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University of Applied Sciences
Groningen

Digital Signal Processing

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Today

- Some additional stuff regarding last time
  - Negative frequencies
  - Spectral leakage
- Nyquist Frequency and Aliasing
- Exercises

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Negative Frequencies

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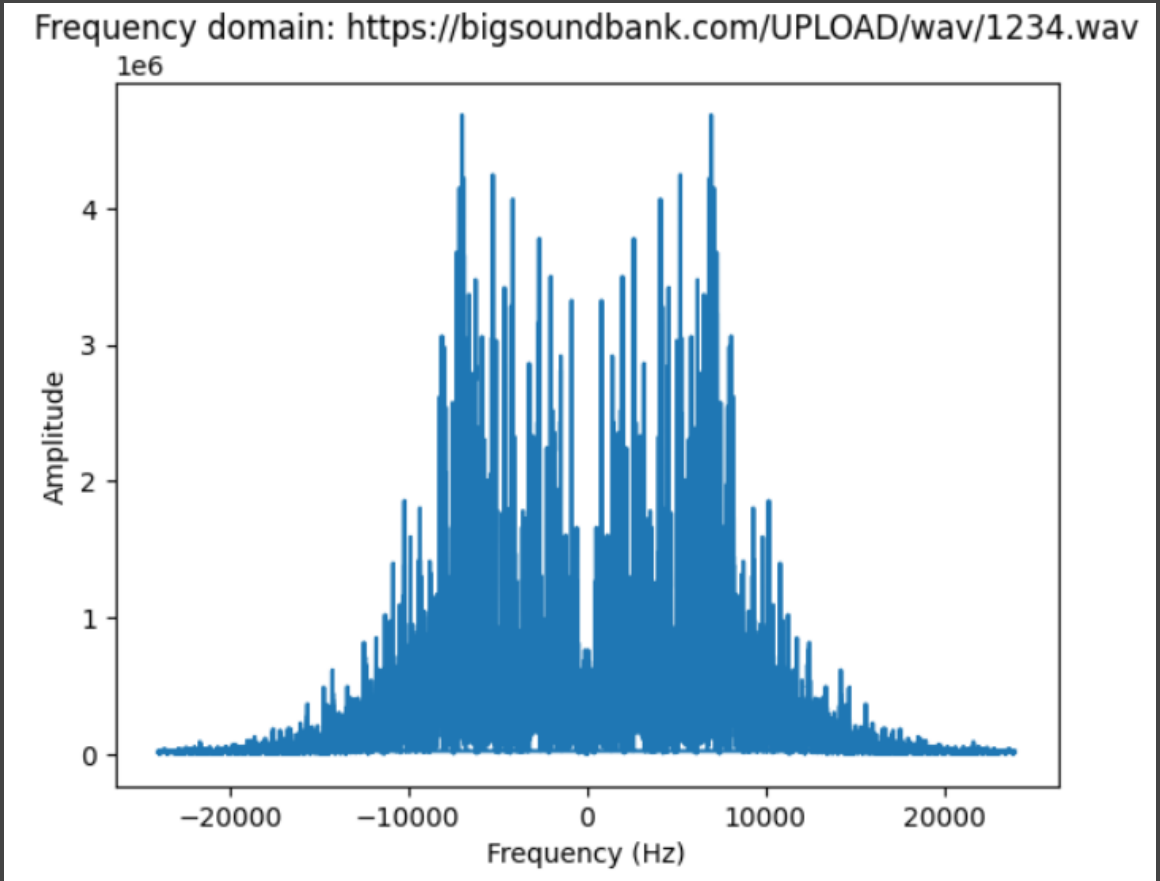
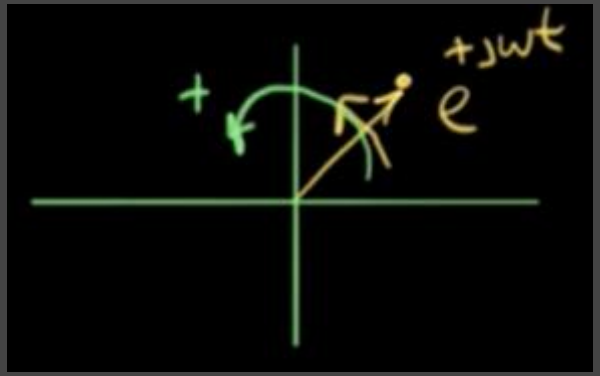

Frequency domain: https://bigsoundbank.com/UPLOAD/wav/1234.wav
1e6
4
3
2
1
0
Amplitude
−20000
−10000
0
10000
20000
Frequency (Hz)

