Midterm Project 4

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I. PROOFS FOR THEOREMS 5.12 AND 5.13

Theorem 1 (Generalized Mean-Value Theorem(5.12)). *Let f and g be two functions, each having a derivative (finite or infinite) at each point of an open interval (a, b) and each continuous at the endpoints a and b. Assume also that there is no interior point x at which both $f'(x)$ and $g'(x)$ are infinite. Then for some interior point c we have*

$$f'(c)[g(b) - g(a)] = g'(c)[f(b) - f(a)]$$

Proof. proof here □

Theorem 2 (5.12 in our book). *Let f and g be two functions, each having a derivative (finite or infinite) at each point of (a, b). At the endpoints assume that the limits $f(a+)$,$g(a+)$,$f(b-)$, and $g(b-)$ exist as finite values. Assume further that there is no interior point x at which both $f'(x)$ and $g'(x)$ are infinite. Then for some interior point c we have*

$$f'(c)[g(b-) - g(a+)] = g'(c)[f(b-) - f(a+)]$$

Proof. proof here □

II. PROOFS FOR THEOREMS 5.11, 5.14, AND 5.16 AS EXAMPLES

Theorem 3 (Mean-Value Theorem(5.11)). *Assume that f has a derivative (finite or infinite) at each point of (a, b), and assume that f is continuous at both endpoints a and b. Then $\exists c \in (a, b)$ such that*

$$f(b) - f(a) = f'(c)(b - a)$$

Proof. proof here □

Theorem 4 (5.14 in our book)**.** *Assume that f has a derivative (finite or infinite) at each point of an open interval (a, b), and that f is continuous at both endpoints a and b.*

(a) *if f' takes only positive values (finite or infinite) in (a, b), then f is strictly increasing on $[a, b]$.*

(b) *if f' takes only negative values (finite or infinite) in (a, b), then f is strictly decreasing on $[a, b]$.*

(c) *if f' is zero everywhere in (a, b), then f is constant on $[a, b]$.*

Proof. proof here □

Theorem 5 (Intermediate-value theorem for derivatives(5.16))**.** *Assume that f is defined on a compact interval $[a, b]$ and that f has a derivative(finite or infinite) at each interior point. Assume also that f has finite one-sided derivatives $f'_+(a)$ and $f'_-(b)$ at the endpoints, with $f'_+(a) \neq f'_-(b)$. Then, if c is a real number between $f'_+(a)$ and $f'_-(b)$, there exists at least one interior point x such that $f'(x) = c$*

Proof. proof here □