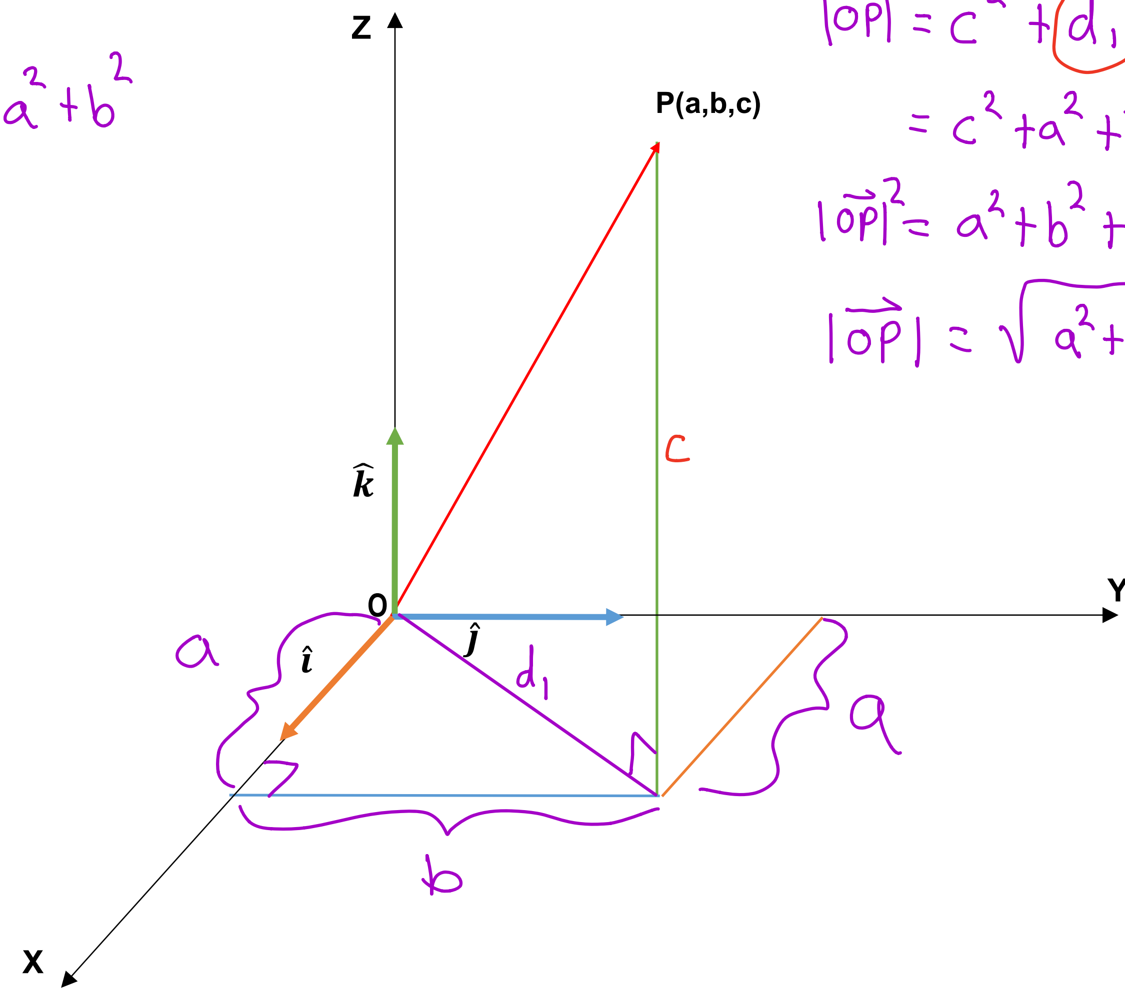
***Magnitude of a Position Vector

Warmup

$d_1^2 = a^2 + b^2$



$|\vec{OP}|^2 = c^2 + d_1^2$

$= c^2 + a^2 + b^2$

$|\vec{OP}|^2 = a^2 + b^2 + c^2$

$|\vec{OP}| = \sqrt{a^2 + b^2 + c^2}$

Operations With Vectors in R^3

UNIT VECTORS

- Let $\hat{\imath}$, $\hat{\jmath}$ and $\hat{k}$ represent the *basis* unit vectors *(*directed along the positive *x,* y *and z-axis)*

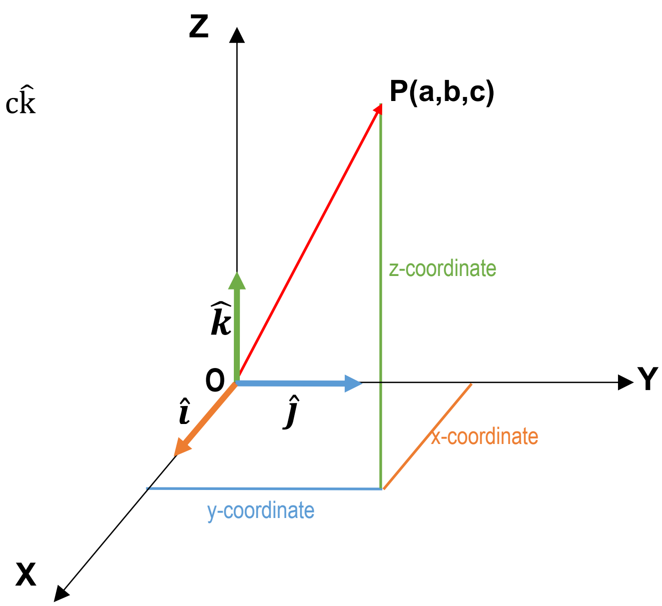
- Every vector $\overrightarrow{OP}$ in the plane can be written:
  - in component form $\overrightarrow{OP} = (a, b, c)$ or
  - using unit vectors such that $\overrightarrow{OP} = a\hat{\imath} + b\hat{\jmath} + c\hat{k}$

