Reliability

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Hazard function

The failure rate or the probability that the system will fail.

$$\lambda(t) = \frac{f(t)}{R(t)}$$

Other important formulas

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- The life time corresponding to a reliability of 0.99 is called B1 life. Corresponding to $R = 0.999$, the life of the system is called B.1

Reliability of Weibull distribution

The pdf of a Weibull distribution is

$$f(t) = \alpha\beta t^{\beta-1} e^{-\alpha t^{\beta}}, \qquad t \geq 0$$

Mean$= (\frac{1}{\alpha})^{\frac{1}{\beta}} \Gamma(\frac{1}{\beta} + 1)$

Variance$= (\frac{1}{\alpha})^{\frac{2}{\beta}} [\Gamma(\frac{2}{\beta} + 1) - [\Gamma(\frac{1}{\beta} + 1)]^2]$

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- $R(t) = e^{-\alpha t^{\beta}}$
- $MTTF = (\frac{1}{\alpha})^{\frac{1}{\beta}} \Gamma(\frac{1}{\beta} + 1)$
- Hazard function is $\lambda(t) = \alpha\beta t^{\beta-1}$

Exponential distribution

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- $R(t) = e^{-\lambda t}$
- Hazard function is $\lambda(t) = \lambda$
- $MTTF = \frac{1}{\lambda}$
- $R(t|T_0) = e^{-\lambda t}$

Problems

1. The density function of the time to failure in years of gizmos (for use on widgets) manufactured by a certain company is given by
$f(t) = \frac{200}{(t+10)^3}, t \geq 0.$
(a) Derive the reliability function and determine the reliability for the first year of operation.
(b) Compute MTTF
(c) What is the design life (time to failure that corresponds to a specified reliability) for a reliability 0.95?

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Solution

(a) $R(t) = \int_t^\infty f(t)dt = \int_t^\infty \frac{200}{(t+10)^3} dt = \frac{100}{(t+10)^2}$
Reliability for the first year of operation is $R(1) = 0.8264$

Solution

(b) $MTTF = \int_0^\infty R(t)dt = \int_0^\infty \frac{100}{(t+10)^2} dt = 10$ years.

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(c) Design life: The time to failure corresponding to given reliability

$$\frac{100}{(t+10)^2} = 0.95$$

$$t = 0.2598 \quad \text{year or} \quad 95 \quad \text{days}$$

Problems

2. Given that $R(t) = e^{-\sqrt{0.001t}}, t \geq 0$

(a) Compute the reliability for a 50 hours mission

(b) Given a 10 hour wear in period, compute the reliability for a 50 hour mission.

(c) What is the design life for a reliability of 0.95?

(d) What is the design life for a reliability of 0.95, given a 10 hour wear in period?

3. The time to failure in operating hours of a critical solid state power unit has hazard rate function $\lambda(t) = 0.003(\frac{t}{500})^{0.5}, \quad t \geq 0.$

(i) What is the reliability if the power unit must operate continuously for 50 hours?

(ii) Determine the design life if reliability of 0.90 is desired.

4. The reliability of a turbine blade is given by $R(t) = (1 - \frac{t}{t_0})^2, 0 \leq t \leq t_0$ where t_0 is the maximum life of the blade.

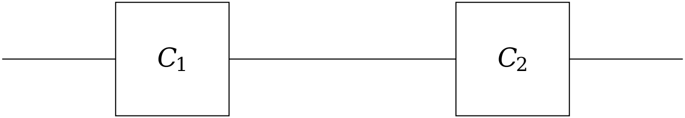
(a) Show that the blades are experiencing wear out.

(b) Compute MTTF as a function of the maximum life.

(c) If the maximum life is 2000 operating hours, what is the design life for a reliability of 0.90?

Reliability of Systems

(i) Series system:



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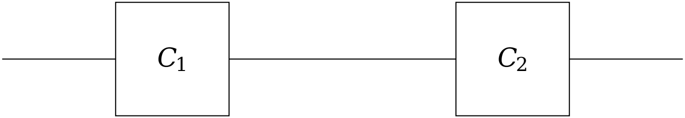
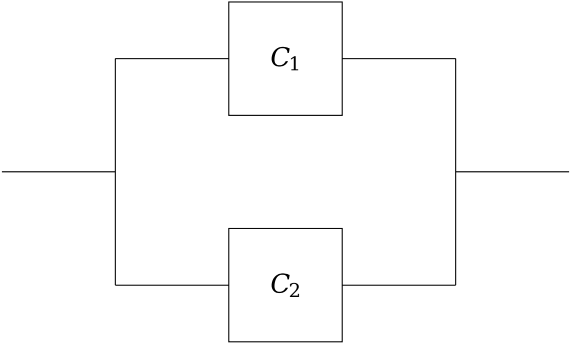


