NYU

Comparative Analysis of Two Strategies in Rates Trading

PCA Butterfly Trading v.s Cointegration Butterfly Trading: Two Historical Factor Models

12.28.23

Key Takeaways

Comparative Analysis of PCA and Cointegration Butterfly Trading

- Examined 3-Year, 5-Year, 10-Year Treasury spreads that replicated the alpha factor of PCA and Canonical Correlation Analysis (CCA) Model.

Metrics Assessing Stationarity, Stability and Predictability

- Utilized the Augmented Dickey-Fuller (ADF) Test and Variance Ratio (VR) Test to assess the stationarity and mean-reversion.
- Evaluated predictability using signals' back-tested hit rate, measuring how often the strategies correctly predicted the market's direction.

Superiority of PCA Butterfly Trading

- PCA Butterfly demonstrates greater stability and predictability in spread rates compared to Cointegration Butterfly.

Introduction

- Latent-factor-based passive rate-trading strategies are increasingly prominent in the fixed income market.
- Two main classes of models: Historical Factor Models (HFM) and Term Structure Models (TSM).
    - TSM Example: Affine Model, Nelson-Siegel Model
    - **HFM Examples: PCA, CCA.**
- PCA and CCA can be used to identify alpha factors that are neutralized to the systematic factors' temporal dynamics. These factors can be replicated through butterfly spread trading.
    - Mean-reversion is preferred for profitable passive mean-reversion trading
    - How stationery? How stable? How predictable?

PCA Butterfly Trading

- Decomposes the movements of yield curves into a set of principal components
- Weakly associated the yield curve's level, slope, and curvature factors
- PC-neutralized butterfly: A butterfly spread that is effectively "neutralized" to changes in the yield curve's level and slope. Remaining factors are typically stable.

The methodology we adopted, inspired by the Salomon Smith Barney report[1], integrates PCA into butterfly trading through a series of systematic steps. First, we compute rate changes at each time point over the estimation period, represented mathematically as $\Delta \mathbf{y}_t = \mathbf{y}_{t+1} - \mathbf{y}_t$, where $T \times n$ are the dimensions. Unlike some approaches that standardize data at this stage, our methodology bypasses this step. Following this, we construct the covariance matrix of the rate changes. We then perform eigenvalue decomposition on the covariance matrix:

$$\mathrm{Cov}(\Delta \mathbf{y}_t) = \mathbf{E} \cdot \mathbf{\Lambda} \cdot \mathbf{E}^{\intercal} \tag{1}$$

where $\mathbf{E}$ represents the eigenvectors, $\mathbf{E}^{(2)}$ represents the first two eigenvectors, which serve as the factor loadings in the model with dimensions $n \times 2$, and $\mathbf{\Lambda}$ is the diagonal matrix of eigenvalues (for two PCs, its dimensions are 2×2). The weights of our PC-Neutralized Butterfly are determined by solving the equation

$$\mathbf{E}^{(2)} \cdot \mathbf{w}_{\mathrm{pca}} = \mathbf{0} \tag{2}$$

$$w_{\mathrm{pca}}^{\mathrm{belly}} = -1 \tag{3}$$

Here, $w_{\mathrm{pca}}^{\mathrm{belly}}$ refers to the second element of $\mathbf{w}_{\mathrm{pca}}$, which corresponds to the "belly" position.

Cointegration Butterfly Trading

- Use the CCA method to extract the cointegration relationship and form a stationary linear combination of multiple non-stationary time-series.

