$Ax+By+C=0$ $Ax+By+Cz+D=0$

Vector and Parametric Equations of a Line

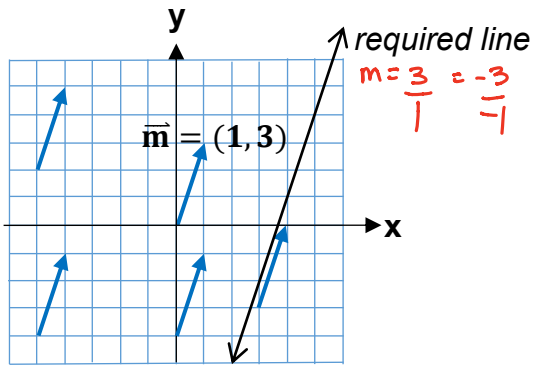
Describing a line using a vector equation:

- We will use a model similar to y = mx + b, i.e. we will need to define a slope and a point
- We will use a "**direction**" vector to describe the slope of the line and a "**position**" vector to indicate a point that can be found on the line

slope point

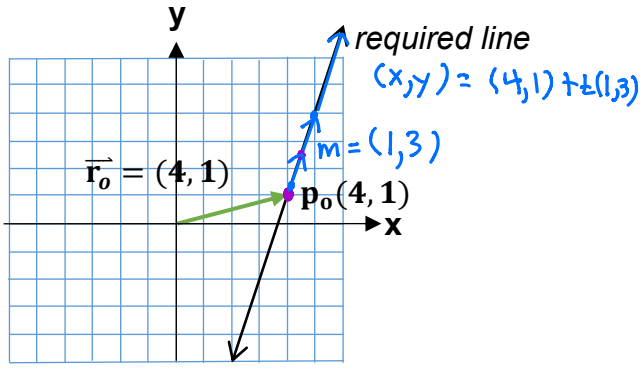
$m = \frac{\text{rise}}{\text{run}}$

Direction Vector



- same vector in many positons.
- the x and y components of the direction vector are called "direction numbers" and are used describe the vector.

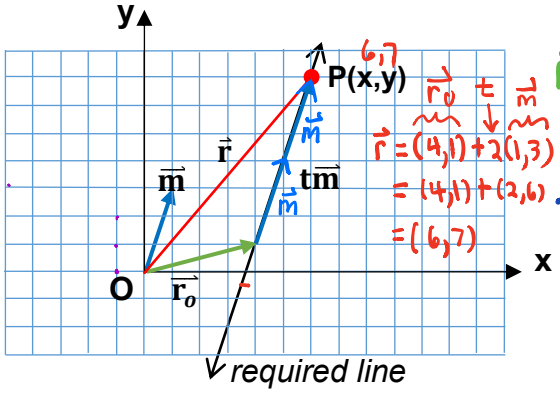
Position Vector



- a vector that runs from the origin to any point on the line

Vector Equation

$$\vec{r} = \vec{r_o} + t\vec{m}$$



$\vec{r} = (4,1) + 2(1,3)$
$= (4,1) + (2,6)$
$= (6,7)$

$\vec{r_o}$ – shows us how to get from the origin to any point on the line in question

$t\vec{m}$ – $\vec{m}$ is in the same orientation as the line, and t allows for travel along the line to any desired point P(x,y)

Vector Equation of a Line:

$$\vec{r} = \vec{r_o} + t\vec{m}, t \in R$$

Vector Equation of a Line (component form):

$$(x, y) = (x_0, y_0) + t(a, b)$$

initial point, parameter, "slope direction"

Parametric Equation of a line:

x-value of any point on the line → $x = x_0 + ta$

y-value of any point on the line → $y = y_0 + tb, t \in R$

P(x,y)

Where r_0 is the vector from (0,0) and the point (x_0,y_0) and $\vec{m}$ is the direction vector with components (a,b)

Ex 1.

a) Given a line through (–1,3) and (3,5). Write two different vector equations of the line.

W V

$\vec{m} = \overrightarrow{WV}$ or $\overrightarrow{VW}$

$(a,b) = (3,5) - (-1,3)$

$(a,b) = (4,2)$

① $(x,y) = (-1,3) + t(4,2)$ ← $t=1$

or

② $(x,y) = (3,5) + t(4,2)$ ← $t=2 \rightarrow t(2,1)$

$t \in \mathbb{R}$

b) Write a parametric equation and name **one** other point on the line.

