INDIAN STATISTICAL INSTITUTE
CHENNAI CENTER

PORTFOLIO OPTIMIZATION USING MARKOWITZ MODEL AND EFFICIENT FRONTIER

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TABLE OF CONTENTS

1. ABSTRACT

This project on Portfolio Optimization is structured into two main parts.

Part 1: Comparative Study of Portfolio Optimization Methods

The first part focuses on a comparative analysis between two different portfolio optimization strategies: the Equally-Weighted portfolio and the Markowitz (mean-variance) portfolio optimization. In this section, both methods of weight allocation are evaluated based on their performance metrics, including expected return, risk (volatility), and the Sharpe Ratio. The equally weighted portfolio assigns the same weight to each asset, while the Markowitz approach optimizes the weights to maximize the Sharpe Ratio.

By comparing these two methods, we aim to understand their effectiveness in achieving desired investment outcomes.

Part 2: Efficient Frontier Analysis

The second part delves into the concept of the Efficient Frontier, which represents a set of optimal investment portfolios expected to yield the highest returns for a given level of risk. A portfolio is deemed efficient if no other portfolio can offer higher returns for the same or a lower level of risk. The position of portfolios on the Efficient Frontier is determined by the investor's risk tolerance, with more risk-averse investors selecting portfolios with lower risk and more aggressive investors opting for higher-risk, higher-return portfolios. This part of the project illustrates how the Efficient Frontier helps investors make informed decisions by balancing return and risk according to their individual preferences.

This comprehensive project provides valuable insights into portfolio optimization. By comparing Equally-Weighted and Markowitz portfolio optimization, and by exploring the Efficient Frontier, it equips investors with the knowledge to construct portfolios that align with their risk tolerance and return expectations. The project underscores the importance of strategic weight allocation in enhancing portfolio performance and achieving efficient investment outcomes.

2. PRELIMINARIES

2.1.1. RISK

Risk can be defined as the variability or uncertainty associated with an investment, market, or company performance. Investors seek returns, but risks can lead to lower-than-expected returns or even losses.

Investments involve various types of risk and return factors, including market risk, speculative risk, industry-specific risk, volatility risk, and inflation risk. However, thorough market analysis can aid investors in making informed decisions. By, analysing trends and forecasting potential outcomes, investors can better manage these risks.

Risk (denoted as "σ") quantifies the uncertainty or variability of returns. One common statistical method for calculating risk is the standard deviation, which measures the dispersion of returns around the mean. The formula for calculating risk using standard deviation is given by,

$$\sigma = \sqrt{\sum \frac{[x_i - \mu]^2}{N-1}}$$

Where:

σ= Risk

x_i= Individual return for each period

μ= Average return

N= Number of periods

2.1.2. RETURN

A portfolio return refers to the amount an investment portfolio gains or loses over a specific period of time. Return (denoted as "r") is the profit or loss generated from an investment, expressed as a percentage of the original investment. The formula for calculating return is:

$$r = \frac{(\mathrm{P}_{t+1} - \mathrm{P}_t)}{\mathrm{P}_t} \equiv \frac{\mathrm{P}_{t+1}}{\mathrm{P}_t} - 1$$

Where P_t and P_{t+1} is the Price of the stock at time t and at time t+1 respectively.

When an investor is considering an investment, they generally expect a certain average return, known as the mean return. By proceeding with the investment, the investor implicitly accepts the risk, measured by the variance or standard deviation, that the actual return may differ from the expected mean return. This risk represents the uncertainty that the realized return could be higher or lower than anticipated. Consequently, it is essential for investors to evaluate both the expected mean return and the associated risk before making investment decisions.

2.1.3. RELATIONSHIP BETWEEN RISK AND RETURN

The relationship between risk and return is a fundamental principle in financial theory. The higher the risk an investor is willing to take, the higher the potential return they expect, as investors need compensation for taking on additional risk.

Consequently, investments with higher potential returns typically come with greater risk. Therefore, it is crucial for investors to thoroughly evaluate both the risk and return aspects of any investment before making decisions.

