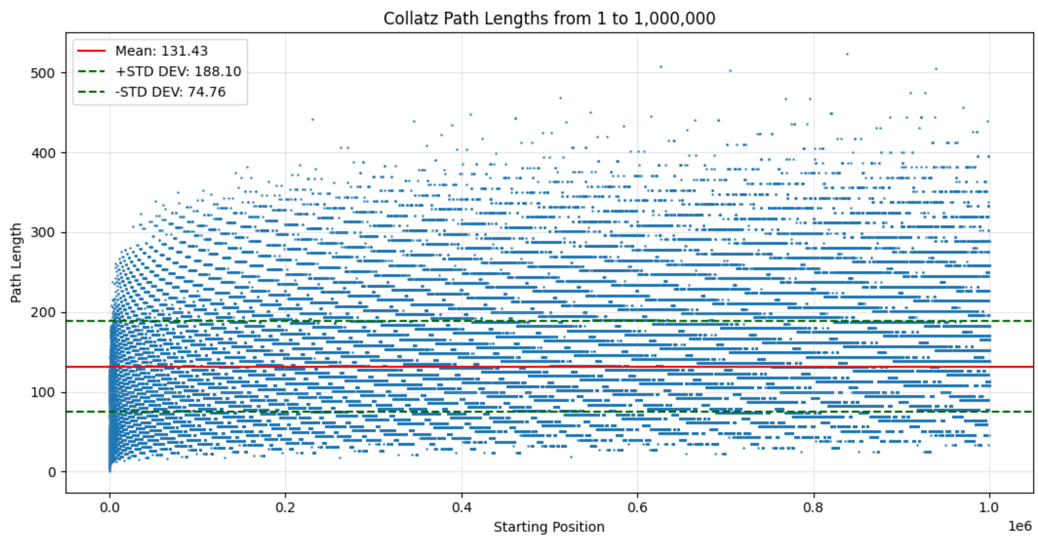
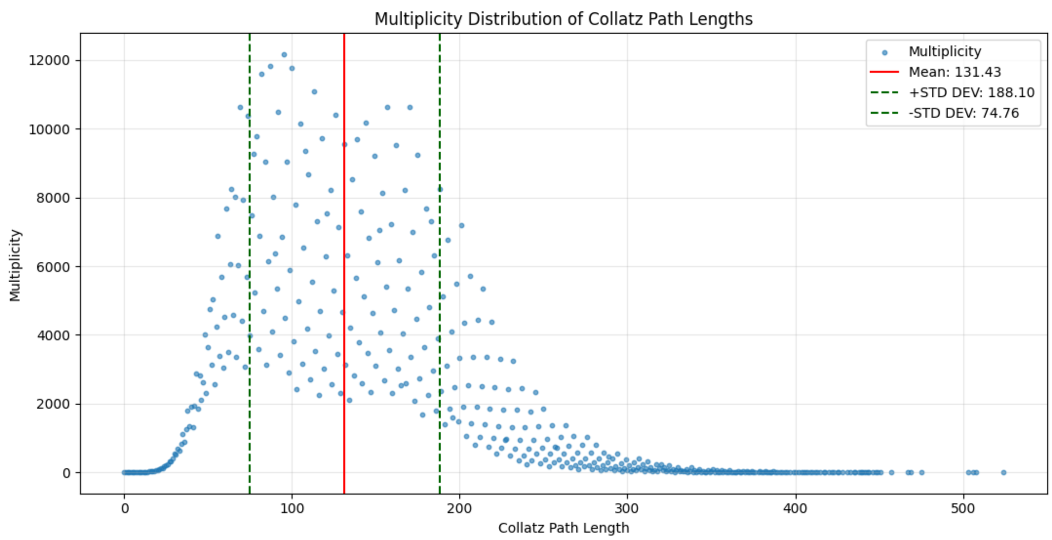
Challenge 1: (original Collatz conjecture)

Task 1a:



The plot shows a clear concentration of path lengths around the mean, which is roughly 130. Most of the values fall between around 75 and 188, indicating that most paths are close to the average. There are, however, a few prominent outliers with comparatively longer trajectories, which result in a less densely populated area at the top. According to these outliers, certain initial values result in far more complex sequences. The uneven distribution of path lengths, with rare dense clusters, shows that specific starting numbers yield paths of comparable lengths, potentially due to shared properties in how the sequences evolve.

