

# Lecture 14:

# Reinforcement Learning

# Administrative

## Grades:

- Midterm grades released last night, see Piazza for more information and statistics
- A2 and milestone grades scheduled for later this week

# Administrative

## Projects:

- All teams must register their project, see Piazza for registration form
- Tiny ImageNet evaluation server is online

# Administrative

## Survey:

- Please fill out the course survey!
- Link on Piazza or <https://goo.gl/forms/eQpVW7IPjqapsDkB2>

# So far... Supervised Learning

**Data:**  $(x, y)$

$x$  is data,  $y$  is label

**Goal:** Learn a *function* to map  $x \rightarrow y$

**Examples:** Classification,  
regression, object detection,  
semantic segmentation, image  
captioning, etc.



→ Cat

Classification

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# So far... Unsupervised Learning

**Data:**  $x$

Just data, no labels!

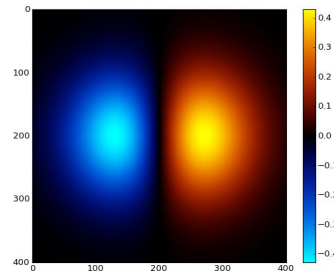
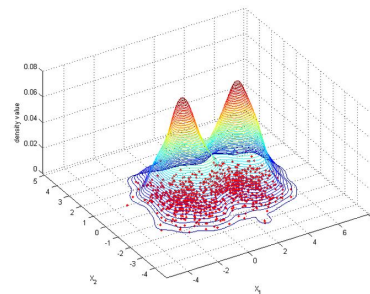
**Goal:** Learn some underlying hidden *structure* of the data

**Examples:** Clustering, dimensionality reduction, feature learning, density estimation, etc.



Figure copyright Ian Goodfellow, 2016. Reproduced with permission.

1-d density estimation



2-d density estimation

2-d density images [left](#) and [right](#)  
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# Today: Reinforcement Learning

Problems involving an **agent** interacting with an **environment**, which provides numeric **reward** signals

**Goal:** Learn how to take actions in order to maximize reward



Atari games figure copyright Volodymyr Mnih et al., 2013. Reproduced with permission.

# Overview

- What is Reinforcement Learning?
- Markov Decision Processes
- Q-Learning
- Policy Gradients



# Reinforcement Learning

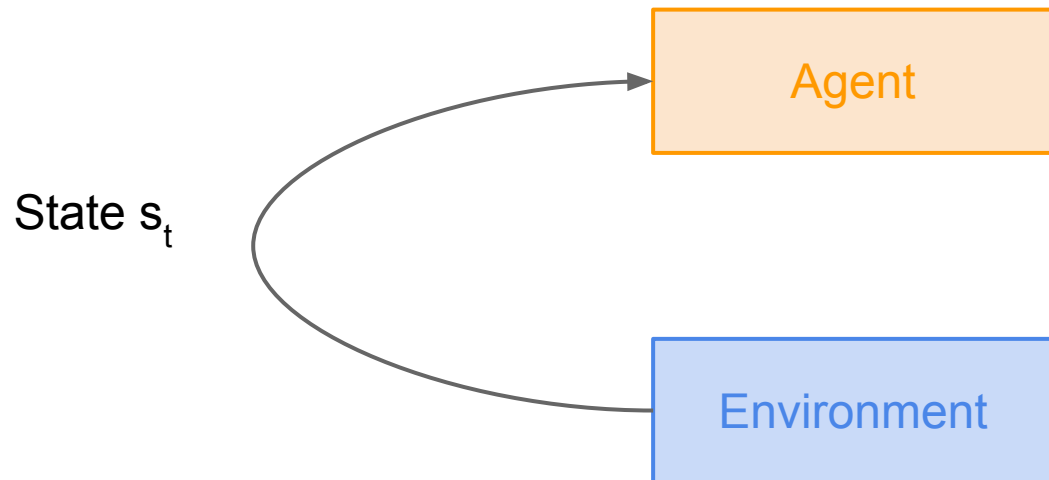
Agent



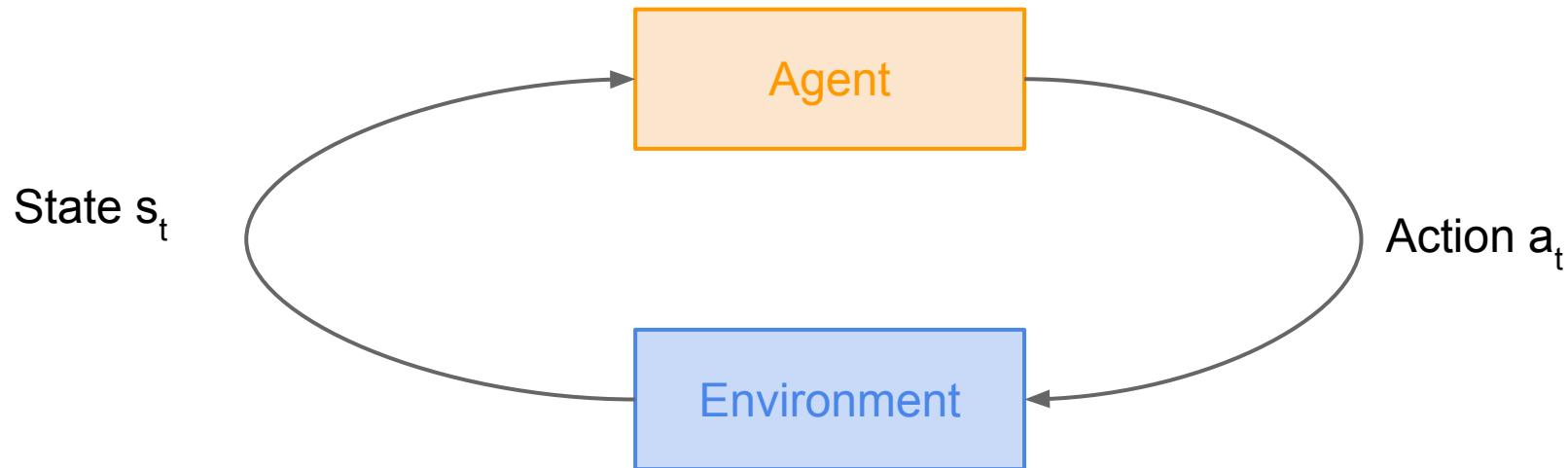
The diagram illustrates the basic components of Reinforcement Learning. It consists of two rectangular boxes arranged vertically. The top box is light orange with an orange border and contains the word 'Agent' in orange text. The bottom box is light blue with a blue border and contains the word 'Environment' in blue text. There are no arrows or other graphical elements connecting the two boxes.

