Understanding the Information Content in Diverse Observations of Forest Carbon Stocks and Fluxes for Data Assimilation and Ecological Modeling
NERC case partnership with Forest Research

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July 16, 2014

The DALEC Model

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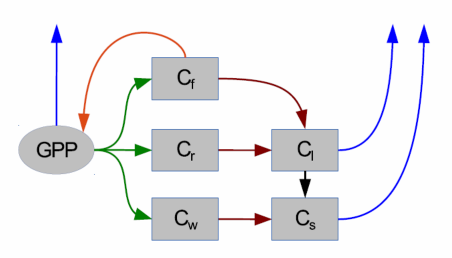

Figure : DALEC carbon balance model, (foliage (C_f), fine roots (C_r), woody biomass (C_w), litter (C_l) and soil organic matter (C_s)). The gross primary production function (GPP) uses meteorological driving data and the site's leaf area index (a function of C_f) to calculate the total amount of carbon to be allocated at a daily time step.

DALEC model equations

The model equations for the carbon pools at day $t+1$ are as follows:

$$C_f(t+1) = (1-p_5)C_f(t) + p_3(1-p_2)GPP(C_f(t), \phi), \tag{1}$$
$$C_r(t+1) = (1-p_7)C_r(t) + p_4(1-p_3)(1-p_2)GPP(C_f(t), \phi), \tag{2}$$
$$C_w(t+1) = (1-p_6)C_w(t) + (1-p_4)(1-p_3)(1-p_2)GPP(C_f(t), \phi), \tag{3}$$
$$C_l(t+1) = (1-(p_1+p_8)T(t))C_l(t) + p_5 C_f(t) + p_7 C_r(t), \tag{4}$$
$$C_s(t+1) = (1-p_9 T(t))C_s + p_6 C_w(t) + p_1 T(t) C_l(t), \tag{5}$$

where $T(t) = \frac{1}{2}exp(p_{10} T_m(t))$, T_m is daily mean temperature, $p_1, \ldots, p_{10}$ are rate parameters and ϕ represents the meteorological driving data used in the *GPP* function.

DALEC Tangent Linear Model

To compute the tangent linear model for DALEC ($\mathbf{M}_i = \frac{\delta \underline{m}_i}{\delta \underline{x}_i}$) using a state vector, $\underline{x} = (C_f, C_r, C_w, C_l, C_s)$, we first need to compute the first derivative of GPP.

We then have,

$$\mathbf{M}_i =$$

$$\begin{pmatrix} (1-p_5)+p_3(1-p_2)\zeta & 0 & 0 & 0 & 0 \\ p_4(1-p_3)(1-p_2)\zeta & (1-p_7) & 0 & 0 & 0 \\ (1-p_4)(1-p_3)(1-p_2)\zeta & 0 & (1-p_6) & 0 & 0 \\ p_5 & p_7 & 0 & (1-(p_1+p_8)T(t)) & 0 \\ 0 & 0 & p_6 & p_1 T(t) & (1-p_9 T(t)) \end{pmatrix},$$

where $\zeta = GPP'(C_f(t), \phi)$.

4DVAR with DALEC

We can now code up a 4DVAR cost function and gradient where,

$$J = \frac{1}{2}(\mathbf{x}_0 - \mathbf{x}_B)^T \mathbf{B}^{-1}(\mathbf{x}_0 - \mathbf{x}_B) + \frac{1}{2}\sum_{i=0}^{n}(\mathbf{y}_i - \underline{h}_i(\mathbf{x}_i))^T \mathbf{R}_i^{-1}(\mathbf{y}_i - \underline{h}_i(\mathbf{x}_i))$$

$$\bigtriangledown J = \mathbf{B}^{-1}(\mathbf{x}_0 - \mathbf{x}_B) - \sum_{i=0}^{n} \mathbf{M}_{i,0}^T \mathbf{H}_i^T \mathbf{R}_i^{-1}(\mathbf{y}_i - \underline{h}_i(\mathbf{x}_i)).$$

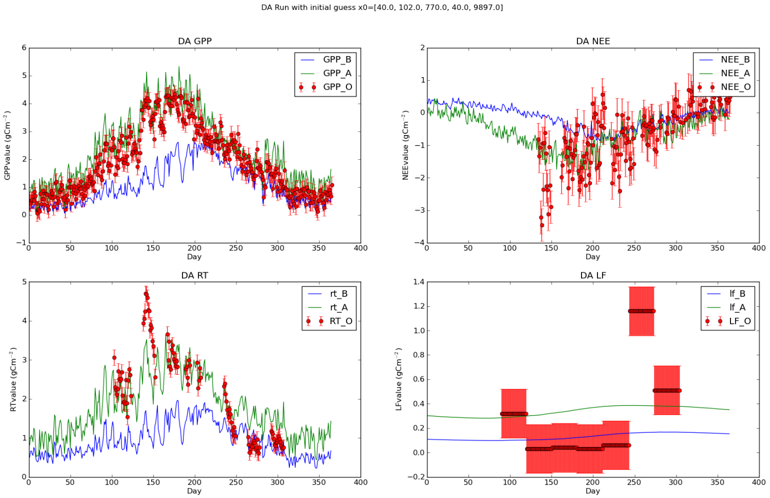

Figure : 4DVAR DALEC, 365 day assimilation window, only NEE observations assimilated.

