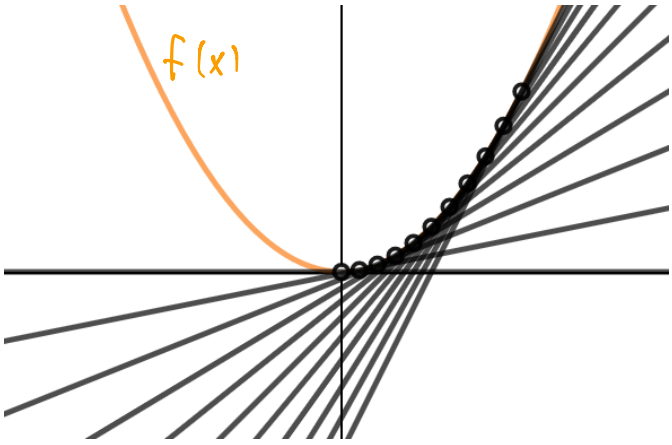
The Derivative Function

Thought Experiment

- For a given function, f(x), imagine determining the instantaneous rate of change (tangent slope or m_{tan}) of the function for every "x" value of the function.



$m_{tan} = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$

$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$

$f(x) = x^2$

$f'(x) = 2x$

- The relationship between "x" and its corresponding tangent slope m_{tan} creates a new function called the "derivative function"
- This derivative function is denoted as:

$f(x) =$ → $y =$ y'

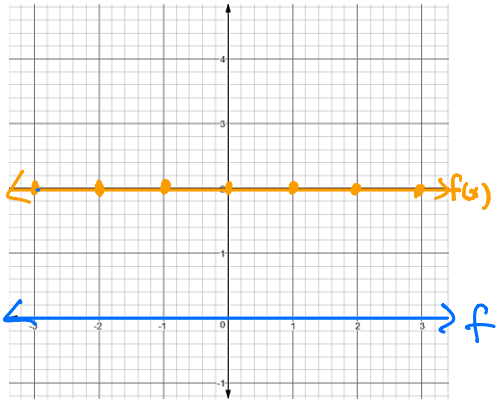
$f'(x)$	or	$\frac{dy}{dx}$	or	y'
"f prime at x" or "f prime of x"		**"d y by d x"**		**"y prime"**

What do derivative functions look like?

- Determine the value of the slope of the tangent for different values of x
- Sketch the derivative functions… if possible write the equation of f'(x)

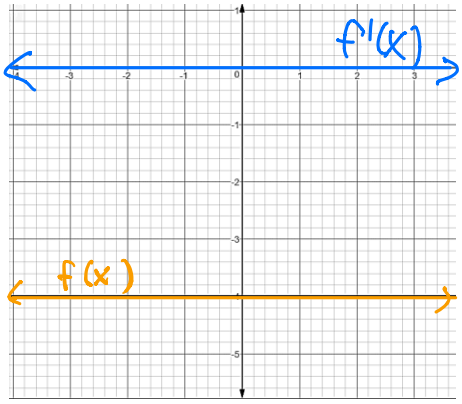
1. CONSTANT FUNCTIONS

$\mathbf{f(x) = 2}$ $\mathbf{f'(x) =}$ 0



x	f(x)	f'(x)
-3	2	0
-2	2	0
-1	2	0
0	2	0
1	2	0
2	2	0
3	2	0

$\mathbf{f(x) = -4}$ $\mathbf{f'(x) =}$ 0



x	f(x)	f'(x)
-3	-4	0
-2	-4	0
-1	-4	0
0	-4	0
1	-4	0
2	-4	0
3	-4	0

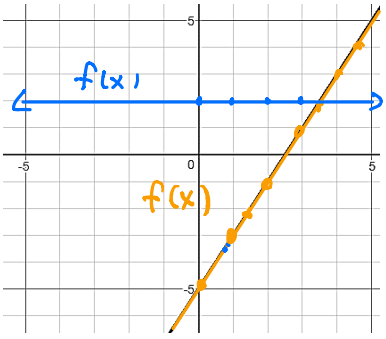
$f'(x) = \lim_{h\to 0} \frac{2(x+h)-5-2x+5}{h}$

