National University of Computer and Emerging Sciences, Lahore Campus

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	Practice Problem:	FDs & NFS - SOLUTION		

Q1. [Find FDs]
List all FDs.

R

X	Y	Z
x1	y1	z1
x1	y1	z2
x2	y1	z1
x2	y1	z3

ANSWER: *X →Y; Z →Y.*

Q2. [Verify FDs]
Which of the following FDs may or may not hold over schema S?
a) $A \rightarrow B$, **b)**$BC \rightarrow A$, **c)**$B \rightarrow C$, ***d)***$BC \rightarrow D$, **e)**$CD \rightarrow B$

S

A	B	C	D
1	2	3	4
4	2	3	4
5	3	3	4

ANSWER: FDs: (a), (c), (d) HOLD and FDs: (b), (e) NOT HOLD.

Q3. [Verify FDs]
Which of the following FDs may or may not hold over schema R. Give valid reason.
a) A → CD, **b)** B → C, **c)** D → E, **d)** CD → E, **e)** E → CA

R

A	B	C	D	E	*Tuple#*
A1	B1	C1	D1	E1	*1*
A1	B2	C1	D1	E1	*2*
A2	B2	C1	D2	E3	*3*
A2	B3	C3	D2	E2	*4*

ANSWER: a. Not Hold   b. Hold   c. Not Hold   d. Hold   e. Hold

Q4. [Prove Inference rules for FDs]
Prove or disprove the following inference rules for functional dependencies. A proof can be made either by a proof argument or by using inference rules IR1 through IR6. A disproof should be done by demonstrating a relation instance that satisfies the conditions and functional dependencies in the left hand side of the inference rule but do not satisfy the conditions or dependencies in the right hand side.

a) {W →Y, X →Z} |= {WX →Y}
b) {X →Y} and Z subset-of Y |= {X →Z}
c) {X →Y, X →W, WY →Z} |= {X →Z}
d) {XY →Z, Y →W} |= {XW →Z}
e) {X →Z, Y →Z} |= {X →Y}
f) {X →Y, XY →Z} |= {X →Z}

ANSWER:
a) Proof:

(1) W →Y (given)
(2) X →Z (given)
(3) WX →YZ (using IR5 (union) on (1) and (2))

(4) WX →Y (using IR4 (decomposition) on (3))

b) Proof:

(1) X →Y (given)
(2) Y →Z (using IR1 (reflexivity), given that Z subset-of Y)
(3) X →Z (using IR3 (transitivity) on (1) and (2))

c) Proof:

(1) X →Y (given)
(2) X →W (given)
(3) WY →Z (given)
(4) X →WY (using IR5 (union) on (1) and (2))
(5) X →Z (using IR3 (transitivity) on (4) and (3))

d) Disproof:

	X	Y	Z	W
t 1 =	x1	y1	z1	w1
t 2 =	x1	y2	z2	w1

The above two tuples satisfy XY ->Z and Y ->W but do not satisfy XW ->Z

e) Disproof:

	X	Y	Z
t 1 =	x1	y1	z1
t 2 =	x1	y2	z1

The above two tuples satisfy X ->Z and Y ->Z but do not satisfy X ->Y

f) Proof:

(1) X →Y (given)
(2) XY →Z (given)
(3) X →XY (using IR2 (augmentation) to augment (1) with X)
(4) X →Z (using IR3 (transitivity) on (3) and (2))

Q5. [Closure]
Consider the following relation and compute the closure of $\{A\}^+$, $\{B\}^+$, $\{C\}^+$, $\{D\}^+$, and $\{CD\}^+$. Show your work.

R

A	B	C	D
1	2	3	4
4	2	3	4
5	3	3	4

ANSWER:
A^+={ABCD}, B^+={BCD}, C^+={CD}, D^+={CD}, and CD^+={CD}.

Q6. [Closure+Key]
Consider the relation R (A, B, C, D, E, F) and the set F = {A→B, C→DF, AC→E, D→F}.
Find the closure of A and C (i.e. A^+ and C^+). What is the KEY of this relation? Prove it.
ANSWER: A^+= {A, B}; C^+= {C, D, F}; Key= {AC}.

Q7. [Closure+Key]
Consider the relation R (A, B, C, D, E, F, G, H, K) and the set F = {A→BC, CD→H, CG→AE, H→G, B→D, F→G}.
Find the closure of A and BC (i.e. A^+ and $\{BC\}^+$). What is the KEY of this relation? Prove it.
ANSWER: A^+= {A, B, C, D, E, G, H}; $\{BC\}^+$= {A, B, C, D, E, G, H}; Key1= {AFK} and Key2= {BCFK}.

