

1주차 세션

이다현 최하경

목차

#01 Why Graphs?

#02 Applications of Graph ML

#03 Choice of Graph Representation



01. Why Graphs?



01. Why Graphs?

📌 Graph란?

- “관계(relation)”와 “상호작용(interaction)”으로 객체들을 분석하고 설명하는 언어

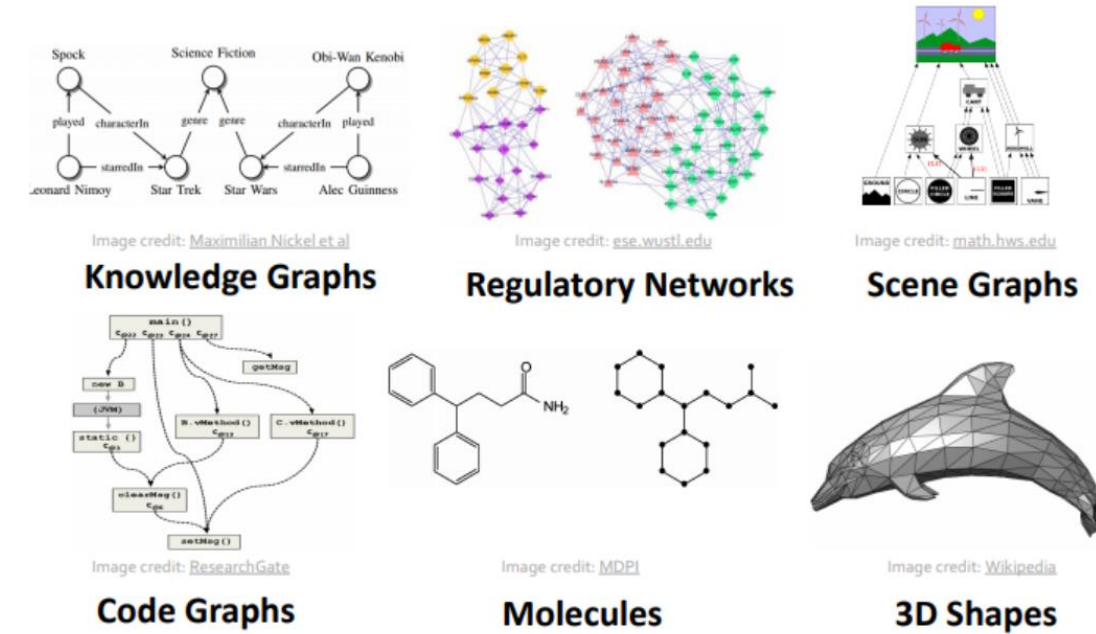
- Graph로 표현하고 모델링할 수 있는 도메인들이 다양함

- Graphical structure을 갖는 도메인은 두 가지로 표현

① Network (Natural graphs)

② Graph (representation)

-> 가끔 네트워크와 그래프 간 경계가 흐려지기도 함



📌 이 수업에서의 main question

- 더 나은 prediction을 위해 relational structure의 이점을 어떻게 사용해야 하는가

- 복잡한 도메인들은 rich relational structure을 갖기 때문에, 명확하게 relational modeling을 하는 것이 중요!!

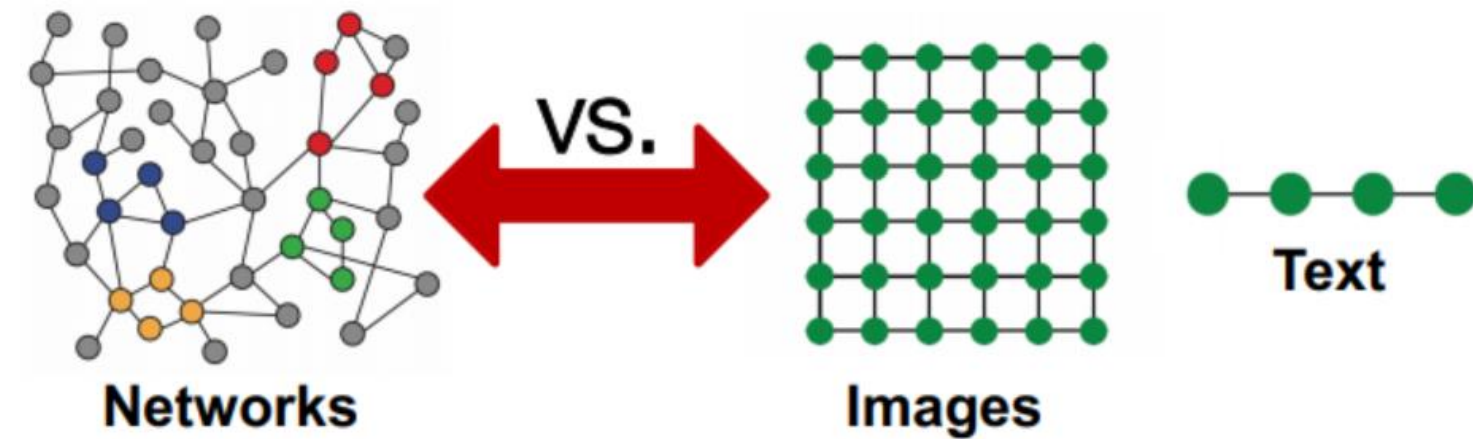
01. Why Graphs?

✂ Modern ML Toolbox

- 간단한 sequence(ex. 텍스트, 음성 데이터)와 grid(ex. 이미지 데이터)를 위해 디자인됨
- graph에 적용하기 힘들

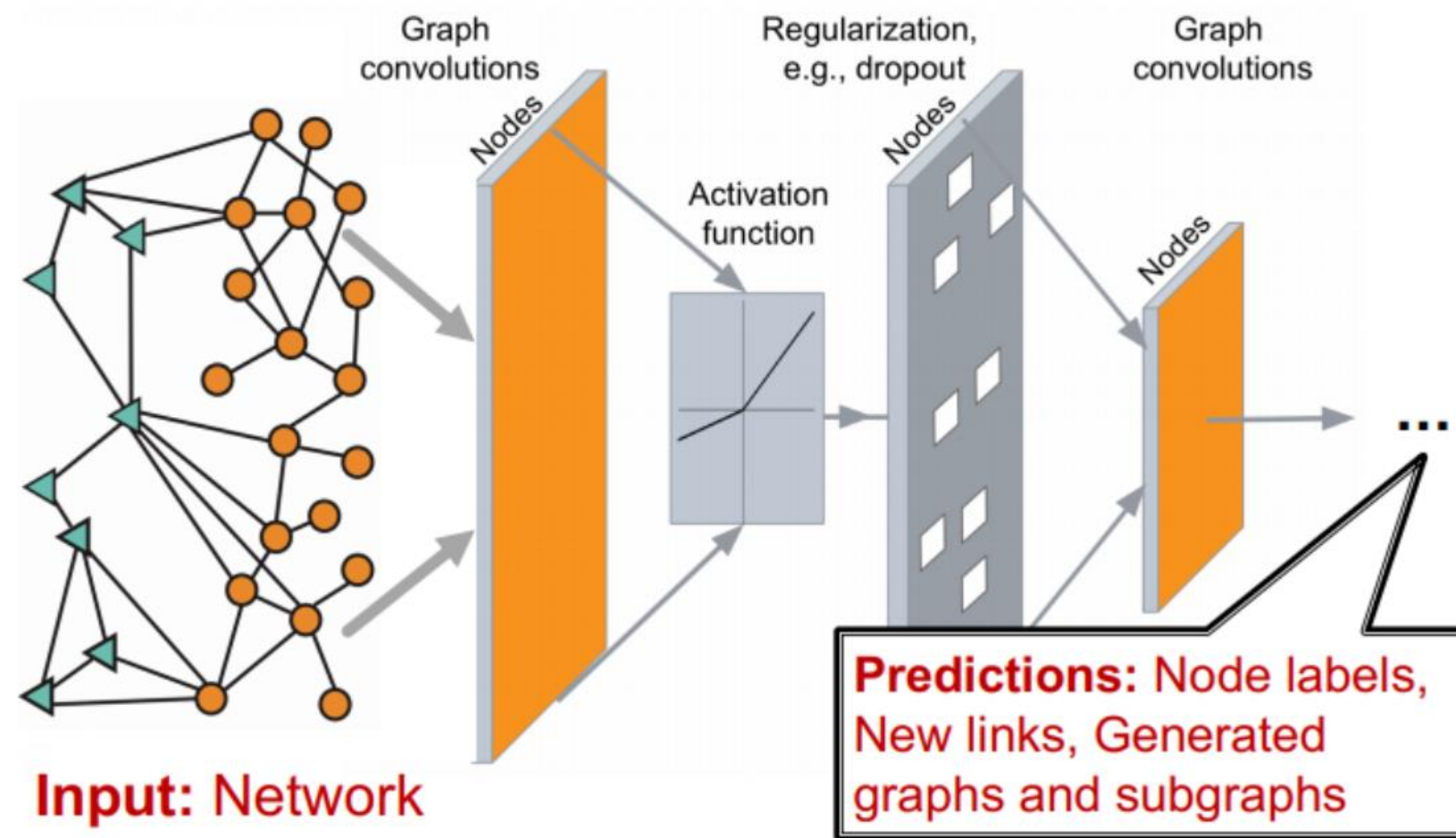
(because) 네트워크의 복잡성

- ① arbitrary size와 복잡한 topological structure
: grid와 텍스트 데이터처럼 공간적 기준점이 없음
- ② 고정적인 노드의 순서가 없음
- ③ 가끔 dynamic하고 multimodal한 feature들이 존재



01. Why Graphs?

📌 Deep Learning in Graph

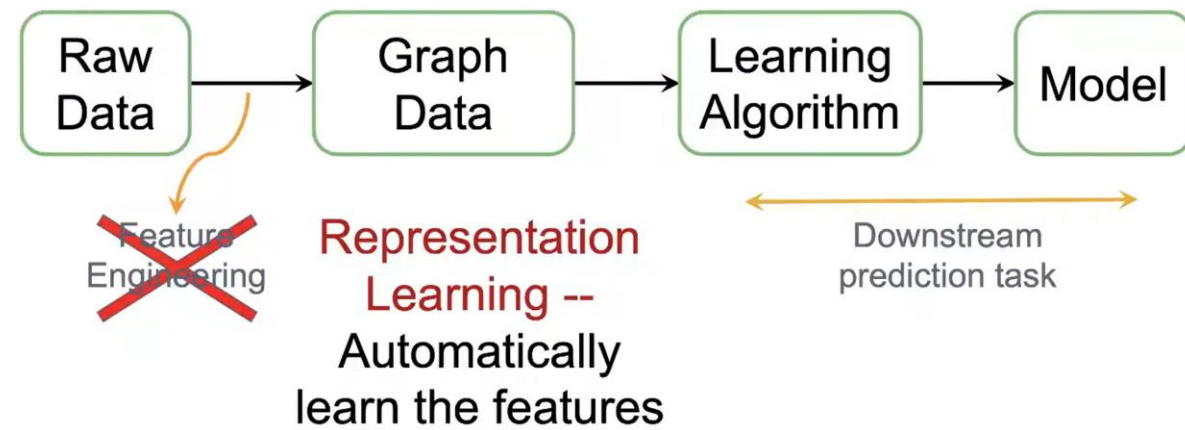


=> Question : 어떻게 이러한 모델을 만들 수 있는가?

