JAModel模拟器

JAModel数学描述：

$$\frac{\partial M}{\partial H} = \frac{c\frac{\partial Man}{\partial He} + (Man - M)\left(k\delta - \frac{\alpha\,(Man - M)}{1 - c}\right)}{1 - \alpha c\frac{\partial Man}{\partial He}}$$

$$Man = Ms\left(\coth\left(\frac{H + \alpha M}{a}\right) - \frac{a}{H + \alpha M}\right)$$

$$\frac{\partial Man}{\partial He} = \frac{Ms\left(\left(\frac{a}{H + \alpha M}\right)^2 - \coth^2\left(\frac{H + \alpha M}{a}\right) + 1\right)}{a}$$

式中：
H为输入量,M为输出量,a,k,α,c,Ms为参数;
δ在dH/dt>0时为1，小于0时为-1。

模拟原理:

记某一时刻H,M的大小为H_t,M_t，下一时刻为
H_{t+1},M_{t+1}，则原数学表达式转化为:

$$\frac{M_{t+1}-M_t}{H_{t+1}-H_t}=\frac{\left(Man_t-M_t\right)\left(k\,sgn\left(H_{t+1}-H_t\right)-\frac{\alpha\left(Man-M_t\right)}{1-c}\right)+c\frac{\partial Man}{\partial He_t}}{1-\alpha c\frac{\partial Man}{\partial He_t}}$$

用f代表右边的表达式，于是有递推关系式:

$$M_{t+1}=\left(H_{t+1}-H_t\right)f\left(M_t,H_t,H_{t+1}\right)+M_t=g\left(M_t,H_t,H_{t+1}\right)$$

