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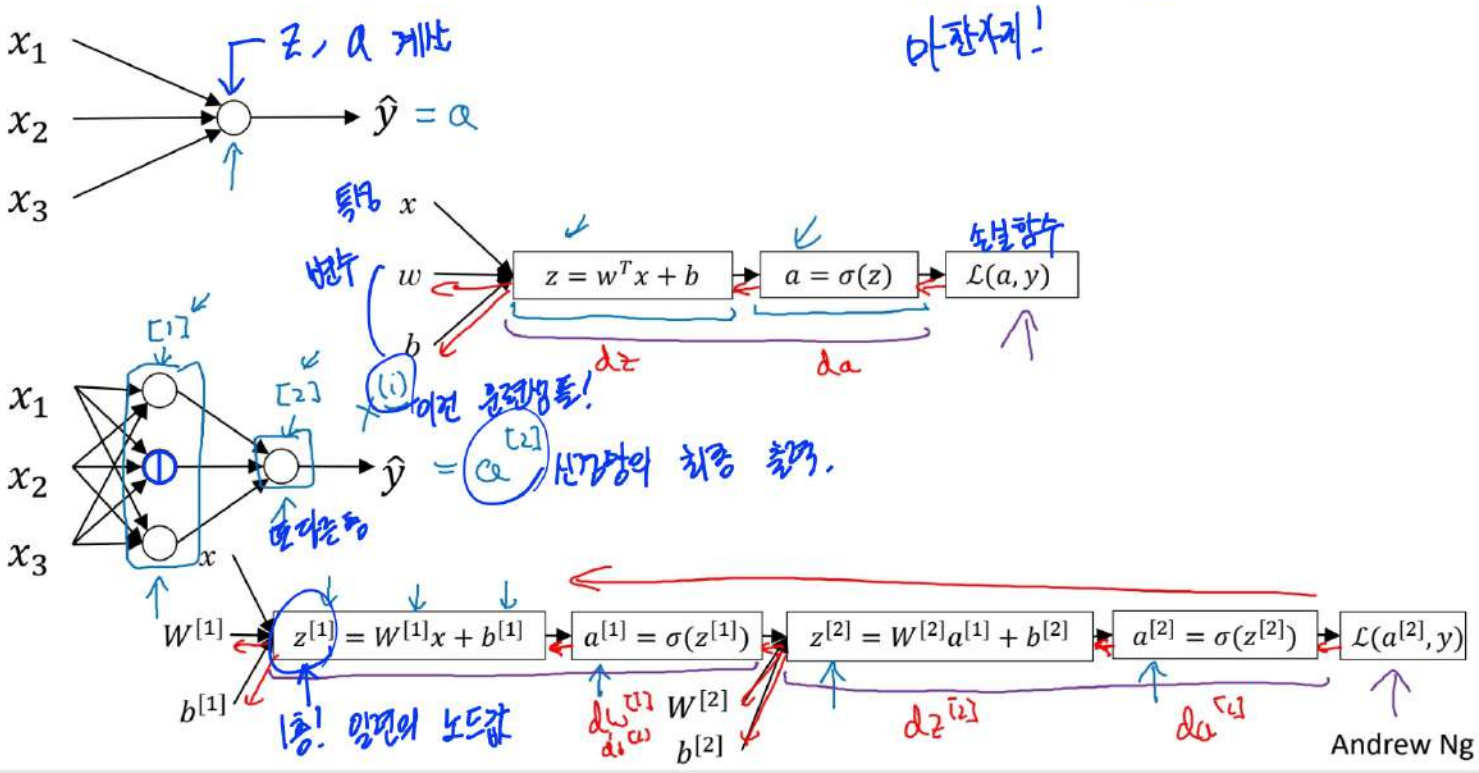
One hidden layer Neural Network

Neural Networks Overview

로지스틱 회귀에서는 도함수 계산을 위해 역방향 계산을 했었다.
마찬가지!

What is a Neural Network?



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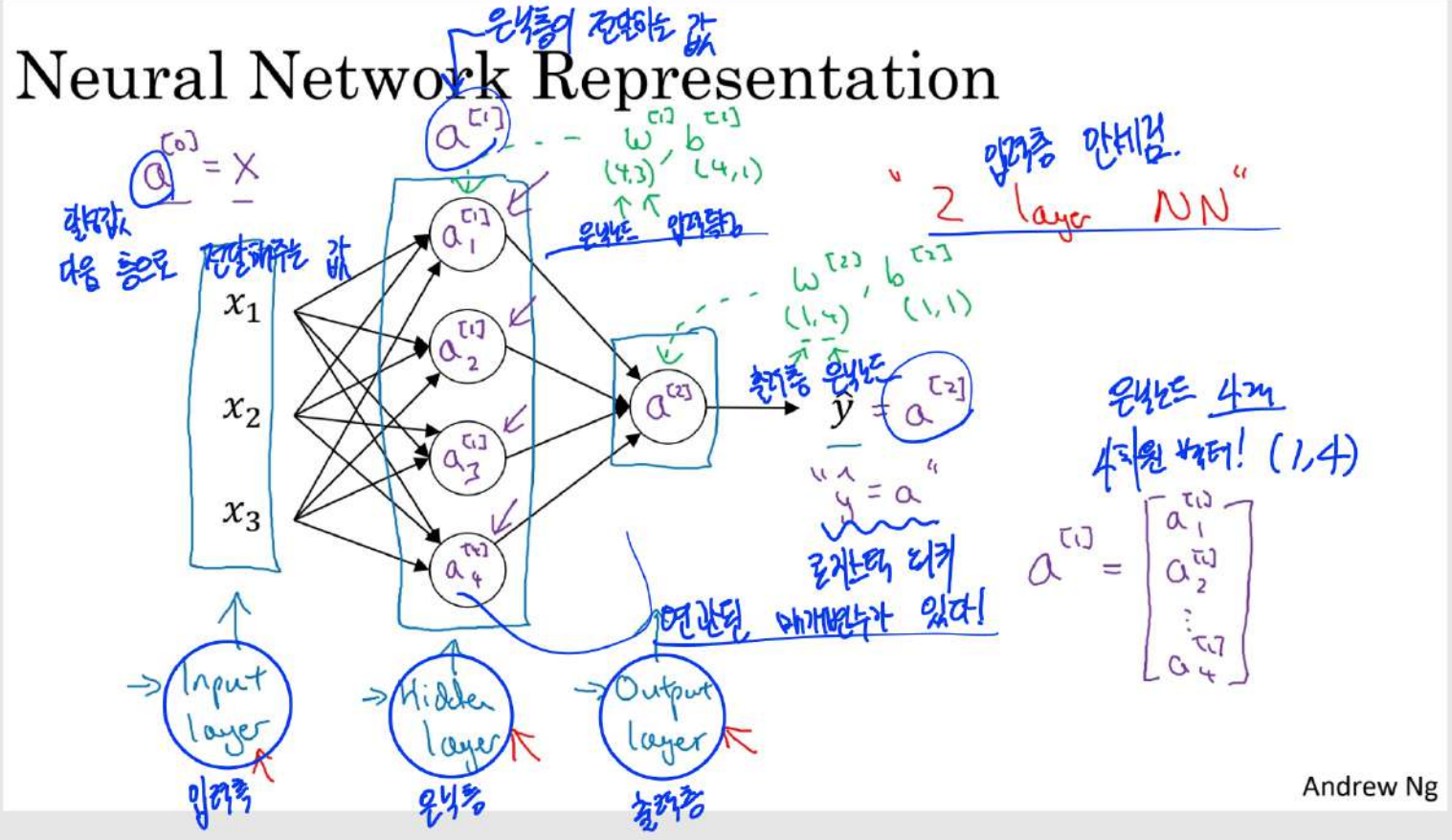
Neural Network Representation

