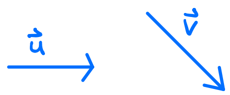
SCALAR MULTIPLICATION OF A VECTOR

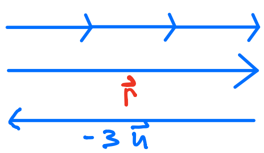
For the vector $k\vec{a}$, where k is a scalar and $\vec{a}$ is a nonzero vector:

- If k > 0, then $k\vec{a}$ is in the same direction as $\vec{a}$ with a magnitude $k|\vec{a}|$
- If k < 0, then $k\vec{a}$ is in the opposite direction as $\vec{a}$ with a magnitude

Ex 1.
Given the vectors $\vec{u}$ and $\vec{v}$ as shown, draw a vector for each of the following.

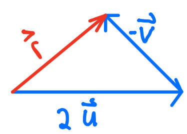


a) $3\vec{u}$

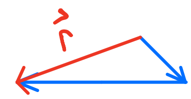


b) $-0.5\vec{v}$

c) $2\vec{u}-\vec{v}$



d) $\frac{2}{3}\vec{v}-2\vec{u}$



The zero vector: multiplication of the zero vector by a scalar gives the zero vector. ie. $k\vec{0}=\vec{0}$

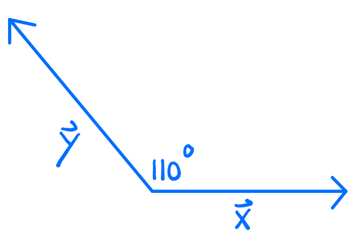
The unit vector: dividing any vector by its magnitude gives the unit vector… a vector in the same direction that is 1 unit long.

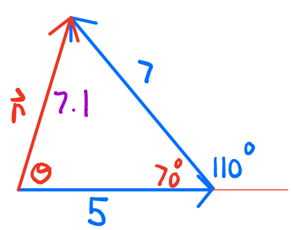
$$\boxed{\hat{a}=\frac{\vec{a}}{|\vec{a}|}}$$

$\frac{\vec{a}}{|\vec{a}|}$ is a vector in the same direction as $\vec{a}$ with a magnitude of 1

$\vec{v}=700\text{km [E]}$ $\quad \hat{v}=\frac{700\text{km [E]}}{700}=1\text{km [E]}$

Ex 2.
The angle between $\vec{x}$ and $\vec{y}$ is 110°. If $|\vec{x}|=5$ and $|\vec{y}|=7$, determine the unit vector in the same direction as $\vec{x}+\vec{y}$.





$\vec{r}=\vec{x}+\vec{y}$

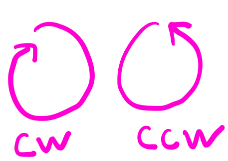
$|\vec{r}|^2=5^2+7^2-2(5)(7)\cos 70$

$|\vec{r}|\doteq 7.1$

$\frac{\sin 70}{7.1}=\frac{\sin\theta}{7}$

$\theta=\sin^{-1}\left[\frac{7\sin 70}{7.1}\right]$

$\theta\doteq 68°$



$\vec{r}=7.1\,[68°\text{ ccw from }\vec{x}]$

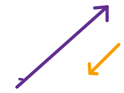
$\hat{r}=\frac{\vec{r}}{|\vec{r}|}=\frac{7.1}{7.1}[68°\text{ ccw from }\vec{x}]$

$\hat{r}$ = "a unit vector in the same direction as $\vec{r}$"

$\hat{r}=1\,[68°\text{ ccw from }\vec{x}]$

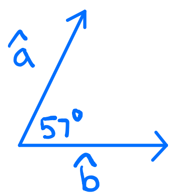
Collinear Vectors:

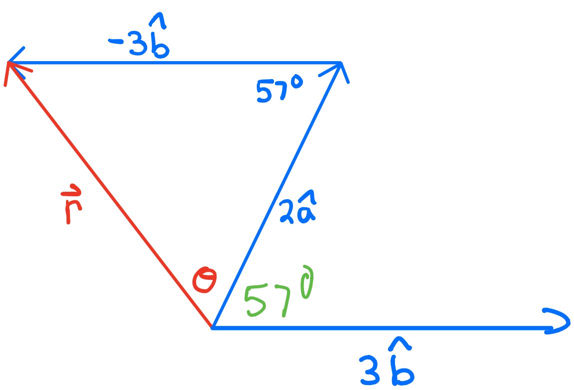
- Two vectors are collinear if they are parallel or can be translated to the same straight line (ie. same direction)
- Two vectors $\vec{u}$ and $\vec{v}$ are collinear if and only if it is possible to find a nonzero scalar k such that $\vec{u} = k\vec{v}$
- Note: “parallel” and “collinear” are used interchangeably



Ex 3.

