

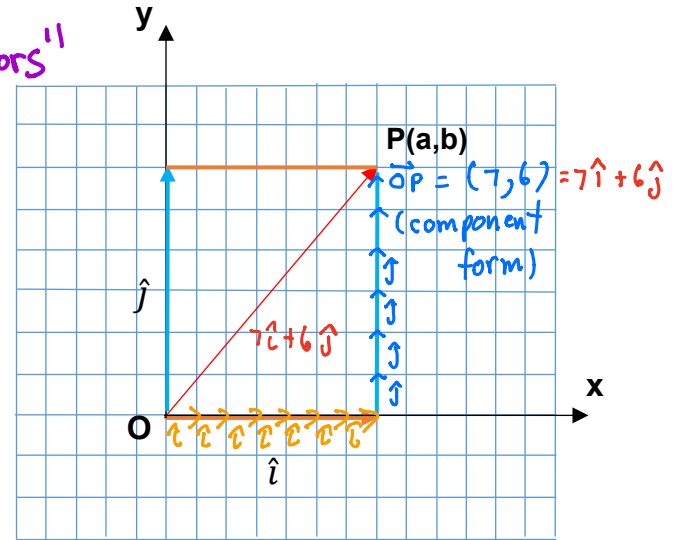
## Operations of Vectors in 2-D

**Focus:** Use component vectors and "basis unit vectors" interchangeably when performing vector operations such as addition and subtraction.

### UNIT VECTORS

"Standard unit vectors"

- Let  $\hat{i}$  and  $\hat{j}$  represent the basis unit vectors directed along the positive x-axis and positive y-axis respectively
- Every vector  $\vec{OP}$  in the plane can be written:
  - in component form  $\vec{OP} = (a, b)$  or
  - using unit vectors such that  $\vec{OP} = a\hat{i} + b\hat{j}$
- $(a, b) = a\hat{i} + b\hat{j}$   
 $|\vec{OP}|^2 = a^2 + b^2$
- $|\vec{OP}| = \sqrt{a^2 + b^2}$



Ex 1.

- a) Write the following using the unit vectors  $\hat{i}$  and  $\hat{j}$ .

$$\vec{OA} = (2, -3) \\ = 2\hat{i} - 3\hat{j}$$

$$\vec{OB} = (0, 5) \\ = 5\hat{j}$$

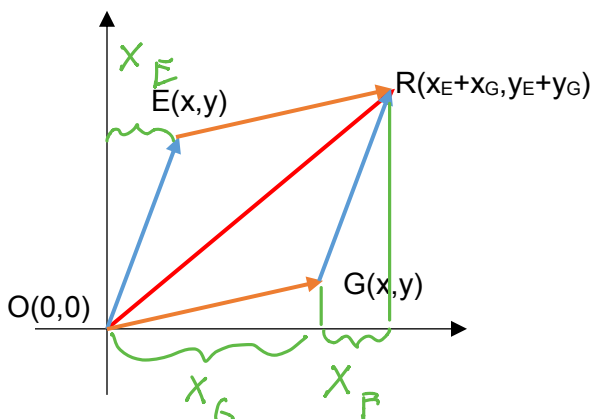
- b) The following vectors are written using unit vectors. Write them in component form.

$$\vec{OC} = -2\hat{i} - 5\hat{j} \\ = (-2, -5)$$

$$\vec{OD} = -4\hat{i} \\ = (-4, 0)$$

### POSITION VECTOR ADDITION AND SUBTRACTION

- Add (or subtract) the corresponding x and y-components of each vector.
- Add (or subtract) the corresponding  $\hat{i}$  and  $\hat{j}$  unit vectors of each vector.



$$\vec{OR} = \vec{OE} + \vec{OG} = (x_E + x_G, y_E + y_G)$$

Ex 2.