지도학습
: 훈련 세트가 입력값 X와 출력값 Y.
(은닉층의 실제 값은 훈련세트에 X)

은닉층이 하나인 신경망!



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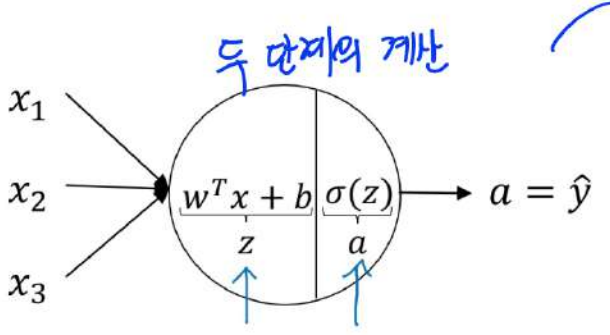
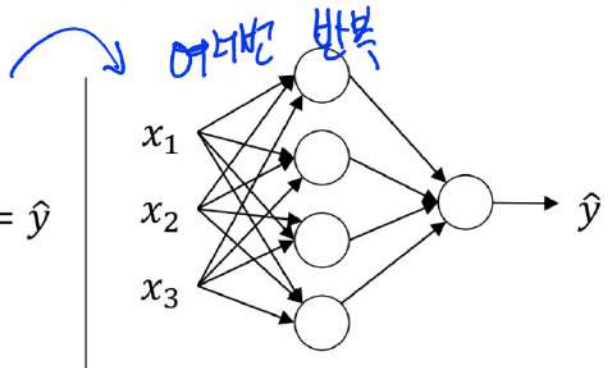
Computing a Neural Network's Output

Neural Network Representation





$$z = w^T x + b$$

$$a = \sigma(z)$$

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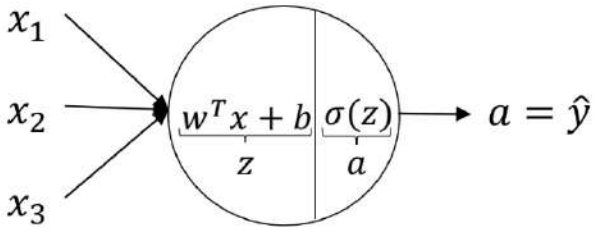
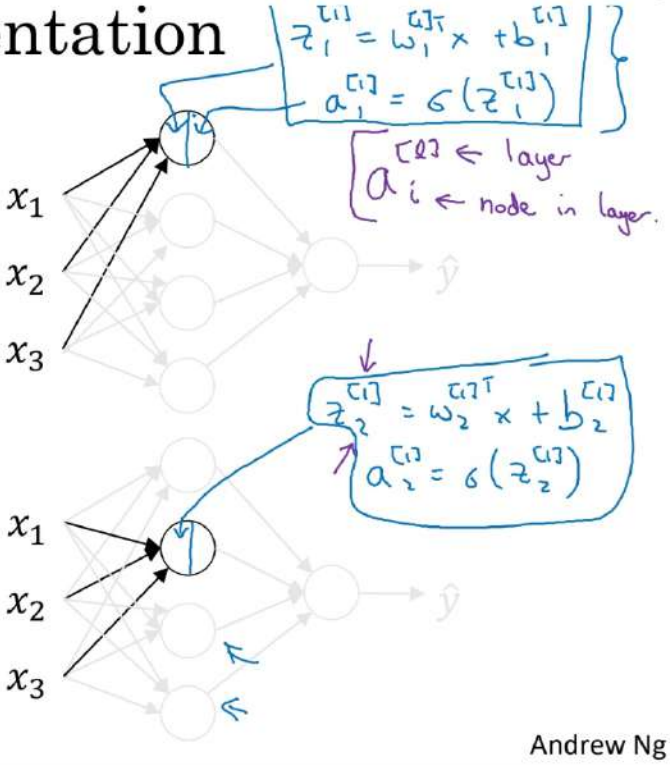
Neural Network Representation





$$z = w^T x + b$$

$$a = \sigma(z)$$

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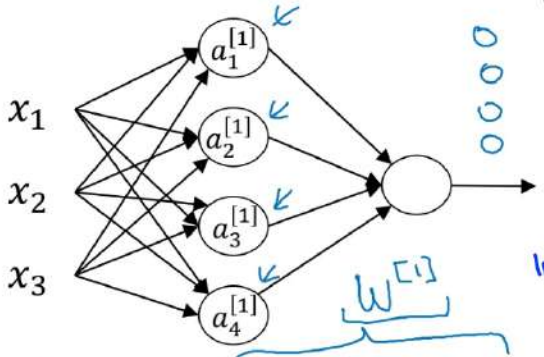
Neural Network Representation



$$z_1^{[1]} = w_1^{[1]T} x + b_1^{[1]}, \quad a_1^{[1]} = \sigma(z_1^{[1]})$$
$$z_2^{[1]} = w_2^{[1]T} x + b_2^{[1]}, \quad a_2^{[1]} = \sigma(z_2^{[1]})$$
$$z_3^{[1]} = w_3^{[1]T} x + b_3^{[1]}, \quad a_3^{[1]} = \sigma(z_3^{[1]})$$
$$z_4^{[1]} = w_4^{[1]T} x + b_4^{[1]}, \quad a_4^{[1]} = \sigma(z_4^{[1]})$$

$$z^{[1]} = \begin{bmatrix} w_1^{[1]T} \\ w_2^{[1]T} \\ w_3^{[1]T} \\ w_4^{[1]T} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} b_1^{[1]} \\ b_2^{[1]} \\ b_3^{[1]} \\ b_4^{[1]} \end{bmatrix} = \begin{bmatrix} w_1^{[1]T} x + b_1^{[1]} \\ w_2^{[1]T} x + b_2^{[1]} \\ w_3^{[1]T} x + b_3^{[1]} \\ w_4^{[1]T} x + b_4^{[1]} \end{bmatrix} = \begin{bmatrix} z_1^{[1]} \\ z_2^{[1]} \\ z_3^{[1]} \\ z_4^{[1]} \end{bmatrix}$$

$$a^{[1]} = \begin{bmatrix} a_1^{[1]} \\ \vdots \\ a_4^{[1]} \end{bmatrix} = \sigma(z^{[1]})$$

