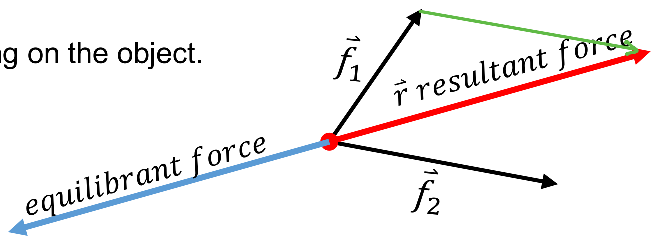
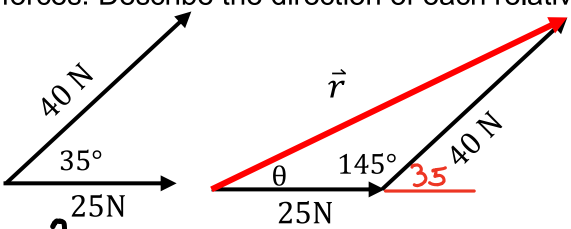
Forces As Vectors

The Resultant Vector
- When two or more forces are applied to an object, the net effect of the forces can be represented by the resultant vector.
- The resultant vector is the sum of all force vectors acting on the object.



The Equilibrant Vector
- The single force that opposes the resultant of the forces acting on the object
- When the equilibrant is applied to the object it maintains the object in a state of equilibrium (ie. No movement of the object)
- **Equilibrant = – Resultant**

Ex 1.
A 25 N force and a 40 N force act at an angle of 35° to each other. Determine the resultant and equilibrant forces. Describe the direction of each relative to BOTH original forces.



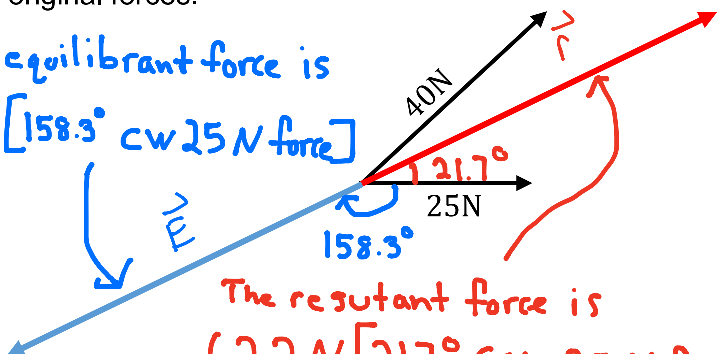
$|\vec{r}|^2 = 40^2 + 25^2 - 2(40)(25)\cos 145$

$|\vec{r}| = 62.2$

$\frac{\sin\theta}{40} = \frac{\sin 145°}{62.2}$

$\theta = 21.7°$

The equilibrant force is 62.2N[158.3° cw 25N force]

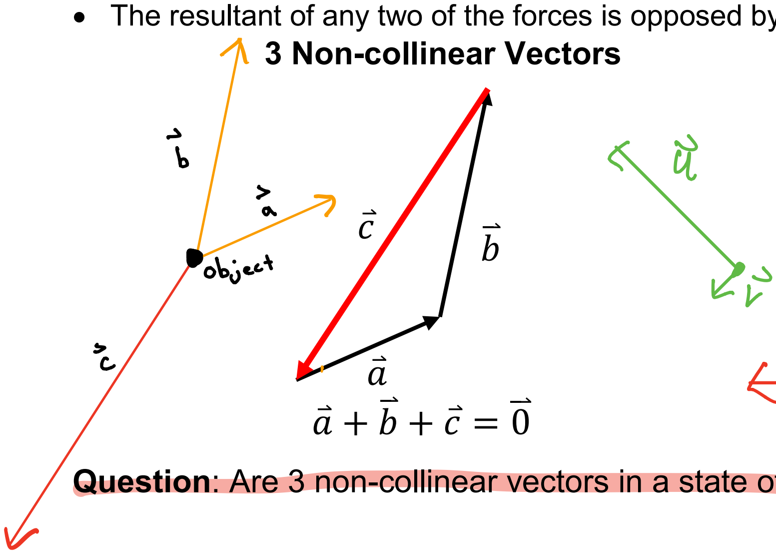


The resutant force is 62.2N[21.7° ccw 25N force]

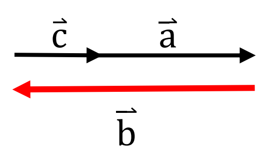
$\vec{r} = 62.2N[21.7°$ ccw 25N force]

Vectors in a State of Equilibrium
- When 3 non-collinear vectors are in a state of equilibrium and are arranged head to tail they form a <u>triangle</u>.
- The resultant of any two of the forces is opposed by the third force.