- Use a fewer rates; more likely to overfit

In our cointegration butterfly trading strategy, we adopt the simplest Box-Tiao procedure, following d'Aspremont's work[4]. This approach focuses on replicating a stationary factor using a butterfly spread. The core of this methodology is encapsulated in a series of equations that describe the time series dynamics. We define our time series (the panel data of m rates) as $\mathbf{y}_t^{(m)}$, with $\mathbf{A}$ as the evolution matrix, $\hat{\mathbf{A}}$ as the estimated evolution matrix, and ϵ_t representing noise. The dynamics are governed by the state evolution equation.

$$\mathbf{y}_t^{(m)} = \mathbf{y}_{t-1}^{(m)}\mathbf{A} + \epsilon_t \tag{4}$$

The evolution matrix can be estimated using the OLS estimator, specifically,

$$\hat{\mathbf{y}}_t^{(m)} = \mathbf{y}_{t-1}^{(m)}\hat{\mathbf{A}} \tag{5}$$

$$\hat{\mathbf{A}} = \left(\mathbf{y}_{t-1}^{(m)\intercal}\mathbf{y}_{t-1}^{(m)}\right)^{-1}\mathbf{y}_{t-1}^{(m)\intercal}\mathbf{y}_t^{(m)} \tag{6}$$

Assuming the positiveness of $\mathbf{y}^{(m)\intercal}_t\mathbf{y}_t^{(m)}$ and $\mathbf{A} \approx \hat{\mathbf{A}}$, the predictability ratio matrix $\mathbf{Q}$ is computed as follows:

$$\mathbf{Q} = \mathbf{\Sigma}^{-1}\hat{\mathbf{A}}^\intercal\mathbf{\Sigma}\hat{\mathbf{A}} \tag{7}$$

where $\mathbf{\Sigma} = \mathrm{Cov}(\mathbf{y}_t^{(m)})$. The eigenvalues and eigenvectors decomposed from the predictability ratio matrix are crucial. The smallest eigenvalue indicates the most mean-reverting direction, while the largest eigenvalue suggests the least mean-reverting direction. The weight vector of the cointegrated butterfly is proportional to the eigenvector corresponding to the smallest eigenvalue.

$$\mathbf{w}_{\text{coint}} = -\frac{\mathbf{v}_1}{w_{\text{coint}}^{\text{belly}}} \tag{8}$$

Here, $\mathbf{v}_1$ represents the eigenvector of $\mathbf{Q}$ associated with the smallest eigenvalue, and $w_{\text{coint}}^{\text{belly}}$ refers to the second element of $\mathbf{v}_1$, which corresponds to the "belly" position.

NYU

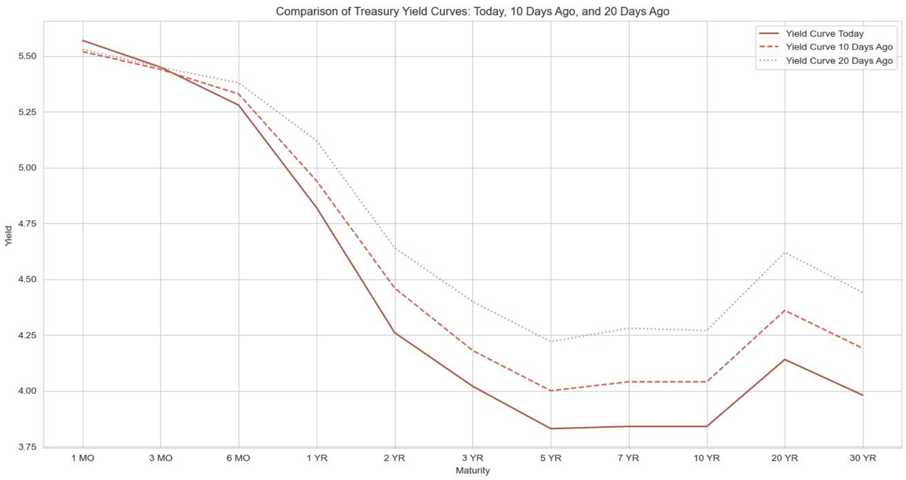
Methodology: Data

Constant Maturity Treasury Rates

- Utilized CMT rates spanning various maturities (1-30 years), excluding 2-month rates due to data gaps. The CMT rates, indicative of the yield curve, were interpolated from daily par **yield curves** and used for constructing the butterfly spreads.
- **Estimation Period**: 1/1/2019 to 12/31/2022
- **Testing Period**: 12/31/2022 to 12/28/2023

Engineering Comparable Butterfly Spreads

- Created two butterfly spreads using 3-YR 5-YR, and 10-YR CMTs. Both spreads consistently positioned the 5-YR as the belly (-1) and adjusted the 3-YR and 10-YR wings based on hedge ratios derived from PCA and CCA models.
- This standardized approach enabled direct comparison and uniform evaluation of both strategies.



