Comparative Analysis of Two Historical Factor Model-Based Strategies in Rates Trading*

Kaichong Wang
kw3181@nyu.edu

December 28, 2023

Abstract

In this study, we evaluated two simple and widely-used rate trading strategies: **PCA butterfly trading**[1] and **cointegrated butterfly trading**[4]. Both strategies use Historical Factor Model and aim to identify mean-reverting stationary alpha factors and construct a replicating portfolio, typically a butterfly spreads. We tested the stationarity, stability, and predictability of these two models' alpha factor (butterfly spread rate) using empirical data from the past five years. Our study finds that the butterfly spreads on 3-Year, 5-Year, and 10-Year CMTs constructed based on the PCA model and CCA model show effectiveness under market conditions in 2023. The factors exhibit significant stationarity and mean-reversion against the unit root process and random walk process under a 95% confidence level, and the dynamic mean-reversion signals on the butterfly are predictive with hit rates significantly above 50%. Additionally, the PC-Neutralized butterfly outperforms the cointegrated butterfly considerably.

1 Introduction

Rate trading strategies have been a core interest for investors and institutions. Securities in the U.S. interest rate market include US Treasury Instruments and SoFR futures, among others. Unlike the stock market, the fixed-income market has fewer instruments, higher collinearity, and greater liquidity. As a result, latent-factor-based quantitative strategies are generally more prominent in this market.

In the realm of fixed-income quantitative trading, two prominent model classes are the Historical Factor Model (HFM) and the Term Structure Model (TSM). HFMs use statistical analysis of historical market data to identify systematic factors in the cross-section of rates. It relies on historical patterns to predict future rate movements. However, this model may be limited by the resilience of historical patterns. Examples of the HFMs include Principal Components Analysis and Canonical Correlation Analysis. On the other hand, the TSMs connects the cross-sectional and time series dynamics of rates consistently. It focuses on the yield curve and employs mathematical models to describe the systematic function of the yield curve as well as the temporal dynamics of the factors in the systematic function. Examples of the Term Structure Model include the affine model[3] and the Nelson-Siegel Model[12]. This project focused on HFM. This is because the alpha factor of HFM is usually a function of rates and can be directly replicated by a portfolio of fixed-income instruments. The replicating portfolio is in the form of rates that can be easily computed and analyzed. On the other hand, the alpha factor of TSM is usually in the form of pricing errors, which is less straightforward.

In this study, we compare two popular strategies that uses two different HFMs: Principal Component Analysis (PCA) and Canonical Correlation Analysis (CCA). PCA is a technique that decomposes the movements of yield curves into a set of principal components, which represent the major sources of variation in interest rates. The first three principal components of the yield curve are widely recognized and weakly associated the yield curve's level, slope, and curvature factors[10]. Notably, the PCA factor loadings of the first two principal components allow us to construct a butterfly spread that is effectively "neutralized" to changes in the yield curve's level and slope, meaning it exhibits zero sensitivity to these risk factors. We

*This document is submitted as the final project report for the course FRE-GY.7801 Real Time Risk Management, Fall 2023.

refer to this spread as **PC-neutralized butterfly**, and the strategy as **PCA butterfly trading**. CCA, in multiple time series, can generate a cointegrated vector. The vector associated with the smallest eigenvalue of the CCA predictivity ratio matrix corresponds to the direction of least persistence (indicating the most mean-reversion) factor. It is also the most stationary and is referred to as the ”stationary vector”. Similarly, we can use CCA on any three rates and extract the ”stationary vector”: a linear combination if the three rates that can be replicated with a butterfly spread. We refer to this spread as **cointegrated butterfly** and the strategy as **cointegration butterfly trading**. Since both strategies involve trading a butterfly, we can directly compare them by fixing the trio of instruments of the butterflies and uniforming the belly position which the wings hedge against.

While investors can actively trade butterfly spreads to express their views on market dynamics, such as the curvature movement using the PC-Neutralized Butterfly, many also identify these spreads as mean-reverting and adopt a passive strategy to capitalize on their deviation from their means. The mean-reversion of the spreads can be explained intuitively. In PCA butterfly trading, the remaining principal component, such as the curvature, of the yield curve is typically stable over time. In cointegrated butterfly trading, the cointegration relationship of the long-term equilibrium of rates is relatively resilient within a short period. As stated by Aygün, Showers, and Cherpelis, mean-reverting spreads are preferred over trending spreads[1]. In this context, the effectiveness of these strategies can be reflected by examining the stationarity, stability (mean-reversion), and predictability of the spread rates. These characteristics can be measured using statistical tests and back-testing.