Environment

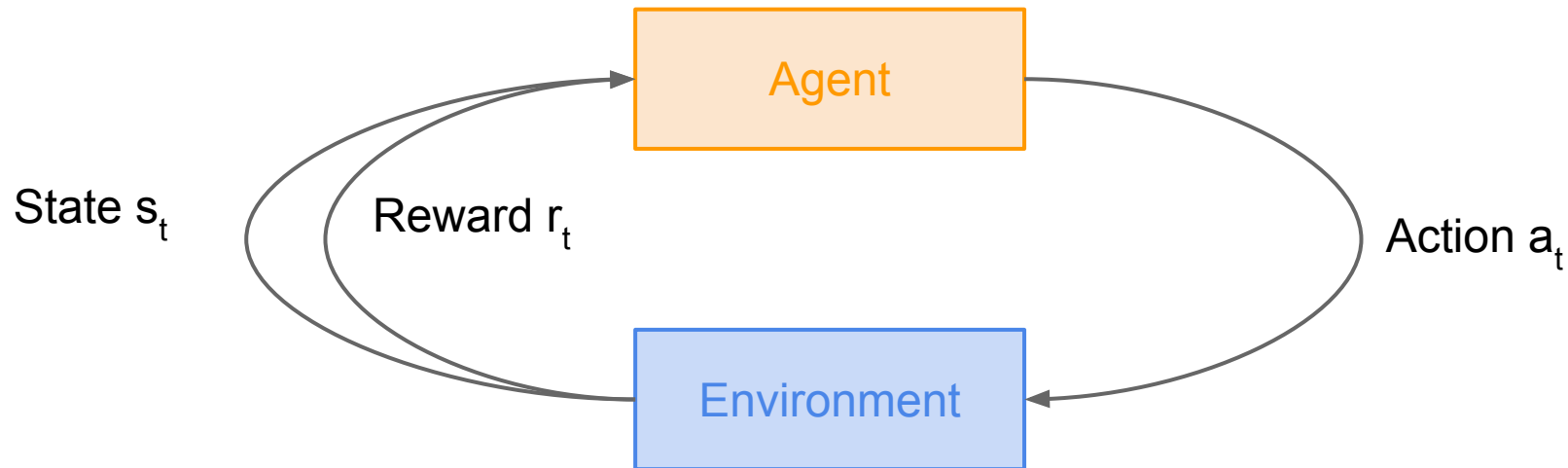
# Reinforcement Learning



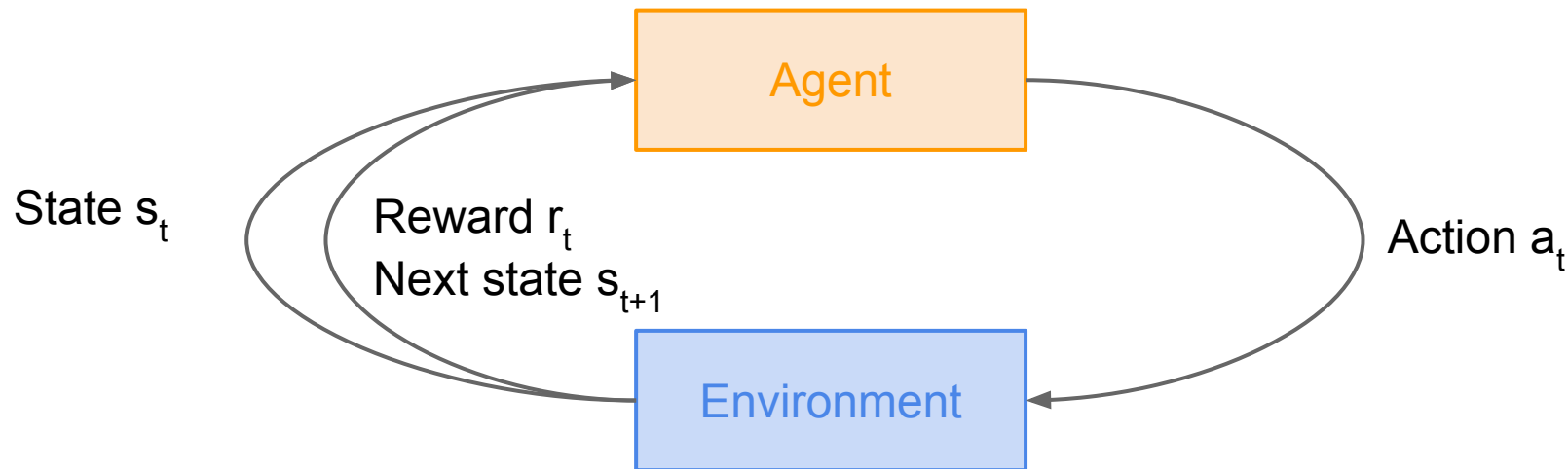
# Reinforcement Learning



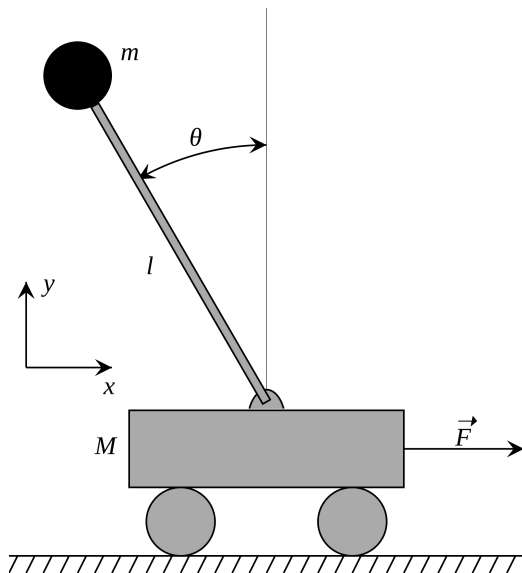
# Reinforcement Learning



# Reinforcement Learning



# Cart-Pole Problem



가능한한 balance 유지

**Objective:** Balance a pole on top of a movable cart

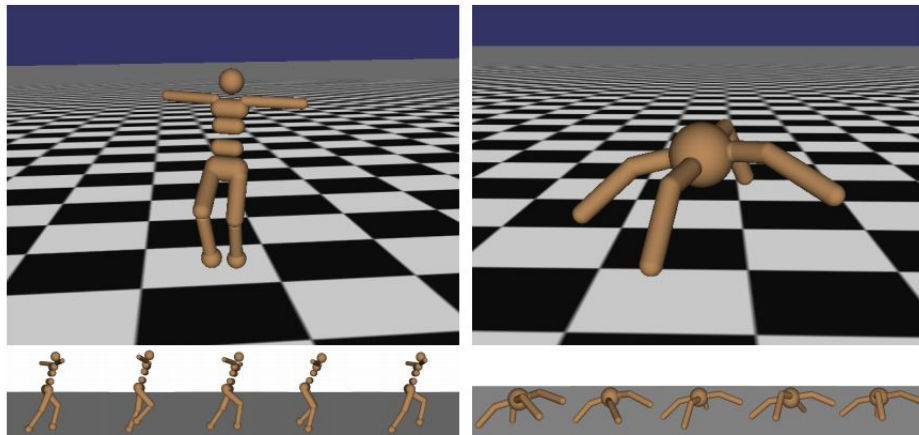
**State:** angle, angular speed, position, horizontal velocity

**Action:** horizontal force applied on the cart

**Reward:** 1 at each time step if the pole is upright

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# Robot Locomotion



**Objective:** Make the robot move forward

**State:** Angle and position of the joints

**Action:** Torques applied on joints

**Reward:** 1 at each time step upright + forward movement

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# Atari Games



**Objective:** Complete the game with the highest score

**State:** Raw pixel inputs of the game state

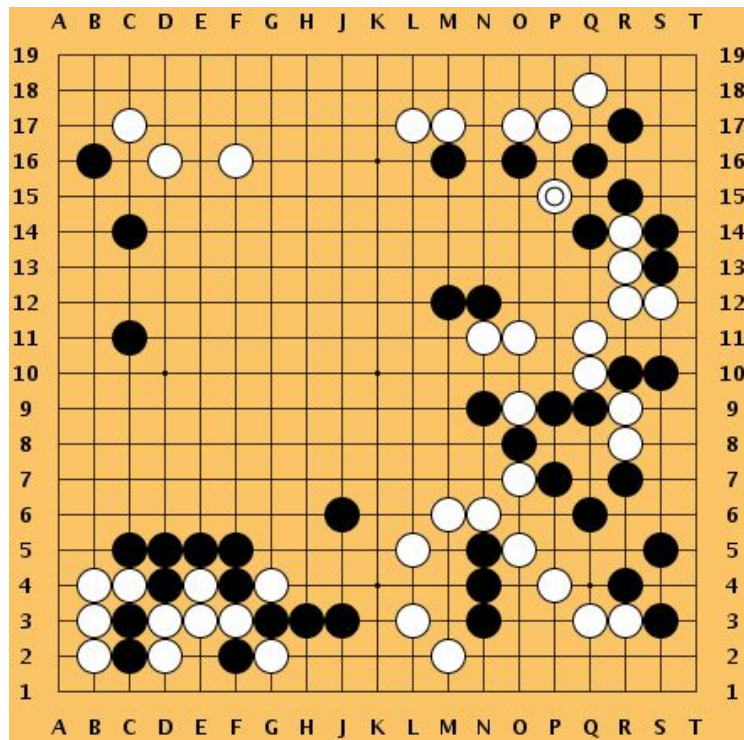
**Action:** Game controls e.g. Left, Right, Up, Down