Source: https://www.youtube.com/watch?v=SvFfVtRT1UI



Source: https://www.youtube.com/watch?v=SvFfVtRT1UI

https://www.khanacademy.org/math/algebra-home/alg-complex-numbers

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Spectral Leakage

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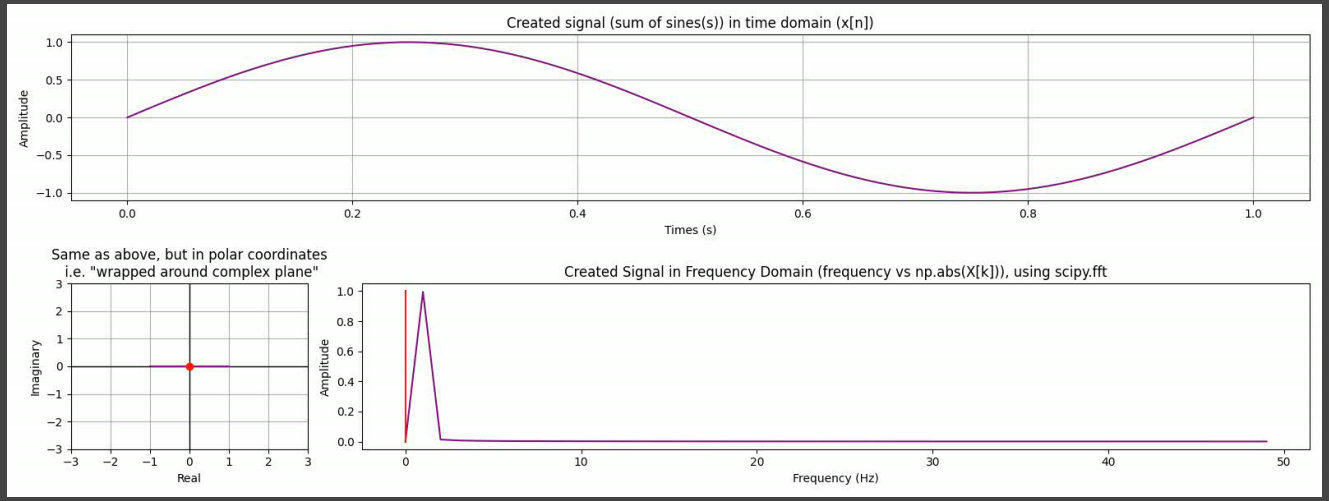
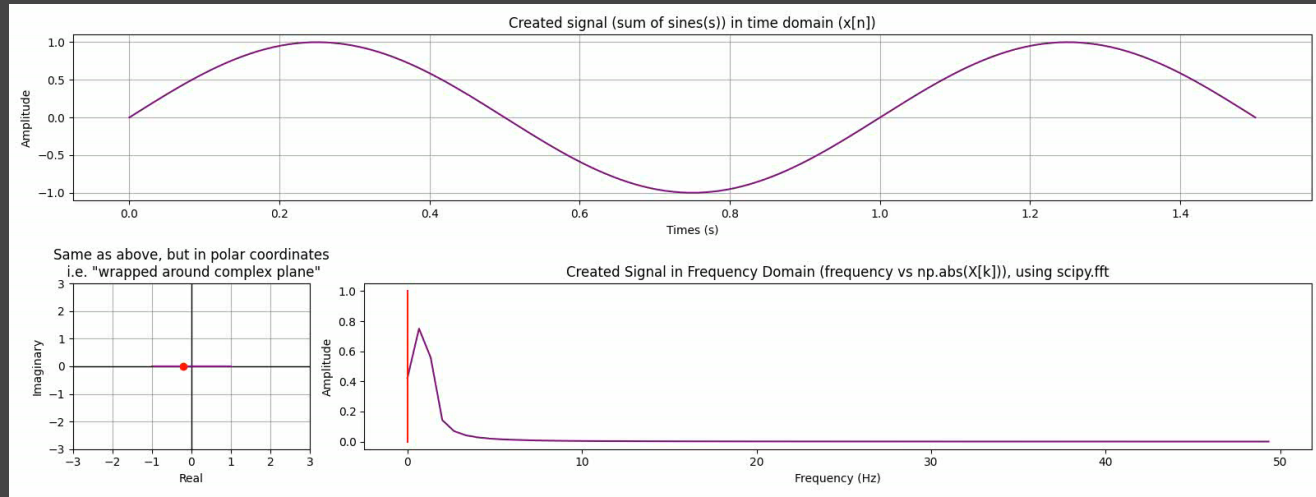

