Differentiation Rules

Recall the power rule...

$$f(x) = x^n$$

$$f'(x) = nx^{n-1}$$

What happens when we try to take the derivative of $f(x) = 3x^2$?

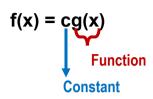
PROOF:

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{cg(x+h)^n - cg(x)}{h}$$

$$= c\lim_{h \to 0} \frac{g(x+h)^n - g(x)}{h}$$

$$= cg'(x)$$



THE CONSTANT MULTIPLE RULE

If
$$f(x) = cg(x)$$
 for any constant c, then $f'(x) = cg'(x)$

Ex 1.

Determine the derivative.

a)
$$2x^{-2}$$
 b) $-3\sqrt[3]{x}$

$$= cx^{-3} \text{ where } c = 2$$

$$= c\sqrt[3]{x} \text{ where } c = -3$$

$$f'(x) = 2 (-2)x^{-3}$$

$$= -4x^{-3}$$

$$= -4$$

$$= -x^{-\frac{3}{3}}$$
Investigate the sum and difference rules...

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Given
$$f(x) = p(x) + g(x)$$

PROOF:

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{[p(x+h) + g(x+h)] - [p(x) + g(x)]}{h}$$

$$= \lim_{h \to 0} \left\{ \frac{[p(x+h) - p(x)]}{h} + \frac{[g(x+h) - g(x)]}{h} \right\}$$

$$= \lim_{h \to 0} \left\{ \frac{[p(x+h) - p(x)]}{h} + \lim_{h \to 0} \left\{ \frac{[g(x+h) - g(x)]}{h} \right\}$$

$$= p'(x) + g'(x)$$

THE SUM RULE & DIFFERENCE RULE

If functions p(x) and q(x) are differentiable, and $f(x) = p(x) \pm q(x)$, then $f'(x) = p'(x) \pm q'(x)$

Ex 2.

Determine the derivative.

a)
$$f(x) = -2x^4 - 3x^2 + 5$$

$$f'(x) = -8x^3 - 6x$$

b)
$$s = t^2(t-1)$$

c)
$$f(x) = (2x - 3)(x + 1)$$

d)
$$f(x) = \sqrt[3]{27x^6}$$

$$S = t^3 - t^2$$

 $S' = 3t^2 - 2t$

$$f(x) = 2x^2 - x - 3$$

 $f'(x) = 4x - i$

$$f(x) = \sqrt[3]{3} x^{6}$$

$$= 3 (x^{6})^{\frac{1}{3}}$$

$$= 2 x^{2}$$

(2.10)

(-1,14)

Ex 3.

Determine the equation of the tangent of $y = \frac{\sqrt{x}-2}{\sqrt[3]{x}}$ at the point (1, -1).

- Omake & differentiable
- using power rule $y = \frac{x^{\frac{1}{2}}}{x^{\frac{1}{3}}} - \frac{2}{x^{\frac{1}{3}}}$

$$= X^{\frac{1}{2} - \frac{1}{3}} - 2X^{\frac{1}{3}}$$
$$= X^{\frac{1}{6}} - 2X^{\frac{1}{3}}$$

$$y' = \frac{1}{6}x^{-\frac{5}{6}} + \frac{2}{3}x^{-\frac{4}{3}}$$

Determine eqⁿ of the tangent line (need slope of line + a point on the line)

$$\frac{M_{tan}@x=1}{M_{tan}=\frac{1}{6}(1)^{\frac{5}{6}}+\frac{3}{3}(1)^{\frac{14}{3}}}$$

$$=\frac{1}{6}+\frac{3}{3}$$

point (1,-1)

$$y = mx + b$$
-sub in m,x,y

$$-1 = \frac{5}{6}(1) + b$$

$$y = \frac{5}{6}x - \frac{11}{6}$$

Ex 4.

Determine the values of a & b for the function $f(x) = ax^3 + bx^2 + 3x - 2$ given f(2) = 10 and f'(-1) = 14.

$$f(x) = 10$$

$$|0 = a(x)^{3} + b(x)^{2} + 3(x) - 2$$

$$|0 = 8a + 4b + 4$$

$$|0 = 8a + 4b$$

$$|1 = 3a(-1)^{2} + 2b(-1) + 4$$

$$|1 = 3a - 2b(2)$$

$$f'(x) = 3\alpha x^{2} + 2bx + 3$$

$$f'(-1) = 14$$

$$14 = 3\alpha (-1)^{2} + 2b(-1) + 3$$

$$11 = 3\alpha - 2b(2)$$

$$\begin{array}{c} 2 = a \\ \rightarrow \text{Sub into } D \\ 3 = 4(2) + 2b \\ \hline \begin{pmatrix} -\frac{5}{2} & =b \end{pmatrix} \end{array}$$

$$f(x) = 2x^3 - \frac{5}{2}x^2 + 3x - 2$$