for popularity and social or economic advantages, and predict that one language will eventually dominate while all others become extinct, and the co-existence of languages is unstable and hard to sustain.

While successfully applied to some language populations, the previous models have not explicitly distinguished language competition from possible societal interventions that may reverse the course of language changes. By combining language competition with possible societal interventions, this paper shows how a multilingual population may evolve under these two different sources of influences. It shows in particular that the co-existence of languages can be made stable and language extinction can be prevented with appropriate interventional measures, as seen in many multilingual communities across the world (Shin & Kominski, 2010; Grosjean, 2012; Batalova & Zong, 2016; China, 2017).

## 2. The Evolutionary Dynamics Model

Consider a bilingual population, the simplest yet the most common multilingual population. Assume it is large and well mixed, i.e., every individual speaker can interact with all others in the population. If an individual speaker uses two languages  $A$  and  $B$  with frequencies  $x_A$  and  $x_B$ , respectively, this individual is called an  $(x_A, x_B)$ -speaker, where  $0 \leq x_A, x_B \leq 1$ , and  $x_A + x_B = 1$ . Likewise, if language  $A$  and  $B$  are used with frequencies  $y_A$  and  $y_B$  in average in the whole population, this population is called a  $(y_A, y_B)$ -population, where  $0 \leq y_A, y_B \leq 1$ , and  $y_A + y_B = 1$ .

First, consider a competition-only population. Let  $P_A(y_A)$  and  $P_B(y_B)$  be the payoff functions for  $A$  and  $B$  speakers in a  $(y_A, y_B)$ -population, respectively, defined in terms of the population sizes  $y_A$  and  $y_B$  and some other parameters for social or economic impacts, with  $P_A$  increasing in  $y_A$  and  $P_B$  in  $y_B$ , meaning that the larger the population size of a language, the more benefit the language provides for its speakers. Then, the payoff function for an  $(x_A, x_B)$ -speaker in a  $(y_A, y_B)$ -population can be defined in terms of the average use of  $A$  and  $B$  by this speaker:

$$\pi((x_A, x_B), (y_A, y_B)) = x_A P_A(y_A) + x_B P_B(y_B). \quad (1)$$

Now consider the situation where the use of languages is influenced by some societal decisions. Assume that the societal interventions are implemented to

counter the arbitrary increase or decrease of either language. Let  $\bar{P}_A(y_A)$  and  $\bar{P}_B(y_B)$  be the payoff functions for  $A$  and  $B$  speakers in a  $(y_A, y_B)$ -population, respectively, defined in terms of the population sizes  $y_A$  and  $y_B$  and some parameters for language reversing, with  $\bar{P}_A$  decreasing in  $y_A$  and  $\bar{P}_B$  in  $y_B$ , meaning that the smaller the population size of a language, the more incentive or less penalty for the speakers of the language. Then, the payoff function for an  $(x_A, x_B)$ -speaker in a  $(y_A, y_B)$ -population can be defined in terms of the average use of  $A$  and  $B$  by this speaker:

$$\bar{\pi}((x_A, x_B), (y_A, y_B)) = x_A \bar{P}_A(y_A) + x_B \bar{P}_B(y_B). \quad (2)$$

The above two types of payoff functions can be combined to obtain an evolutionary dynamics model for bilingual competition under societal intervention:

$$\begin{cases} \dot{y}_A = y_A y_B (\tilde{P}_A(y_A) - \tilde{P}_B(y_B)) \\ \dot{y}_B = y_B y_A (\tilde{P}_B(y_B) - \tilde{P}_A(y_A)), \end{cases} \quad (3)$$

where

$$\tilde{P}_A(y_A) = \lambda P_A(y_A) + (1 - \lambda) \bar{P}_A(y_A), \quad (4)$$

$$\tilde{P}_B(y_B) = \lambda P_B(y_B) + (1 - \lambda) \bar{P}_B(y_B). \quad (5)$$

where  $0 \leq \lambda \leq 1$ . The model is reduced to competition-only when  $\lambda = 1$ , and to intervention-only when  $\lambda = 0$ .