Created signal (sum of sines(s)) in time domain (x[n])
Amplitude
Times (s)
Same as above, but in polar coordinates
i.e. "wrapped around complex plane"
Imaginary
Real
Created Signal in Frequency Domain (frequency vs np.abs(X[k])), using scipy.fft
Amplitude
Frequency (Hz)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Amplitude
Times (s)
Same as above, but in polar coordinates
i.e. "wrapped around complex plane"
Imaginary
Real
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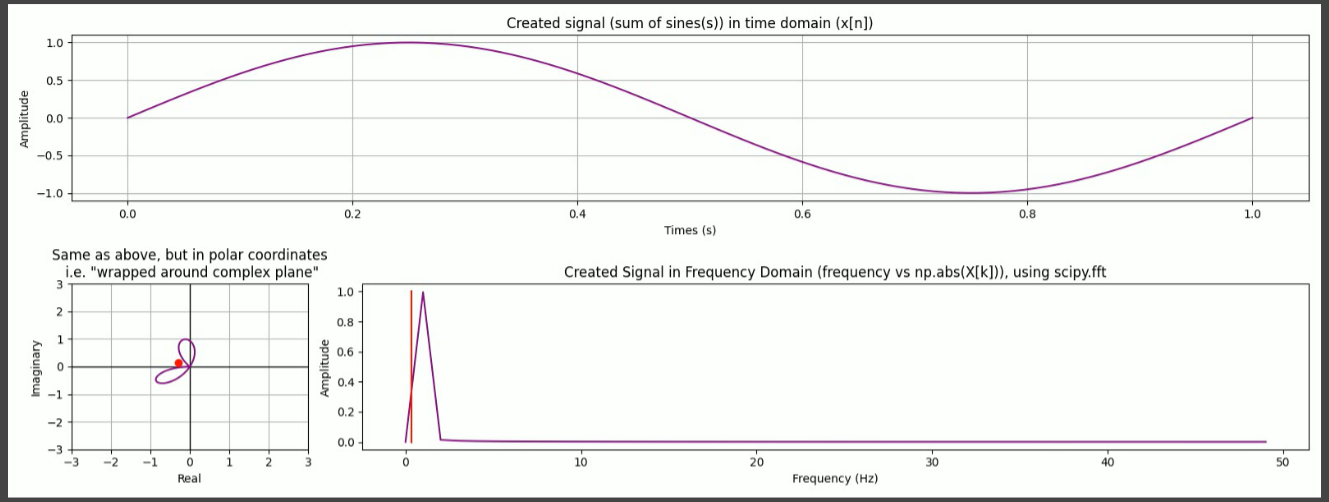
Amplitude
Frequency (Hz)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Amplitude
Times (s)
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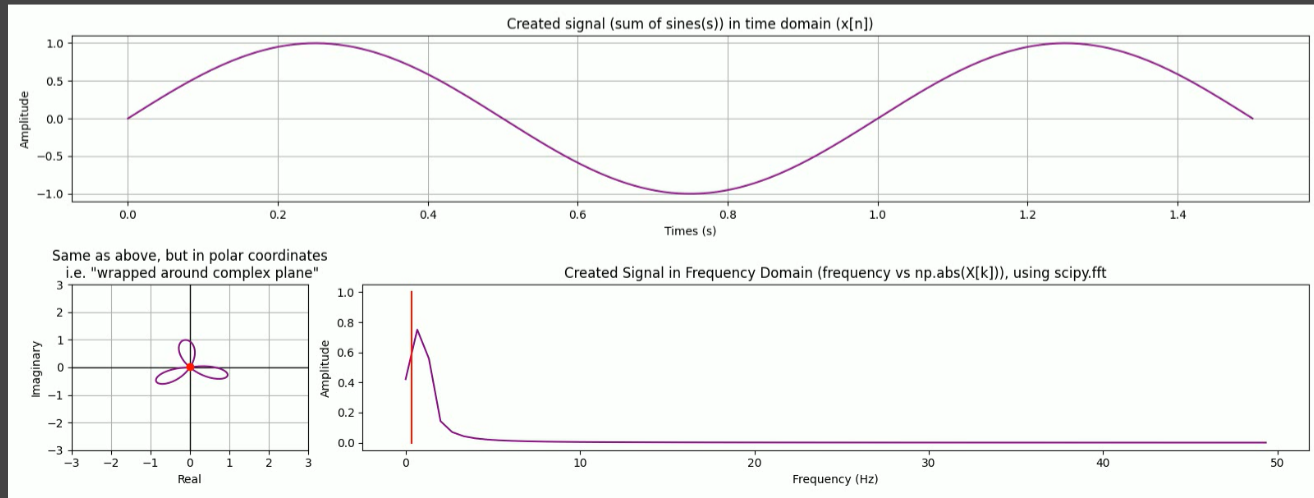
Amplitude
Frequency (Hz)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Amplitude
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Amplitude
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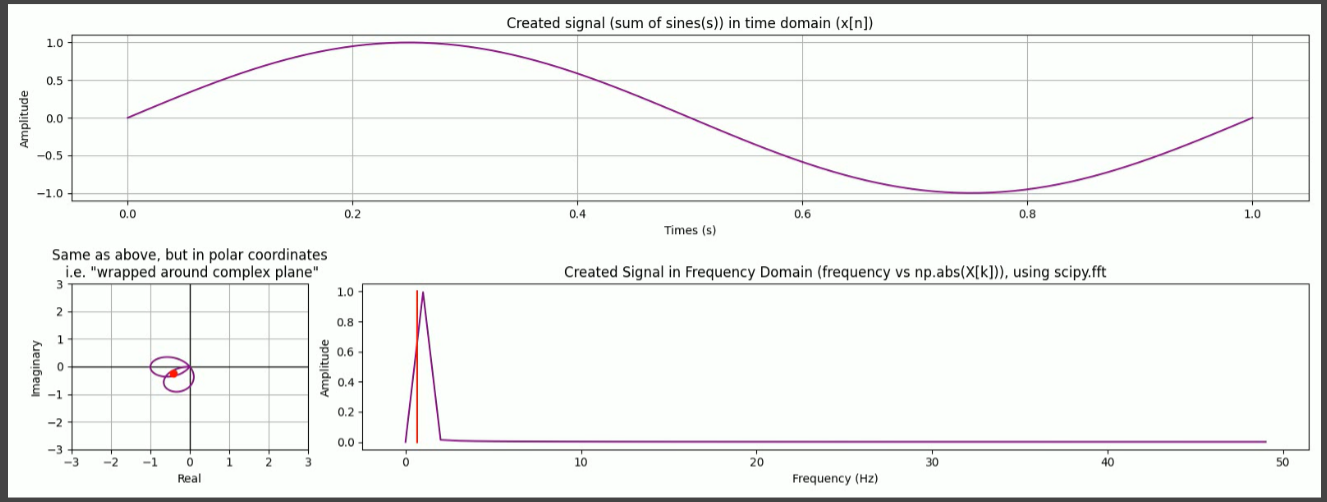
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Amplitude
Times (s)
Same as above, but in polar coordinates
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Real
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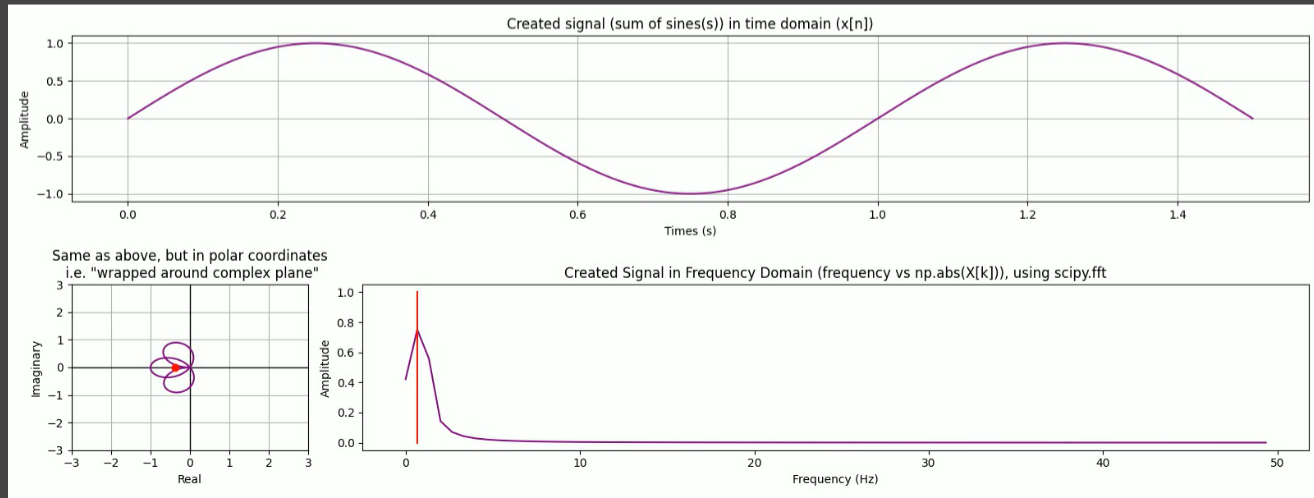
Amplitude
Frequency (Hz)