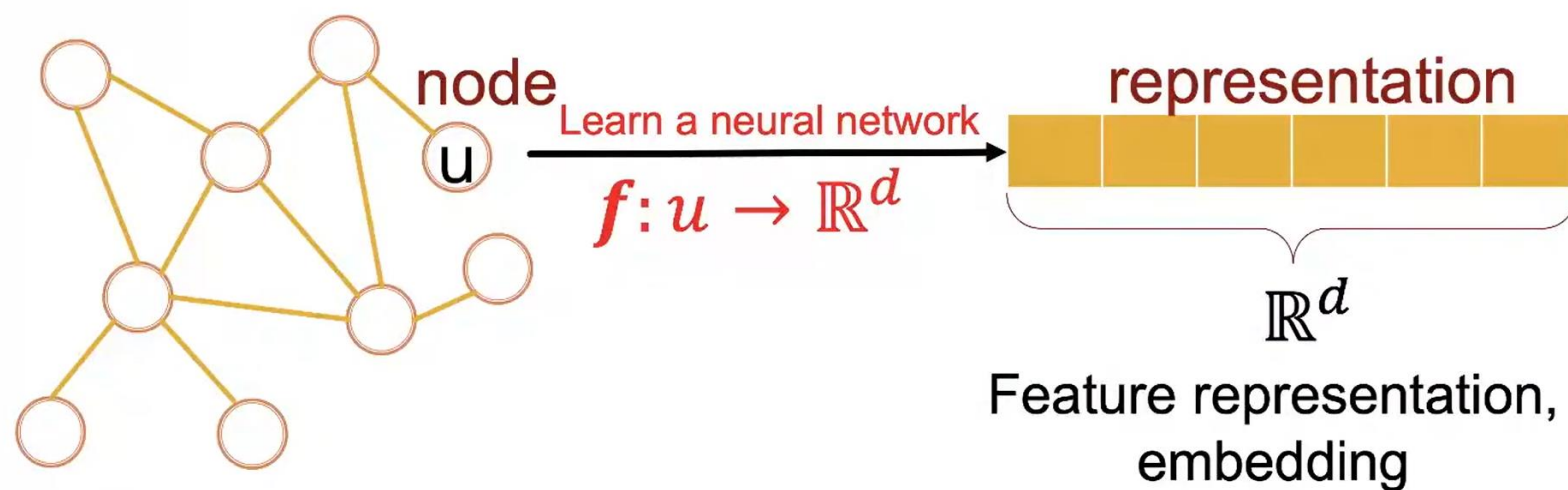
01. Why Graphs?

✧ Representative Learning (표현 학습)

- feature engineering 단계가 없어 그래프에서 특징을 자동으로 추출하거나 학습하도록 함



- 그래프의 노드를 d차원의 벡터로 임베딩하는 방법을 이용
 - > 네트워크에서 비슷한 노드들은 임베딩 공간에 가깝게 mapping됨



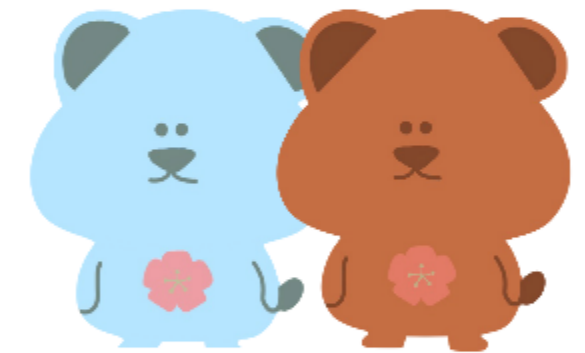
➡ 목표 : mapping 함수 f 를 학습하는 것

01. Why Graphs?

📌 이 수업에서 배울 내용

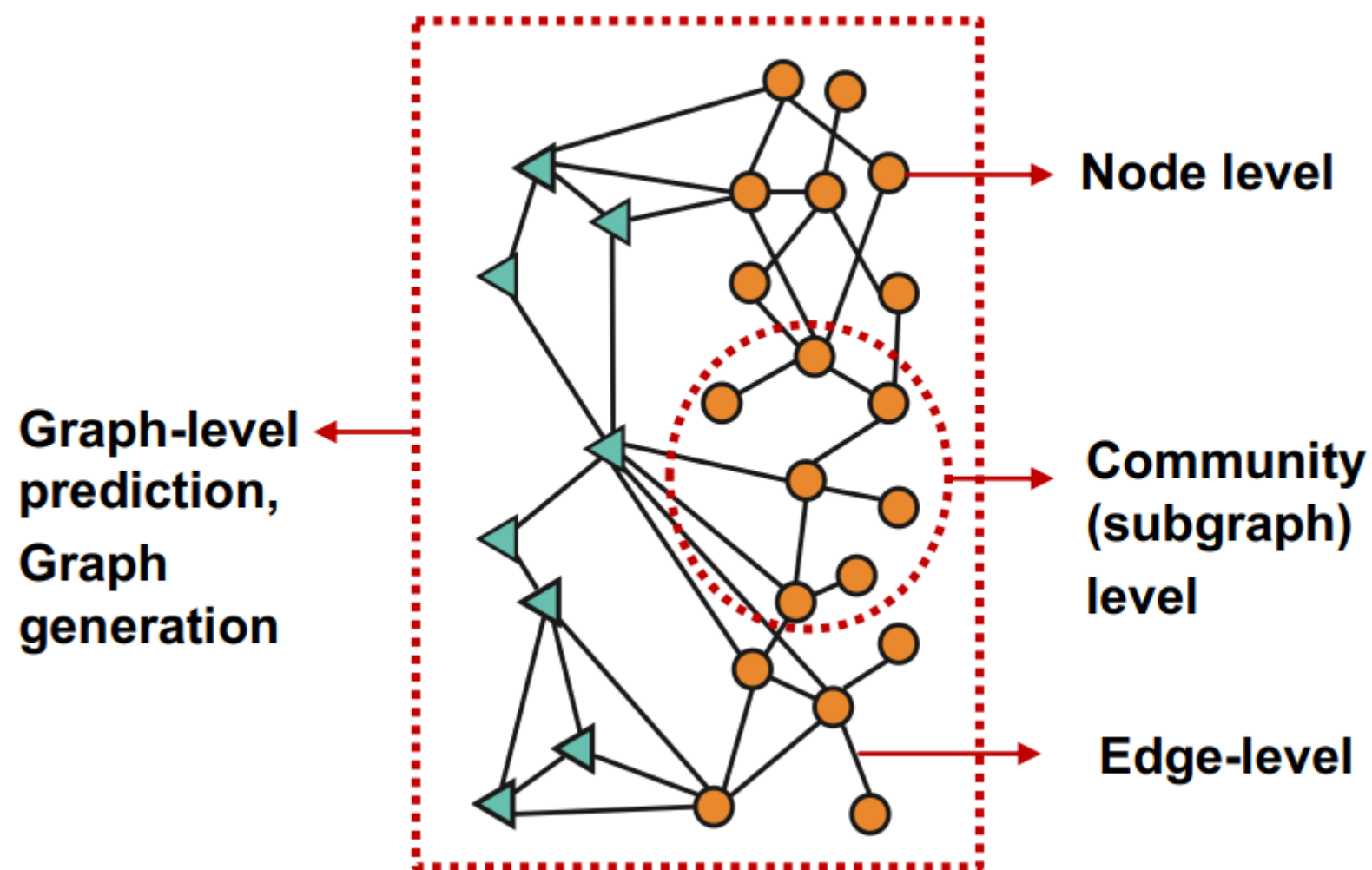
- graph structured data를 이용한 machine learning & representative learning
 - ① 그래프를 나타내는 전통적인 방법 : Graphlets, Graph Kernels
 - ② node embedding 방법들 : DeepWalk, Node2Vec
 - ③ Graph Neural Networks : GCN, GraphSAGE, GAT, Theory of GNNs
 - ④ Knowledge graphs and reasoning : TransE, BetaE
 - ⑤ Deep generative models for Graphs
 - ⑥ Biomedicine과 과학, 산업에서의 응용

02. Applications of Graph ML



02. Applications of Graph ML

1 GNN TASK 종류



Node Edge Graph

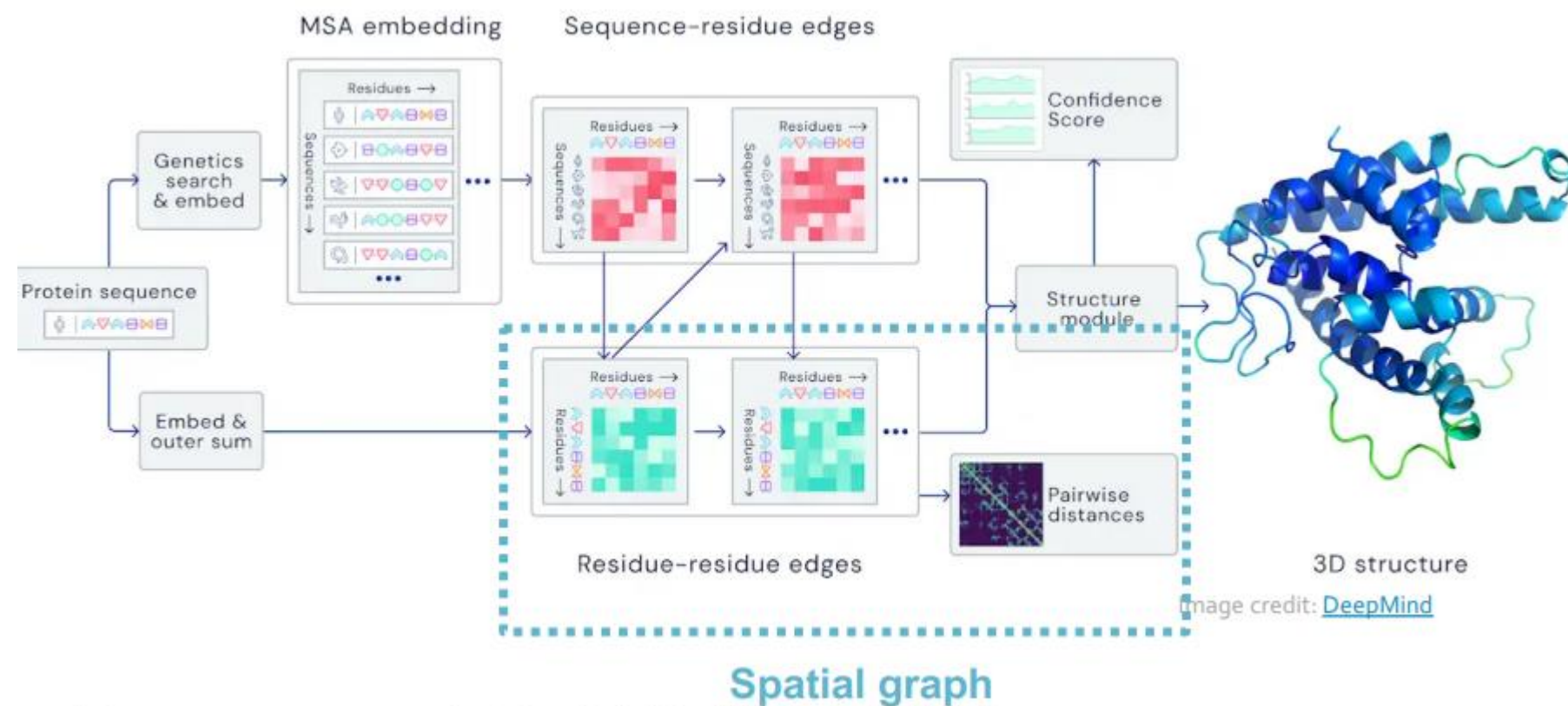
- ① Node classification
- ② Line prediction
- ③ Graph classification
- ④ Clustering
- ⑤ Graph generation, Graph evolution