用上一时刻的M，H和当前的H即可计算当前的M

给定一组参数：

a: 632.6

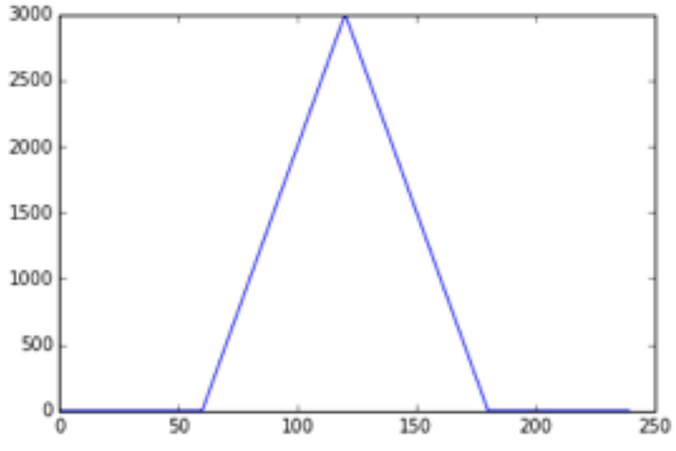
k: 60.16

α: 0.002397

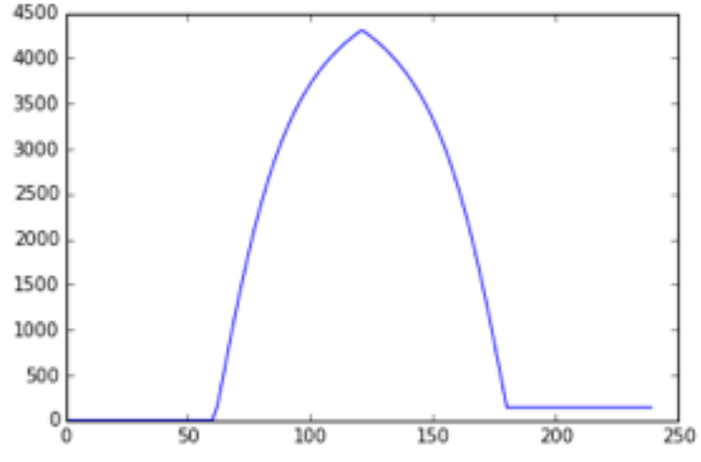
c: 0.1855

Ms: 5460

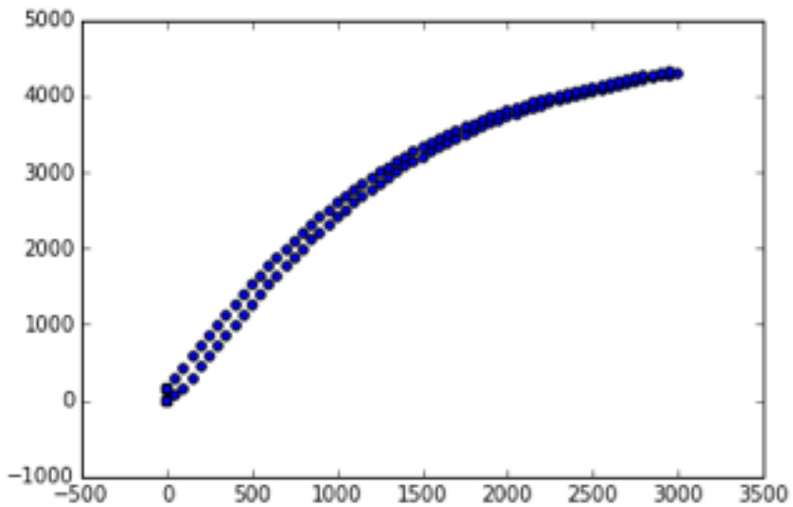
输入输出如图：



H输入



M输出



H－M图

数据拟合原理

记θ=(a,k,α,c,Ms),为参数向量,递推关系式记为

$$M_{t+1} = g_{\theta}\left(M_t, H_t, H_{t+1}\right)$$

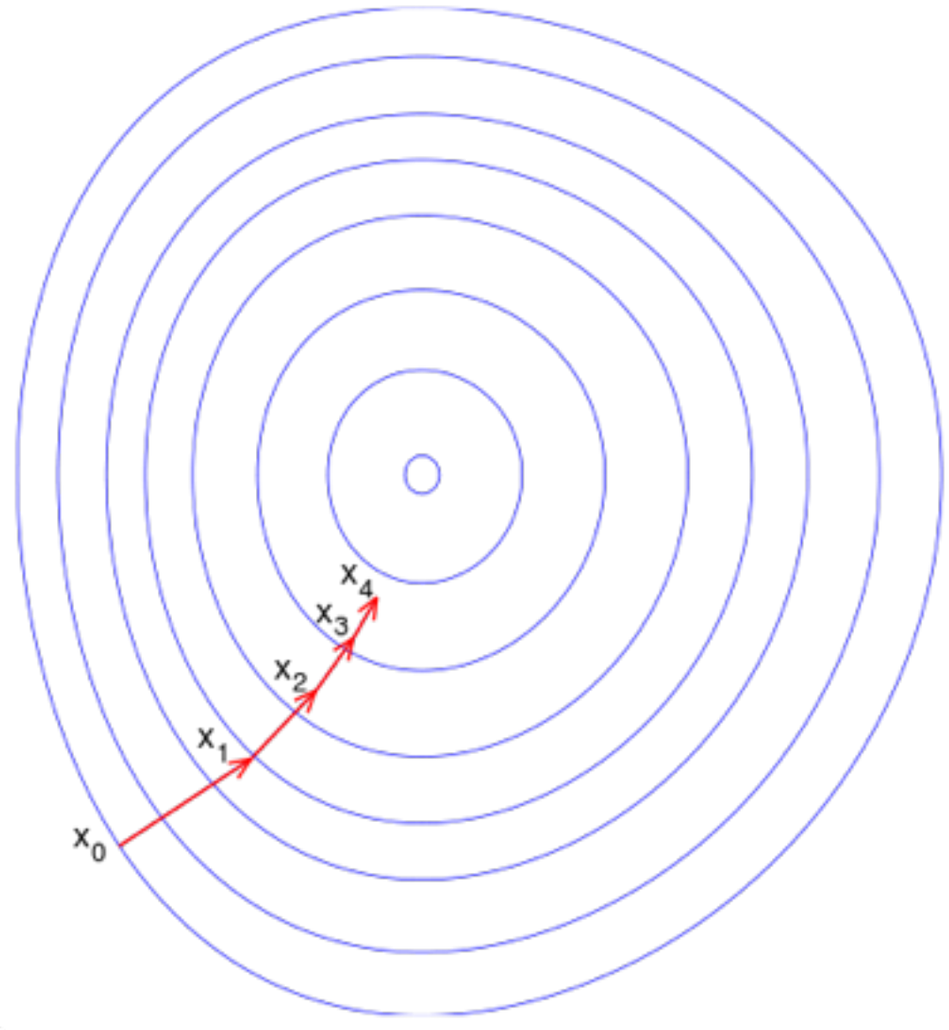
给定要拟合的实验数据M(t)和H(t),则我们最终想要得到的参数可表示为:

$$\theta_{optim} = \underset{\theta}{\operatorname{argmin}} \left(\sum_{t} \left(M\left(t+1\right) - g_{\theta}\left(M\left(t\right), H\left(t\right), H\left(t+1\right)\right) \right)^{2} \right)$$

梯度递降拟合

$$new\_\theta_{opt} = old\_\theta_{opt} - step\_length \nabla_\theta \left(M(t+1) - g_\theta (M(t), H(t), H(t+1)) \right)^2$$

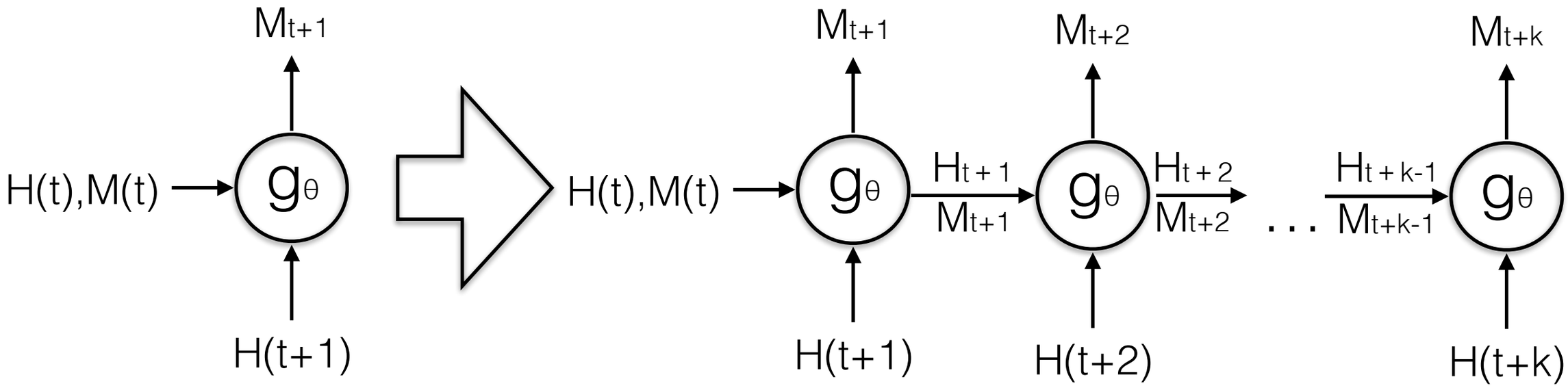
实验结果，拟合效果不佳，因为每次更新参数仅使用了短短2个相邻采样时间点的信息



模仿BPTT算法

更新步： $\nabla_\theta (M(t+1)-M_{t+1})^2$

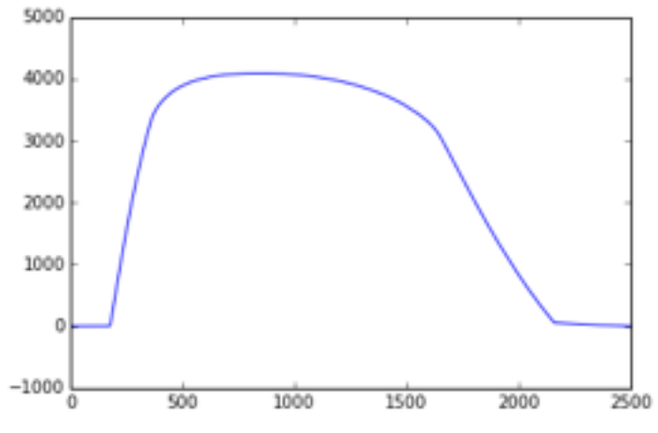
$\nabla_\theta [(M(t+1)-M_{t+1})^2+\ldots+(M(t+k)-M_{k+1})^2]$



