Quantum Field Theory Equation Sheet

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Here are some useful equations

Table 1: Equation Sheet

Noether conserved current $ j^{\mu} = \partial_{\mu\nu} \mathcal{L} \delta \phi - F^{\mu} $ here $\mathcal{L}(x + \delta x) = \mathcal{L} + \delta x \partial_{\mu} F^{\mu}, \partial_{\mu} j^{\mu} = 0 $ The conserved current $ The \text{ conserved current} $ The Energy-Momentum Tensor $ T^{\mu}_{\nu} = \partial_{\sigma} \phi \mathcal{L} \partial_{\nu} \phi - \delta^{\mu}_{\nu} \mathcal{L} $ This is the Noether current under translation. This tensor can always be chosen to be symmetric. It is a Noether current, so conserved as $\partial_{\mu} T^{\mu\nu} = 0 $ Momentum Operator	Name/Description	Equation	Remarks
The conserved charge arising from a conserved current $T_{\nu}^{\mu} = \partial_{0,\rho} \mathcal{L} \partial_{\nu} \phi - \delta_{\nu}^{\mu} \mathcal{L}$ This is the Noether current under translation. This tensor can always be chosen to be symmetric. It is a Noether current, so conserved as $\partial_{\mu} T^{\mu\nu} = 0$ conserved as $\partial_{\mu} T^$		$j^{\mu} = \partial_{\partial_{\mu}\phi} \mathcal{L} \delta \phi - F^{\mu}$	
The Energy-Momentum Tensor $T^{\nu}_{\nu} = \partial_{\theta_{\mu}\phi}\mathcal{L}\partial_{\nu}\phi - \delta^{\nu}_{\nu}\mathcal{L}$ This is the Noether current under translation. This tensor can always be chosen to be symmetric. It is a Noether current, so conserved as $\partial_{\mu}T^{\mu\nu} = 0$ Eadder Operators $[a_{p}, a_{q}^{\dagger}] = (2\pi)^{3}\delta(p - q)$	The conserved charge arising	$Q = \int d^3x j^0$, ,
$ \begin{array}{c} \text{der translation. This tensor can always be chosen to be symmetric. It is a Noether current, so conserved as } \partial_{\mu}T^{\mu\nu}=0 \\ \text{Eidd Operator} & [a_p,a_q^{\dagger}]=(2\pi)^3\delta(p-q) \\ \text{Field Operator} & \phi=\int \frac{d^3p}{(2\pi)^3}\frac{1}{\sqrt{2\omega_p}}(a_pe^{ip\cdot x}+a_p^{\dagger}e^{-ip\cdot x}) \\ \text{Momentum Operator} & \pi=\int \frac{d^3p}{(2\pi)^3}(-i)\sqrt{\frac{\omega_p}{2}}(a_pe^{ip\cdot x}-a_p^{\dagger}e^{-ip\cdot x}) \\ \text{Dirac Equation} & (i\gamma^{\mu}\partial_{\mu}-m)\cdot\psi=(i\not{\partial}-m)\cdot\psi=0 \\ \text{Dirac Adjoint} & \psi=\psi^{\dagger,\gamma}0 \\ \text{Chiral represntation and Clifford} & \{\gamma^{\mu},\gamma^{\nu}\}=2\eta^{\mu\nu}I,\{\gamma^{5},\gamma^{\mu}\}=0,(\gamma)^2=0,\gamma^5=i\gamma^0\gamma^1\gamma^2\gamma^3,\gamma^0=\{I-I\},\gamma^i=\left(\sigma^i\right)^j,\gamma^i=\left(\sigma^i\right)^j,\gamma^5=\left(I-I\right) \\ \text{General Solution to Dirac Equation} & \text{positive