The butterflies are constructed using constant maturity treasury (CMT). CMT rates are the yields on a U.S. Treasury security with multiple maturities, usually ranging from 1 to 30 years, that are adjusted daily to reflect prevailing market conditions. The CMTs represent the observed term structure, also known as the yield curve, at any given time. *Figure 1* displays the yield curve for today, 10 days ago, and 20 days ago. In practice, CMT rates are not directly tradable. However, there do exist instruments that are affected by changes in CMT rates, such as actual U.S. Treasury securities with maturities that are close to those represented by the CMT rates, or interest rate swaps that are indexed by CMTs.

Figure 1: Comparison of Treasury Yield Curves: Today (Dec. 28, 2023), 10 Days and 20 Days Ago

In this comparative analysis, we use PCA and CCA to construct the PC-neutralized butterfly and cointegrated butterfly. The butterfly is constructed using the 3-year, 5-year, and 10-year daily CMTs as the left wing, belly, and right wing, respectively. This represents the portfolio of three hypothetical CMTs. The *estimation period* ranges from the 2019 to 2022. We then test the stationarity, stability (mean-reversion), and predictability of the two constructed butterfly spreads over the *testing period* of 2023.

The remaining paper is structured as follows. Section 2 reviews the literature on our PCA butterfly trading strategy and cointegrated butterfly trading strategy and provides a detailed estimation process. Section 3 explains the methodology of the metrics we use to evaluate the alpha factors of these two strategies. Section 4 presents the results. Section 5 lists the limitations of this study and suggests areas for further research. Section 6 concludes.

2 Literature Review

2.1 PCA Butterfly Trading

The seminal work by Litterman and Scheinkman (1991)[10] laid the foundation for Principal Component Analysis (PCA) in finance. They demonstrated its effectiveness in analyzing the yield curve and identified that a few principal components - level, slope, and curvature - explain most of the yield curve movements. These components are crucial in butterfly trading strategies. Knez, Litterman, and Scheinkman (1994)[9] further expanded on this research, showcasing the practical application of PCA in dissecting bond returns.

Butterfly trades involve taking positions in bonds with three different maturities, creating a 'butterfly spread'. This butterfly trading strategy, based on PCA, is very popular and has been extensively discussed in the literature. Tsay (2005)[13], Fabozzi et al. (2005)[6], and James and Webber (2000)[7] delve deeply into the practical aspects of implementing PCA-based butterfly trades. Most literature highlights the key challenge of accurately interpreting and acting upon the insights provided by PCA, especially considering the often rapidly changing market conditions.

The methodology we adopted, inspired by the Salomon Smith Barney report[1], integrates PCA into butterfly trading through a series of systematic steps. First, we compute rate changes at each time point over the estimation period, represented mathematically as $\Delta \mathbf{y}_t = \mathbf{y}_{t+1} - \mathbf{y}_t$, where $T \times n$ are the dimensions. Unlike some approaches that standardize data at this stage, our methodology bypasses this step. Following this, we construct the covariance matrix of the rate changes. We then perform eigenvalue decomposition on the covariance matrix:

$$\mathrm{Cov}(\Delta \mathbf{y}_t) = \mathbf{E} \cdot \mathbf{\Lambda} \cdot \mathbf{E}^{\intercal} \tag{1}$$

where $\mathbf{E}$ represents the eigenvectors, $\mathbf{E}^{(2)}$ represents the first two eigenvectors, which serve as the factor loadings in the model with dimensions $n \times 2$, and $\mathbf{\Lambda}$ is the diagonal matrix of eigenvalues (for two PCs, its dimensions are 2×2). The weights of our PC-Neutralized Butterfly are determined by solving the equation

$$\mathbf{E}^{(2)} \cdot \mathbf{w}_{\text{pca}} = \mathbf{0} \tag{2}$$

$$w_{\text{pca}}^{\text{belly}} = -1 \tag{3}$$

Here, $w_{\text{pca}}^{\text{belly}}$ refers to the second element of $\mathbf{w}_{\text{pca}}$, which corresponds to the "belly" position.

2.2 Cointegration Butterfly Trading

Cointegration of multiple non-stationary time series was first introduced by Engle and Granger in 1987[5]. This approach involves identifying combinations of non-stationary financial instruments that together form a stationary series, exploiting pricing inefficiencies through these relationships. The essence of this strategy lies in decomposing a panel dataset, typically represented as $\mathbf{y}_t$, with a covariance matrix $\mathbf{\Sigma}$ in dimension $(N \times N)$. This process distinguishes between M non-stationary and $N - M$ stationary, or cointegrated, directions. The variable nature of M throughout economic cycles adds a dynamic element to the strategy, necessitating continuous adaptation to changing market conditions.

