Correlation and Regression

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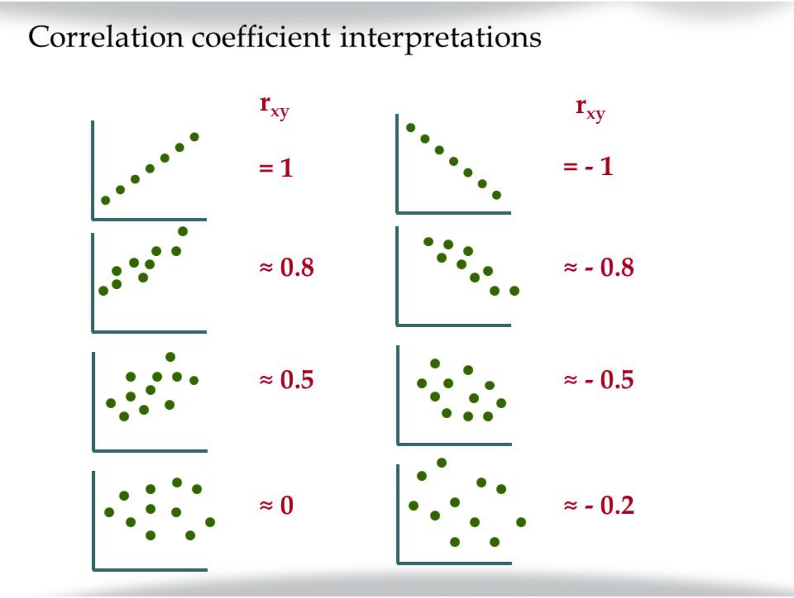
- Graphical - Scatter plot
- Correlation coefficient (due to Karl Pearson)
- Rank correlation


Correlation coefficient interpretations
r_{xy}
= 1
≈ 0.8
≈ 0.5
≈ 0
r_{xy}
= - 1
≈ - 0.8
≈ - 0.5
≈ - 0.2

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$$r_{XY} = \frac{N\sum XY - \sum X \sum Y}{\sqrt{(N\sum X^2 - [\sum X]^2)(N\sum Y^2 - [\sum Y]^2)}}$$

Properties

1. $-1 \leq r_{XY} \leq 1$
   If $r_{XY} = -1 \implies$ perfect negative correlation
   If $r_{XY} = 1 \implies$ perfect positive correlation
   If $r_{XY} = 0 \implies$ Uncorrelated (no linear relationship bet X and Y)

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2. $r_{XY} = r_{UV} = \frac{N\sum UV - \sum U \sum V}{\sqrt{(N\sum U^2 - [\sum U]^2)(N\sum V^2 - [\sum V]^2)}}$
   where $U = \frac{X-a}{h}$ and $V = \frac{Y-b}{k}$

Regression

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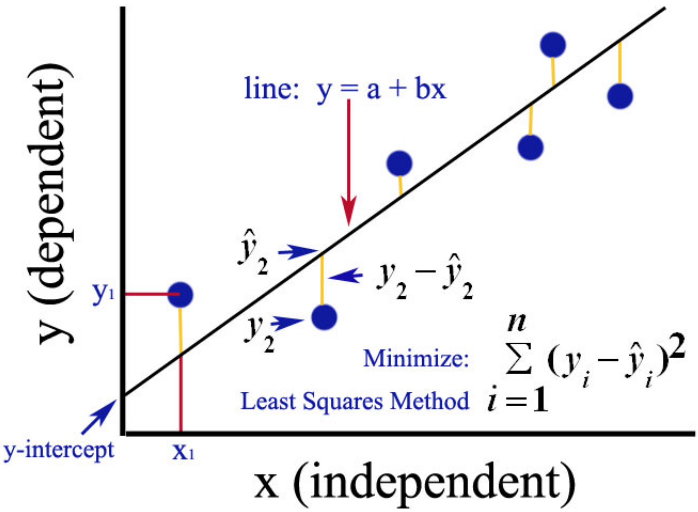
Regression line

Line which gives the best estimate to the value of one variable for any specific value of the other variable.

Regression Line



Lines of regression

Regression line of y on x

$$y - \bar{y} = r_{xy}\frac{\sigma_y}{\sigma_x}(x - \bar{x})$$

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Regression coefficients

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1. Calculate the correlation coefficient for the following heights (in inches) of fathers' (x) and their sons' (y):

x	65	66	67	67	68	69	70	72
y	67	68	65	68	72	72	69	71

Obtain the lines of regression for the above data and find the estimate of x for $y = 70$

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Regression equation of x on y: $x = 0.5454y + 30.3674$

Regression equation of y on x: $y = 0.6666x + 23.6712$

Try!!!!

