MIET2419: Mechanics and Materials 1

Stress & Strain

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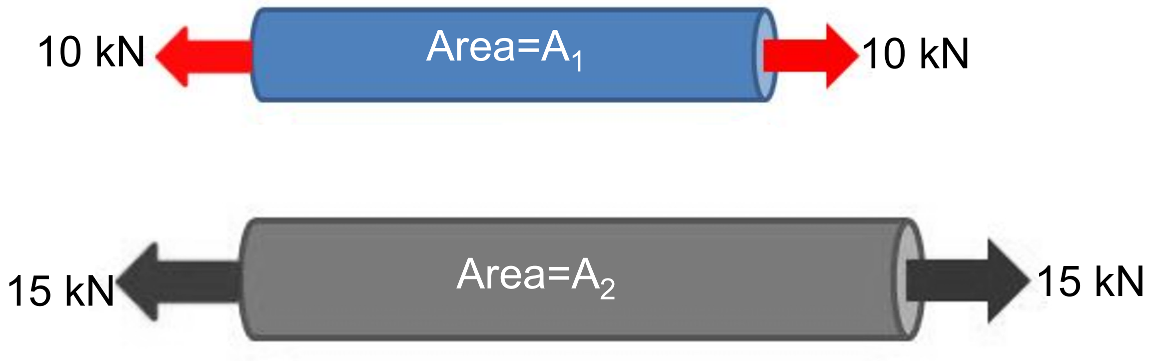
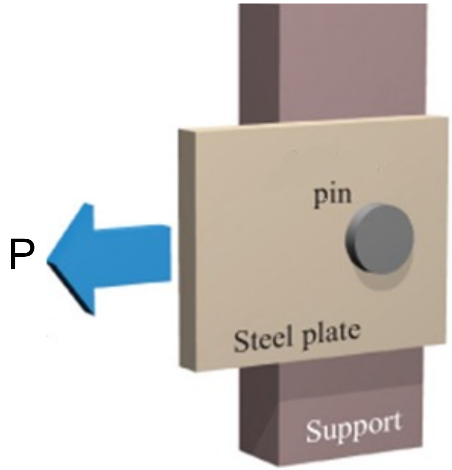
Introduction

- How to compare two loaded rods under different external loads?



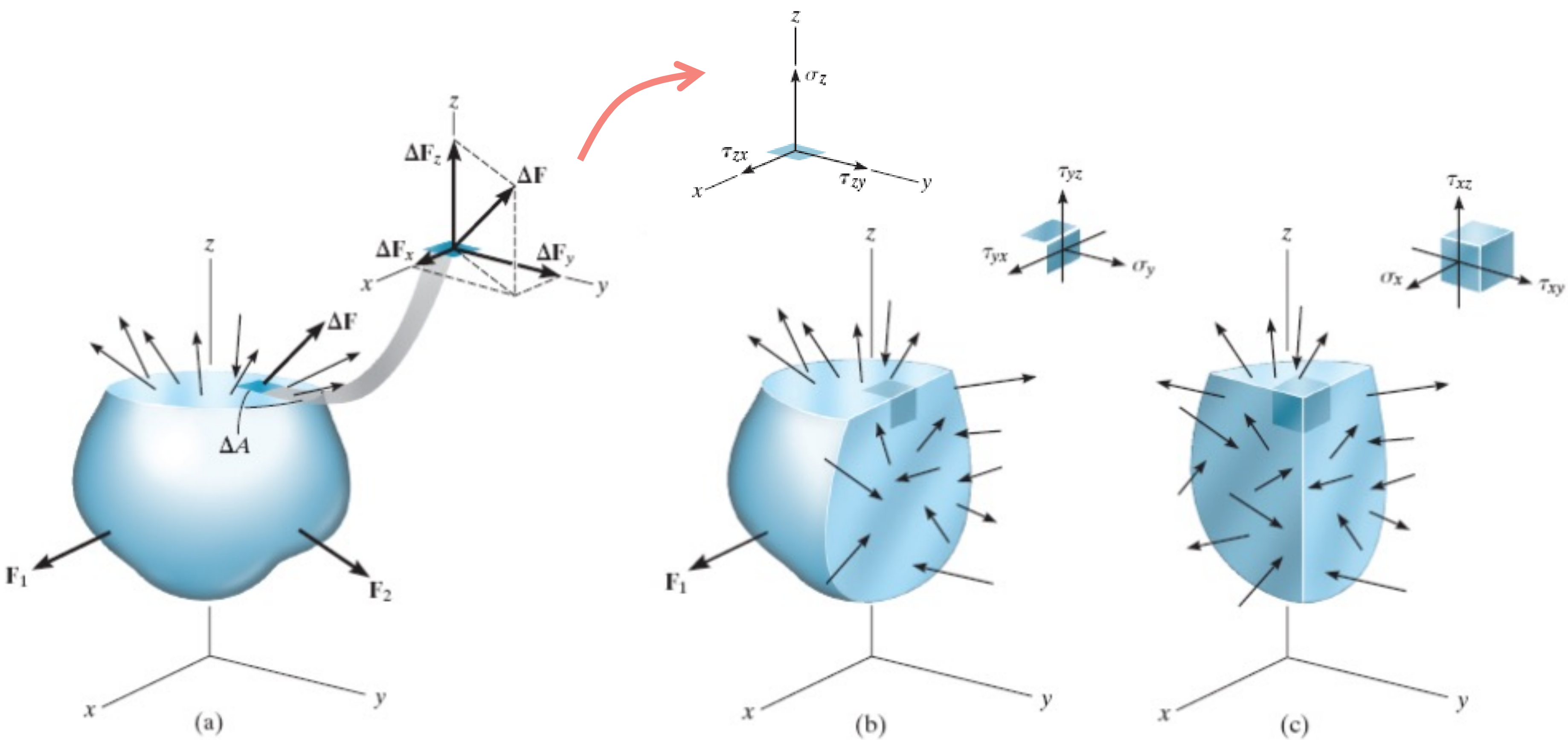
Which rod will fail?
What information do we need to find the answer?
How can we define comparable parameters?



