Differentiation Rules

Recall the power rule… **f(x) = x^n**

f'(x) = nx^{n-1}

What happens when we try to take the derivative of **f(x) = 3x^2** ?

$$\mathbf{f(x)} = \underset{\text{Constant}}{\mathbf{c}}\underbrace{\mathbf{g(x)}}_{\text{Function}}$$

PROOF:

$$f'(x) = \lim_{h\to 0}\frac{\mathrm{f(x+h)-f(x)}}{h}$$

$$= \lim_{h\to 0}\frac{\mathrm{cg(x+h)}^{n}-\mathrm{cg(x)}}{h}$$

$$= c\lim_{h\to 0}\frac{\mathrm{g(x+h)}^{n}-\mathrm{g(x)}}{h}$$

$$= \mathrm{cg'(x)}$$

THE CONSTANT MULTIPLE RULE
If f(x) = cg(x) for any constant c, then
f'(x) = cg'(x)

Ex 1.
Determine the derivative.

a) $2x^{-2}$

$= cx^{-2}$ where $c=2$

$f'(x) = 2(-2)x^{-3}$

$= -4x^{-3}$

$= \frac{-4}{x^3}$

b) $-3\sqrt[3]{x}$

$= c\sqrt[3]{x}$ where $c=-3$

$= cx^{\frac{1}{3}}$

$f'(x) = c(\frac{1}{3})x^{-\frac{2}{3}}$

$= -x^{-\frac{2}{3}}$ $\rightarrow$ $= \frac{-1}{x^{\frac{2}{3}}}$

Investigate the sum and difference rules…

Given f(x) = p(x) + g(x)

PROOF:

$$f'(x) = \lim_{h\to 0}\frac{\mathrm{f(x+h)-f(x)}}{\mathrm{h}}$$

$$= \lim_{\mathrm{h}\to 0}\frac{\mathrm{[p(x+h)+g(x+h)]-[p(x)+g(x)]}}{\mathrm{h}}$$

$$= \lim_{\mathrm{h}\to 0}\left\{\frac{\mathrm{[p(x+h)-p(x)]}}{\mathrm{h}}+\frac{\mathrm{[g(x+h)-g(x)]}}{\mathrm{h}}\right\}$$

$$= \lim_{\mathrm{h}\to 0}\left\{\frac{\mathrm{[p(x+h)-p(x)]}}{\mathrm{h}}\right\}+\lim_{\mathrm{h}\to 0}\left\{\frac{\mathrm{[g(x+h)-g(x)]}}{\mathrm{h}}\right\}$$

$$= \mathrm{p'(x)+g'(x)}$$

THE SUM RULE & DIFFERENCE RULE
If functions p(x) and q(x) are differentiable, and f(x) = p(x) $\pm$ q(x), then
f'(x) = p'(x) $\pm$ g'(x)

Ex 2.
Determine the derivative.

a) $f(x) = -2x^4 - 3x^2 + 5$

$f'(x) = -8x^3 - 6x$

b) $s = t^2(t-1)$

$s = t^3 - t^2$

$s' = 3t^2 - 2t$

c) $f(x) = (2x-3)(x+1)$

$f(x) = 2x^2 - x - 3$

$f'(x) = 4x - 1$

d) $f(x) = \sqrt[3]{27x^6}$

$f(x) = \sqrt[3]{27}\sqrt[3]{x^6}$

$= 3(x^6)^{\frac{1}{3}}$

$= 3x^2$

$f'(x) = 6x$

Ex 3.
Determine the equation of the tangent of $y = \frac{\sqrt{x}-2}{\sqrt[3]{x}}$ at the point (1, –1).

① make y differentiable using power rule

$y = \frac{x^{\frac{1}{2}}}{x^{\frac{1}{3}}} - \frac{2}{x^{\frac{1}{3}}}$

$= x^{\frac{1}{2}-\frac{1}{3}} - 2x^{-\frac{1}{3}}$

$= x^{\frac{1}{6}} - 2x^{-\frac{1}{3}}$

$y' = \frac{1}{6}x^{-\frac{5}{6}} + \frac{2}{3}x^{-\frac{4}{3}}$

② Determine eq^n of the tangent line (need slope of line + a point on the line)

m_{tan} @ $x = 1$

$m_{tan} = \frac{1}{6}(1)^{-\frac{5}{6}} + \frac{2}{3}(1)^{-\frac{4}{3}}$

$= \frac{1}{6} + \frac{2}{3}$

$m_{tan} = \frac{5}{6}$

point (1, –1)

$y = mx + b$
- sub in m, x, y

$-1 = \frac{5}{6}(1) + b$

$-\frac{11}{6} = b$

$y = \frac{5}{6}x - \frac{11}{6}$

Ex 4.
Determine the values of a & b for the function $f(x) = ax^3 + bx^2 + 3x - 2$ given $f(2) = 10$ and $f'(-1) = 14$.

(2,10) (–1,14)

$f(2) = 10$

$10 = a(2)^3 + b(2)^2 + 3(2) - 2$

$10 = 8a + 4b + 4$

$6 = 8a + 4b$

$3 = 4a + 2b$ ①

$f'(x) = 3ax^2 + 2bx + 3$

$f'(-1) = 14$

$14 = 3a(-1)^2 + 2b(-1) + 3$

$11 = 3a - 2b$ ②

① + ②

$3 = 4a + 2b$

$11 = 3a - 2b$

$14 = 7a$

$2 = a$ → sub into ①

$3 = 4(2) + 2b$

$-\frac{5}{2} = b$

$f(x) = 2x^3 - \frac{5}{2}x^2 + 3x - 2$