Several methodologies have been pivotal in the development and implementation of cointegration butterfly trading. Johansen's Multivariate Maximum Likelihood Estimation (MLE) in 1988[8] enhanced the Engle-Granger approach, enabling a more comprehensive multivariate analysis of cointegration. This method is particularly adept at identifying multiple cointegrating relationships, making it a staple in financial research.

Canonical Correlation Analysis (CCA) in multiple time series was introduced by Box-Tiao in 1977[2]. It provides a simplified cointegration analysis method and facilitates the identification of pairs or groups of bonds that exhibit stable long-term relationships. This approach has been integral in pinpointing cointegrated combinations in the bond market. Chou and Ng's work in 1994[16], combining multivariate level regressions with CCA, offered an improved methodology for identifying cointegrating vectors, particularly beneficial in complex market environments where multiple relationships are present. Furthermore, the introduction of Regularized CCA[15] marked a significant advancement in handling large datasets. Traditional CCA approaches, when applied to high-dimensional financial data, often face challenges like overfitting or computational inefficiencies. Regularization techniques in CCA address these issues, refining the cointegration analysis for more accurate and applicable results in modern financial contexts.

In our cointegration butterfly trading strategy, we adopt the simplest Box-Tiao procedure, following d'Aspremont's work[4]. This approach focuses on replicating a stationary factor using a butterfly spread. The core of this methodology is encapsulated in a series of equations that describe the time series dynamics. We define our time series (the panel data of m rates) as $\mathbf{y}_t^{(m)}$, with $\mathbf{A}$ as the evolution matrix, $\hat{\mathbf{A}}$ as the estimated evolution matrix, and ϵ_t representing noise. The dynamics are governed by the state evolution equation.

$$\mathbf{y}_t^{(m)} = \mathbf{y}_{t-1}^{(m)}\mathbf{A} + \epsilon_t \tag{4}$$

The evolution matrix can be estimated using the OLS estimator, specifically,

$$\hat{\mathbf{y}}_t^{(m)} = \mathbf{y}_{t-1}^{(m)}\hat{\mathbf{A}} \tag{5}$$

$$\hat{\mathbf{A}} = \left(\mathbf{y}_{t-1}^{(m)\intercal}\mathbf{y}_{t-1}^{(m)}\right)^{-1}\mathbf{y}_{t-1}^{(m)\intercal}\mathbf{y}_t^{(m)} \tag{6}$$

Assuming the positiveness of $\mathbf{y}{}^{(m)}{}_t^{\intercal}\mathbf{y}_t^{(m)}$ and $\mathbf{A} \approx \hat{\mathbf{A}}$, the predictability ratio matrix $\mathbf{Q}$ is computed as follows:

$$\mathbf{Q} = \mathbf{\Sigma}^{-1}\hat{\mathbf{A}}^{\intercal}\mathbf{\Sigma}\hat{\mathbf{A}} \tag{7}$$

where $\mathbf{\Sigma} = \mathrm{Cov}(\mathbf{y}_t^{(m)})$. The eigenvalues and eigenvectors decomposed from the predictability ratio matrix are crucial. The smallest eigenvalue indicates the most mean-reverting direction, while the largest eigenvalue suggests the least mean-reverting direction. The weight vector of the cointegrated butterfly is proportional to the eigenvector corresponding to the smallest eigenvalue.

$$\mathbf{w}_{\text{coint}} = -\frac{\mathbf{v}_1}{w_{\text{coint}}^{\text{belly}}} \tag{8}$$

Here, $\mathbf{v}_1$ represents the eigenvector of $\mathbf{Q}$ associated with the smallest eigenvalue, and $w_{\text{coint}}^{\text{belly}}$ refers to the second element of $\mathbf{v}_1$, which corresponds to the "belly" position.

3 Methodology

3.1 Data

Our study utilizes Constant Maturity Treasury (CMT) rates sourced from **Quandl**, a prominent provider of alternative data for institutional investors. The CMT rates are based on a specific methodology: they are interpolated by the U.S. Treasury from the daily par yield curve, which correlates a security's yield with its maturity time. This curve is constructed using the closing market bid prices of the most recently auctioned Treasury securities in the over-the-counter market. The indicative par yields are derived from bid-side market price quotations (not actual transactions) collected by the Federal Reserve Bank of New York around 3:30 PM each trading day. The CMT yield values are then extracted from the par yield curve at predetermined maturities[11][14]. Our dataset encompasses a total of 11 different maturities – 1, 2, 3, 6 months, and 1, 2, 3, 5, 7, 10, 20, 30 years – after excluding the 2-month rates due to excessive data gaps[1].