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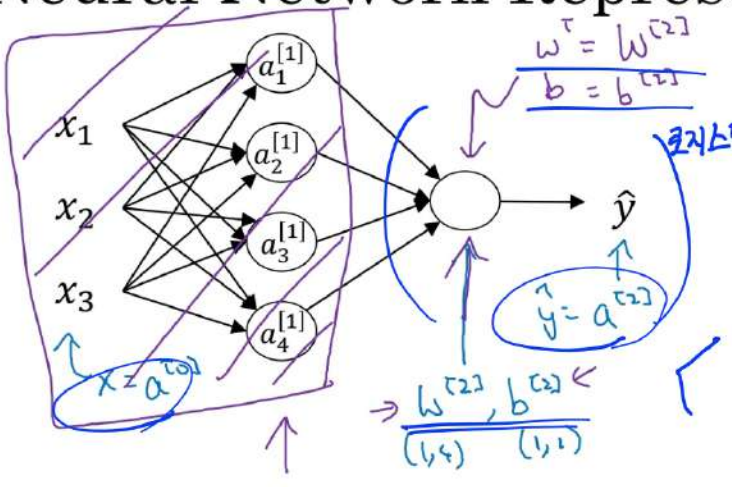
Neural Network Representation learning



Given input x:

$$z^{[1]} = W^{[1]} a^{[0]} + b^{[1]} \quad (4,1) \quad (4,3) \quad (3,1) \quad (4,1)$$
$$a^{[1]} = \sigma(z^{[1]}) \quad (4,1) \quad (4,1)$$
$$z^{[2]} = W^{[2]} a^{[1]} + b^{[2]} \quad (1,1) \quad (1,4) \quad (4,1) \quad (1,1)$$
$$a^{[2]} = \sigma(z^{[2]}) \quad (1,1) \quad (1,1)$$

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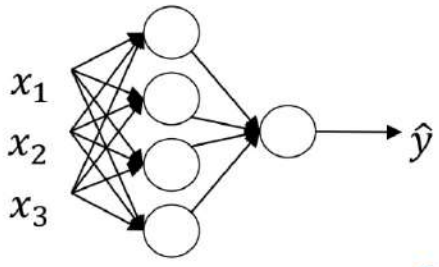
One hidden layer Neural Network

Vectorizing across multiple examples

Vectorizing across multiple examples



$$z^{[1]} = W^{[1]}x + b^{[1]}$$
$$a^{[1]} = \sigma(z^{[1]})$$
$$z^{[2]} = W^{[2]}a^{[1]} + b^{[2]}$$
$$a^{[2]} = \sigma(z^{[2]})$$

$X \rightarrow a^{[2]} = \hat{y}$

$x^{(1)} \rightarrow a^{[2](1)} = \hat{y}^{(1)}$

$x^{(2)} \rightarrow a^{[2](2)} = \hat{y}^{(2)}$

$x^{(m)} \rightarrow a^{[2](m)} = \hat{y}^{(m)}$

$a^{[2](i)}$ ← example i 번째 출력값들

layer 2

for $i = 1$ to m:

$z^{[1](i)} = W^{[1]}x^{(i)} + b^{[1]}$

$a^{[1](i)} = \sigma(z^{[1](i)})$

$z^{[2](i)} = W^{[2]}a^{[1](i)} + b^{[2]}$

$a^{[2](i)} = \sigma(z^{[2](i)})$

$Z^{[1]} = W^{[1]}X + b^{[1]}$

$A^{[1]} = \sigma(Z^{[1]})$

$Z^{[2]} = W^{[2]}A^{[1]} + b^{[2]}$

$A^{[2]} = \sigma(Z^{[2]})$

$X = [x^{(1)}\ x^{(2)}\ \cdots\ x^{(m)}]$ (대문자 입력값들)

n_x, m (입력 개수)