- By increasing P, what types of failure may happen in the structure?

Stress

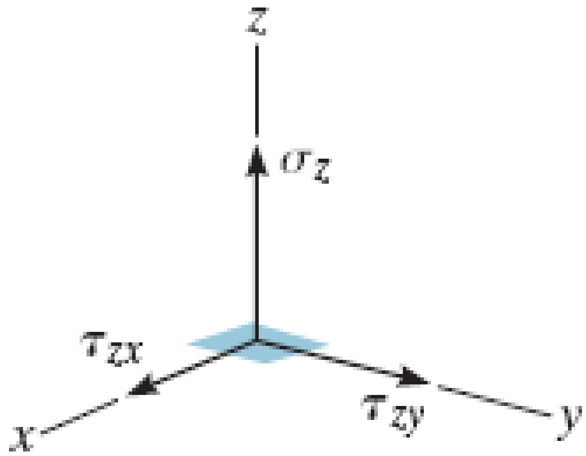
- ***Distribution*** of internal loading is important in mechanics of materials.
- We will consider the material to be ***continuous***.
- The ***intensity*** of **internal force** at a point is called ***stress*** = Force/Area



Stress

- **Normal Stress σ:**
  - Force per unit area acting normal to ΔA

$$\sigma_z = \lim_{\Delta A \to 0} \frac{\Delta F_z}{\Delta A}$$



What's the unit of stress?

Keep the units consistent:

- N, mm^2, MPa
- N, m^2, Pa

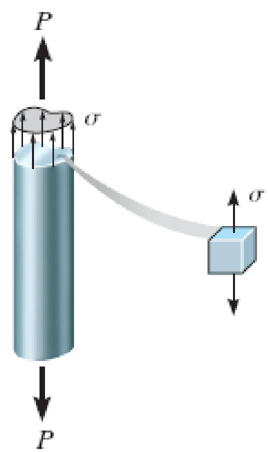
- **Shear Stress τ :**
  - Force per unit area acting tangent to ΔA

$$\tau_{zx} = \lim_{\Delta A \to 0} \frac{\Delta F_x}{\Delta A}$$

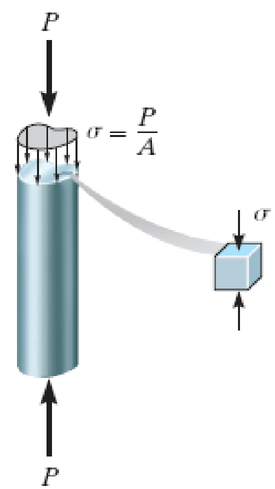
$$\tau_{zy} = \lim_{\Delta A \to 0} \frac{\Delta F_y}{\Delta A}$$

