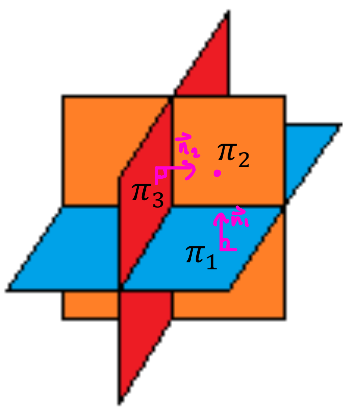
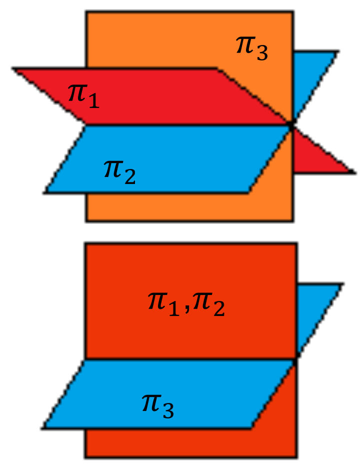
Intersection of Three Planes

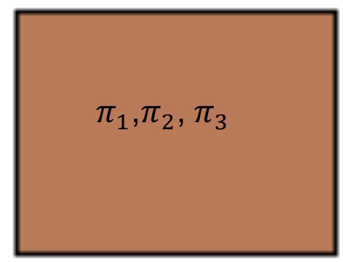
Given Three planes in R3, there are six possible geometric models for the intersection of the planes.

CONSISTENT SYSTEMS

CASE 1	CASE 2	CASE 3
• Two planes intersect at a point • There is exactly one solution	• The planes intersect in a line • There are an infinite # of solutions • Requires the use of 1 parameter	• The planes intersect in a plane (3 coincident planes) • There are an infinite number of solutions • Requires the use of 2 parameters.
Normals	**Normals**	**Normals**
• Not parallel, not coplanar.	• Not parallel, coplanar.	• Parallel, coplanar

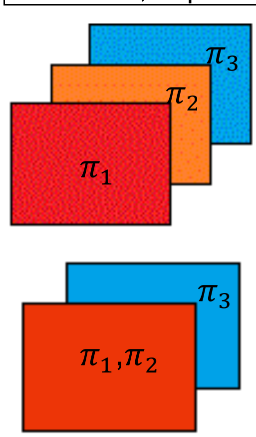


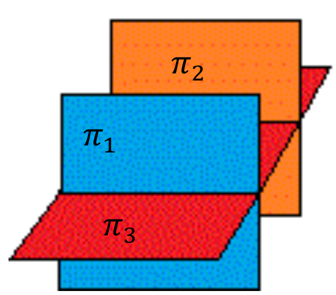


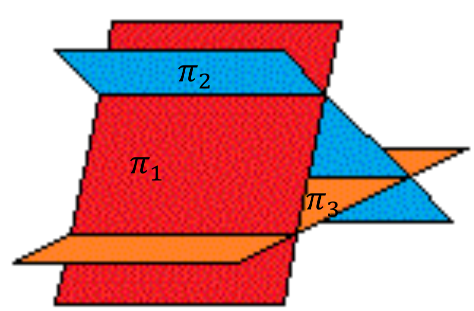


INCONSISTENT SYSTEMS

CASE 4	CASE 5	CASE 6
• Three planes are parallel and at least 2 are distinct	• Two planes are parallel and distinct • The third plane is not parallel to the other two	• The planes intersect in pairs • The pairs intersect in lines that are parallel and distinct
Normals	**Normals**	**Normals**
• Parallel, coplanar	• Not parallel, coplanar.	• Not parallel, coplanar