frequencies: } \psi(x)=u(p)e^{-ip\cdot x},u^s(p)=\left(\sqrt{p\cdot\sigma}\xi^s\right),\text{ for negative frequencies: } \psi(x)=u(p)e^{-ip\cdot x},v^s(p)=\left(\sqrt{p\cdot\sigma}\xi^s\right),\text{ for negative frequencies: } \psi(x)=v(p)e^{ip\cdot x},v^s(p)=\left(\sqrt{p\cdot\sigma}\eta^s\right) \\ \text{Plane wave solutions satisfy} & (\not{p}-m)u=0=(\not{p}+m)v \\ \text{Steps for Calculating Scattering} & 0) \text{ Receive and quantise the Lagrangian 1) try to use the Dyson formula 2) calculate the time ordered product 3) find a version of Wick's formula 4) develop Feynman rules \\ \text{Dyson formula} & A_{i\rightarrow f}=\langle f S i\rangle,S=T\left(e^{\frac{1}{i}\int_{-\infty}^{\infty}H_I(t)dt}\right) \text{ for interaction Hamiltonian } H_I \\ \text{Standard spin-1/2 Fermion Ex-pansion} & \psi_{\alpha}=\sum_{s=1}^{2-1}\int \frac{d^3p}{(2\pi)^3}\frac{1}{\sqrt{2E_p}}\left(b_p^su_{\alpha}^s(p)e^{-ip\cdot x}+c_p^sv_{\alpha}^s(p)e^{ip\cdot x}\right) \\ \end{array}$			
$ \begin{array}{c} \text{always be chosen to be symmetric. It is a Noether current, so conserved as } \partial_{\mu}T^{\mu\nu} = 0 \\ \text{Ladder Operators} & [a_{p},a_{q}^{\dagger}] = (2\pi)^{3}\delta(p-q) \\ \text{Field Operator} & \phi = \int \frac{d^{3}p}{(2\pi)^{3}} \frac{1}{\sqrt{2\omega}p} (a_{p}e^{ip\cdot x} + a_{p}^{\dagger}e^{-ip\cdot x}) \\ \text{Momentum Operator} & \pi = \int \frac{d^{3}p}{(2\pi)^{3}} (-i)\sqrt{\frac{a_{p}}{2}} (a_{p}e^{ip\cdot x} - a_{p}^{\dagger}e^{-ip\cdot x}) \\ \text{Dirac Equation} & (i\gamma^{\mu}\partial_{\mu}-m)\cdot\psi=(i\partial^{\prime}-m)\cdot\psi=0 \\ \text{Dirac Adjoint} & \psi=\psi^{\dagger}\gamma^{0} \\ \text{Chiral repressitation and Clifford Algebra} & \{\gamma^{\mu},\gamma^{\nu}\}=2\eta^{\mu\nu}I,\{\gamma^{5},\gamma^{\mu}\}=0,(\gamma)^{2}=0,\gamma^{5}=i\gamma^{0}\gamma^{1}\gamma^{2}\gamma^{3},\gamma^{0}=\begin{pmatrix} I\\I\\I\end{pmatrix},\gamma^{i}=\begin{pmatrix}\sigma^{i}\\\sigma^{i}\end{pmatrix},\gamma^{5}=\begin{pmatrix}I\\I\\I\end{pmatrix} \\ \text{Dirac Equation} & \text{positive frequencies: } \psi(x)=u(p)e^{-ip\cdot x},u^{s}(p)=\begin{pmatrix}\sqrt{p\cdot\sigma}\xi^{s}\\\sqrt{p\cdot\sigma}\xi^{s}\end{pmatrix}, \text{ for meative frequencies: } \psi(x)=u(p)e^{-ip\cdot x},v^{s}(p)=\begin{pmatrix}\sqrt{p\cdot\sigma}\xi^{s}\\\sqrt{p\cdot\sigma}\xi^{s}\end{pmatrix}, \text{ for meative frequencies: } \psi(x)=v(p)e^{ip\cdot x},v^{s}(p)=\begin{pmatrix}\sqrt{p\cdot\sigma}\xi^{s}\\-\sqrt{p\cdot\sigma}\eta^{s}\end{pmatrix} \\ \text{Plane wave solutions satisfy} & (\not{p}-m)u=0=(\not{p}+m)v \\ \text{Steps for Calculating Scattering Amplitudes} & 