Created signal (sum of sines(s)) in time domain (x[n])
Amplitude
Times (s)
Same as above, but in polar coordinates
i.e. "wrapped around complex plane"
Imaginary
Real
Created Signal in Frequency Domain (frequency vs np.abs(X[k])), using scipy.fft
Amplitude
Frequency (Hz)

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Nyquist Frequency & Aliasing

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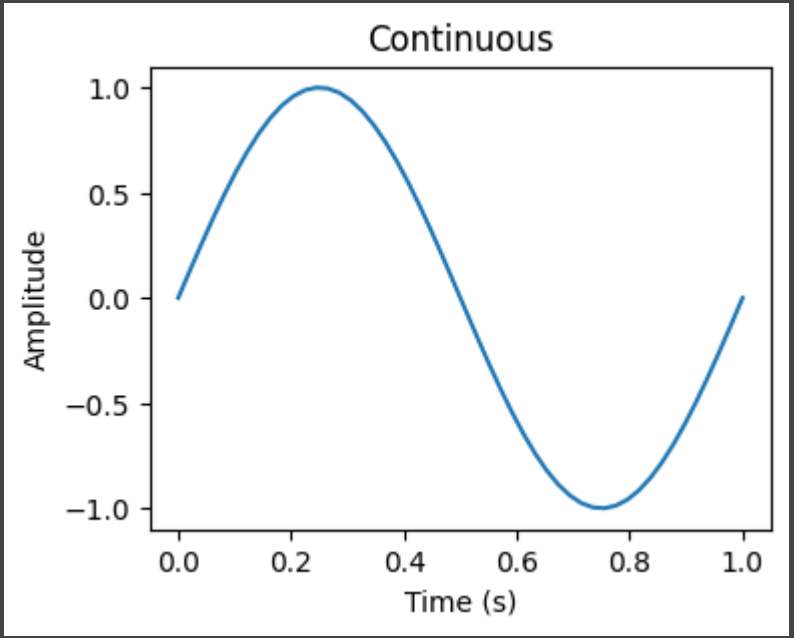
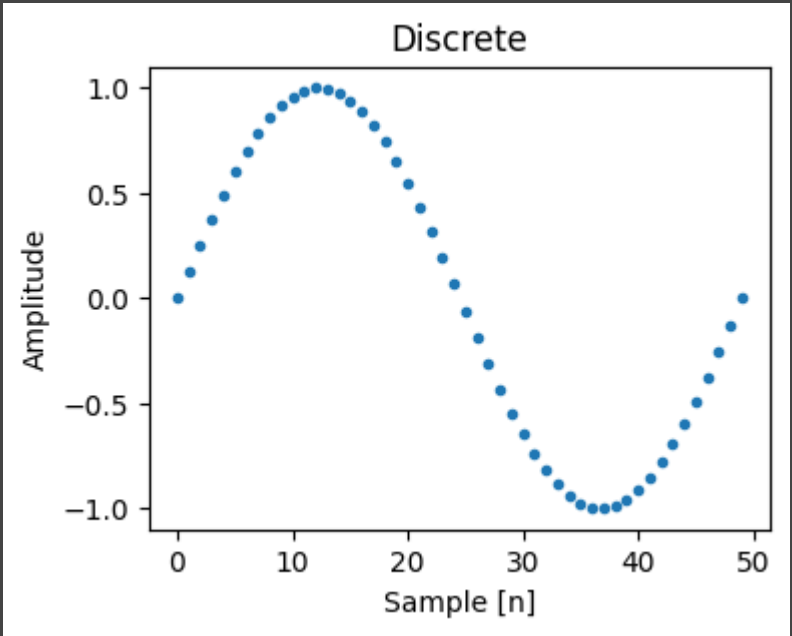
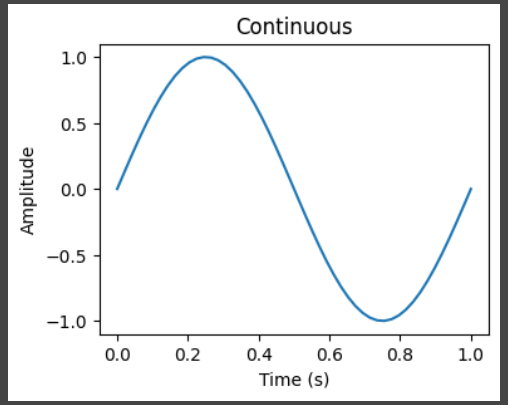
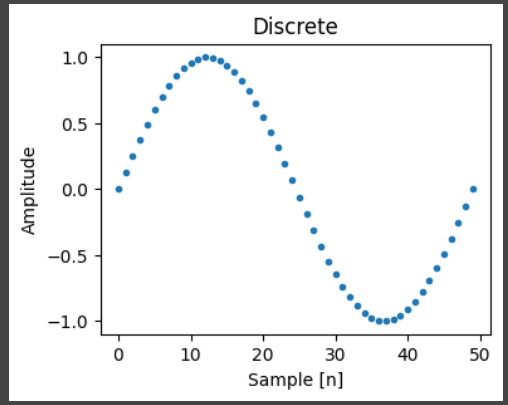

