UNIVERSITAT POLITÈCNICA DE CATALUNYA
BARCELONATECH
Facultat d'Informàtica de Barcelona

Randomized Algorithms

Assigment 1

Ewa Miklewska
ewa.miklewska@estudiantat.upc.edu

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Contents

All the code can be found on Github here.

1 Introduction

Experiments with Galton board can be used to demonstrate the central limit theorem. By simulating various Galton boards with parametrized number of balls N and board size n, we can study the behaviour of binomial distribution and compare it to the expected normal distribution.

2 Simulation

The Galton board of size n was simulated by computing the end position of each of the N balls via virtual triangle matrix. To avoid redundancy, only the the first coordinate i was computed. It was achieved by generating a random boolean variable n times and counting the number of $True$ results.

3 Results

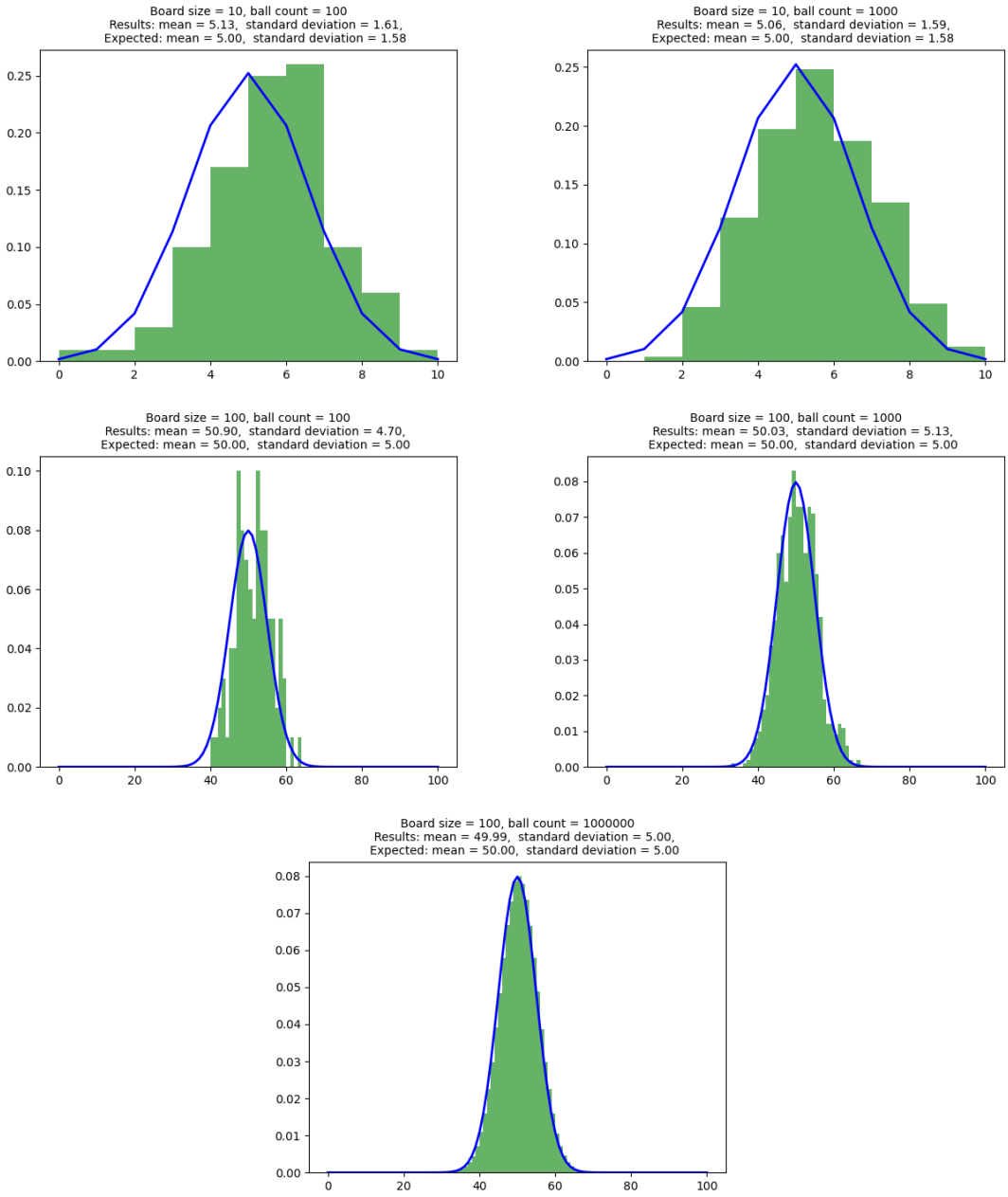
3.1 Visual representation

The end result can be represented using a normalized histogram with the expected standard distribution overlaid (Figure 1). It allows a fast visual assessment of the fit of the distribution. It is clear, that increasing values of both N and n leads to a smaller difference between the binomial and the normal distributions.

3.2 Mean quadratic error

The mean square error for different parameter values has been computed for constant board size (Figure 2) and constant ball count (Figure 3). The relationship between the MSE and parameter is logarithmic, meaning the means square error sharply decreases at the beginning, and slows down after reaching a certain point. Both constant ball count and board size plots behaving in the same manner shows that the decrease defends on number of random variables and not, for example, on the ratio. It further proofs the central limit theorem.