Based on evolutionary game theory (Weibull, 1995; Hofbauer & Sigmund, 1998), the equations in (3) form a so-called system of replicator equations, which corresponds to an evolutionary game, with the Nash equilibrium being a strategy  $(x_A^*, x_B^*)$  such that

$$\tilde{\pi}((x_A^*, x_B^*), (x_A^*, x_B^*)) \geq \tilde{\pi}((x_A, x_B), (x_A^*, x_B^*)) \text{ for all } (x_A, x_B), \quad (6)$$

where  $\tilde{\pi}$  is the payoff function for the game, and for an  $(x_A, x_B)$ -speaker in a  $(y_A, y_B)$ -population

$$\begin{aligned} & \tilde{\pi}((x_A, x_B), (y_A, y_B)) \\ &= \lambda \pi((x_A, x_B), (y_A, y_B)) + (1 - \lambda) \bar{\pi}((x_A, x_B), (y_A, y_B)). \end{aligned} \quad (7)$$

In general,  $P_A$  and  $P_B$  can be some increasing functions and  $\bar{P}_A$  and  $\bar{P}_B$  be some decreasing functions. However, in this study, they are defined using the following empirical functions similar to those in Abrams & Strogatz, 2003:

$$P_A(y_A) = c y_A^{\alpha-1} s_A, \quad P_B(y_B) = c y_B^{\alpha-1} s_B, \quad 1 < \alpha \leq 2, \quad (8)$$

$$\bar{P}_A(y_A) = \bar{c} y_A^{\bar{\alpha}-1} \bar{s}_A, \quad \bar{P}_B(y_B) = \bar{c} y_B^{\bar{\alpha}-1} \bar{s}_B, \quad 0 \leq \bar{\alpha} < 1, \quad (9)$$

where  $c$  and  $\bar{c}$  are scaling constants,  $\alpha, \bar{\alpha}, s_A, s_B, \bar{s}_A, \bar{s}_B$  are all parameters,  $0 \leq s_A, s_B, \bar{s}_A, \bar{s}_B \leq 1$ . The parameters  $\alpha, \bar{\alpha}$  determine the order of dependency of the payoffs on the population sizes. Since  $1 < \alpha \leq 2$ , the payoffs from  $P_A$  and  $P_B$  increase with increasing population sizes. On the other hand, since  $0 \leq \bar{\alpha} < 1$ , the payoffs from  $\bar{P}_A$  and  $\bar{P}_B$  decrease with increasing population sizes. The parameters  $s_A, s_B$  are used to define the payoffs from competition. They are indicators of social or economic impacts on the payoffs. The larger these values, the more benefits for the corresponding language groups. The parameters  $\bar{s}_A, \bar{s}_B$  are used to define the payoffs from intervention. They are rates for language reversing due to interventions. The larger these values, the faster the reversing rates.

### 3. Dynamics Analysis

Several theoretical results can immediately be established following the model given in the previous section: (i) Languages  $A$  and  $B$  can co-exist when the payoffs for speaking  $A$  and  $B$  are balanced. (ii) The co-existence is evolutionarily stable under certain conditions. (iii) Interventional conditions can be found to maintain an evolutionarily stable multilingual population. The following are more detailed analysis.

Without loss of generality, let  $\lambda = 0.5$  and  $c = \bar{c} = 1$ . Consider a  $(y_A^*, y_B^*)$ -population,  $y_A^*, y_B^* \neq 0$ , i.e., languages  $A$  and  $B$  co-exist in the population. Then, it is easy to see that a *necessary and sufficient condition* for  $(y_A^*, y_B^*)$  to be an equilibrium solution to the equations in (3) or in other words, an equilibrium strategy for the game in (6) is  $\tilde{P}_A(y_A^*) = \tilde{P}_B(y_B^*)$ , i.e.,

$$(y_A^*)^{\alpha-1} s_A + (y_A^*)^{\bar{\alpha}-1} \bar{s}_A = (y_B^*)^{\alpha-1} s_B + (y_B^*)^{\bar{\alpha}-1} \bar{s}_B. \quad (10)$$

For a specific population, for example, for  $\alpha = 3/2$  and  $\bar{\alpha} = 0$ , it can be simplified to

$$(y_A^*)^{1/2} s_A + (y_A^*)^{-1} \bar{s}_A = (y_B^*)^{1/2} s_B + (y_B^*)^{-1} \bar{s}_B. \quad (11)$$