The dataset spans from January 1, 2019, to December 28, 2023, providing approximately five years of recent data. We employ the data from January 1, 2019, to December 31, 2023 (referred as the *estimation*

[1] This method calculates a par yield for a 10-year maturity, even if there is no security with exactly 10 years left until maturity. The Treasury par yield curve (CMT rates) is estimated daily using a monotone convex spline method. The model takes indicative bid-side prices for the most recently auctioned nominal Treasury securities as inputs. The Treasury reserves the right to make changes to the yield curve at its discretion. The 4-month constant maturity series began on October 19, 2022.

period), to calibrate our PCA and CCA models, establishing the constant weights for two distinct butterfly spreads. These spreads are subsequently tested over a one-year period, from January 1, 2023, to December 28, 2023 (referred as the *testing period*).

3.2 Engineering Comparable Butterfly Spreads

The construction of two butterfly spreads for analysis are constructed based on the following methodology. Both spreads utilize the 3-Year, 5-Year, and 10-Year CMTs. In both spreads, the 3-Year CMT serves as the left wing (short wing), the 10-Year CMT as the right wing (long wing), and the 5-Year CMT as the belly (bullet) of the butterfly. We standardize the position of the belly at -1 for both butterflies. Consequently, the positions of the short and long wings are determined by the hedge ratios derived from their respective HFM. This consistent structure allows for a direct comparison between the two butterflies, enabling a uniform evaluation of both strategies under the same set of metrics.

The detailed construction methodology is stated in Section 2, specifically equations (1) to (8). Please note that $\mathbf{y}_t$ represents the panel data of all 11 CMTs in the *estimation period*, while $\mathbf{y}_t^{(m)}$ represents the panel data of the three selected CMTs in the *estimation period*.

3.3 Evaluation with Statistical Tests

To assess the effectiveness of the factors replicated the two butterfly spreads, we begin by implementing a few statistical tests. These tests analyze and compare the time-series of the constructed butterfly spreads throughout the *testing period*, focusing on evaluating their stationarity and stability (mean reversion).

3.3.1 Assessing Stationarity: Augmented Dickey-Fuller Test

The Augmented Dickey-Fuller (ADF) test is employed to assess the stationarity of the time series. A time series is stationary if its statistical properties, such as mean and variance, remain constant over time. The ADF test involves the following regression:

$$\Delta f_t = \alpha + \beta t + \gamma f_{t-1} + \sum_{i=1}^{p} \phi_i \Delta f_{t-i} + \epsilon_t \tag{9}$$

Here, f_t represents the factor (spread rate) time series, Δ is the difference operator, α is a constant, β captures a linear time trend, and ϵ_t is the error term. The null hypothesis posits the presence of a unit root ($\gamma = 0$), indicating non-stationarity. A more negative value of the ADF statistic and a lower p-value imply stronger evidence against the null hypothesis, suggesting stationarity.

3.3.2 Assessing Stability (Mean-Reversion): Variance Ratio Test

To evaluate the mean-reversion tendency of the time series, we employ the Variance Ratio Test. This test compares the behavior of the time series to a random walk, where the series' increments are uncorrelated over time. The Variance Ratio at lag k is calculated using the formula:

$$VR(k) = \frac{\mathrm{Var}(f_t - f_{t-k})}{k \cdot \mathrm{Var}(f_t - f_{t-1})} \tag{10}$$

In this equation, $\mathrm{Var}(f_t - f_{t-k})$ denotes the variance of k-period factor changes, while $\mathrm{Var}(f_t - f_{t-1})$ represents the variance of one-period factor changes. A Variance Ratio ($VR(k)$) close to 1 indicates a random walk pattern. Conversely, values significantly lower than 1 are indicative of mean reversion, suggesting that the series tends to revert to its mean. A lower VR statistic combined with a smaller p-value strengthens the evidence against the random walk hypothesis, supporting a robust mean-reversion characteristic. This infers stability in the time series, implying a lower likelihood of prolonged deviations from its historical mean. Here, we arbitrarily set the number of lags to be $k = 2$.

