Auditoría | Consultoría | Impuestos | Estrategia y Transancciones

EY

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Lab 03 - Parte teórica

preg 1 Sea $x[n] \to X(z)$ con ROC $\alpha < |z| < \beta$

a) Por definición $r_{xx}[l] = \sum_n x[n]\,x[n-l]$ (*)

Demostrar : $R_{xx}[z) = X(z)\,X(z^{-1})$?

y su ROC.

(*) Como $r_{xx}[l]$ es par

$$\to \quad r_{xx}[l] = \sum_{n=-\infty}^{\infty} x[n]\,x[n+l]$$

cambio índice

$$r_{xx}[l] = \sum_{n=-\infty}^{\infty} x[-n]\,x[-n+l]$$

es convolución x def

$$r_{xx}[l] = x[-n] * x[n]$$

Z{ } (Convolución en tiempo es producto en z)

$$R_{xx}(z) = Z\{x[-n]\} \cdot Z\{x[n]\}$$

x propiedad

$$R_{xx}(z) = X(z^{-1}) \cdot X(z) \quad \text{lqqd}$$

Continúa cálculo ROC ...

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$X(z) - ROC : \alpha < |z| < \beta$ ... Conjunto 1

$X(z^{-1}) \to ROC = \frac{1}{\beta} < |z| < 1/\alpha$ ... Conjunto 2

$\to$ la ROC de $\underbrace{X(z)X(z^{-1})}_{R_{xx}(z)}$ = Conjunto 1 $\cap$ Conjunto 2

$\to$ ROC $R_{xx}(z) = \left\{ \min\left(\alpha, \frac{1}{\beta}\right) < |z| < \min\left(\beta, \frac{1}{\alpha}\right) \right\}$ //

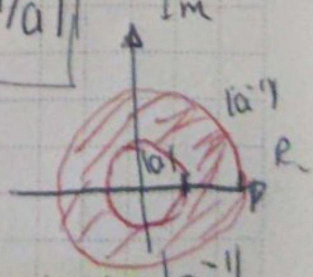
$\angle$ qqd

b) Sea $x[n] = a^n u[n]$, $|a| < 1$, hallar $R_{xx}(z)$, $r_{xx}[l]$ y ROC

$$X(z) = \sum_{n=0}^{\infty} a^n z^{-n} = 1 + az^{-1} + a^2 z^{-2} + a^3 z^{-3} + \dots$$
$$= (az^{-1})^0 + (az^{-1})^1 + (az^{-1})^2 + (az^{-1})^3 + \dots$$

$$\boxed{X(z) = \frac{1}{1 - az^{-1}} \; ; \; ROC \; |z| > a}$$

$$\boxed{X(z^{-1}) = \frac{1}{1 - az} \; ; \; ROC \; |z| < |1/a|}$$



x lo antes demostrado

$$R_{xx}(z) = \frac{1}{(1 - az^{-1})} \frac{1}{(1 - az)} \; ; \text{ con ROC } |a| < |z| < |a^{-1}|$$

Falta $R_{xx}[l] = Z^{-1}\{R_{xx}(z)\}$

Distribuimos

$$R_{xx}(z) = \left(\frac{1}{1-az^{-1}}\right)\left(\frac{1}{1-az}\right) = \frac{z^{-1}}{(1-az^{-1})(z^{-1}-a)}$$

Usando fracciones parciales y $z^{-1} = x$

$$\frac{x}{(1-ax)(x-a)} = \frac{A}{1-ax} + \frac{B}{x-a} \rightarrow \left\{ Ax - aA + B - axB = x \right.$$

$$\boxed{B = aA}$$

$$\boxed{A - aB = 1}$$

$$\boxed{A = \frac{1}{1-a^2}}$$

$$\boxed{B = \frac{a}{1-a^2}}$$

$$\frac{x}{(1-ax)(x-a)} = \frac{1}{1-a^2}\left(\frac{1}{1-ax} + \frac{a}{x-a}\right)$$

$$\rightarrow R_{xx}(z) = \frac{1}{1-a^2}\left(\frac{1}{1-az^{-1}} + \frac{a}{z^{-1}-a}\right)$$