① $(x,y) = (-1,3) + t(4,2)$

$x = -1 + 4t$

$y = 3 + 2t$

Let $t = 3$ ← random t-value

$x = -1 + 12 = 11$

$y = 3 + 6 = 9$

$(11,9)$

c) Find the point where the line crosses the x-axis.

$y = 0$

where does $y=0$?

$0 = 3 + 2t$

$t = -\frac{3}{2}$ → sub into t in parametric eqn → $x = -1 + 4\left(-\frac{3}{2}\right)$

$x = -7$ x-int

$(-7, 0)$

Ex 2.

a) Determine a vector equation for a line that is perpendicular to $\vec{r} = (2,3) + s(-1,4), s \in \mathbf{R}$ and passes through the point P (–3,6).

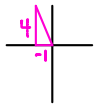
$\vec{m} = (-1,4)$ (x, y)

Slope $= \frac{4}{-1}$ $\frac{\text{rise}}{\text{run}}$

$\perp$ slope $= \frac{1}{4}$ $\frac{y}{x}$

$\vec{m}_\perp = (4,1) = (a,b)$

$(x,y) = (-3,6) + t(4,1)$



b) Is the point (–4,5) on the new perpendicular line?

$(-4,5) = (-3,6) + s(4,1)$

⊗ $-4 = -3 + 4s$

Ⓨ $5 = 6 + s$

$5 - 6 = s$

$-1 = s$

Sub $s = -1$ into ⊗

$-4 = -3 + 4(-1)$

$-4 \neq -7$

$(-4,5)$ is not on the $\perp$ line

Ex 3. Can I do this?

a) Given $\vec{r} = (2,3) + t(\frac{1}{2}, \frac{2}{3})$, get rid of the fractions and write $\vec{r} = (2,3) + t(3,4)$?

yes!
blc (3,4) and $(\frac{1}{2}, \frac{2}{3})$ are pll
blc reference point (2,3) is the same

b) Given $\vec{r} = (\frac{1}{2}, \frac{2}{3}) + t(1,2)$, get rid of the fractions and write $\vec{r} = (3,4) + t(1,2)$?

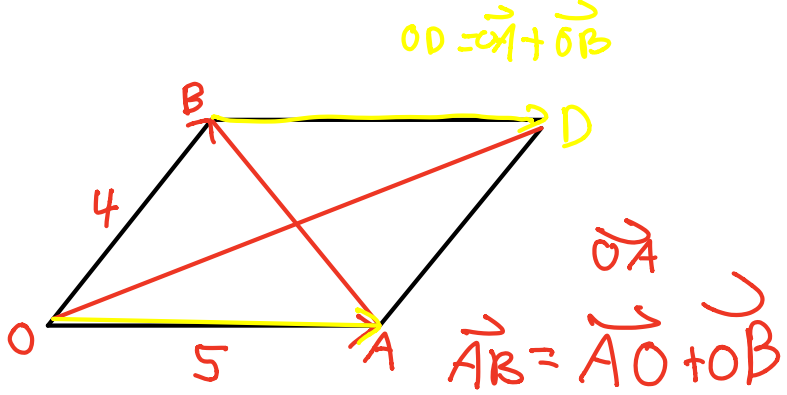
- check if (3,4) is on the line
- (3,4) will satisfy the eqⁿ if it is on the line

OPTIONAL
Investigation Pg 432

$(3,4) = (\frac{1}{2}, \frac{2}{3}) + t(1,2)$

$3 = \frac{1}{2} + t$
$4 = \frac{2}{3} + 2t$
} Can't solve for t
So (3,4) is not on the line.