**Reward:** Score increase/decrease at each time step

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# Go



**Objective:** Win the game!

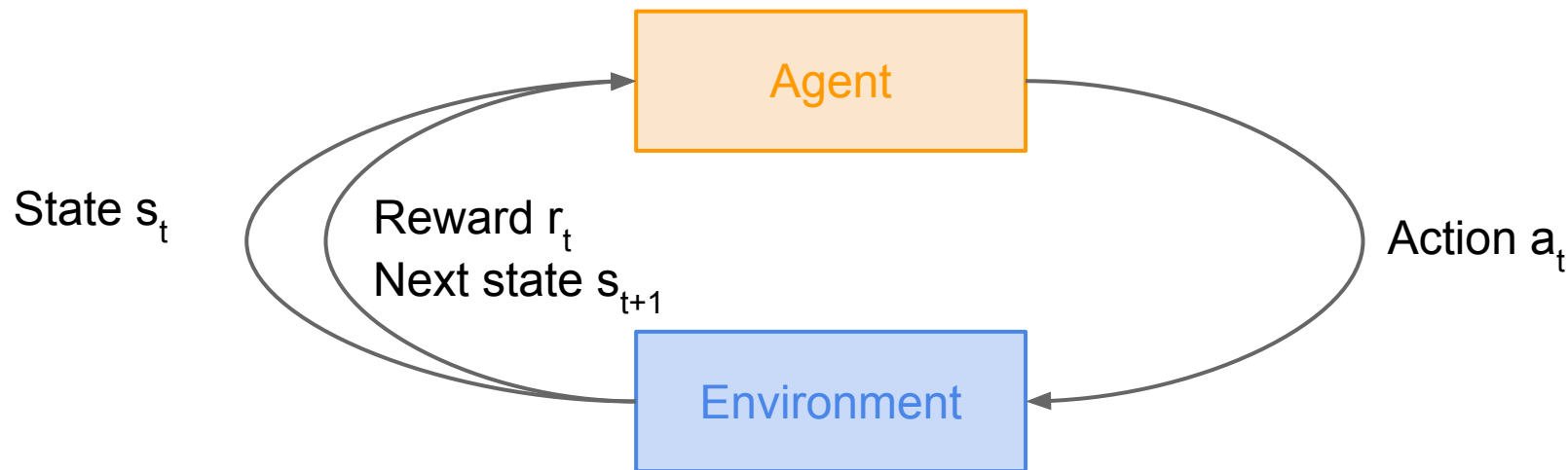
**State:** Position of all pieces

**Action:** Where to put the next piece down

**Reward:** 1 if win at the end of the game, 0 otherwise

[This image is CC0 public domain](#)

# How can we mathematically formalize the RL problem?



# Markov Decision Process

- Mathematical formulation of the RL problem
- **Markov property**: Current state completely characterises the state of the world

Defined by:  $(\mathcal{S}, \mathcal{A}, \mathcal{R}, \mathbb{P}, \gamma)$

$\mathcal{S}$  : set of possible states

$\mathcal{A}$  : set of possible actions

$\mathcal{R}$  : distribution of reward given (state, action) pair

$\mathbb{P}$  : transition probability i.e. distribution over next state given (state, action) pair

$\gamma$  : discount factor

# Markov Decision Process

- At time step  $t=0$ , environment samples initial state  $s_0 \sim p(s_0)$
- Then, for  $t=0$  until done:
  - Agent selects action  $a_t$
  - Environment samples reward  $r_t \sim R(\cdot | s_t, a_t)$
  - Environment samples next state  $s_{t+1} \sim P(\cdot | s_t, a_t)$
  - Agent receives reward  $r_t$  and next state  $s_{t+1}$

states      actions

- A policy  $\pi$  is a function from  $S$  to  $A$  that specifies what action to take in each state
- **Objective:** find policy  $\pi^*$  that maximizes cumulative discounted reward:  $\sum_{t \geq 0} \gamma^t r_t$

# A simple MDP: Grid World

actions = {

1. right →

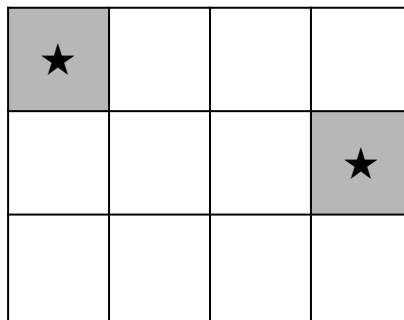
2. left ←

3. up ↑

4. down ↓

}

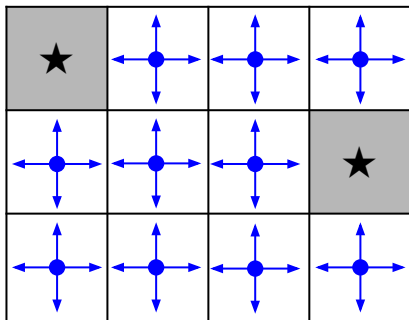
states



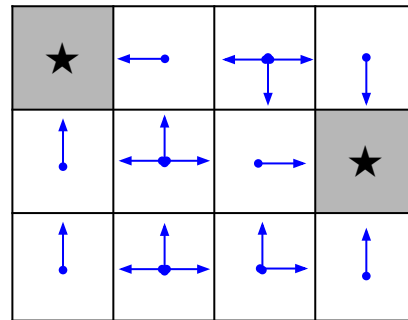
Set a negative “reward”  
for each transition  
(e.g.  $r = -1$ )

**Objective:** reach one of terminal states (greyed out) in  
least number of actions

# A simple MDP: Grid World



Random Policy



Optimal Policy

# The optimal policy $\pi^*$

We want to find optimal policy  $\pi^*$  that maximizes the sum of rewards.

How do we handle the randomness (initial state, transition probability...)?

*in the MDP*

# The optimal policy $\pi^*$

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How do we handle the randomness (initial state, transition probability...)?

Maximize the **expected sum of rewards!**

$$\text{Formally: } \pi^* = \arg \max_{\pi} \mathbb{E} \left[ \sum_{t \geq 0} \gamma^t r_t | \pi \right] \quad \text{with } s_0 \sim p(s_0), a_t \sim \pi(\cdot | s_t), s_{t+1} \sim p(\cdot | s_t, a_t)$$



# Definitions: Value function and Q-value function

Following a policy produces sample trajectories (or paths)  $s_0, a_0, r_0, s_1, a_1, r_1, \dots$

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How good is a state?

The **value function** at state  $s$ , is the expected cumulative reward from following the policy from state  $s$ :

$$V^\pi(s) = \mathbb{E} \left[ \sum_{t \geq 0} \gamma^t r_t \mid s_0 = s, \pi \right]$$

# Definitions: Value function and Q-value function

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## How good is a state-action pair?

The **Q-value function** at state  $s$  and action  $a$ , is the expected cumulative reward from taking action  $a$  in state  $s$  and then following the policy:

$$Q^\pi(s, a) = \mathbb{E} \left[ \sum_{t \geq 0} \gamma^t r_t \mid s_0 = s, a_0 = a, \pi \right]$$

# Bellman equation

The optimal Q-value function  $Q^*$  is the maximum expected cumulative reward achievable from a given (state, action) pair:

$$Q^*(s, a) = \max_{\pi} \mathbb{E} \left[ \sum_{t \geq 0} \gamma^t r_t \mid s_0 = s, a_0 = a, \pi \right]$$

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$Q^*$  satisfies the following **Bellman equation**:

$$Q^*(s, a) = \mathbb{E}_{s' \sim \mathcal{E}} \left[ r + \gamma \max_{a'} Q^*(s', a') \mid s, a \right]$$

**Intuition:** if the optimal state-action values for the next time-step  $Q^*(s', a')$  are known, then the optimal strategy is to take the action that maximizes the expected value of  $r + \gamma Q^*(s', a')$

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The optimal policy  $\pi^*$  corresponds to taking the best action in any state as specified by  $Q^*$

# Solving for the optimal policy

**Value iteration** algorithm: Use Bellman equation as an iterative update

$$Q_{i+1}(s, a) = \mathbb{E} \left[ r + \gamma \max_{a'} Q_i(s', a') | s, a \right]$$

$Q_i$  will converge to  $Q^*$  as  $i \rightarrow \infty$

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$Q_i$  will converge to  $Q^*$  as  $i \rightarrow \text{infinity}$

What's the problem with this?

Not scalable. Must compute  $Q(s, a)$  for every state-action pair. If state is e.g. current game state pixels, computationally infeasible to compute for entire state space!

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**Solution:** use a function approximator to estimate  $Q(s, a)$ . E.g. a neural network!