$= \lim_{h\to 0} \frac{2h}{h}$

$= 2$

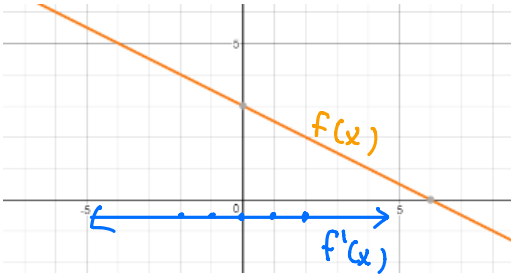
2. LINEAR FUNCTIONS

$f(x) = 2x^1 - 5 \quad f'(x) = 2x^0$



x	f(x)	f'(x)
-2	-9	2
-1	-7	2
0	-5	2
1	-3	2
2	-1	2

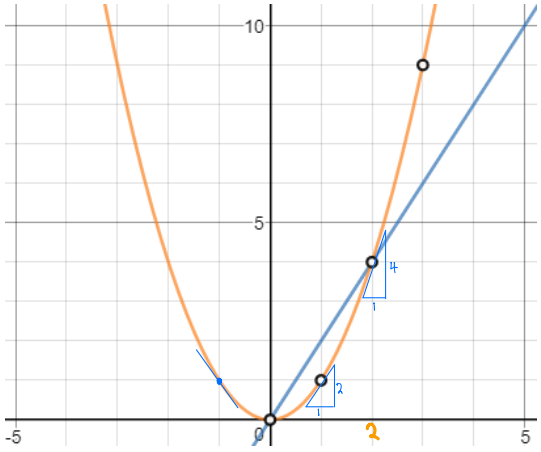
$f(x) = -0.5x + 3 \quad f'(x) = -0.5$



x	f(x)	f'(x)
-2	4	-0.5
-1	3.5	-0.5
0	3	-0.5
1	2.5	-0.5
2	2	-0.5

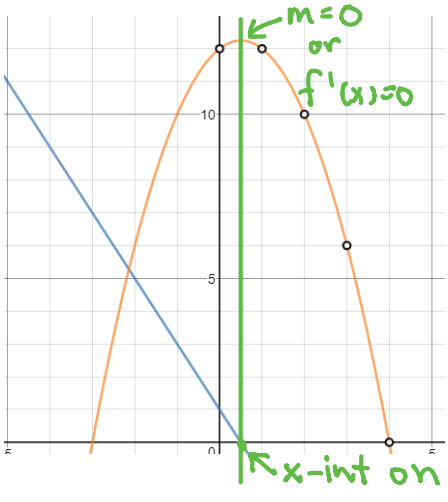
3. QUADRATIC FUNCTIONS

$f(x) = x^2 \quad f'(x) = 2x$



x	f(x)	f'(x)
-2	4	-4
-1	1	-2
0	0	0
1	1	2
2	4	4

$f(x) = -(x+3)(x-4) \quad f'(x) = -2x+1$

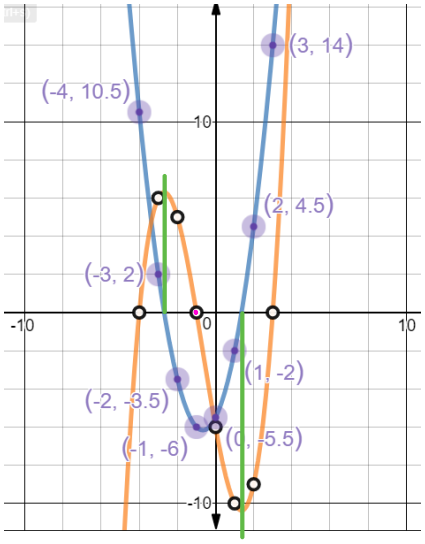


x	f(x)	f'(x)
-3	0	7
-2	6	5
-1	10	3
0	12	1
1	10	-1
2	6	-3

3. CUBIC FUNCTIONS

$f(x) = 0.5(x+4)(x+1)(x-3)$

$f'(x) =$ order 2



x	f(x)	f'(x)
-3	6	2
-2	5	-3.5
-1	0	-6
0	-6	-5.5
1	-10	-2
2	-9	4.5
3	0	14

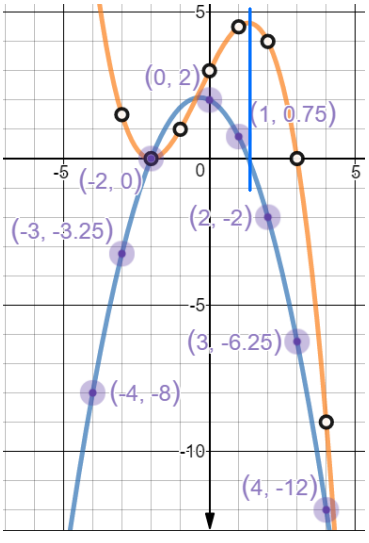
$f'(x) = ax^2 + bx + c$

$2 = 9a - 3b + c$

$f'(x) = 1.5x^2 + 2x - 5.5$

$f(x) = -0.25(x+2)^2(x-3)$

$f'(x) =$ order 2



x	f(x)	f'(x)
-4	7	-8
-3	1.5	-3.25
-2	0	0
-1	1	1.75
0	3	2
1	4.5	0.75
2	4	-2
3	0	-6.25
4	-9	-12

$f'(x) = -\frac{3}{4}x^2 - \frac{1}{2}x + 2$

SUMMARY

The derivative function of a:

constant function is a zero function. $f(x) = \text{constant} \rightarrow f'(x) = 0$

linear function is a constant function. $f(x) = ax^1 \rightarrow f'(x) = a$

quadratic function is a linear function. $f(x) = ax^2 \rightarrow f'(x) = x$

cubic function is a quadratic function. $f(x) = ax^3 \rightarrow f'(x) = x^2$