Equations and Derivations

0.1 Irrep of de Sitter Thermodynamic Space

The original expression [1] for the Riemann curvature tensor is given by:

$$R_{\mu\nu\alpha\beta} = \frac{1}{12}\left(g_{\mu\alpha}g_{\nu\beta} - g_{\mu\beta}g_{\nu\alpha}\right)\mathscr{R} \tag{1}$$

where $\mathscr{R}$ is the Ricci scalar, and $g_{\mu\nu}$ is the metric tensor.

In flat Cartesian space, we replace the metric tensor $g_{\mu\nu}$ with the Kronecker delta $\delta_{\mu\nu}$, and introduce a perturbation $\mathscr{R}'$ to the Ricci scalar which itself must be zero, $\mathscr{R} = 0$:

$$R_{\mu\nu\alpha\beta} = \frac{1}{12}\left(\delta_{\mu\alpha}\delta_{\nu\beta} - \delta_{\mu\beta}\delta_{\nu\alpha}\right)\left(\mathscr{R} + \mathscr{R}'\right) \tag{2}$$

Using the identity involving the contraction of two Levi-Civita tensors:

$$\epsilon_{\alpha\beta\lambda}\epsilon_{\mu\nu\lambda} = \delta_{\alpha\mu}\delta_{\beta\nu} - \delta_{\alpha\nu}\delta_{\mu\beta} \tag{3}$$

By substituting the above identity into the expression for $R_{\mu\nu\alpha\beta}$, we get:

$$R_{\mu\nu\alpha\beta} = \frac{1}{12}\epsilon_{\alpha\beta\lambda}\epsilon_{\mu\nu\lambda}(\mathscr{R}') \tag{4}$$

which also implies the symmetry, $R_{\mu\nu\alpha\beta} = R_{\alpha\beta\mu\nu}$. The two covariant rank 3 Levi Civita tensors effectively perform a pair of rotations in the planes $\beta\lambda$ and $\mu\lambda$. Both $g_{\alpha\beta}$ and $g_{\mu\nu}$ are 5D with the compactified form of $g_{\mu\nu} = \mathbf{diag}(1,1,1,|\sigma|)$ where σ is the polarization tensor in the form of the usual Pauli matrices, defined on compact and transverse components with $c = 1$.

$$\sigma = \begin{pmatrix} \frac{1}{t_\parallel}\frac{1}{t_\parallel} & \frac{1}{t_\parallel}\frac{1}{t_\perp} \\ \frac{1}{t_\perp}\frac{1}{t_\parallel} & \frac{1}{t_\perp}\frac{1}{t_\perp} \end{pmatrix} = \begin{cases} \sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \\ \sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \\ \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \end{cases} \tag{5}$$

Clearly there is a dimensional missmatch. de Sitter space is 5D and $SO(3,2)$:

$$R_{\mu\nu\alpha\beta} = \frac{1}{6}\epsilon_{\alpha\beta\lambda\gamma\phi}\epsilon_{\mu\nu}^{\lambda\gamma\phi}(\mathscr{R}') \tag{6}$$

The 5D radial gauge can be written,

$$A_\rho = \frac{\epsilon_{\rho\mu\nu\lambda\tau}x^\mu x^\nu x^\lambda}{\rho^3}A^\tau = 0, \tag{7}$$

where μ, ν, λ are summed over the transverse space $\{1, 2, 3, 5\}$ and $\rho = \sqrt{x_1^2 + x_2^2 + x_3^2 + x_\perp^2}$. The de Sitter metric is compact, simply connected and has a singularity horizon at R. At this singularity horizon the vector potential is subject to the radial gauge. If we set the transverse gauge contribution to zero and look only at the stress energy contribution of the vector potential under radial gauge, we can equate the second terms of the RHS's of (6) and (8).