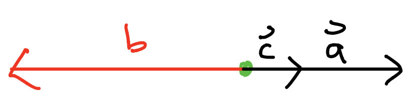
3 Non-collinear Vectors **3 Collinear Vectors**



$\vec{a} + \vec{b} + \vec{c} = \vec{0}$



$\vec{a} + \vec{b} + \vec{c} = \vec{0}$

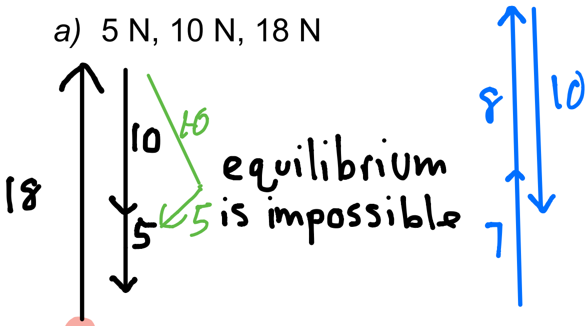


Question: Are 3 non-collinear vectors in a state of equilibrium coplanar?

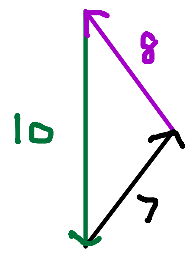
Yes ∵ A triangle in space <u>must</u> be coplanar.

Ex 2.
Which of the following sets of forces acting on an object could produce a state of equilibrium?

a) 5 N, 10 N, 18 N



b) 7 N, 8 N, 10 N

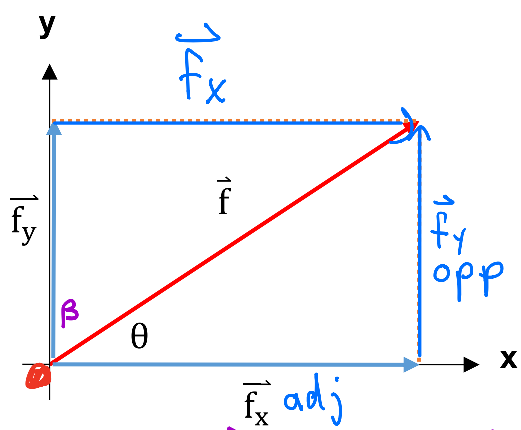


The largest force must be less than or equal to the sum of the other two.

Resolution of Vectors into Perpendicular Components

- Takes a single vector/force and decomposes it into 2 component vectors
- There are an infinite # of ways to resolve any vector, but it is most useful to use horizontal and vertical components

$\sin\theta = \dfrac{|\vec{f_y}|}{|\vec{f}|}$



$\sin\beta = \dfrac{|\vec{f_x}|}{|\vec{f}|}$ | $\cos\beta = \dfrac{|\vec{f_y}|}{|\vec{f}|}$

$|\vec{f}|\sin\beta = |\vec{f_x}|$ | $|\vec{f}|\cos\beta = |\vec{f_y}|$

From the diagram:

$\vec{f}$ is a force vector
$\vec{f_x}$ is the horizontal component of $\vec{f}$
$\vec{f_y}$ is the vertical component of $\vec{f}$

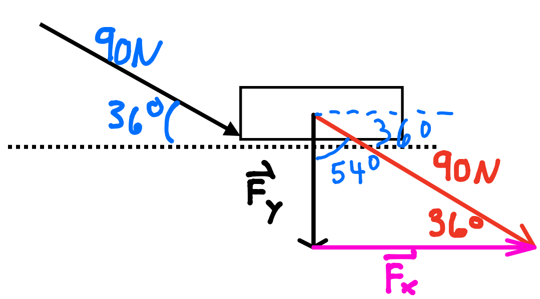
It follows that:

$\sin\theta = \dfrac{|\vec{f_y}|}{|\vec{f}|}$ | $\cos\theta = \dfrac{|\vec{f_x}|}{|\vec{f}|}$

$|\vec{f}|\sin\theta = |\vec{f_y}|$ | $|\vec{f}|\cos\theta = |\vec{f_x}|$

In general:
If $\vec{f}$ is resolved into horizontal and vertical components, $\vec{f_x}$ and $\vec{f_y}$ then $|\vec{f_x}| = |\vec{f}|\cos\theta$ and $|\vec{f_y}| = |\vec{f}|\sin\theta$, where θ is the angle that $\vec{f}$ makes with the x-axis.

Ex 3.
A lawnmower is pushed with a force of 90 N directed along the handle, which makes an angle of 36° with the ground. Determine the horizontal and vertical components of the force on the mower.