NYU

Methodology: Statistical Tests

Assessing Stationarity: Augmented Dickey-Fuller Test

- A more negative value of the ADF statistic and a lower p-value imply stronger evidence against the null hypothesis, suggesting stationarity.

Assessing Stability (Mean-Reversion): Variance Ratio Test

- A lower VR statistic combined with a smaller p-value strengthens the evidence against the random walk hypothesis, supporting a robust mean-reversion characteristic.
- This infers stability in the time series, implying a lower likelihood of prolonged deviations from its historical mean.

Methodology: Back-testing

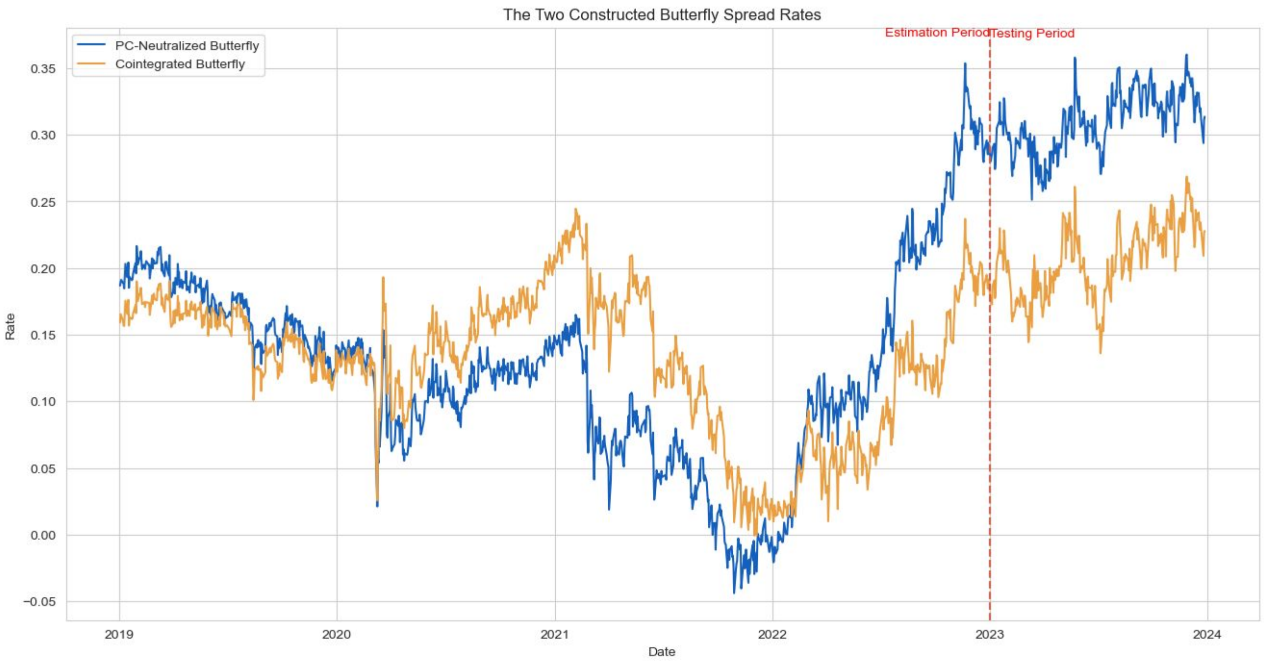
Assessing Predictability: Hit Rate

- Signals is active when the spreads diverges from its threshold (z std. away from its mean)
- Hit Rate: the frequency of successful predictions against the market direction, demonstrating the reliability of the strategies

Static Model and Dynamic Model

- **Static Model:** constant mean and volatility, estimated using historical data.
- **Dynamic Model**: dynamic mean and volatility estimates with the exponential weighted moving average (EWMA), adjusting to market changes.

