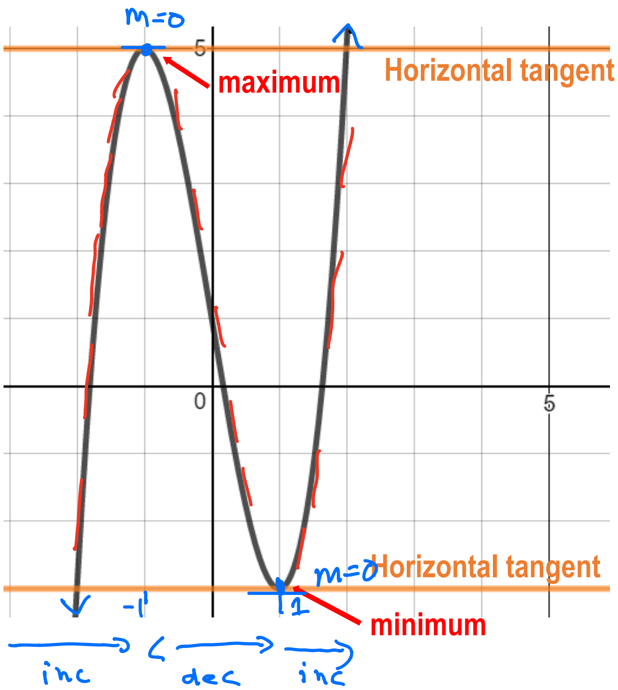
Increasing/Decreasing Functions

LOCAL EXTREMA:



Local Maximum:

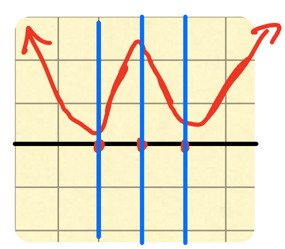
Occurs when f'(x) changes from a __+__ to a __−__.

Local Minimum:

Occurs when f'(x) changes from a __−__ to a __+__.

Q: Why do we care about local maxima and minima?
A: Local maxima and minima will mark where a function changes from an interval where it is increasing to an interval where it is decreasing, and vise versa.

INTERVAL OF INCREASE	INTERVAL OF DECREASE
• The graph rises from left to right • For any x interval such that $x_1 < x_2$, it follows that $f(x_1) < f(x_2)$ • The slope of the tangent is positive • $f'(x) > 0$	• The graph falls from left to right • For any x interval such that $x_1 < x_2$, it follows that $f(x_1) > f(x_2)$ • The slope of the tangent is negative • $f'(x) < 0$



Ex 1.
Determine the local extrema, and the intervals of increase and decrease.

a) $f(x) = x^4 - 4x^3 + 4x^2$

$f'(x) = 4x^3 - 12x^2 + 8x$

Let $f'(x) = 0$ to solve for critical numbers

$0 = 4x^3 - 12x^2 + 8x$

$0 = 4x(x^2 - 3x + 2)$

$0 = 4x(x-2)(x-1)$

extrema @ $x = 0, 1, 2$

points $(0,0)$ $(1,1)$ $(2,0)$

*Sub x into f(x) to determine points

Test any value of x within each interval to determine the sign of the slope.

	$-\infty$ to 0	0 to 1	1 to 2	2 to $+\infty$
f'(x)	−	+	−	+

$-\infty$ 0 1 2 $+\infty$

min $(0,0)$ max $(1,1)$ min $(2,0)$

$0<x<1$ $2<x$

Increasing $(0,1) + (2,\infty)$

Decreasing $(-\infty,0) + (1,2)$

$0 \leq x < 1$

$[0,1)$

b) $g(x) = \frac{4x}{x^2+2}$ ← $f(x)$, ← $h(x)$ $\frac{f'(x)h(x) - f(x)h'(x)}{[h(x)]^2}$

