Quantum Field Theory Equation Sheet

quinten tupker

October 7 2020 - December 6, 2020

Here are some useful equations

Table 1: Equation Sheet

Name/Description	Equation	Remarks
Noether conserved current	$j^\mu = \partial_{\partial_\mu \phi}\mathcal{L}\delta\phi - F^\mu$	here $\mathcal{L}(x+\delta x) = \mathcal{L} + \delta x \partial_\mu F^\mu$, $\partial_\mu j^\mu = 0$
The conserved charge arising from a conserved current	$Q = \int d^3x j^0$	
The Energy-Momentum Tensor	$T^\mu_\nu = \partial_{\partial_\mu \phi}\mathcal{L}\partial_\nu\phi - \delta^\mu_\nu \mathcal{L}$	This is the Noether current under translation. This tensor can always be chosen to be symmetric. It is a Noether current, so conserved as $\partial_\mu T^{\mu\nu} = 0$
Ladder Operators	$[a_p, a_q^\dagger] = (2\pi)^3\delta(p-q)$	
Field Operator	$\phi = \int \frac{d^3p}{(2\pi)^3}\frac{1}{\sqrt{2\omega_p}}(a_p e^{ip\cdot x} + a_p^\dagger e^{-ip\cdot x})$	
Momentum Operator	$\pi = \int \frac{d^3p}{(2\pi)^3}(-i)\sqrt{\frac{\omega_p}{2}}(a_p e^{ip\cdot x} - a_p^\dagger e^{-ip\cdot x})$	
Dirac Equation	$(i\gamma^\mu\partial_\mu - m)\cdot\psi = (i\not\partial - m)\cdot\psi = 0$	
Chiral represntation and Clifford Algebra	$\{\gamma^\mu, \gamma^\nu\} = 2\eta^{\mu\nu}I, \{\gamma^5, \gamma^\mu\} = 0, (\gamma)^2 = 0, \gamma^5 = i\gamma^0\gamma^1\gamma^2\gamma^3$, $\gamma^0 = \begin{pmatrix} & I \\ I & \end{pmatrix}$, $\gamma^i = \begin{pmatrix} & \sigma^i \\ \sigma^i & \end{pmatrix}$, $\gamma^5 = \begin{pmatrix} I & \\ & -I \end{pmatrix}$	
General Solution to Dirac Equation	positive frequencies: $\psi(x) = u(p)e^{-ip\cdot x}, u^s(p) = \begin{pmatrix}\sqrt{p\cdot\sigma}\xi^s \\ \sqrt{p\cdot\bar{\sigma}}\xi^s\end{pmatrix}$, for negative frequencies $\psi(x) = v(p)e^{ip\cdot x}, v^s(p) = \begin{pmatrix}\sqrt{p\cdot\sigma}\eta^s \\ -\sqrt{p\cdot\bar{\sigma}}\eta^s\end{pmatrix}$	Here ξ^r, η^s form orthonormal bases for $\mathbb{C}^2$ and $\sqrt{p\cdot\sigma} = \sqrt{m}e^{\chi\cdot\sigma/2}, \sqrt{p\cdot\bar{\sigma}} = \sqrt{m}e^{-\chi\cdot\sigma}$