# Solving for the optimal policy: Q-learning

Q-learning: Use a function approximator to estimate the action-value function

$$Q(s, a; \theta) \approx Q^*(s, a)$$

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If the function approximator is a deep neural network => **deep q-learning!**

# Solving for the optimal policy: Q-learning

Q-learning: Use a function approximator to estimate the action-value function

$$Q(s, a; \theta) \approx Q^*(s, a)$$

function parameters (weights)

If the function approximator is a deep neural network => **deep q-learning!**

# Solving for the optimal policy: Q-learning

Remember: want to find a Q-function that satisfies the Bellman Equation:

$$Q^*(s, a) = \mathbb{E}_{s' \sim \mathcal{E}} \left[ r + \gamma \max_{a'} Q^*(s', a') | s, a \right]$$

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## Forward Pass

Loss function:  $L_i(\theta_i) = \mathbb{E}_{s, a \sim \rho(\cdot)} [(y_i - Q(s, a; \theta_i))^2]$

where  $y_i = \mathbb{E}_{s' \sim \mathcal{E}} \left[ r + \gamma \max_{a'} Q(s', a'; \theta_{i-1}) | s, a \right]$

*loss function of Bellman equation error  $\hat{=}$  minimize  $\hat{=}$  model train*

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## Backward Pass

Gradient update (with respect to Q-function parameters  $\theta$ ):

$$\nabla_{\theta_i} L_i(\theta_i) = \mathbb{E}_{s, a \sim \rho(\cdot); s' \sim \mathcal{E}} \left[ r + \gamma \max_{a'} Q(s', a'; \theta_{i-1}) - Q(s, a; \theta_i) \right] \nabla_{\theta_i} Q(s, a; \theta_i)$$



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# Case Study: Playing Atari Games



**Objective:** Complete the game with the highest score

**State:** Raw pixel inputs of the game state

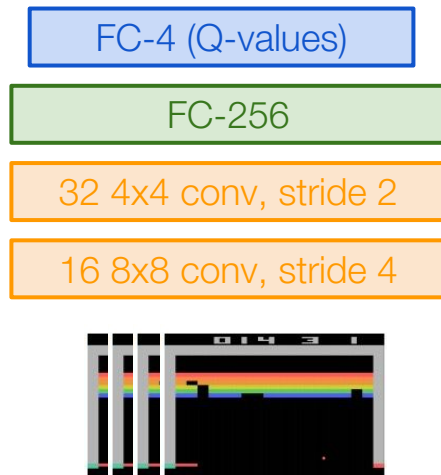
**Action:** Game controls e.g. Left, Right, Up, Down

**Reward:** Score increase/decrease at each time step

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# Q-network Architecture

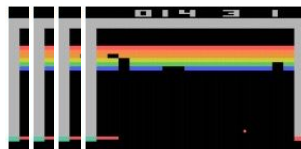
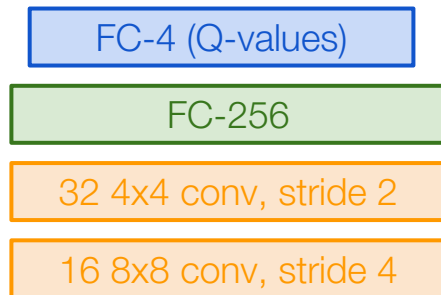
$Q(s, a; \theta)$  :  
neural network  
with weights  $\theta$



**Current state  $s_t$ : 84x84x4 stack of last 4 frames**  
(after RGB->grayscale conversion, downsampling, and cropping)

# Q-network Architecture

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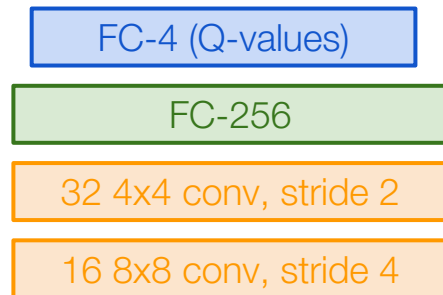


← Input: state  $s_t$

**Current state  $s_t$ : 84x84x4 stack of last 4 frames**  
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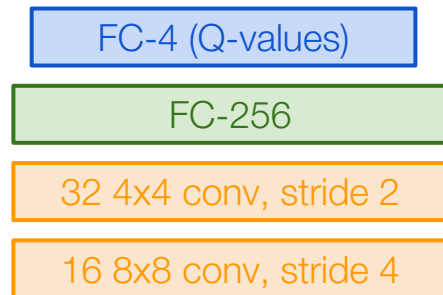
← Familiar conv layers,  
FC layer



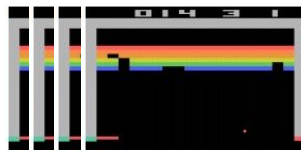
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# Q-network Architecture

$Q(s, a; \theta)$  :  
neural network  
with weights  $\theta$



← Last FC layer has 4-d output (if 4 actions), corresponding to  $Q(s_t, a_1)$ ,  $Q(s_t, a_2)$ ,  $Q(s_t, a_3)$ ,  $Q(s_t, a_4)$

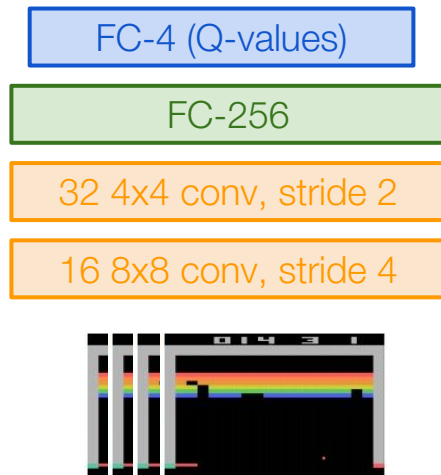


**Current state  $s_t$ : 84x84x4 stack of last 4 frames**  
(after RGB->grayscale conversion, downsampling, and cropping)

no Softmax layer after connected  
∴ goal is to directly predict  
our Q-value functions

# Q-network Architecture

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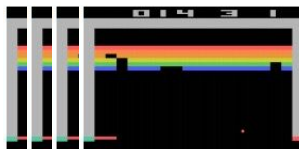
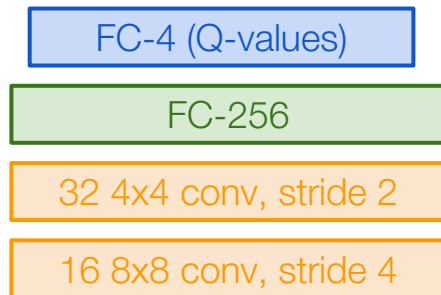
Number of actions between 4-18 depending on Atari game

**Current state  $s_t$ : 84x84x4 stack of last 4 frames**  
(after RGB->grayscale conversion, downsampling, and cropping)

# Q-network Architecture

$Q(s, a; \theta)$  :  
neural network  
with weights  $\theta$

A single feedforward pass  
to compute Q-values for all  
actions from the current  
state => efficient!



**Current state  $s_t$ : 84x84x4 stack of last 4 frames**  
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Number of actions between 4-18  
depending on Atari game



# Training the Q-network: Loss function (from before)

Remember: want to find a Q-function that satisfies the Bellman Equation:

$$Q^*(s, a) = \mathbb{E}_{s' \sim \mathcal{E}} \left[ r + \gamma \max_{a'} Q^*(s', a') | s, a \right]$$

## Forward Pass

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## Backward Pass

Gradient update (with respect to Q-function parameters  $\theta$ ):

$$\nabla_{\theta_i} L_i(\theta_i) = \mathbb{E}_{s, a \sim \rho(\cdot); s' \sim \mathcal{E}} \left[ (r + \gamma \max_{a'} Q(s', a'; \theta_{i-1}) - Q(s, a; \theta_i)) \nabla_{\theta_i} Q(s, a; \theta_i) \right]$$

# Training the Q-network: Experience Replay

Learning from batches of consecutive samples is problematic:

- Samples are correlated => inefficient learning
- Current Q-network parameters determines next training samples (e.g. if maximizing action is to move left, training samples will be dominated by samples from left-hand size) => can lead to bad feedback loops

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Address these problems using **experience replay**

- Continually update a **replay memory** table of transitions ( $s_t, a_t, r_t, s_{t+1}$ ) as game (experience) episodes are played
- Train Q-network on random minibatches of transitions from the replay memory, instead of consecutive samples

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Each transition can also contribute  
to multiple weight updates  
=> greater data efficiency

# Putting it together: Deep Q-Learning with Experience Replay

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**Algorithm 1** Deep Q-learning with Experience Replay
 