High – impact applications

02. Applications of Graph ML

2 Node Level

(1) 단백질 생성 (Protein Folding)



- **Node** : Amino Acids in protein sequence
- **Edges** : Proximity between amino acids (residues)



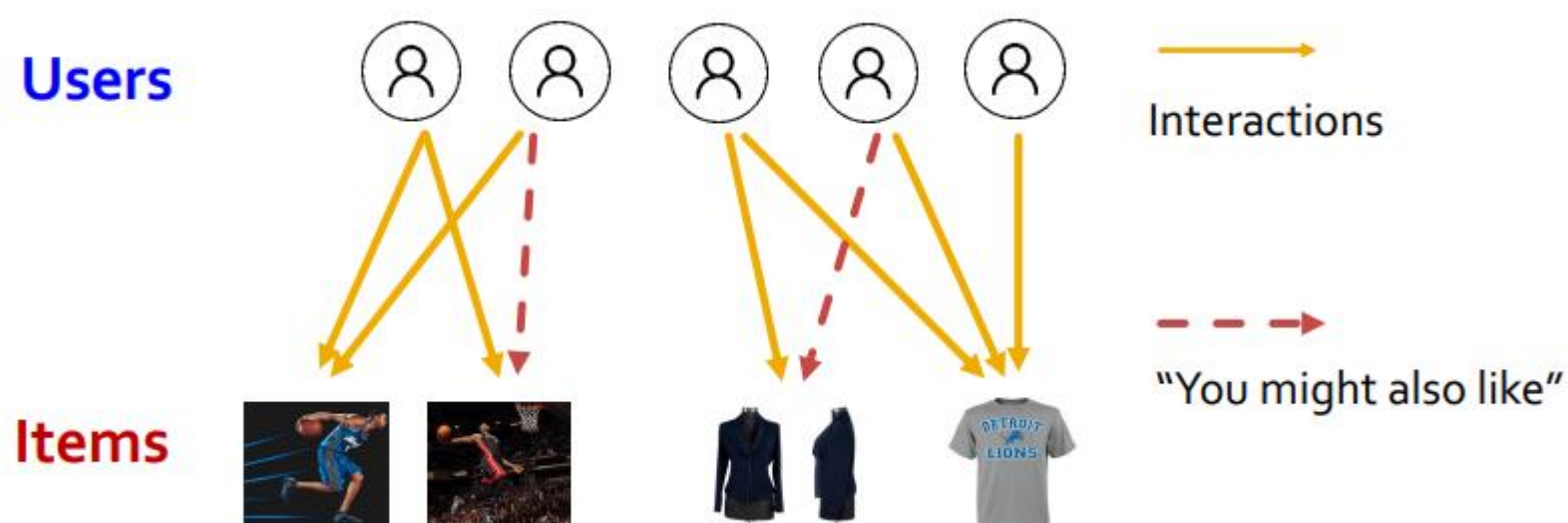
단백질 구조 예측대회 : 단백질의 3D 구조를 AI가 예측한다.

<https://velog.io/@hewas1230/AlphaFold>

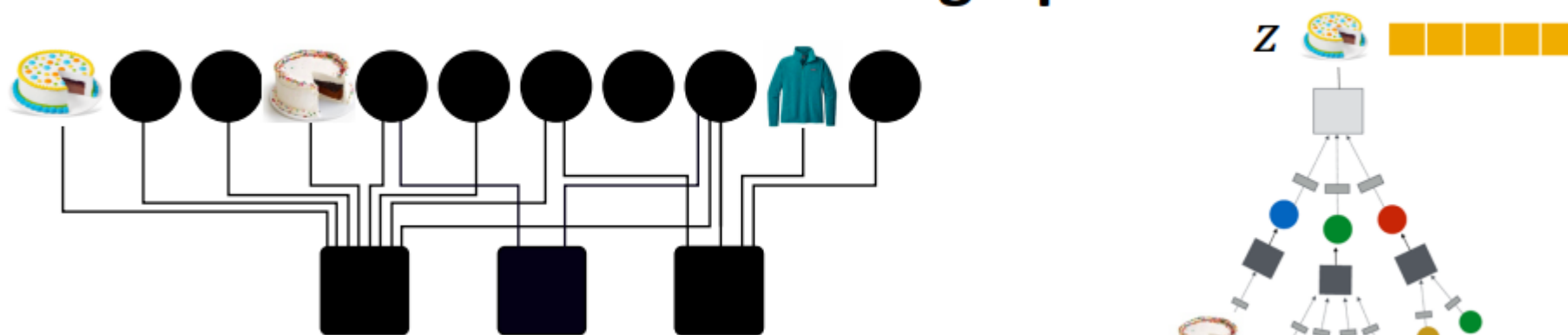
02. Applications of Graph ML

3 Edge Level

(1) 추천시스템



Predict whether two nodes in a graph are related



- Node : Users and items
- Edges : User-item interactions

유저가 좋아할 법한 상품을 추천

Task: Learn node embeddings z_i such that

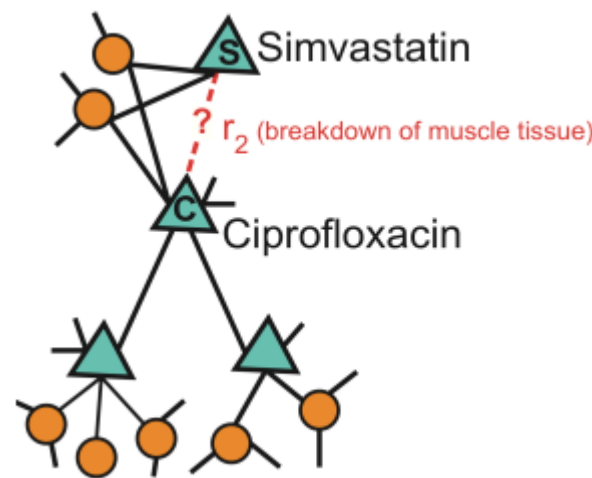
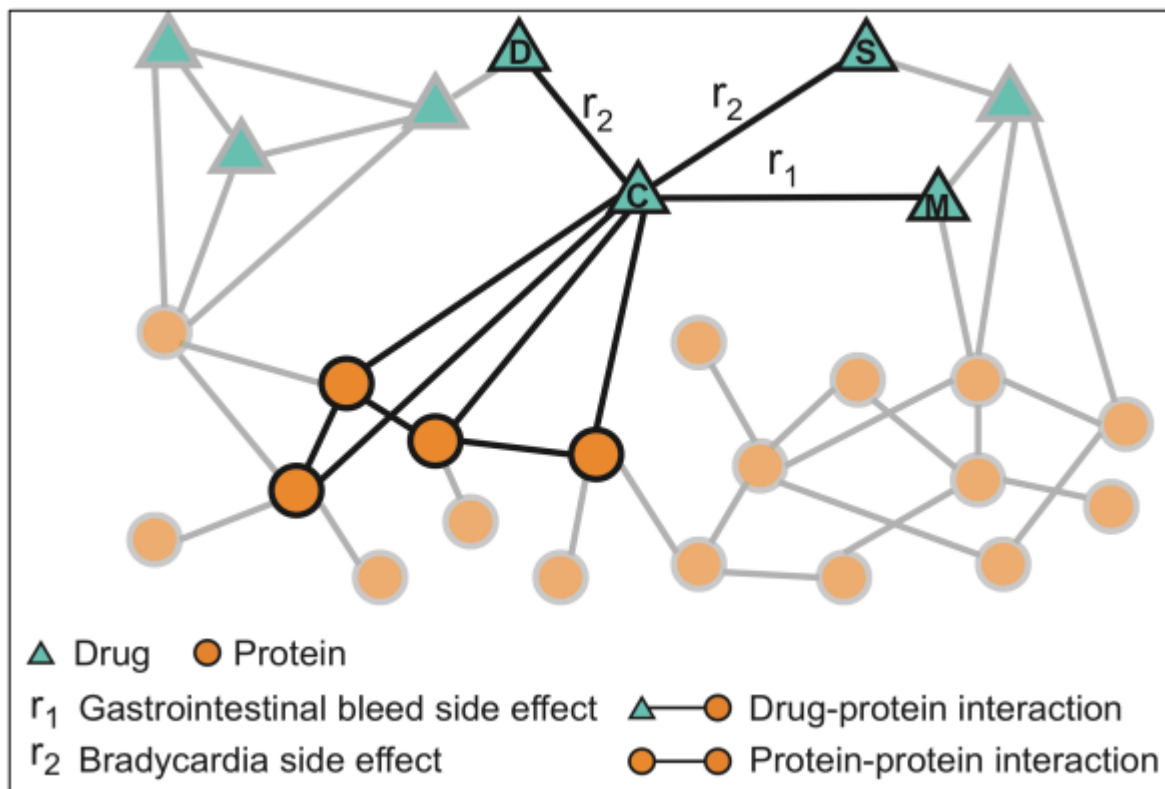
$$d(z_{cake1}, z_{cake2}) < d(z_{cake1}, z_{sweater})$$

