Acceleration

Acceleration: Rate of change of velocity per unit of time. It is the second derivative of position.

$s(t)$ $v(t)$

$s'(t) = v(t)$ $v'(t) = a(t)$

Average acceleration: $\frac{\Delta v}{\Delta t}$ $\frac{\Delta m/s}{\Delta s} \to \frac{\frac{m}{s}}{s} = m/s^2$

Instantaneous acceleration: $a = \frac{d}{dt}v = \frac{d^2 s}{dt^2}$ displacement or $a(t) = v'(t) = s''(t)$

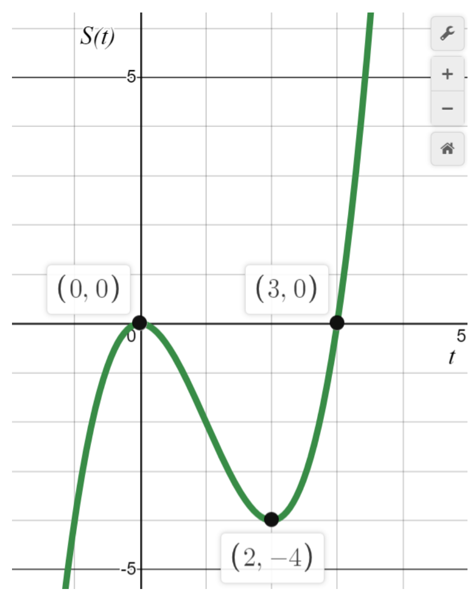
REMINDER:

- a(t) > 0 → object is accelerating upward or to the right (in this course)
- a(t) < 0 → object is accelerating downward or to the left (in this course)
- If velocity and acceleration are acting in the same direction the object is speeding up.
- If the velocity and acceleration are acting in opposite directions, the object is slowing down

Ex 1.

The position, in m, relative to a fixed point O of an object moving in a straight line is **s(t) = t³ – 3t²**.

a) Determine the average acceleration from t = 1 to t = 2 seconds
b) Determine the instantaneous acceleration at t = 1 second
c) When does the object change direction?
d) When does the object have a negative accelleration (i.e. accelerating in the negative direction)?



$s'(t) = v(t) = 3t^2 - 6t$ $s''(t) = v'(t) = a(t) = 6t - 6$

a) $a_{avg} = \frac{\Delta v}{\Delta t}$ $1 \to 2$ sec

$= \frac{v(2) - v(1)}{2-1}$ $\frac{v(1) - v(2)}{1-2}$

$= \frac{0-(-3)}{1}$ $\frac{-3-(0)}{-1}$

$= \frac{3\ m/s}{1s}$ $\frac{-3}{-1}$

$= 3\ m/s^2$ 3

b) $a(1) = s''(1) = 6(1) - 6 = 0\ m/s^2$

c) Direction changes when $v(t) = 0$

Let $s'(t) = 0$

$3t^2 - 6t = 0$

$3t(t-2) = 0$

$t = 0s$, $t = 2s$

d) Determine when $a(t) < 0$

$s''(t) < 0$ $a(t) = 0$

$6t - 6 < 0$ $6t - 6 = 0$

$6t < 6$ $6t = 6$

$t < 1s$ $t = 1$

a(t)	−	+
		1

t < 1 sec

***Ex 2. Challenge**

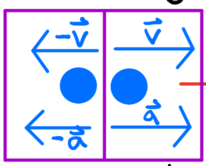
Determine when the particle whose position is described by **$s(t) = t^3 - 12t^2 + 36t - 20$** is speeding up.

- If velocity and acceleration are acting in the same direction, the particle is "speeding up"

$a(t)\cdot v(t) > 0$

$(+)(+) > 0$

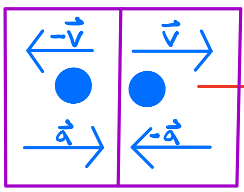
$(-)(-) > 0$



"speed is increasing"

- If velocity and acceleration are acting in opposite directions, the particle is "slowing down"

$a(t)\cdot v(t) < 0$



"speed is decreasing"

$S'(t) = V(t) = 3t^2 - 24t + 36$

$= 3(t^2 - 8t + 12)$

$= 3(t-2)(t-6)$

$S''(t) = V'(t) = a(t) = 6t - 24$

$= 6(t-4)$

Let $a(t)\cdot v(t) = 0$ (determine positive intervals)

$3(t-2)(t-6)\cdot 6(t-4) = 0$

$18(t-2)(t-6)(t-4) = 0$

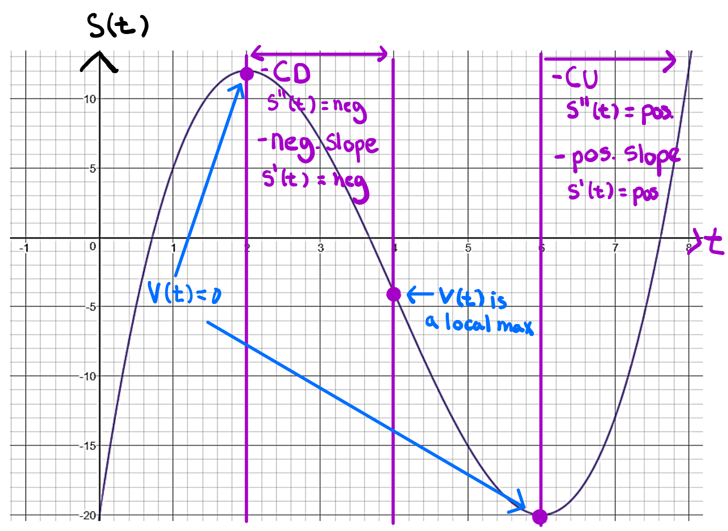
$t = 2, 6, 4$

		2		4		6	
$a(t)\cdot v(t)$	−		+		−		+

speeding up

$2 < t < 4$

$t > 6$



HW

A particle moves along a vertical line. Its position in ***m*** at time t in ***s*** is given by **$s(t) = t^3 - 9t^2 + 24$**.

a) Determine the distance the particle travels between t = 0 & t = 6

b) When is the particle speeding up?

Ans:

a) 108 m

b) 0 < t < 3 and t > 6 seconds

$f(x) = \sqrt{x} + x^2$

$f'(x) = \frac{1}{2}x^{-\frac{1}{2}} + 2x$

$f''(x) = -\frac{1}{4}x^{-\frac{3}{2}} + 2$

$= \frac{-1}{4x^{\frac{3}{2}}} + 2$

let $f''(x) = 0$

$8 = x^{-\frac{3}{2}}$

$8x^{\frac{3}{2}} = 1$

$x^{\frac{3}{2}} = \frac{1}{8}$

$x^{\frac{3}{2}} = 8^{-1}$

$x = (8)^{-\frac{2}{3}}$

$= \frac{1}{8^{\frac{2}{3}}}$

$= \frac{1}{4} = 0.25$

Test 1 Test 2

W → 12 W → 25

60% 70%

$$\text{Mark} = \frac{60(12) + 70(25)}{1 - \;}$$