$Z^{[1]} = [z^{[1](1)}\ z^{[1](2)}\ \cdots\ z^{[1](m)}]$

$A^{[1]} = [a^{[1](1)}\ a^{[1](2)}\ \cdots\ a^{[1](m)}]$

training examples

hidden units

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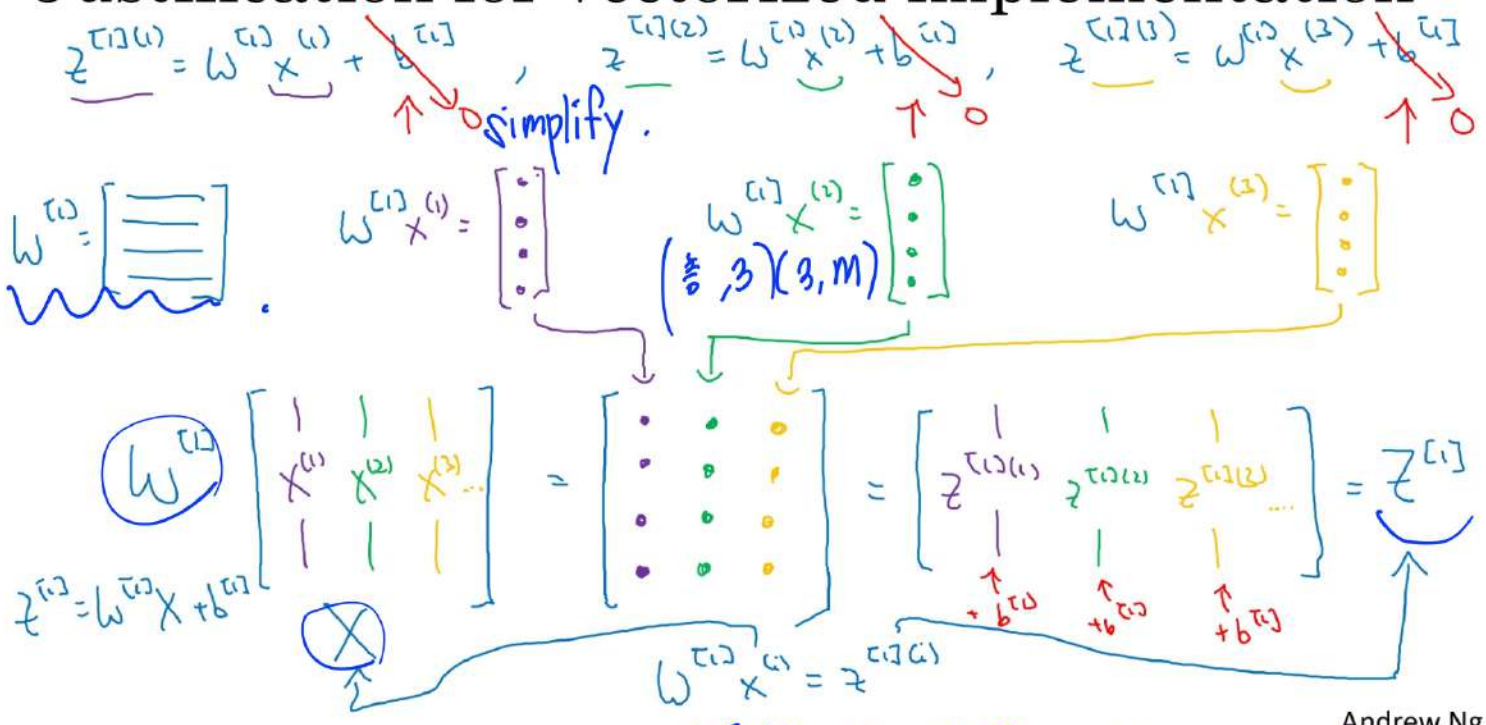
One hidden layer Neural Network

Explanation for vectorized implementation

Justification for vectorized implementation



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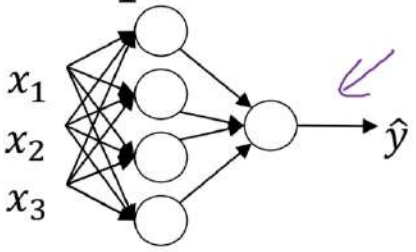
Recap of vectorizing across multiple examples



$$X = \begin{bmatrix} | & | & & | \\ x^{(1)} & x^{(2)} & \cdots & x^{(m)} \\ | & | & & | \end{bmatrix}$$

$$A^{[1]} = \begin{bmatrix} | & | & & | \\ a^{[1](1)} & a^{[1](2)} & \cdots & a^{[1](m)} \\ | & | & & | \end{bmatrix}$$

```
for i = 1 to m
```

$$z^{[1](i)} = W^{[1]}x^{(i)} + b^{[1]}$$
$$a^{[1](i)} = \sigma(z^{[1](i)})$$
$$z^{[2](i)} = W^{[2]}a^{[1](i)} + b^{[2]}$$
$$a^{[2](i)} = \sigma(z^{[2](i)})$$

$$Z^{[1]} = W^{[1]}X + b^{[1]}$$
$$A^{[1]} = \sigma(Z^{[1]})$$
$$Z^{[2]} = W^{[2]}A^{[1]} + b^{[2]}$$
$$A^{[2]} = \sigma(Z^{[2]})$$

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One hidden layer Neural Network

Activation functions

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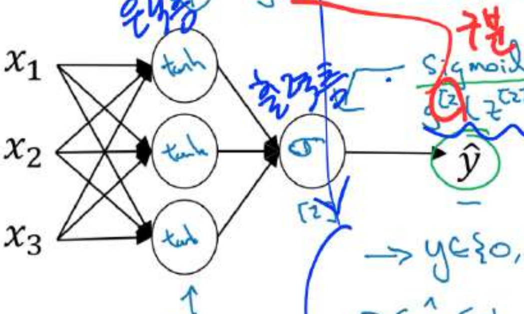
→ 시그모이드 & tanh: z ↑↓ 이면 도함수 굉장히 작아짐 → 0.

Activation functions

시그모이드 활성화 함수를 쓰는 한 가지 예외! 이진 분류할 때 출력층의 σ

은닉 유닛에 대해! 시그모이드보다 대부분의 경우 좋다.



→ $y \in \{0, 1\}$

$0 \le \hat{y} \le 1$

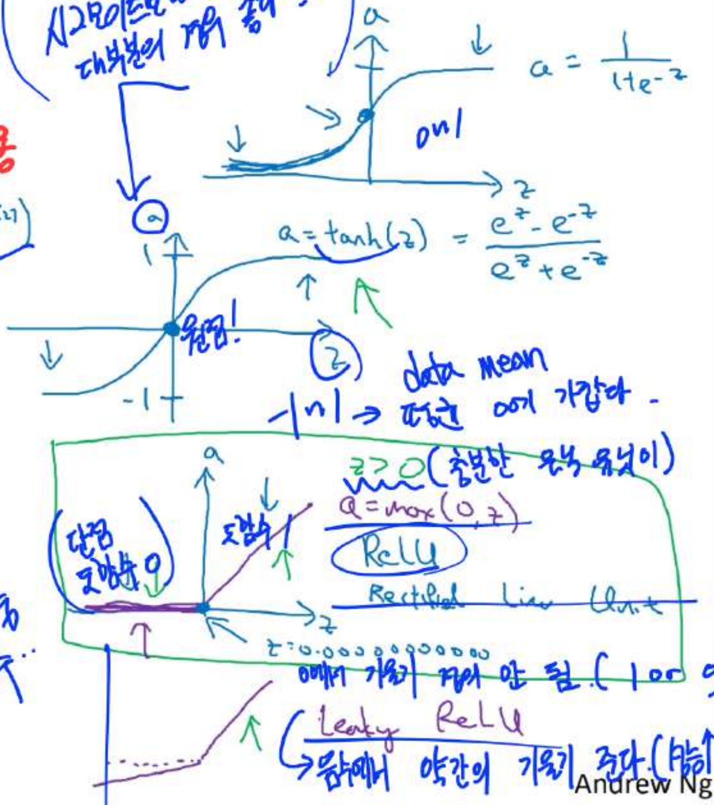
Given x:

$$z^{[1]} = W^{[1]}x + b^{[1]}$$

$$a^{[1]} = \sigma(z^{[1]}) \quad g^{[1]}(z^{[1]})$$ 비선형 함수…

$$z^{[2]} = W^{[2]}a^{[1]} + b^{[2]}$$

$$a^{[2]} = \sigma(z^{[2]}) \quad g^{[2]}(z^{[2]})$$



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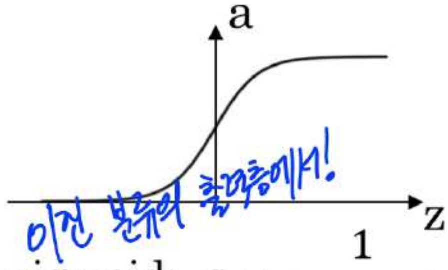
선택… (은닉층 수, 활성화 함수의 종류, 가중치 초기화)