a) Calculate  $\vec{a} + \vec{b}$  a.k.a  $\vec{OA} + \vec{OB}$

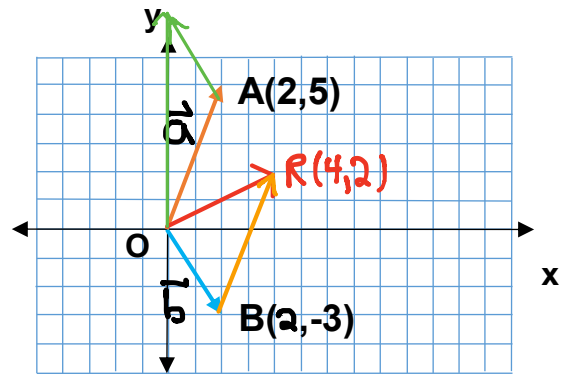
$$\vec{OA} + \vec{OB} = (2, 5) + (2, -3)$$

$$\vec{R} = (4, 2)$$

b) Calculate  $\vec{OA} - \vec{OB}$

$$\vec{OA} - \vec{OB} = 2\hat{i} + 5\hat{j} - (2\hat{i} - 3\hat{j})$$

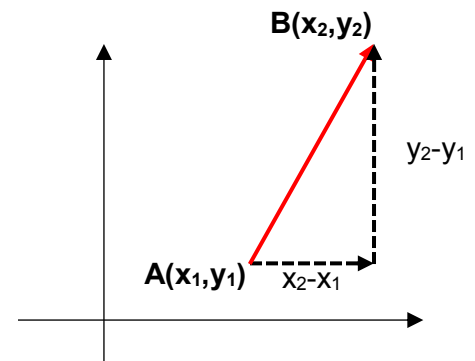
$$= 8\hat{j}$$



## VECTORS DEFINED BY TWO POINTS (THAT ARE NOT POSITION VECTORS)

If a vector is defined by two points  $A(x_1, y_1)$  and  $B(x_2, y_2)$  then

- $\vec{AB} = (x_2 - x_1, y_2 - y_1)$
- $|\vec{AB}| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$



Ex 3.

Given points  $A(2, 3)$  and  $B(-5, -2)$ , determine  $\vec{AB}$  and  $\vec{BA}$ .

$$\vec{AB} = B - A$$

$$= (-5, -2) - (2, 3)$$

$$\vec{AB} = (-5 - 2, -2 - 3)$$

$$\vec{AB} = (-7, -5)$$

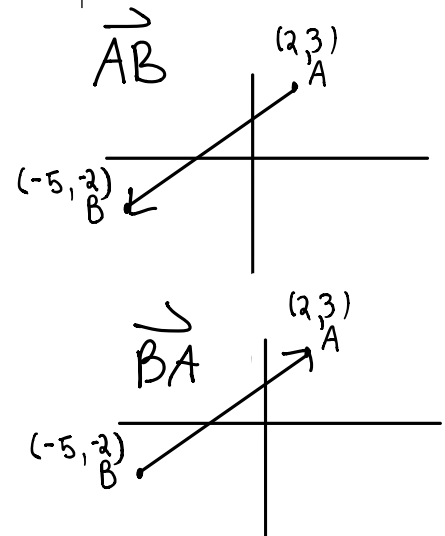
$$\vec{BA} = -(-7, -5)$$

$$\vec{BA} = A - B$$

$$\vec{BA} = (2 - (-5), 3 - (-2))$$

$$= (7, 5)$$

$$\vec{AB} = -\vec{BA}$$



Ex 4.  $\vec{r} = (7, 5)$

Given  $\vec{u} = (-2, 4)$  and  $\vec{v} = (4, -3)$ , calculate

a)  $|\vec{u}|$

$$|\vec{u}| = \sqrt{(-2)^2 + 4^2}$$

$$= \sqrt{20} = \sqrt{4(5)}$$

$$= 2\sqrt{5}$$

$$\sqrt{20} = \sqrt{4 \times 5}$$

$$= \sqrt{4} \sqrt{5}$$

$$= 2\sqrt{5}$$

b)  $|\vec{u} - \vec{v}| = |\vec{r}|$

$$\vec{u} - \vec{v} = (-2 - 4, 4 - (-3))$$

$$\vec{r} = (-6, 7)$$

$$|\vec{u} - \vec{v}| = \sqrt{36 + 49}$$

$$|\vec{r}| = \sqrt{85}$$