Continuous
1.0
0.5
0.0
−0.5
−1.0
Amplitude
0.0
0.2
0.4
0.6
0.8
1.0
Time (s)

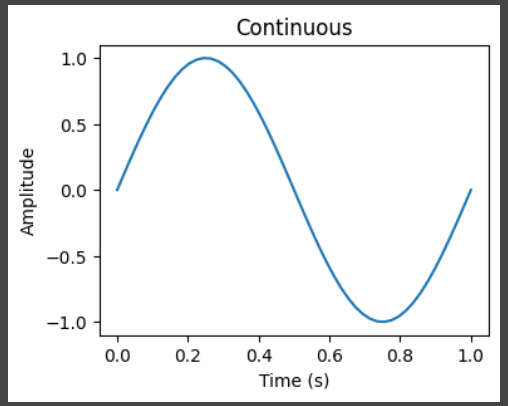

Discrete
1.0
0.5
0.0
−0.5
−1.0
Amplitude
0
10
20
30
40
50
Sample [n]



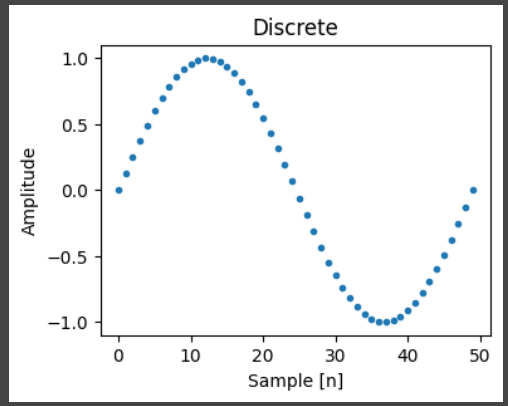
Analog to Digital Converter (ADC)



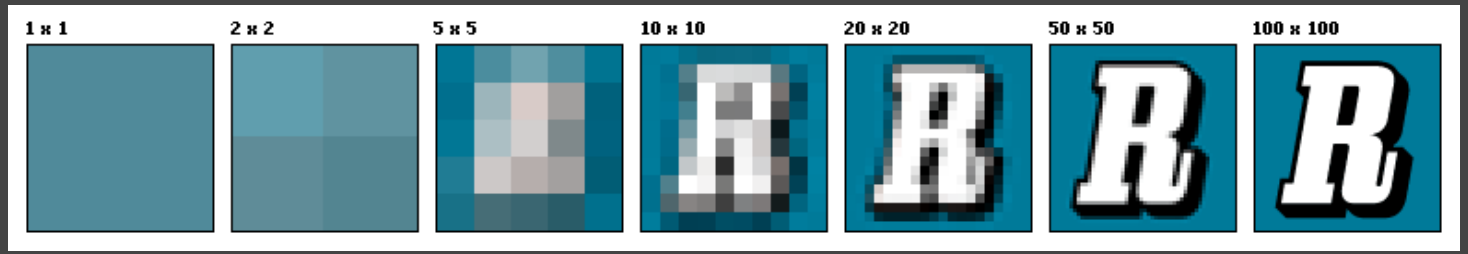
Sample rate



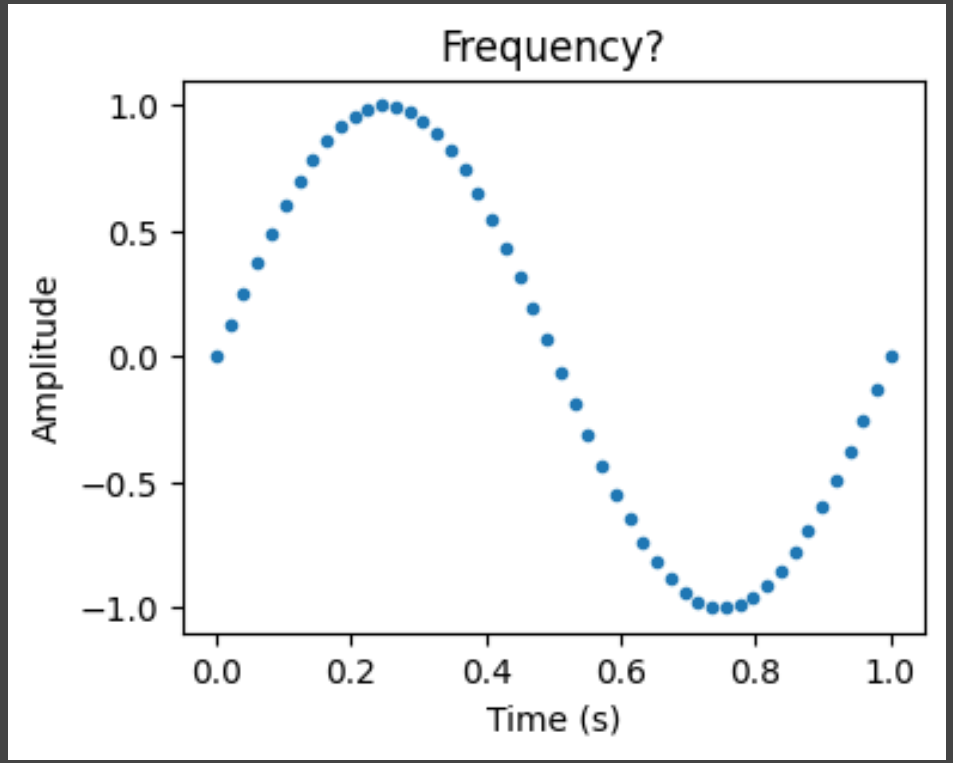
Analog to Digital Converter (ADC)