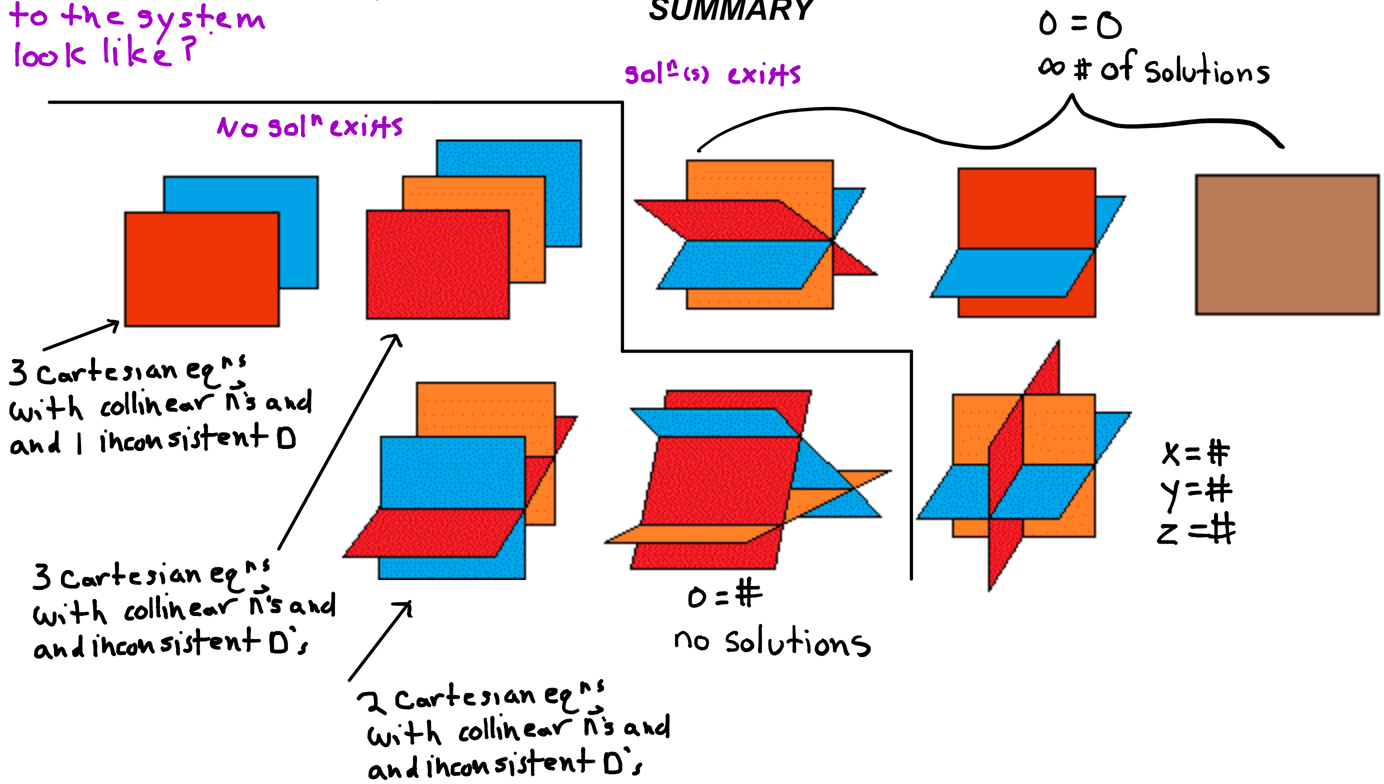






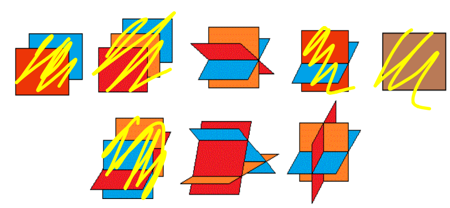
What will the solution to the system look like?

SUMMARY



Ex 1.