Graphical Representation of Risk – Return Relationship

2.1.4. PORTFOLIO AND IT’S OPTIMIZATION

Portfolio is a collection of a wide range of assets that are owned by investors. The said collection of financial assets may also be valuables ranging from gold, stocks, funds, derivatives, property, cash equivalents, bonds, etc. However, the decision to invest solely in a single asset or stock can expose investors to substantial risk, given the volatility inherent in individual assets. Diversification, achieved by spreading investments across a variety of assets with differing risk and return profiles, emerges as a prudent strategy to mitigate this risk.

Yet, the mere act of assembling a diverse collection of assets doesn't suffice. The construction of an optimized portfolio stands as a critical step in achieving a balanced risk-return trade-off. An optimized portfolio is finely tuned to minimize risk while maximizing returns. Without optimization, a portfolio might inadvertently concentrate too heavily on a specific asset or sector, heightening exposure to risk and potentially limiting returns.

Portfolio optimization furnishes investors with a structured methodology for creating portfolios that strike an ideal balance between diversification and optimization. This process involves a meticulous analysis of each asset's expected returns, variances, and correlations within the portfolio. By leveraging statistical measures such as covariance, investors can craft portfolios tailored to offer the highest expected return for a given level of risk or conversely, minimize risk for a desired level of expected return.

In essence, portfolio optimization empowers investors to tailor their investment strategies to meet their financial objectives while prudently managing risk. By integrating a disciplined approach to asset allocation, investors can navigate market uncertainties with greater confidence, driving towards their long-term financial goals.

3. MARKOWITZ PORTFOLIO OPTIMIZATION

Markowitz portfolio optimization stands as a prominent investment strategy widely embraced in financial markets. Introduced by Harry Markowitz in 1952, this strategy aims to strike a balance between risk and return by leveraging the principle of diversification.

At its core, Markowitz portfolio optimization revolves around the concept of selecting a mix of diversified assets. This strategic selection allows for the construction of a well-diversified portfolio, where the movements of individual assets offset each other to a certain extent. Consequently, this diversification helps mitigate the overall risk of the portfolio without compromising potential returns.

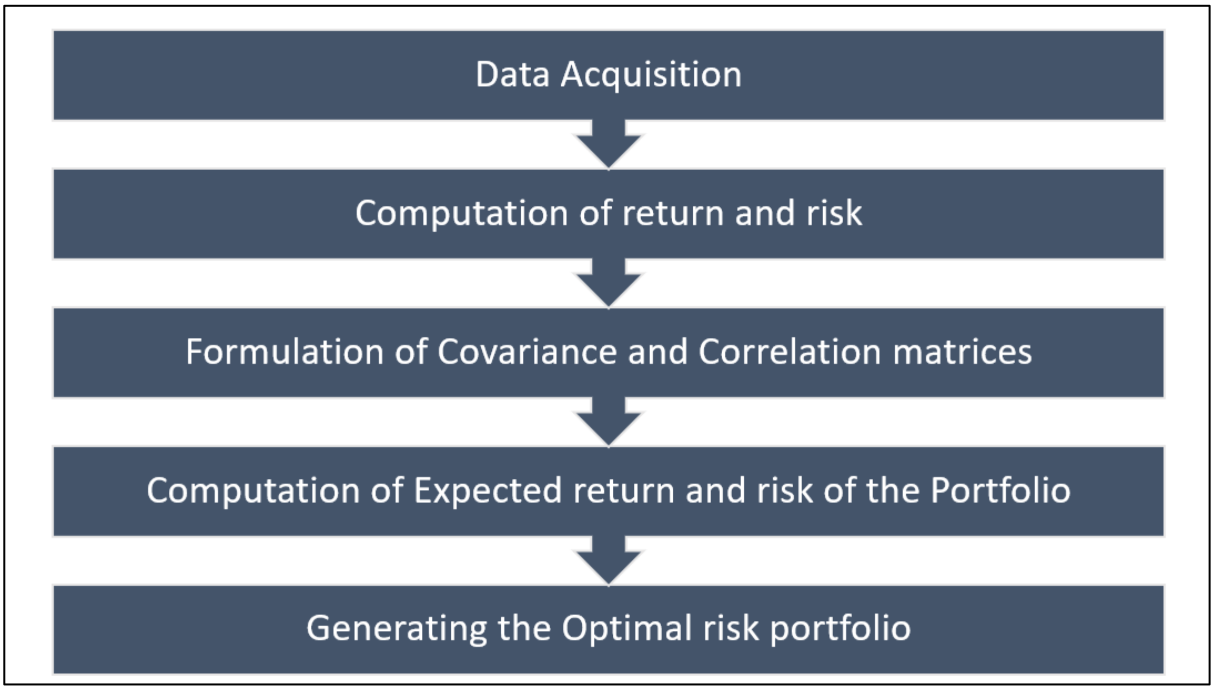
The optimization process unfolds in following steps:

1. **Estimating Expected Returns and Variances**: Initially, the expected returns and variances of each asset in the portfolio are calculated. This step involves analyzing historical data, market trends, and other relevant factors to gauge the anticipated performance and volatility of each asset.

2. **Selecting an Optimal Portfolio**: Subsequently, the portfolio that minimizes risk for a specified level of expected return is identified. This entails evaluating various asset combinations and weightings to determine the optimal mix that achieves the desired risk-return trade-off.

By adhering to the principles of Markowitz portfolio optimization, investors can construct portfolios that strike an optimal balance between risk and return. This approach empowers investors to make informed decisions and effectively manage their investment portfolios in line with their financial objectives and risk tolerance levels.

BUILDING AN OPTIMAL RISK PORTFOLIO:

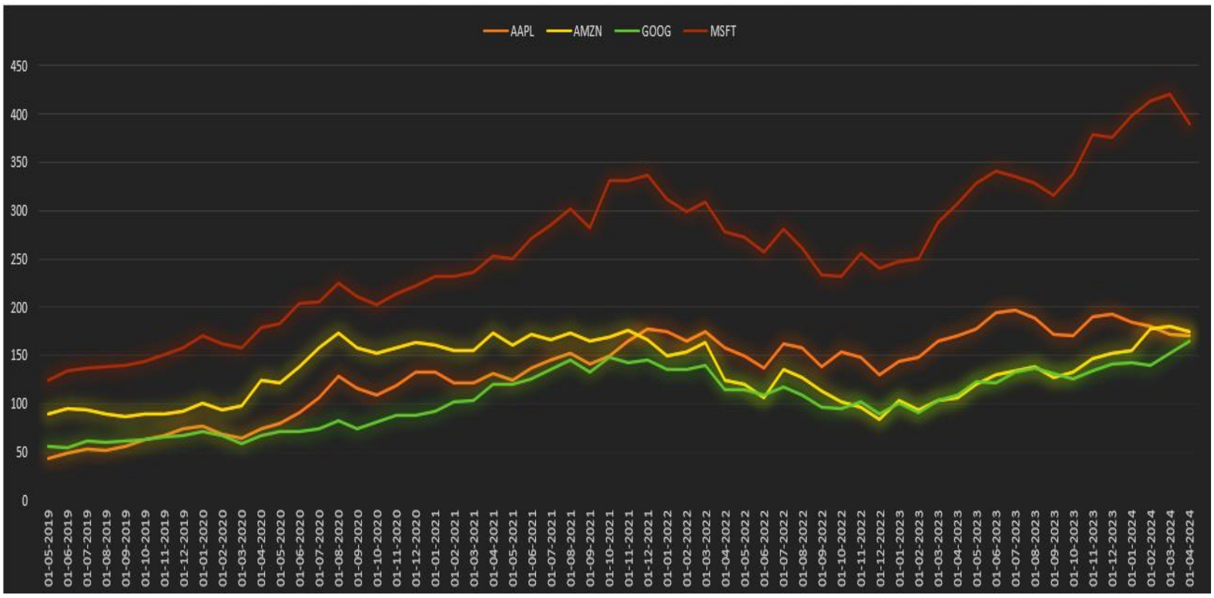


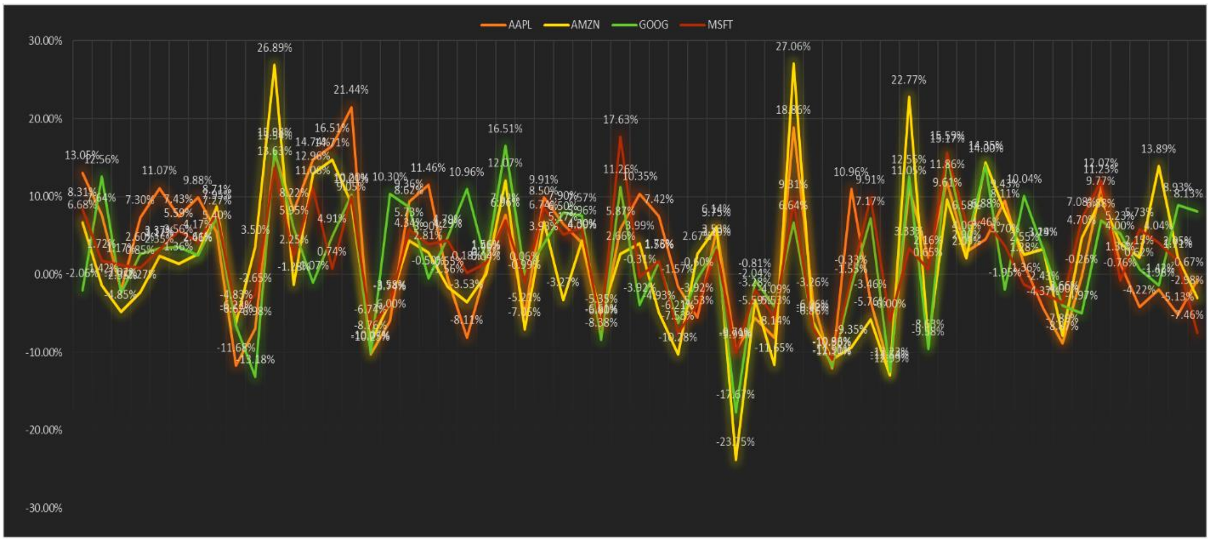
DATA ACQUISITION

The stock prices of Microsoft (MSFT), Alphabet Inc. (GOOG), Amazon (AMZN), and Apple Inc. (AAPL) are retrieved from the Yahoo Finance website covering the period from May 1, 2019, to May 1, 2024. The stock data encompass attributes such as open, high, low, close, volume, and adjusted close. Given that the analysis focuses on a single variable, the monthly closing price is selected as the primary variable of interest, while the other attributes are disregarded.