Q8. [Key]
Consider the relation SALES (transno, itemno, price, qty, seller, sregion)
and the set F = {{transno, itemno} → qty, itemno → price, transno → seller, seller → sregion}.
What is the KEY of this relation? Prove it.
Ans: {transno, itemno}

Q9. [Key]
Consider the relation R (A, B, C) and the set F = {A→C, C→A}.
What is the KEY of this relation? Prove it.
ANSWER: {AB} & {BC} are keys.

Q10. [Key]
Given relation R(A,B,C,D,E) with dependencies AB → C, CD → E, DE → B
Is AB a candidate key of this relation?
If not, is ABD? Explain your answer.
Ans: No, AB+ = {A,B,C}, a proper subset of {A,B,C,D,E} i.e. R
Yes, ABD+ = {A,B,C,D,E}

Q11. [Minimal Cover]

Find the <u>minimal cover</u> for the following set of FDs for a relation R (A, B, C, D):

$F = \{A \rightarrow BC, B \rightarrow C, A \rightarrow B, AB \rightarrow C, AC \rightarrow D\}$

Ans:

$F_c = \{A \rightarrow B\cancel{C}, B \rightarrow C, \cancel{A \rightarrow B}, \cancel{AB \rightarrow C}, A\cancel{C} \rightarrow D\}$ OR

$\mathbf{F_c = \{A \rightarrow B, B \rightarrow C, A \rightarrow D\}}$

Q12. [Minimal Cover]

Find the <u>minimal cover</u> for the following set of FDs for a relation R (A, B, C, D, E, F):

$F = \{A \rightarrow BC, E \rightarrow C, D \rightarrow AEF, ABF \rightarrow BD\}$

Ans: $\mathbf{F_c = \{A \rightarrow B, A \rightarrow C, E \rightarrow C, D \rightarrow A, D \rightarrow E, D \rightarrow F, A\cancel{B}F \rightarrow \cancel{B}D\}}$

OR

$\mathbf{F_c = \{A \rightarrow BC, E \rightarrow C, D \rightarrow AEF, AF \rightarrow D\}}$

Q13. [Minimal Cover]

Find the <u>minimal cover</u> for the following set of FDs for a relation R (A, B, C, D):

$F = \{C \rightarrow BD, BC \rightarrow AD\}$

Ans:

$F_c = \{C \rightarrow ABD\}$ OR

$F_c = \{C \rightarrow B, C \rightarrow D, \cancel{B}C \rightarrow A, \cancel{BC \rightarrow D}\}$

Q14. [Minimal Cover]

Find the <u>minimal cover</u> for the following set of FDs for a relation R (A, B, C, D, E, G, H):

$F = \{AB \rightarrow C, DEG \rightarrow H, A \rightarrow C, DE \rightarrow G\}$

Ans: Key={ABDE}

$F_c = \{\cancel{AB \rightarrow C}, DE\cancel{G} \rightarrow H, A \rightarrow C, DE \rightarrow G\}$ OR

$F_c = \{DE \rightarrow GH, A \rightarrow C\}$

Q15. [Minimal Cover]

Consider the relation schema *R(A, B, C, D),* with FDs *F = {AB →CD, C →A, AD→C, CD →AB, D →B}*. Find a minimal cover of *F* (i.e. F_c).

ANSWER:

$F_c = \{AB \rightarrow CD, C \rightarrow A, \cancel{AD \rightarrow C}, \cancel{CD \rightarrow AB}, D \rightarrow B\}$

i.e. $F_c = \{AB \rightarrow CD, C \rightarrow A, D \rightarrow B\}$

OR

$F_c = \{AB \rightarrow \cancel{C}D, C \rightarrow A, AD \rightarrow C, \cancel{CD \rightarrow AB}, D \rightarrow B\}$

i.e. $F_c = \{AB \rightarrow D, C \rightarrow A, AD \rightarrow C, D \rightarrow B\}$

Q16. [Minimal Cover]

Consider the relation schema *R(A B C D E F G H)* with FDs *F = { A →BCD, AD →E, EFG→H, F →GH }*. Find a minimal cover of *F* (i.e. F_c).

ANSWER:

$F_c = \{A \rightarrow BCD, A\cancel{D} \rightarrow E, \cancel{EFG \rightarrow H}, F \rightarrow GH\}$

i.e. $F_c = \{A \rightarrow BCDE, F \rightarrow GH\}$

Q17. [Minimal Cover]

Find two different minimal cover of *F= {A → BC, B → AC, C → AB}*. Show your work. Also find all possible keys of R.