Average Normal Stress in an Axially Loaded Bar

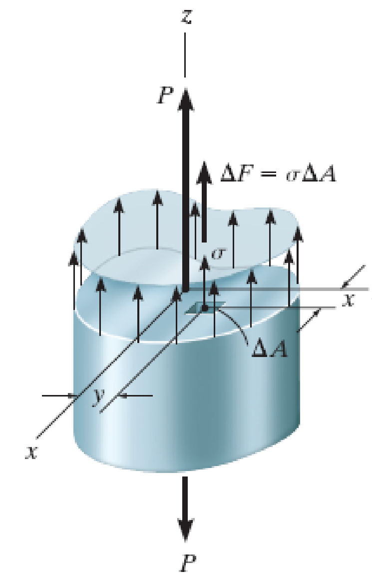
- When a ***cross-sectional area*** bar is subjected to axial force through the centroid, it is only subjected to normal stress.
- Stress is assumed to be averaged over the area.



Tension



Compression



$$\int dF = \int_A \sigma dA$$

$$P = \sigma A$$

Average Stress

Force

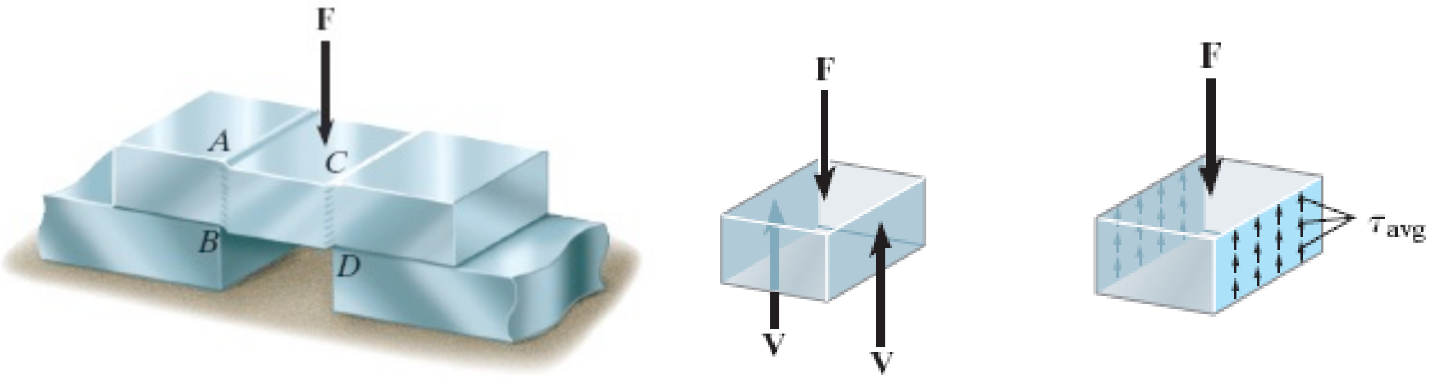
$$\sigma = \frac{P}{A}$$

Area

- **Force distribution diagrams**

Average Shear Stress

- The ***average shear stress*** distributed over each sectioned area that develops a shear force.

