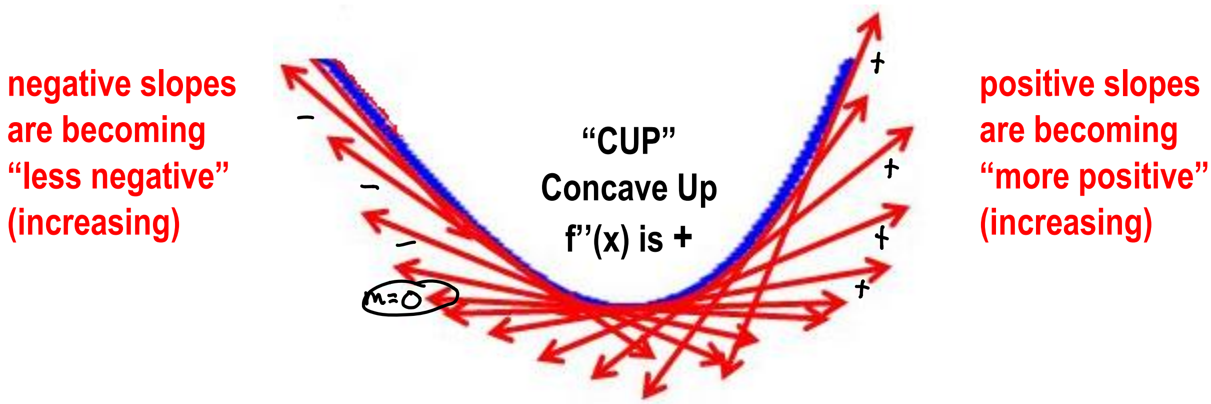
Concavity and Points of Inflection

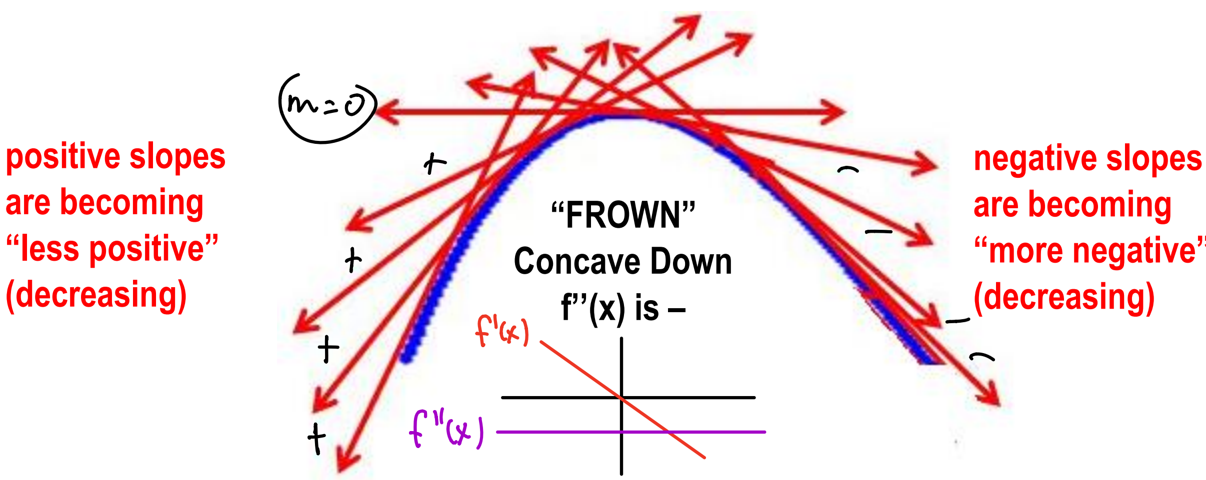
CONCAVITY

Case 1: Consider the shape of f(x) when the slopes of the tangents are increasing. (ie. getting larger)



f'(x) is increasing and concave up (ie. the slopes of the tangents are increasing)