Distance

02. Applications of Graph ML

3 Edge Level

(2) 약물 복합 섭취 시 부작용 예측



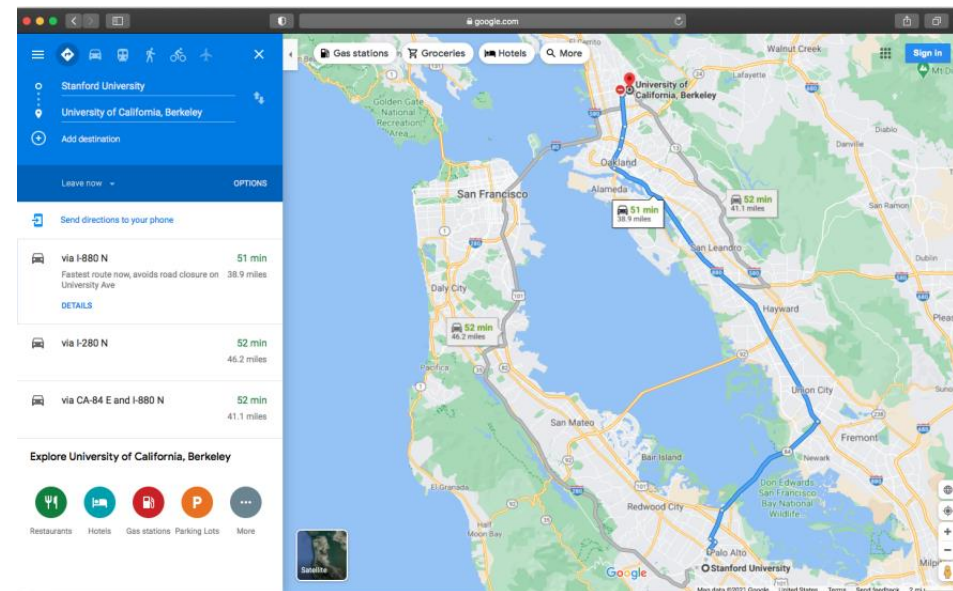
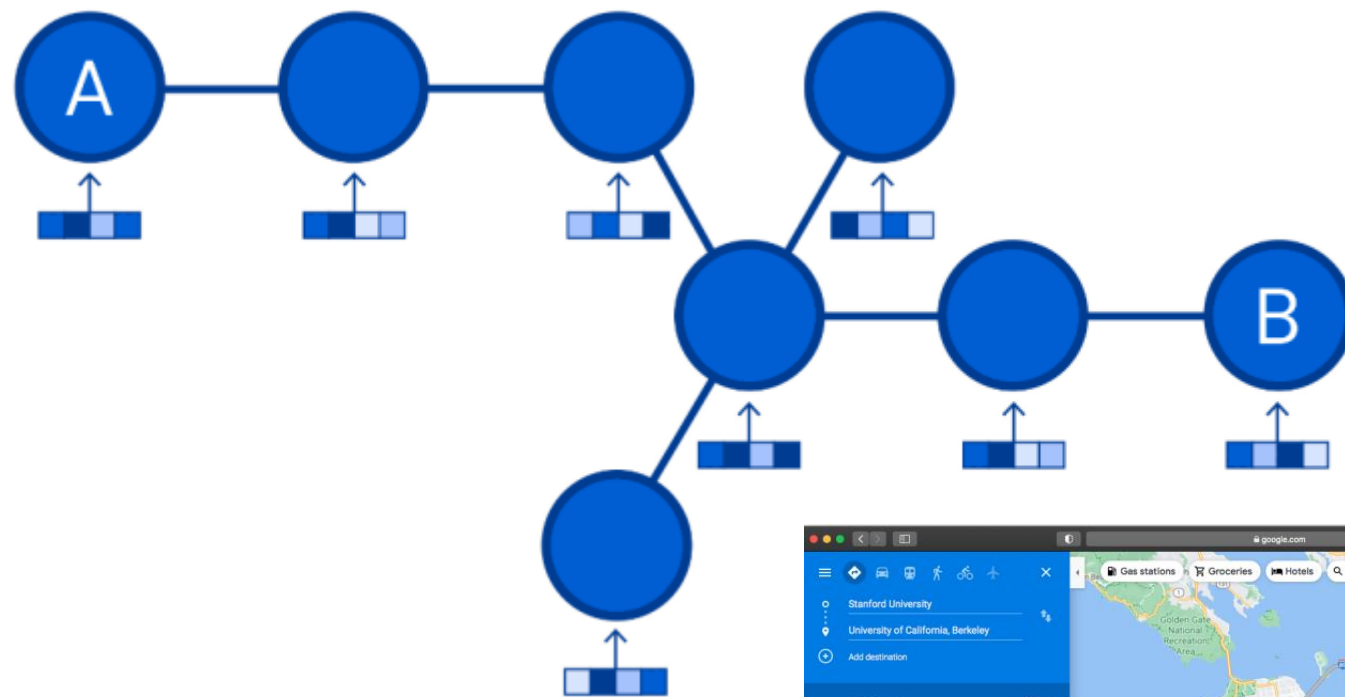
- Node : Drugs & Protein
- Edges : Interactions

A 약과 B 약을 함께 복용했을 때,
근육조직을 파괴하는가

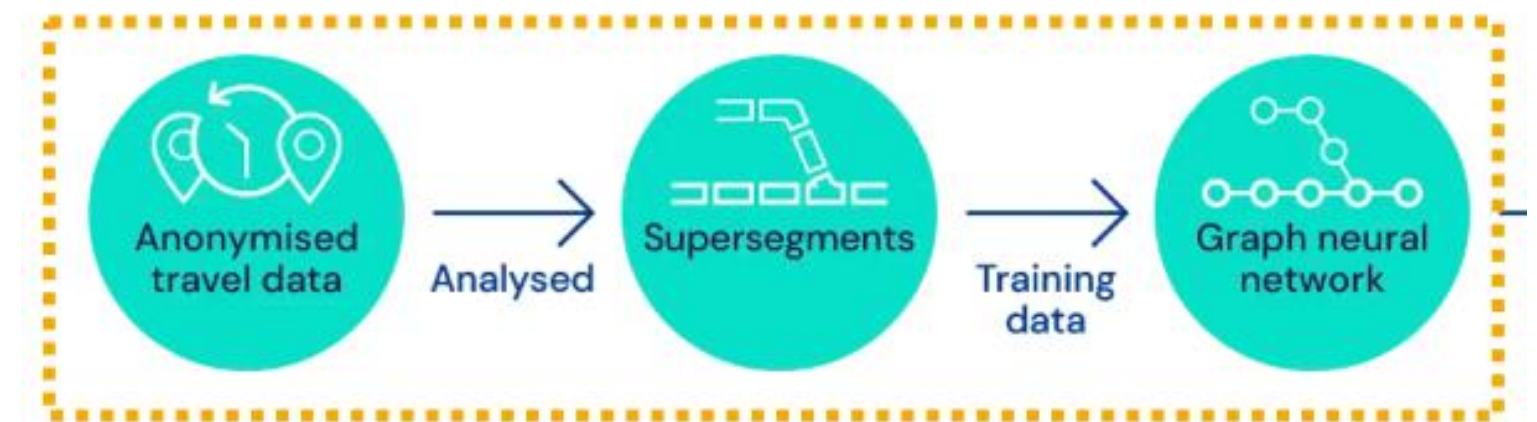
02. Applications of Graph ML

4 Subgraph Level

(1) 지도 경로 예측 (구글맵)



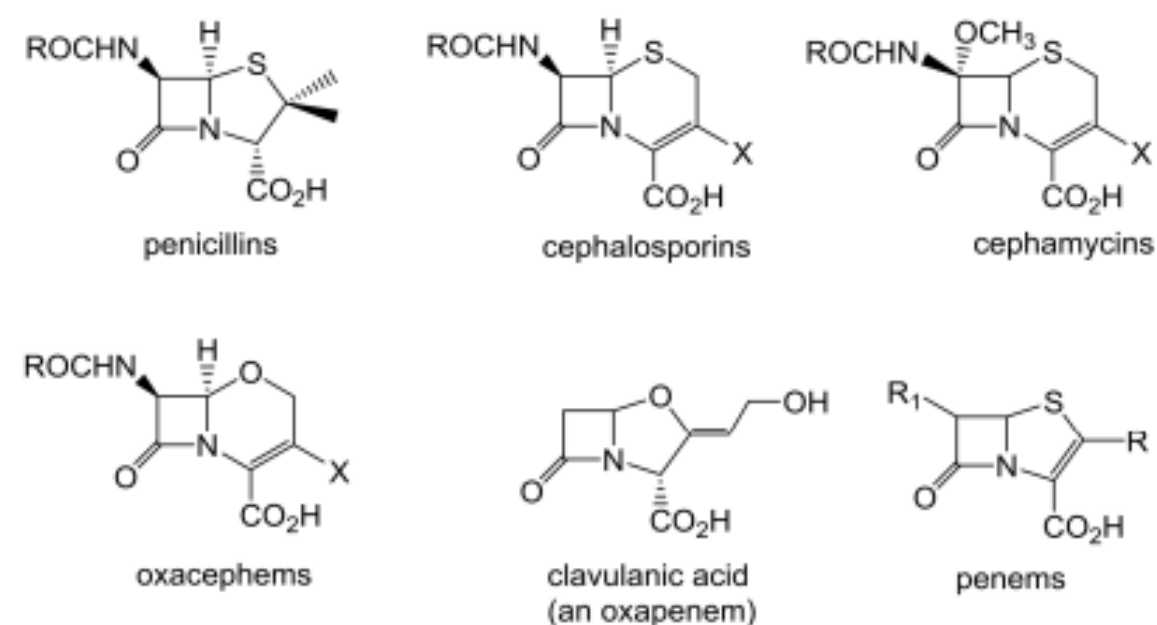
- Node : Road segments
- Edges : Connectivity between road segments