Determine the solution. Describe the planes.



a) π_1: $2x + y + 6z - 7 = 0$ $\quad \vec{n}_1 = (2,1,6)$

π_2: $3x + 4y + 3z + 8 = 0$ $\quad \vec{n}_2 = (3,4,3)$

π_3: $x - 2y - 4z - 9 = 0$ $\quad \vec{n}_3 = (1,-2,-4)$

No multiples

$(3, -5, 1)$

$$\left[\begin{array}{ccc|c} 2 & 1 & 6 & 7 \\ 3 & 4 & 3 & -8 \\ 1 & -2 & -4 & 9 \end{array}\right] \quad \left[\begin{array}{ccc|c} 1 & -2 & -4 & 9 \\ 3 & 4 & 3 & -8 \\ 2 & 1 & 6 & 7 \end{array}\right] \begin{array}{l} \\ -3R1 \\ -2R1 \end{array} \quad \left[\begin{array}{ccc|c} 1 & -2 & -4 & 9 \\ 0 & 10 & 15 & -35 \\ 0 & 5 & 14 & -11 \end{array}\right] \begin{array}{l} \\ R_2 \div 10 \\ R_3 \div 5 \end{array}$$

$$\left[\begin{array}{ccc|c} 1 & -2 & -4 & 9 \\ 0 & 1 & \frac{3}{2} & -\frac{7}{2} \\ 0 & 1 & \frac{14}{5} & -\frac{11}{5} \end{array}\right] \begin{array}{l} \\ \\ R_3 - R_2 \end{array} \quad \left[\begin{array}{ccc|c} 1 & -2 & -4 & 9 \\ 0 & 1 & \frac{3}{2} & -\frac{7}{2} \\ 0 & 0 & \frac{13}{10} & \frac{13}{10} \end{array}\right]$$

<u>Solve Row 3</u>

$\frac{13}{10} z = \frac{13}{10}$

$z = 1$

<u>Solve Row 2</u>

$y + \frac{3}{2}(1) = -\frac{7}{2}$

$y = -\frac{10}{2}$

$y = -5$

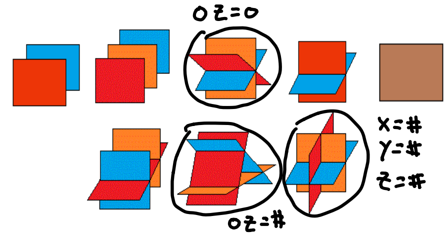
<u>Solve Row 1</u>

$x - 2(-5) - 4(1) = 9$

$x + 10 - 4 = 9$

$x = 3$

b) ① $x - 5y + 2z - 10 = 0$ $\vec{n}_1 = (1,-5,2)$
② $x + 7y - 2z + 6 = 0$ $\vec{n}_2 = (1,7,-2)$
③ $8x + 5y + z - 20 = 0$ $\vec{n}_3 = (8,5,1)$



$$\left[\begin{array}{ccc|c} 1 & -5 & 2 & 10 \\ 1 & 7 & -2 & -6 \\ 8 & 5 & 1 & 20 \end{array}\right] \begin{array}{l} \\ R_2 - R_1 \\ R_3 - 8R_1 \end{array} \left[\begin{array}{ccc|c} 1 & -5 & 2 & 10 \\ 0 & 12 & -4 & -16 \\ 0 & 45 & -15 & -60 \end{array}\right] \begin{array}{l} \\ R_2 \div 12 \\ R_2 \div 45 \end{array} \left[\begin{array}{ccc|c} 1 & -5 & 2 & 10 \\ 0 & 1 & -\frac{1}{3} & -\frac{4}{3} \\ 0 & 1 & -\frac{1}{3} & -\frac{4}{3} \end{array}\right] \begin{array}{l} \\ \\ R_3 - R_2 \end{array}$$

$$\left[\begin{array}{ccc|c} 1 & -5 & 2 & 10 \\ 0 & 1 & -\frac{1}{3} & -\frac{4}{3} \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$0x + 0y + 0z = 0$
infinite $soln^s$