Figure 1: Visualization of the best fitting model over the dataset



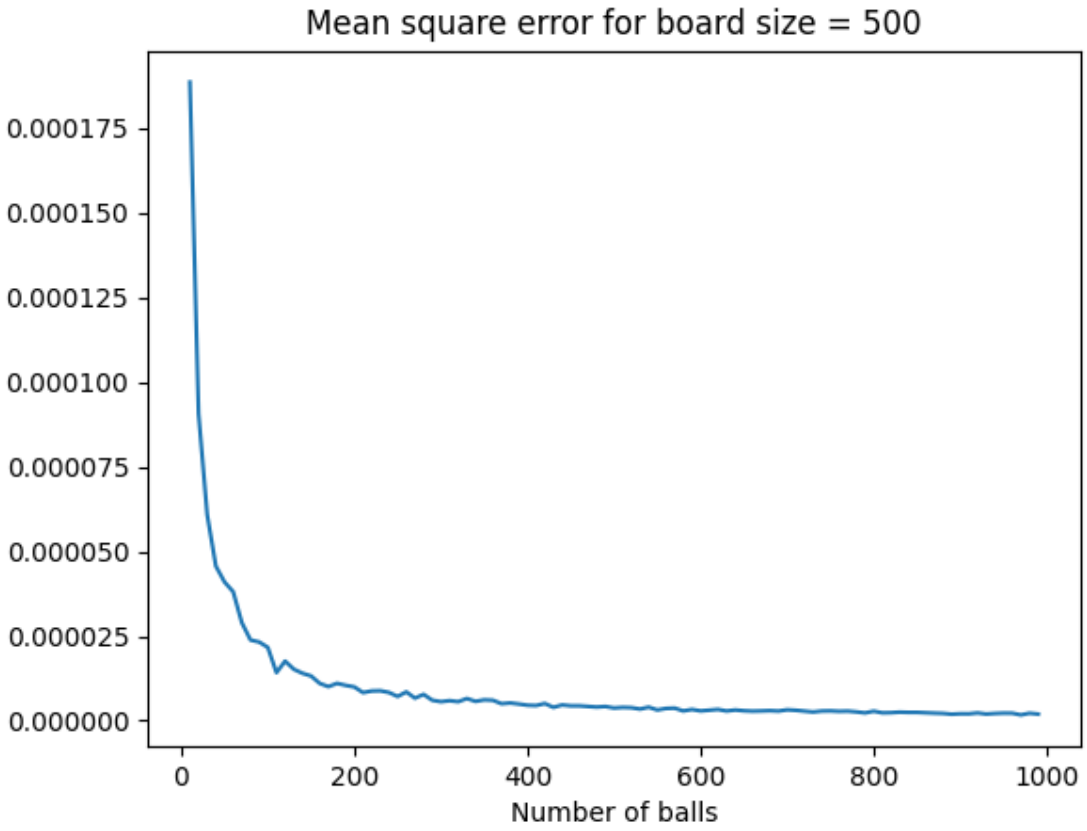


Figure 2: Mean square error depending on number of balls N. For each N the experiment was repeated 10 times and average result was computed.

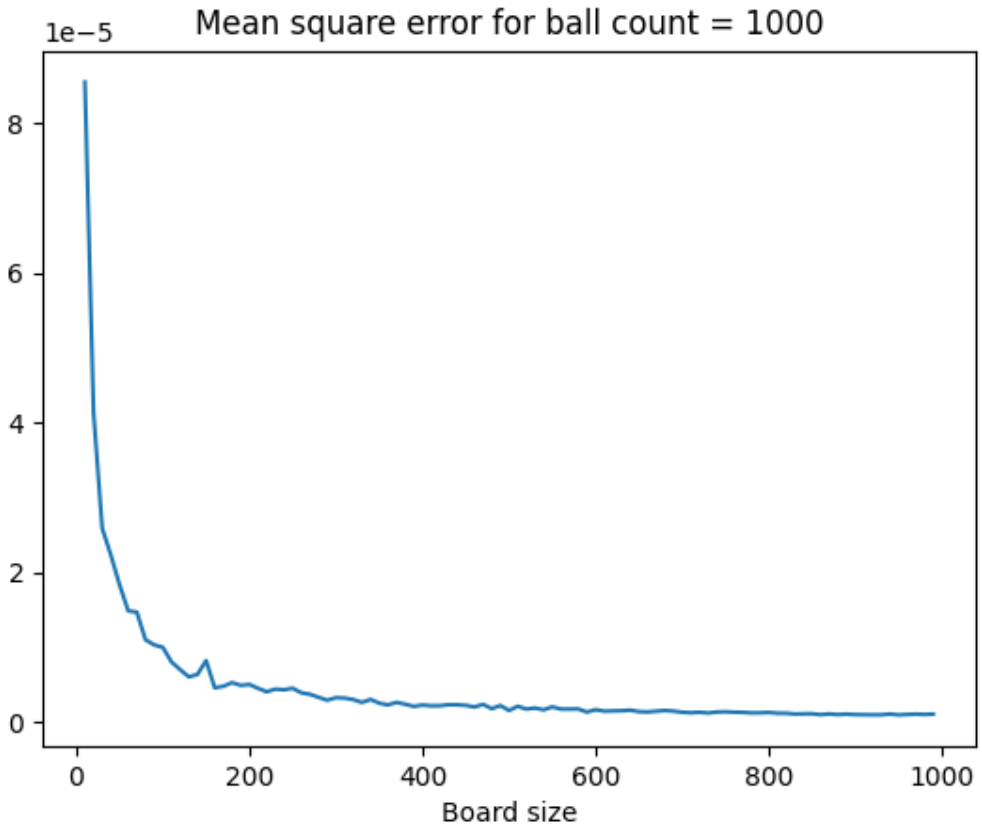


Figure 3: Mean square error depending on board size n. For each n the experiment was repeated 10 times and average result was computed.