Results: Visualization



Positions

[3YR, 5YR, 10TR]

PCA: [0.6149, -1, 0.4354]
Coint: [0.5142, -1, 0.5184]

NYU

Results: Statistical Test

Findings

- The ADF test results indicate stationarity in both butterflies at a 5% significance level. PC-Neutralized Butterfly demonstrates stronger stationarity compared to the Cointegrated Butterfly.
- The Variance Ratio Test results suggest a higher degree of mean reversion or stability in the PC-Neutralized Butterfly. The Cointegrated Butterfly also shows mean reversion, but to a lesser extent compared to the PC-Neutralized Butterfly in the testing period.
- The statistical tests conclusively demonstrate that both the PCA and CCA factors, replicated by the spreads, were stationary and stable in 2023, with the PCA model's factor showing a higher degree of both stationarity and stability.

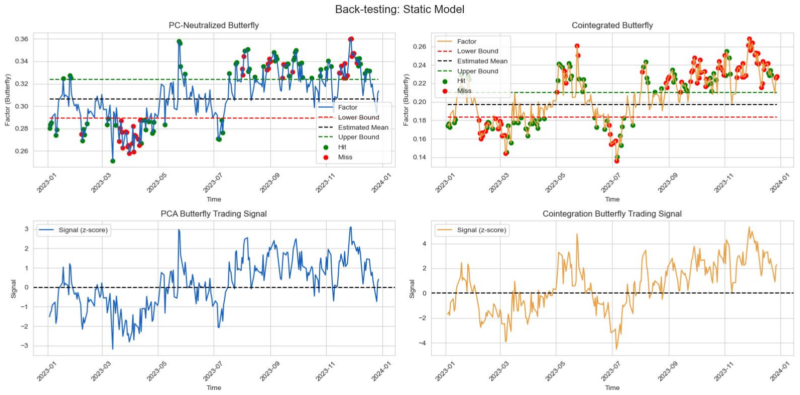
	PC-Neutralized Butterfly	Cointegrated Butterfly
ADF Statistic	**-3.1216**	-2.8778
ADF p-value	**0.0250**	0.4509
VR Statistic	**-3.4069**	-3.3182
VR p-value	**0.0007**	0.0009

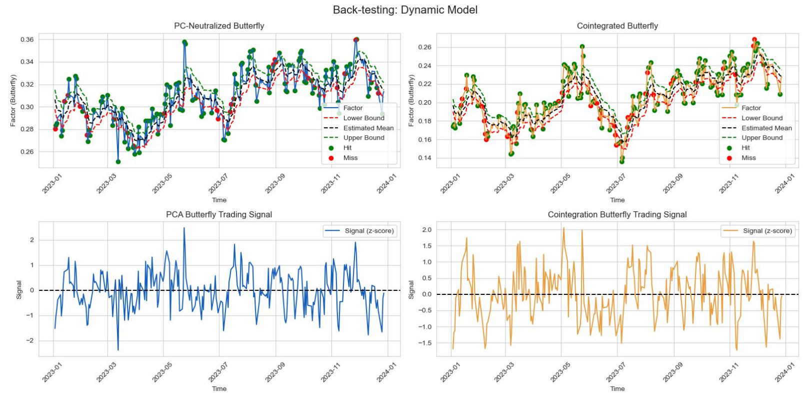
NYU

Results: Back-testing

Findings

- The static model backtesting reveals the **PC-Neutralized Butterfly** had a Hit Rate of **61.95%**. The **Cointegrated Butterfly**, however, demonstrated a lower Hit Rate of **51.23%**. Initially, the Cointegrated Butterfly's predictability was modest but diminished as the factor began an upward trend in the last quarter.
- The dynamic model results indicates that both strategies performed effectively. The **PC-Neutralized Butterfly** achieved a higher Hit Rate of **83.20%**, suggesting superior predictability over the **Cointegrated Butterfly**, which had a Hit Rate of **76.47%**. The dynamic model, incorporating an Exponential Weighted Moving Average, adeptly captured the mean and volatility changes, enhancing factor predictability.





