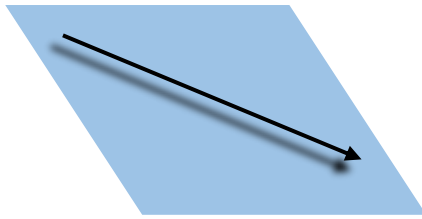
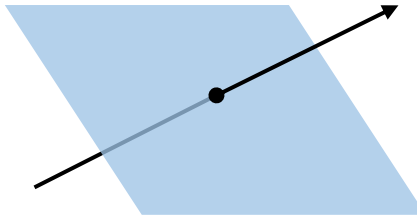


1. Intersection of a Line with a Plane

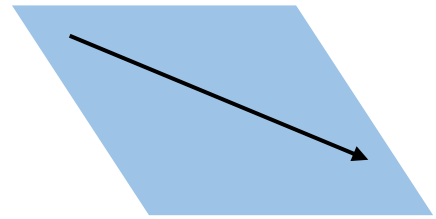
No Intersection (Parallel)



Point



Line Contained in Plane



Ex 1.

Determine the point of intersection of the line and plane.

a) L: $x = 3 + 4t$
 $y = 2 + 3t$
 $z = -1 + 2t$ } sub in \rightarrow $\pi: x - y - 2z = 0$

$$(3 + 4t) - (2 + 3t) - 2(-1 + 2t) = 0$$

$$3 + 4t - 2 - 3t + 2 - 4t = 0$$

$$3 - 3t = 0$$

$$t = 1 \leftarrow \text{a single } t \text{ value proves an intersection point}$$

where?

$$\begin{aligned} x &= 3 + 4(1) = 7 \\ y &= 2 + 3(1) = 5 \\ z &= -1 + 2(1) = 1 \end{aligned} \left. \vphantom{\begin{aligned} x &= 3 + 4(1) = 7 \\ y &= 2 + 3(1) = 5 \\ z &= -1 + 2(1) = 1 \end{aligned}} \right\} \text{intersect at } (7, 5, 1)$$

b) L: $\vec{r} = (2, -5, 6) + t(3, 1, 8)$ $\pi: 5x + y - 2z = 0$

$$\begin{aligned} x &= 2 + 3t \\ y &= -5 + t \\ z &= 6 + 8t \end{aligned} \left. \vphantom{\begin{aligned} x &= 2 + 3t \\ y &= -5 + t \\ z &= 6 + 8t \end{aligned}} \right\} \text{sub in}$$

$$5(2 + 3t) + (-5 + t) - 2(6 + 8t) = 0$$

$$10 + 15t - 5 + t - 12 - 16t = 0$$

$$0t = 7$$

$$LS \neq RS$$

- No solⁿ
- No intersection (p||)

c) L: $x = 2 + t$
 $y = -1 - 2t$
 $z = -3 - 5t$ $\pi: 3x + 19y - 7z - 8 = 0$

$$3(2 + t) + 19(-1 - 2t) - 7(-3 - 5t) - 8 = 0$$

$$6 + 3t - 19 - 38t + 21 + 35t - 8 = 0$$

$$0t = 0$$

$\rightarrow LS = RS$ but cannot solve for t

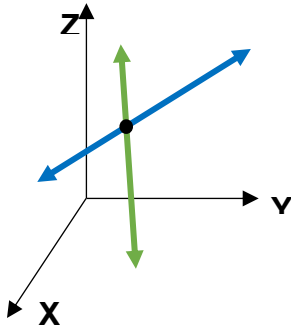
\rightarrow the line is in the plane

$$\left[\begin{array}{cccc|c} 3 & 19 & -7 & 0 & 8 \\ 1 & 0 & 0 & -1 & 2 \\ 0 & 1 & 0 & 2 & -1 \\ 0 & 0 & 1 & 5 & -3 \end{array} \right]$$