0) \text{ Receive and quantise the Lagrangian 1) try to use the Dyson formula 2) calculate the time ordered product 3) find a version of Wick's formula 4) develop Feynman rules \\ \text{Dyson formula} & A_{i\rightarrow f}=\langle f S i\rangle, S=T\left(e^{\frac{i}{i}\int^{\infty}_{\infty}H_{I}(i)dt}\right) \text{ for interaction Hamiltonian } H_{I} \\ \text{Standard spin-1/2 Fermion Ex-pansion} & \psi_{\alpha}=\sum_{s=1}^{2}\int\frac{d^{3}p}{(2\pi)^{3}}\frac{1}{\sqrt{2E_{p}}}\left(b_{p}^{s}u_{\alpha}^{s}(p)e^{-ip\cdot x}+c_{p}^{s}v_{\alpha}^{s}(p)e^{ip\cdot x}\right) \end{array}$	The Energy-Momentum Tensor	$T^{\mu}_{\nu} = \partial_{\partial_{\mu}\phi} \mathcal{L} \partial_{\nu} \phi - \delta^{\mu}_{\nu} \mathcal{L}$	
Ladder Operators $[a_p,a_q^{\dagger}] = (2\pi)^3\delta(p-q)$ Field Operator $\phi = \int \frac{d^3p}{(2\pi)^3} \frac{1}{\sqrt{2\omega_p}} (a_p e^{ip\cdot x} + a_p^{\dagger} e^{-ip\cdot x})$ Momentum Operator $\pi = \int \frac{d^3p}{(2\pi)^3} \frac{1}{\sqrt{2\omega_p}} (a_p e^{ip\cdot x} - a_p^{\dagger} e^{-ip\cdot x})$ Dirac Equation $(i\gamma^{\mu}\partial_{\mu} - m) \cdot \psi = (i\partial - m) \cdot \psi = 0$ Dirac Adjoint $\psi = \psi^{\dagger} \gamma^{0}$ Chiral representation and Clifford Algebra $\begin{cases} \gamma^{\mu}, \gamma^{\nu} \} = 2\eta^{\mu\nu} I, \{\gamma^{5}, \gamma^{\mu}\} = 0, (\gamma)^{2} = 0, \gamma^{5} = i\gamma^{0}\gamma^{1}\gamma^{2}\gamma^{3}, \gamma^{0} = (I) - I \end{pmatrix}$ General Solution to Dirac Equation $(i\gamma^{\mu}\partial_{\mu} - m) \cdot \psi = (i\partial - m) \cdot \psi = 0$ Dositive frequencies: $\psi(x) = u(p)e^{-ip\cdot x}, u^{s}(p) = (\sqrt{p\cdot \sigma}\xi^{s}), \text{ for } m \text{ or } m or$			
$ \begin{array}{c} \text{Ladder Operators} & [a_p,a_q^\dagger] = (2\pi)^3\delta(p-q) \\ \text{Field Operator} & \phi = \int \frac{d^3p}{(2\pi)^3} \frac{1}{\sqrt{2\omega_p}} (a_p e^{ip\cdot x} + a_p^\dagger e^{-ip\cdot x}) \\ \text{Momentum Operator} & \pi = \int \frac{d^3p}{(2\pi)^3} \frac{1}{\sqrt{2\omega_p}} (-i)\sqrt{\frac{\omega_p}{2}} (a_p e^{ip\cdot x} - a_p^\dagger e^{-ip\cdot x}) \\ \text{Dirac Equation} & (i\gamma^\mu \partial_\mu - m) \cdot \psi = (i\not\!\!\partial - m) \cdot \psi = 0 \\ \text{Dirac Adjoint} & \psi = \psi^\dagger \gamma^0 \\ \text{Chiral represntation and Clifford} & \{\gamma^\mu, \gamma^\nu\} = 2\eta^{\mu\nu}I, \{\gamma^5, \gamma^\mu\} = 0, (\gamma)^2 = 0, \gamma^5 = i\gamma^0\gamma^1\gamma^2\gamma^3, \gamma^0 = \\ I \end{pmatrix}, \gamma^i = \begin{pmatrix} I \\ \sigma^i \end{pmatrix}, \gamma^i = \begin{pmatrix} \sigma^i \\ \sigma^i \end{pmatrix}, \gamma^5 = \begin{pmatrix} I \\ -I \end{pmatrix} \\ \text{General Solution to Dirac Equation} & \text{positive frequencies: } \psi(x) = u(p)e^{-ip\cdot x}, u^s(p) = \begin{pmatrix} \sqrt{p\cdot\sigma}\xi^s \\ \sqrt{p\cdot\sigma}\xi^s \end{pmatrix}, \text{ for } \\ \text{negative frequencies: } \psi(x) = v(p)e^{ip\cdot x}, v^s(p) = \begin{pmatrix} \sqrt{p\cdot\sigma}\xi^s \\ \sqrt{p\cdot\sigma}\eta^s \end{pmatrix} & \text{Here } \xi^r, \eta^s \text{ form orthonormal bases for } \mathbb{C}^2 \text{ and } \sqrt{p\cdot\sigma} = \\ \sqrt{m}e^{\chi\cdot\sigma/2}, \sqrt{p\cdot\sigma} = \sqrt{m}e$, v
$\begin{array}{lll} \begin{tabular}{lll} Field Operator & \phi = \int \frac{d^3p}{(2\pi)^3} \frac{1}{\sqrt{2\omega_p}} (a_p e^{ip\cdot x} + a_p^\dagger e^{-ip\cdot x}) \\ \hline Momentum Operator & \pi = \int \frac{d^3p}{(2\pi)^3} \frac{1}{\sqrt{2}} \sqrt{\frac{\omega_p}{2}} (a_p e^{ip\cdot x} - a_p^\dagger e^{-ip\cdot x}) \\ \hline Dirac Equation & (i\gamma^\mu\partial_\mu - m) \cdot \psi = (i\not\!\!/\!\!\!/-m) \cdot \psi = 0 \\ \hline Dirac Adjoint & \psi = \psi^\dagger\gamma^0 \\ \hline Chiral representation and Clifford Algebra & \begin{cases} \gamma^\mu, \gamma^\nu \} = 2\eta^{\mu\nu}I, \{\gamma^5, \gamma^\mu\} = 0, (\gamma)^2 = 0, \gamma^5 = i\gamma^0\gamma^1\gamma^2\gamma^3, \gamma^0 = \\ I \\ I \end{pmatrix}, \gamma^i = \begin{pmatrix} \sigma^i \\ \sigma^i \end{pmatrix}, \gamma^5 = \begin{pmatrix} I \\ -I \end{pmatrix} \\ \hline General Solution to Dirac Equation & positive frequencies: \psi(x) = u(p)e^{-ip\cdot x}, u^s(p) = \begin{pmatrix} \sqrt{p\cdot\sigma}\xi^s \\ \sqrt{p\cdot\sigma}\xi^s \end{pmatrix}, \text{ for } \\ negative frequencies: \psi(x) = v(p)e^{ip\cdot x}, v^s(p) = \begin{pmatrix} \sqrt{p\cdot\sigma}\eta^s \\ -\sqrt{p\cdot\sigma}\eta^s \end{pmatrix} & \frac{1}{\sqrt{p\cdot\sigma}} (p)e^{-ip\cdot x} + c_p^*i\gamma^s (p)e^{ip\cdot x} \end{pmatrix} \\ \hline Plane wave solutions satisfy & (p-m)u = 0 = (\not\!\!/\!\!\!/ + m)v \\ \hline Dyson formula & A_{i\to f} = \langle f S i\rangle, S = T\left(e^{\frac{1}{i}\int_{-\infty}^\infty H_I(t)dt}\right) \text{ for interaction Hamiltonian } \\ \hline Dyson formula & A_{i\to f} = \langle f S i\rangle, S = T\left(e^{\frac{1}{i}\int_{-\infty}^\infty H_I(t)dt}\right) \text{ for interaction Hamiltonian } \\ \hline Standard spin-1/2 Fermion Expansion & \psi_\alpha = \sum_{s=1}^2 \int \frac{d^3p}{(2\pi)^3} \frac{1}{\sqrt{2E_p}} \left(b_p^s u_\alpha^s(p)e^{-ip\cdot x} + c_p^si\gamma_\alpha^s(p)e^{ip\cdot x}\right) \\ \hline \end{array}$			conserved as $\partial_{\mu}T^{\mu\nu} = 0$
$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	Ladder Operators	$\left[a_p, a_q^{\dagger}\right] = (2\pi)^3 \delta(p-q)$	
