Velocity

$d(t) \rightarrow$ distance

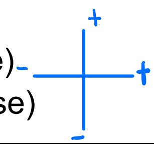
$s(t) \rightarrow$ displacement

Velocity: Rate of change of position per unit of time.

Average velocity: $\frac{\textbf{change in displacement}}{\textbf{change in time}} = \frac{\Delta s}{\Delta t}$ } Secant

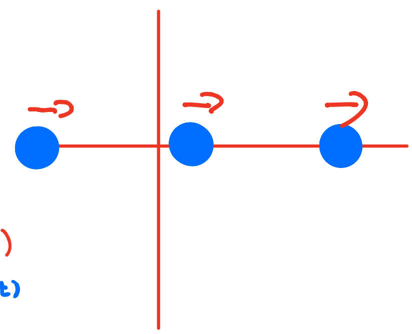
Instantaneous velocity: $v = \frac{d}{dt}s$ } tangent or $v(t) = s'(t)$

> **REMINDER:**
>
> - Velocity is a vector, so it has both a magnitude and direction.
> - Speed is a scalar, so it has only a magnitude
> - speed = |velocity|
>
>   v(t) > 0 → motion is $\boxed{\text{upward}}$ or to the $\boxed{\text{right}}$ (in this course)
>
>   v(t) < 0 → motion is $\boxed{\text{downward}}$ or to the $\boxed{\text{left}}$ (in this course)



Ex 1.
A particle moving along a horizontal axis has **displacement** from the origin **s(t) = t³ – 4t² + 4t – 1** at time **t seconds**. Determine:

a) initial position
b) average velocity $2 \le t \le 5$
c) instantaneous velocity at t = 1 & t = 2 seconds
d) direction in which the particle is moving at t = 3 seconds



a) $s(0) = 0^3 - 4(0)^2 + 4(0) - 1$
$= -1$ unit or $\vec{s} = 1$ unit [left]

b) $s(2) = -1$
$s(5) = 44$

$m = \frac{y_2 - y_1}{x_2 - x_1}$

$V_{avg} = \frac{s(5) - s(2)}{5 - 2}$ $\frac{\Delta s}{\Delta t}$

$= \frac{44 + 1 \text{ units}}{3 \text{ sec}}$

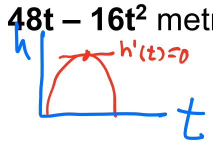
$= 15$ units/sec

c) $v(t) = s'(t)$
$v(t) = \frac{d}{dt} s(t)$
$= 3t^2 - 8t + 4$

$v(1) = -1$ units/sec or $\vec{v} = 1$ u/s [left]
$v(2) = 0$ units/sec

d) $v(3) = 7$
↑ positive velocity so moving to the right

Ex 2.
A model rocket is launched vertically upward and at **t seconds** has a height **h(t) = 48t – 16t²** metres. What is the maximum height?



– rocket will come to a stop for an instant at its maximum height
– $V(x) = 0$

$h(t) = 48t - 16t^2$
$h'(t) = V(t) = 48 - 32t$
Let $v(t) = 0$
$0 = 48 - 32t$
$\frac{3}{2} = t$

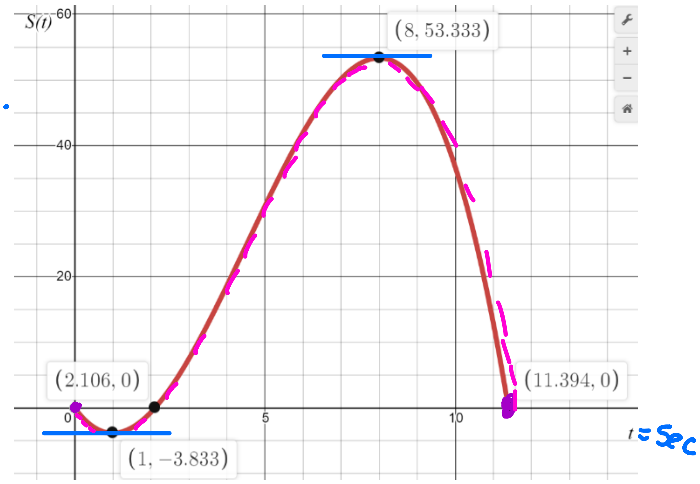
$h(\frac{3}{2}) = 48(\frac{3}{2}) - 16(\frac{3}{2})^2$
$h(\frac{3}{2}) = 36$ m

– max height is 36 m

Ex 3.

The graph shown displays the position of an object moving in a ~~straight~~ line. Answer the following questions.

vertical



a) When is the velocity zero? $s'(t)=0$

m_{tan} = horiz.

$t = 1$ sec | $t = 0$

$t = 8$ sec | $t = 11.394$

b) When is the object moving in a positive direction $s'(t) = pos$

$(1, 8)$ OR $1 < x < 8$

c) When is the object moving in a negative direction? $s'(t) = neg.$

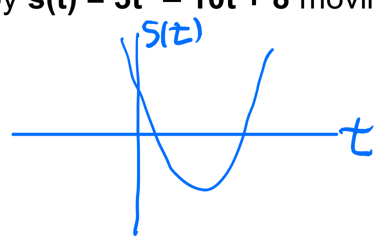
$0 < x < 1$

$8 < x < 11.394$

***Ex 4. CHALLENGE**

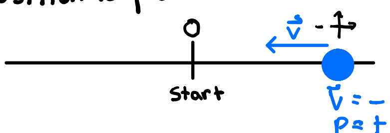
When is the particle whose position is described by **s(t) = 3t² – 10t + 8** moving toward the x-axis? ?

vert. disp.

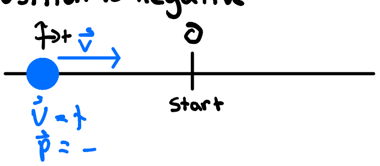


An object is moving towards the x-axis if:

- its velocity is negative and its position is positive



- its velocity is positive and its position is negative



$s(t)v(t) < 0$

Corollary:

$\boxed{\text{if } s(t)v(t) > 0 \text{, then the object is moving away from the x-axis}}$

Where is S(t) positive and negative?

Let $S(t) = 0$

$0 = 3t^2 - 10t + 8$

$0 = (t-2)(3t-4)$

$t = 2$ & $\frac{4}{3}$

where is v(t) positive and negative?

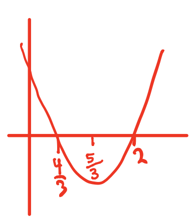
$v(t) = 6t - 10$

Let $v(t) = 0$

$\frac{5}{3} = t$

S(t)	+	−	−	+
		$\frac{4}{3}$		2

S'(t)	−	−	+	+
			$\frac{5}{3}$	



$\ominus = \oplus$ ±

The object is moving towards its starting position when

$0 < t < \frac{4}{3}$, $\frac{5}{3} < t < 2$

$(0, \frac{4}{3})$ $(\frac{5}{3}, 2)$

Where is $S(t)\cdot V(t) < 0$?

Let $S(t)\cdot V(t) = 0$

$(t-2)(3t-4)$ $2(3t-5)$

$2(3t-5)(t-2)(3t-4) = 0$

$t = \frac{5}{3}, 2, \frac{4}{3}$

S(t)·V(t)	−	+	−	+
		$\frac{4}{3}$	$\frac{5}{3}$	2

$0 < t < \frac{4}{3}$

$(0, \frac{4}{3})$

$\frac{5}{3} < t < 2$

$(\frac{5}{3}, 2)$