Horizontal

$\dfrac{|\vec{F_x}|}{90} = \cos 36°$

$|\vec{F_x}| = 90\cos 36°$
$= 72.8 N$

$\vec{F_x} = 72.8 N$ [right]

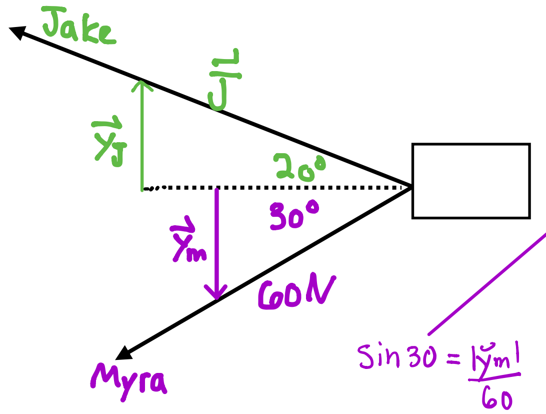
Vertical

$\dfrac{|\vec{F_y}|}{90} = \sin 36°$

$|\vec{F_y}| = 90 \sin 36$
$= 52.9 N$

$\vec{F_y} = 52.9 N$ [down]

Ex 4.
Jake and Myra are pulling their friend on a toboggan. Myra is pulling with a force of 60 N at an angle of 30° with the line of motion. If Jake pulls at an angle of 20° to the line of motion, how much force does he need to exert to keep the toboggan moving straight forward without turning?



$\sin 30 = \frac{|\vec{y}_m|}{60}$

To avoid turning...

$|\vec{y}_m| = |\vec{y}_J|$

$60\sin 30^\circ = |\vec{J}|\sin 20^\circ$

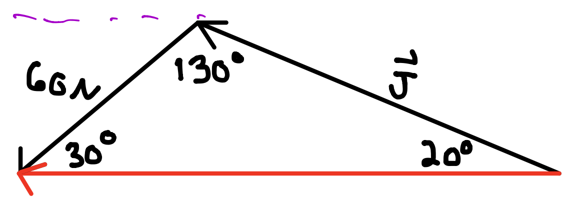
$\frac{60\sin 30^\circ}{\sin 20^\circ} = |\vec{J}|$

$87.7\,N = |\vec{J}|$

$\sin 20 = \frac{|\vec{y}_J|}{|\vec{J}|}$

$\vec{J} = 87.7\,N$ [20° from the line of motion]

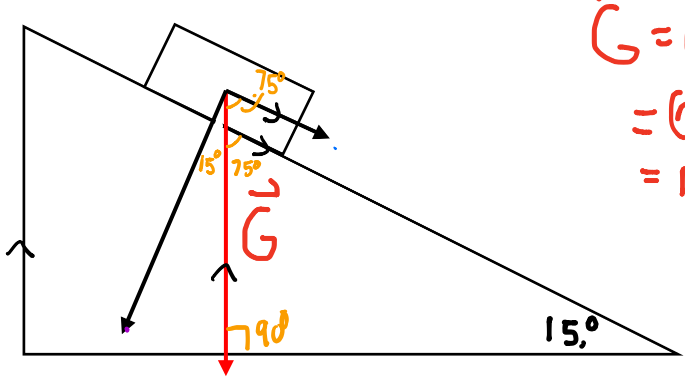
Alt.



$\frac{|\vec{J}|}{\sin 30^\circ} = \frac{60}{\sin 20^\circ}$

$|\vec{J}| = 87.7\,N$

Ex 5.
A 20 kg box is resting on a ramp inclined at an angle of 15°. Calculate the components of the force of gravity on the box that are parallel and perpendicular to the ramp.

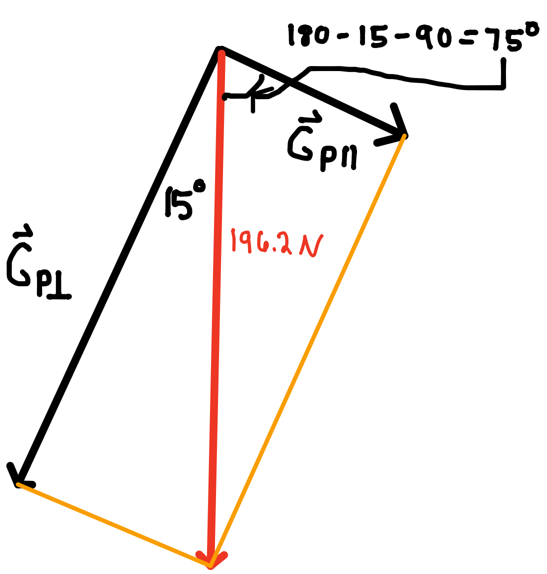


$\vec{G} = (m)(a)$

$= (20\text{ kg})(9.81\text{ m/s}^2)$

$= 196.2\,N$ [down]