3.4 Evaluation with Back-testing

To gauge the predictability of the two alpha factors replicated by the two spreads and potential profitability of a mean-reversion trading strategy applied to the butterfly spreads, we conduct back-testing. Our analysis hinges on the following dynamic model:

$$f_t = \mu_t + \epsilon_t \tag{11}$$

where ϵ_t represents a mean-reverting process, often interpreted as a signal process, and is defined by:

$$d\epsilon_t = -\theta \epsilon_t dt + \sigma_t dW_t \tag{12}$$

If the mean (μ_t) and volatility (σ_t) of a signal process remain constant over time, then the process is not only mean-reverting but also stationary. Otherwise, the signal process needs to be standardized based on its mean and volatility processes to become mean-reverting and stationary. If the signal process based on the estimated μ_t and σ_t are stably mean-reverting against trending, the factor can be well-predicted. To evaluate this predictability, we implement a mean-reversion test on the butterfly spreads over the *testing period*, using a threshold based on the factor levels at time t:

$$\begin{cases} [\hat{\mu} - z \cdot \hat{\sigma}, \hat{\mu} + z \cdot \hat{\sigma}] & \text{in the static model,} \\ [\hat{\mu}_t - z \cdot \hat{\sigma}_t, \hat{\mu}_t + z \cdot \hat{\sigma}_t] & \text{in the dynamic model,} \end{cases}$$

where $\hat{\mu}$ / $\hat{\mu}_t$ and $\hat{\sigma}$ / $\hat{\sigma}_t$ denote the estimated mean and volatility (at time t), respectively, and z is a predetermined z-score defining the threshold band. The signal time series is defined as:

$$\begin{cases} z_t = \frac{(f_t - \hat{\mu})}{\hat{\sigma}} & \text{in the static model,} \\ z_t = \frac{(f_t - \hat{\mu_t})}{\hat{\sigma_t}} & \text{in the dynamic model,} \end{cases}$$

The strategy posits that if the factor value exceeds this threshold, it will revert to the range $[\hat{\mu} \pm z \cdot \hat{\sigma}]$ / $[\hat{\mu}_t \pm z \cdot \hat{\sigma}_t]$ within u days. We define the strategy's *Hit Rate* as the frequency with which the factor value, upon crossing the specified threshold, reverts to its range which it diverged from. Signals are recognized cumulatively and are overlapping; that is, each instance where the factor strays from the threshold is noted and judged as a hit or miss, even if the factor has not yet returned within the set bounds. This approach ensures all divergences are accounted for in the analysis. Essentially, a *Hit Rate* exceeding 50% suggests reliable factor predictability. Conversely, a *Hit Rate* below 50% implies the strategy's ineffectiveness in capitalizing on mean-reversion tendencies.

3.4.1 Static Model

In the static model, $\hat{\mu}$ and $\hat{\sigma}$ are constants. These estimates are derived from the historical mean and volatility over a look-back period of k days. The hyperparameters are set as follows:

- $z = 1$: This sets the confidence level at approximately 68%. A higher z reflects a more conservative approach to the prediction interval.
- $u = 5$: This represents an expected market correction period of 5 days, within which the mean reversion is anticipated to occur.
- $k = 30$: Based on the premise that financial market information typically becomes obsolete after about 30 periods.

These parameters are chosen for their alignment with standard financial market practices and heuristic analysis.

3.4.2 Dynamic Model

The dynamic model updates $\hat{\mu}_t$ and $\hat{\sigma}_t$ throughout the testing period, using an exponential weighted moving average (EWMA). The initial values are set based on historical data over a look-back period of k days. The updating equations are:

$$\hat{\mu}_t = \lambda_\mu f_t + (1 - \lambda_\mu)\hat{\mu}_{t-1}, \tag{13}$$

$$\hat{\sigma}_t = \sqrt{\lambda_\sigma (f_t - \hat{\mu}_{t-1})^2 + (1 - \lambda_\sigma)\hat{\sigma}^2_{t-1}}. \tag{14}$$

The hyperparameters in this model are:

- $z = 0.5$: Adjusted for increased strategy conservativeness.
- $u = 5$: A 5-day market correction period expectation.
- $\lambda_\mu = 0.2$ and $\lambda_\sigma = 0.05$: Arbitrarily chosen for mean and volatility responsiveness.
- $k = 30$: Reflecting the assumed information relevance period in financial markets.

These parameters, while intuitive, conform to the general conventions of dynamic market modeling.

An important note is that the strategy *hits* if the spread reverts to its initial thresholds range, instead of the new one, given that the thresholds are dynamically changing.