$$= \frac{1}{1-a^2}\left(\frac{1}{1-az^{-1}} - \frac{1}{1-a^{-1}z^{-1}}\right)$$

$\downarrow Z^{-1}\{\}$

$$R_{xx}[n] = \frac{1}{1-a^2}\left(a^n u[n] + a^{-n} u[-n-1]\right)$$

Lqqd //

x Fórmula

$$a^n u[n] \xrightarrow{Z\{\}} \frac{1}{1-az^{-1}}$$

$$a^{-n} u[-n-1] \xrightarrow{Z\{\}} \frac{-1}{1-a^{-1}z^{-1}}$$

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Pregunta 2

$$y[n] = \frac{3}{4}y[n-1] - \frac{1}{8}y[n-2] + x[n]$$

a) Hallar $H(z)$

$$Y(z) = \frac{3}{4}z^{-1}Y(z) - \frac{1}{8}z^{-2}Y(z) + X(z) \rightarrow \frac{Y(z)}{X(z)} = \frac{1}{1-\frac{3}{4}z^{-1}+\frac{1}{8}z^{-2}}$$

$$\rightarrow H(z) = 8\left(\frac{1}{8-6z^{-1}+z^{-2}}\right) \rightarrow H(z) = \frac{8}{(z^{-1}-4)(z^{-1}-2)}$$

tomamos $z^{-1}=x$

$$\frac{8}{(x-4)(x-2)} = \frac{A_1}{x-4}+\frac{B_1}{x-2} \rightarrow A_1(x-2)+B_1(x-4)=8$$

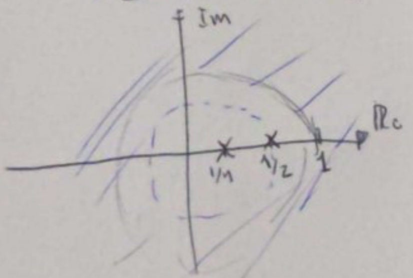
$$A_1(2)=8 \rightarrow \boxed{A_1=4}$$

$$B_1(-2)=8 \rightarrow \boxed{B_2=-4}$$

$$\rightarrow H(z) = 4\left(\frac{1}{z^{-1}-4} - \frac{1}{z^{-1}-2}\right)$$

, Analizamos polos $(z^{-1})=4 \rightarrow z=1/4$, $(z^{-1})=2$, $z=1/2$ $\}$ Polo

∴ Roc



El sistema será estable siempre y cuando consideremos una Roc: $|z|>1/2$ (tomando $h[n]$ causales)

Caso contrario será inestable

Ya que para que sea estable debe incluir $|z|=1$

b) Como se mencionó antes, tomo $h[n]$ causal

$$\rightarrow H(z) = 4\left(\frac{1}{z^{-1}-4}\right) - 4\left(\frac{1}{z^{-1}-2}\right) \rightarrow \cancel{H(z) = 4\left(\frac{z}{1-4z}\right) - 4(}$$

$$= 4\left(\frac{-1/4}{1-z^{-1}/4}\right) - 4\left(\frac{-1/2}{1-z^{-1}/2}\right)$$

$$= \frac{-1}{1-z^{-1}/4} + \frac{2}{1-z^{-1}/2}$$

ambos causal $z^{-1}/4$

$$h[n] = -\left(\frac{1}{4}\right)^n u[n] + 2\left(\frac{1}{2}\right)^n u[n]$$

(respuesta al impulso)

respuesta escalón $= h[n] * u[n]$

$$= \underbrace{-\left(\frac{1}{4}\right)^n u[n] * u[n]}_{\text{tabla acumulador}} + \underbrace{2\left(\frac{1}{2}\right)^n u[n] * u[n]}_{\text{tabla acumulador}}$$

$$= -\left(\frac{1-(1/4)^{n+1}}{1-1/4}\right)u[n] + 2\left(\frac{1-(1/2)^{n+1}}{1-1/2}\right)u[n]$$

$$P[n] = \frac{4}{3}\left((1/4)^{n+1}-1\right)u[n] + 4\left(1-(1/2)^{n+1}\right)u[n]$$

(Respuesta al escalón)

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Pregunta 3

$$y[n] - \frac{5}{2}y[n-1] + y[n-2] = x[n-1]$$

Hallamos $H(z)$: $Y(z)\left(1 - \frac{5}{2}z^{-1} + z^{-2}\right) = X(z)\left(z^{-1}\right)$

$$H(z) = \frac{z^{-1}}{1 - \frac{5}{2}z^{-1} + z^{-2}}$$

tomamos $z^{-1} = x$

$$\frac{2x}{2 - 5x + x^2} = \frac{2x}{(x-2)(2x-1)} = \frac{x}{(x-2)(x-1/2)} = \frac{A}{x-2} + \frac{B}{x-1/2}$$

$$A(x - 1/2) + B(x-2) = x$$

$$\begin{cases} A+B = 1 \\ -A/2 - 2B = 0 \end{cases} \quad \begin{cases} A+B=1 \\ -A-4B=0 \end{cases} \quad \therefore \frac{x}{(x-2)(x-1/2)} = \frac{4}{3}\left(\frac{1}{x-2}\right) + \frac{-1}{3}\left(\frac{1}{x-1/2}\right)$$