$g'(x) = \frac{4(x^2+2) - 4x(2x)}{(x^2+2)^2}$

$= \frac{4x^2+8-8x^2}{(x^2+2)^2}$

$= \frac{-4x^2+8}{(x^2+2)^2}$

Let $g'(x) = 0$

$0 = -4x^2+8$

$4x^2 = 8$

$x^2 = 2$

$x = \pm\sqrt{2}$

Extrema @ $(-\sqrt{2}, -\sqrt{2})$ $(\sqrt{2}, \sqrt{2})$ on $f(x)$

test

$g'(-2) = -\frac{4}{9}$ $g'(0) = 2$ $g'(2) = -\frac{4}{9}$

	$-\infty$ to $-\sqrt{2}$	$-\sqrt{2}$ to $\sqrt{2}$	$\sqrt{2}$ to ∞
$g'(x)$	−	+	−

$-\sqrt{2}$: min $\sqrt{2}$: max

$x < \sqrt{2}$ $x > \sqrt{2}$

Decreasing $(-\infty, -\sqrt{2}) + (\sqrt{2}, \infty)$

$-\sqrt{2} < x < \sqrt{2}$

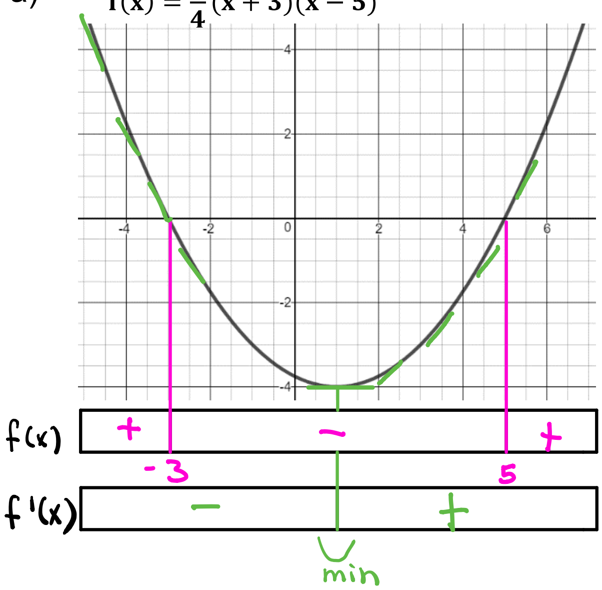
Increasing $(-\sqrt{2}, \sqrt{2})$

note ( ) not including   [ ] including

Ex 2.

Given the graph of f(x), determine the intervals where f(x) is increasing and decreasing.

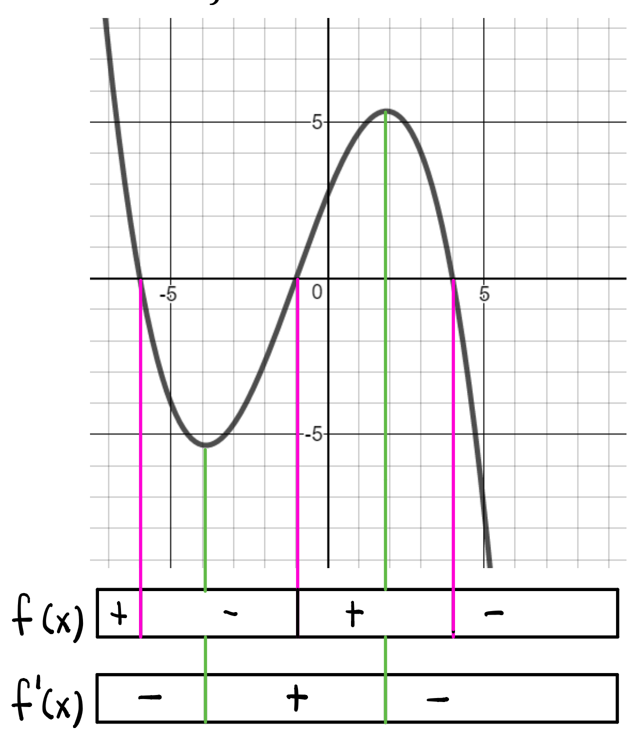
a) $f(x) = \frac{1}{4}(x+3)(x-5)$



Increasing $(1, +\infty)$ } based on $f'(x)$
Decreasing $(-\infty, 1)$

* we examine f(x) in order to create a more accurate "sketch". A strip for f(x) is not necessary to determine intervals of inc./dec.

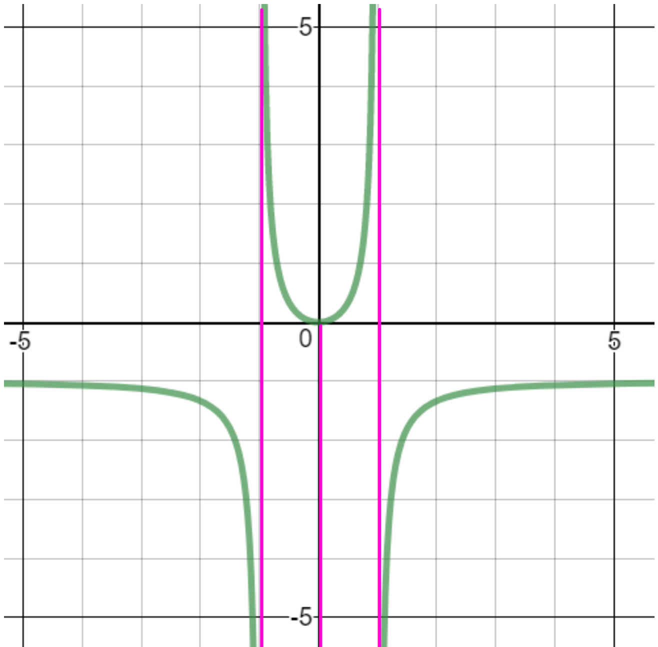
b) $f(x) = -\frac{1}{9}(x+6)(x+1)(x-4)$



based on f'(x) { Increasing $(-3.9, 1.9)$
Decreasing $(-\infty, -3.9) + (1.9, \infty)$