The choice of monthly closing prices for univariate analysis of MSFT, GOOG, AMZN, and AAPL stocks is justified by their stability, representativeness, and capacity to unveil long-term trends. These prices mitigate daily fluctuations, minimizing noise and enhancing trend clarity. Monthly data are pragmatic for analysis, offering manageability and computational efficiency.

Graph showing Historical Close prices of all four stock





Graph showing the fluctuation of returns

CALCULATION OF RETURNS AND VARIANCES:

The Average Monthly Return for each stock is determined using Excel's built-in functions. This monthly return is then annualized by multiplying it by 12, thereby converting it into the Annual Return. Similarly, the Monthly Variance is calculated and then scaled to an annual figure by applying appropriate transformations, resulting in the Annual Variance.

	AAPL	AMZN	GOOG	MSFT
Average Monthly Return	2.68%	1.58%	2.17%	2.16%
Monthly Variance	0.73%	0.89%	0.61%	0.40%
Average Annual Return	32.17%	18.99%	26.06%	25.86%
Annual Variance	8.76%	10.67%	7.36%	4.81%

CALCULATING VARIANCE-COVARIANCE AND CORRELATION MATRIX

After calculating the volatilities and returns of the stocks, the variance-covariance and correlation matrices for the four stocks are derived using historical data. These matrices aid in understanding the strength of the association between pairs of stock prices. A high correlation coefficient for any pair indicates a strong relationship between them.

A well-constructed portfolio aims to minimize risk while optimizing returns. Minimizing portfolio risk involves identifying stocks with low correlations among themselves to achieve greater diversification. Therefore, the computation and analysis of the covariance and correlation matrices of the stocks are crucial.