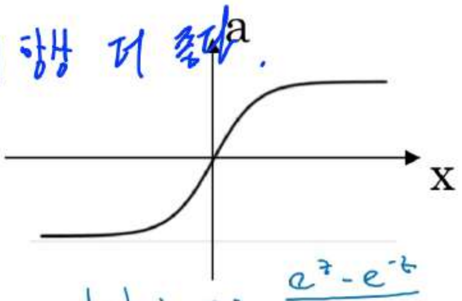
→ 활성화 함수의 기본값! 장점: 대부분의 z에 대해 기울기가 0과 매우 다름 (느린 학습 원인) → 학습 빠름 (tanh, σ 보다)

Pros and cons of activation functions

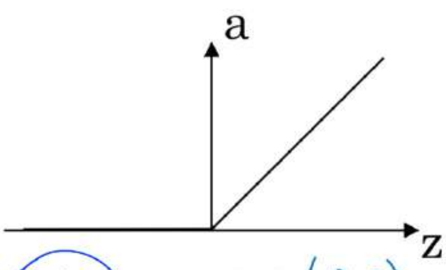


sigmoid: $a = \frac{1}{1 + e^{-z}}$

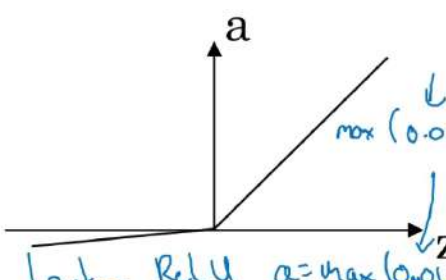
거의 항상 더 좋다.



tanh: $a = \frac{e^z - e^{-z}}{e^z + e^{-z}}$



ReLU $a = \max(0, z)$



Leaky ReLU $a = \max(0.01z, z)$

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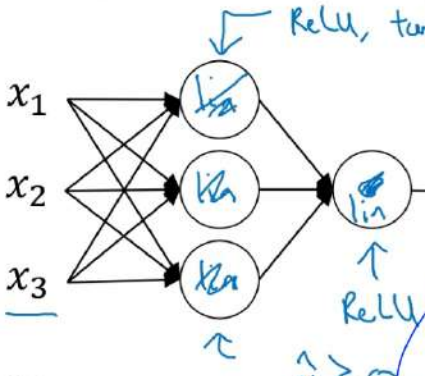
One hidden layer Neural Network

Why do you need non-linear activation functions?

$in \to \sigma$ 가 로지스틱 regression보다 나아지지 않는다!

Activation function

(선형 은닉층은 쓸모가 없다)
선형함수들의 조합은 또 다른 선형함수



$$a^{[1]} = z^{[1]} = W^{[1]}x + b^{[1]}$$

$$a^{[2]} = z^{[2]} = W^{[2]}a^{[1]} + b^{[2]}$$

$$= W^{[2]}\left(W^{[1]}x + b^{[1]}\right) + b^{[2]}$$

$$= \left(W^{[2]}W^{[1]}\right)x + \left(W^{[2]}b^{[1]} + b^{[2]}\right)$$

$$= W'x + b'$$

Given x:

$$z^{[1]} = W^{[1]}x + b^{[1]}$$
$$a^{[1]} = g^{[1]}(z^{[1]}) = z^{[1]}$$
$$z^{[2]} = W^{[2]}a^{[1]} + b^{[2]}$$
$$a^{[2]} = g^{[2]}(z^{[2]}) = z^{[2]}$$

$g(z) = z$
"linear activation function"
항등함수

$g(z) = z$

선형함수 입력이 선형해석만을 출력!

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$\hat{y}$: x에 대한 선형 함수

타겟 문제에 대한 머신러닝을 할 때 y가 실수값이라면 출력층에서 (앞층…?) 선형 activation 함수를 써도 괜찮다!

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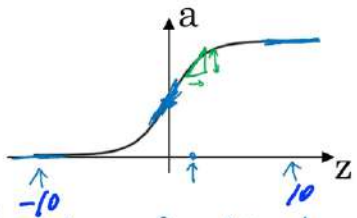
Derivatives of activation functions

Sigmoid activation function



$$g(z) = \frac{1}{1+e^{-z}}$$

$a = g(z) = \frac{1}{1+e^{-z}}$

$g'(z) = \frac{d}{dz} g(z)$ = slope of $g(x)$ at z

$= \frac{1}{1+e^{-z}} \left(1 - \frac{1}{1+e^{-z}}\right)$

$= g(z)\left(1 - g(z)\right)$ ←

$= a(1-a)$

$g'(z) = a(1-a)$

$z = 10$. $g(z) \approx 1$

$\frac{d}{dz} g(z) \approx 1\,(1-1) \approx 0$

$z = -10$ $g(z) \approx 0$

$\frac{d}{dz} g(z) \approx 0 \cdot (1-0) \approx 0$

$z = 0$ $g(z) = \frac{1}{2}$

$\frac{d}{dz} g(z) = \frac{1}{2}\left(1 - \frac{1}{2}\right) = \frac{1}{4}$

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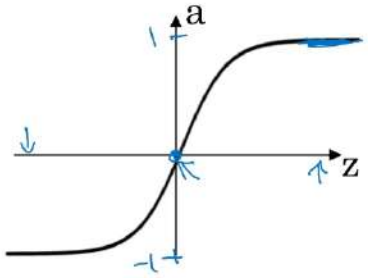
Tanh activation function



$$g(z) = \tanh(z)$$

$= \frac{e^{z} - e^{-z}}{e^{z} + e^{-z}}$

$g'(z) = \frac{d}{dz} g(z)$ = slope of $g(z)$ at z

$= 1 - (\tanh(z))^2$ ←

$a = g(z)$, $g'(z) = 1 - a^2$

$z = 10$ $\tanh(z) \approx 1$

$g'(z) \approx 0$

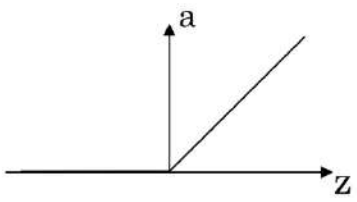
$z = -10$ $\tanh(z) \approx -1$

$g'(z) \approx 0$

$z = 0$ $\tanh(z) = 0$

$g'(z) = 1$

ReLU and Leaky ReLU



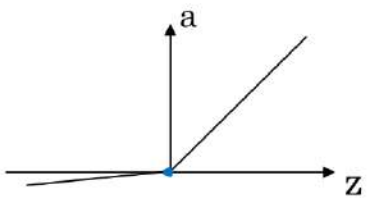
ReLU

$g(z) = \max(0, z)$

$g'(z) = \begin{cases} 0 & \text{if } z < 0 \\ 1 & \text{if } z \geq 0 \end{cases}$