$\begin{array}{lll} \mbox{Dirac Equation} & (i\gamma^{\mu}\partial_{\mu}-m)\cdot\psi=(i\not\!\partial-m)\cdot\psi=0 \\ \mbox{Dirac Adjoint} & \psi=\psi^{\dagger}\gamma^{0} \\ \mbox{Chiral representation and Clifford} & \{\gamma^{\mu},\gamma^{\nu}\}=2\eta^{\mu\nu}I, \{\gamma^{5},\gamma^{\mu}\}=0, (\gamma)^{2}=0, \gamma^{5}=i\gamma^{0}\gamma^{1}\gamma^{2}\gamma^{3}, \gamma^{0}=\\ & \begin{pmatrix} I\\I \end{pmatrix}, \gamma^{i}=\begin{pmatrix} \sigma^{i}\\\sigma^{i} \end{pmatrix}, \gamma^{5}=\begin{pmatrix} I\\-I \end{pmatrix} \\ \mbox{General Solution to Dirac Equation} & positive frequencies: \ \psi(x)=u(p)e^{-ip\cdot x}, u^{s}(p)=\begin{pmatrix} \sqrt{p\cdot\sigma}\xi^{s}\\\sqrt{p\cdot\sigma}\xi^{s} \end{pmatrix}, \ \text{for}\\ \mbox{negative frequencies:} \ \psi(x)=v(p)e^{ip\cdot x}, v^{s}(p)=\begin{pmatrix} \sqrt{p\cdot\sigma}\eta^{s}\\\sqrt{p\cdot\sigma}\xi^{s} \end{pmatrix}, \ \text{for}\\ \mbox{negative frequencies:} \ \psi(x)=v(p)e^{ip\cdot x}, v^{s}(p)=\begin{pmatrix} \sqrt{p\cdot\sigma}\eta^{s}\\\sqrt{p\cdot\sigma}\xi^{s} \end{pmatrix}, \ \text{for}\\ \mbox{negative frequencies:} \ \psi(x)=v(p)e^{ip\cdot x}, v^{s}(p)=\begin{pmatrix} \sqrt{p\cdot\sigma}\eta^{s}\\\sqrt{p\cdot\sigma}\xi^{s} \end{pmatrix}, \ \text{for}\\ \mbox{negative frequencies:} \ \psi(x)=v(p)e^{ip\cdot x}, v^{s}(p)=\begin{pmatrix} \sqrt{p\cdot\sigma}\eta^{s}\\\sqrt{p\cdot\sigma}\xi^{s} \end{pmatrix}, \ \text{for}\\ \mbox{negative frequencies:} \ \psi(x)=v(p)e^{ip\cdot x}, v^{s}(p)=\begin{pmatrix} \sqrt{p\cdot\sigma}\eta^{s}\\\sqrt{p\cdot\sigma}\xi^{s} \end{pmatrix}, \ \text{for}\\ \mbox{negative frequencies:} \ \psi(x)=v(p)e^{ip\cdot x}, v^{s}(p)=\begin{pmatrix} \sqrt{p\cdot\sigma}\eta^{s}\\\sqrt{p\cdot\sigma}\xi^{s} \end{pmatrix}, \ \text{for}\\ \mbox{negative frequencies:} \ \psi(x)=v(p)e^{ip\cdot x}, v^{s}(p)=\begin{pmatrix} \sqrt{p\cdot\sigma}\eta^{s}\\\sqrt{p\cdot\sigma}\xi^{s} \end{pmatrix}, \ \text{for}\\ \mbox{negative frequencies:} \ \psi(x)=v(p)e^{ip\cdot x}, v^{s}(p)=\begin{pmatrix} \sqrt{p\cdot\sigma}\eta^{s}\\\sqrt{p\cdot\sigma}\xi^{s} \end{pmatrix}, \ \text{for}\\ \mbox{negative frequencies:} \ \psi(x)=v(p)e^{ip\cdot x}, v^{s}(p)=\begin{pmatrix} \sqrt{p\cdot\sigma}\eta^{s}\\\sqrt{p\cdot\sigma}\eta^{s} \end{pmatrix}, \ \text{for}\\ \mbox{negative frequencies:} \ \psi(x)=v(p)e^{ip\cdot x}, v^{s}(p)=\begin{pmatrix} \sqrt{p\cdot\sigma}\eta^{s}\\\sqrt{p\cdot\sigma}\eta^{s} \end{pmatrix}, \ \text{for}\\ \mbox{negative frequencies:} \ \psi(x)=v(p)e^{ip\cdot x}, v^{s}(p)=\begin{pmatrix} \sqrt{p\cdot\sigma}\eta^{s}\\\sqrt{p\cdot\sigma}\eta^{s} \end{pmatrix}, \ \text{for}\\ \mbox{negative frequencies:} \ \psi(x)=v(p)e^{ip\cdot x}, v^{s}(p)=\begin{pmatrix} \sqrt{p\cdot\sigma}\eta^{s}\\\sqrt{p\cdot\sigma}\eta^{s} \end{pmatrix}, \ \text{for}\\ \mbox{negative frequencies:} \ \psi(x)=v(p)e^{ip\cdot x}, v^{s}(p)=\begin{pmatrix} \sqrt{p\cdot\sigma}\eta^{s}\\\sqrt{p\cdot\sigma}\eta^{s} \end{pmatrix}, \ \text{for}\\ \mbox{negative frequencies:} \ \psi(x)=v(p)e^{ip\cdot x}, v^{s}(p)=\begin{pmatrix} \sqrt{p\cdot\sigma}\eta^{s}\\\sqrt{p\cdot\sigma}\eta^{s} \end{pmatrix}, \ \text{for}\\ \mbox{negative frequencies:} \ \psi(x)=v(p)e^{ip\cdot x}, v^{s}(p)=\begin{pmatrix} \sqrt{p\cdot\sigma}\eta^{s}\\\sqrt{p\cdot\sigma}\eta^{s} \end{pmatrix}, \ \text{for}\\ \mbox{negative frequencies:} \ \psi(x)=v(p)e^{ip\cdot x}, v^{s}(p)=\begin{pmatrix} \sqrt{p\cdot\sigma}\eta^{s}\\\sqrt{p\cdot\sigma}\eta^{s} \end{pmatrix}, \ \text{for}\\ \mbox{negative frequencies:} \ \psi(x)=v$	Field Operator	· · _	
Chiral representation and Clifford Algebra $\begin{cases} \{\gamma^{\mu}, \gamma^{\nu}\} = 2\eta^{\mu\nu}I, \{\gamma^{5}, \gamma^{\mu}\} = 0, (\gamma)^{2} = 0, \gamma^{5} = i\gamma^{0}\gamma^{1}\gamma^{2}\gamma^{3}, \gamma^{0} = (I - I) \end{cases}$ General Solution to Dirac Equation positive frequencies: $\psi(x) = u(p)e^{-ip \cdot x}, u^{s}(p) = (\sqrt{p \cdot \sigma}\xi^{s})$, for negative frequencies: $\psi(x) = v(p)e^{ip \cdot x}, v^{s}(p) = (\sqrt{p \cdot \sigma}\eta^{s})$ has so for \mathbb{C}^{2} and $\sqrt{p \cdot \sigma} = \sqrt{m}e^{\chi \cdot \sigma/2}, \sqrt{p \cdot \sigma}e^{\chi \cdot \sigma/2}, p \cdot \sigma$	Momentum Operator		
Chiral representation and Clifford Algebra $\begin{cases} \{\gamma^{\mu}, \gamma^{\nu}\} = 2\eta^{\mu\nu}I, \{\gamma^{5}, \gamma^{\mu}\} = 0, (\gamma)^{2} = 0, \gamma^{5} = i\gamma^{0}\gamma^{1}\gamma^{2}\gamma^{3}, \gamma^{0} = (I - I) \end{cases}$ General Solution to Dirac Equation positive frequencies: $\psi(x) = u(p)e^{-ip \cdot x}, u^{s}(p) = (\sqrt{p \cdot \sigma}\xi^{s})$, for negative frequencies: $\psi(x) = v(p)e^{ip \cdot x}, v^{s}(p) = (\sqrt{p \cdot \sigma}\eta^{s})$ has so for \mathbb{C}^{2} and $\sqrt{p \cdot \sigma} = \sqrt{m}e^{\chi \cdot \sigma/2}, \sqrt{p \cdot \sigma}e^{\chi \cdot \sigma/2}, p \cdot \sigma$	Dirac Equation	$(i\gamma^{\mu}\partial_{\mu} - m) \cdot \psi = (i\partial \!\!\!/ - m) \cdot \psi = 0$	