VARIANCE-COVARIANCE MATRIX				
	AAPL	AMZN	GOOG	MSFT
AAPL	0.086136612	0.065367158	0.041928194	0.044789756
AMZN	0.065367158	0.104919743	0.052884129	0.047842397
GOOG	0.041928194	0.052884129	0.072335977	0.037293266
MSFT	0.044789756	0.047842397	0.037293266	0.047260905

	AAPL	AMZN	GOOG	MSFT
AAPL	1	0.687601959	0.531171901	0.701994628
AMZN	0.687601959	1	0.607043163	0.679412135
GOOG	0.531171901	0.607043163	1	0.637826007
MSFT	0.701994628	0.679412135	0.637826007	1

Correlation Heat Map

CALCULATION OF EXPECTED RETURN FOR THE PORTFOLIO:

The Expected return for a portfolio is calculated as:

$$\mathbf{E(r_p)} = \sum_{i=0}^{n} w_i \, E(r_i)$$

The variance of a two asset (x and y) portfolio is calculated as:

$$\sigma_p^2 = w_x^2\sigma_x^2 + w_y^2\sigma_y^2 + 2w_xw_y \, Cov(r_x,r_y)$$

Generalizing the equation to accommodate more than two assets results in the equation:

$$\sigma_p^2 = \sum_{i=1}^{n}\sum_{j=1}^{n} w_i w_j \, Cov(r_i, r_j)$$

As we move past a two-asset portfolio it is necessary to use matrix multiplication to determine the optimal asset weights in the portfolio.

The Expected return for the portfolio is calculated as, $\mathbf{E(r_p) = W^TR} = \begin{bmatrix} w_1 \dots w_j \end{bmatrix} \begin{bmatrix} E(r_1) \\ \vdots \\ E(r_j) \end{bmatrix}$

Where:

W is the vector of weights of the individual assets in the portfolio

R is the vector of expected returns of the individual assets in the portfolio

CALCULATION OF STANDARD DEVIATION:

The variance of the portfolio is calculated as $\sigma_p^2 = W^TSW$

The standard deviation of the portfolio is calculated as

$$\sigma_p = \sqrt{W^TSW} = \left\{ \begin{bmatrix} w_1 & \dots & w_j \end{bmatrix} \begin{bmatrix} \sigma_{11} & \cdots & \sigma_{1j} \\ \vdots & \ddots & \vdots \\ \sigma_{j1} & \cdots & \sigma_{jj} \end{bmatrix} \begin{bmatrix} w_1 \\ \vdots \\ w_j \end{bmatrix} \right\}^{\frac{1}{2}}$$

Where **S** is the variance-covariance matrix of the covariances between each of the two asset's returns in the portfolio. And **W** is the vector of the weights of the individual assets in the portfolio.

RISK FREE RATE:

It is the theoretical rate of return of an investment with no loss. It is used as a bench mark to measure other investment that have some risk. The US 10-year Treasury rate is favoured as the risk-free rate due to its strong government backing, high liquidity, relevance to long-term investments, widespread acceptance in the financial community, and the availability of transparent market data.

SHARPE RATIO:

The Sharpe Ratio, also referred to as the modified Sharpe ratio or the Sharpe index, quantitatively evaluates the performance of an investment by accounting for risk. This metric can be applied to individual securities or entire investment portfolios. A higher Sharpe ratio indicates superior investment performance in terms of risk-adjusted returns.

By comparing the return on an investment to the additional risk taken beyond that of a risk-free asset, the Sharpe ratio provides investors with a clear assessment of whether higher returns justify the increased risk. The ratio offers a score reflecting the risk-adjusted returns, which can be used to evaluate both historical performance and anticipated future performance. This essential financial ratio helps investors discern if returns are the result of prudent decision-making or merely the consequence of assuming excessive risk. In the latter scenario, investors might incur losses greater than their risk tolerance during adverse market conditions.

To calculate the Sharpe Ratio, one determines the "excess return" of an asset or portfolio over a specified period. This excess return is then divided by the portfolio's standard deviation, which quantifies its volatility.

$$\textbf{Sharpe Ratio} = S_p = \frac{E(r_p) - r_f}{\sigma_p}$$

Where $E(r_p)$ is the Expected return of the portfolio, r_f is the risk-free rate and σ_p is the standard deviation of the portfolio.

