The Power Rule

FOCUS: Using a formula (shortcut) to determine the derivative that eliminates the tedious and time consuming method that requires calculating limits from first principles.

What Does $\lim\limits_{h \to 0} \frac{f(a+h)-f(a)}{h}$ represent?

→ Represents the slope of the tangent to the curve f(x) where x = a.
→ Represents the instantaneous rate of change of f(x) where x = a

OR

→ **The derivative of f(x) at x = a, written as f'(a) or $\frac{d}{dx}f(a)$**

Recall, the derivative of f(x) with respect to x is $f'(x) = \lim\limits_{h \to 0} \frac{f(a+h)-f(a)}{h}$, provided that the limit exists.

Lets derive a formula that can be used to determine the derivative function f(x) = x^n

$$f'(x) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$

$$= \lim_{h \to 0} \frac{(x+h)^n - x^n}{h}$$

$$= \lim_{h \to 0} \frac{(x+h-x)[(x+h)^{n-1}\cancel{a}x + (x+h)^{n-2}x^{1} + \cdots (x+h)x^{n-2} + x^{n-1}]}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{(h)}[(x+h)^{n-1} + (x+h)^{n-2}x + \cdots (x+h)x^{n-2} + x^{n-1}]}{\cancel{h}}$$

$$= \lim_{h \to 0}(x+h)^{n-1} + (x+h)^{n-2}x + \cdots (x+h)x^{n-2} + x^{n-1}$$

$$= x^{n-1} + x^{n-2}x^{1} + \cdots (x)x^{n-2} + x^{n-1}$$

$$= x^{n-1} + x^{n-1} + \cdots x^{n-1} + x^{n-1}$$

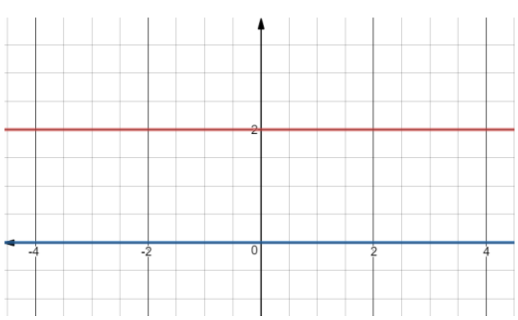
$$= nx^{n-1}$$

We will use this formula:
$x^n - a^n = (x-a)(x^{n-1} + x^{n-2}a + \ldots + xa^{n-2} + a^{n-1})$,
where ***x* is *x + h*** and ***a* is *x***

$(x-a)^n \neq x^n - a^n$

THE POWER RULE
If f(x) = cx^n, where n is a real number, then
Coefficient **f'(x) = cnx^{n-1}**

What is the derivative of a constant function? Ex. f(x) = 2



Thought Experiment:
Are the derivatives of constant functions unique if the constant functions themselves are unique?

$m = \frac{\Delta y}{\Delta x}$ $y = x^2$ a(b)

$y' = 2x$ $\frac{dy}{dx}$ $\frac{d}{dx}y$ f(x)

Ex 1.
Determine the derivative of the following.

a) f(x) = –7 $f'(x) = 0$

b) y = 23π $\frac{dy}{dx} = \frac{d}{dx}y = y' = 0$

If f is a constant funtion, f(x) = c, then f'(x) = 0 or $\frac{d}{dx}C = 0$

$\boxed{\sqrt[n]{x} = x^{\frac{1}{n}}}$ $\boxed{\sqrt[n]{x^a} = x^{\frac{a}{n}}}$ $\boxed{\frac{x^n}{1} = \frac{1}{x^{-n}} \text{ or } \frac{x^{-n}}{1} = \frac{1}{x^n}}$

Ex 2.
Determine the derivative.

a) f(x) = x^4

$f'(x) = 4x^{4-1}$

$= 4x^3$

b) s = t^{-3}

$s' = -3t^{-3-1}$

$= -3t^{-4}$

c) y = 3

$\frac{d}{dx}y = 0$

$y' = 0$

d) $v = t^{\frac{5}{2}}$

$\frac{d}{dt}v = \frac{5}{2}t^{\frac{5}{2}-1}$

$= \frac{5}{2}t^{\frac{3}{2}}$

e) $f(x) = \frac{-1}{x^2}$ $\frac{3^{1}a^{3}b^{2}}{c^{5}d^{-4}} = \frac{b^{2}c^{-5}d^{4}}{3^{-1}a^{-3}}$

$f(x) = -1x^{-2}$

$f'(x) = (-1)(-2)x^{-2-1}$

$= 2x^{-3} = \frac{2}{1} x^{+3}$

$= \frac{2}{x^3}$

Thought Experiment:

1. Determine the values of x so that the tangent to the function $y = \frac{3}{\sqrt[3]{x}}$ is parallel to the line x + 16y + 3 = 0. $m = -\frac{1}{16}$

Rewrite y using exponent rules

$y = 3(x)^{-\frac{1}{3}}$

Calculate y'

$y' = 3(-\frac{1}{3})x^{-\frac{4}{3}}$

$y' = -x^{-\frac{4}{3}}$

Set y' equal to the desired slope

$m = -\frac{1}{16}$

$-\frac{1}{16} = -x^{-\frac{4}{3}}$

Solve for x

$\frac{1}{16} = x^{-\frac{4}{3}}$

$\frac{1}{16} = \frac{1}{x^{\frac{4}{3}}}$

$16 = x^{\frac{4}{3}}$

$(16)^{\frac{1}{4}} = (x^{\frac{4}{3}})^{\frac{1}{4}}$

$\pm 2 = x^{\frac{1}{3}}$

$(\pm 2)^3 = x$

$\pm 8 = x$

$\boxed{(x^a)^b = x^{ab}}$

$16^3 = (x^{\frac{4}{3}})^3$

$4096 = x^4$

$4096^{\frac{1}{4}} = x$

$\pm 8 = x$

2. Do the functions $h = \frac{1}{x}$ and y = x^3 ever have the same slope?

Determine the derivatives of each fn

$h = x^{-1}$

$\frac{d}{dx}h = -1x^{-2}$

$\frac{d}{dx}h = \frac{-1}{x^2}$ $\frac{d}{dx}y = 3x^2$

The functions will have the same slope where h' = y'

$\frac{-1}{x^2} = 3x^2$

$-1 = 3x^4$

$-\frac{1}{3} = x^4$

$\sqrt[4]{-\frac{1}{3}} = x$

complex

no real solutions, so the slopes of h + y will never be equal