In addition, a *sufficient condition* for  $(y_A^*, y_B^*)$  to be evolutionarily stable is

$$(1 - \bar{\alpha})[(y_A^*)^{\bar{\alpha}-2} \bar{s}_A + (y_B^*)^{\bar{\alpha}-2} \bar{s}_B] > (\alpha - 1)[(y_A^*)^{\alpha-2} s_A + (y_B^*)^{\alpha-2} s_B]. \quad (12)$$

For a specific population, for example, for  $\alpha = 3/2$  and  $\bar{\alpha} = 0$ , the condition can be simplified to:

$$(y_A^*)^{-2} \bar{s}_A + (y_B^*)^{-2} \bar{s}_B > [(y_A^*)^{-1/2} s_A + (y_B^*)^{-1/2} s_B]/2. \quad (13)$$

Assume that  $(y_A^*, y_B^*), y_A^*, y_B^* \neq 0$ , is an equilibrium solution to the system of equations in (3) satisfying the equilibrium condition in (10). Then, several

interventional conditions can be obtained to make the solution to be evolutionarily stable when the interventional parameters  $\bar{s}_A$  and  $\bar{s}_B$  fall in certain ranges:

**Condition 1:** If  $1 - \bar{\alpha} \geq \alpha - 1$ , i.e.,  $\alpha + \bar{\alpha} \leq 2$ , the stability condition in (12) can be satisfied easily when the reversing rates  $\bar{s}_A$  and  $\bar{s}_B$  are in certain ranges: Since  $\bar{\alpha} - 2 < \alpha - 2$ ,  $(y_A^*)^{\bar{\alpha}-2} > (y_A^*)^{\alpha-2}$  and  $(y_B^*)^{\bar{\alpha}-2} > (y_B^*)^{\alpha-2}$ , and therefore, the stability condition in (12) is satisfied if  $\bar{s}_A$  and  $\bar{s}_B$  are sufficiently large, say  $\bar{s}_A = ts_A$  and  $\bar{s}_B = ts_B$ , where  $1 \leq t \leq \min\{1/s_A, 1/s_B\}$ . With such a choice of  $\bar{s}_A$  and  $\bar{s}_B$ , one can prove that  $(y_A^*, y_B^*)$  is also unique (see an example in Figure 1 (a)).

**Condition 2:** If in particular,  $1 - \bar{\alpha} = \alpha - 1$ , i.e.,  $\alpha + \bar{\alpha} = 2$ , and  $\bar{s}_A = ts_B$  and  $\bar{s}_B = ts_A$  for any  $1 \leq t \leq \min\{1/s_A, 1/s_B\}$ , then it is easy to verify that  $P_A(y_A^*) = P_B(y_B^*)$  and  $\bar{P}_A(y_A^*) = \bar{P}_B(y_B^*)$  if and only if  $\tilde{P}_A(y_A^*) = \tilde{P}_B(y_B^*)$ , which implies that  $(y_A^*, y_B^*)$  is an equilibrium solution to the system of equations in (3) for competition-only and intervention-only and both combined. For the combined one, one can prove that  $(y_A^*, y_B^*)$  is also evolutionarily stable.

**Condition 3:** In general, given a desired solution  $(y_A^*, y_B^*)$ ,  $y_A^*, y_B^* \neq 0$ , it is possible to make it to be an equilibrium solution if the reversing rates  $\bar{s}_A$  and  $\bar{s}_B$  satisfy the following conditions: For  $1 - \bar{\alpha} \geq \alpha - 1$ ,

$$\bar{s}_A = (y_A^*)^{1-\bar{\alpha}}(y_B^*)^{\alpha-1}s_B, \quad \bar{s}_B = (y_B^*)^{1-\bar{\alpha}}(y_A^*)^{\alpha-1}s_A. \quad (14)$$

Then,  $\bar{P}_A(y_A^*) = P_B(y_B^*)$  and  $\bar{P}_B(y_B^*) = P_A(y_A^*)$ . It follows that  $\tilde{P}_A(y_A^*) = \tilde{P}_B(y_B^*)$ , and  $(y_A^*, y_B^*)$  becomes an equilibrium solution. In addition, let  $y_A^o = 1/(1 + (s_A/s_B)^{1/(\alpha-1)})$ , and assume that  $(y_A^*, y_B^*)$  is selected such that  $y_A^* \geq \max\{y_A^o, y_B^*\}$  or  $y_A^* \leq \min\{y_A^o, y_B^*\}$ . Then, the condition in (12) is satisfied at  $(y_A^*, y_B^*)$ , and the solution is also evolutionarily stable (see an example in Figure 1 (b)).

