Understanding the Information Content in Diverse Observations of Forest Carbon Stocks and Fluxes for Data Assimilation and Ecological Modeling
NERC case partnership with Forest Research

Ewan Pinnington

March 2, 2015

The DALEC Model

The DALEC model is a simple process-based model describing the carbon balance of an evergreen forest ecosystem . The model is constructed of five carbon pools linked via fluxes.

The DALEC Model

The DALEC model is a simple process-based model describing the carbon balance of an evergreen forest ecosystem . The model is constructed of five carbon pools linked via fluxes.

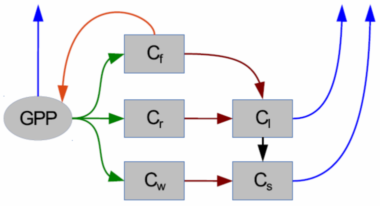

Figure: DALEC carbon balance model, (foliage (C_f), fine roots (C_r), woody biomass (C_w), litter (C_l) and soil organic matter (C_s)). The gross primary production function (GPP) uses meteorological driving data and the site's leaf area index (a function of C_f) to calculate the total amount of carbon to be allocated at a daily time step.

DALEC model equations

The model equations for the carbon pools at day $t+1$ are as follows:

$$C_f(t+1) = (1-p_5)C_f(t) + p_3(1-p_2)GPP(C_f(t), \phi), \tag{1}$$

$$C_r(t+1) = (1-p_7)C_r(t) + p_4(1-p_3)(1-p_2)GPP(C_f(t), \phi), \tag{2}$$

$$C_w(t+1) = (1-p_6)C_w(t) + (1-p_4)(1-p_3)(1-p_2)GPP(C_f(t), \phi \tag{3}$$

$$C_l(t+1) = (1-(p_1+p_8)T(t))C_l(t) + p_5 C_f(t) + p_7 C_r(t), \tag{4}$$

$$C_s(t+1) = (1-p_9 T(t))C_s + p_6 C_w(t) + p_1 T(t) C_l(t), \tag{5}$$

where $T(t) = \frac{1}{2}exp(p_{10} T_m(t))$, T_m is daily mean temperature, $p_1, \ldots, p_{10}$ are rate parameters and ϕ represents the meteorological driving data used in the GPP function.

DALEC Tangent Linear Model

To compute the tangent linear model for DALEC ($\mathbf{M}_i = \frac{\delta \underline{m}_i}{\delta \underline{x}_i}$) using a state vector, $\underline{x} = (C_f, C_r, C_w, C_l, C_s)$, we first need to compute the first derivative of GPP.

We then have,

$$\mathbf{M}_i =$$

$$\begin{pmatrix} (1-p_5)+p_3(1-p_2)\zeta & 0 & 0 & 0 & 0 \\ p_4(1-p_3)(1-p_2)\zeta & (1-p_7) & 0 & 0 & 0 \\ (1-p_4)(1-p_3)(1-p_2)\zeta & 0 & (1-p_6) & 0 & 0 \\ p_5 & p_7 & 0 & (1-(p_1+p_8)T(t)) & 0 \\ 0 & 0 & p_6 & p_1 T(t) & (1-p_9 T(t)) \end{pmatrix},$$

where $\zeta = GPP'(C_f(t), \phi)$.

4DVAR with DALEC

We can now code up a 4DVAR cost function and gradient where,

$$J = \frac{1}{2}(\mathbf{x}_0 - \mathbf{x}_B)^T \mathbf{B}^{-1}(\mathbf{x}_0 - \mathbf{x}_B) + \frac{1}{2}\sum_{i=0}^{n}(\mathbf{y}_i - \underline{h}_i(\mathbf{x}_i))^T \mathbf{R}_i^{-1}(\mathbf{y}_i - \underline{h}_i($$

$$\bigtriangledown J = \mathbf{B}^{-1}(\mathbf{x}_0 - \mathbf{x}_B) - \sum_{i=0}^{n} \mathbf{M}_{i,0}^T \mathbf{H}_i^T \mathbf{R}_i^{-1}(\mathbf{y}_i - \underline{h}_i(\mathbf{x}_i)).$$