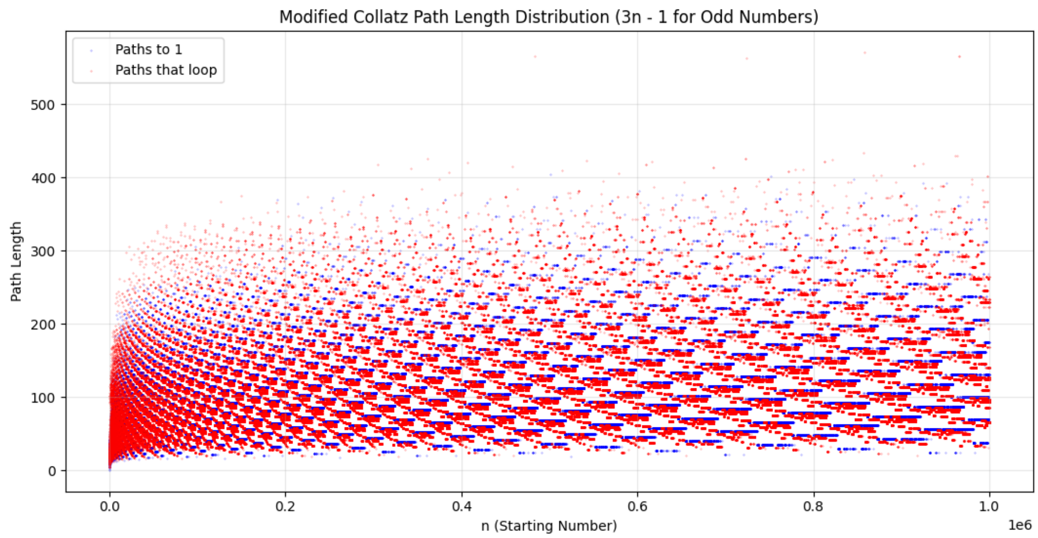
Task 1b:



The plot demonstrates the unequal distribution of Collatz path lengths. The most frequent path lengths are those close to the mean (around 130), with the number of occurrences decreasing as the path length increases. Higher lengths cause the distribution to narrow down, indicating that long pathways are not as common but frequently exhibit recurring patterns. On the other hand, the larger clusters surrounding shorter path lengths imply that the sequences of some initial values exhibit comparable traits.

Challenge 2: (modified Collatz conjecture)

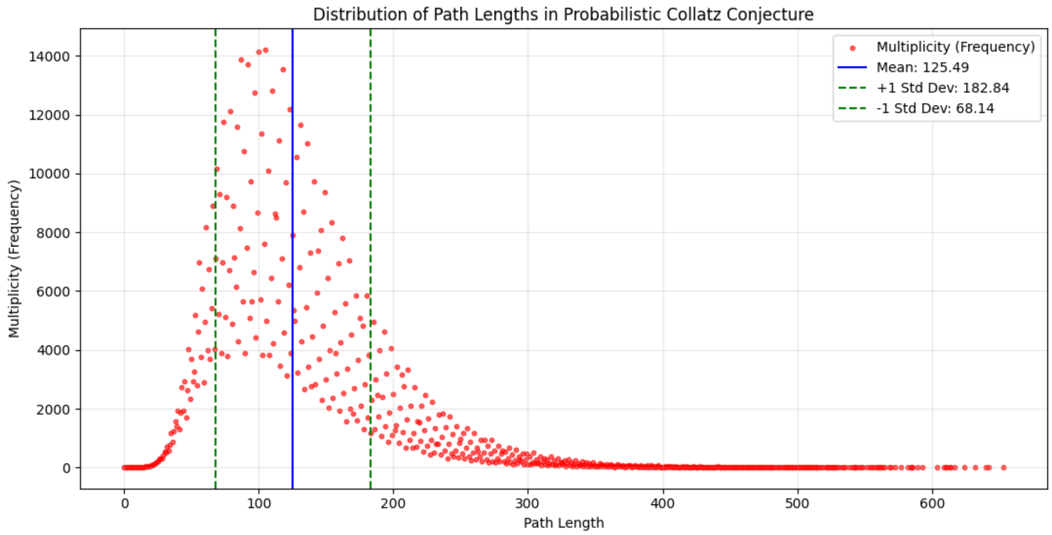
Task 2:



The plot shows two groups: the blue group (paths that reach 1) and the red group (paths that loop). The behaviour of the blue group is more predictable; after a certain number of steps, the trajectories usually approach 1. Although there is still some variety, the majority of the numbers in this group show a similar pattern. On the other hand, there are more abnormalities and a larger variation of path lengths in the red group, which represents paths that loop. This illustrates how the new rule for odd numbers adds complexity and unpredictability.

