

## Numerical on Hebbian-Associative Memory

Q For the given input vector  $S = (s_1, s_2, s_3, s_4)$  and output vector  $T = (T_1, T_2)$ , find the weight matrix using hebbian associative training algo.

$$S = (s_1, s_2, s_3, s_4)$$

$$T = (T_1, T_2)$$

$$I = (1, 1, 0, 0)$$

$$(0, 1)$$

$$II = (1, 0, 0, 1)$$

$$(1, 1)$$

$$III = (1, 1, 0, 0)$$

$$(1, 0)$$

$$IV = (0, 1, 0, 1)$$

$$(0, 0)$$

Sol<sup>n</sup>

$$\text{weight matrix} = S^T \cdot T$$

$$S = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 \\ 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix}$$

$$S^T = \begin{bmatrix} 1 & 1 & 1 & 0 \\ 1 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix}$$

$$w = S^T T = \begin{bmatrix} 1 & 1 & 1 & 0 \\ 1 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 1 \\ 1 & 1 \\ 1 & 0 \\ 0 & 0 \end{bmatrix}$$

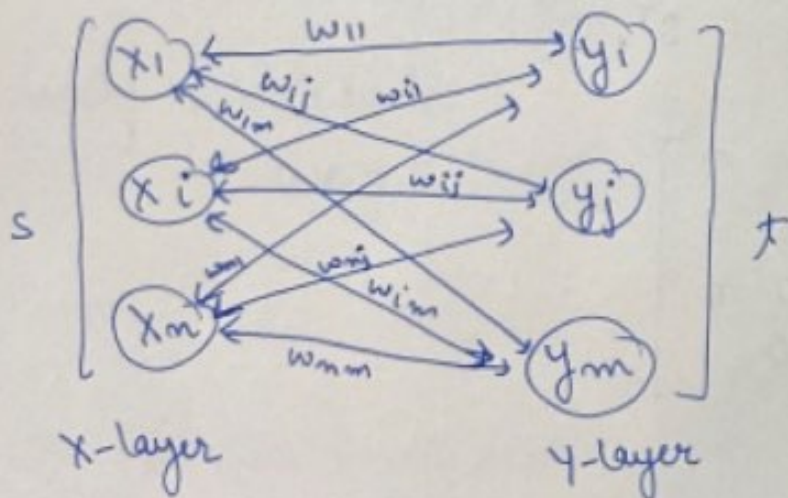
$$w = \begin{bmatrix} 2 & 2 \\ 1 & 1 \\ 0 & 0 \\ 1 & 1 \end{bmatrix} \text{ Ans}$$

## Bi-Directional Associative Memory (BAM)

It is a hetero associative recurrent neural network consisting of two layers:

- i) X-layer
- ii) Y-layer

which are connected by means of directional weighted connection paths.



BAM Architecture

- The network iterates by sending a signal back and forth between the two layers until all neurons reach equilibrium.
- The network can respond to input on ~~the~~ either layer.



If the weight matrix for signal sent from the X-layer to Y-layer is  $w$ , then weight matrix for signal sent from Y-layer to X-layer is  $w^T$ .

### Types of BAM

- Discrete BAM
  - ↙ Binary form (0,1)
  - ↘ Bipolar form (-1,+1)
- Continuous BAM

for the set of input & output vector

$$s(p) : t(p), p=1, 2, 3, \dots, P$$

using Hebb's rule

$$\text{weight } (w_{ij}) = \sum_{p=1}^P [2s_i(p) - 1] [2t_j(p) - 1]$$

$$= [2s^T - 1] [2t - 1]$$

for Bipolar input vector

$$w_j = \sum_{p=1}^P s_i(p) t_j(p) = s^T \cdot t$$

→ Activation function for Binary form (step function)  
for X-layer

$$x_i = \begin{cases} 1, & \text{if } x_{ini} > 0 \\ x_i, & \text{if } x_{ini} = 0 \\ 0, & \text{if } x_{ini} < 0 \end{cases}$$

for y-layer

$$y_j = \begin{cases} 1, & \text{if } y_{in} > 0 \\ y_j, & \text{if } y_{in} = 0 \\ 0, & \text{if } y_{in} < 0 \end{cases}$$

Activation function for Bipolar form

for x-layer

$$x_i = \begin{cases} 1, & \text{if } x_{in} > \theta \\ x_i, & \text{if } x_{in} = \theta \\ -1, & \text{if } x_{in} < \theta \end{cases}$$

for y-layer

$$y_i = \begin{cases} 1, & \text{if } y_{in} > \theta \\ y_i, & \text{if } y_{in} = \theta \\ -1, & \text{if } y_{in} < \theta \end{cases}$$

Continuous BAM

A Continuous BAM has the capability to transfer the input smoothly & continuously into the respective output.

→ The Continuous BAM uses logistic sigmoid activation function for all the units.



For binary input vector

weights are determined using Hebb's rule

$$w_{ij} = \sum_{p=1}^P [2s_i(p) - 1][2t_j(p) - 1]$$

Activation function

for binary "logistic function"

$$f(y_{in}) = \left[ \frac{1}{1 + e^{-y_{in}}} \right]$$

for Bipolar logistic function

$$f(y_{in}) = \frac{1 - e^{-y_{in}}}{1 + e^{-y_{in}}}$$

These activation function are applied over the input to calculate the output

$$y_{in} = b_j + \sum x_i w_{ij}$$

$\downarrow$   
Output

$\downarrow$   
bias

$\downarrow$   
input

$\rightarrow$  weight