Limit Laws

$$\lim_{x\to c} f(x) \pm g(x) = \lim_{x\to c} f(x) \pm \lim_{x\to c} g(x)$$
$$\lim_{x\to c} f(x) \cdot g(x) = \lim_{x\to c} f(x) \cdot \lim_{x\to c} g(x)$$
$$\lim_{x\to c} \frac{f(x)}{g(x)} = \frac{\lim_{x\to c} f(x)}{\lim_{x\to c} g(x)}$$
$$\lim_{x\to c} (f(x))^n = (\lim_{x\to c} f(x))^n$$

One Sided Limits

$$x \to c : \lim_{x\to c^-} f(x)$$
$$c \leftarrow x : \lim_{x\to c^+} f(x)$$

L'Hospital's Rule

if $\frac{f(x)}{g(x)} = \frac{0}{0} \text{or} \frac{\infty}{\infty}$, then

$$\lim_{x\to c} \frac{f(x)}{g(x)} = \lim_{x\to c} \frac{f'(x)}{g'(x)}$$

Derivative Rules

Constant Rule	$\frac{d}{dx} c \cdot f(x) = c \cdot f'(x)$
Power Rule	$\frac{d}{dx} x^n = nx^{n-1}$
Sum Rule	$\frac{d}{dx}(f+g) = \frac{d}{dx} f + \frac{d}{dx} g$
Product Rule	$\frac{d}{dx}(f \cdot g) = \frac{d}{dx} f \cdot g + f \cdot \frac{d}{dx} g$
Quotient Rule	$\frac{d}{dx}\frac{f}{g} = \frac{\frac{d}{dx} f \cdot g - f \cdot \frac{d}{dx} g}{g^2}$
Chain Rule	$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$
Inverse	$(\frac{dy}{dx})^{-1} = \frac{dx}{dy}$

Derivatives of Polynomials

$$\frac{d}{dx} c = 0$$
$$\frac{d}{dx} x^n = nx^{n-1}$$

Derivatives of Trig Functions

$$\frac{d}{dx} \sin(x) = \cos(x)$$
$$\frac{d}{dx} \cos(x) = -\sin(x)$$
$$\frac{d}{dx} \tan(x) = \sec^2(x)$$
$$\frac{d}{dx} \cot(x) = -\csc^2(x)$$
$$\frac{d}{dx} \sec(x) = \sec(x)\tan(x)$$
$$\frac{d}{dx} \csc(x) = -\csc(x)\cot(x)$$

Derivatives of Inverse Trig Functions

$$\frac{d}{dx} \sin^{-1}(x) = \frac{1}{\sqrt{1-x^2}}$$
$$\frac{d}{dx} \cos^{-1}(x) = -\frac{1}{\sqrt{1-x^2}}$$
$$\frac{d}{dx} \tan^{-1}(x) = \frac{1}{1+x^2}$$
$$\frac{d}{dx} \cot^{-1}(x) = -\frac{1}{1+x^2}$$

Derivatives of Transcendental Functions

$$\frac{d}{dx} a^x = a^x \ln(a)$$
$$\frac{d}{dx} e^x = e^x$$
$$\frac{d}{dx} \ln(x) = \frac{1}{x}$$
$$\frac{d}{dx} \log_a(x) = \frac{1}{x \ln(a)}$$
$$\frac{d}{dx} f^g = \frac{d}{dx} e^{g \ln(f)}$$

Derivatives of Implicit Functions

Find $\frac{dy}{dx}$ of $x^3 + y^3 - 9xy = 0$

$$\frac{d}{dx}(x^3 + y^3 - 9xy) = 0$$
$$\frac{d}{dx} x^3 + \frac{d}{dx} y^3 - 9\frac{d}{dx}(xy) = 0$$
$$3x^2 + \frac{d}{dy} y^3 \frac{dy}{dx} - 9(\frac{d}{dx} x \cdot y + x \frac{d}{dx} y) = 0$$
$$3x^2 + 3y^2 \frac{dy}{dx} - 9(y + x\frac{dy}{dx}) = 0$$
$$\frac{dy}{dx} = \frac{-x^2 + 3y}{-3x + y^2}$$