$\vec{G} = \vec{F}_g$



$|\vec{G}_{p\parallel}| = 196.2\sin 15^\circ$

or

$= 196.2\cos 75^\circ$

$= 50.8\,N$

$|\vec{G}_{p\perp}| = 196.2\cos 15$

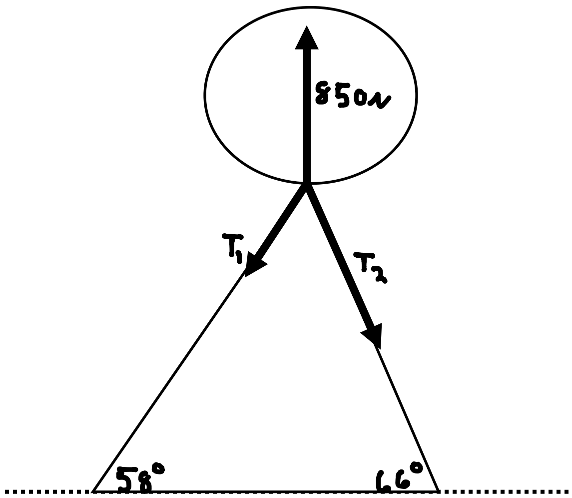
or

$= 196.2\sin 75^\circ$

$= 189.5\,N$

***Ex 6.**

A *large promotional balloon is tethered to the top of a building by two guy wires attached at 20 m apart. If the buoyant force is 850 N, and the two guy wires make angles of 58° and 66° with the horizontal, find the tension in each of the wires.*



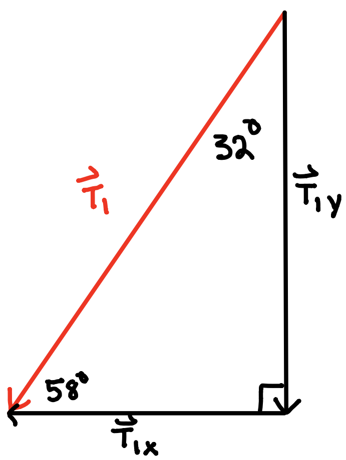
balloon is stationary so:

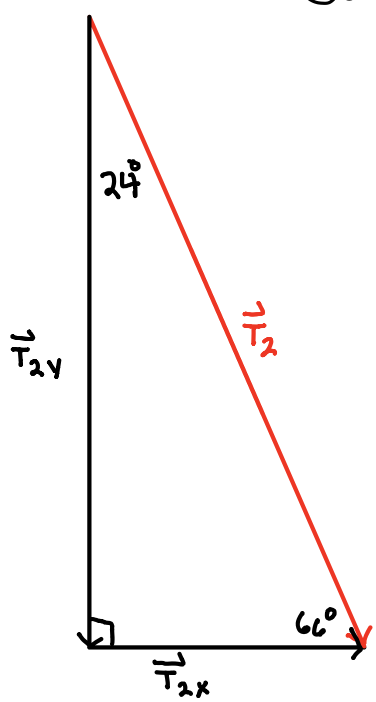
(1) $|\vec{T}_{1x}| = |\vec{T}_{2x}|$

and

(2) $|\vec{T}_{1y}| + |\vec{T}_{2y}| = 850N$

Goal → create a system of equations using $|\vec{T}_1|$ and $|\vec{T}_2|$





$|\vec{T}_{1y}| = |\vec{T}_1| \cos 32$

or

$= |\vec{T}_1| \sin 58$

$|\vec{T}_{1x}| = |\vec{T}_1| \cos 58$

or

$= |\vec{T}_1| \sin 32$

$|\vec{T}_{2y}| = |\vec{T}_2| \cos 24$

or

$= |\vec{T}_2| \sin 66$

$|\vec{T}_{2x}| = |\vec{T}_2| \cos 66$

or

$= |\vec{T}_2| \sin 24$

① $|\vec{T}_{1x}| = |\vec{T}_{2x}|$

$$|\vec{T}_1|\sin 32 = |\vec{T}_2|\sin 24$$

$$|\vec{T}_1| = \frac{|\vec{T}_2|\sin 24}{\sin 32}$$

② $|\vec{T}_{1y}| + |\vec{T}_{2y}| = 850\,N$

$$\underbrace{|\vec{T}_1|}_{①}\sin 58 + |\vec{T}_2|\sin 66 = 850$$

$$\left(\frac{|\vec{T}_2|\sin 24}{\sin 32}\right)\sin 58 + |\vec{T}_2|\sin 66 = 850$$

– factor out $|\vec{T}_2|$

$$|\vec{T}_2|\left(\frac{(\sin 24)(\sin 58)}{\sin 32} + \sin 66\right) = 850$$

$$|\vec{T}_2| = \frac{850}{\frac{(\sin 24)(\sin 58)}{\sin 32} + \sin 66}$$

$$|\vec{T}_2| = 543.3\,N$$

① $|\vec{T}_1| = \frac{543.3\sin 24}{\sin 32}$

$$|\vec{T}_1| = 417\,N$$