2. Calculate the coefficient of correlation between X and Y by Karl Pearson's method:

X	25	30	28	29	32	24	36	28	27	21
Y	18	20	21	16	14	13	22	15	19	12

Also, obtain the regression equations.

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Also, obtain the regression equations. $r_{XY} = 0.5955$, positive correlation

Problems

3. A computer while calculating the correlation coefficient between x and y from 25 pairs of observations, obtained the following: $n = 25, \sum x = 125, \sum x^2 = 650, \sum y = 100, \sum y^2 = 460, \sum xy = 508$. It was later discovered that they had copied two pairs as $(6, 14)$ and $(8, 6)$ while the correct values were $(8, 12)$ and $(6, 8)$. Obtain the correct value of the correlation coefficient.

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5. Out of two lines of regression, which is the regression line of X on Y.

$$X + 2Y - 5 = 0, 2X + 3Y - 8 = 0$$

Also, obtain (i) the value of correlation coefficient, (ii) mean values of X and Y, (iii) if the variance of X is 12, find σ_Y.

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$r_{XY} = -0.866, b_{XY} = -1.5, b_{YX} = -0.5$

$\bar{X} = 1, \bar{Y} = 2, \sigma_Y = 2$

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First-order partial correlation coefficient :

$$r_{xy.z} = \frac{r_{xy} - r_{xz} r_{yz}}{\sqrt{(1 - r_{xz}^2)(1 - r_{yz}^2)}}$$

Problems

1. The simple correlation coefficients between temperature (X_1), corn yield (X_2), and rainfall (X_3) are

$$r_{12} = 0.59, r_{13} = 0.46, \quad \text{and} \quad r_{23} = 0.77$$

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3. The correlation between a general intelligence test and school achievement in a group of children from 6 to 15 years is 0.8. The correlation between the general intelligence test and age in the same group is 0.7 and the correlation between school achievement and age is 0.6. What is the correlation between general intelligence and school achievement in children of the same age?
(Hint: $X_1 =$ General intelligence, $X_2 =$ School achievement, $X_3 =$ Age. Given $r_{12} = 0.8, r_{13} = 0.7, r_{23} = 0.6$. Calculate $r_{12.3}$)

Multiple correlation

We study the effects of all the independent variables simultaneously on a dependent variable. For example, to study the correlation coefficient between the yied of paddy (X_1) and the other independent variables namely, manure (X_2), humidity (X_3), type of seedlings (X_4), rainfall (X_5), we use multiple correlation, denoted by $R_{1.2345}$

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Properties

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- $R_{1.23} \geq r_{12}, r_{13}, r_{23}$

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Multiple Regression

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$X_1 = 0.3899X_2 - 0.6233X_3 + 16.4776$

Multiple Regression

For a multivariate data, the regression equation of X on Y and Z is

$$(X-\bar{X})\frac{\omega_{11}}{\sigma_1}+(Y-\bar{Y})\frac{\omega_{12}}{\sigma_2}+(Z-\bar{Z})\frac{\omega_{13}}{\sigma_3}=0$$

where

$$\omega = det\begin{bmatrix} 1 & r_{12} & r_{13} \\ r_{12} & 1 & r_{23} \\ r_{13} & r_{23} & 1 \end{bmatrix}$$

$$\omega_{11} = det\begin{bmatrix} 1 & r_{23} \\ r_{23} & 1 \end{bmatrix}$$

$$\omega_{12} = -det\begin{bmatrix} r_{12} & r_{23} \\ r_{13} & 1 \end{bmatrix}$$

$$\omega_{13} = det\begin{bmatrix} r_{12} & 1 \\ r_{13} & r_{23} \end{bmatrix}$$

Problems

1. Find the regression equation of X on Y and Z given the following results:

Variables	Mean	SD	r_{12}	r_{23}	r_{31}
X	35.8	4.2	0.6	-	-
Y	52.4	5.3	-	0.7	-
Z	48.8	6.1	-	-	0.8

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$$\omega = det \begin{bmatrix} 1 & 0.6 & 0.8 \\ 0.6 & 1 & 0.7 \\ 0.8 & 0.7 & 1 \end{bmatrix}$$

$$X = 0.062Y + 0.513Z + 7.6$$