02. Applications of Graph ML

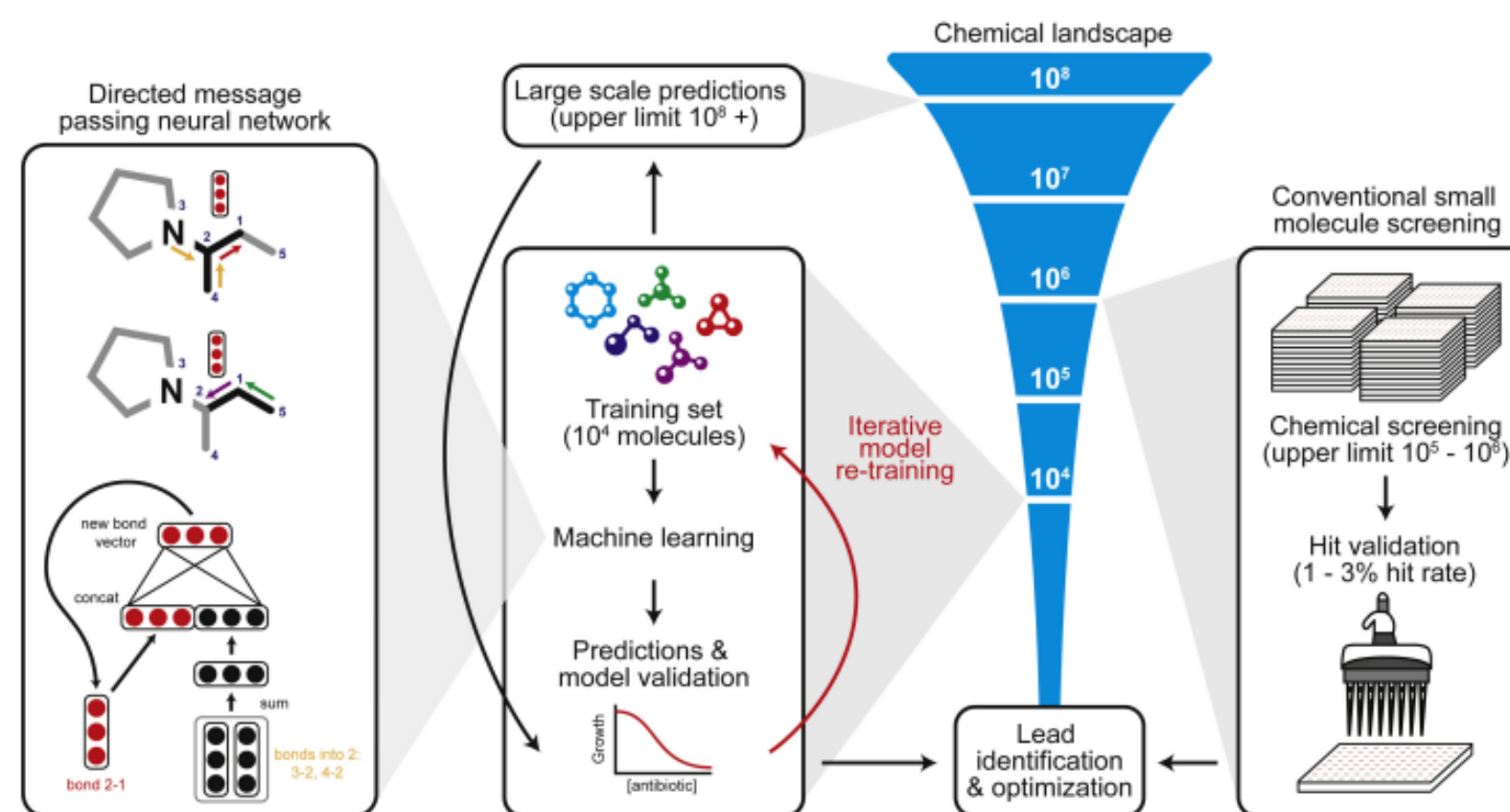
5 Graph Level

(1) 신약 개발



- Node : Atoms
- Edges : Chemical bonds

(2) 항생제 개발

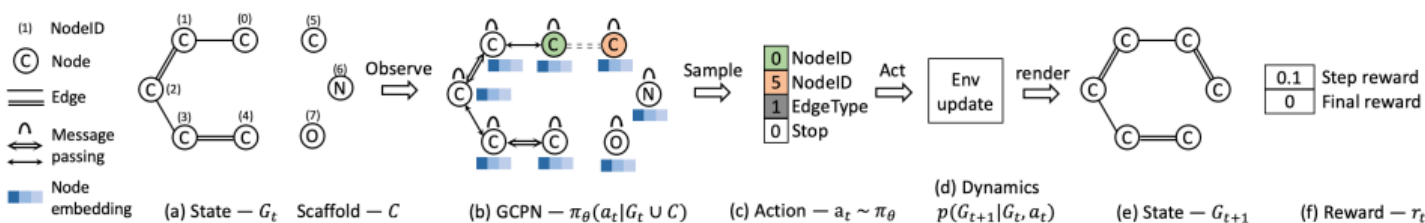


- Graph classification model

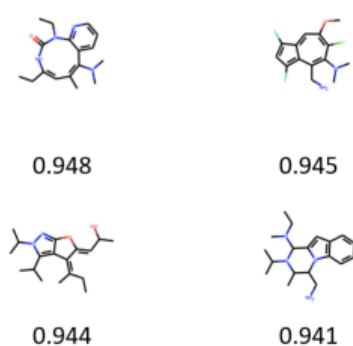
02. Applications of Graph ML

5 Graph Level

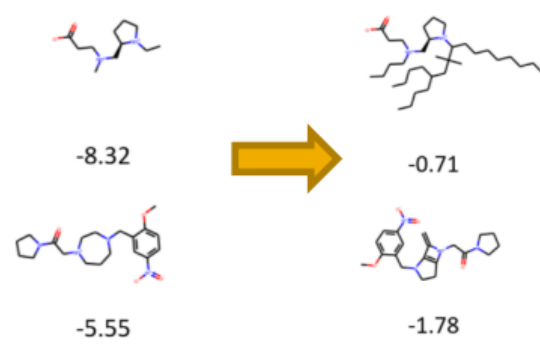
(3) 분자 생성/최적화



Use case 1: Generate novel molecules with high drug likeness

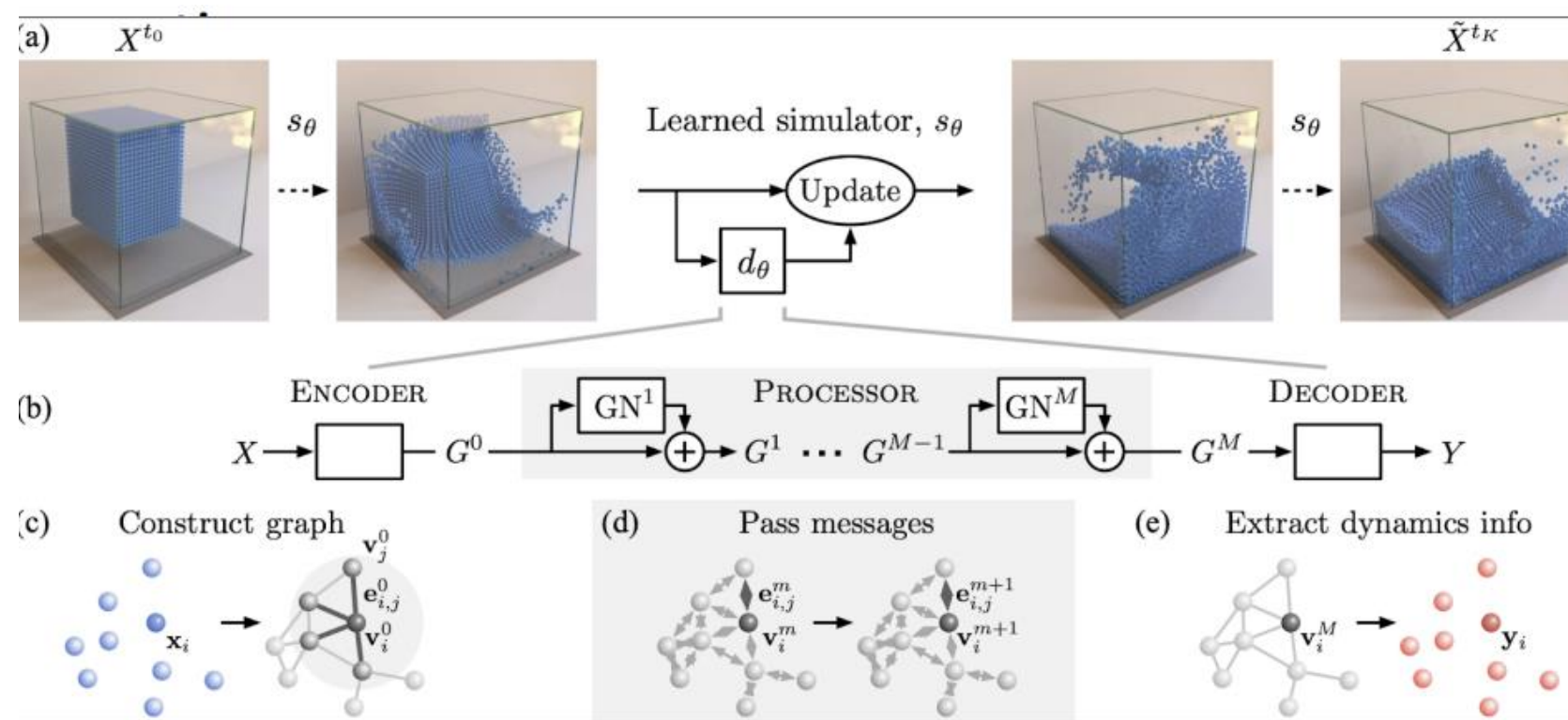


Use case 2: Optimize existing molecules to have desirable properties



- Graph generation

(4) 시뮬레이션 (물리학)



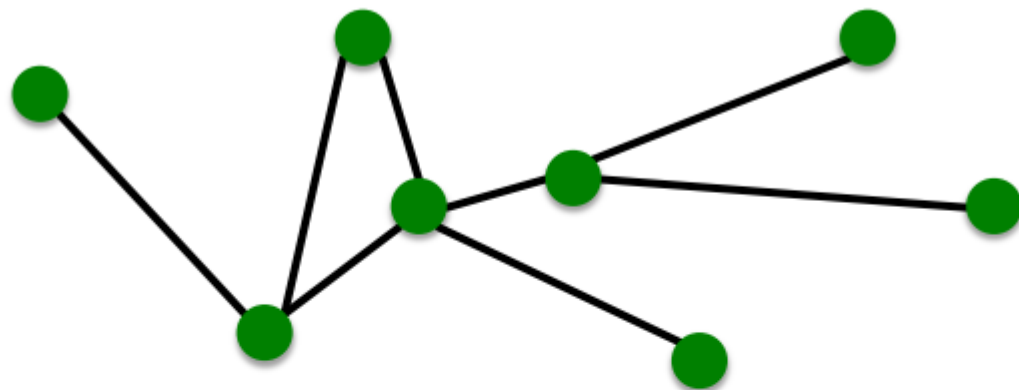
- Graph evolution
- Nodes : 입자
- Edges : 입자 간 상호작용

03. Choice of Graph Representation



03. Choice of Graph Representation

💡 그래프 구조를 어떻게 수학적으로 표현할까



- **Objects:** nodes, vertices
- **Interactions:** links, edges
- **System:** network, graph