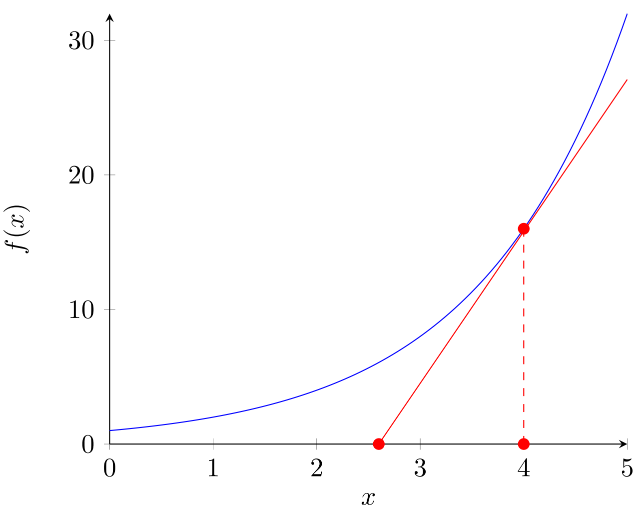
Multiple Derivative

$$\frac{d^k}{dx^k}x^n = \frac{n!}{(n-k)!}x^{n-k}$$
$$\frac{d^k}{dx^k}e^{\lambda x} = \lambda^k e^{\lambda x}$$
$$\frac{d^k}{dx^k}a^x = (\ln(a))^k a^x$$

Newton's Method

To find the root of $f(x) = 0$ using Newton's Method, start with an initial guess x_0 and then iterate

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$



Exponential Substitution

To find the derivative of $y = x^{\ln x}$, use exponential substitution due to $x = e^{\ln x}$, then $x^{\ln x} = e^{(\ln x)^2}$

$$\begin{aligned}
y &= x^{\ln x} \\
\frac{dy}{dx} &= \frac{d}{dx}e^{(\ln x)^2} \\
&= \frac{d}{du}e^u \frac{d}{dx}(\ln x)^2 \\
&= \frac{d}{du}e^u \frac{d}{dv}v^2 \frac{d}{dx}\ln x \\
&= e^u \cdot 2v \cdot \frac{1}{x} \\
&= x^{\ln x} \cdot 2\ln^2 x \cdot \frac{1}{x}
\end{aligned}$$

Implicit Differentiation using Exponential Substitution

To find the derivative of $x = y^{xy}$, use exponential substitution due to $y = e^{\ln y}$, then $y^{xy} = e^{xy \ln y}$

$$\begin{aligned}
x &= y^{xy} \\
x &= e^{xy\ln y} \\
\frac{d}{dx}x &= \frac{d}{dx}e^{xy\ln y} \\
1 &= \frac{d}{du}e^u \frac{d}{dx}xy\ln y \\
1 &= e^u(y\ln y + x \cdot \frac{d}{dx}y\ln y) \\
1 &= x(y\ln y + x \cdot \frac{d}{dy}y\ln y\frac{dy}{dx}) \\
1 &= xy\ln y + x^2(\ln y + 1)\frac{dy}{dx} \\
\frac{dy}{dx} &= \frac{1 - xy\ln y}{x^2(\ln y + 1)}
\end{aligned}$$

Derivative of Inverse Trigonometric Functions

To find the derivative of $\theta = \sin^{-1}(x)$, we have $x = \sin\theta$, since $\sin^2\theta + \cos^2\theta = 1$

$$\begin{aligned}
x &= \sin\theta \\
\frac{dx}{d\theta} &= \cos\theta \\
\frac{d\theta}{dx} &= \frac{1}{\cos\theta} \\
\frac{d\theta}{dx} &= \frac{1}{\sqrt{1-\sin^2\theta}} \\
\frac{d\theta}{dx} &= \frac{1}{\sqrt{1-x^2}}
\end{aligned}$$

Series

$f^{(k)}(x)$ is the kth derivative of $f(x)$

$$\text{Taylor Series} \quad f(x) = \sum_{k=0}^{\infty} \frac{f^{(k)}(a)}{k!}(x-a)^k$$
$$\text{Maclaurin Series} \quad f(x) = \sum_{k=0}^{\infty} \frac{f^{(k)}(0)}{k!}x^k$$