Velocity as a Vector

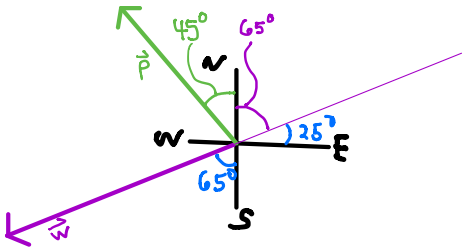
Focus: Understand that the velocity of an object is stated relative to a frame of reference, i.e. the frame of reference used influences the velocity.

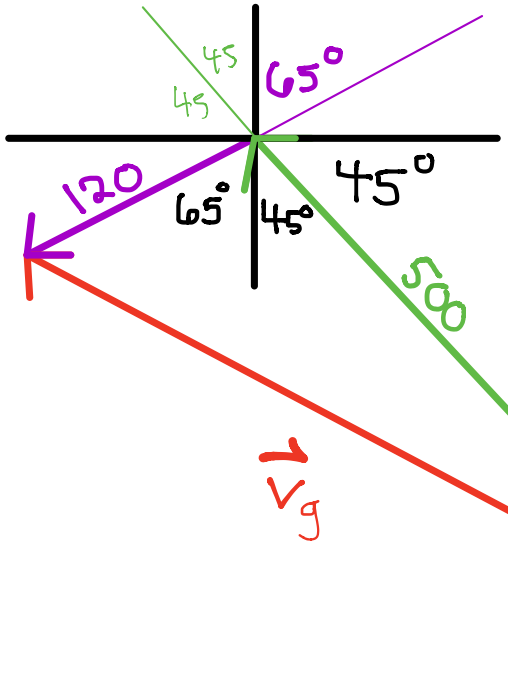
NOTE:

Air Speed/ Water Speed
The speed of an object relative to the frame of reference of the air current or water current.
i.e. The speed of an object such as a plane/boat as measured (observed) by a person being carried by the current.

Ground Speed
The speed of an object moving relative to the frame of reference of the ground.
i.e. The speed of an object such as a plane or boat as measured (observed) by a person on the ground… it includes the effect of the wind or current.

Ex 1.
An airplane heading northwest at 500 km/h encounters a wind of 120 km/h from N65°E. Determine the resultant ground velocity of the plane (determine the velocity with respect to the ground frame of reference).





$90^\circ - 45^\circ - 12^\circ = 33^\circ$

$|\vec{r}|^2 = 500^2 + 120^2 - 2(500)(120)\cos 110$

$|\vec{r}| = 552.7$

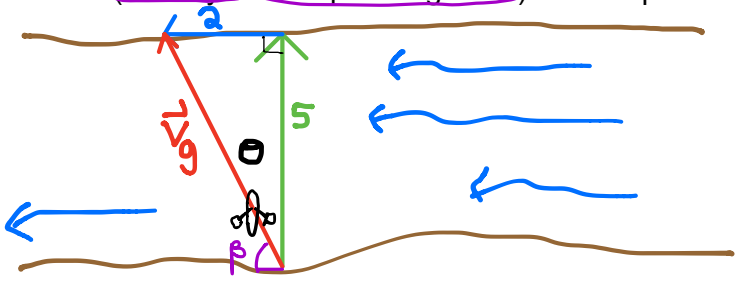
$\frac{\sin\theta}{120} = \frac{\sin 110^\circ}{552.7}$

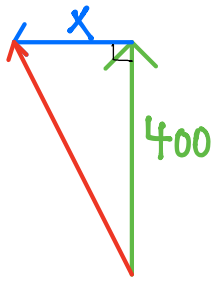
$\theta = 12^\circ$

$\vec{v}_g = 552.7\ \text{km/h}\ [N57^\circ W]$
$[W33^\circ N]$

Ex 2.
A canoeist who can paddle at a speed of 5 km/h in still water will cross a river 400 m wide that has a 2 km/h current.

a) If she steers the canoe in a direction perpendicular to the current, determine the resultant velocity (velocity with respect to ground) and the point on the opposite bank where the canoe arrives.





$\frac{x}{2} = \frac{400}{5}$

$x = 160$ m downstream

$|\vec{r}| = \sqrt{5^2+2^2}$ $\tan\theta = \frac{2}{5}$

$= \sqrt{29}$ $\theta = 21.8°$

$\vec{v}_g = \sqrt{29}$ km/h [21.8° downstream from the original heading]

[68.2° from the shore]

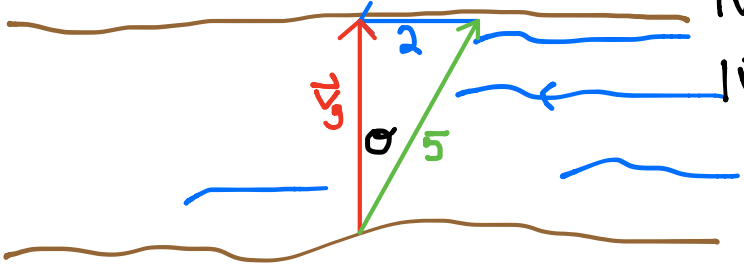
$t_{cross} = \frac{0.4}{5} = 0.08$ h

$d = 2 \times 0.08$

$= 0.16$ km

$= 160$ m

b) If she would like to travel in straight line perpendicular to the shore, determine the direction she must head and the time that it will take her to cross the river.



$|\vec{v}_g|^2 + 2^2 = 5^2$

$|\vec{v}_g| = \sqrt{21}$

$= 4.6$ km/h

$\sin\theta = \frac{2}{5}$

$\theta = 23.6°$

$t = \frac{d}{5}$ (400 m river)

$= \frac{0.4 \text{ km}}{4.6 \text{ km/h}}$

$= 0.09$ h

∴ She should head 23.6° upstream.
The trip will take 5 min 24 sec
or
if using $\frac{0.4}{4.6}$
5 min 13 sec