NO



$\vec{AB} = \vec{AO} + \vec{OB}$

$= -\vec{OA} + \vec{OB}$

$|\vec{OA}| + |\vec{OB}| \neq |\vec{OD}|$

$\vec{p} = a\vec{q} + b\vec{r}$ | $\vec{q} = \vec{p} + \vec{r}$ | $\vec{r} = \vec{q} + p$

$(-11,7) = a(-3,1) + b(-1,2)$

X

$-11 = -3a - b$

Y

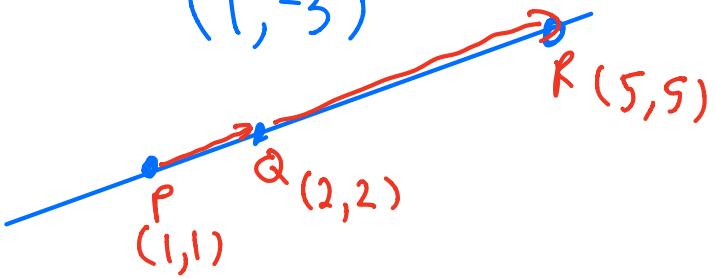
$7 = a + 2b$

$a = ?$

$b = ?$

$(-1,3) \parallel$

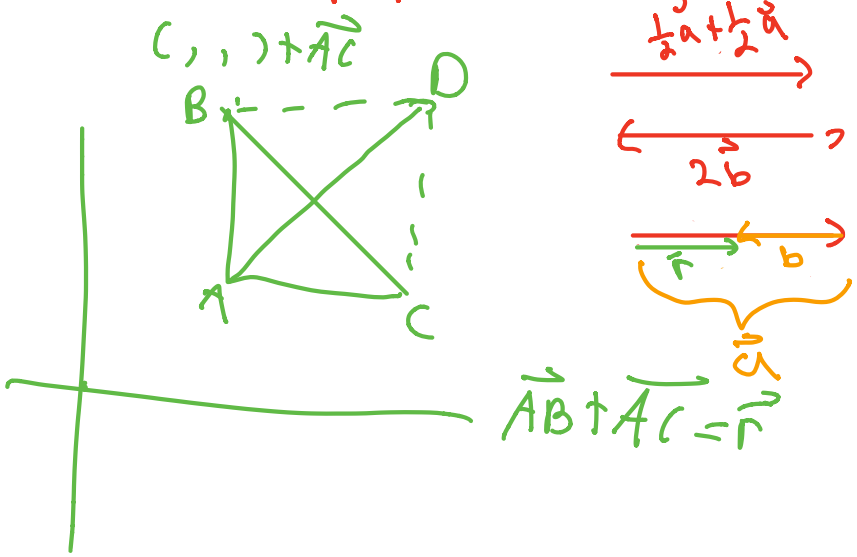
$(1,-3)$



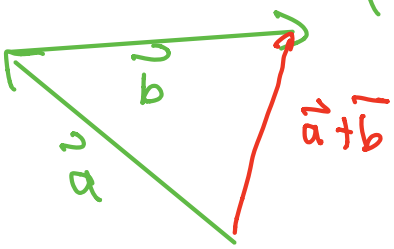
$\vec{PQ} = (1,1)$

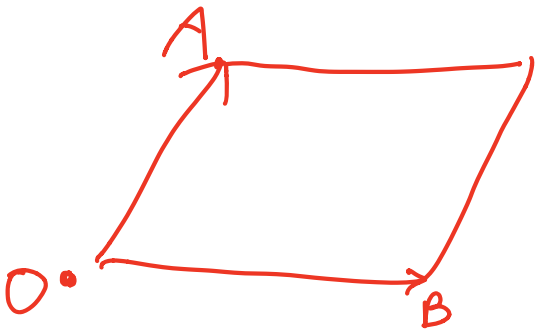
$\vec{QR} = (3,3) = 3(1,1)$

$|\vec{b}| = |\vec{a}+\vec{b}|$

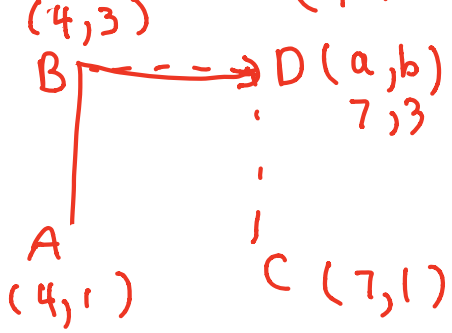


$|\vec{a}+\vec{b}|$





(4,3)+(3,0)



$\vec{AC} = (3,0)$

8.

$\vec{x} + \vec{y} = -\hat{i} + 2\hat{j} + 5\hat{k}$

$\vec{x} - \vec{y} = 3\hat{i} + 6\hat{j} - 7\hat{k}$ ⊕ to eliminate $\vec{y}$

$2\vec{x} = 2\hat{i} + 8\hat{j} - 2\hat{k}$