Let $z = t$

ignore

$y - \frac{1}{3}t = -\frac{4}{3}$

$y = \frac{1}{3}t - \frac{4}{3}$

$x - 5y + 2z = 10$

$x - 5\left(\frac{1}{3}t - \frac{4}{3}\right) + 2t = 10$

$x - \frac{5}{3}t + \frac{20}{3} + 2t = 10$

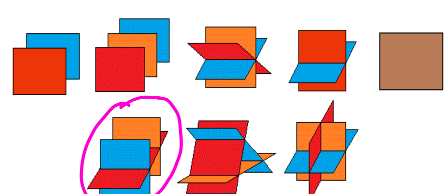
$x + \frac{1}{3}t + \frac{20}{3} = \frac{30}{3}$

parametric sol^n of line

$x = -\frac{1}{3}t + \frac{10}{3}$

c) ① $3x + y - 2z = 7$ $\vec{n}_1 = (3,1,-2)$
② $x - 5y + z = 8$ $\vec{n}_2 = (1,-5,1)$
③ $12x + 4y - 8z = -4$ $\vec{n}_3 = (12,4,-8)$

×4 2 pll planes



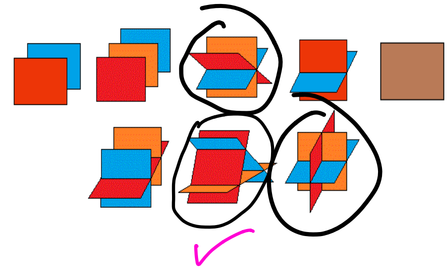
Check if ① + ③ are the same π

$D_1 = -7$
$D_4 = 4$
×4? No!

No Solution

d) ① $x + 3y - z = -10$ $\quad \vec{n}_1 = (1,3,-1)$
② $2x + y + z = 8$ $\quad \vec{n}_2 = (2,1,1)$
③ $x - 2y + 2z = -4$ $\quad \vec{n}_3 = (1,-2,1)$

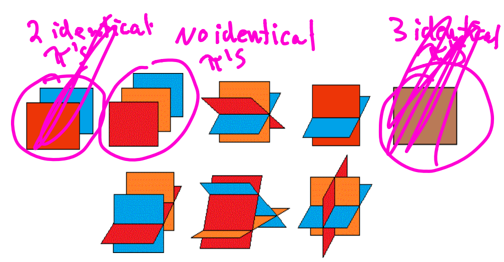
$\left.\begin{matrix} \\ \\ \\ \end{matrix}\right\}$ No multiples



$$\left[\begin{array}{ccc|c} 1 & 3 & -1 & -10 \\ 2 & 1 & 1 & 8 \\ 1 & -2 & 2 & -4 \end{array}\right] \quad \left[\begin{array}{ccc|c} 1 & 3 & -1 & 10 \\ 0 & -5 & 3 & 28 \\ 0 & -5 & 3 & 6 \end{array}\right] \begin{array}{l} \\ R_2 - 2R_1 \\ R_3 - R_1 \end{array} \quad \left[\begin{array}{ccc|c} 1 & 3 & -1 & 10 \\ 0 & -5 & 3 & 28 \\ 0 & 0 & 0 & -22 \end{array}\right] \begin{array}{l} \\ \\ R_3 - R_2 \end{array}$$

$0x + 0y + 0z = -22$

no solution



e) ① $4x - 2y + 6z = 35$ $\quad \vec{n}_1 = (4,-2,6)$ $\quad \times -\frac{5}{2}$
② $-10x + 5y - 15z = 20$ $\quad \vec{n}_2 = (-10,5,-15)$
③ $6x - 3y + 9z = -50$ $\quad \vec{n}_3 = (6,-3,9)$ $\quad \times -\frac{3}{5}$

Are all 3 π's the same?

$D_1 = -35$
$\quad \times -\frac{5}{2}$ no!
$D_2 = -20$
$\quad \times -\frac{3}{5}$ no!
$D_3 = 50$

3 pll but distinct planes