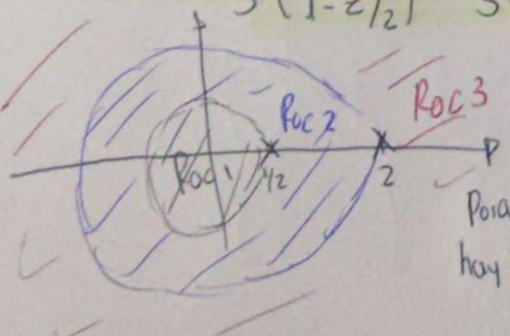
$$-1-3B=0$$

$$\boxed{B=-1/3}$$

$$\boxed{A=4/3}$$

$$\therefore H(z) = \frac{4}{3}\left(\frac{1}{z^{-1}-2}\right) - \frac{1}{3}\left(\frac{1}{z^{-1}-1/2}\right) = \frac{4}{3}\left(\frac{(-1/2)}{1-z^{-1}/2}\right) - \frac{1}{3}\left(\frac{(-2)}{1-2z^{-1}}\right)$$

$$H(z) = \frac{-2}{3}\left(\frac{1}{1-z^{-1}/2}\right) + \frac{2}{3}\left(\frac{1}{1-2z^{-1}}\right)$$



Ahora analizamos polos

$z^{-1} = 2$ $\quad$ $z^{-1} = 1/2$

$\boxed{z = 1/2}$ $\quad$ $\boxed{z = 2}$

polos

Para cada Roc hay un $h[n]$ distinto

Para Roc 1 : $|z| < 1/2$ (Anticausal)

Aplicando tabla

$$h[n] = -\frac{2}{3}\left(-\left(\frac{1}{2}\right)^n u[-n-1]\right) + \left(\frac{2}{3}\right)(-1)(2)^n u[-n-1] \rightarrow h[n] = \frac{2}{3}\left(\left(\frac{1}{2}\right)^n - (2)^n\right)u[-n-1]$$

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Para ROC 2 $\frac{1}{2} < |z| < 2$ (una causal y otra anticausal)

$$h[n] = -\frac{2}{3}\left(\left(\frac{1}{2}\right)^n u[n]\right) + \frac{2}{3}\left(-(2)^n u[-n-1]\right)$$

Para ROC 3 $|z| > 2$

$$h[n] = -\frac{2}{3}\left(\left(1/2\right)^n u[n]\right) + \frac{2}{3}\left(2^n u[n]\right)$$

Pregunta (4)

Sea $y[n] - \frac{1}{4} y[n-1] = x[n] + 3x[n-1]$

Sea $\underbrace{x[n] = e^{j\pi n/4} u[n] \text{ y } y[-1] = 2}$ $\qquad X(z) = \frac{1}{1 - z^{-1} e^{j\pi/4}}$

$$\xrightarrow{Zh} Y(z) - \frac{1}{4}\left(z^{-1} Y(z) + \underbrace{y[-1]}_{2}\right) = X(z) + 3\left(z^{-1} X(z) + \cancel{x[-1]}\right)$$

$$\Rightarrow Y(z)\left(1 - \frac{1}{4} z^{-1}\right) - \frac{1}{2} = X(z)\left(1 + 3z^{-1}\right)$$

$$Y(z)\left(1 - \frac{1}{4} z^{-1}\right) = \underbrace{\frac{1}{2}}_{\text{Condición inicial}} + \underbrace{X(z)\left(1 + 3z^{-1}\right)}_{\text{Entrada}}$$

$$\rightarrow Y(z) = \underbrace{\frac{1}{2}\left(\frac{1}{1 - \frac{1}{4} z^{-1}}\right)}_{ZIR(z)} + \underbrace{\frac{X(z)\left(1 + 3z^{-1}\right)}{\left(1 - \frac{1}{4} z^{-1}\right)}}_{ZSR(z)}$$

z^{-1} ↓

$$\rightarrow ZIR: \ y_{ZIR}[n] = \frac{1}{2}\left(\frac{1}{4}\right)^n u[n]$$

Zero Input Response

Continúa ZSR

$$Y_{ZSR} = Z^{-1}\left\{ \frac{1}{1-z^{-1}e^{j\pi/4}} \cdot \frac{1+z^{-1}(3)}{\left(1-\frac{1}{4}z^{-1}\right)} \right\}$$

