**Problem 1:**

1. False, a random variable that is discrete has PMF (Probability Mass Function). It is a statistical function that describes the probabilities of discrete random variables assigning a probability to each possible outcome.
2. True, an individual probably value in a probability mass function must be between 0 and 1 inclusive. However, the sum of all the probabilities must equal 1.
3. False, the probability density function is the describing the likelihood of a continuous random variable taking on a particular value or range of values. The set of all possible realizations (outcomes) is the sample space.
4. False, the expected value E[X] of a discrete random variable is no part of its support Rx​, which is the set of values that the random variable can take with nonzero probability.
5. True, you can imagine a continuous random variable X representing height of a random person. X maps people to real number (their height). The continuous aspect is that the mapped height could be between a range with an infinite number of measured heights.
6. False, the statement is reversed.
7. True, the is because in a continuous random variable there are an infinite number of possibilities.

**Problem 2:**

1. My current expectation is that the market will go up. Personally, I would not invest in this market. If we do it based on expected value, we find that the resulting value is -1.2. We can calculate this by using the discrete values that we are given. There are only two probabilities with their given outcome.

One limitation of expectations is that it doesn’t account for risk. You could most likely make a return on your investment, however if you end up taking a loss there is no guarantee of recuperation of your losses in the future. We also do not know have variable the chances are.

**Problem 3:**

1. Prove
2. Prove
3. Affine transformation

**Problem 4**

**Problem 5**

**The Case of the Missing Blueprint**

Professor Alden had been working on groundbreaking energy-efficient power system design, but one morning, the key blueprint vanished from his secure lab. Only three people had access:

* Dr. Evelyn Carter – A meticulous researcher who was last seen in the lab at 7:00 PM.
* Mark Dawson – A struggling graduate student who left at 5:30 PM.
* Sarah Lin – A lab technician who left at 6:15 PM.

The only evidence available: The lab’s security system detected unauthorized access at 8:00 PM.

**Prior Distribution**

Before considering the security log, we assign prior probabilities based on access and potential motivation:

* Dr. Evelyn Carter: 30% (She had access and expertise.)
* Mark Dawson: 50% (He was struggling financially and may have had a motive.)
* Sarah Lin: 20% (She had access but no known motive.)

**Likelihood (Security Log Evidence)**

We define the likelihood P (Evidence | Suspect) as the probability that a suspect could have accessed the lab at 8:00 PM:

* Dr. Evelyn Carter: 0.8 (She was the last one seen near the lab.)
* Mark Dawson: 0.3 (He left much earlier but could have returned.)
* Sarah Lin: 0.4 (She had access but was not seen near the lab.)