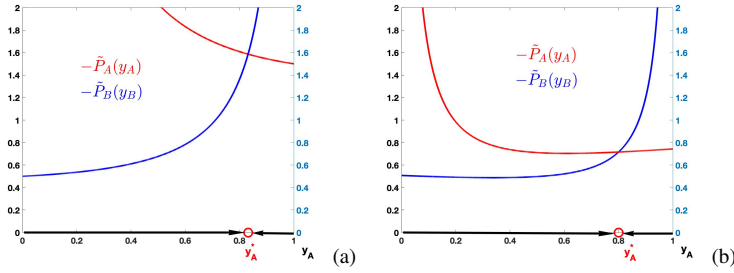


Figure 1. Dynamic Behaviors with Competition and Intervention. Payoff functions  $\tilde{P}_A$  and  $\tilde{P}_B$  are plotted against  $y_A$  and the changing directions of  $y_A$  are pointed with arrows. In (a),  $\alpha = 3/2$  and  $\bar{\alpha} = 0$ ,  $\bar{s}_A = s_A = 0.75$  and  $\bar{s}_B = s_B = 0.25$ , and  $y_A^* = 0.8315$  and  $y_B^* = 0.1685$ . In (b),  $\alpha = 3/2$  and  $\bar{\alpha} = 0$ ,  $s_A = 0.6$  and  $s_B = 0.4$ ,  $y_A^* = 0.8$  and  $y_B^* = 0.2$ , and  $\bar{s}_A = (y_A^*)(y_B^*)^{1/2}s_B = 0.1431$  and  $\bar{s}_B = (y_B^*)(y_A^*)^{1/2}s_A = 0.1073$ .

#### 4. Computer Simulation

A dynamic simulation is carried out to track the changes of the bilingual level of a population across time and space. A 2D torus-shaped lattice of  $n \times n$  cells is constructed first, with each cell assumed to be occupied by an individual speaker. An individual can then be selected repeatedly from the lattice, and a game is played for the individual against the population of the lattice: Let  $(x_A, x_B)$  be the current strategy for the individual, and  $(y_A, y_B)$  the strategy for the population. Let  $p_A = \tilde{P}_A(y_A)$  and  $p_B = \tilde{P}_B(y_B)$  be the payoffs for  $A$  and  $B$  speakers, respectively. Then, the payoff for the individual,  $\pi = x_A \tilde{P}_A(y_A) + x_B \tilde{P}_B(y_B)$ , is computed. If  $p_A > \pi$ ,  $x_A$  is increased by setting  $x_A = y_A$  if  $x_A < y_A$ . On the other hand, if  $p_A < \pi$ ,  $x_A$  is reduced by setting  $x_A = y_A$  if  $x_A > y_A$ .

Initially, each individual is assigned with a random strategy. The game is played  $n^2$  times for the population to complete a generation. The game is repeated for 100 generations to make sure the population reaches its equilibrium. In general, the game can be played in a neighborhood of each selected individual. Let the neighborhood be an  $m \times m$  sub-lattice, with the selected individual located at the center. Then, the game can be carried out for each selected individual only against the population in its neighborhood of this size, with the population strategy  $(y_A, y_B)$  computed from the population in the neighborhood. Such a game may in fact be more realistic, as people usually interact only with a small group of others around them.

Figure 2 demonstrates a typical set of results obtained from the simulation for a given population. The population is distributed on a  $75 \times 75$  lattice, with  $\alpha = 2$  and  $\bar{\alpha} = 0$ , and  $\bar{s}_A = s_B = 0.75$  and  $\bar{s}_B = s_A = 0.25$ . The simulation is done three times with the neighborhood size equal to  $75 \times 75$ ,  $25 \times 25$ , and  $5 \times 5$ , respectively. The final distribution of the individual frequency  $x_A^*$  in the population is displayed for each simulation in the corresponding order.