- $(a, b, c) = a\hat{\imath} + b\hat{\jmath} + c\hat{k}$
- $|\overrightarrow{OP}| = \sqrt{a^2 + b^2 + c^2}$

$\hat{\imath}$ = (1,0,0)

$\hat{\jmath}$ = (0,1,0)

$\hat{k}$ = (0,0,1)



Ex 1.
Given $\vec{u} = 2\hat{\imath} - 3\hat{\jmath} + 4\hat{k}$ and $\vec{v} = 5\hat{\imath} - 3\hat{\jmath} - 4\hat{k}$, determine $4\vec{u} - 2\vec{v}$.

$$4\vec{u} - 2\vec{v} = 4(2\hat{\imath} - 3\hat{\jmath} + 4\hat{k}) - 2(5\hat{\imath} - 3\hat{\jmath} - 4\hat{k})$$
$$= 8\hat{\imath} - 12\hat{\jmath} + 16\hat{k} - 10\hat{\imath} + 6\hat{\jmath} + 8\hat{k}$$
$$= -2\hat{\imath} - 6\hat{\jmath} + 24\hat{k}$$

$$= 4(2,-3,4) - 2(5,-3,-4)$$
$$= (8,-12,16) + (-10,6,8)$$
$$= (-2,-6,24)$$

VECTORS DEFINED BY TWO POINTS (THAT ARE NOT POSITION VECTORS)

Recall that if a vector is defined by two points A(x_1,y_1) and B(x_2,y_2) then:

$B(x_2,y_2) - A(x_1,y_1)$

$\overrightarrow{AB}$ **= ($x_2 - x_1$, $y_2 - y_1$)** and $|\overrightarrow{AB}| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Therefore, if a vector is defined by two points A(x_1,y_1,z_1) and B(x_2,y_2,$z_{\cancel{1}}$ 2) then:

$\overrightarrow{AB}$ **= ($x_2 - x_1$, $y_2 - y_1$, $z_2 - z_1$)** and $|\overrightarrow{AB}| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$

Note: vector $\overrightarrow{AB}$ is not a position vector because it is not connected to the origin. A position vector must always have its tail at point (0,0,0)

Ex 2.

Given the points A(3, –1, –4) and B(–3, 1, 5) and C(–7, –4,0), determine:

a) $\overrightarrow{CA}$

$\overrightarrow{CA} = A - C$

$= (3,-1,-4) - (-7,-4,0)$

$= [3-(-7), -1-(-4), -4-0]$

$= [10, 3, -4]$

b) $|\overrightarrow{AB}|$

$\overrightarrow{AB} = [\overset{x_B - x_A}{-3-3}, \overset{y_B - y_A}{1-(-1)}, \overset{z_B - z_A}{5-(-4)}]$

$= [-6, 2, 9]$

$|\overrightarrow{AB}| = \sqrt{(-6)^2 + 2^2 + 9^2}$

$= \sqrt{121}$

$= 11$

c) the perimeter of triangle ABC

$P = |\overrightarrow{AB}| + |\overrightarrow{BC}| + |\overrightarrow{CA}|$

$= |[-6,2,9]| + |[-4,-5,-5]| + |[10,3,-4]|$

$= 11 + \sqrt{66} + \sqrt{125}$

$= 11 + \sqrt{66} + 5\sqrt{5}$

$\doteq$ 30.3 units

$\sqrt{125}$

$\sqrt{25 \cdot 5}$

$\sqrt{25} \cdot \sqrt{5}$

$5\sqrt{5} \neq \sqrt[5]{5}$

$5(\sqrt{5})$

$\sqrt[3]{}$

$\sqrt[4]{}$

$\sqrt[5]{5}$

Ex 3.

Given $\vec{v} = (-2,2,5)$, determine the unit vector with the opposite direction to $\vec{v}$.

Opposite

$-\vec{v} = -1(-2,2,5)$

$= (2,-2,-5)$

$\hat{a} = \frac{\vec{a}}{|\vec{a}|} \longrightarrow$

Unit vector

$-\hat{v} = \frac{(2,-2,-5)}{\sqrt{(2)^2 + (-2)^2 + (-5)^2}}$

$= \frac{(2,-2,-5)}{\sqrt{33}}$

$-\hat{v} = \frac{1}{\sqrt{33}}(2,-2,-5)$

$= \left(\frac{2}{\sqrt{33}}, \frac{-2}{\sqrt{33}}, \frac{-5}{\sqrt{33}}\right)$