4 Results and Findings

4.1 Butterflies Estimation and Visualization

The two models result in slightly distinct allocations for the butterfly spreads using [3-YR, 5-YR, 10-YR] CMTs. The computed positions of the PC-neutralized butterfly are [0.61, -1, 0.44], while the computed positions of the cointegrated butterfly are [0.51, -1, 0.52], as detailed in *Table 1*. *Figure 2* shows the time series of the two spread rates for both the estimation and testing periods. *Table 2* provides the descriptive statistics of the two spread rates over the estimation for both the estimation and testing period.

A visual examination of the data from the 4-year estimation period preceding the final year reveals noticeable non-stationarity in both spreads. This suggests a turbulent economic and Treasury environment influenced by factors like COVID-19, geopolitical incidents, and a recessionary economy. Notably, the cointegrated butterfly displayed a lesser degree of trending behavior compared to the PC-neutralized butterfly during this period. In other words, the cointegrated butterfly maintained a more stable mean and stronger mean-reversion properties. This observation aligns with the CCA model's objective of identifying the most stationary vector. In the testing period, the PC-neutralized butterfly's mean-reversion became more pronounced. Meanwhile, the Cointegrated Butterfly's mean began to show an upward trajectory, especially from late 2023, suggesting a shifting behavior.

Strategy/Position	**3-Year CMT**	**5-Year CMT**	**10-Year CMT**
PC-Neutralized Butterfly	0.6149	-1	0.4354
Cointegrated Butterfly	0.5142	-1	0.5184

Table 1: Positions of Butterfly Spreads on CMTs

4.2 Statistical Tests Results

The ADF test results indicate stationarity in both butterflies at a 5% significance level. Specifically, the PC-neutralized butterfly, with an ADF Statistic of -3.1216 and a p-value of 0.0250, demonstrates stronger stationarity compared to the cointegrated butterfly, which has an ADF Statistic of -2.8778 and a p-value of 0.0480.

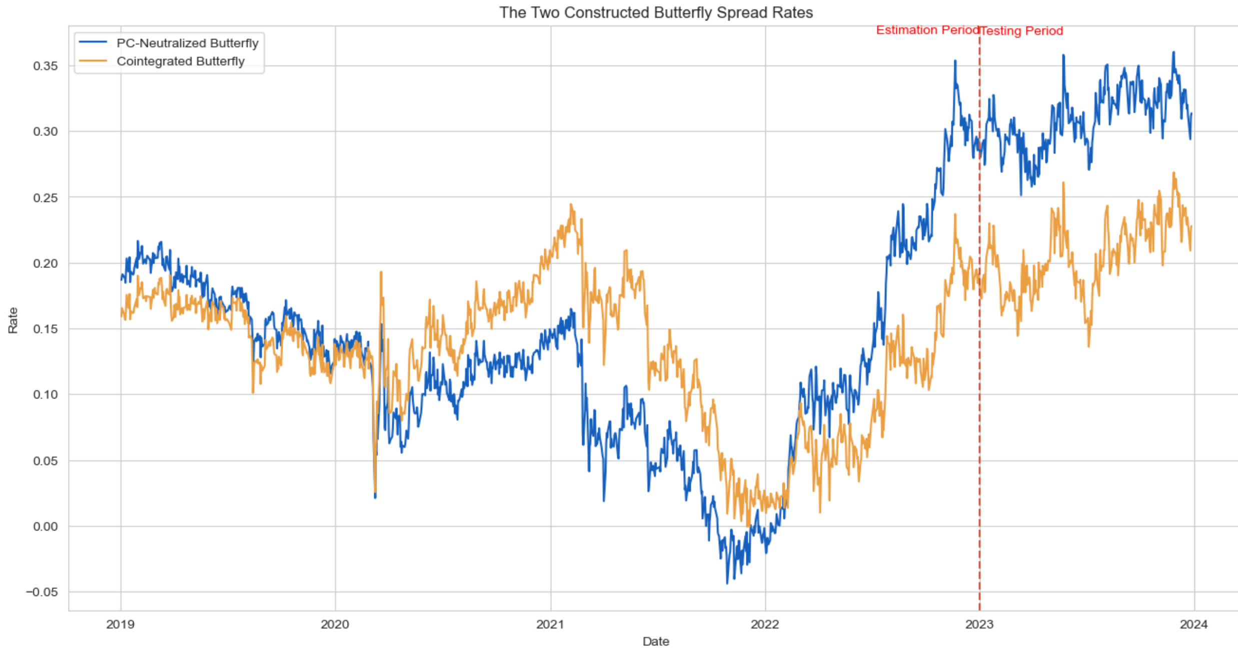

Figure 2: The Two Constructed Butterfly Spread Rates for the *Estimation Period* and the *Testing Period*

