Quantum Information Theory Info Sheet

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Handy definitions:

- The **Shannon entropy** of rand var X is $H(X) = -\sum_{x\in J} p(x)\log_2(p(x))$
- **Typical sets** contain sequences $u = (u_1, \dots, u_n)$ satisfying $2^{-n(H(U)+\epsilon)} \leq \mathbb{P}(u) \leq 2^{-n(H(U)-\epsilon)}$. Here U describes a single letter in the sequence.
- The **joint entropy** of X, Y is $H(X,Y) = -\sum_{x\in J_X, y\in J_Y} p(x,y)\log_2(p(x,y))$
- The **relative entropy** or **Kullback-Leibler divergence** of $p << q$ (p **absolutely continuous** wrt q meaning $\mathrm{supp}(p) \subseteq \mathrm{supp}(q)$) is $D(p||q) = \sum_x p(x)\log(p(x)/q(x))$
- The **mutual information** is $I(X:Y) = H(X) + H(Y) - H(X,Y) = H(X) - H(Y|X)$.
- The **conditional entropy** (weirdly) is $H(Y|X) = -\sum_{x,y} p(x,y)\log(p(x,y)) = \sum_x p(x)H(Y|X=x)$

Handy equations:

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Handy theorems:

- Jensen's inequality states that if ϕ convex, then $\phi(\mathbb{E}(X)) \leq \mathbb{E}(\phi(X))$. Moreover, equality occurs iff ϕ is linear almost everywhere.