System reliability $R_s = R_1 \times R_2$

System Reliability

(ii) Parallel system:



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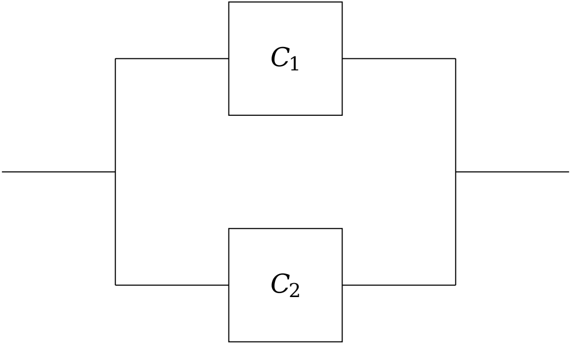
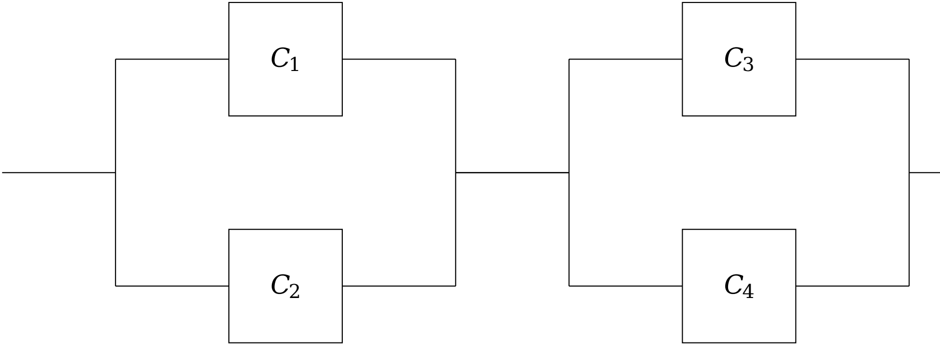


System reliability $R_s = 1 - [(1 - R_1)(1 - R_2)]$

System Reliability

(iii) Both series and parallel



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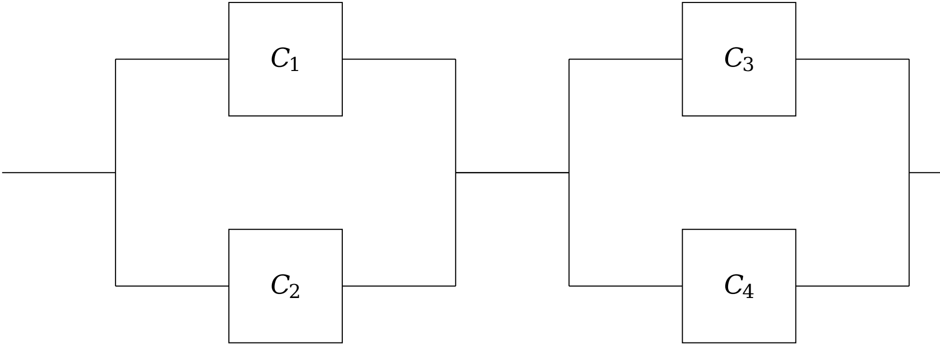
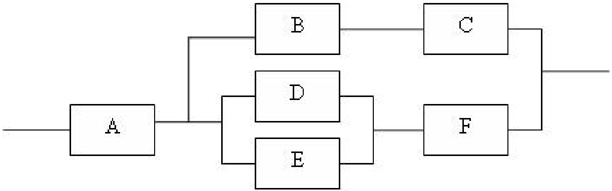


System reliability

$R_s = (1 - [(1 - R_1)(1 - R_2)]) \times (1 - [(1 - R_3)(1 - R_4)])$

Problems

1. Find the system reliability for the network



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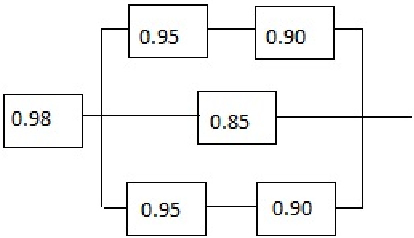

