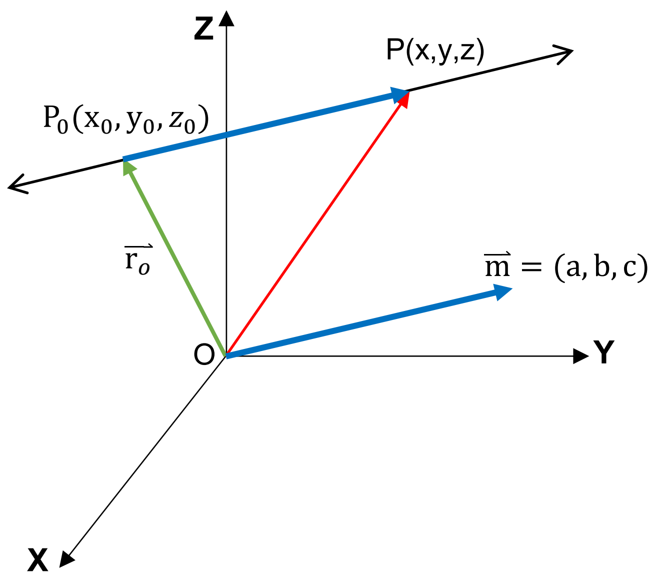
Vector and Parametric Equations of a Line in R3

Focus: Derive vector, parametric, and Cartesian equations of a line.



Vector Equation of a Line:

$$\vec{r} = \vec{r_0} + t\vec{m}, t \in R$$

Vector Equation of a Line (component form)

$$(x, y, z) = (x_0, y_0, z_0) + t(a, b, c)$$

Where r_0 is the vector from (0,0,0) and the point (x_0,y_0,z_0) and $\vec{m}$ is the direction vector with components (a,b,c)

Parametric Equation of a line:

$$x = x_0 + ta$$
$$y = y_0 + tb$$
$$z = z_0 + tc, t \in R$$

Ex 1.
Write the vector and parametric equations of the line through the given points.

a) J(–1,2,4) and K(–3, –2,1)

$\vec{m} = K - J$
$= (-3,-2,1) - (-1,2,4)$
$= (-2,-4,-3)$

OR
$\vec{m} = J - K$
$= (2,4,3)$

V $(x,y,z) = (-1,2,4) + t(2,4,3), t \in \mathbb{R}$

P $x = -1 + 2t$
$y = 2 + 4t$
$z = 4 + 3t$

b) C(–3,2, –1) and D(0,2,4)

$\vec{m} = (3,0,5)$

$(x,y,z) = (0,2,4) + t(3,0,5), t \in \mathbb{R}$

$x = 3t$
$y = 2$
$z = 4 + 5t$

Ex 2.
Given the parametric equations of a line in R3, determine the vector equations.

x = 3 + 4t
y = –1 + 2t
z = 1 – 3t

$(x,y,z) = (3,-1,1) + t(4,2,-3), t \in \mathbb{R}$

Ex 3. * intersection of lines

Determine the angle between the lines.

x = –5 – 3s
y = 2
z = 1 + s

$\vec{r} = (3, -2, 4) + t(2, -1, 1)$

$x = 3 + 2t = -5 - 3s$
$y = -2 - t = 2$
$z = 4 + t = 1 + s$

In R^2, any two non-pll lines MUST intersect.

In R^3, two lines are unlikely to intersect (skew lines)

How can we determine if lines intersect in R^3?

① Use two eq^{ns} to solve for unknowns

② Use LS/RS check on third eq^{n} to check for consistency

Lets use both $\underline{y}$ eq^{ns} to solve for $\underline{t}$

$2 = -2 - t$

$-4 = t$

Sub t into z eq^{ns}

$1 + s = 4 + t$

$1 + s = 4 - 4$

$s = -1$

Sub $\underline{t}$ + $\underline{s}$ into $\underline{x}$-eq^{ns} + check for consistency

$-5 - 3s = 3 + 2t$

LS	RS
$-5-3(-1)$	$3+2(-4)$
-2	-5

$LS \neq RS$

- Do not intersect (skew)
- No angle

Ex 4.

Determine a vector equation for each line:

a) Parallel to the z-axis and passes through (1,5,10)

$\vec{m} = (0, 0, 1)$
(0, 0, 2)
(0, 0, 3)

$(x, y, z) = (1, 5, 10) + t(0, 0, 1), t \in \mathbb{R}$

b) Has the same x-intercept as **(x,y,z) = (3,0,0) + t(4, –4,1)** and the same z-intercept as **(x,y,z) = (6,12, –1) + s(3,6, –2)**

x-int (3,0,0)

z-int $(0, 0, z) = (6, 12, -1) + s(3, 6, -2)$

$0 = 6 + 3s \longrightarrow -2 = s$

$0 = 12 + 6s$

$z = -1 - 2s$

$z = -1 - 2(-2)$

$z = 3$

z-int (0, 0, 3)

$\vec{m} = (3, 0, 0) - (0, 0, 3)$
$= (3, 0, -3)$

$(x, y, z) = (3, 0, 0) + t(3, 0, -3)$
$t \in \mathbb{R}$

$(x, y, z) = (3, 0, 0) + t(1, 0, -1)$

Ex 5.
Are these lines the same? (A.K.A coincident)

(x,y,z) = (11, –2,17) + $\boxed{s(3, -1,4)}$
(x,y,z) = (–13,6, –10) + $\boxed{t(-3, 1, -4)}$

P∥?

$(3,-1,4) = -1(-3,1,-4)$

- Scalar multiples of each other, so P∥

Same line?

- check if a point on one line satisfies the eqⁿ of the other line

$(11,-2,17) = (-13,6,-10) + t(-3,1,-4)$

x	y	z
$11 = -13 - 3t$	$-2 = 6 + t$	$17 = -10 - 4t$
$24 = -3t$	$-8 = t$	$27 = -4t$
$-8 = t$		$\frac{27}{-4} = t$

Inconsistent!
∴ Different lines

OR

Check if difference vector is a multiple of direction vector

$(11,-2,17) - (-13,6,-10)$

$(24,-8,27) \longrightarrow (-3,1,-4)$

$\frac{24}{-3} = -8$

$-\frac{8}{1} = -8$

$\frac{27}{-4}$

inconsistent!