Critical Points, Local Maxima/Minima

Critical values are x-values where $\boxed{f'(x) = 0 \text{ OR } f'(x) \text{ is undefined}}$. This implies that all local max/mins occur at a critical value. Keep in mind however:

cusp
corner

- Not all critical values yield a max/min (do the 1st derivative test)
- Critical values where f'(x) is undefined are the locations of cusps, corners, vertical tangents and discontinuities

max/min

1st Derivative Test
If x = c is a critical value, then there is a: • Local maximum if f'(x) changes from a positive to a negative at c • Local minimum if f'(x) changes from a negative to a positive at c

Ex 1.
Given this strip for f'(x), identifiy:

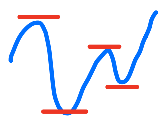
double zero
Note:
V. Asymptote exists on f(x) as well
f'(x)=0

f'(x)	+	‖	+	⁞	−	|	+
		-2		3		5	

Critical values:	$x = -2, 3, 5$
Intervals of increase:	$x < -2$, $-2 < x < 3$, $x > 5$
Intervals of decrease:	$3 < x < 5$
Local max:	none
Local min:	@ $x = 5$

Ex 2.
Determine the critical values and local extrema.

$y = \frac{1}{2}x^4 - \frac{16}{3}x^3 + 16x^2$



$y' = 2x^3 - 16x^2 + 32x$

Let $y' = 0$

$0 = 2x(x^2 - 8x + 16)$

$0 = 2x(x-4)^2$

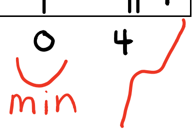
can y' be undef.?
-no!

$\frac{\#}{0}$ or $\frac{0}{0}$ or $\sqrt{-\#}$

asym. hole

C.N. $\rightarrow x = 0, 4$

f'(x)	−	|	+	‖	+
		0		4	



min
(0,0)

a) $y = x^3$

$y' = 3x^2$

Let $y' = 0$

$0 = 3x^2$

can y' be undef?
no!

C.N. $\rightarrow x = 0$

f'(x)	+	‖	+
		0	

no max/min

c) $f(x) = (3x-6)^{\frac{2}{3}}$

$f'(x) = \frac{2}{3}(3x-6)^{-\frac{1}{3}}(3)$

$f'(x) = 0$

$0 = \frac{2}{\sqrt[3]{3x-6}}$

impossible

$f'(x) = \text{und.}$

$f'(2) = \frac{2}{\sqrt[3]{0}}$

$= \frac{2}{0}$

V. Asym @ $x=2$

f(x)	+	+
f'(x)	−	+

2

min

→ $f'(2)$ is not defined, but a minimum exists

d) $f(x) = \sqrt[3]{x-6}$

$f'(x) = \frac{1}{3}(x-6)^{-\frac{2}{3}}$

$= \frac{1}{3\sqrt[3]{(x-6)^2}}$

C.N → no zeros

→ undef. → V. Asymp. @ $x=6$ {for $f'(x)$}

$\frac{\#}{0}$

f'(x)	+	+

6

→ no max/min

e) $f(x) = \frac{x^2-9}{x-3}$

* hole @ $x=3$ $\frac{0}{0}$

$f(x) = \frac{(x-3)(x+3)}{(x-3)}$

$= x+3, \ x \neq 3$

$f'(x) = 1, \ x \neq 3$

C.N → hole @ $x=3$

discontinuity

f'(x)	+	o	+

3

no max/min

f) $f(x) = |x-3|$

$= -(x-3) \quad x \leq 3$

$= x-3 \quad x \geq 3$

$f'(x) = -1 \quad x < 3$

$= 1 \quad x > 3$

C.N → discontinuity @ $x=3$

f(x)			
f'(x)	−	o	+

3

min

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- Local maximum if f'(x) changes from a positive to a negative at c
- Local minimum if f'(x) changes from a negative to a positive at c

Ex 1.

Given this strip for f'(x), identifiy:

f'(x)

Critical values:	
Intervals of increase:	
Intervals of decrease:	
Local max:	
Local min:	

Ex 2.

Determine the critical values and local extrema.

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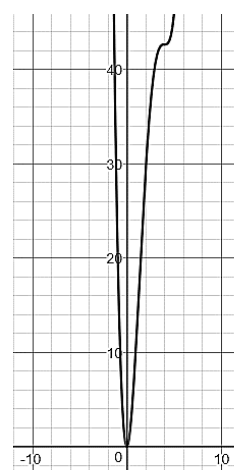
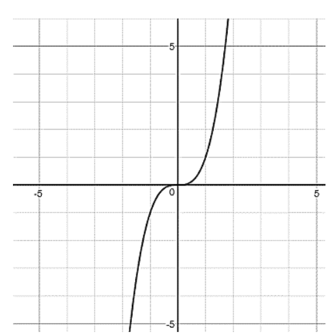
f'(x)



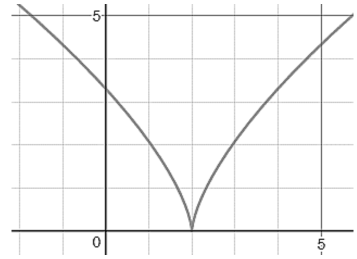
b) $y = x^3$

f'(x)



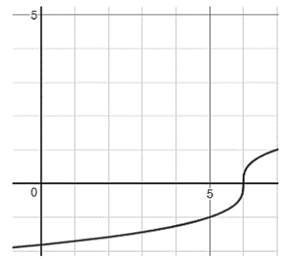
c) $f(x) = (3x - 6)^{\frac{2}{3}}$

f'(x)



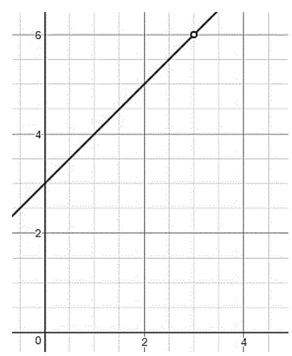
d) $f(x) = \sqrt[3]{x - 6}$

f'(x)



e) $f(x) = \frac{x^2-9}{x-3}$

f'(x)



f) $f(x) = |x - 3|$

f'(x)