~~undef if $z = 0$~~

$z = 0.000\ldots0$



Leaky ReLU

$g(z) = \max(0.01z, z)$

$g'(z) = \begin{cases} 0.01 & \text{if } z < 0 \\ 1 & \text{if } z \geq 0 \end{cases}$

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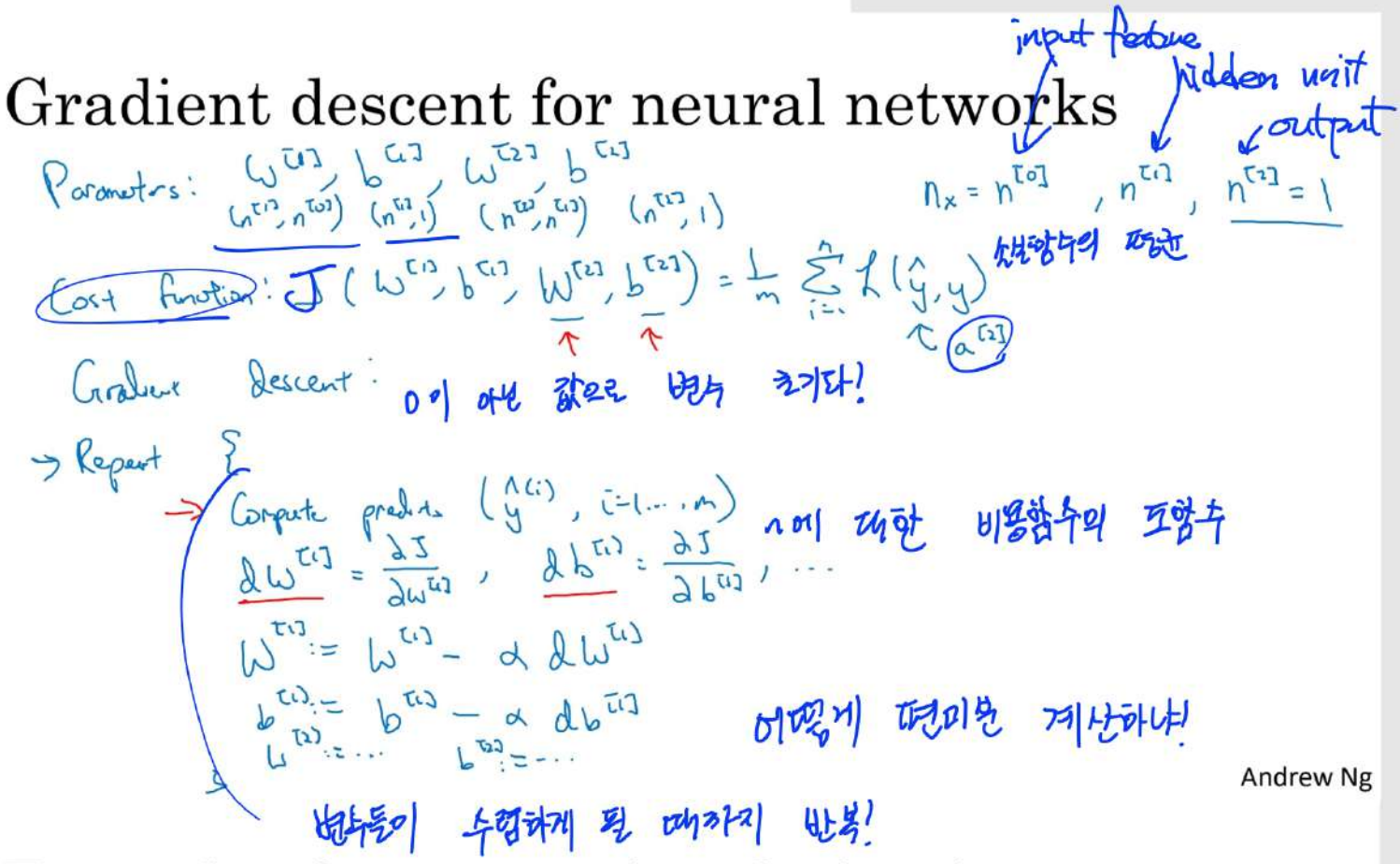
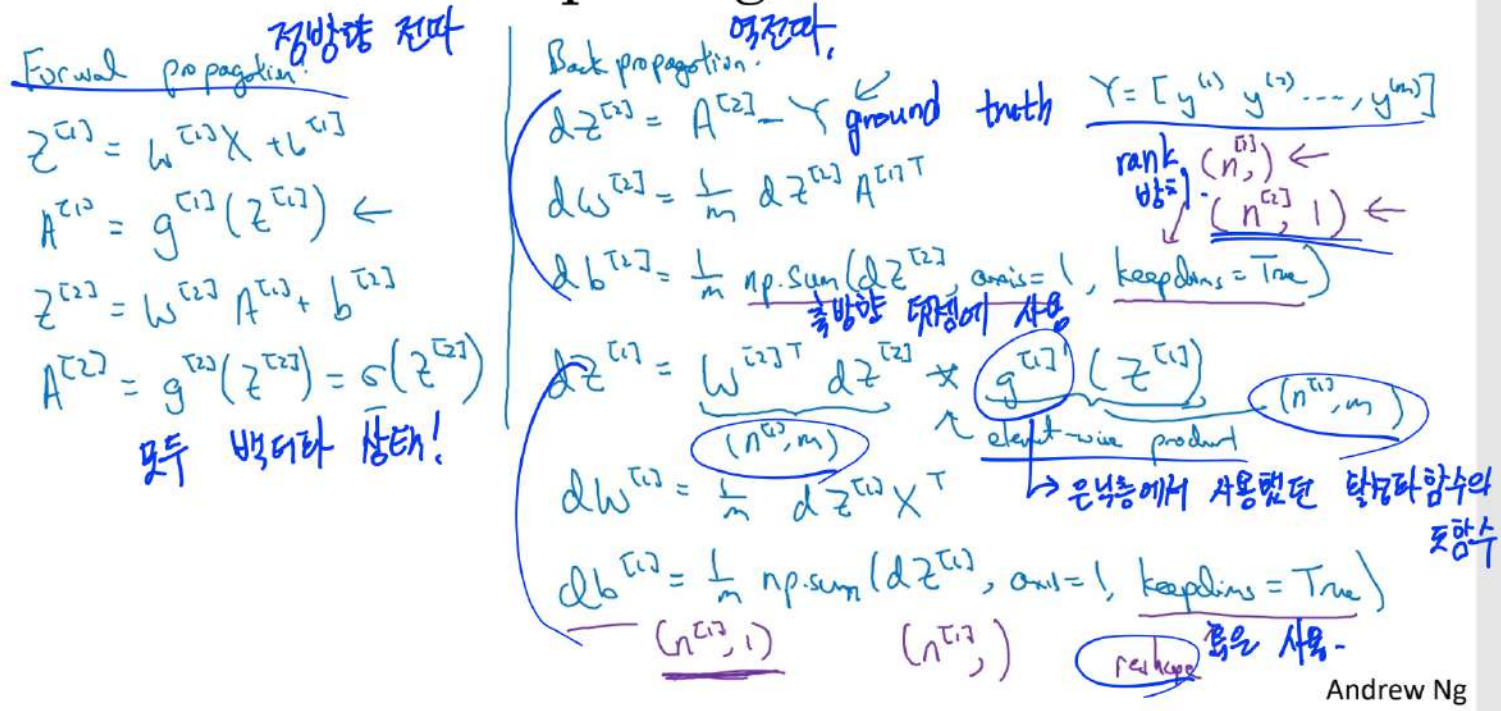
Gradient descent for neural networks