N

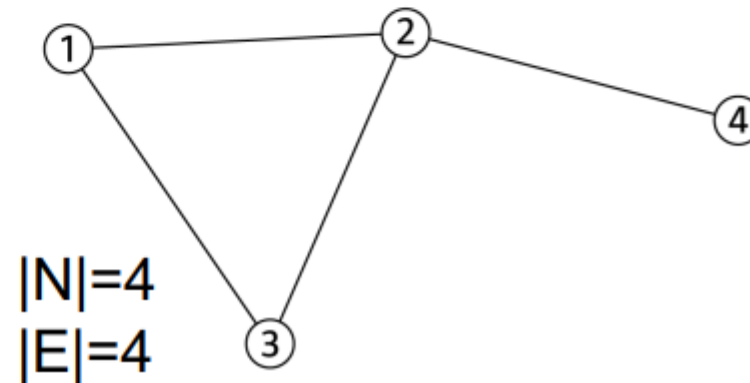
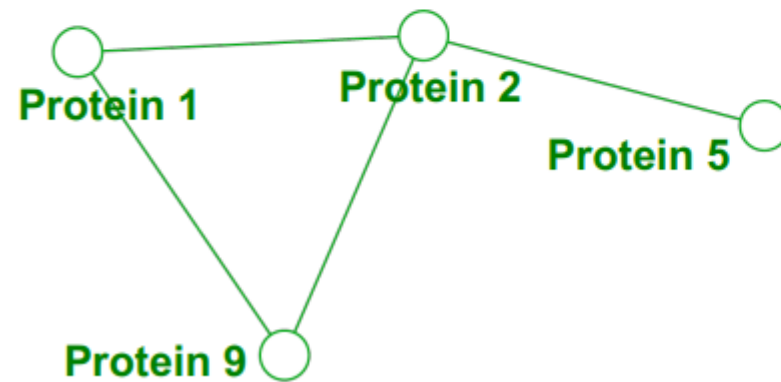
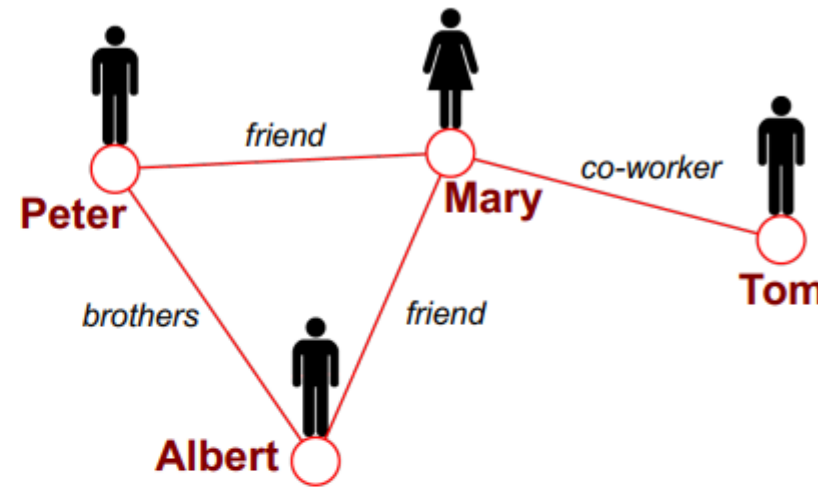
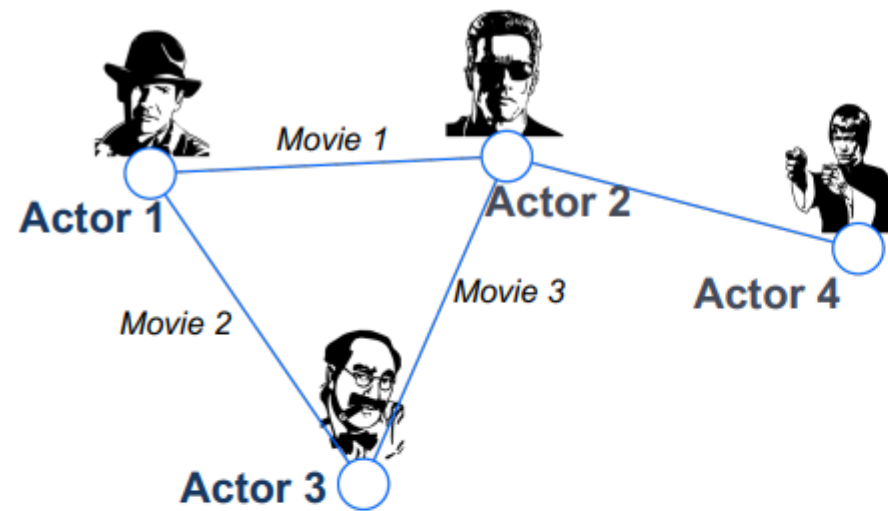
E

$G(N,E)$

- 대상 : nodes (N)
- 상호작용 : edges (E)
- 체계 : graph $G(N,E)$

03. Choice of Graph Representation

💡 그래프는 일반적인 상황들을 표현할 수 있는 수단이다.



- 배우 – 작품
- 사람 – 관계 (지인, 친구)
- 단백질 – 연결관계

→ 모두 동일한 그래프 구조로
표현 가능

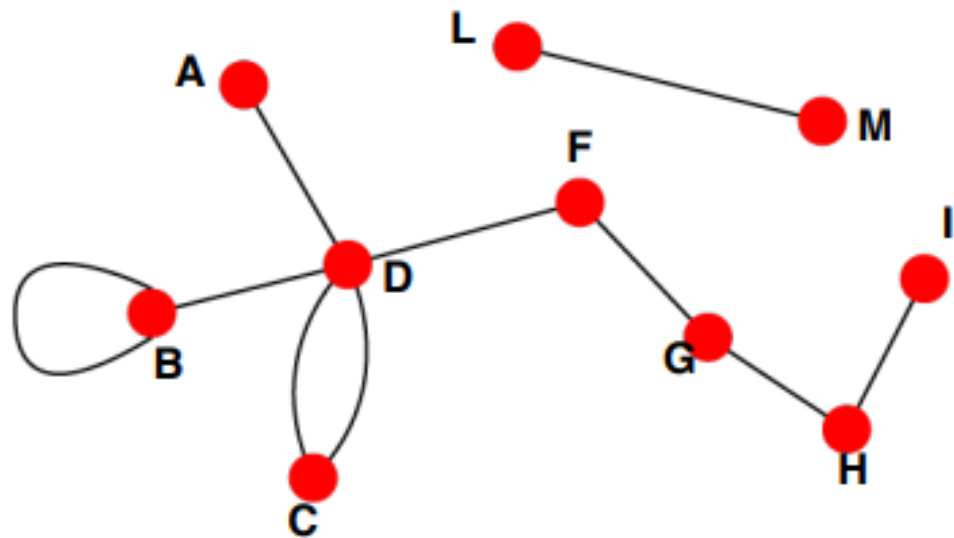
도메인, 정의한 문제에 따라 적절한 그래프 표현법을 선택해야 한다.

03. Choice of Graph Representation

① Direction of Edges

Undirected

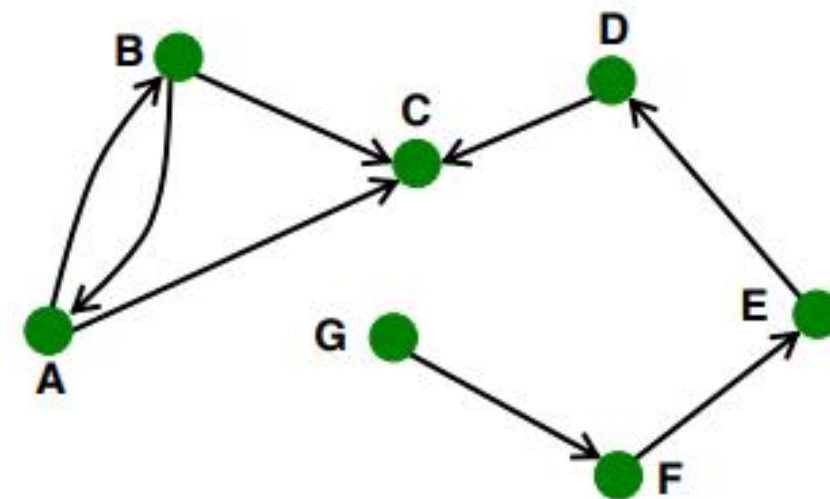
- **Links:** undirected (symmetrical, reciprocal)



- 연결이 양방향인 그래프
- collaboration, frendship

Directed

- **Links:** directed (arcs)

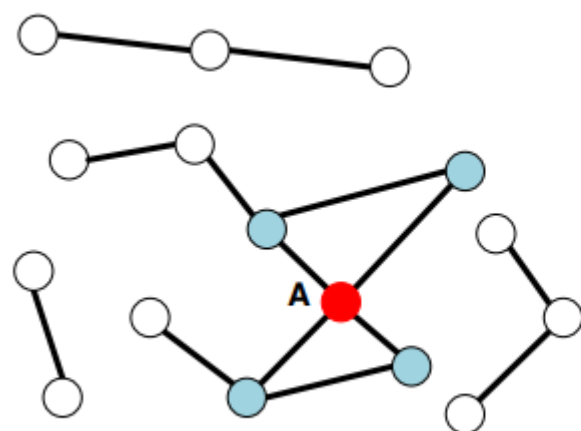


- 연결이 특정 방향으로 생성되는 그래프
- citation/인용, 인스타, DM, 팔로잉

03. Choice of Graph Representation

② Node Degrees

Undirected



Node degree, k_i : the number of edges adjacent to node i

$k_A = 4 \rightarrow$ node A 에 연결된 링크는 4개

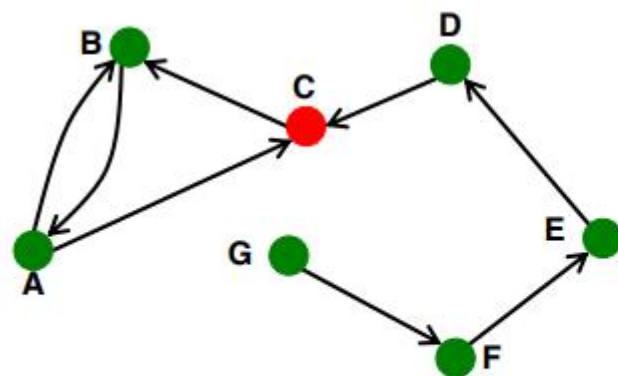
Avg. degree: $\bar{k} = \langle k \rangle = \frac{1}{N} \sum_{i=1}^N k_i = \frac{2E}{N}$

- Degree

: 노드에 부착된 edge 의 개수

\rightarrow Undirected graph 평균 차수

Directed



Source: Node with $k^{in} = 0$

Sink: Node with $k^{out} = 0$

In directed networks we define an **in-degree** and **out-degree**. The (total) degree of a node is the sum of in- and out-degrees.