Ans:

$F_{c1} = \{A \rightarrow B, B \rightarrow C, C \rightarrow A\}$

$F_{c2} = \{A \rightarrow C, C \rightarrow B, B \rightarrow A\}$

$F_{c3} = \{A \rightarrow C, B \rightarrow C, C \rightarrow AB\}$

$F_{c4} = \{A \rightarrow B, B \rightarrow AC, C \rightarrow B\}$

$F_{c5} = \{A \rightarrow BC, B \rightarrow A, C \rightarrow A\}$

Keys are {A}, {B}, and {C}.

Q18. [Minimal Cover]

Consider the relation schema *R (A, B, C, D, E, F),* with a set of FDs *F =* {A→ BC , FC→ D , D→ B , AB→ F , F→ C, AD→ E*}*. Compute the minimal cover for *F* (i.e. F_c). Show your work! Also find all possible keys of R.

Ans:

***F_C* = { ~~A→BC~~ , F~~C~~→ D , D→ B , AB→ F , F→ C, A~~D~~→ E }**
or
***F_C* = { A→ EF, F→ CD, D→ B }**

Key is {A}.

Q19. [Equivalent Sets]
Consider the following two sets of FDs. Check whether or not they are equivalent. Provide proper reason.
F1 = {A→B, B→C, C→A} and *F2 = {A→C, C→B, B→A}.*

ANSWER: *They are equivalent.*

Q20. [Equivalent Sets]
Consider the following two sets of FDs. Check whether or not they are equivalent. Provide proper reason.
F1 = {A→C, B→C, C→AB} and *F2 = {A→BC, B→A, C→A}.*

ANSWER: *They are equivalent.*

Q21. [Equivalent Sets]
Consider the following two sets of FDs:
F = {A→C, AC→D, E→AD, E→H} and G = {A→CD, E→AH}. Check whether they are equivalent.
Ans: Yes
Proof:

IN F:

(1)	A→C	(given)
(2)	AC→D	(given)
(3)	AA→D	(IR6 on 2, replace A with C)
(4)	A→D	(simplification of 3)
(5)	**A→CD**	**(IR5 on 1 and 4)**
(6)	E→AD	(given)
(7)	E→H	(given)
(8)	E→A	(IR4 on 6)
(9)	**E→AH**	**(IR5 on 7 and 8)**

<u>Hence F COVERS G.</u>

IN G:

1.	A→CD	(given)
2.	**A→C**	**(IR4 on 1)**
3.	A→D	(IR4 on 1)
4.	**AC→D**	**(IR2 to augment 3 with C on LHS)**
5.	E→AH	(given)
6.	**E→H**	**(IR4 on 5)**
7.	E→A	(IR4 on 5)
8.	E→D	(IR3 on 7 and 3)
9.	**E→AD**	**(IR5 on 7 and 8)**

<u>Hence G COVERS F.</u>

<u>So F and G are equivalent.</u>

OR with closure method:
Answer:
To show equivalence, we prove that G is covered by F and F is covered by G.
Proof that G is covered by F:
{A} + = {A, C, D} (with respect to F), which covers A ->CD in G
{E} + = {E, A, D, H, C} (with respect to F), which covers E ->AH in G

Proof that F is covered by G:
{A} + = {A, C, D} (with respect to G), which covers A ->C in F
{A, C} + = {A, C, D} (with respect to G), which covers AC ->D in F
{E} + = {E, A, H, C, D} (with respect to G), which covers E ->AD and E ->H in F

Q22. [Key+NF]
Consider the relation R (A, B, C, D, E, F) and the set F = {A→B, C→DF, AC→E, D→F}.
a. What is the <u>KEY</u> of this relation? Prove it.
b. What is the highest normal form of this relation? Give reason.
c. If it is not in 3NF find a decomposition that is lossless and dependency preserving.
Ans:
a. AC
b. I NF (FD1 & FD2 are PFDs)
c. R1 (<u>A, C</u>, E), R2(<u>C</u>, D), R3 (<u>D</u>, F), R4 (<u>A</u>, B)

Q23. [Key+NF]
Consider the relation SALES (transno, itemno, price, qty, seller, sregion) and the set
F = {{transno, itemno} → qty, itemno → price, transno → seller, seller → sregion}.

a. What is the KEY of this relation? Prove it.
b. What is the highest normal form of this relation? Give reason.
c. If it is not in 3NF find a decomposition that is lossless and dependency preserving.