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```

Initialize replay memory  $\mathcal{D}$  to capacity  $N$ 
Initialize action-value function  $Q$  with random weights
for episode = 1,  $M$  do
  Initialise sequence  $s_1 = \{x_1\}$  and preprocessed sequenced  $\phi_1 = \phi(s_1)$ 
  for  $t = 1, T$  do
    With probability  $\epsilon$  select a random action  $a_t$ 
    otherwise select  $a_t = \max_a Q^*(\phi(s_t), a; \theta)$ 
    Execute action  $a_t$  in emulator and observe reward  $r_t$  and image  $x_{t+1}$ 
    Set  $s_{t+1} = s_t, a_t, x_{t+1}$  and preprocess  $\phi_{t+1} = \phi(s_{t+1})$ 
    Store transition  $(\phi_t, a_t, r_t, \phi_{t+1})$  in  $\mathcal{D}$ 
    Sample random minibatch of transitions  $(\phi_j, a_j, r_j, \phi_{j+1})$  from  $\mathcal{D}$ 
    Set  $y_j = \begin{cases} r_j & \text{for terminal } \phi_{j+1} \\ r_j + \gamma \max_{a'} Q(\phi_{j+1}, a'; \theta) & \text{for non-terminal } \phi_{j+1} \end{cases}$ 
    Perform a gradient descent step on  $(y_j - Q(\phi_j, a_j; \theta))^2$  according to equation 3
  end for
end for
  
```

---

# Putting it together: Deep Q-Learning with Experience Replay

---

**Algorithm 1** Deep Q-learning with Experience Replay
 

---

Initialize replay memory  $\mathcal{D}$  to capacity  $N$

Initialize action-value function  $Q$  with random weights

← Initialize replay memory, Q-network

**for** episode = 1,  $M$  **do**

    Initialize sequence  $s_1 = \{x_1\}$  and preprocessed sequenced  $\phi_1 = \phi(s_1)$

**for**  $t = 1, T$  **do**

        With probability  $\epsilon$  select a random action  $a_t$

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Initialize action-value function  $Q$  with random weights

**for** episode = 1,  $M$  **do**

← Play  $M$  episodes (full games)

    Initialize sequence  $s_1 = \{x_1\}$  and preprocessed sequence  $\phi_1 = \phi(s_1)$

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**end for**

**end for**

---

← Initialize state  
(starting game  
screen pixels) at the  
beginning of each  
episode



# Putting it together: Deep Q-Learning with Experience Replay

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**Algorithm 1** Deep Q-learning with Experience Replay
 

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**end for**

**end for**

---



For each timestep  $t$   
of the game

# Putting it together: Deep Q-Learning with Experience Replay

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**end for**

---

← With small probability, select a random action (explore), otherwise select greedy action from current policy

# Putting it together: Deep Q-Learning with Experience Replay

---

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        Perform a gradient descent step on  $(y_j - Q(\phi_j, a_j; \theta))^2$  according to equation 3

**end for**

**end for**

---

← Take the action ( $a_t$ ),  
and observe the  
reward  $r_t$  and next  
state  $s_{t+1}$

# Putting it together: Deep Q-Learning with Experience Replay

---

**Algorithm 1** Deep Q-learning with Experience Replay
 

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**end for**

**end for**

---

← Store transition in  
replay memory



# Putting it together: Deep Q-Learning with Experience Replay

---

**Algorithm 1** Deep Q-learning with Experience Replay
 

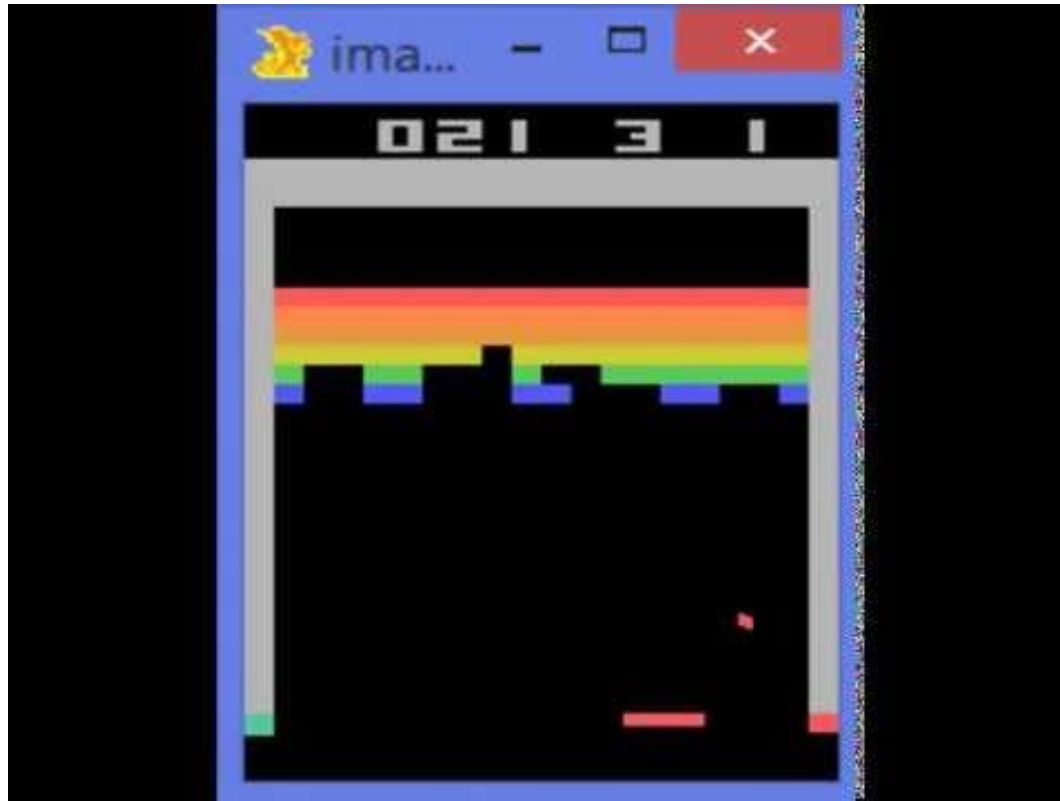
---