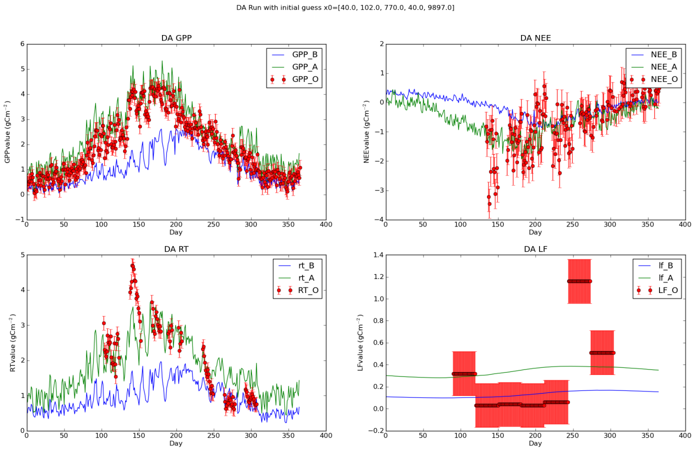

Figure: 4DVAR DALEC, 365 day assimilation window, only NEE observations assimilated.

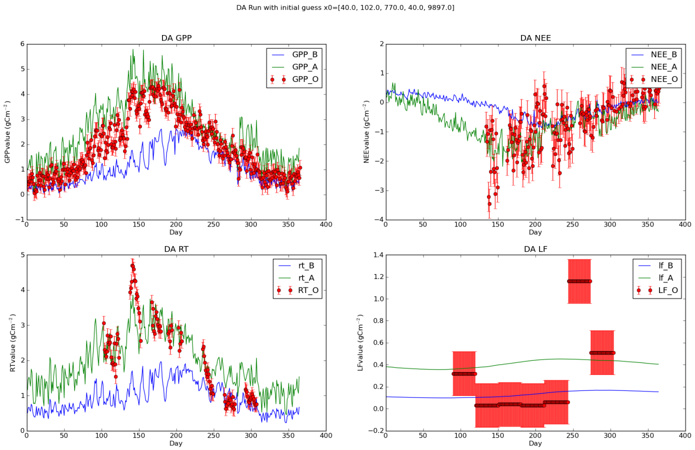

Figure: 4DVAR DALEC, 365 day assimilation window, NEE and RT observations assimilated.

Test for DA code

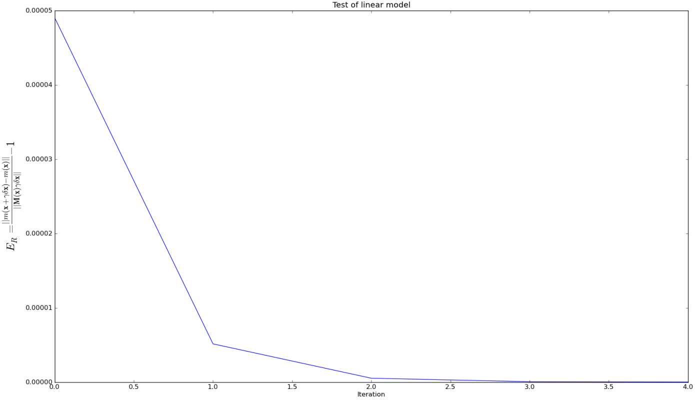

Figure: Linear model test for a 365 day run,
$E_R = \frac{||m(\mathbf{x}+\gamma\delta\mathbf{x})-m(\mathbf{x})||}{||\mathbf{M}(\mathbf{x})\gamma\delta\mathbf{x}||} - 1.$

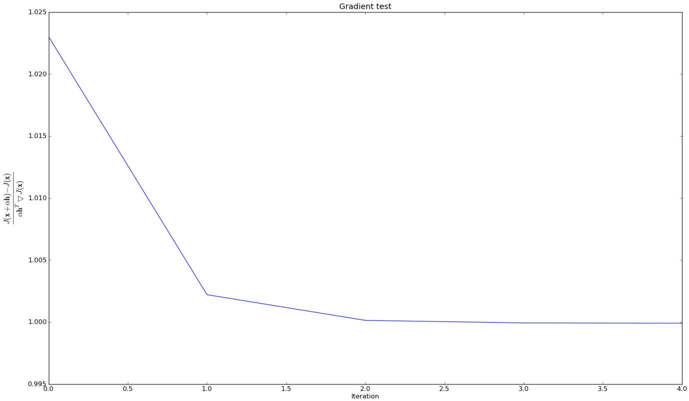

Figure: Gradient test, $\frac{J(\mathbf{x}+\alpha\mathbf{h})-J(\mathbf{x})}{\alpha\mathbf{h}^T \triangledown J(\mathbf{x})}$.