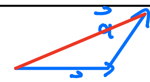
Properties of Vectors

Properties of Vector Addition and Scalar Multiplication

Commutative Property: $\vec{a} + \vec{b} = \vec{b} + \vec{a}$



Associative Property: $\underbrace{(\underbrace{\vec{a} + \vec{b}}_{\vec{r}_1}) + \vec{c}}_{\vec{r}_2} = \vec{a} + (\vec{b} + \vec{c})$

Summary:
The order in which vectors are added will not change the resultant.

Adding $\vec{0}$: $\vec{a} + \vec{0} = \vec{a}$

Summary:
Adding the zero vector has no effect on the resultant.

Distributive Property: $k(\vec{a} + \vec{b}) = k\vec{a} + k\vec{b}$

Associative Law for Scalars: $m(n\vec{a}) = (mn)\vec{a} = mn\vec{a}$

$3(2\vec{a}) = (3)(2)\vec{a}$

Distributive Law for Scalars: $(m + n)\vec{a} = m\vec{a} + n\vec{a}$

$\vec{a}(m+n)$

Summary:
The distributive property behaves the same with vectors as it does scalars.

Ex 1.
Write the following vector in simplified form. $2(1\vec{u} - 3\vec{v} - \vec{w}) - 3(2\vec{u} + 4\vec{v} + \vec{w})$

$= 2\vec{u} - 6\vec{v} - 2\vec{w} - 6\vec{u} - 12\vec{v} - 3\vec{w}$

$= -4\vec{u} - 18\vec{v} - 5\vec{w}$

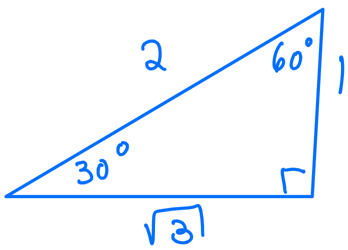
Ex 2.
If $\vec{x} = 3\vec{a} - 4\vec{b} + \vec{c}$ and $\vec{y} = 2\vec{b} + 3\vec{c}$ express $-\vec{x} + 3\,\vec{y}$ in terms of $\vec{a}, \vec{b}$, and $\vec{c}$.

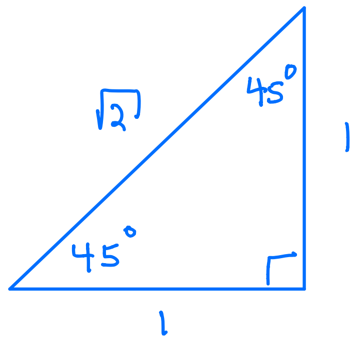
$-\vec{x} + 3\vec{y}$

$= -1(3\vec{a} - 4\vec{b} + \vec{c}) + 3(2\vec{b} + 3\vec{c})$

$= -3\vec{a} + 4\vec{b} - \vec{c} + 6\vec{b} + 9\vec{c}$

$= -3\vec{a} + 10\vec{b} + 8\vec{c}$





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