$$k_C^{in} = 2 \quad k_C^{out} = 1 \quad k_C = 3$$

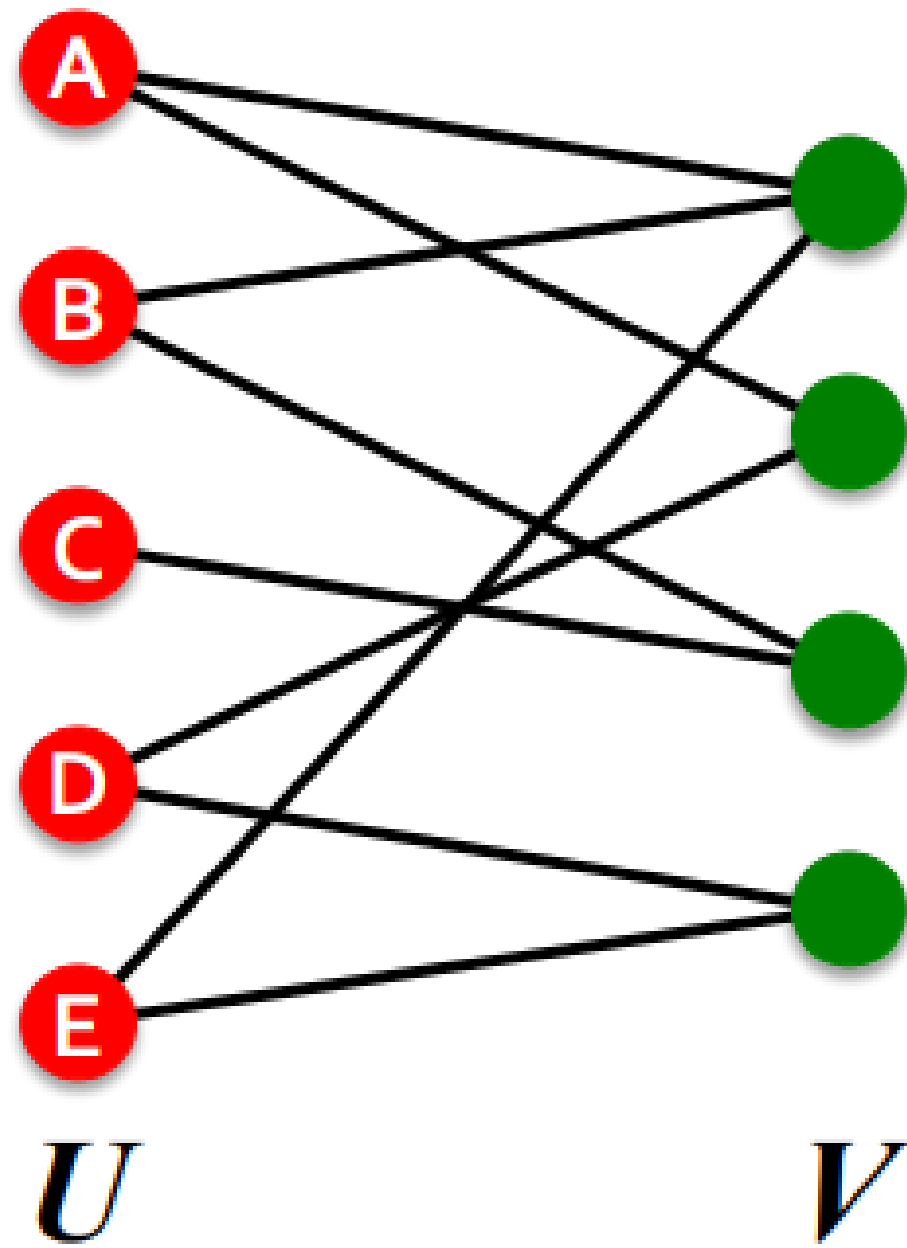
$$\bar{k} = \frac{E}{N}$$

$$\bar{k}^{in} = \bar{k}^{out}$$

\rightarrow Directed 의 경우 in-degree 와 out-degree 가 구분된다.

03. Choice of Graph Representation

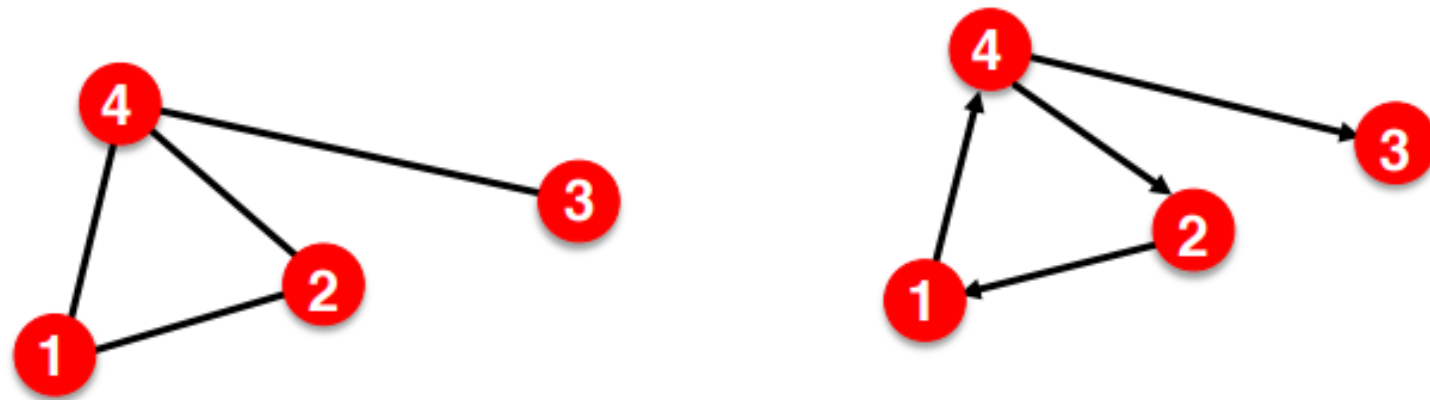
③ Bipartite Graph



- Bipartite graph
: 서로 다른 종류의 독립된 노드들로 구성된 그래프
- EX. U 에 속하는 노드는
: V 에 속한 노드에게만 연결되고 U 끼리는 독립
(V 도 반대로 마찬가지)
- 이는 실제 도메인에서 많이 나타나는 구조이며 특히 추천시스템에서 많이 사용되는 개념임

03. Choice of Graph Representation

④ Graph 표현 방법 – (1) 인접행렬



$A_{ij} = 1$ if there is a link from node i to node j

$A_{ij} = 0$ otherwise

$$A = \begin{pmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{pmatrix}$$

$$A = \begin{pmatrix} 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 \end{pmatrix}$$

Note that for a directed graph (right) the matrix is not symmetric.

- Adjacency Matrix

: 그래프 연결 유무를 1 과 0으로 나눠 행렬을 표현한 것

→ Undirected 의 경우 행렬이 대칭으로 나타나는데, Undirected 는 단방향도 존재하므로 대칭이 아닐 수도 있다.

🗨 노드가 많아질수록 (행렬의 차원이 커질수록) 행렬이 Sparse 해지는 단점

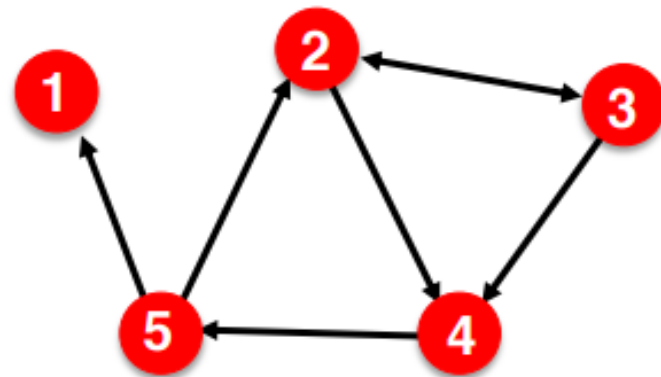
Most real-world networks are sparse

$$E \ll E_{\max} \text{ (or } k \ll N-1)$$

03. Choice of Graph Representation

④ Graph 표현 방법 – (2) Edge list

- (2, 3)
- (2, 4)
- (3, 2)
- (3, 4)
- (4, 5)
- (5, 2)
- (5, 1)



- Edge list

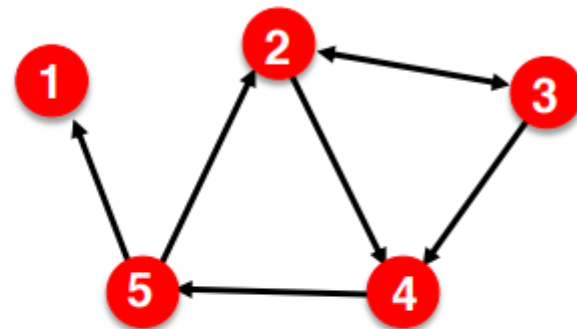
: Edge 를 연결된 노드 쌍으로 표현한 것

03. Choice of Graph Representation

④ Graph 표현 방법 – (3) 인접 list

Adjacency list:

- Easier to work with if network is
 - Large
 - Sparse
- Allows us to quickly retrieve all neighbors of a given node
 - 1:
 - 2: 3, 4
 - 3: 2, 4
 - 4: 5
 - 5: 1, 2



- Adjacency list

: 출발 방향의 노드를 Key 값으로 도착 노드를 Value 값으로 가지는 Dictionary 형태

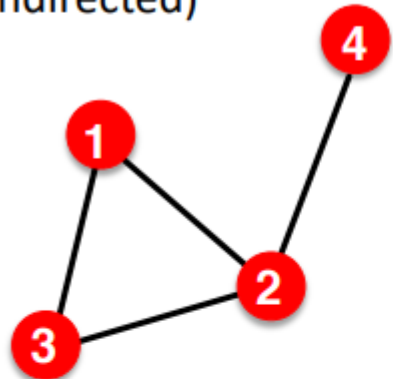
: 단방향이거나 거대한 그래프에서 효율이 좋다.

03. Choice of Graph Representation

⑤ Edge 특성

■ Unweighted

(undirected)



$$A_{ij} = \begin{pmatrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}$$

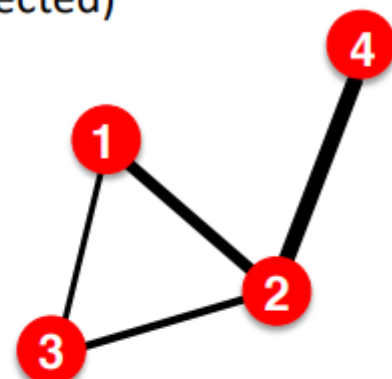
$$A_{ii} = 0 \quad A_{ij} = A_{ji}$$

$$E = \frac{1}{2} \sum_{i,j=1}^N A_{ij} \quad \bar{k} = \frac{2E}{N}$$

Examples: Friendship, Hyperlink

■ Weighted

(undirected)



$$A_{ij} = \begin{pmatrix} 0 & 2 & 0.5 & 0 \\ 2 & 0 & 1 & 4 \\ 0.5 & 1 & 0 & 0 \\ 0 & 4 & 0 & 0 \end{pmatrix}$$

$$A_{ii} = 0 \quad A_{ij} = A_{ji}$$

$$E = \frac{1}{2} \sum_{i,j=1}^N \text{nonzero}(A_{ij}) \quad \bar{k} = \frac{2E}{N}$$

Examples: Collaboration, Internet, Roads

Possible options:

- Weight (e.g., frequency of communication)
- Ranking (best friend, second best friend...)
- Type (friend, relative, co-worker)
- Sign: Friend vs. Foe, Trust vs. Distrust
- Properties depending on the structure of the rest of the graph: Number of common friends

• Weight

: edge 에 weight 를 줄 수 있음

• Ranking

: 연결 관계에 순위를 부여 (짱친 > 절친 > ...)