Strategy/Statistics	**Mean**	**Std**	**Min**	**Median**	**Max**
Estimation Period					
PC-Neutralized Butterfly	0.1242	0.0729	-0.0440	0.1244	0.3537
Cointegrated Butterfly	0.1308	0.0524	-0.0008	0.1375	0.2446
Testing Period					
PC-Neutralized Butterfly	0.3099	0.0225	0.2513	0.3105	0.3603
Cointegrated Butterfly	0.2057	0.0269	0.1361	0.2068	0.2687

Table 2: Descriptive Statistics of Butterfly Spreads Rate Time Series

The Variance Ratio Test results suggest a higher degree of mean reversion or stability in the PC-neutralized butterfly, as evidenced by its VR Statistic of -3.4069 and a p-value of 0.0007. The cointegrated butterfly, with a VR Statistic of -3.3182 and a p-value of 0.0009, also shows mean reversion, but to a lesser extent compared to the PC-neutralized butterfly in the *testing period*.

Strategy	**ADF Statistic**	**ADF p-value**	**VR Statistic**	**VR p-value**
PC-Neutralized Butterfly	-3.1216	0.0250	-3.4069	0.0007
Cointegrated Butterfly	-2.8778	0.0480	-3.3182	0.0009

Table 3: ADF and VR Statistics for Butterfly Spreads for the *Testing Period*

The statistical tests conclusively demonstrate that both the PCA and CCA factors, replicated by the spreads, were stationary and stable in 2023, with the PCA model's factor showing a higher degree of both stationarity and stability.

4.3 Back-Testing Results

The static model backtesting, depicted in *Figure 3*, reveals the PC-neutralized butterfly had a hit rate of 0.6195 with 70 hits and 43 misses. The cointegrated butterfly, however, demonstrated a lower hit rate of 0.5123, with 83 hits and 79 misses. Initially, the cointegrated butterfly's predictability was modest but diminished as the factor began an upward trend in the last quarter.

Strategy	Hit Rate	#Hit	#Miss	#Active Signals
Static Model				
PC-Neutralized Butterfly	61.95%	70	43	113
Cointegrated Butterfly	51.23%	83	79	162
Dynamic Model				
PC-Neutralized Butterfly	83.20%	104	21	125
Cointegrated Butterfly	76.47%	104	32	136

Table 4: Backtesting Results of Butterfly Spreads

The dynamic model results are showcased *Figure 4*, which indicates that the strategies are likely to be effective with smoothing mean and volatility signals. The PC-neutralized butterfly achieved a higher hit rate of 0.8320, with 104 hits and 21 misses, suggesting superior predictability over the cointegrated butterfly, which had a hit rate of 0.7647 with 104 hits and 32 misses. The dynamic model, incorporating EWMA models, adeptly captured the mean and volatility changes and produced more predictive signals.

