Quantum Field Theory Equation Sheet

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Here are some useful equations

Table 1: Equation Sheet

Name/Description	Equation	Remarks
Noether conserved current	$j^\mu = \partial_{\partial_\mu \phi}\mathcal{L}\delta\phi - F^\mu$	here $\mathcal{L}(x+\delta x) = \mathcal{L} + \delta x \partial_\mu F^\mu$, $\partial_\mu j^\mu = 0$
The conserved charge arising from a conserved current	$Q = \int d^3 x j^0$	
The Energy-Momentum Tensor	$T^\mu_\nu = \partial_{\partial_\mu \phi}\mathcal{L}\partial_\nu\phi - \delta^\mu_\nu \mathcal{L}$	This is the Noether current under translation. This tensor can always be chosen to be symmetric. It is a Noether current, so conserved as $\partial_\mu T^{\mu\nu} = 0$
Ladder Operators	$[a_p, a_q^\dagger] = (2\pi)^3\delta(p-q)$	
Field Operator	$\phi = \int \frac{d^3p}{(2\pi)^3}\frac{1}{\sqrt{2\omega_p}}(a_p e^{ip\cdot x} + a_p^\dagger e^{-ip\cdot x})$	
Momentum Operator	$\pi = \int \frac{d^3p}{(2\pi)^3}(-i)\sqrt{\frac{\omega_p}{2}}(a_p e^{ip\cdot x} - a_p^\dagger e^{-ip\cdot x})$	
Dirac Equation	$(i\gamma^\mu\partial_\mu - m)\cdot\psi = (i\not\partial - m)\cdot\psi = 0$	
Dirac Adjoint	$\bar\psi = \psi^\dagger\gamma^0$	
Chiral represntation and Clifford Algebra	$\{\gamma^\mu,\gamma^\nu\} = 2\eta^{\mu\nu}I, \{\gamma^5,\gamma^\mu\} = 0, (\gamma)^2 = 0, \gamma^5 = i\gamma^0\gamma^1\gamma^2\gamma^3$, $\gamma^0 = \begin{pmatrix} & I \\ I & \end{pmatrix}$, $\gamma^i = \begin{pmatrix} & \sigma^i \\ \sigma^i & \end{pmatrix}$, $\gamma^5 = \begin{pmatrix} I & \\ & -I\end{pmatrix}$	
General Solution to Dirac Equation	positive frequencies: $\psi(x) = u(p)e^{-ip\cdot x}, u^s(p) = \begin{pmatrix}\sqrt{p\cdot\sigma}\xi^s \\ \sqrt{p\cdot\bar\sigma}\xi^s\end{pmatrix}$, for negative frequencies $\psi(x) = v(p)e^{ip\cdot x}, v^s(p) = \begin{pmatrix}\sqrt{p\cdot\sigma}\eta^s \\ -\sqrt{p\cdot\bar\sigma}\eta^s\end{pmatrix}$	Here ξ^r, η^s form orthonormal bases for $\mathbb{C}^2$ and $\sqrt{p\cdot\sigma} = \sqrt{m}e^{\chi\cdot\sigma/2}$, $\sqrt{p\cdot\bar\sigma} = \sqrt{m}e^{-\chi\cdot\sigma}$
Plane wave solutions satisfy	$(\not p - m)u = 0 = (\not p + m)v$	
Steps for Calculating Scattering Amplitudes	0) Receive and quantise the Lagrangian 1) try to use the Dyson formula 2) calculate the time ordered product 3) find a version of Wick's formula 4) develop Feynman rules	
Dyson formula	$A_{i\to f} = \langle f\| S \|i\rangle, S = T\left(e^{\frac{1}{i}\int_{-\infty}^{\infty} H_I(t)dt}\right)$ for interaction Hamiltonian H_I	
Standard spin-1/2 Fermion Expansion	$\psi_\alpha = \sum_{s=1}^2 \int \frac{d^3p}{(2\pi)^3}\frac{1}{\sqrt{2E_p}}\left(b_p^s u_\alpha^s(p)e^{-ip\cdot x} + c_p^{s\dagger} v_\alpha^s(p)e^{ip\cdot x}\right)$	
Scalar Contraction	$\Delta_F(x-y) = \lim_{\epsilon\to 0^+}\frac{d^4p}{(2\pi)^4}\frac{ie^{-p_\mu(x-y)^\mu}}{p^2-m^2+i\epsilon}$	the scalar contraction in higher dimension takes this and multiplies by $\eta_{\mu\nu}$
spin-1/2 Fermian contraction	$S_F(x-y) = \int\frac{d^4p}{(2\pi)^4}\frac{i(\not p + mI)_{\alpha\beta}}{p^2-m^2+i\epsilon}e^{-ip\cdot(x-y)},\ \epsilon\to 0^+$	
4-vector mode expansion	$A_\mu(x) = \int\frac{d^3p}{(2\pi)^3}\frac{1}{\sqrt{2\|p\|}}\sum_{\lambda=0}^3 \epsilon_\mu^{(\lambda)}(p)(a_p^\lambda e^{-ip\cdot x} + a_p^{\lambda\dagger}e^{ip\cdot x})$	this is used for photons in QED. Note $[a_p^\lambda, a_q^{\rho\dagger}] = -\eta^{\lambda\rho}(2\pi)^3\delta(p-q)$
QED Lagrangian coupling a Gauge theory to a scalar field	$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \bar\psi(i\gamma^\mu\partial_\mu - m)\psi - e\bar\psi\gamma^\mu\psi A_\mu$	