The vectors $\hat{a}$ and $\hat{b}$ make an angle of 57° with each other. Determine the magnitude and direction of $2\vec{a} - 3\vec{b}$.





$|\vec{r}|^2 = 2^2 + 3^2 - 2(3)(2)\cos 57^\circ$

$|\vec{r}| \doteq 2.54$

$\frac{\sin\theta}{3} = \frac{\sin 57}{2.54}$

$\theta = \sin^{-1}\left[\frac{3\sin 57}{2.54}\right]$

$\theta = 82^\circ$

$\vec{r} = 2.54$ units [82° ccw of $\hat{a}$]

Ex 4.

Three collinear vectors $\vec{u}, \vec{v}$, and $\vec{w}$ are related to each other such that $\vec{u} = 2\vec{v}$ and $\vec{v} = 3\vec{w}$. Determine the integer values for **a** and **b** such that $a\vec{v} + b\vec{w} = \vec{0}$.

$a\vec{v} + b\vec{w} = \vec{0}$

$a(3\vec{w}) + b\vec{w} = \vec{0}$

$3a\vec{w} + b\vec{w} = \vec{0}$

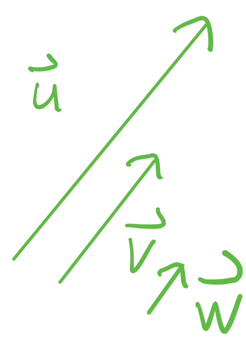
$\vec{w}(3a+b) = \vec{0}$

$3a + b = 0$

$b = -3a$ → ∞ number of sol^{ns}

eq^{ns}

a	b
1	-3
2	-6
⋮	⋮

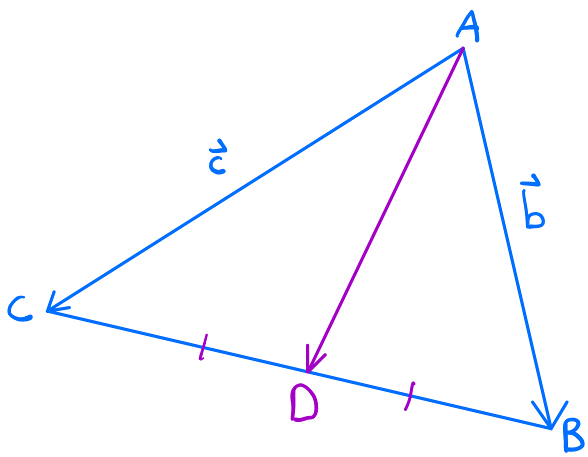


$d \cdot e = 0$

d=0 or e=0 or d and e=0

Ex 5.

In ABC, a median is drawn from A to the midpoint of BC which is labelled D. If $\overrightarrow{AB} = \vec{b}$ and $\overrightarrow{AC} = \vec{c}$, prove that $\overrightarrow{AD} = \frac{1}{2}\vec{b} + \frac{1}{2}\vec{c}$



What we know

$$\overrightarrow{AD} = \vec{b} + \overrightarrow{BD} = \vec{c} + \overrightarrow{CD}$$

$$\overrightarrow{AD} + \overrightarrow{AD} = \vec{b} + \overrightarrow{BD} + c + \overrightarrow{CD}$$

$-\overrightarrow{BD}$

$$2\overrightarrow{AD} = \vec{b} + \vec{c} + \overrightarrow{BD} - \overrightarrow{BD}$$

$$2\overrightarrow{AD} = \vec{b} + \vec{c}$$

$$\overrightarrow{AD} = \frac{1}{2}\vec{b} + \frac{1}{2}\vec{c}$$

Alternative

$$\overrightarrow{CD} = -\vec{c} + \overrightarrow{AD}$$

$$\overrightarrow{DB} = -\overrightarrow{AD} + \vec{b}$$

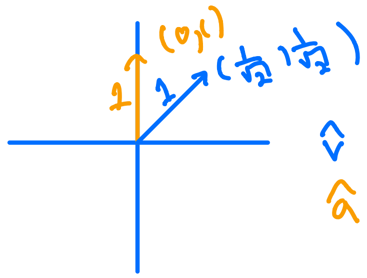
$$\overrightarrow{CD} = \overrightarrow{DB}$$

$$-\vec{c} + \overrightarrow{AD} = -\overrightarrow{AD} + \vec{b}$$

$$\overrightarrow{AD} + \overrightarrow{AD} = \vec{c} + \vec{b}$$

$$2\overrightarrow{AD} = \vec{c} + \vec{b}$$

$$\overrightarrow{AD} = \frac{1}{2}\vec{c} + \frac{1}{2}\vec{b}$$



(0,1)
1
1
$(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}})$
$\hat{v}$
$\hat{a}$