Simplification of Regular Expression. RE

Arden's theorem:

if P and Q are RE over Σ, if P does not contain ε then the equation in R is given by

$$\boxed{R = Q + RP}$$

has unique solution Ces

$$\boxed{R = QP^*}$$

$$R = Q + RP \longrightarrow (1) \qquad R = QP^*$$

$$= Q + [QP^*]\, P$$

$$R = Q(\varepsilon + P^* P) \qquad \therefore [\varepsilon + R^* R] = R^*$$

$$\boxed{R = QP^*} \qquad \boxed{R = QP^*}$$

RE→3

Prove $R = \varepsilon + \underbrace{1^*(011)^*}_{R_1}(\underbrace{1^*(011)^*}_{R_1})^* = (1+011)^*$

LHS

$$R = \varepsilon + \underbrace{1^*(011)^*}_{R_1}(\underbrace{1^*(011)^*}_{R_1})^*$$

Now As per Identity

$$\varepsilon + RR^* = R^*$$

$R = (1^*(011)^*)^*$ Now change the equation for to reach the outcome.

$R = (R_1^* R_2^*)^*$ where $R_1 = 1^*$

$R_2 = (011)^*$

As per identity

$$(P^* Q^*)^* = (P^* + Q^*)^* = (P+Q)^*$$

$\Rightarrow$ We can write

$$R = (1^*(011)^*)^* = (1+011)^*$$

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RE → 2

(3) Prove that $(1+00^*1)+(1+00^*1)(0+10^*1)^*$ $(0+10^*1)$ is equal to $0^*1(0+10^*1)^*$

$$LHS = (\underline{1+00^*1}) + (\underline{1+00^*1})(0+10^*1)^*(0+10^*1)$$

$$= (1+00^*1)\left[\varepsilon + (\underbrace{0+10^*1}_{R})^*(\underbrace{0+10^*1}_{R})\right]$$

⇓ Identity

$$(\varepsilon + R^*R) = R^*$$

$$\Rightarrow (1+00^*1)\left[0+10^*1\right]^*$$

for Simplification purpose we can multiply with ε,

$$\Rightarrow (1.\varepsilon + 00^*1)(0+10^*1)^*$$

$$\Rightarrow (\underline{\varepsilon+00^*})\,1\,(0+10^*1)^*$$

$$= 0^*1(0+10^*1)^*$$

Sample Problems :

$$(a^* a b + ba)^* a^* = (a + ab + ba)^*$$