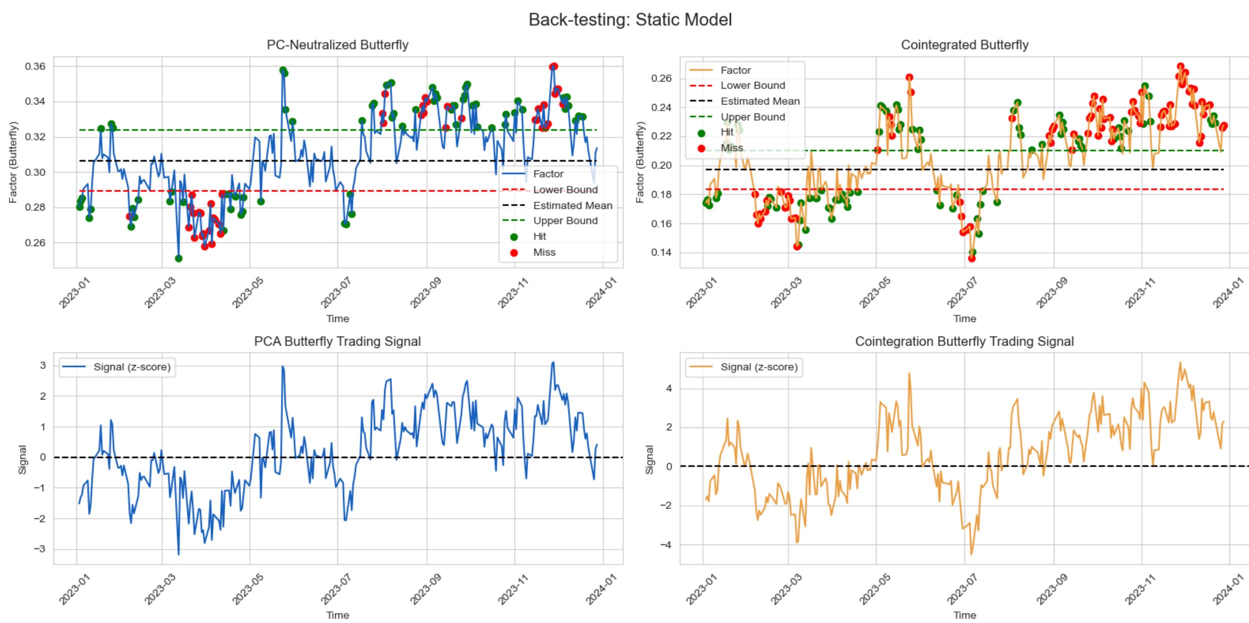

Figure 3: Back-testing: Static Model

5 Limitations and Further Study

This study demonstrates a methodology applicable to a trio of securities rate time series, yet it's crucial to note that the CMTs used are not directly tradable. Future work should adapt the derived butterfly spreads and signals for tradable securities. Moreover, the selection of hyperparameters was somewhat arbitrary; their sensitivity and impact on the model's performance require further analysis. Lastly, while the stationarity, stability, and predictability of the spreads have been tested, translating these factors into real-world effectiveness and profitability necessitates simulated trading and a thorough evaluation of the resulting profits and losses.

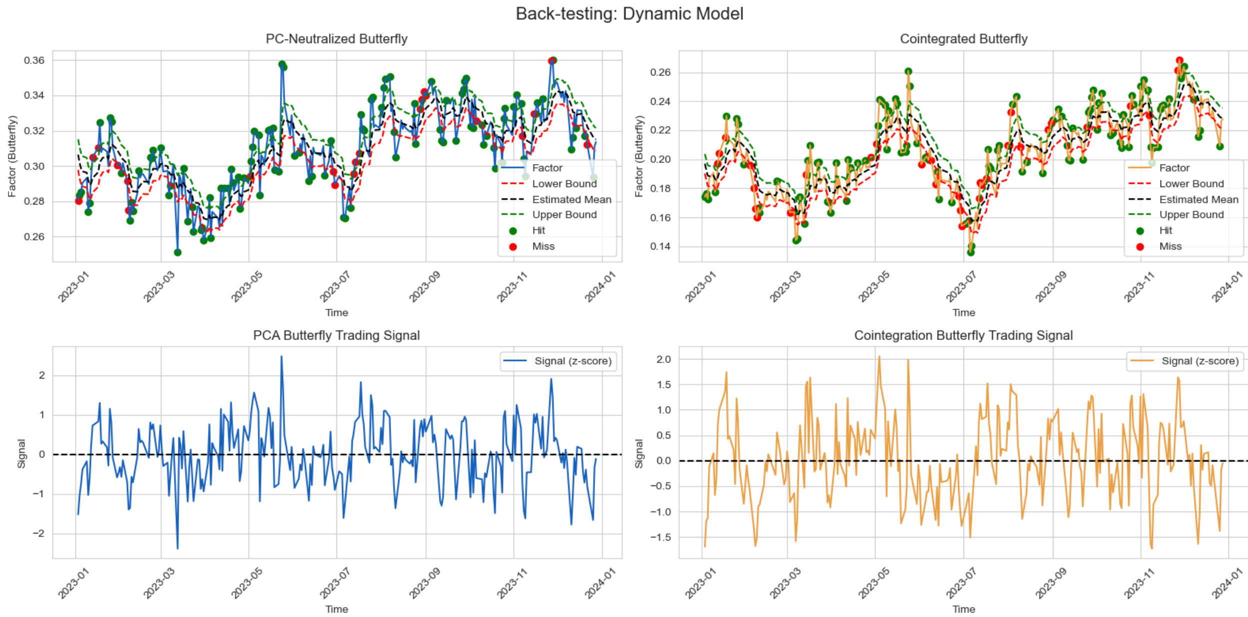

Figure 4: Back-testing: Dynamic Model

6 Conclusion

The study has successfully employed PCA and CCA models to construct butterfly spreads and assess their stationarity, stability, and predictability. Despite revealing that both strategies are stationary and stable, with the PCA model considerably outperforms, there remains a substantial leap from these theoretical constructs to actual trading viability. Future research should bridge this gap by applying the strategies to tradable instruments and rigorously testing their profitability through simulated trading environments.

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