```

Initialize replay memory  $\mathcal{D}$  to capacity  $N$ 
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for episode = 1,  $M$  do
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  end for
end for
  
```

---

Experience Replay:  
 Sample a random  
 minibatch of transitions  
 from replay memory  
 and perform a gradient  
 descent step



<https://www.youtube.com/watch?v=V1eYniJ0Rnk>

Video by Károly Zsolnai-Fehér. Reproduced with permission.

# Policy Gradients

What is a problem with Q-learning?

The Q-function can be very complicated!

Example: a robot grasping an object has a very high-dimensional state => hard to learn exact value of every (state, action) pair

# Policy Gradients

What is a problem with Q-learning?

The Q-function can be very complicated!

Example: a robot grasping an object has a very high-dimensional state => hard to learn exact value of every (state, action) pair

But the policy can be much simpler: just close your hand

Can we learn a policy directly, e.g. finding the best policy from a collection of policies?



# Policy Gradients

Formally, let's define a class of parametrized policies:  $\Pi = \{\pi_\theta, \theta \in \mathbb{R}^m\}$

For each policy, define its value:

$$J(\theta) = \mathbb{E} \left[ \sum_{t \geq 0} \gamma^t r_t | \pi_\theta \right]$$

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We want to find the optimal policy  $\theta^* = \arg \max_{\theta} J(\theta)$

How can we do this?

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We want to find the optimal policy  $\theta^* = \arg \max_{\theta} J(\theta)$

How can we do this?

Gradient ascent on policy parameters!

# REINFORCE algorithm

Mathematically, we can write:

$$\begin{aligned} J(\theta) &= \mathbb{E}_{\tau \sim p(\tau; \theta)} [r(\tau)] \\ &= \int_{\tau} r(\tau) p(\tau; \theta) d\tau \end{aligned}$$

Where  $r(\tau)$  is the reward of a trajectory  $\tau = (s_0, a_0, r_0, s_1, \dots)$

# REINFORCE algorithm

Expected reward:

$$\begin{aligned} J(\theta) &= \mathbb{E}_{\tau \sim p(\tau; \theta)} [r(\tau)] \\ &= \int_{\tau} r(\tau) p(\tau; \theta) d\tau \end{aligned}$$

# REINFORCE algorithm

Expected reward:  $J(\theta) = \mathbb{E}_{\tau \sim p(\tau; \theta)} [r(\tau)]$

$$= \int_{\tau} r(\tau) p(\tau; \theta) d\tau$$

Now let's differentiate this:  $\nabla_{\theta} J(\theta) = \int_{\tau} r(\tau) \nabla_{\theta} p(\tau; \theta) d\tau$

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Intractable! Gradient of an expectation is problematic when  $p$  depends on  $\theta$

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However, we can use a nice trick:  $\nabla_{\theta} p(\tau; \theta) = p(\tau; \theta) \frac{\nabla_{\theta} p(\tau; \theta)}{p(\tau; \theta)} = p(\tau; \theta) \nabla_{\theta} \log p(\tau; \theta)$



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If we inject this back:

$$\begin{aligned} \nabla_{\theta} J(\theta) &= \int_{\tau} (r(\tau) \nabla_{\theta} \log p(\tau; \theta)) p(\tau; \theta) d\tau \\ &= \mathbb{E}_{\tau \sim p(\tau; \theta)} [r(\tau) \nabla_{\theta} \log p(\tau; \theta)] \end{aligned}$$

Can estimate with  
Monte Carlo sampling

# REINFORCE algorithm

Can we compute those quantities without knowing the transition probabilities?

We have:  $p(\tau; \theta) = \prod_{t \geq 0} p(s_{t+1} | s_t, a_t) \pi_{\theta}(a_t | s_t)$

# REINFORCE algorithm

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We have:  $p(\tau; \theta) = \prod_{t \geq 0} p(s_{t+1} | s_t, a_t) \pi_{\theta}(a_t | s_t)$

Thus:  $\log p(\tau; \theta) = \sum_{t \geq 0} \log p(s_{t+1} | s_t, a_t) + \log \pi_{\theta}(a_t | s_t)$

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Doesn't depend on  
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# REINFORCE algorithm

$$\begin{aligned}\nabla_{\theta} J(\theta) &= \int_{\tau} (r(\tau) \nabla_{\theta} \log p(\tau; \theta)) p(\tau; \theta) d\tau \\ &= \mathbb{E}_{\tau \sim p(\tau; \theta)} [r(\tau) \nabla_{\theta} \log p(\tau; \theta)]\end{aligned}$$

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And when differentiating:  $\nabla_{\theta} \log p(\tau; \theta) = \sum_{t \geq 0} \nabla_{\theta} \log \pi_{\theta}(a_t | s_t)$  Doesn't depend on transition probabilities!

Therefore when sampling a trajectory  $\tau$ , we can estimate  $J(\theta)$  with

$$\nabla_{\theta} J(\theta) \approx \sum_{t \geq 0} r(\tau) \nabla_{\theta} \log \pi_{\theta}(a_t | s_t)$$

# Intuition

Gradient estimator:  $\nabla_{\theta} J(\theta) \approx \sum_{t \geq 0} r(\tau) \nabla_{\theta} \log \pi_{\theta}(a_t | s_t)$