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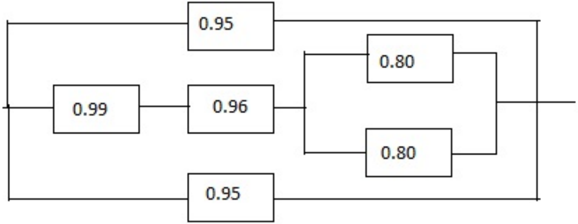
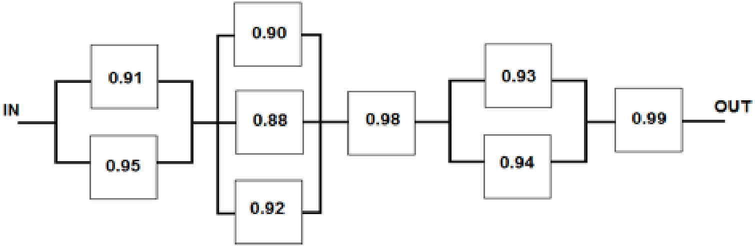


Problems

. A block diagram representation of a system is shown below. Determine the overall system reliability.



Problems

5. There are 16 components in a non-redundant system. The average reliability of each component is 0.99. In order to achieve at least this system reliability by using a redundant system with 4 identical new components, what should be the least reliability of each new component?

Maintainability

Two types of maintenance - Preventive or Proactive and Repair or Reactive

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Preventive maintenance

$R(t)$- reliability of system without maintenance

$R_M(t)$- with maintenance

T-time period, n- number of services

After n services, the reliability is given by

$$R_M(t) = (R(T))^n R(t - nT)$$

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$$R_M(t) = (R(T))^n R(t - nT)$$

$$MTTF = \frac{\int_0^T R(t)dt}{1 - R(T)}$$

Problems

1. If $\lambda(t) = (0.015 + 0.02t)$ per year, where t is in years, (a) calculate the reliability for a 5 year design life, assuming that no maintenance is performed. (b) calculate the reliability for a 5 year design life, assuming that annual preventive maintenance restores the device to an as-good-as new condition.

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Hint: (a) Find $R(t) = e^{-\int_0^t \lambda(t)dt}$ and then substitute $t = 5$, we get $R(5) = 0.7225$,
(b) $T = 1$, $n = 4$, substitute T and n in $R_M(t)$. Then substitute $t = 5$, we get $R_M(5) = 0.8825$

Repair Maintenance

Let T be a continuous random variable representing the time to repair a failed unit having a pdf $h(t)$, then

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- Repair rate is $\mu = \frac{h(t)}{1-H(t)}$ where $H(t) = \int_0^t h(t)dt$ is the cummulative distribution function
- For an exponential distribution or when repair rate is constant, then $\mu = \frac{1}{MTTR}$

Problem

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Solution

$P(T < 3) = \int_1^3 h(t)dt = 0.333$

$MTTR = \int_1^5 t \quad h(t)dt = 3.44hr$

Availability $A(t)$

The probability that a system is performing its intended function at a given time t on the assumption that it is operated and maintained as per the prescribed conditions.

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Availability function of a single component

Point availability

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Interval availability over $(0, T)$

$$A_I(T) = \frac{\mu}{\lambda + \mu} + \frac{\lambda}{(\lambda + \mu)^2 T}\left(1 - e^{-(\lambda+\mu)T}\right)$$

Steady state availability

$$A(\infty) = \frac{\mu}{\lambda + \mu} = \frac{MTTF}{MTTF + MTTR}$$

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Problem

1. A critical communications relay has a constant failure rate of 0.1 per day. Once it has failed, the mean time to repair is 2.5 days (the repair rate is constant).

- What are the point availability at the end of 2 days, the interval availability over a 2-day mission, starting from zero and the steady state availability?
- If two communication relays operate in series, compute the availability at the end of 2 days.
- If they operate in parallel, compute the steady state availability of the system.

Solution

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(ii) $A_s(2) = (A_p(2))^2 = 0.7632$

(iii) $A_s(\infty) = 1 - (1 - A(\infty))^2 = 0.96$

Problems

2. Reliability testing has indicated that a voltage inverter has a 6 month reliability of 0.87 without repair facility. If repair facility is made available with an MTTR of 2.2 months, compute the availability over the 6-month period by assuming constant failure and repair rate.

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Solution

Since it is a system with constant failure rate, the distribution of the failure rate is exponential.

$R(t) = e^{-\lambda t}$

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Hence, $\mu = 0.4545$

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Calculate $A_p(6)$.