$$Y_{ZSR}(z) = \frac{z^2(1+3z^{-1})}{(z-e^{j\pi/4})(z-1/4)} \rightarrow \frac{Y_{ZSR}(z)}{z} = \frac{(z+3)}{(z-e^{j\pi/4})(z-1/4)} = \frac{A}{z-e^{j\pi/4}} + \frac{B}{z-1/4}$$

$$z+3 = A(z-1/4) + B(z-e^{j\pi/4}) \xrightarrow{z=1/4} 3+\frac{1}{4} = 0 + B\left(\frac{1}{4}-e^{j\pi/4}\right)$$

$$\boxed{B = \left(\frac{13}{4}\right)\left(\frac{1}{\frac{1}{4}-e^{j\pi/4}}\right)}$$

$$\xrightarrow{z=e^{j\pi/4}} e^{j\pi/4}+3 = A(e^{j\pi/4}-1/4)$$

$$\boxed{A = \frac{e^{j\pi/4}+3}{e^{j\pi/4}-1/4}}$$

$$\rightarrow Y_{ZSR}(z) = \left(\frac{e^{j\pi/4}+3}{e^{j\pi/4}-1/4}\right)\left(\frac{z}{z-e^{j\pi/4}}\right) + \frac{13}{4}\left(\frac{1}{\frac{1}{4}-e^{j\pi/4}}\right)\left(\frac{z}{z-1/4}\right)$$

$$Y_{ZSR}[n] = \left(\frac{e^{j\pi/4}+3}{e^{j\pi/4}-1/4}\right)\left(e^{j\pi/4}\right)^n u[n] + \left(\frac{13}{1-4e^{j\pi/4}}\right)\left(\frac{1}{4}\right)^n u[n]$$

↳ Zero State Response

Analizamos

$$Y_{total}[n] = \underbrace{\frac{1}{2}\left(\frac{1}{4}\right)^n u[n]}_{\text{decae en el tiempo}} + \underbrace{\left(\frac{e^{j\pi/4}+3}{e^{j\pi/4}-1/4}\right)\left(e^{j\pi/4}\right)^n u[n]}_{\text{No decae en el tiempo}} + \underbrace{\left(\frac{13}{1-4e^{j\pi/4}}\right)\left(1/4\right)^n u[n]}_{\text{decae en el tiempo}}$$

La Respuesta ~~estable~~ estable se define como una que perdura en el tiempo

$$\rightarrow Y_{estable}[n] = \left(\frac{e^{j\pi/4}+3}{e^{j\pi/4}-1/4}\right)\left(e^{j\pi/4}\right)^n u[n]$$

La Respuesta transitoria decae en el tiempo

Entonces $Y_{transitoria}[n] = \frac{1}{2}\left(\frac{1}{4}\right)^n u[n] + \left(\frac{13}{1-4e^{j\pi/4}}\right)\left(1/4\right)^n u[n]$

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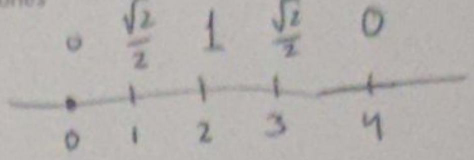


Pregunta 5

Hallar Z trans:

a) $$x[n] = \left(1/3\right)^n \sin\left(\frac{n\pi}{4}\right) u[n]$$

$$= \left(1/3\right)^n \left(\frac{e^{j\pi n/4} - e^{-j\pi n/4}}{2j}\right) u[n]$$

$$= \frac{1}{2j}\left(\left(\frac{e^{j\pi/4}}{3}\right)^n - \left(\frac{e^{-j\pi/4}}{3}\right)^n\right) u[n]$$

$$x[n] = \frac{1}{2j}\left(\underbrace{\left(\frac{e^{j\pi/4}}{3}\right)^n u[n]}_{(I)} - \underbrace{\left(\frac{e^{-j\pi/4}}{3}\right)^n u[n]}_{(II)}\right)$$

(I) $$X_I(z) = \sum_{n=0}^{\infty}\left(\frac{e^{j\pi/4} z^{-1}}{3}\right)^n$$

$\left|\frac{e^{j\pi/4}}{3}\right| < 1$ $\quad \left|\frac{z^{-1}}{3}\right| < 1$