Challenge 3: (probabilistic Collatz conjecture)

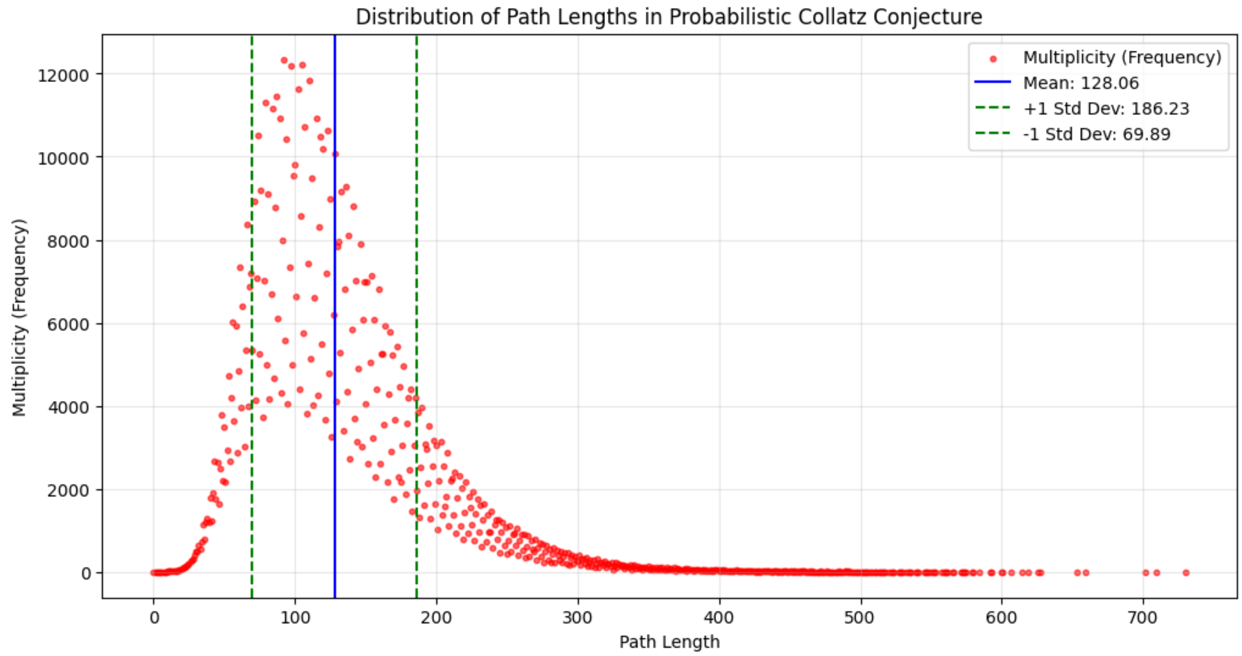
Task 3:



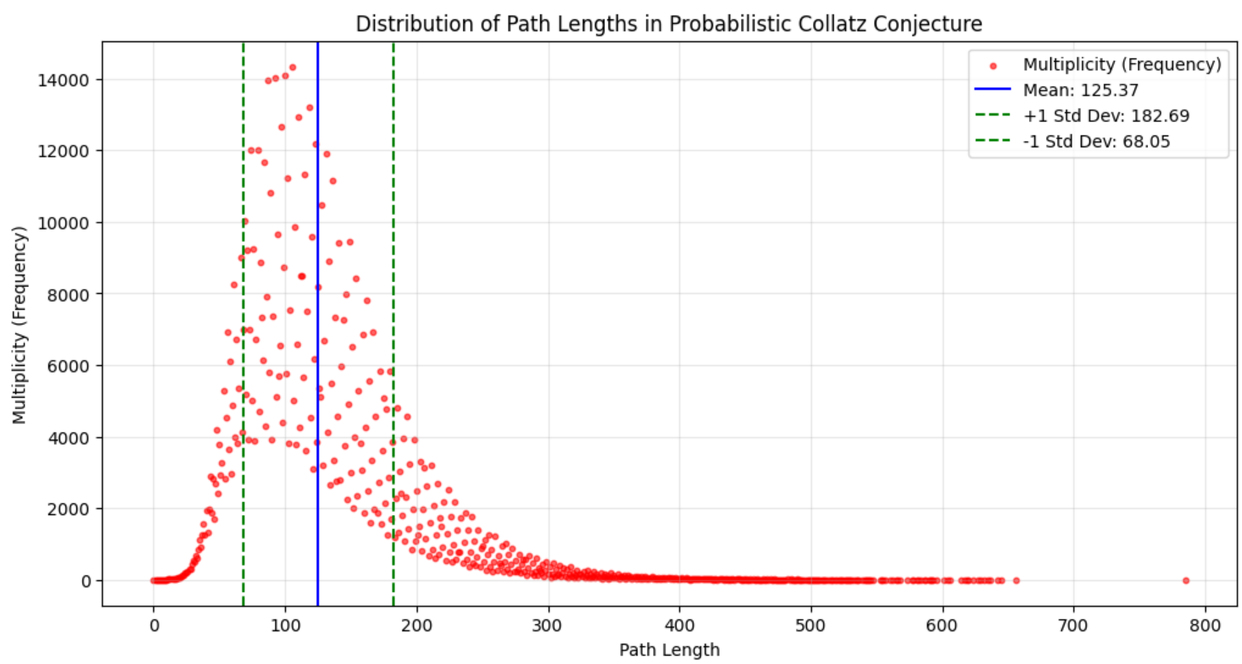
For a probabilistic variation with p=0.5, the plot shows the multiplicity distribution of Collatz path lengths, where odd numbers have a 50% chance of executing the n = 3n + 1 rule and a 50% chance of following the n = 3n - 1 rule. This randomness adds more unpredictability and results in a larger range of path lengths than in the deterministic form, where 100% of odd integers follow n = 3n + 1. Because the rule application is unpredictable, the path lengths in this probabilistic form are more varied, with some paths requiring fewer or more steps. The lengths' central tendency and dispersion are indicated by the mean and standard deviation lines. In contrast to the more predictable behavior in the deterministic version, the distribution demonstrates that the probabilistic rules generate significant variation in the number of steps, even though many paths eventually reach 1.