$$\epsilon_{\mu\nu}^{\lambda\gamma\tau} R^2 = \frac{x^\lambda x^\gamma x^\tau}{\rho^3} A^\rho \tag{8}$$

there are only 10 surviving positive values:

$$\epsilon_{51}^{234}, \epsilon_{52}^{134}, \epsilon_{53}^{124}, \epsilon_{54}^{123}, \epsilon_{12}^{534}, \epsilon_{13}^{524}, \epsilon_{14}^{523}, \epsilon_{23}^{514}, \epsilon_{24}^{513}, \epsilon_{34}^{512}. \tag{9}$$

Only three permutations of the octupole tensor exist on the RHS of (8) for an oscillator with space-like dim(2):

$$\Omega^{ij\tau}, \Omega^{jk\tau}, \Omega^{ik\tau}. \tag{10}$$

In this case, both the upper four Levi-Civita tensors and the lower right group of three. [1]

$$\begin{array}{lll} \epsilon_{51}^{234} & \epsilon_{54}^{123} & \\ \epsilon_{52}^{134} & & \\ \epsilon_{53}^{124} & & \\ & & \\ \epsilon_{12}^{534} & & \\ \epsilon_{13}^{524} & \epsilon_{14}^{523} & \\ \epsilon_{23}^{514} & \epsilon_{24}^{513} & \epsilon_{34}^{512} \end{array} \tag{11}$$

The 5D energy surface distributes scalar charge between transverse and compact (axial) components according to the upper and lower portions of (11). Likewise, the Ricci scalar perturbation has real values which correspond to the upper four tensors of (11) while the imaginary components of the Ricci scalar perturbation transform via the lower six components.

On inspection of the lower indices, the three covariant time-like groups are $(\epsilon_{51}^{234}, \epsilon_{52}^{134}, \epsilon_{53}^{124})$, $(\epsilon_{14}^{523}, \epsilon_{24}^{513}, \epsilon_{34}^{512})$ and (ϵ_{54}^{123}) leaving only one space-like group, $(\epsilon_{12}^{534}, \epsilon_{13}^{524}, \epsilon_{23}^{514})$. The contraction of the covariant space-like tensors $(\epsilon_{12}^{534}, \epsilon_{13}^{524}, \epsilon_{23}^{514})$ in (6) leaves only the lower indices. Invoking the equivalence of the identity (3) lets us identify the irreducible de Sitter thermodynamic space since the singularity horizon at R is defined under the radial gauge. The octupole space-like 2D representation is the irreducible thermodynamic inertial representation is shown in fig 1 and forms an enveloping toroidal de Sitter space.

[1] We can only make this comparison because of the equality of the identity (3) and (6).

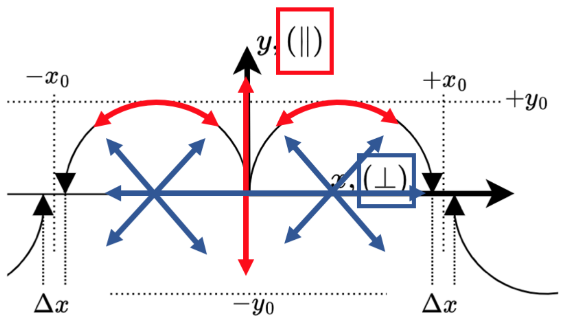

Figure 1: Two spatial dimensions $\hat{y} \equiv (ct_{\|})$, $\hat{x} \equiv (ct_{\perp})$ correspond to the compact (red) and transverse (blue) components. There are three orthogonal such spaces covering three-space.