**Interpretation:**

- If  $r(\tau)$  is high, push up the probabilities of the actions seen
- If  $r(\tau)$  is low, push down the probabilities of the actions seen

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Might seem simplistic to say that if a trajectory is good then all its actions were good. **But in expectation, it averages out!**

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Might seem simplistic to say that if a trajectory is good then all its actions were good. **But in expectation, it averages out!**

However, this also suffers from high variance because credit assignment is really hard. Can we help the estimator?



# Variance reduction

Gradient estimator:  $\nabla_{\theta} J(\theta) \approx \sum_{t \geq 0} r(\tau) \nabla_{\theta} \log \pi_{\theta}(a_t | s_t)$

# Variance reduction

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**First idea:** Push up probabilities of an action seen, only by the cumulative future reward from that state

$$\nabla_{\theta} J(\theta) \approx \sum_{t \geq 0} \left( \sum_{t' \geq t} r_{t'} \right) \nabla_{\theta} \log \pi_{\theta}(a_t | s_t)$$

# Variance reduction

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$$\nabla_{\theta} J(\theta) \approx \sum_{t \geq 0} \left( \sum_{t' \geq t} r_{t'} \right) \nabla_{\theta} \log \pi_{\theta}(a_t | s_t)$$

**Second idea:** Use discount factor  $\gamma$  to ignore delayed effects

$$\nabla_{\theta} J(\theta) \approx \sum_{t \geq 0} \left( \sum_{t' \geq t} \gamma^{t'-t} r_{t'} \right) \nabla_{\theta} \log \pi_{\theta}(a_t | s_t)$$

# Variance reduction: Baseline

**Problem:** The raw value of a trajectory isn't necessarily meaningful. For example, if rewards are all positive, you keep pushing up probabilities of actions.

**What is important then?** Whether a reward is better or worse than what you expect to get

**Idea:** Introduce a baseline function dependent on the state.  
Concretely, estimator is now:

$$\nabla_{\theta} J(\theta) \approx \sum_{t \geq 0} \left( \sum_{t' \geq t} \gamma^{t'-t} r_{t'} - b(s_t) \right) \nabla_{\theta} \log \pi_{\theta}(a_t | s_t)$$

# How to choose the baseline?

$$\nabla_{\theta} J(\theta) \approx \sum_{t \geq 0} \left( \sum_{t' \geq t} \gamma^{t'-t} r_{t'} - b(s_t) \right) \nabla_{\theta} \log \pi_{\theta}(a_t | s_t)$$

A simple baseline: constant moving average of rewards experienced so far from all trajectories

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$$\nabla_{\theta} J(\theta) \approx \sum_{t \geq 0} \left( \sum_{t' \geq t} \gamma^{t'-t} r_{t'} - b(s_t) \right) \nabla_{\theta} \log \pi_{\theta}(a_t | s_t)$$

A simple baseline: constant moving average of rewards experienced so far from all trajectories

Variance reduction techniques seen so far are typically used in “Vanilla REINFORCE”

# How to choose the baseline?

A better baseline: Want to push up the probability of an action from a state, if this action was better than the **expected value of what we should get from that state**.

Q: What does this remind you of?

# How to choose the baseline?

A better baseline: Want to push up the probability of an action from a state, if this action was better than the **expected value of what we should get from that state**.

Q: What does this remind you of?

A: Q-function and value function!



# How to choose the baseline?

A better baseline: Want to push up the probability of an action from a state, if this action was better than the **expected value of what we should get from that state**.

Q: What does this remind you of?

A: Q-function and value function!

Intuitively, we are happy with an action  $a_t$  in a state  $s_t$  if  $Q^\pi(s_t, a_t) - V^\pi(s_t)$  is large. On the contrary, we are unhappy with an action if it's small.

# How to choose the baseline?

A better baseline: Want to push up the probability of an action from a state, if this action was better than the **expected value of what we should get from that state**.

Q: What does this remind you of?

A: Q-function and value function!

Intuitively, we are happy with an action  $a_t$  in a state  $s_t$  if  $Q^\pi(s_t, a_t) - V^\pi(s_t)$  is large. On the contrary, we are unhappy with an action if it's small.

Using this, we get the estimator: 
$$\nabla_\theta J(\theta) \approx \sum_{t \geq 0} (Q^{\pi_\theta}(s_t, a_t) - V^{\pi_\theta}(s_t)) \nabla_\theta \log \pi_\theta(a_t | s_t)$$