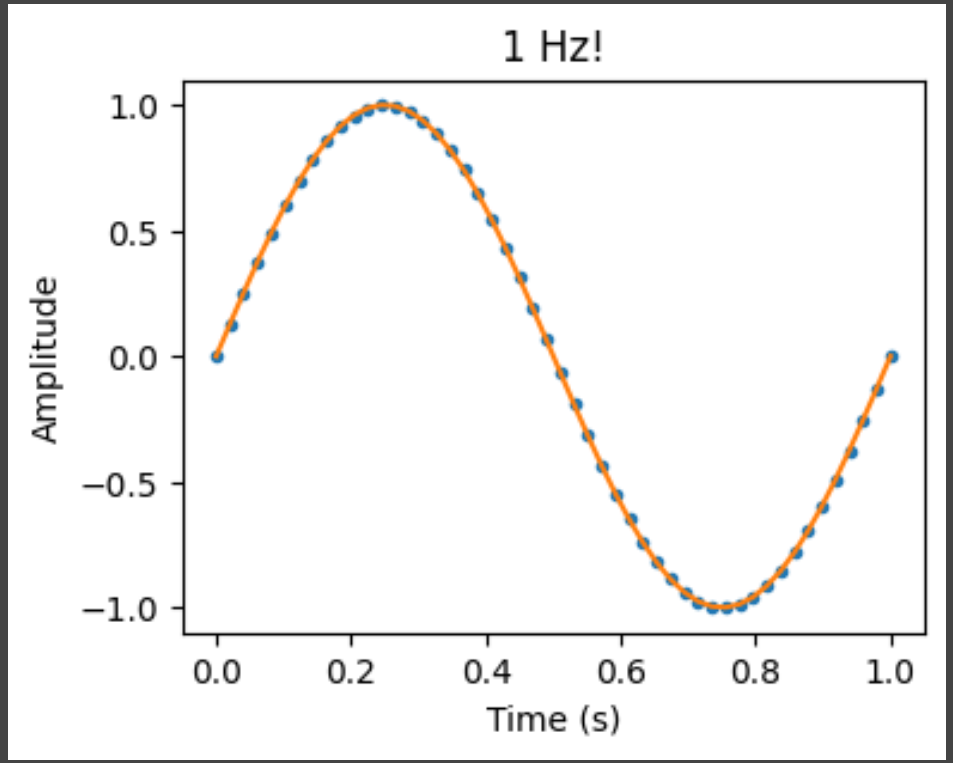


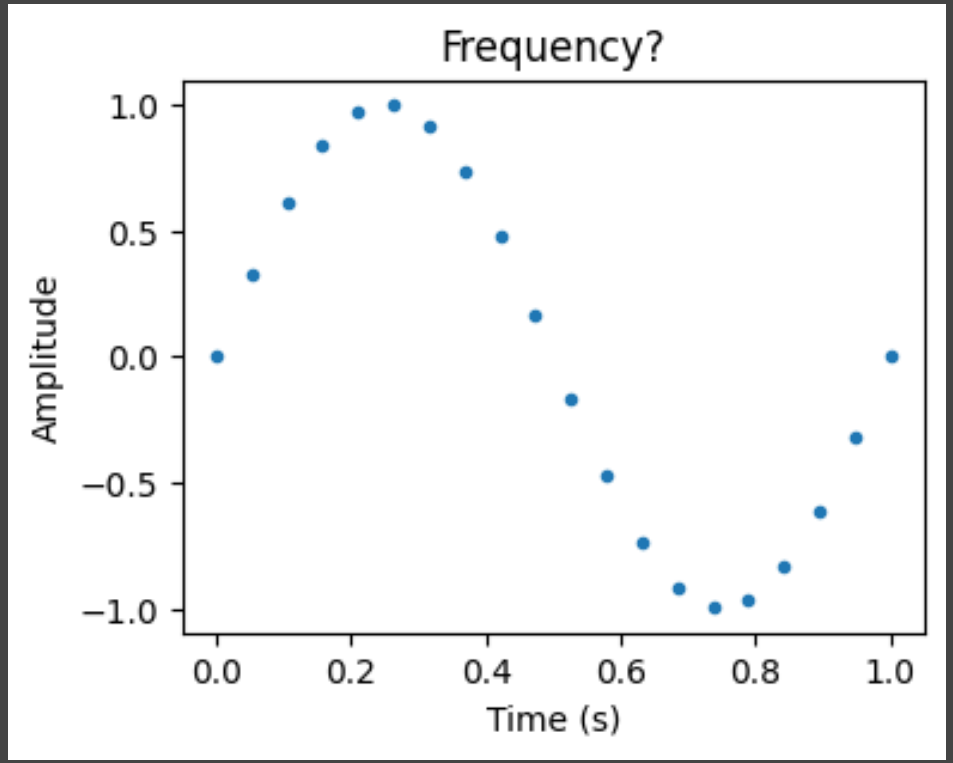
Resolution

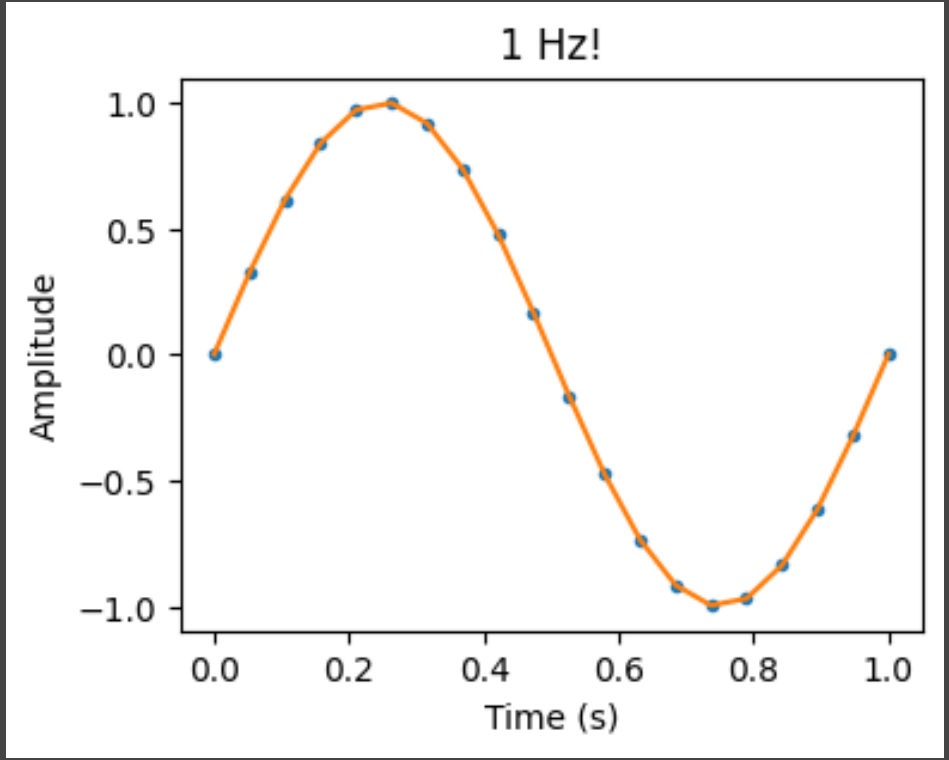


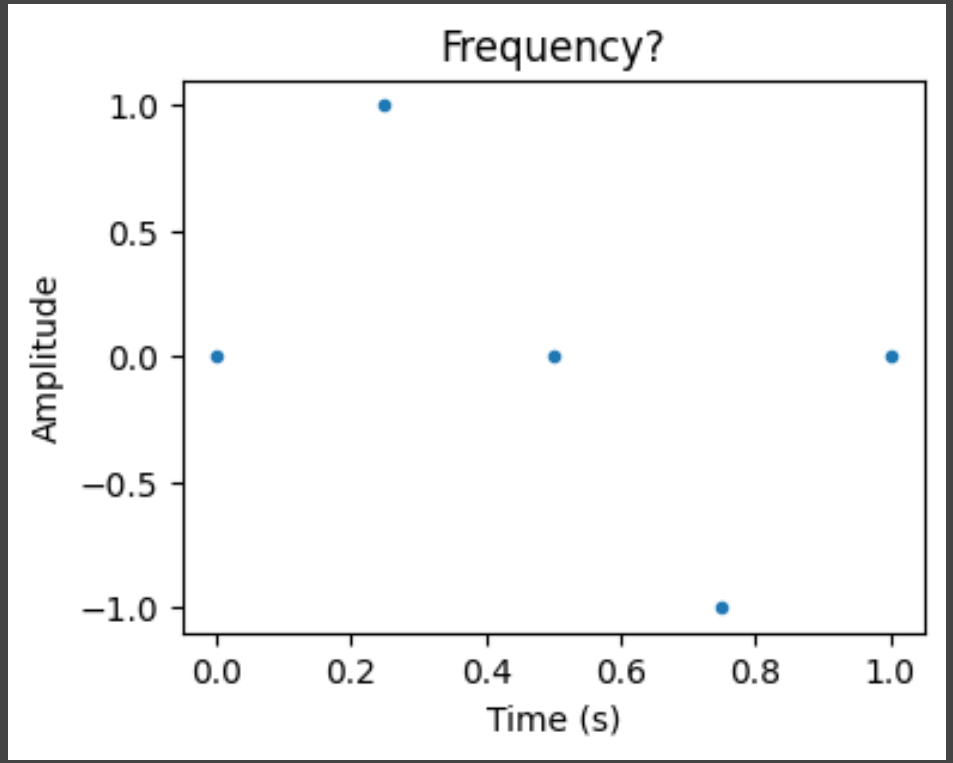