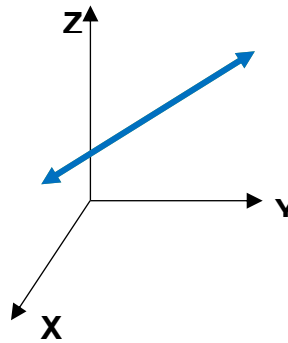


2. Intersection of Two Lines

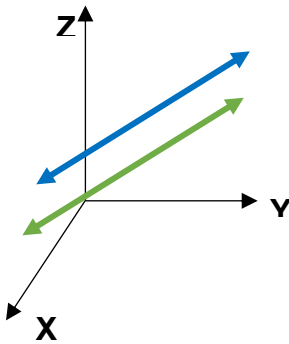
Intersect at One Point



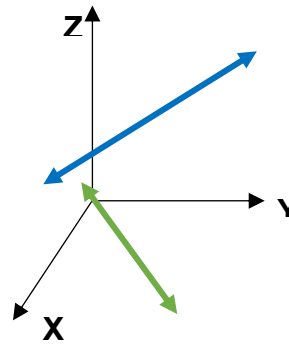
Intersect at Every Point (Coincident Lines)



Parallel Lines



Skew Lines



Ex 2.

Determine the point of intersection of the two lines.

a) $L1: \begin{cases} x = -5 + 3s \\ y = 2 + 2s \\ z = -7 + 6s \end{cases} \vec{m}_1 = (3, 2, 6)$ $L2: \begin{cases} x = t \\ y = -6 - 5t \\ z = -3 - t \end{cases} \vec{m}_2 = (1, -5, -1)$

not p11

→ either intersect at a point or skew

1st Set $L1 = L2$

$$\begin{cases} ① -5 + 3s = t \\ ② 2 + 2s = -6 - 5t \\ ③ -7 + 6s = -3 - t \end{cases}$$

2nd Solve for s or t using 2 eqⁿs

Sub t from ① into ②

$$2 + 2s = -6 - 5(-5 + 3s)$$

$$2 + 2s = -6 + 25 - 15s$$

$$17s = 17$$

$$s = 1$$

Sub s into ①

$$-5 + 3(1) = t$$

$$-2 = t$$

3rd Check s or t in 3rd eqⁿ

LS	RS
$-7 + 6(1)$	$-3 - (-2)$
-1	-1

$LS = RS \rightarrow$ intersect
- must sub s or t into $L1$ or $L2$

$L1: \begin{cases} x = -5 + 3(1) = -2 \\ y = 2 + 2(1) = 4 \\ z = -7 + 6(1) = -1 \end{cases}$
- the P.O.I is $(-2, 4, -1)$

b) L1: $\vec{OP} = (-2, 1, 0) + s(1, 3, 7)$ L2: $\vec{r} = (3, -3, 4) + t(5, -4, -2)$

- must have a P.O.I or be skew

① $-2 + s = 3 + 5t$

② $1 + 3s = -3 - 4t$

③ $7s = 4 - 2t$

① $\rightarrow s = 5 + 5t$

Sub s into ②

$1 + 3(5 + 5t) = -3 - 4t$

$19t = -19$

$t = -1$

Sub back into ①

$-2 + s = 3 + 5(-1)$

$s = 0$

Check $s + t$ in ③

LS	RS
$7(0)$	$4 - 2(-1)$
0	6

$LS \neq RS$

- lines are skew

$$\left[\begin{array}{cc|c} 1 & -1 & 5 \\ 3 & 4 & -4 \\ 7 & 2 & 4 \end{array} \right]$$

$$= \left[\begin{array}{cc|c} 1 & -1 & 5 \\ 0 & 7 & 11 \\ 0 & 9 & -31 \end{array} \right]$$

$7s = 11 \rightarrow s = \frac{11}{7}$
 $9s = -31 \rightarrow s = -\frac{31}{9}$ } inconsistent
 \therefore no Solⁿ
 skew

c) L1: $\begin{cases} x = 3 + 2t \\ y = -1 - 3t \\ z = 1 + t \end{cases} \vec{m}_1 = (2, -3, 1)$

L2: $\vec{r} = (1, -1, 3) + s(-4, 6, -2)$

$\vec{m}_2 = (-4, 6, -2)$

$= -2(2, -3, 1)$

$\vec{m}_2 = -2\vec{m}_1$

- direction vectors are collinear

\rightarrow lines are parallel or coincident

- Check for coincidence using any point on either line

Sub $(1, -1, 3)$ into L1

$$\begin{array}{l|l|l} 1 = 3 + 2t & -1 = -1 - 3t & 3 = 1 + t \\ -1 = t & 0 = t & 2 = t \end{array}$$

- inconsistent!

- they are pll but not coincident