4. EQUALLY-WEIGHTED PORTFOLIO:

An equal weight portfolio represents a distinctive investment strategy characterized by an even distribution of funds across all securities or asset classes within the portfolio.

Unlike traditional market-capitalization-weighted portfolios, where investments are allocated based on the market value of each security, an equal weight portfolio assigns the same investment amount to each security regardless of its market size.

Weight of each security =1 / Number of securities in the portfolio

5. OPTIMAL RISKY PORTFOLIO:

The model calculates the optimal portfolio by solving the following quadratic optimization problem:

minimize: $w^T S w$

- subject to: $w^T \mu \geq r$
- $\Sigma w = 1$

where: w = vector of portfolio weights

μ = vector of expected returns of assets

S = covariance matrix of asset returns

r = target return

The objective function represents the portfolio risk, which is the weighted sum of the variances and covariances of the assets in the portfolio. The first constraint ensures that the portfolio achieves a target expected return, while the second constraint ensures that the portfolio weights sum to one.

The investors in the stock markets are usually not interested in the minimum risk portfolios as the return values are usually low. In most cases, the investors are ready to incur some amount of risk if the associated return values are high. For computing the optimum risk portfolio, the metric Sharpe Ratio of a portfolio is used.

$$\textbf{Sharpe Ratio} = S_p = \frac{E(r_p) - r_f}{\sigma_p}$$

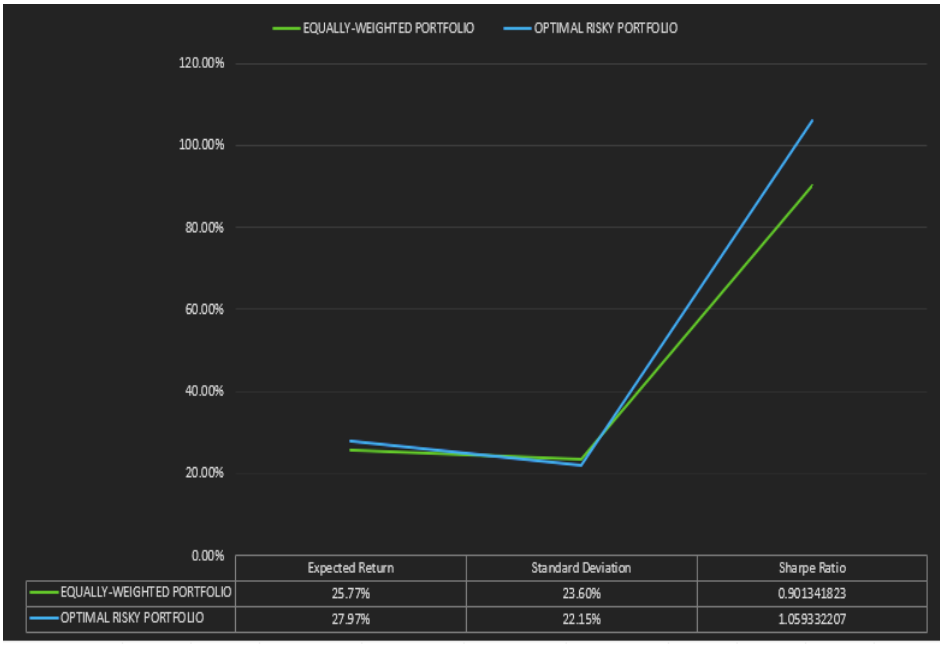
Where $E(r_p)$ is the Expected return of the portfolio, r_f is the risk-free rate and σ_p is the standard deviation of the portfolio.