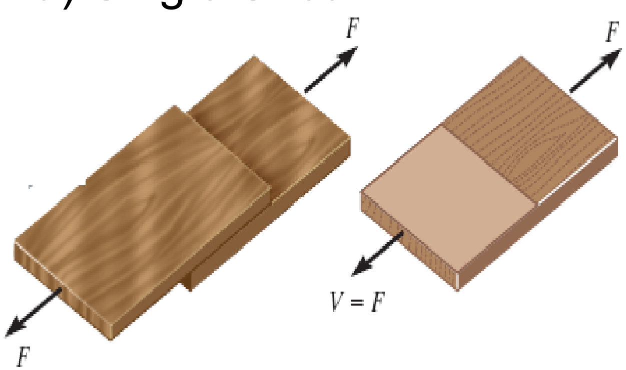


Internal resultant shear force

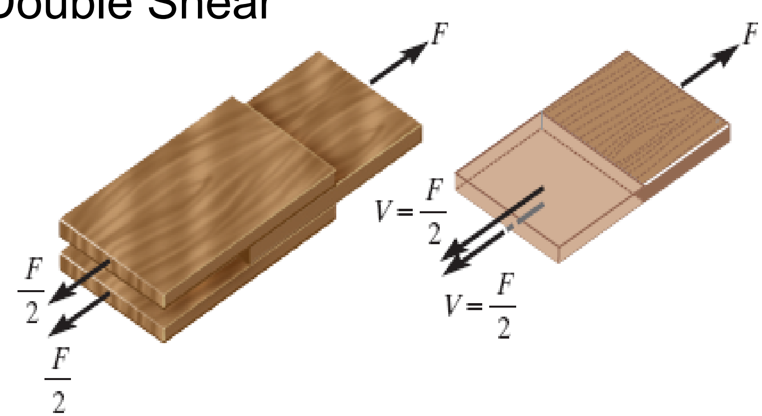
$$\tau_{avg} = \frac{V}{A_V}$$

Cross-sectional area under shear

a) Single Shear



b) Double Shear



Allowable Stress

- Many unknown factors that influence the actual stress in a member.
- A *factor of safety* is needed to obtain the allowable load.
- The ***factor of safety*** (F.S.) is a ratio of the failure load divided by the allowable load:

$$F.S = \frac{F_{fail}}{F_{allow}}$$

$$F.S = \frac{\sigma_{fail}}{\sigma_{allow}}$$

$$F.S = \frac{\tau_{fail}}{\tau_{allow}}$$

F.S. is always greater than 1 to avoid the potential failure.

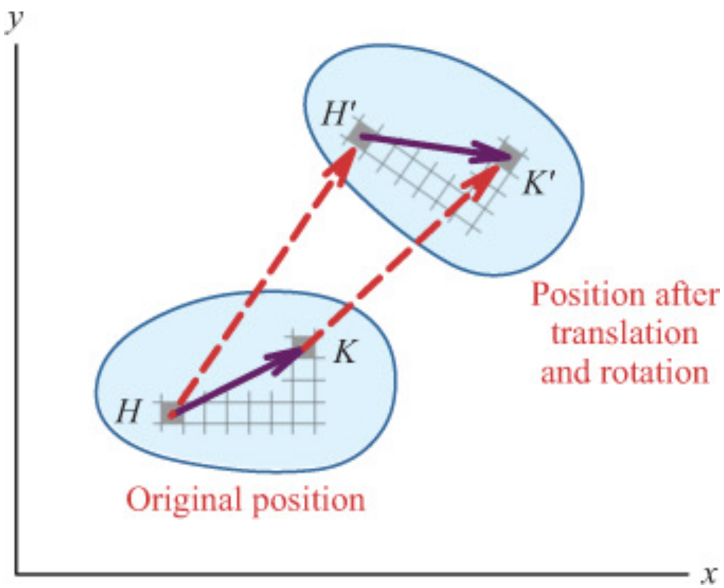
- To size the cross-sectional area of a simple connection:

$$\text{Assume a } F.S. \quad \Rightarrow \quad \sigma_{allow} = \frac{\sigma_{fail}}{F.S.} \quad \Rightarrow \quad A = \frac{P}{\sigma_{allow}}$$