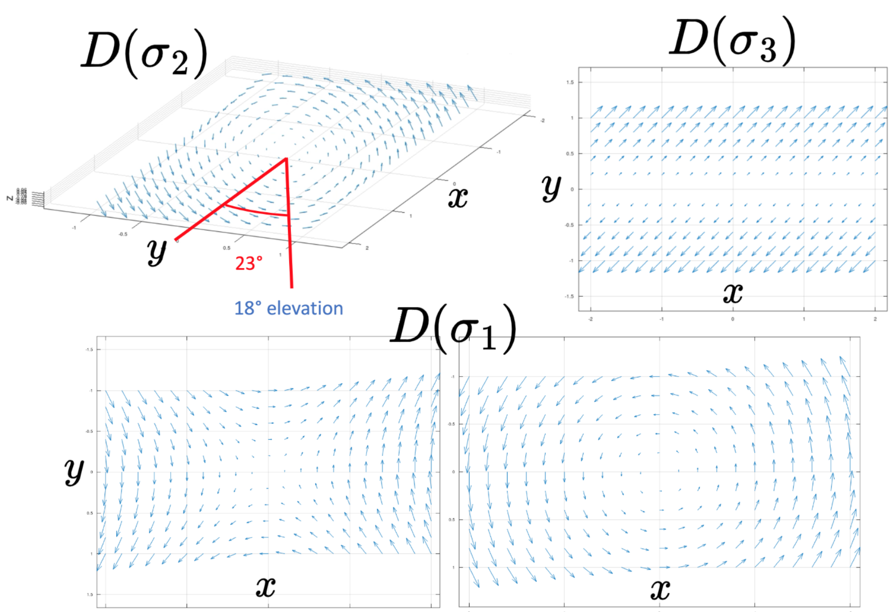

Figure 2: Electric displacement vector fields as the solutions to the polarization PDEs from section 0.2. These map to the irreducible inertial space shown in fig 1.

0.2 Polarization Tensor

In (5) we explicitly defined the Pauli matrices with both compact and transverse time-like components. The infintesimal path in the inertial frame of $(1, 1, 1, \sigma)$ is three sets of PDE's which correspond to the complete set of polarizations. Using the notation of the specific domain of the irrep space in fig 1, $y \equiv ct_{\parallel}, x \equiv ct_{\perp}$ we can find the PDEs for each of the matrices. Starting with $\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ we can rewrite as,

$$(\nabla u \cdot \nabla v) = 1$$

where $u(x, y)$ and $v(x, y)$ are scalar functions, and $D(x, y) = (u(x, y), v(x, y))$ is a vector-valued function.

$$\nabla u = \left(\frac{\partial u}{\partial x}, \frac{\partial u}{\partial y}\right), \quad \nabla v = \left(\frac{\partial v}{\partial x}, \frac{\partial v}{\partial y}\right)$$

$$(\nabla u \cdot \nabla v) = \frac{\partial u}{\partial x}\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}\frac{\partial v}{\partial y} = 1$$

Since in the inertial frame both coordinates are of equal magnitude and c is constant, both terms contribute equally, giving the PDEs for σ_1:

$$x = 0, \pm 2, \quad v_y = x, \quad y = 0, \pm 2, \quad v_x = y,$$

Following a similar procedure for σ_2 gives:

$$u = x + iy, \quad v = x - iy,$$

where importantly, the real components are defined in the 2D plane and the imaginary component is defined in the orthogonal spatial dimension, $\hat{z}$ in this case. Finally, σ_3 has the solutions:

$$u = -y, \quad v = x$$

Plotting the above fields on the irrep thermodynamic space with the domain $x \in [-2R, 2R], y \in [-R, R], R = 1$ gives a classical interpretation of total currents implied by the use of the $SU(2)$ generators in Field Theory.

0.3 Heat Death Boundary Condition

Find the maximum viscosity condition & identify the invariant form factors. See arxiv.org/pdf/2407.13065.

References

[1] G. E. Volovik. "De Sitter Local Thermodynamics in f(R) Gravity". en. In: *JETP Letters* 119.7 (Apr. 2024), pp. 564–571. ISSN: 0021-3640, 1090-6487. DOI: 10.1134/S0021364024600526. URL: https://link.springer.com/10.1134/S0021364024600526 (visited on 06/02/2024).