Source: https://commons.wikimedia.org/wiki/File:Resolution_illustration.png

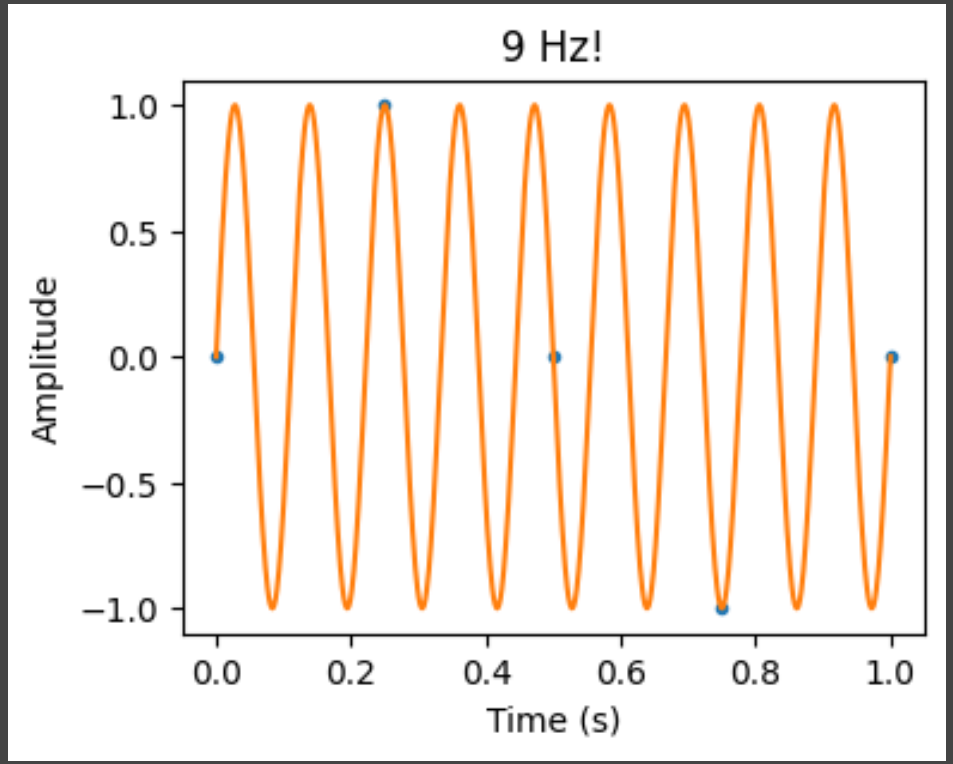

Frequency?
1.0
0.5
0.0
−0.5
−1.0
Amplitude
0.0
0.2
0.4
0.6
0.8
1.0
Time (s)