Formulas for computing derivatives



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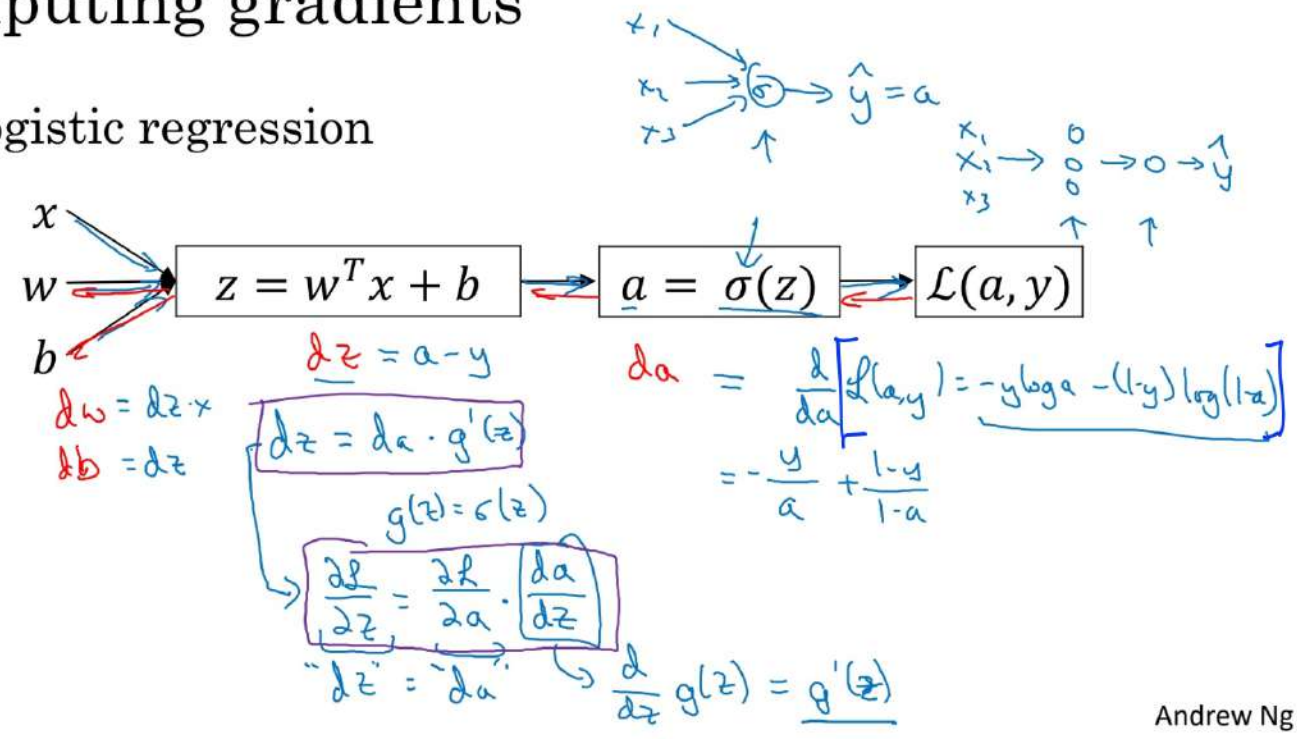
Backpropagation intuition (Optional)