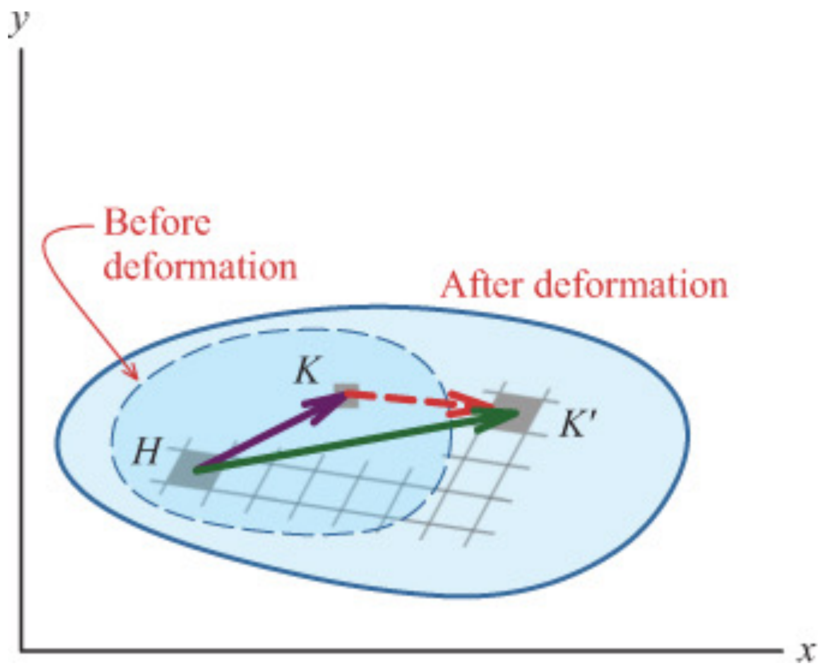
- How to estimate F.S.?
- How to determine σ_{fail}?

Displacement, Deformation & Strain

Rigid-body displacement: no changes in the size and shape of a body



Deformation: changes in the size and shape of a body due to load- or temperature-induced displacement



Strain: intensity of a deformation

Normal Strain

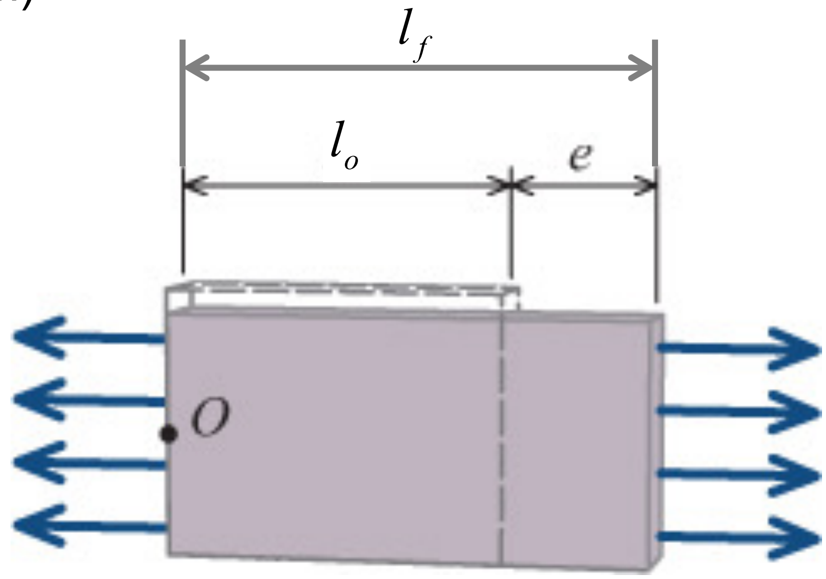
The elongation / contraction of a line segment per unit of length is referred to as ***normal strain***.

- Average normal strain: $$\varepsilon_{avg} = \frac{l_f - l_o}{l_o} = \frac{e}{l_o}$$

- If the normal strain is known, then the approximate final length is: $l_f \approx (1+\varepsilon)l_o$

- Normal strain unit: dimensionless (mm/mm)
- Micro-strain: $1\,\mu\varepsilon = 1\times10^{-6}\ \text{m/m}$