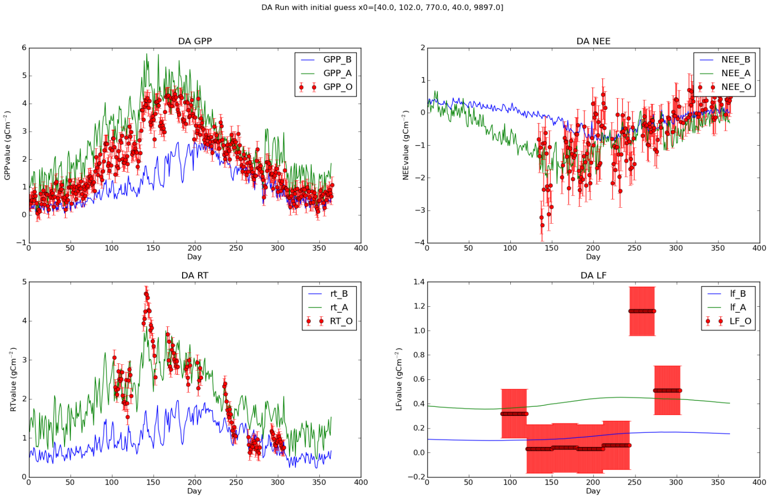

Figure : 4DVAR DALEC, 365 day assimilation window, NEE and RT observations assimilated.

Test for DA code

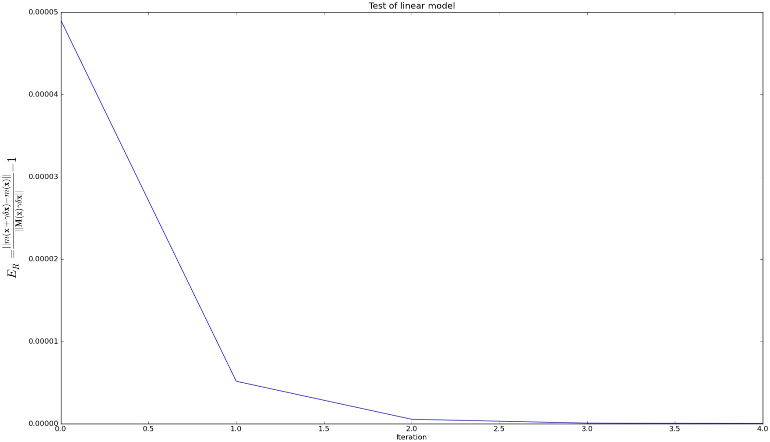

Figure : Linear model test for a 365 day run, $E_R = \frac{||m(\mathbf{x}+\gamma\delta\mathbf{x})-m(\mathbf{x})||}{||\mathbf{M}(\mathbf{x})\gamma\delta\mathbf{x}||} - 1$.

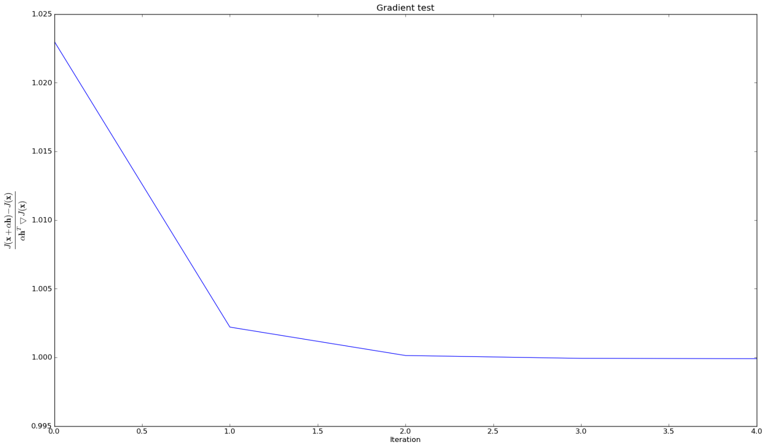

Figure : Gradient test, $\frac{J(\mathbf{x}+\alpha\mathbf{h})-J(\mathbf{x})}{\alpha\mathbf{h}^T\bigtriangledown J(\mathbf{x})}$.