$f(x) = x^5$

$f'(x) = 5x^4$

$f''(x) = 5(4)x^3 = 20x^3$

$m = \frac{\Delta y}{\Delta x}$ $\quad \frac{d}{dx} y$

Higher Order Derivatives

- The derivative of a derivative is called the ***second derivative*** and can be represented by :

$$\mathbf{f''(x)} \leftarrow \text{NEWTON} \quad \text{or} \quad \frac{\mathbf{d^2y}}{\mathbf{dx^2}} \leftarrow \text{LEIBNIZ} \quad \frac{d}{dx}\left(\frac{d}{dx}y\right) = \frac{d^2y}{(dx)^2}$$

- The derivative of the second derivative is called the ***third derivative*** and can be represented by:

$$\mathbf{f'''(x)} \leftarrow \text{NEWTON} \quad \text{or} \quad \frac{\mathbf{d^3y}}{\mathbf{dx^3}} \leftarrow \text{LEIBNIZ} \quad \frac{d}{dx}\left(\frac{d}{dx}\left(\frac{d}{dx}y\right)\right)$$

Ex 1.
Determine the second derivitave of

a) $y = (2x-1)^4$

$y' = 4(2x-1)^3(2)$

$= 8(2x-1)^3$

$y'' = 8(3)(2x-1)^2(2)$

$y'' = 48(2x-1)^2$

b) $y = \frac{x}{x^2-1} = x(x^2-1)^{-1}$

$\frac{dy}{dx} = \frac{1(x^2-1) - x(2x)}{(x^2-1)^2}$

$= \frac{-x^2-1}{(x^2-1)^2}$ $\quad \frac{f'g - fg'}{g^2}$

$\frac{d^2y}{dx^2} = \frac{(-2x)(x^2-1)^2 - (-x^2-1)(2)(x^2-1)(2x)}{(x^2-1)^4}$

$= \frac{(-2x)(x^2-1)^2 + 4x(x^2+1)(x^2-1)}{(x^2-1)^4}$

$= \frac{-2x(x^2-1)[x^2-1-2(x^2+1)]}{(x^2-1)^4}$

$y'' = \frac{-2x[-x^2-3]}{(x^2-1)^3}$

$y'' = \frac{2x[x^2+3]}{(x^2-1)^3}$

Ex 2.
Determine f''(1), if $f(x) = (2-x^2)^{10}$.

$f'(x) = 10(2-x^2)^9(-2x)$

$= \underbrace{-20x}_{f}\underbrace{(2-x^2)^9}_{g}$ $\quad f'g + fg'$

$f''(x) = \underbrace{-20}_{f'}\underbrace{(2-x^2)^9}_{g} + \underbrace{(-20x)}_{f}\underbrace{(9)(2-x^2)^8(-2x)}_{g'}$

$= -20(2-x^2)^9 + 360x^2(2-x^2)^8$

$f''(x) = -20(2-x^2)^8(2-x^2-18x^2)$

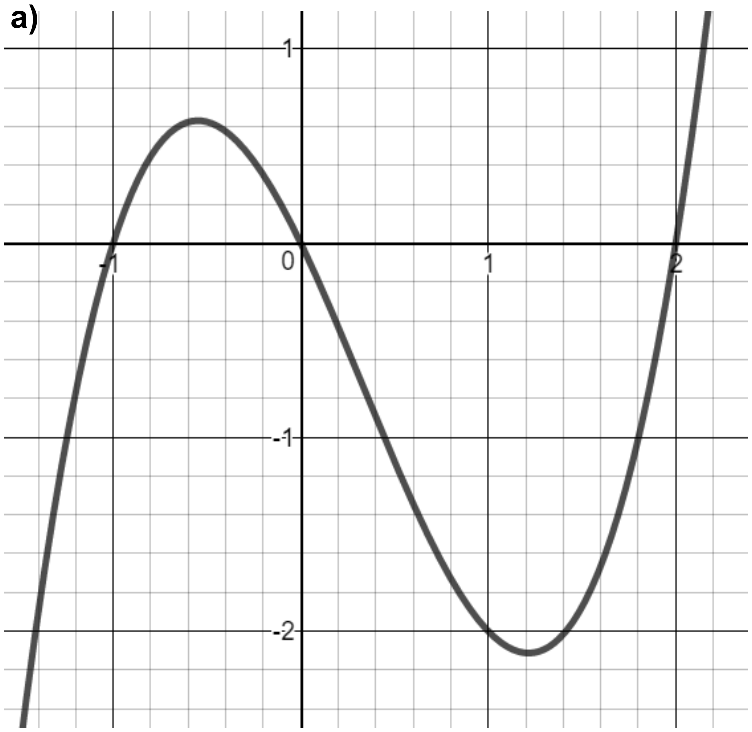
$f''(1) = -20(1)(2-1-18)$

$= 340$

Thought Experiment:
Since a derivative can be described as a rate of change ***or*** the slope of the tangent line…
What would the derivative of a derivative represent?

Ex 3.
Given the following functions, sketch the graphs of the first and second derivative. You may work in pairs if you wish.

a)



b)