↓ Suma Converge

$$X_I(z) = \frac{1}{1 - z^{-1}\left(\frac{e^{j\pi/4}}{3}\right)}, \quad Roc\ |z| > 1/3$$

(II) $$X_{II}(z) = \sum_{n=0}^{\infty}\left(\frac{e^{-j\pi/4} z^{-1}}{3}\right)^n$$

$\left|\frac{e^{-j\pi/4}}{3}\right| < 1$

↓ Suma converg

$$X_{II}(z) = \frac{1}{1 - z^{-1}\left(\frac{e^{-j\pi/4}}{3}\right)}$$

Roc $|z| > 1/3$

$$\rightarrow X(z) = \frac{1}{2j}\left(X_I(z) - X_{II}(z)\right)$$

$$X(z) = \frac{1}{2j}\left(\frac{1}{1 - z^{-1}\left(\frac{e^{j\pi/4}}{3}\right)} - \frac{1}{1 - z^{-1}\left(\frac{e^{-j\pi/4}}{3}\right)}\right)$$

Rpta a)
Roc $|z| > 1/3$

b) $x[n] = (1/2)^n u[n+1] + 3^n u[-n-1]$

$n+1 \geq 0$ $\quad -n-1 \geq 0$

$n \geq -1$ $\quad -1 \geq n$

xdot

$$X(z) = \sum_{n=-1}^{\infty} (1/2)^n z^{-n} + \sum_{n=-1}^{-\infty} (3)^n z^{-n}$$

$$= \sum_{n=-1}^{\infty} \left((1/2) z^{-1}\right)^n + \sum_{n=-1}^{-\infty} (3z^{-1})^n$$

$$= \left(\tfrac{1}{2} z^{-1}\right)^{-1} + \sum_{n=0}^{\infty} \left(\tfrac{1}{2} z^{-1}\right)^n + \sum_{n=1}^{+\infty} (3z^{-1})^{-n}$$

$$= (2z) + \sum_{n=0}^{\infty} \left(\tfrac{1}{2} z^{-1}\right)^n + \sum_{n=1}^{\infty} (z/3)^n$$

$$= 2z + \sum_{n=0}^{\infty} \left(\tfrac{1}{2} z^{-1}\right)^n + \sum_{n=0}^{\infty} (z/3)^n - (z/3)^0$$

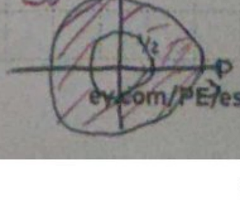
$$= 2z - 1 + \sum_{n=0}^{\infty} \left(\frac{z^{-1}}{2}\right)^n + \sum_{n=0}^{\infty} (z/3)^n$$

$\left|\frac{z^{-1}}{2}\right| < 1$ $\quad \left|\frac{z^{+1}}{3}\right| < 1$

$Roc_1 : |z| > 1/2$ $\quad Roc_2 : |z| < 3$

$$X(z) = 2z - 1 + \frac{1}{1 - z^{-1}/2} + \frac{1}{1 - z/3}, \text{ con } Roc = Roc_1 \cap Roc_2$$

$\therefore 1/2 < |z| < 3$



Pregunta ⑤

c) $Z\{x[n]\}$ con $x[n] = |n|(1/2)^{|n|} = -n(1/2)^{-n}\mu[-n-1] + n(1/2)^{n}\mu[n]$

$= -n(2)^{n}\mu[-n-1] + n(1/2)^{n}\mu[n]$

Por propiedades

$$(1/2)^n \mu[n] \xleftrightarrow{Z\{\}} \frac{1}{1-z^{-1}/2} \quad |z| > 1/2$$

$$-(2)^n \mu[-n-1] \xleftrightarrow{Z\{\}} \frac{1}{1-z^{-1}2} \quad |z| < 2$$

y por propiedad

$$n\,x[n] \longleftrightarrow -z\frac{d}{dz}(X(z))$$

Entonces usando fórmulas y propiedades

$$= -z\frac{d}{dz}\left(\frac{1}{1-z^{-1}(2)}\right) + (-z)\frac{d}{dz}\left(\frac{1}{1-z^{-1}/2}\right)$$

$$= -z\left(\frac{d}{dz}\left(\frac{z}{z-2}\right)\right) - z\frac{d}{dz}\left(\frac{z}{z-1/2}\right)$$

$$X(z) = -z\left(\frac{(1)(z-2)-(z)(1)}{(z-2)^2} + \frac{(z-1/2)-(z)(1)}{(z-1/2)^2}\right)$$

