

1 极限篇

$$(1) \lim_{x \rightarrow \alpha} \frac{\sin x - \sin \alpha}{x - \alpha}$$

$$(2) \lim_{x \rightarrow 0} (1 + \tan^2 x)^{\cot^2 x}$$

$$(3) \lim_{x \rightarrow 0} \frac{\sqrt{1 + \tan x} - \sqrt{1 + \sin x}}{x\sqrt{1 + \sin^2 x} - x}$$

$$(4) \lim_{x \rightarrow 0} \frac{\tan x - \sin x}{x^3}$$

$$(5) \lim_{x \rightarrow 0} \left(\frac{a^x + b^x + c^x}{3} \right)^{\frac{1}{x}} (a > 0, b > 0, c > 0)$$

$$(6) \lim_{x \rightarrow -\infty} x(\sqrt{x^2 + 100} + x)$$

$$(7) \lim_{x \rightarrow \infty} \frac{\sqrt{4x^2 + x - 1} + x + 1}{\sqrt{x^2 + \sin x}}$$

$$(8) \lim_{x \rightarrow 0} \frac{1}{x^3} \left[\left(\frac{2 + \cos x}{3} \right)^x - 1 \right]$$

$$(9) \lim_{x \rightarrow 0} \frac{(1 + x)^{\frac{1}{x}} - e}{x}$$

$$(10) \lim_{n \rightarrow \infty} n^2 (2^{\frac{1}{n}} - 2^{\frac{1}{n+1}})$$

$$(11) \lim_{n \rightarrow \infty} \left(\frac{1}{n^2 + n + 1} + \frac{2}{n^2 + n + 2} + \cdots + \frac{n}{n^2 + n + n} \right)$$

$$(12) \lim_{x \rightarrow 0} \frac{\sqrt{1+x} \arctan 3x - 1}{\sin(1-\cos x)}$$

$$(13) \lim_{x \rightarrow 0} \frac{\sin[\sin(\sin x)]}{\sqrt{1+x} \sqrt{1+x-1}}$$

$$(14) \lim_{x \rightarrow +\infty} \left(\sin \frac{1}{x} + \cos \frac{1}{x} \right)^x$$

$$(15) \lim_{x \rightarrow 0} \frac{\cos x - e^{-\frac{x}{2}}}{x^2 \sin x \ln(1+2x)}$$

$$(16) \lim_{x \rightarrow \infty} (\sqrt[3]{x^3+3x^2} - \sqrt{x^4-2x^3})$$

$$(17) \lim_{n \rightarrow \infty} [(n^3 - n^2 + \frac{n}{2})e^{\frac{1}{n}} - \sqrt{1+n^6}]$$

$$(18) \lim_{x \rightarrow \frac{\pi}{3}} \frac{\tan^3 x - 3 \tan x}{\cos(x + \frac{\pi}{6})}$$

$$(19) \lim_{x \rightarrow +\infty} \frac{\ln(1 + \frac{1}{x})}{\arccos x}$$

$$(20) \lim_{n \rightarrow \infty} \sin(\sqrt{n^2+1}\pi)$$

$$(21) \lim_{n \rightarrow \infty} n^2 \left[\arctan \frac{1}{n} - \arctan \frac{1}{n+1} \right]$$

$$(22) \lim_{x \rightarrow 0} \frac{(1+\tan x)^{\frac{1}{\tan x}} - (1+\sin x)^{\frac{1}{\sin x}}}{\tan x - \sin x}$$

2 积分篇

$$(1) \int \frac{\ln \tan x}{\cos x \sin x} dx \quad (2) \int \sin 5x \sin 7x dx \quad (3) \int \frac{dx}{(x+1)(x-2)} \quad (4) \int \frac{x^2 dx}{\sqrt{a^2 - x^2}}$$

$$(5) \int \frac{dx}{x\sqrt{x^2-1}} \quad (6) \int \frac{\sqrt{x^2-9}}{x} dx \quad (7) \int \frac{dx}{1+\sqrt{1-x^2}} \quad (8) \int \frac{dx}{x+\sqrt{1-x^2}}$$

$$(9) \int \frac{x-1}{x^2+2x+3} dx \quad (10) \int \frac{x^3+1}{(x^2+1)^2} dx \quad (11) \int x^2 \cos^2 \frac{x}{2} dx \quad (12) \int x \ln(x-1) dx$$

$$(13) \int \frac{\ln^3 x}{x^2} dx \quad (14) \int \cos \ln x dx \quad (15) \int (\arcsin x)^2 dx \quad (16) \int e^x \sin^2 x dx$$

$$(17) \int \frac{1 + \sin x}{\sin x(1 + \cos x)} dx$$

$$(18) \int \frac{-x^2 - 2}{(x^2 + x + 1)^2} dx$$

$$(19) \int \frac{dx}{2 + \sin x}$$

$$(20) \int \frac{dx}{2\sin x - \cos x + 5}$$

$$(21) \int \frac{\sqrt{x+1} - 1}{\sqrt{x+1} + 1} dx$$

$$(22) \int \frac{dx}{\sqrt[3]{(x+1)^2(x-1)^4}}$$

$$(23) \int \frac{dx}{\sqrt{x(1+x)}}$$

$$(24) \int \frac{x^3 \arccos x}{\sqrt{1-x^2}} dx$$

$$(25) \int \frac{\sin x \cos x}{\sin x + \cos x} dx$$

$$(26) \int \frac{dx}{\sin 2x + 2\sin x} dx$$

$$(27) \int e^{2x} (\tan x + 1)^2 dx$$

$$(28) \int \frac{\arctan \sqrt{x} + \ln x}{\sqrt{x}} dx$$

$$(29) \int \frac{2x^4 - x^3 - x + 1}{x^3 - 1} dx$$

$$(30) \int \frac{\sin x}{\sin x + \cos x} dx$$