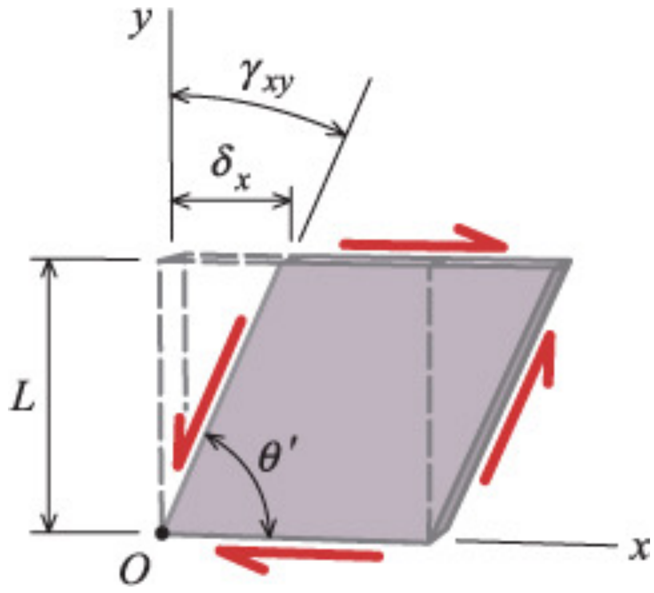
+ε ➔line elongate
-ε ➔line contracts



Shear strain

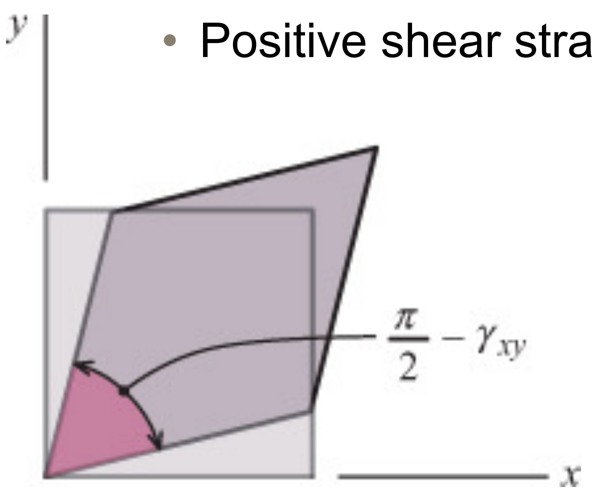
- Change in angle between 2 line segments that were *perpendicular* to one another

$$\gamma_{avg} = \frac{\delta_x}{L} \qquad \gamma_{xy}(O) = \frac{\pi}{2} - \theta'$$



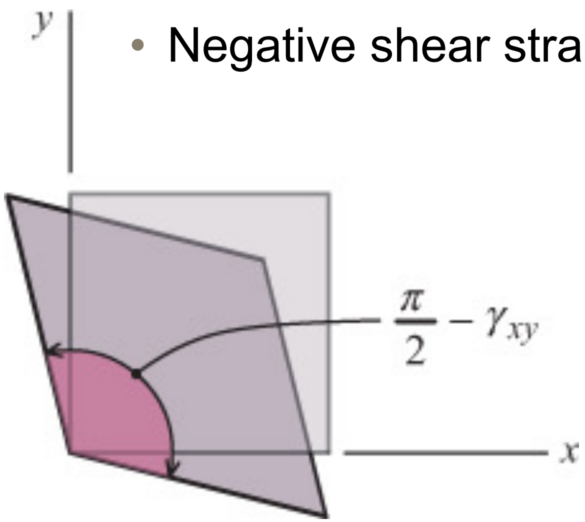
- Shear strain units: dimensionless (rad or μrad)

- Positive shear strain

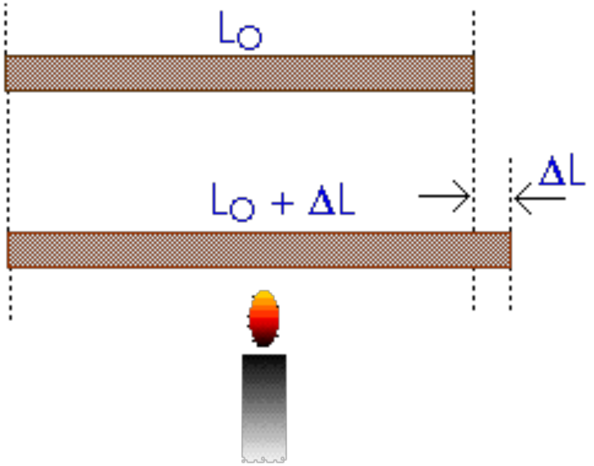


θ'<90 ➔+shear strain
θ'>90 ➔-shear strain

- Negative shear strain



Thermal strain



- Coefficient of thermal expansion:
- α: The amount of **expansion** (or contraction) per unit length of a material resulting from one degree change in temperature

- Behaviour of material subjected to temperature change (ΔT)
- Thermal strain: $\varepsilon_T = \alpha \Delta T$
- Total strain: $\varepsilon_{\text{total}} = \varepsilon_\sigma + \varepsilon_T$

Strain due to applied load

Strain due to change in temperature

Further reading: Chapter 7