c) $f(x) = \dfrac{x^2}{1 - x^2}$

| zero   ¦ asymptote



f(x)	−	+	+	−
	-1	0	1	

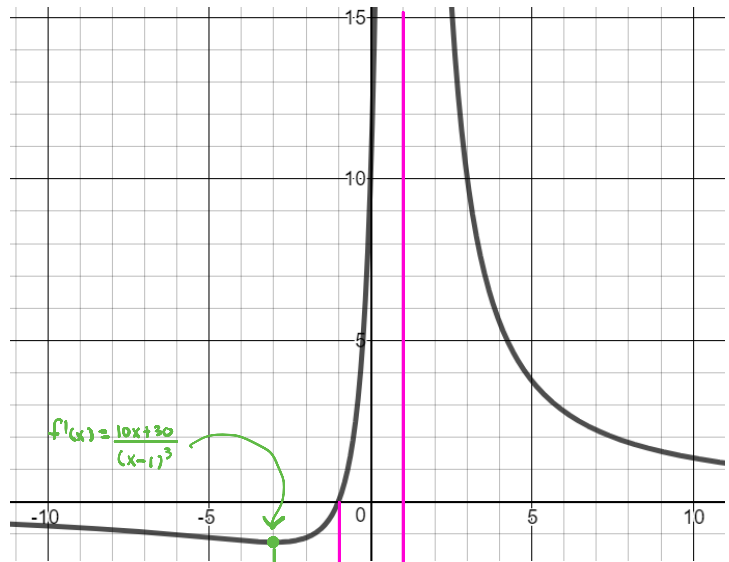
0 ← double root

f'(x)	−	−	+	+

Increasing $(0,1) + (1,\infty)$

Decreasing $(-\infty,-1) + (-1,0)$

d) $f(x) = \dfrac{10(x+1)}{(x-1)^2}$



f(x)	−	+	+
f'(x)	−	+	−

Increasing $(-3,1)$

Decreasing $(-\infty,-3) + (1,+\infty)$

Ex 3.

Sketch a graph that has the following properties:

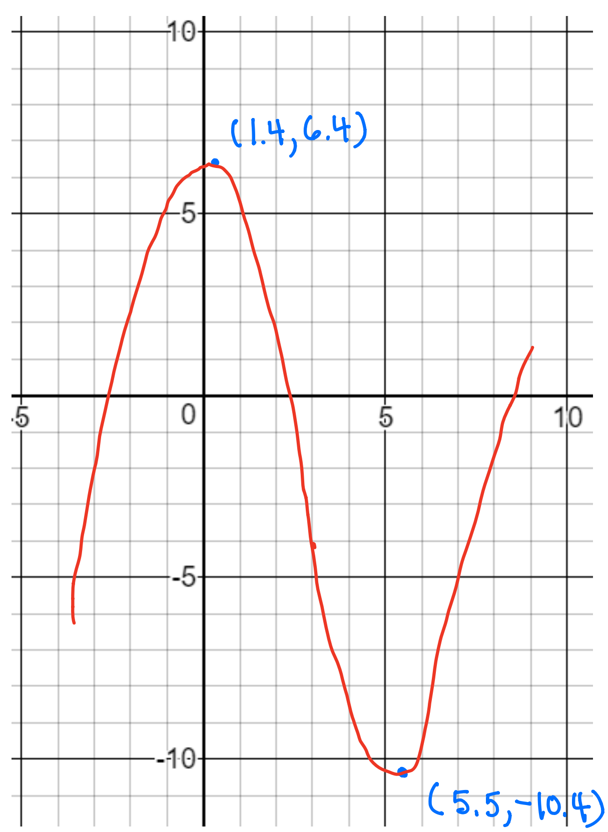
f'(x) > 0 for x < 1.4 and x > 5.5 (positive slopes)

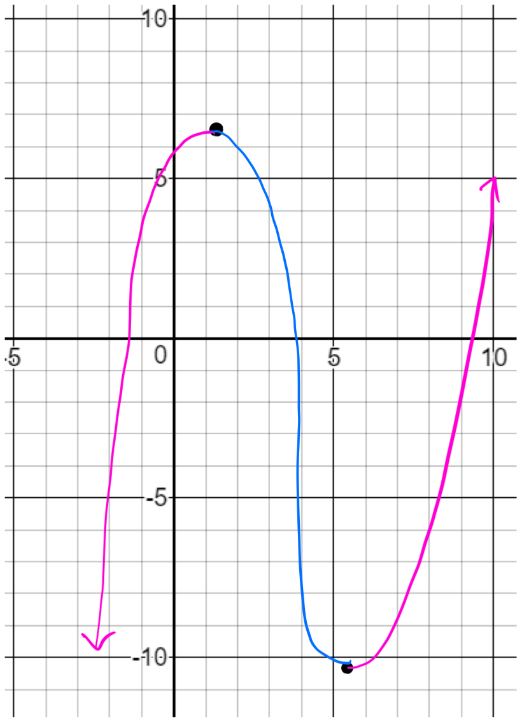
f'(x) < 0 for 1.4 < x < 5.5 (negative slopes)

f'(1.4) = f'(5.5) = 0

f(1.4) = 6.4 and f(5.5) = –10.4

f'(x)	+	−	+
	1.4	5.5	





10
5
-5
0
5
10
-5
-10