• Type

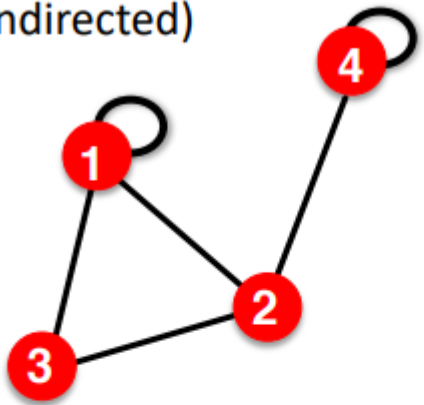
: 연결 관계의 유형을 부여 (친척, 직장동료)

03. Choice of Graph Representation

⑥ 그래프 유형 +

■ Self-edges (self-loops)

(undirected)



$$A_{ij} = \begin{pmatrix} 1 & 1 & 1 & 0 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 \end{pmatrix}$$

$$A_{ii} \neq 0$$

$$A_{ij} = A_{ji}$$

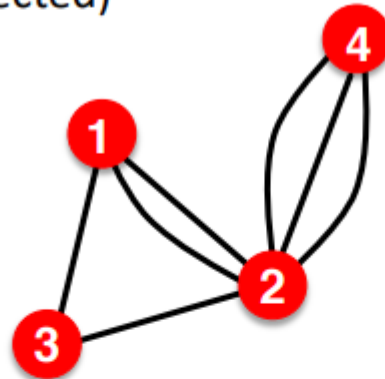
$$E = \frac{1}{2} \sum_{i,j=1, i \neq j}^N A_{ij} + \sum_{i=1}^N A_{ii}$$

Examples: Proteins, Hyperlinks

- 자기 자신과 연결된 형태

■ Multigraph

(undirected)



$$A_{ij} = \begin{pmatrix} 0 & 2 & 1 & 0 \\ 2 & 0 & 1 & 3 \\ 1 & 1 & 0 & 0 \\ 0 & 3 & 0 & 0 \end{pmatrix}$$

$$A_{ii} = 0$$

$$A_{ij} = A_{ji}$$

$$E = \frac{1}{2} \sum_{i,j=1}^N \text{nonzero}(A_{ij}) \quad \bar{k} = \frac{2E}{N}$$

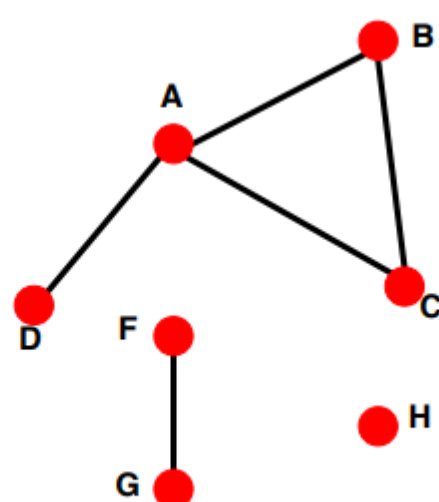
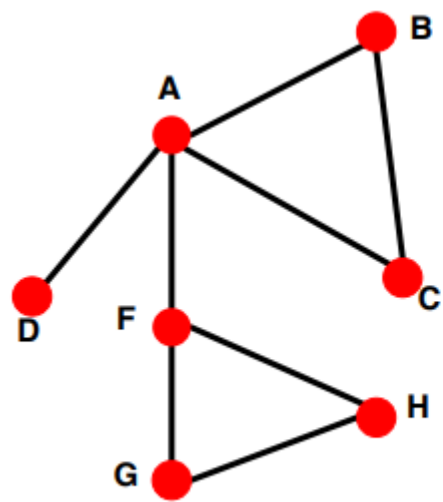
Examples: Communication, Collaboration

- 다중 연결

03. Choice of Graph Representation

⑦ Connectivity of Undirected/Directed Graphs

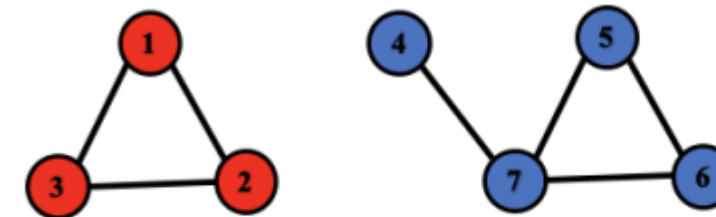
- Connected Graph (undirected) : 어떤 노드에서 출발하든지 다른 모든 노드로 도착할 수 있음
- Disconnected Graph : 최소 2개 이상의 connected graph 로 구성됨
- Bridge edge : 삭제되면 connected 에서 disconnected 로 바꿀 수 있는 Edge
- Articulation node : 삭제되면 connected 에서 disconnected 로 바꿀 수 있는 Node



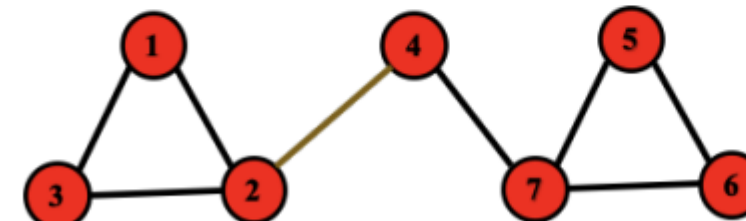
Largest Component:
Giant Component

Isolated node (node H)

Disconnected



Connected



Block diagonal

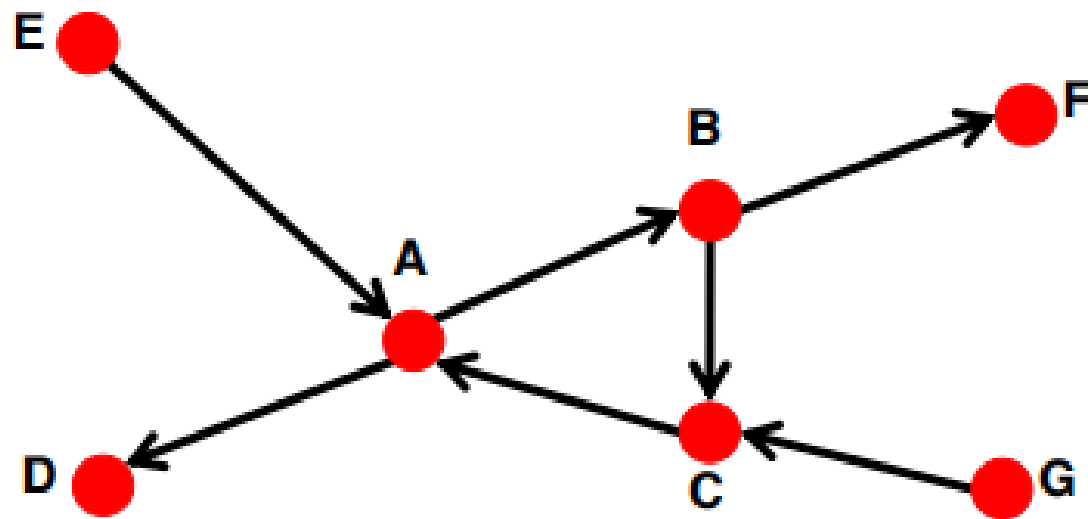
0	1	1	0	0	0	0
1	0	1	0	0	0	0
1	1	0	0	0	0	0
0	0	0	0	0	0	1
0	0	0	0	0	1	1
0	0	0	0	1	0	1
0	0	0	1	1	1	0

0	1	1	0	0	0	0
1	0	1	1	0	0	0
1	1	0	0	0	0	0
0	1	0	0	0	0	1
0	0	0	0	0	1	1
0	0	0	0	1	0	1
0	0	0	1	1	1	0

03. Choice of Graph Representation

⑦ Connectivity of Undirected/Directed Graphs

- Connected Directed Graph



- ✓ Strong Connected Directed Graph

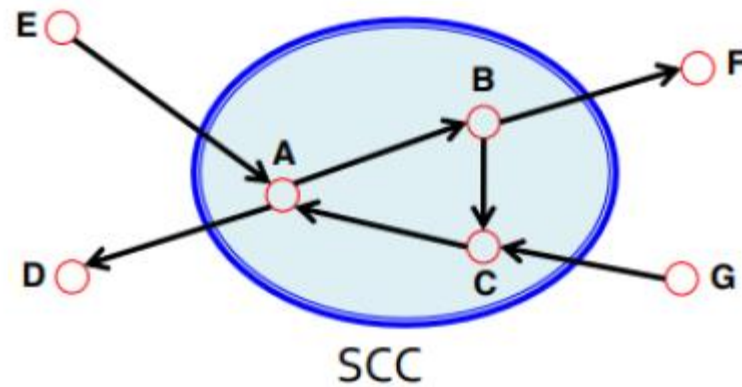
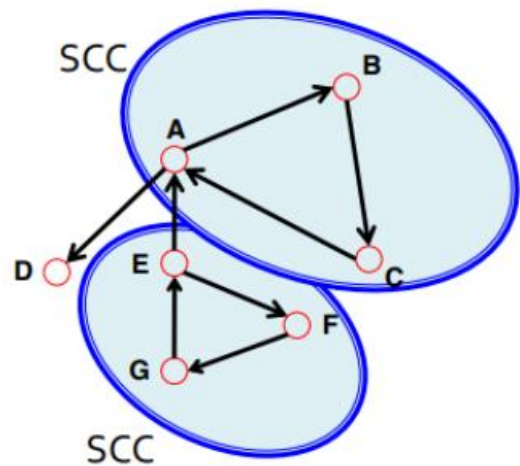
: 어떤 노드에서 출발하던지 edge 방향을 지키면서 다른 모든 노드로 도착 가능

- ✓ Weakly Connected Directed Graph

: 어떤 노드에서 출발하던지 edge 방향을 무시한다면 다른 모든 노드로 도착 가능

- ✓ SCCs (Strongly)

: 그래프에서 부분적으로 나타나는 connected subgraphs



SUMMARY

(1) Machine learning with Graphs

- 그래프 구조란
- 응용사례

(2) Different types of tasks

- Node level
- Edge level
- Graph level

(3) Choice of a graph representation

- Directed , Undirected, Bipartite, Weighted, Adjacency matrix

(4) 복습과제 안내

예제를 통해 알아보는 pytorch Geometric

<https://baeseongsu.github.io/posts/pytorch-geometric-introduction/>

→ 필사하며 기본 메서드, 라이브러리 익히기

SUMMARY

GNN 라이브러리

- (1) PyG : GNN 을 위한 파이썬 라이브러리 (https://github.com/pyg-team/pytorch_geometric)
- (2) GraphGym (<https://github.com/snap-stanford/GraphGym>)
- (3) Deepsnap (<https://github.com/snap-stanford/deepsnap/tree/master/examples>)
- (4) Tensorflow GNN (<https://github.com/tensorflow/gnn>)

+) 읽기자료 (GNN 을 활용한 요기요의 추천시스템 2022.07)

THANK YOU

