The Derivative of a Polynomial Function

Recall: We have the ability to determine the slope of a curve (the derivative) at any given point.

$$\boxed{\lim_{h\to 0}\frac{f(a+h)-f(a)}{h}}$$

→ Represents the slope of the tangent to the curve f(x) where x = a.

→ Represents the instantaneous rate of change of f(x) where x = a

Ex 1.

Determine the slope of the tangent to $f(x) = 3x^2 - 5x + 4$ where $x = 2$.

$a = 2$

$$m_{tangent} = \lim_{h\to 0}\frac{f(a+h)-f(a)}{h}$$

$$= \frac{f(2+h)-f(2)}{h}$$

$$= \left[3(2+h)^2 - 5(2+h) + 4 - (3(2)^2 - 5(2) + 4)\right]\cdot\frac{1}{h}$$

$$= \left[3(4+4h+h^2) - 10 - 5h + 4 - 12 + 10 - 4\right]\cdot\frac{1}{h}$$

$$= \left[12 + 12h + 3h^2 - 5h - 12\right]\cdot\frac{1}{h}$$

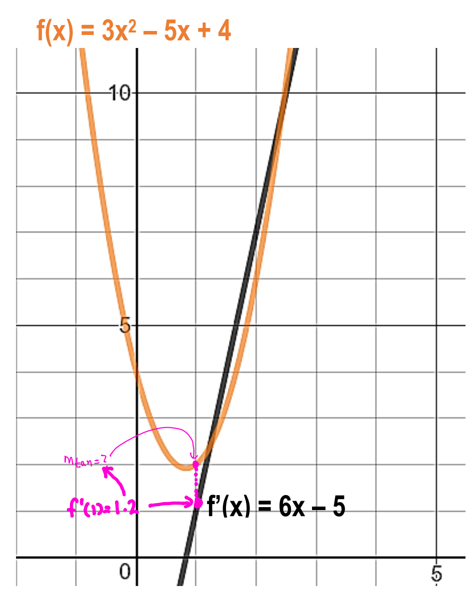
$$= \left[7h + 3h^2\right]\cdot\frac{1}{h}$$

$$= \lim_{h\to 0}\left[7 + 3h\right]$$

$$= 7$$

Thought Experiment

- The derivative function f'(x) produces the slope of the tangent for all x-values of the function.
- How can we determine the equation of the derivative function?



Ex 2.

Determine the derivative function f'(x) for $f(x) = 3x^2 - 5x + 4$.

$$f'(x) = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$

$$= \left[3(x+h)^2 - 5(x+h) + 4 - 3x^2 + 5x - 4\right]\cdot\frac{1}{h}$$

$$= \left[\cancel{3x^2} + 6xh + 3h^2 \cancel{-5x} - 5h \cancel{+4} \cancel{-3x^2} \cancel{+5x} \cancel{-4}\right]\cdot\frac{1}{h}$$

$$= \left[6xh + 3h^2 - 5h\right]\cdot\frac{1}{h}$$

$$= \lim_{h\to 0}\cancel{\frac{1}{h}\cdot h}\left[6x + 3h - 5\right]$$

$$\boxed{f'(x) = 6x - 5}$$

$$\frac{\left[6xh + 3h^2 - 5h\right]}{h}$$

$$\frac{\cancel{h}\left[6x + 3h - 5\right]}{\cancel{h}}$$

$$m_{tan} = \lim_{h\to 0}\frac{f(a+h)-f(a)}{h}$$ for a specific value of x. (x = a)

$$f'(x) = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$ for all values of x. (derivative functions)

Ex 3.

Determine f'(x) for each of the following.

a) f(x) = -2x + 7

$$f'(x) = \lim_{h\to 0}\frac{f(x+h)\boxed{-f(x)}}{h}$$

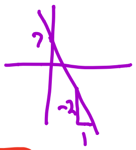
$$= \lim_{h\to 0}\frac{-2(x+h)+7\boxed{+2x-7}}{h}$$

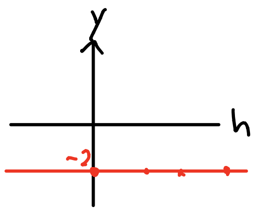
$$= \lim_{h\to 0}\frac{-2x-2h+2x}{h}$$

$$= \lim_{h\to 0}\frac{-2\cancel{h}}{\cancel{h}}$$

$$= \lim_{h\to 0} -2$$

$$f'(x) = -2$$





b) $f(x) = 2x^3 + x - 3$

$$f'(x) = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$

$$= \left[2(x+h)^3 + (x+h) \cancel{-3} - 2x^3 \cancel{-x} \cancel{+3}\right]\cdot\frac{1}{h}$$

$$= \left[2(x^3+3x^2h+3xh^2+h^3)+h-2x^3\right]\cdot\frac{1}{h}$$

$$= \left[6x^2h+6xh^2+2h^3+h\right]\cdot\frac{1}{h}$$

$$\left[6x^2+6xh+2h^2+1\right]\cancel{h}\cdot\frac{1}{\cancel{h}}$$

$$= \lim_{h\to 0} 6x^2+6x\underset{0}{h}+2\underset{0}{h^2}+1$$

$$f'(x) = 6x^2+1$$

x = 1

$f'(1) = 6(1)^2+1$

$= 7$

Ex 4.

Determine where the slope of the tangent to f(x) = $(4x – 5)^2$ is 2.

$$f'(x) = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$

$= 16x^2-40x+25$

$$= \left[16(x+h)^2-40(x+h)+25-16x^2+40x-25\right]\cdot\frac{1}{h}$$

$$= \left[\cancel{16x^2}+32xh+16h^2\cancel{-40x}-40h\cancel{+25}\cancel{-16x^2}\cancel{+40x}\cancel{-25}\right]\cdot\frac{1}{h}$$

$$= \lim_{h\to 0}\frac{1}{\cancel{h}}\cdot\left[\cancel{h}(32x+16h-40)\right]$$

$$f'(x) = 32x-40$$

→ slope of tan @ x

Where is the slope = 2 ? (where is $f'(x) = 2$

$2 = 32x - 40$

$\frac{42}{32} = x$

$\frac{21}{16} = x$

$m_{tan} = 2 @ x = \frac{21}{16}$

$$(x-1)(x-1)(x-1)$$

$$(x^2-2x+1)(x-1)$$

$$x^3-2x^2+x-x^2+2x-1$$

$$x^3-3x^2+3x-1$$

$$3(x-1)^2=6$$

$$(x-1)^2=\frac{6}{3}$$

$$x-1=\sqrt{\frac{6}{3}}$$

$$x=\sqrt{\frac{6}{3}}+1$$