Ans:
a. {transno, itemno}
b. I NF (PFD exist.)
c. S1 (item, price), S2 (transno, itemno, qty), S3 (transno, seller), S4 (seller, region)

Q24. [Key+NF]
Consider the relation SCHEDULE (stdid, classno, stdname, stdmajor, classtime, room, instructor) and the set
F = {stdid → {stdname, stdmajor}, classno → {classtime, room, instructor}}

a. What is the KEY of this relation? Prove it.
b. What is the highest normal form of this relation? Give reason.
c. What type of anomalies does this relation have?
d. Transfer this relation to its next higher form.

Ans:
a. {stdid, classno}
b. I NF (PFD exist.)
c. All (insert, update, delete) anomalies
d. S1 (stdid, classno), S2 (stdid, stdname, stdmajor), S3 (classno, classtime, room, instructor)

Q25. [Key+NF]
Consider the relation PROGRAMMER TASK (prog-id, programming-package-id, programming-package-name, total-hours-worked-on-package) and the set
F = { programming-package-id → programming-package-name,
{prog-id, programming-package-id} → total-hours-worked-on-package }

a. What is the KEY of this relation? Prove it.
b. What is the highest normal form of this relation? Give reason.
c. Transfer this relation to its next higher form.
d. Can the information if the given relation be recovered?
e. What operation is necessary to recover it?

Ans:
a. {prog-id, programming-package-id}
b. I NF (PFD exist.)
c. P1 (programming-package-id, programming-package-name),
P2 (prog-id, programming-package-id, total-hours-worked-on-package)
d. Yes
e. Natural Join Operation

Q26. [Key+NF]
Consider the relation TEACH (student, course, instructor) and the set
F = {{student, course} → instructor, instructor → course}.
a. What is the KEY of this relation? Prove it.
b. What is the highest normal form of this relation? Give reason.
c. If it is not in BCNF find a decomposition that is lossless.

Ans:
a. {student, course} and {student, instructor}
b. 3 NF
c. T1 (instructor, course), T2 (instructor, student)

Q27. [Key+NF]
Consider a relation R(A, B, C) and set of functional dependencies F = {AB→ C, B→A, C→B}.
Find all possible candidate keys of R. Prove it. What is the highest normal form that relation R is in? Justify your answer. Decompose it into BCNF, if it is not.

Ans:
{B} & {C} are keys and it is in BCNF

Q28. [NF]
Consider a relation schema R(A, B, C, D) and set of functional dependencies F = {AB→C, C→A, D→B, AB→D}.
{A,B}, {B,C}, {A,D}, and {C,D} are the candidate keys of R. What is the highest normal form that relation R is in? Justify your answer. Decompose it into BCNF, if it is not.

Ans:
It is in 3NF. Lossless decomposition is R1(C, A), R2(D, B), R3(C, D)

Q29. [Closure+Key+NF]
Consider the relation R (A, B, C, D, E), with FDs {AB → C, C → D, D → B, D → E}.
a) Find the closure of C and AB (i.e. C^+ and $\{AB\}^+$).
b) Find all the keys for this relation R. (you don't need to list superkeys that are not keys.)
c) Is this relation in BCNF? If your answer is yes, explain why. If your answer is no, decompose the relation into BCNF. Show your decomposition steps.
Ans:
a) C+= {BCDE}, {AB}+ = {ABCDE}
b) Keys are {AB}, {AC}, and {AD}.
c) Highest NF is 1NF (FD4 is PFD).
Set of BCNF relations are:
R1(A, C),
R2(C, D); C→D
R3(B, D, E); D→BE

Q30. [Key+NF]
Consider the relation R (A, B, C, D), with FDs {C→BD, BC→AD}.
What is the highest normal form of this relation? Give reason. If it is not in BCNF find a decomposition that is lossless.
Ans:
Key is {C}. Highest NF is BCNF.

Q31. [Key+NF]
Consider the relation R (A, B, C, D, E, G, H), with FDs F = {AB→C, DEG→H, A→C, DE→G}.
What is the highest normal form of this relation? Give reason. If it is not in BCNF find a decomposition that is lossless.
Ans:
Key is {ABDE}.
F_c = {~~AB → C~~, DE~~G~~→H, A→C, DE→G} OR
F_c = {DE→GH, A→C}
Highest NF is 1NF (both are PFDs).
Set of BCNF relations:
R1(A, B, D, E),
R2(D, E, G, H); DE→GH
R3(A, C); A→C

Q32. [Key+NF]
Consider a relation with schema *R(A, B,C,D)*, with FDs *F* = {BC→A, AD→B, CD→B, AC→D}.
Identify the best normal form that R satisfies (1NF, 2NF, 3NF, or BCNF). Justify your answer. If R is not in BCNF, decompose it into a set of BCNF relations and show your steps. Indicate which dependencies if any are not preserved by the BCNF decomposition.
ANSWER:
Keys are {BC}, {CD}, and {AC}.
Highest NF= 3NF; due to violation of FD2: AD → B
BCNF relations schemas are R1(A, C, D) & R2(A, D, B); but FD1: *BC→A* & FD3: *CD→B* are lost.