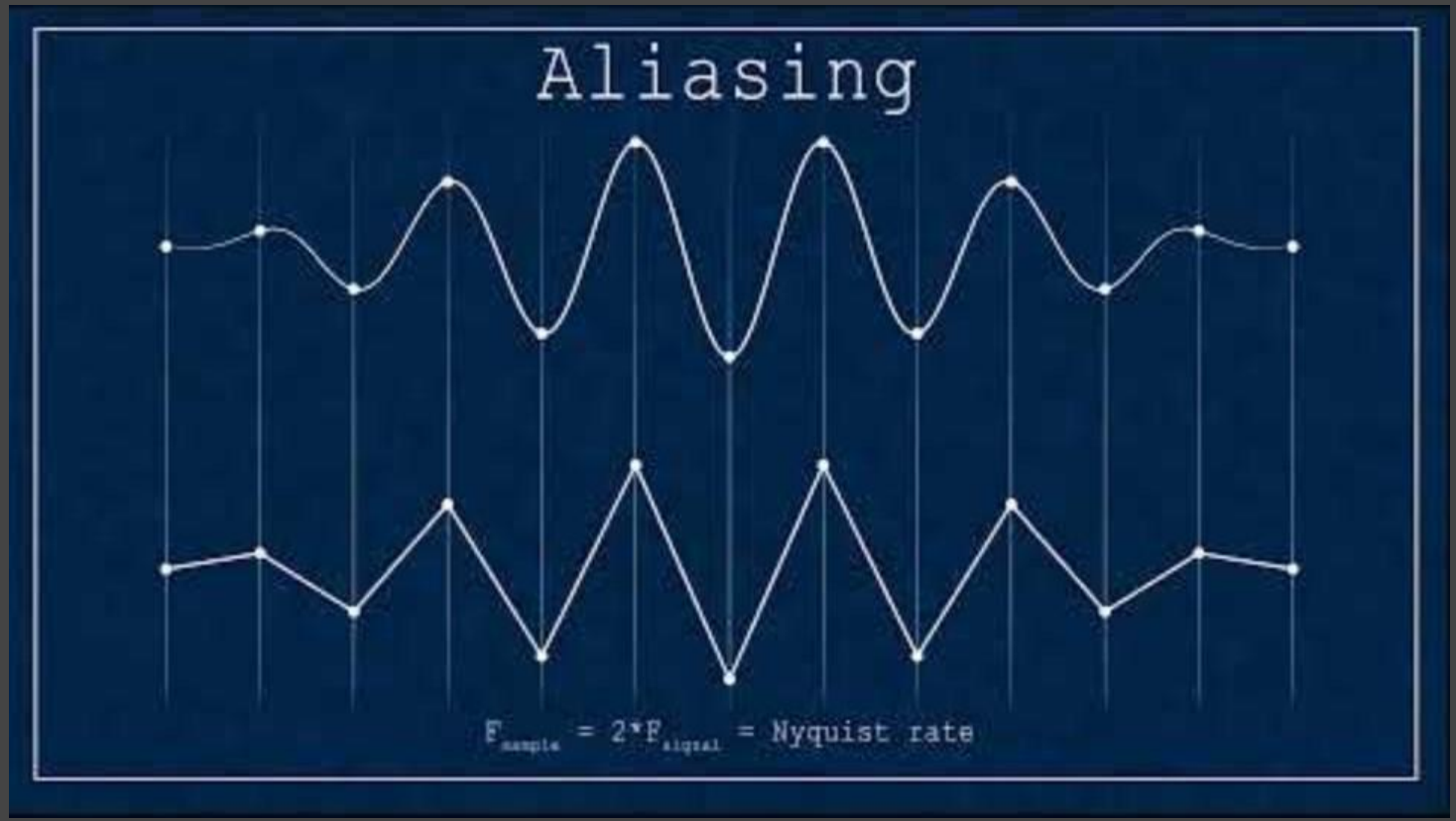

1 Hz!
Amplitude
1.0
0.5
0.0
−0.5
−1.0
0.0
0.2
0.4
0.6
0.8
1.0
Time (s)


Frequency?
Amplitude
1.0
0.5
0.0
−0.5
−1.0
0.0
0.2
0.4
0.6
0.8
1.0
Time (s)


1 Hz!
Amplitude
1.0
0.5
0.0
−0.5
−1.0
0.0
0.2
0.4
0.6
0.8
1.0
Time (s)


Frequency?
1.0
0.5
0.0
−0.5
−1.0
Amplitude
0.0
0.2
0.4
0.6
0.8
1.0
Time (s)


9 Hz!
1.0
0.5
0.0
−0.5
−1.0
Amplitude
0.0
0.2
0.4
0.6
0.8
1.0
Time (s)



Source: https://youtu.be/CTIICx5pHvM?si=TRD44vsEh_-Y7E3u

$$f_{Nyquist} = \frac{1}{2} f_s$$

“The Nyquist frequency is the highest frequency that can be present in a discrete signal, at a given sampling rate, without causing aliasing.”

MP3 default sample rate is 44.1kHz, why?

We found 48kHz previously, but that’s besides the point.

$f_s = 44.1\text{kHz}$

Exercises!