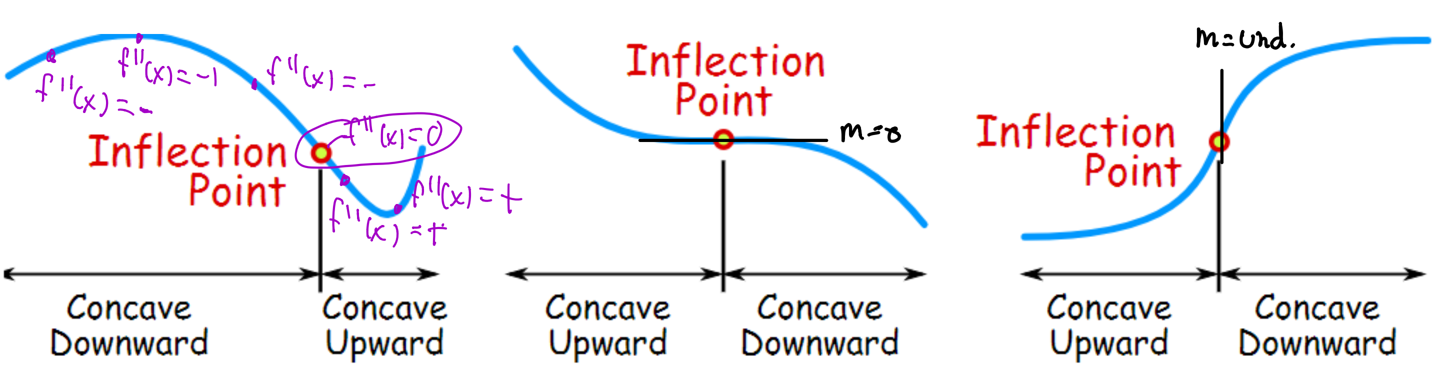
Case 2: Consider the shape of f(x) when the slopes of the tangents are decreasing. (ie. getting smaller)



f'(x) is decreasing → concave down (ie. the slopes of the tangents are decreasing)

POINTS OF INFLECTION:

Occur when f(x) changes from concave up to concave down or from concave down to concave up.



SUMMARY

Concave Up: f'(x) is increasing so f''(x) is positive.

Concave Down: f'(x) is decreasing so f''(x) is negative.

Point of Inflection (POI): - where the function changes from CU to CD or vice versa

- f''(x) = 0 or is undefined (DNE)

* f(x) must exist at the POI

Ex 1.
Determine the intervals of concavity.

a) $f(x) = -2x^3 + 9x^2 + 20$

$f'(x) = -6x^2 + 18x$

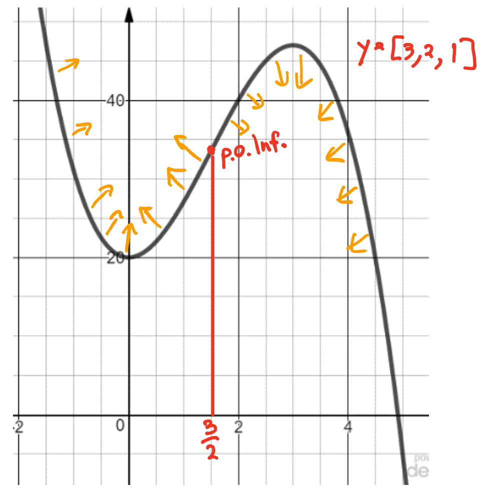
$f''(x) = -12x + 18$

Let $f''(x) = 0$

$0 = -12x + 18$

$\frac{3}{2} = x \leftarrow$ C.N

$f''(x) = $ undef.
- impossible for polynomial



$-\infty < x < \frac{3}{2}$ CU $(-\infty, \frac{3}{2})$

$x > \frac{3}{2}$ CD $(\frac{3}{2}, \infty)$

f''(x)	+	−
	$-\infty$ to $\frac{3}{2}$	$\frac{3}{2}$ to $+\infty$

b) $f(x) = x^4 + 2x^3 - 3x^2 - 4x + 4$

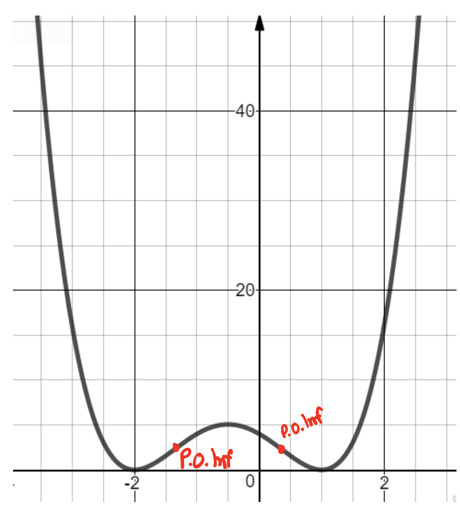
$f'(x) = 4x^3 + 6x^2 - 6x - 4$

$f''(x) = 12x^2 + 12x - 6$

Let $f''(x) = 0$

$0 = 6(2x^2 + 2x - 1)$

C.N $\longrightarrow x = \frac{-1 \pm \sqrt{3}}{2}$ (from quad. formula)



CU $(-\infty, \frac{-1-\sqrt{3}}{2})$ + $(\frac{-1+\sqrt{3}}{2}, \infty)$

CD $(\frac{-1-\sqrt{3}}{2}, \frac{-1+\sqrt{3}}{2})$

f''(x)	+	−	+
		−1.37 to 0.37	

−1.37 0.37

Ex 2.