Q33. [Key+NF]
Consider the relation schema *R(A, B, C, D)*, with FDs *{AB→C, BC→D, CD→A}*. Identify the best normal form that *R* satisfies (*1NF, 2NF, 3NF, or BCNF*). Justify your answer. If *R* is not in *BCNF*, decompose it into a set of BCNF relations. Indicate which dependencies if any are not preserved by the decomposition.
ANSWER:
Keys are {AB} and {BC}.
Highest Normal Form is 3NF. R is not in BCNF since the FD CD->A violates the condition; CD is not a superkey of R.
BCNF relations schemas are R1(B, C, D) & R2(A, C, D); but FD1: AB → C is lost.

Q34. [Key+NF]
Consider the relation *R(A, B. C, D, E)*, with FDs *{AB→C, DE→C, B→D}*. State which of the following decompositions of R relation are lossless decomposition. Justify your answer.
a. R1(A, B, C), R2(C, D, E), and R3(B, D).
b. R1(A, B, C), R2(A, B, E), and R3(B, D).
ANSWER:
Key={ABE}
a. *Not Lossless*
b. *Lossless*

Q35. [Key+NF]
Consider a relation with schema *R(A, B, C, D)*, with FDs *F = {AB→C, BC→D, CD→A}*.

a) We are considering to decompose R into *R1(A, B, C)* and *R2(A, C, D)*. Is this a lossless decomposition? Prove it.
b) Provide BCNF relations for this relation R. Also indicate which dependencies if any are not preserved.
c) Provide 3NF relations for this relation R.

ANSWER:

a) Key1= {AB} & Key2= {BC}. This decomposition is not lossless.

AC ⇏ B (i.e. R1 intersect R2→R1–R2)

AC ⇏ D (i.e. R1 intersect R2→R2–R1)

b) HNF= 3NF

BCNF relation schemas are R1(<u>B</u>, <u>C</u>, D) with FD2: BC → D and R2(<u>C</u>, <u>D</u>, A) with FD3: CD→A. But *AB→C* LOST.

OR Alternate: BCNF schemas are R1(<u>B</u>, <u>C</u>, D) with FD2: BC→D and R2(<u>A</u>, <u>B</u>, C) with FD1: AB→C. But *CD→A* LOST.

c) R is Already in 3NF.

Q36. [Key+NF]

Consider a relation with schema *R(A, B, C, D, E)*, with FDs *F = {AB→C, DE→C, B→D}*.
Identify the best normal form that R satisfies (1NF, 2NF, 3NF, or BCNF). Justify your answer. If R is not in BCNF, decompose it into a set of BCNF relations and show your steps. Indicate which dependencies if any are not preserved by the BCNF decomposition.

ANSWER:

Key={ABE}

HNF= 1NF; PFD1 & PFD3 violate 2NF. Also you may replace FD2 *DE→C* with *BE→C* using IR6. Then FD2 will also violate 2NF.

BCNF relation schemas are R1(<u>A</u>, <u>B</u>, <u>E</u>), R2(<u>A</u>, <u>B</u>, C) with FD1, R3(<u>B</u>, D) with FD3; But FD2: *BE→C OR DE→C* Lost.

OR

BCNF relation schemas are R1(<u>A</u>, <u>B</u>, <u>E</u>), R2(<u>B</u>, <u>E</u>, C) with FD2, R3(<u>B</u>, D) with FD3; But FD1: *AB→C* Lost.

Q37. [Key+NF]

Consider the relation *R (A, B, C, D, E)*, with FDs {A→BC, C→D, E→D, BE→A}. List all the possible keys of *R*. Show the intermediate steps of your derivation. Also Identify the best normal form that R satisfies. If R is not in BCNF, decompose it into a set of BCNF relations and show your steps. Indicate which dependencies if any are not preserved by the BCNF decomposition.

ANSWER:

Keys are {AE} and {BE}.

Best normal form is 1NF. A→C & E→D violate 2NF, C→D violate 3NF, and A→B violate BCNF.

BCNF relation schemas are R1(<u>A, E</u>), (<u>A</u>, B, C), (D, <u>E</u>), (<u>C</u>, D). FD4: BE→A lost.