$$\hookrightarrow X(z) = \frac{2z}{(z-2)^2} + \frac{z/2}{(z-1/2)^2}$$

Con Roc $\boxed{\frac{1}{2} < |z| < 2}$

Rpta

d) $x[n] = (a^n + a^{-n})\mu[n]$

$$x[n] = a^n \mu[n] + a^{-n}\mu[n]$$

$$X(z) = \sum_{n=0}^{\infty} a^n z^{-n} + \sum_{n=0}^{\infty} a^{-n} z^{-n}$$

$$= \sum_{n=0}^{\infty} (a z^{-1})^n + \sum_{n=0}^{\infty} (a^{-1} z^{-1})^n$$

$\downarrow$ Roc I $|az^{-1}|<1$ $\qquad$ $\downarrow$ Roc $|a^{-1}z^{-1}|<1$

$$X(z) = \frac{1}{1-az^{-1}} + \frac{1}{1-a^{-1}z^{-1}}$$

Roc I $|a|<|z|$

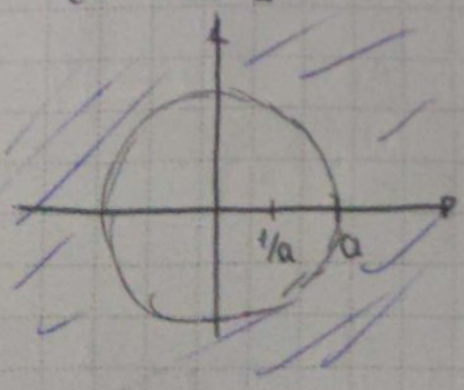
Roc II $\frac{1}{|a|} < |z|$

$$\boxed{X(z) = \frac{1}{1-az^{-1}} + \frac{1}{1-a^{-1}z^{-1}}}$$

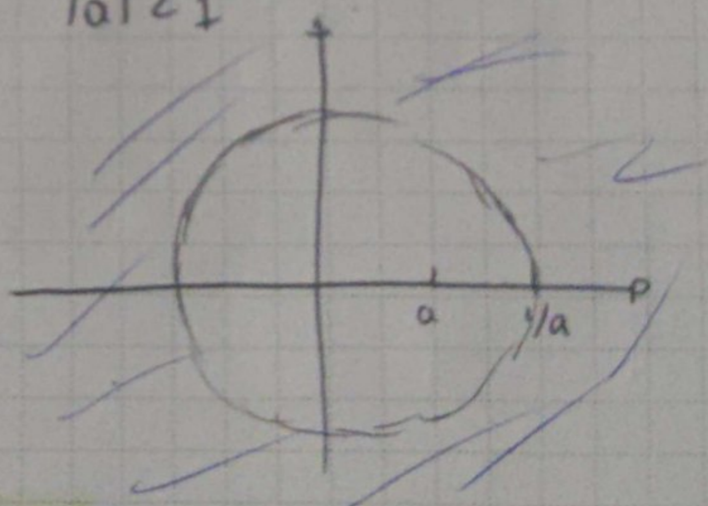
Roc: Roc I $\cap$ Roc II

$\rightarrow \max\{|a|, \frac{1}{|a|}\} < |z|$

con $|a|>1$



$|a|<1$



Roc $= \max\{|a|, \frac{1}{|a|}\} < |z|$

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Pregunta 6

d) $X(z) = \underbrace{e^{z}}_{X_I} + \underbrace{e^{1/z}}_{X_{II}}$

$$e^x = \sum_{n=0}^{\infty} \frac{(x)^n}{n!} \quad (\text{Taylor})$$

$$e^{z^{-1}} = \sum_{n=0}^{\infty} \frac{1}{n!}(z^{-n}) \quad , |z^{-1}| < 1 \rightarrow |z| > 1$$

Por inspección $X_{II}(z) = \sum_{n=0}^{\infty} x_{II}[n] z^{-n}$

$$\rightarrow x_{II}[n] = 1/n! \quad , n \geq 0$$

$$\hookrightarrow x_{II}[n] = \frac{1}{n!} u[n]$$

Ahora

~~Si $x_{II}[n] \leftrightarrow X$~~

Ahor $X_{II}(z) \xrightarrow{Z^{-1}} x_{II}[n]$

$X_{II}(z^{-1}) \xrightarrow{Z^{-1}} x_{II}[-n]$

$$\rightarrow X_{II}(z^{-1}) = X_I(z)$$

$$\rightarrow x[n] = \frac{1}{(-n)!} u[-n] + \frac{1}{n!} u[n]$$

Rpta

$\hookrightarrow$ 0, 0

$-n > 0$, para $n < 0$

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c) $x[n]$: {Causal

$$X(z) = \frac{1+3z^{-1}}{1+3z^{-1}+2z^{-2}} = \frac{1+3z^{-1}}{(1+z^{-1})(1+2z^{-1})}$$