From left to right, the first graph in Figure 2 shows the result from the simulation with the neighborhood size equal to  $75 \times 75$ , when each individual interacts with all others in the whole population. The equilibrium frequency  $y_A^*$  of the population in this case is approximately equal to 0.75, which agrees with the direct prediction from the model described in previous sections. In addition, the distribution of the individual frequency  $x_A^*$  in the population is very homogeneous, with  $x_A^* \approx 0.75$  across the board, suggesting that language  $A$  and  $B$  co-exist in the population in an evenly distributed bilingual form. The second graph shows the result from the simulation with the neighborhood size equal to  $25 \times 25$ , when the interactions among individual speakers are restricted. The equilibrium frequency  $y_A^*$  of the population remains about the same, approximately equal to 0.75. However, the individual frequency  $x_A^*$  becomes less constant. Some regions have higher individual frequencies than others, and local groups are formed with varying individual frequencies, as shown in the graph. The third graph shows the result

from the simulation with the neighborhood size further reduced to  $5 \times 5$ . While the population frequency  $y_A^*$  is not significantly changed, the individual frequency  $x_A^*$  shows even bigger variations, with even smaller local spots formed with higher or lower individual frequencies than average. The dynamic behaviors shown from these simulations agree with the general experience in language development: Indeed, when communications are restricted to local groups, language variations often remain.

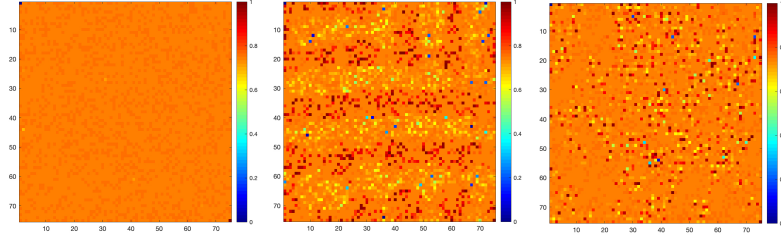


Figure 2. Dynamic Simulation Results. The distributions of color-coded  $A$  speaking frequencies in the 2D lattice at equilibrium are displayed in graphs from left to right, with corresponding neighborhood sizes equal to  $75 \times 75$ ,  $25 \times 25$ , and  $5 \times 5$ . Each of the graphs is a  $75 \times 75$  2D lattice. The  $x$ -axis and  $y$ -axis of the graph represent the 75 units of the lattice in the horizontal and vertical directions. The population is assumed to have  $\alpha = 2$  and  $\bar{\alpha} = 0$ ,  $\bar{s}_A = s_B = 0.75$  and  $\bar{s}_B = s_A = 0.25$ .

## 5. Conclusion

An evolutionary dynamics model for multilingual competition with societal intervention is proposed and analyzed. The model consists of two separate parts corresponding to two evolutionary games, one for the evolution of multilingualism with “natural” competition, and the other for the evolution with “artificial” intervention. Both games may have a multilingual co-existing equilibrium state, but the one for the competition-only game is evolutionarily unstable, which leads to the conclusion that multiple languages cannot co-exist, and one of them will eventually dominate while all others become extinct, as stated in many previous studies.

However, multiple languages do exist in many language communities, often in multilingual forms. By combining competition with intervention, the proposed model provides a more general theoretical framework for the study of language competition than those previously investigated. The model shows how multiple languages may co-evolve when appropriate interventions are introduced, and why they may co-exist in stable equilibrium states, at least in theory. The computer simulation on the dynamic behaviors of bilingual populations further validates the model, and also demonstrates how local bilingual groups may be formed when the

interactions among the speakers are restricted.

Some experimental work needs to be done to connect the theory to the reality: The parameters in the model need to be refined, denoted, and determined with real-world language data, while their values may vary with varying language populations. For simplicity, the model is defined and discussed only for bilingual populations, but it can in fact be extended to populations with more than two languages, although the analysis may be more mathematically involved (Wu, 2020).

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