Determine the intervals of concavity and points of inflection.

a) $f(x) = 3x^4$

$f'(x) = 12x^3$

$f''(x) = 36x^2$

Let $f''(x) = 0$

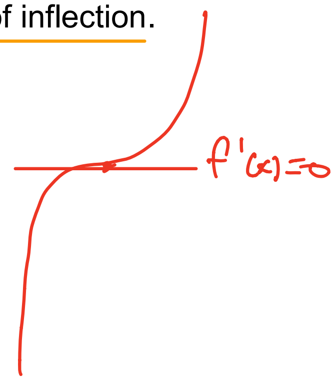
$0 = 36x^2$ $\quad 0 = 36xx$

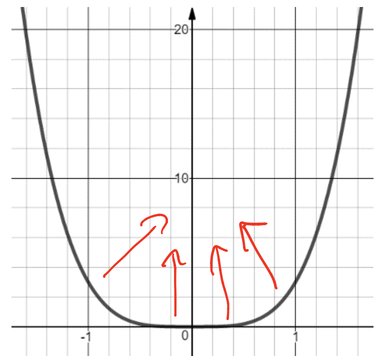
$0 = x^2$

$\sqrt{0} = x$

$0 = x$

C.N → $x = 0, 0$





CU $(-\infty, 0) + (0, \infty)$

no change in concavity, so no P.O. Inf.

f''(x)	+	0	+

b) $f(x) = \sqrt[3]{x} = x^{\frac{1}{3}}$

$f'(x) = \frac{1}{3}x^{-\frac{2}{3}}$

$f''(x) = \frac{-2}{9}x^{-\frac{5}{3}}$

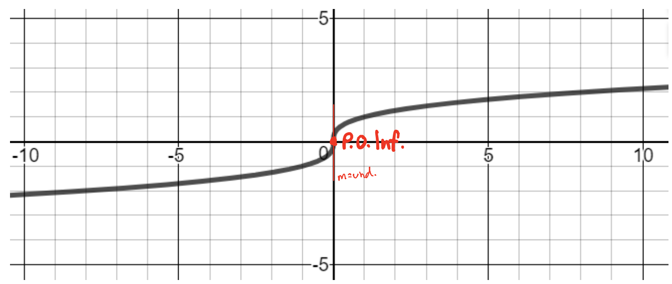
Let $f''(x) = 0$

$0 = \dfrac{-2}{9x^{\frac{5}{3}}}$

$f''(x) = $ undef.

$f''(0) = \frac{-2}{0}$

C.N → V. Asymptote $f''(x)$ @ $x = 0$



CU $(-\infty, 0)$

CD $(0, \infty)$

P.O. Inf @ $x = 0$ $(0,0)$ b/c concavity changes + $f(0)$ exists

f''(x)	+	0	−

c) $f(x) = \frac{1}{x^2+3}$, $f'(x) = \frac{-2x}{(x^2+3)^2}$, $f''(x) = \frac{6(x^2-1)}{(x^2+3)^3}$

Let $f''(x) = 0$

$0 = 6(x^2 - 1)$

$x^2 = 1$

$x = \pm 1$

C.N → $x = \pm 1$

$f''(x) = $ undef.

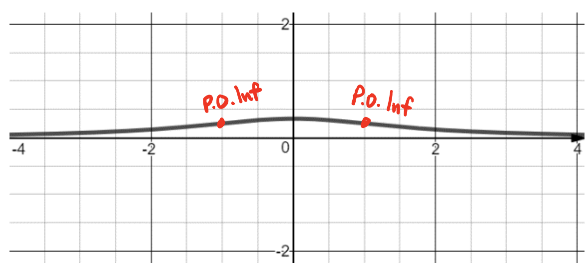
Can $(x^2+3)^3 = 0$?

$x^2 + 3 = 0$

$x^2 = -3$

$x = \pm\sqrt{-3}$

n/A



- CU $(-\infty, -1) + (1, \infty)$
- CD $(-1, 1)$
- P.O. Inf. @ $x = \pm 1$ → $(-1, \frac{1}{4})$ point $+ (1, \frac{1}{4})$ point

f''(x)	+	−1	−	1	+