Sea $z^{-1} = x$ Usamos F.P.

$$\frac{1+3x}{(1+x)(1+2x)} = \frac{A}{(1+x)} + \frac{B}{(1+2x)} \rightarrow A + 2Ax + B + Bx = 1+3x$$

$$\begin{cases} A+B = 1 \\ 2A+B = 3 \end{cases}$$

$$\begin{cases} A = 2 \\ B = -1 \end{cases}$$

$$\rightarrow X(z) = \frac{2}{1+z^{-1}} - \frac{1}{1+2z^{-1}}$$

para que sea causal

$|-z^{-1}| < 1 \rightarrow 1 < |z|$

$\downarrow z^{-1}\{\}$

$$x[n] = 2(-1)^n u[n] - (-2)^n u[n]$$

$|-2z^{-1}| < 1 \quad 2 < |z|$

$\rightarrow$ Roc $2 < |z|$

Roc

Causal

Roc

Prog 6 a)

$$X(z) = \frac{1+z^{-1}+z^{-2}}{1+\frac{1}{2}z^{-1}-\frac{z^{-2}}{4}} = 4\left(\frac{1+z^{-1}+z^{-2}}{4+2z^{-1}-z^{-2}}\right) = -4\left(\frac{z^{-2}+z^{-1}+1}{z^{-2}-2z^{-1}-4}\right)$$

$$= -4\left(\frac{z^{-2}-2z^{-1}-4+3z^{-1}+5}{z^{-2}-2z^{-1}-4}\right)$$

$$= -4\left(1+\frac{3z^{-1}+5}{z^{-2}-2z^{-1}-4}\right)$$

Usomos F.P.

$$\frac{3z^{-1}+5}{z^{-2}-2z^{-1}-4} = \frac{A}{(z^{-1}-1-\sqrt{5})} + \frac{B}{z^{-1}-1+\sqrt{5}}$$

$$3z^{-1}+5 = A(z^{-1}-1+\sqrt{5}) + B(z^{-1}-1-\sqrt{5})$$

$$z^{-1} = 1-\sqrt{5} \rightarrow 3(1-\sqrt{5})+5 = B(1-\sqrt{5}-1-\sqrt{5})$$

$$\cancel{B = 8\ 2\sqrt{5}}$$

$$B = \frac{8-3\sqrt{5}}{-2\sqrt{5}} = \boxed{\frac{3\sqrt{5}-8}{2\sqrt{5}}}$$

$$z^{-1} = 1+\sqrt{5} \rightarrow 3(1+\sqrt{5})+5 = A(1+\sqrt{5}-1+\sqrt{5})$$

$$\boxed{A = \frac{8+3\sqrt{5}}{2\sqrt{5}}}$$

$$\therefore X(z) = -4 - 4A\left(\frac{1}{z^{-1}-1-\sqrt{5}}\right) - 4B\left(\frac{1}{z^{-1}-1+\sqrt{5}}\right)$$

$$= -4 - 4A\left(\frac{-1}{1+\sqrt{5}}\right)\left(\frac{1}{1-\frac{z^{-1}}{1+\sqrt{5}}}\right) - 4B\,\frac{\frac{1}{\sqrt{5}-1}}{1+\frac{z^{-1}}{\sqrt{5}-1}}$$

$$= -4 + 4A\left(\frac{+1}{1+\sqrt{5}}\right)\left(\frac{1}{1-\frac{z^{-1}}{1+\sqrt{5}}}\right) - \frac{4B}{\sqrt{5}-1}\left(\frac{1}{1-\frac{z^{-1}}{1-\sqrt{5}}}\right)$$

Continúa

$$\underline{X}(z) = -4 + 4\left(\frac{8+3\sqrt{5}}{2\sqrt{5}}\right)\left(\frac{1}{1+\sqrt{5}}\right)\left(\frac{1}{1-\frac{z^{-1}}{1+\sqrt{5}}}\right) - \left(\frac{4}{\sqrt{5}-1}\right)\left(\frac{3\sqrt{5}-8}{2\sqrt{5}}\right)\left(\frac{1}{1-\frac{z^{-1}}{1-\sqrt{5}}}\right)$$