# Actor-Critic Algorithm

**Problem:** we don't know Q and V. Can we learn them?

**Yes**, using Q-learning! We can combine Policy Gradients and Q-learning by training both an **actor** (the policy) and a **critic** (the Q-function).

- The actor decides which action to take, and the critic tells the actor how good its action was and how it should adjust
- Also alleviates the task of the critic as it only has to learn the values of (state, action) pairs generated by the policy
- Can also incorporate Q-learning tricks e.g. experience replay
- **Remark:** we can define by the **advantage function** how much an action was better than expected

$$A^\pi(s, a) = Q^\pi(s, a) - V^\pi(s)$$

# Actor-Critic Algorithm

Initialize policy parameters  $\theta$ , critic parameters  $\phi$

**For** iteration=1, 2 ... **do**

    Sample  $m$  trajectories under the current policy

$\Delta\theta \leftarrow 0$

**For**  $i=1, \dots, m$  **do**

**For**  $t=1, \dots, T$  **do**

$$A_t = \sum_{t' \geq t} \gamma^{t'-t} r_{t'}^i - V_{\phi}(s_t^i)$$

$$\Delta\theta \leftarrow \Delta\theta + A_t \nabla_{\theta} \log(a_t^i | s_t^i)$$

$$\Delta\phi \leftarrow \sum_i \sum_t \nabla_{\phi} \|A_t^i\|^2$$

$$\theta \leftarrow \theta + \alpha \Delta\theta$$

$$\phi \leftarrow \phi + \beta \Delta\phi$$

**End for**

# REINFORCE in action: Recurrent Attention Model (RAM)

**Objective:** Image Classification

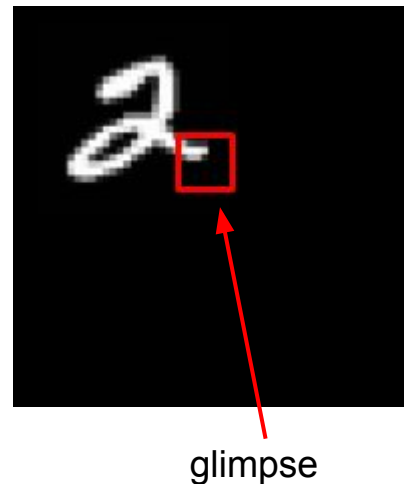
Take a sequence of “glimpses” selectively focusing on regions of the image, to predict class

- Inspiration from human perception and eye movements
- Saves computational resources => scalability
- Able to ignore clutter / irrelevant parts of image

**State:** Glimpses seen so far

**Action:** (x,y) coordinates (center of glimpse) of where to look next in image

**Reward:** 1 at the final timestep if image correctly classified, 0 otherwise



*[Mnih et al. 2014]*

# REINFORCE in action: Recurrent Attention Model (RAM)

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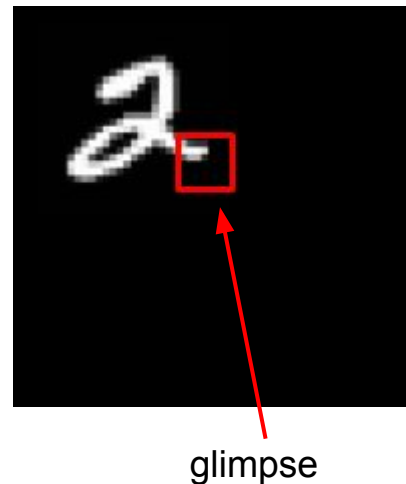
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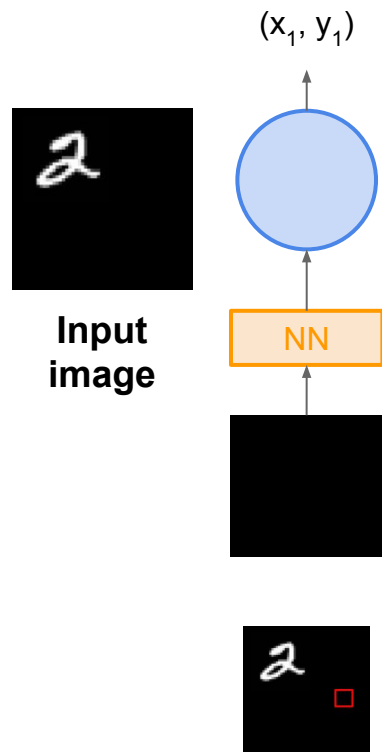
**Reward:** 1 at the final timestep if image correctly classified, 0 otherwise



Glimpsing is a non-differentiable operation => learn policy for how to take glimpse actions using REINFORCE  
Given state of glimpses seen so far, use RNN to model the state and output next action

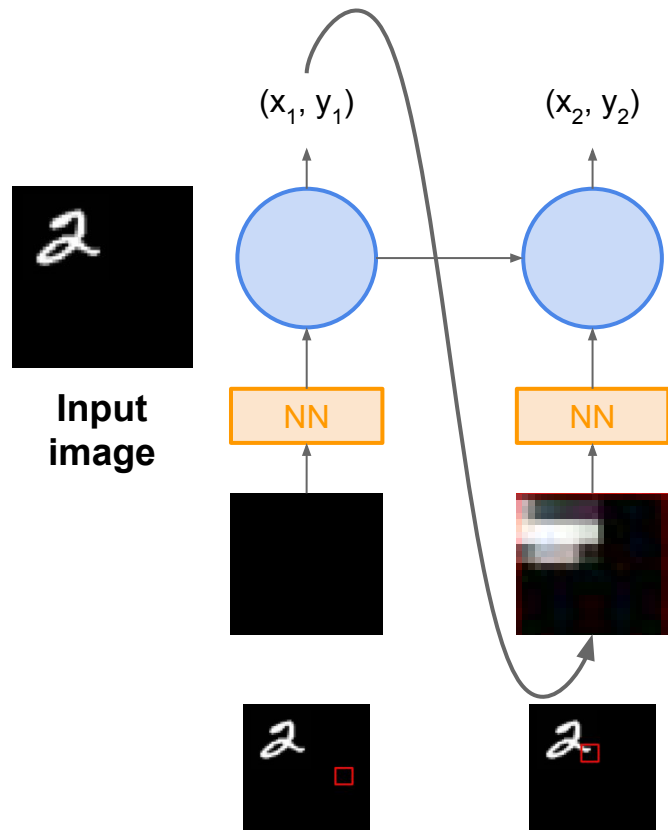
*[Mnih et al. 2014]*

# REINFORCE in action: Recurrent Attention Model (RAM)



[Mnih et al. 2014]

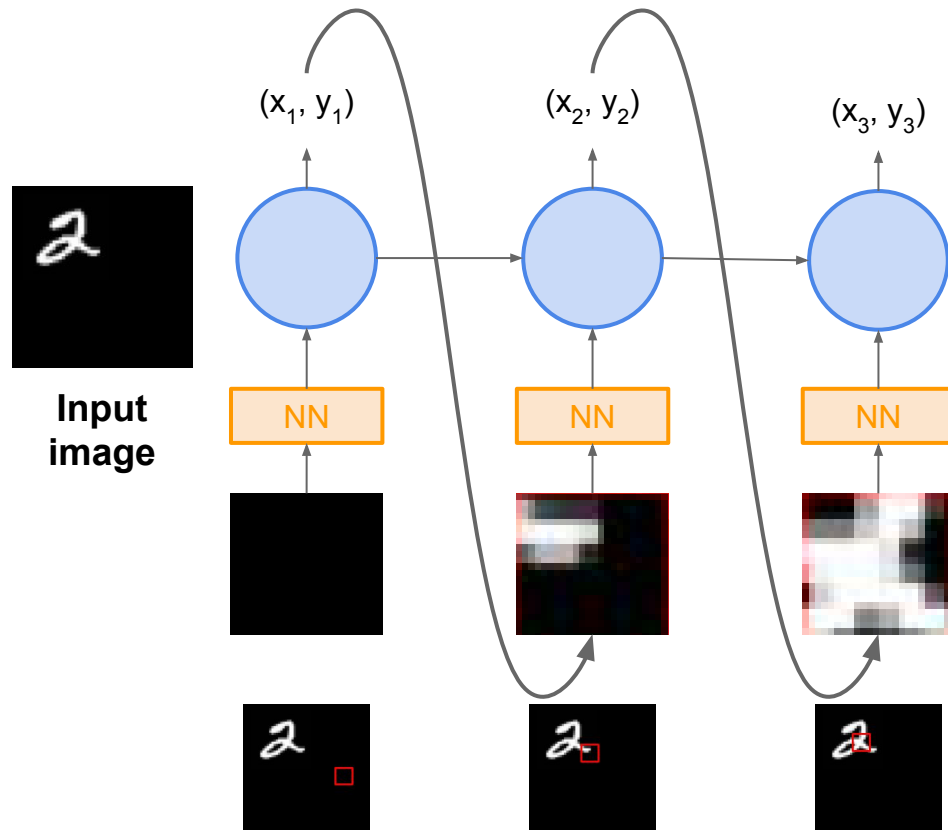
# REINFORCE in action: Recurrent Attention Model (RAM)



[Mnih et al. 2014]

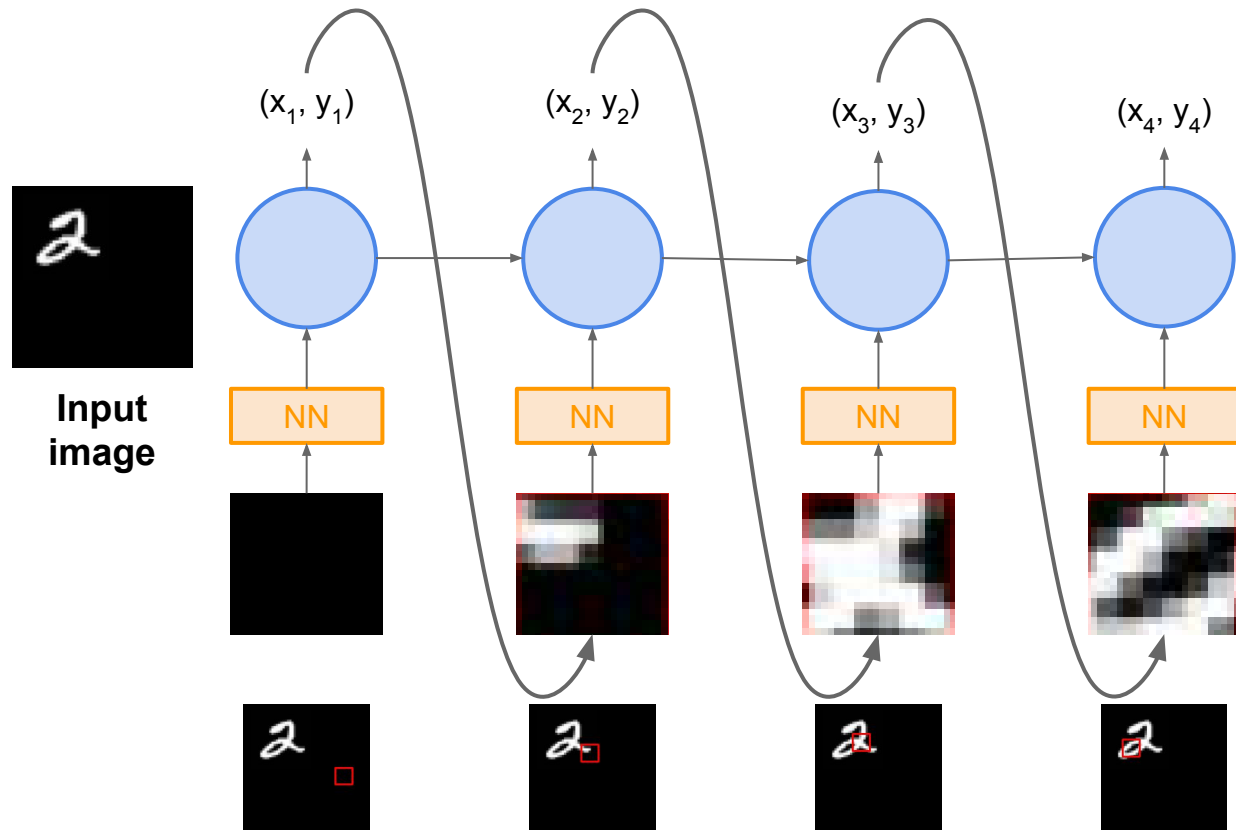


# REINFORCE in action: Recurrent Attention Model (RAM)



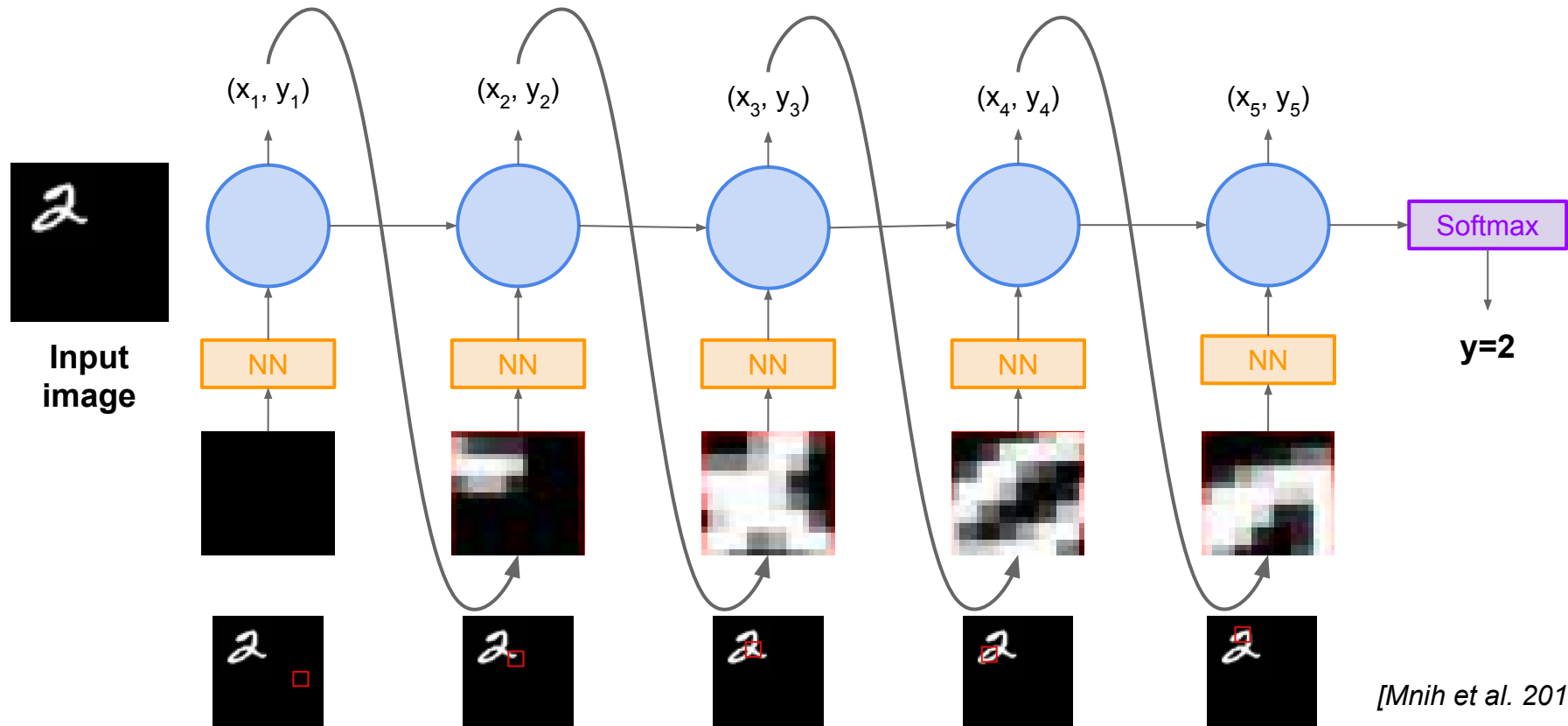
[Mnih et al. 2014]

# REINFORCE in action: Recurrent Attention Model (RAM)

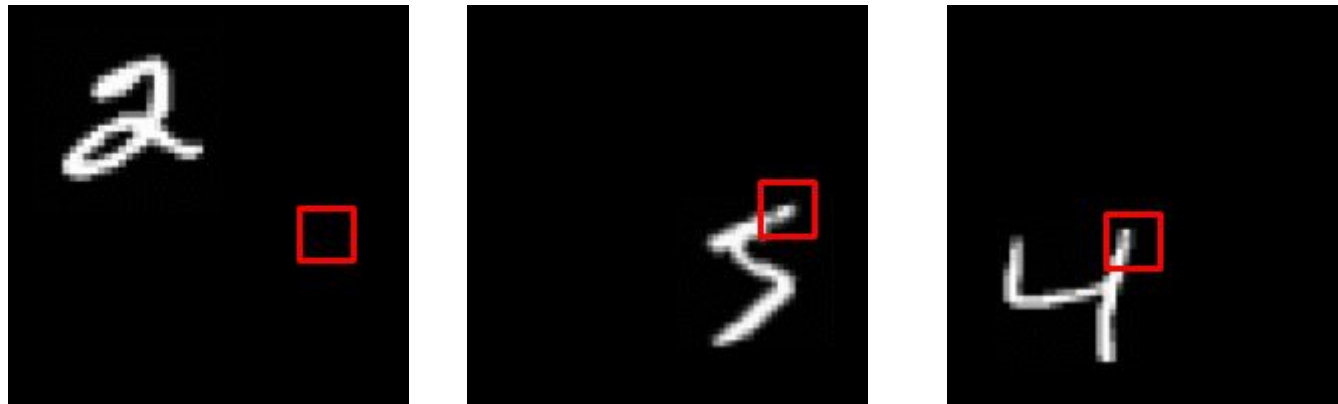


[Mnih et al. 2014]

# REINFORCE in action: Recurrent Attention Model (RAM)



# REINFORCE in action: Recurrent Attention Model (RAM)



Has also been used in many other tasks including fine-grained image recognition, image captioning, and visual question-answering!

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*[Mnih et al. 2014]*

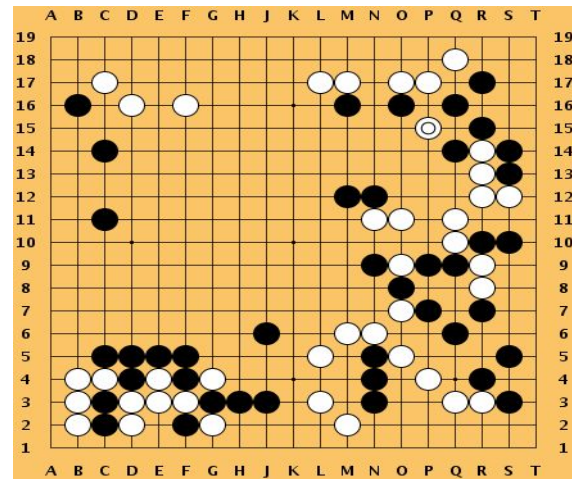
# More policy gradients: AlphaGo

## Overview:

- Mix of supervised learning and reinforcement learning
- Mix of old methods (Monte Carlo Tree Search) and recent ones (deep RL)

## How to beat the Go world champion:

- Featurize the board (stone color, move legality, bias, ...)
- Initialize policy network with supervised training from professional go games, then continue training using policy gradient (play against itself from random previous iterations, +1 / -1 reward for winning / losing)
- Also learn value network (critic)
- Finally, combine combine policy and value networks in a Monte Carlo Tree Search algorithm to select actions by lookahead search



[Silver et al.,  
Nature 2016]

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# Summary

- **Policy gradients**: very general but suffer from high variance so requires a lot of samples. **Challenge**: sample-efficiency
- **Q-learning**: does not always work but when it works, usually more sample-efficient. **Challenge**: exploration
- Guarantees:
  - **Policy Gradients**: Converges to a local minima of  $J(\theta)$ , often good enough!
  - **Q-learning**: Zero guarantees since you are approximating Bellman equation with a complicated function approximator

# Next Time

## **Guest Lecture: Song Han**

- Energy-efficient deep learning
- Deep learning hardware
- Model compression
- Embedded systems
- And more...