Task 4:

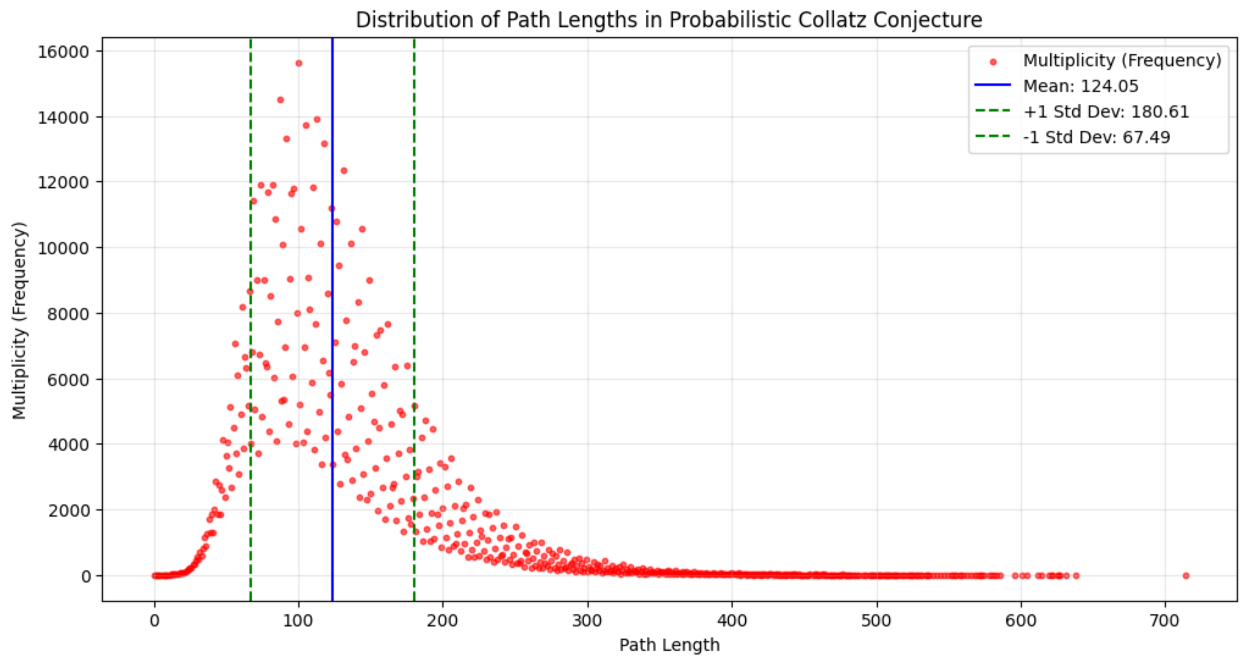
p = 0.2 mean = 128.06 std dev = 58.17



p = 0.5 mean = 125.37 std dev = 57.32



p = 0.8 mean = 124.05 std dev = 56.56



To demonstrate how changing the probability affects the Collatz path lengths, three charts were provided for probabilities p = 0.2, p = 0.5, and p = 0.8. The n=3n−1 rule is more likely to apply for p=0.2, resulting in more loops and generally shorter path lengths.

Applying n=3n+1 is more likely as p rises to 0.5 and 0.8, which tends to lengthen the paths and decrease the frequency of loops.

A higher chance of n=3n+1 yields a lower mean and less variability (lower standard deviation). Lowering the likelihood of n=3n+1 increases the mean and variety of path lengths, whereas the n=3n−1 rule causes more looping and unpredictable behaviour.

Submit zipped codes and this answer sheet with your plots and comments via Canvas by

Monday December 2nd 23:59 hours, 2024.