**Online Supplementary Materials for**

**“Cooperative Coevolution for Non-Separable Large-Scale Black-Box Optimization: Convergence Analyses and Distributed Accelerations”**

(First Version)

1. **Convergence to a Pure Nash Equilibrium for Cooperative Coevolution (CC):**

***Theorem* 1:** Given *any* partition of the objective function , where , we say that the convergence point of CC under this partition is also one *pure Nash equilibrium (PNE).*

Proof: If is a convergence point of CC under the partition , then is one of the global optima of the function

If is not one *pure Nash equilibrium* (w.r.t. ), according to the definition of **Pure Nash Equilibrium (PNE),** we have that , , such that , namely, then is not the global optimum of It is a contradiction.

Note that we assume that for CC, the suboptimizer could obtain the global optimum for each subproblem given a *limited* number of function evaluations (but which could be any large number). We admit that such an assumption appears to be difficult to satisfy in practice. However, it helps to understand the convergence behavior of CC and capture the *game-theoretical* essence of CC.

1. **Convergence Analyses on Four Representative Test Functions:**

Since its Hessian matrix is positive definite, is differentiable strictly convex, then it has a unique global optimum .

Given is a PNE, by the definition of PNE, is the unique global optimum of differentiable strictly convex and .

Then , we have , , we have . So,

Since its Hessian matrix is positive definite, and its rotation variant are all strictly convex and have the unique global optimum .

Like the above proof, they have only one PNE, namely global optimum .

Obviously, and

So, it has a unique global optimum .

Suppose is a PNE, then are the global minima of differentiable and . Then

we have

Since is one of solutions of equation , we conclude that

For any , the function and obtain their global minima at and , respectively. So, the set is the set of PNEs.

1. **Convergence Analysis on A Function (with Loss of Gradients):**

*Corollary 3*: For the *Schwefel’s Problem 2.21*, , defined on an open , the set of global pure Nash equilibria w.r.t. any partition is . There is a unique strict global Nash equilibrium w.r.t. any partition set, which equals the global optimum, and vice versa.

Proof: Given is a PNEw.r.t. any partition of , owing to the definition of PNE, we have for each , satisfies

, ,

namely, for any , , *max*{} *max*{}, only if, *max*{} *min*{}, only if,

owing to the definition of PNE,

so, we only need for any ,

we have .

Since is a PNE, and , is the uniqueglobal optimum, thus it is a unique strict global Nash equilibrium, and vice versa.

1Here it is *implicitly* assumed that there is (at least) one global optimum in this open set.