Para asegurar estabilidad, $x[n]$ causal

$$x[n] = -4\delta[n] + 4\left(\frac{8+3\sqrt{5}}{2\sqrt{5}}\right)\left(\frac{1}{1+\sqrt{5}}\right)\left(\frac{1}{1+\sqrt{5}}\right)^{n} u[n] - \left(\frac{4}{\sqrt{5}-1}\right)\left(\frac{3\sqrt{5}-8}{2\sqrt{5}}\right)\left(\frac{1}{1-\sqrt{5}}\right)^{n} u[n]$$

Rpta

(6) b) $X(z) = \dfrac{z^2-1}{(z-3)^2} = \dfrac{z^2-6z+9+6z-10}{z^2-6z+9} = 1+6\dfrac{(z-10/6)}{(z-3)^2}$

$(a+b)^2 = a^2+2ab+b^2$

$$= 1+6\left(\frac{z-3}{(z-3)^2}+\frac{3-10/6}{(z-3)^2}\right)$$

$$= 1+6\left(\frac{1}{z-3}+\frac{3-10/6}{(z-3)^2}\right)$$

$$= 1+6\,\frac{z^{-1}}{1-3z^{-1}}+8\,\frac{z^{-2}}{(1-3z^{-1})^2}$$

$$= 1+6\left(-\frac{1}{3}+\frac{1/3}{1-3z^{-1}}\right)+8\left(-\frac{1}{3}+\frac{1/3}{1-3z^{-1}}\right)^2$$

$$= 1-2+\frac{2}{1-3z^{-1}}+8\left(\frac{1}{9}+\left(\frac{1/3}{1-3z^{-1}}\right)^2-\frac{2(1/9)}{1-3z^{-1}}\right)$$

$$= -\frac{1}{9}+\frac{2}{1-3z^{-1}}+\frac{8}{9}\frac{1}{(1-3z^{-1})^2}-\frac{16}{9}\left(\frac{1}{1-3z^{-1}}\right)$$

$$= -\frac{1}{9}+\left(2-\frac{16}{9}\right)\left(\frac{1}{1-3z^{-1}}\right)+\frac{8}{9}\frac{1}{(1-3z^{-1})^2}$$

$$= -\frac{1}{9}+\frac{(2/9)}{(1-3z^{-1})}+\frac{(8/9)}{(1-3z^{-1})^2}$$

Ahora

$$\frac{1}{-3}\left(\frac{-3z^{-1}}{1-3z^{-1}}\right) = -\frac{1}{3}\left(\frac{1-3z^{-1}}{1-3z^{-1}}+\frac{-1}{1-3z^{-1}}\right)$$

$$\frac{z^{-1}}{1-3z^{-1}} = -\frac{1}{3}\left(1+\frac{-1}{1-3z^{-1}}\right)$$

$$\boxed{\frac{z^{-1}}{1-3z^{-1}} = -\frac{1}{3}+\frac{1/3}{1-3z^{-1}}}$$

$$-\frac{1}{9}+\frac{\cancel{1}0/9}{(1-3z^{-1})}-\frac{8/9}{(1-3z^{-1})}+\frac{8/9}{(1-3z^{-1})^2}$$

$$\cancel{\frac{z}{z-a} = \frac{z-a+a}{}} = 1+\frac{a}{z-a}$$

$$-\frac{1}{9}+\frac{10/9}{(1-3z^{-1})}+\frac{8}{9}\left(\frac{-1}{(1-3z^{-1})}+\frac{1}{(1-3z^{-1})^2}\right)$$

tabla

$$=-\frac{1}{9}+\frac{10/9}{(1-3z^{-1})}+\frac{8}{9}\left(\frac{3z^{-1}-1+1}{(1-3z^{-1})^2}\right)$$

$$-\alpha^n u[-n-1] \xleftrightarrow{ZT} \frac{1}{1-\alpha z^{-1}} \quad |z|<|\alpha|$$

$$=-\frac{1}{9}+\frac{10/9}{(1-3z^{-1})}+\frac{8}{9}\left(\frac{3z^{-1}}{(1-3z^{-1})^2}\right)$$ usamo tabla

$$-n\alpha^n u[-n-1] \xrightarrow{ZT} \frac{\alpha z^{-1}}{(1-\alpha z^{-1})^2} \quad |z|<\alpha$$

$$x[n]=-\frac{\delta[n]}{9}+\frac{10}{9}\left(-(3)^n u[-n-1]\right)+\frac{8}{9}\left(-n3^n u[-n-1]\right)$$ // Rota b)