Limitations and Conclusion

Limitations

- **Tradable Securities**: Notes that used Constant Maturity Treasuries (CMTs) are not directly tradable; adaptation for tradable securities is a future scope.
- **Hyperparameter Selection**: The arbitrary selection of hyperparameters and the need for further analysis of their impact and sensitivity.
- **More Informative Backtesting**: While stationarity, stability, and predictability are tested, translating these into real-world profitability requires simulated trading and profit-loss evaluation.

Conclusion

- The study has successfully employed PCA and CCA models to construct butterfly spreads and assess their stationarity, stability, and predictability. It reveals that both strategies' factor are stationary and stable, with the PCA model considerably outperforms

References

- 1. Baygün, B., Showers, J., & Cherpelis, G. (2000). Principles of Principal Components: A fresh look at Risk, Hedging, and Relative Value. Salomon Smith Barney.
- 2. Box, G. E., & Tiao, G. C. (1977). A canonical analysis of multiple time series. Biometrika, 64(2), 355-365.
- 3. Dai, Q., & Singleton, K. J. (2000). Specification analysis of affine term structure models. The Journal of Finance, 55(5), 1943-1978.
- 4. d'Aspremont, A. (2011). Identifying small mean-reverting portfolios. Quantitative Finance, 11(3), 351-364.
- 5. Engle, R. F., & Granger, C. W. J. (1987). Co-integration and error correction: representation, estimation, and testing. Econometrica: Journal of the Econometric Society, 251-276.
- 6. Fabozzi, F. J., & Mann, S. V. (2005). The Handbook of Fixed Income Securities. New York.
- 7. Eurex Frankfurt AG. (2007). Interest Rate Derivatives Fixed Income Trading Strategies. Frankfurt/Main, Bundesländer.
- 8. James, J., & Webber, N. (2000). Interest Rate Modelling. Wiley-Blackwell Publishing Ltd.
- 9. Johansen, S. (1988). Statistical analysis of cointegration vectors. Journal of Economic Dynamics and Control, 12(2-3), 231-254.
- 10. Knez, P. J., Litterman, R., & Scheinkman, J. (1994). Explorations into factors explaining money market returns. The Journal of Finance, 49(5), 1861-1882. Wiley Online Library.
- 11. Litterman, R. B., & Scheinkman, J. (1991). Common factors affecting bond returns. The Journal of Fixed Income, 1(1), 54-61.
- 12. Nasdaq Data Link. (n.d.). Treasury Yield Curve Rates. Retrieved December 14, 2023, from https://data.nasdaq.com/data/USTREASURY/YIELD-treasury-yield-curve-rates.
- 13. Nelson, C. R., & Siegel, A. F. (1987). Parsimonious modeling of yield curves. Journal of Business, 473-489.
- 14. Tsay, R. S. (2005). Analysis of Financial Time Series. John Wiley & Sons.
- 15. U.S. Department of the Treasury. (2023). Treasury Yield Curve Methodology. Retrieved December 14, 2023, from https://home.treasury.gov/resource-center/data-chart-center/interest-rates/TextView?type=daily_treasury_yield_curve&field_tdr_date_value=all&data=yieldAll.
- 16. Vinod, H. D. (1976). Canonical ridge and econometrics of joint production. Journal of Econometrics, 4(2), 147-166.
- 17. Zhou, H., Choong, P., McCarthy, R., Chou, S. T., Martin, T. J., & Ng, K. W. (1994). In situ hybridization to show sequential expression of osteoblast gene markers during bone formation in vivo. Journal of Bone and Mineral Research, 9(9), 1489-1499.