Computing gradients

Logistic regression



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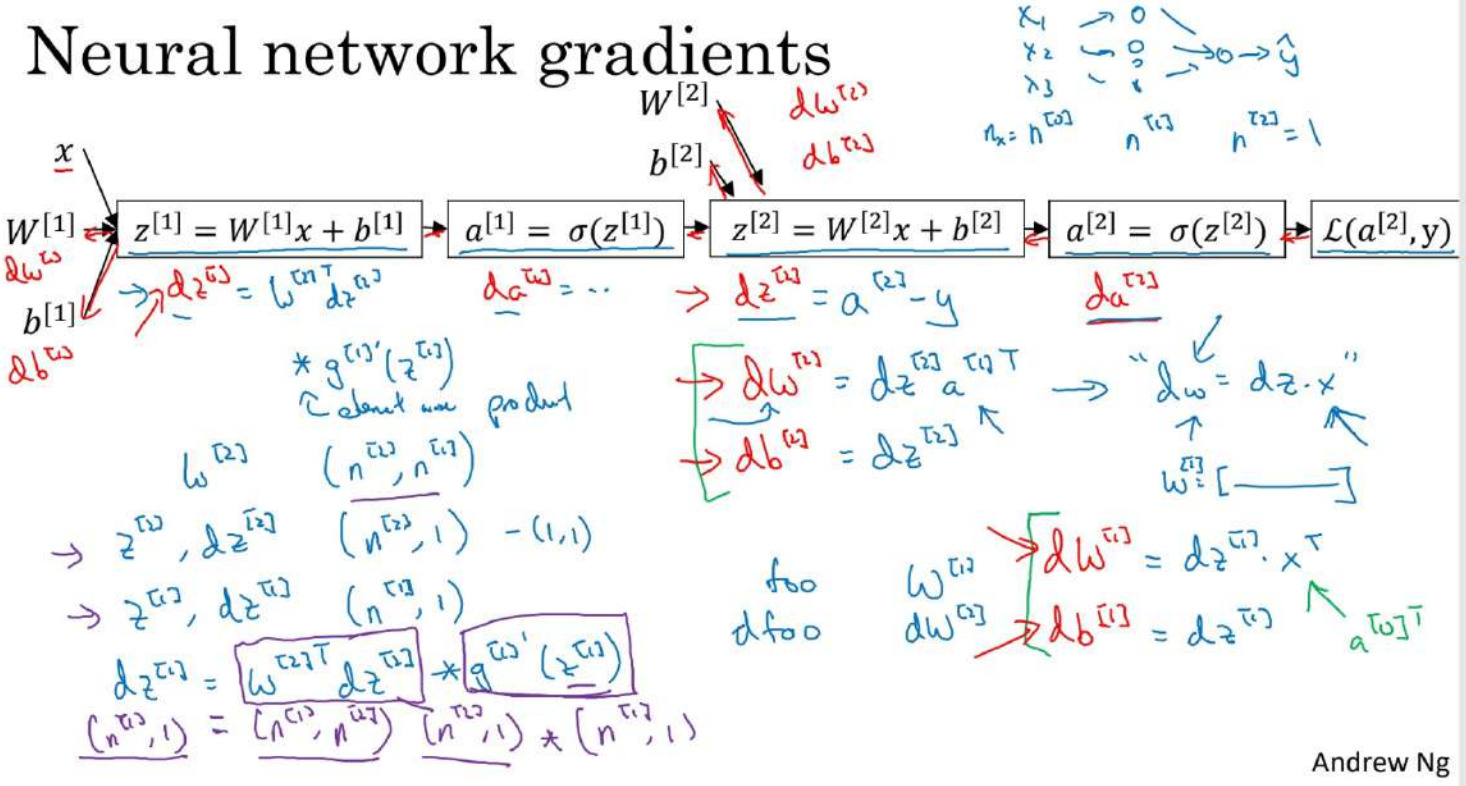
Neural network gradients



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Summary of gradient descent

$dz^{[2]} = a^{[2]} - y$

$dW^{[2]} = dz^{[2]}a^{[1]^T}$

$db^{[2]} = dz^{[2]}$

$dz^{[1]} = W^{[2]T}dz^{[2]} * g^{[1]\prime}(z^{[1]})$

$dW^{[1]} = dz^{[1]}x^T$

$db^{[1]} = dz^{[1]}$

Vectorized Implementation:

$Z^{[1]} = W^{[1]}X + b^{[1]}$

$A^{[1]} = g^{[1]}(Z^{[1]})$

$Z^{[1]} = \begin{bmatrix} | & | & & | \\ z^{[1](1)} & z^{[1](2)} & \cdots & z^{[1](m)} \\ | & | & & | \end{bmatrix}$

$Z^{[1]} = W^{[1]}X + b^{[1]}$

$A^{[1]} = g^{[1]}(Z^{[1]})$

$da = -\frac{y}{a} + \frac{1-y}{1-a}$

$dz = a - y$

$dw = dz\,x$

$db = dz$

x 고정 → dx 계산 완료

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Summary of gradient descent

$dz^{[2]} = a^{[2]} - y$

$dW^{[2]} = dz^{[2]}a^{[1]^T}$

$db^{[2]} = dz^{[2]}$

$dz^{[1]} = W^{[2]T}dz^{[2]} * g^{[1]\prime}(z^{[1]})$

$(n^{[1]}, 1)$

$dW^{[1]} = dz^{[1]}x^T$

$db^{[1]} = dz^{[1]}$

$dZ^{[2]} = A^{[2]} - Y$

$J(\cdot) = \frac{1}{m}\sum_{i=1}^{m}\mathcal{L}(\hat{y}, y)$

$dW^{[2]} = \frac{1}{m}dZ^{[2]}A^{[1]^T}$

$db^{[2]} = \frac{1}{m}np.sum(dZ^{[2]}, axis = 1, keepdims = True)$

$dZ^{[1]} = W^{[2]T}dZ^{[2]} * g^{[1]\prime}(Z^{[1]})$

$(n^{[1]}, m)$ $(n^{[1]}, m)$ $(n^{[1]}, m)$

elementwise product

$dW^{[1]} = \frac{1}{m}dZ^{[1]}X^T$

$db^{[1]} = \frac{1}{m}np.sum(dZ^{[1]}, axis = 1, keepdims = True)$

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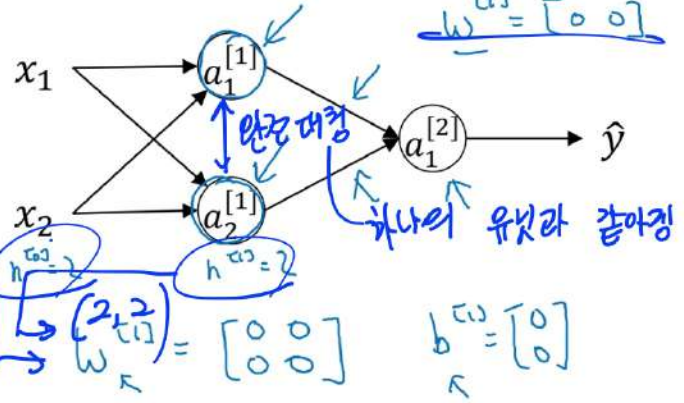
One hidden layer Neural Network

Random Initialization

What happens if you initialize weights to zero?



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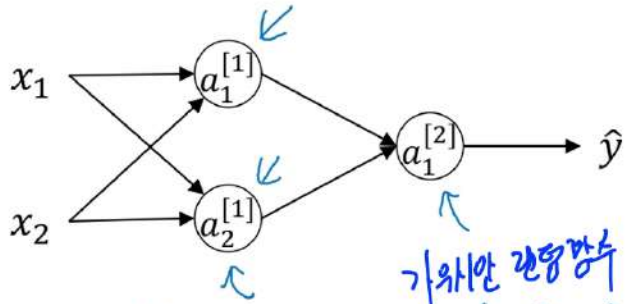
Random initialization



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