Algebra $\begin{pmatrix} I \\ I \end{pmatrix}, \gamma^i = \begin{pmatrix} \sigma^i \\ \sigma^i \end{pmatrix}, \gamma^5 = \begin{pmatrix} I \\ -I \end{pmatrix}$ General Solution to Dirac Equation positive frequencies: $\psi(x) = u(p)e^{-ip\cdot x}, u^s(p) = \begin{pmatrix} \sqrt{p\cdot\sigma}\xi^s \\ \sqrt{p\cdot\bar{\sigma}}\xi^s \end{pmatrix}$, for negative frequencies $\psi(x) = v(p)e^{ip\cdot x}, v^s(p) = \begin{pmatrix} \sqrt{p\cdot\sigma}\eta^s \\ -\sqrt{p\cdot\bar{\sigma}}\eta^s \end{pmatrix}$ Here ξ^r, η^s form orthonormal bases for \mathbb{C}^2 and $\sqrt{p\cdot\sigma} = \sqrt{m}e^{\chi\cdot\sigma/2}, \sqrt{p\cdot\bar{\sigma}} = \sqrt{m}e^{\chi\cdot\sigma$			
General Solution to Dirac Equation positive frequencies: $\psi(x) = u(p)e^{-ip \cdot x}, u^s(p) = \left(\frac{\sqrt{p \cdot \sigma} \xi^s}{\sqrt{p \cdot \sigma} \xi^s}\right)$, for negative frequencies $\psi(x) = v(p)e^{ip \cdot x}, v^s(p) = \left(\frac{\sqrt{p \cdot \sigma} \eta^s}{\sqrt{p \cdot \sigma} \eta^s}\right)$ Here ξ^r, η^s form orthonormal bases for \mathbb{C}^2 and $\sqrt{p \cdot \sigma} = \sqrt{m}e^{\chi \cdot \sigma/2}, \sqrt{p \cdot \overline{\sigma}} = \sqrt{m}e^{\chi \cdot \sigma/2}, p $	1	$\left\{ \gamma^{\mu}, \gamma^{\nu} \right\} = 2\eta^{\mu\nu} I, \{ \gamma^5, \gamma^{\mu} \} = 0, (\gamma)^2 = 0, \gamma^5 = i\gamma^0 \gamma^1 \gamma^2 \gamma^3, \ \gamma^0 = 0, (\gamma)^2 = 0, \gamma^5 = 0, $	
negative frequencies $\psi(x) = v(p)e^{ip\cdot x}, v^s(p) = \begin{pmatrix} \sqrt{p\cdot \sigma}\eta^s \\ -\sqrt{p\cdot \bar{\sigma}}\eta^s \end{pmatrix}$ $\sqrt{m}e^{\chi\cdot \sigma/2}, \sqrt{p\cdot \bar{\sigma}} = \sqrt{m}e^{-\chi\cdot \sigma}$ Plane wave solutions satisfy Steps for Calculating Scattering Amplitudes O) Receive and quantise the Lagrangian 1) try to use the Dyson formula 2) calculate the time ordered product 3) find a version of Wick's formula 4) develop Feynman rules Dyson formula $A_{i\to f} = \langle f S i \rangle, S = T \left(e^{\frac{1}{i} \int_{-\infty}^{\infty} H_I(t) dt} \right) \text{ for interaction Hamiltonian } H_I$ Standard spin-1/2 Fermion Expansion $\psi_{\alpha} = \sum_{s=1}^2 \int \frac{d^3p}{(2\pi)^3} \frac{1}{\sqrt{2E_p}} \left(b_p^s u_{\alpha}^s(p) e^{-ip\cdot x} + c_p^{\dagger} v_{\alpha}^s(p) e^{ip\cdot x} \right)$	Algebra		
negative frequencies $\psi(x) = v(p)e^{ip\cdot x}, v^s(p) = \begin{pmatrix} \sqrt{p\cdot \sigma}\eta^s \\ -\sqrt{p\cdot \bar{\sigma}}\eta^s \end{pmatrix}$ $\sqrt{m}e^{\chi\cdot \sigma/2}, \sqrt{p\cdot \bar{\sigma}} = \sqrt{m}e^{-\chi\cdot \sigma}$ Plane wave solutions satisfy Steps for Calculating Scattering Amplitudes O) Receive and quantise the Lagrangian 1) try to use the Dyson formula 2) calculate the time ordered product 3) find a version of Wick's formula 4) develop Feynman rules Dyson formula $A_{i\to f} = \langle f S i \rangle, S = T \left(e^{\frac{1}{i} \int_{-\infty}^{\infty} H_I(t) dt} \right) \text{ for interaction Hamiltonian } H_I$ Standard spin-1/2 Fermion Expansion $\psi_{\alpha} = \sum_{s=1}^2 \int \frac{d^3p}{(2\pi)^3} \frac{1}{\sqrt{2E_p}} \left(b_p^s u_{\alpha}^s(p) e^{-ip\cdot x} + c_p^s \dagger v_{\alpha}^s(p) e^{ip\cdot x} \right)$		positive frequencies: $\psi(x) = u(p)e^{-ip\cdot x}, u^s(p) = \begin{pmatrix} \sqrt{p\cdot\sigma}\xi^s \\ \sqrt{p\cdot\overline{\sigma}}\xi^s \end{pmatrix}$, for	Here ξ^r, η^s form orthonormal
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Amplitudes formula 2) calculate the time ordered product 3) find a version of Wick's formula 4) develop Feynman rules	Plane wave solutions satisfy	$(\not p - m)u = 0 = (\not p + m)v$	
$\begin{array}{c} \text{Wick's formula 4) develop Feynman rules} \\ \text{Dyson formula} & A_{i \to f} = \left\langle f \middle S \middle i \right\rangle, S = T \left(e^{\frac{1}{i} \int_{-\infty}^{\infty} H_I(t) dt} \right) \text{ for interaction Hamiltonian } H_I \\ \text{Standard spin-1/2 Fermion Expansion} & \psi_{\alpha} = \sum_{s=1}^2 \int \frac{d^3p}{(2\pi)^3} \frac{1}{\sqrt{2E_p}} \left(b_p^s u_{\alpha}^s(p) e^{-ip \cdot x} + c_p^{s\dagger} v_{\alpha}^s(p) e^{ip \cdot x} \right) \end{array}$, , , , , , , , , , , , , , , , , , , ,	
Dyson formula $A_{i\to f} = \langle f S i \rangle, S = T \left(e^{\frac{1}{i} \int_{-\infty}^{\infty} H_I(t) dt} \right) \text{ for interaction Hamiltonian } H_I$ Standard spin-1/2 Fermion Expansion $\psi_{\alpha} = \sum_{s=1}^2 \int \frac{d^3p}{(2\pi)^3} \frac{1}{\sqrt{2E_p}} \left(b_p^s u_{\alpha}^s(p) e^{-ip \cdot x} + c_p^{s\dagger} v_{\alpha}^s(p) e^{ip \cdot x} \right)$	Amplitudes	, , , , , , , , , , , , , , , , , , , ,	
$\begin{array}{c c} & \text{nian } H_I \\ \hline \text{Standard spin-1/2 Fermion Expansion} & \psi_\alpha = \sum_{s=1}^2 \int \frac{d^3p}{(2\pi)^3} \frac{1}{\sqrt{2E_p}} \left(b_p^s u_\alpha^s(p) e^{-ip\cdot x} + c_p^{s\dagger} v_\alpha^s(p) e^{ip\cdot x} \right) \end{array}$			
Standard spin-1/2 Fermion Expansion $\psi_{\alpha} = \sum_{s=1}^{2} \int \frac{d^{3}p}{(2\pi)^{3}} \frac{1}{\sqrt{2E_{p}}} \left(b_{p}^{s} u_{\alpha}^{s}(p) e^{-ip \cdot x} + c_{p}^{s\dagger} v_{\alpha}^{s}(p) e^{ip \cdot x} \right)$	Dyson formula	$A_{i\to f} = \langle f S i \rangle, S = T \left(e^{\frac{1}{i} \int_{-\infty}^{\infty} H_I(t) dt} \right)$ for interaction Hamilto-	
pansion			
Scalar Contraction $\Delta_F(x-y) = \lim_{\epsilon \to 0^+} \frac{d^4 p}{(2\pi)^4} \frac{i e^{-p_\mu (x-y)^\mu}}{p^2 - m^2 + i\epsilon}$,	` ' V P	
	Scalar Contraction	$\Delta_F(x-y) = \lim_{\epsilon \to 0^+} \frac{d^4p}{(2\pi)^4} \frac{ie^{-p_\mu(x-y)^\mu}}{p^2 - m^2 + i\epsilon}$	