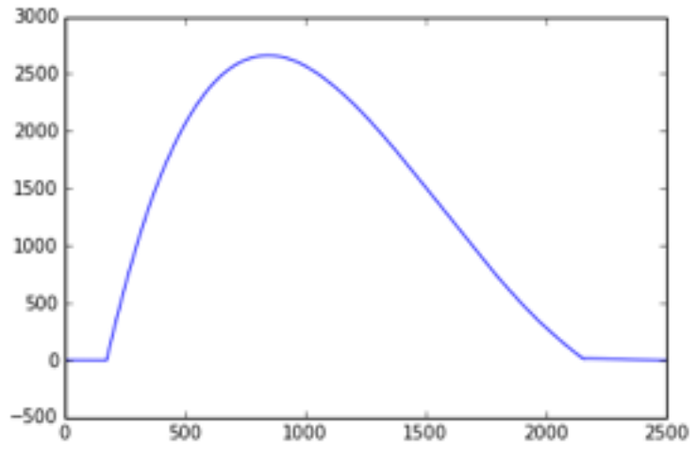
原

新

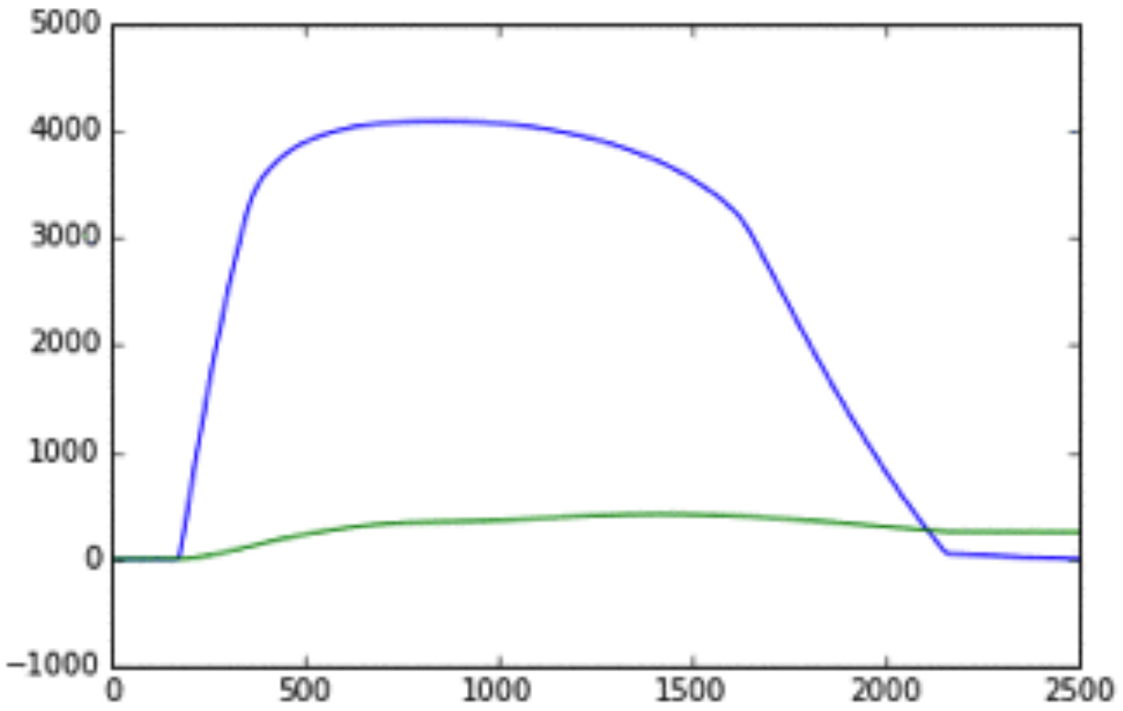
拟合效果演示



M(t)



H(t)



参数θ不断更新时，
模型在输入H(t)下
输出结果(绿线)的变化