6. COMPARISON OF EQUALLY-WEIGHTED PORTFOLIO AND OPTIMAL RISKY PORTFOLIO:

	Expected Return	Standard Deviation	Sharpe Ratio
EQUALLY-WEIGHTED PORTFOLIO	25.77%	23.60%	0.90134
OPTIMAL RISKY PORTFOLIO	27.97%	22.15%	1.05933

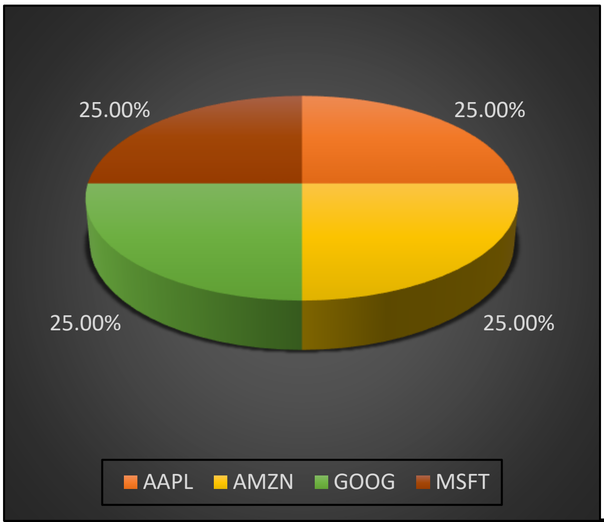
Based on the comparison of the equally-weighted portfolio and the optimal risky portfolio, several key insights can be drawn:

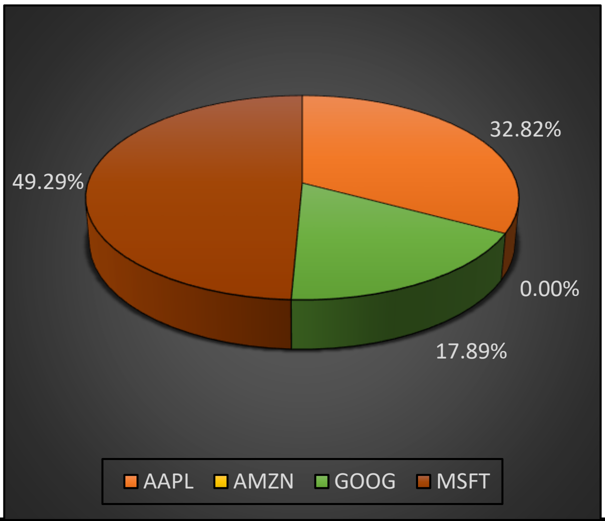
- **Expected Return**:
  The optimal risky portfolio has a higher expected return (27.97%) compared to the equally-weighted portfolio (25.77%). This indicates that the optimal risky portfolio is expected to generate more returns.
- **Standard Deviation**:
  The standard deviation, which measures the portfolio's risk, is lower for the optimal risky portfolio (22.15%) compared to the equally-weighted portfolio (23.60%). This suggests that the optimal risky portfolio is less volatile and carries less risk.
- **Sharpe Ratio**:
  The Sharpe ratio, which measures the risk-adjusted return, is higher for the optimal risky portfolio (1.05933) compared to the equally-weighted portfolio (0.90134).
  A higher Sharpe ratio indicates that the optimal risky portfolio offers better returns per unit of risk.



6.1.1. WEIGHT ALLOCATION:

The allocation of weights (% Allocation) to the four stocks by the Equally-Weighted and the Optimal Risky portfolios based on stock price data from 01-May- 2019 to 01-May-2024 is given as:





EQUALLY-WEIGHTED PORTFOLIO		***OPTIMAL RISKY PORTFOLIO***	
ASSET	**WEIGHT**	**ASSET**	**WEIGHT**
AAPL	***0.25***	**AAPL**	***0.3282***
AMZN	***0.25***	**AMZN**	***0.00***
GOOG	***0.25***	**GOOG**	***0.1789***
MSFT	***0.25***	**MSFT**	***0.4929***
Sum	***1***	**Sum**	***1***

❖ **Overall Insights:**

Both portfolios exhibit varying allocations across the four stocks, reflecting differences in their construction methodologies and optimization objectives.

- While **AAPL** and **GOOG** maintain substantial positions in both portfolios.
- **AMZN** is excluded entirely from the Optimal Risky Portfolio.
- **MSFT** emerges as a significant component of the Optimal Risky Portfolio, indicating its importance in maximizing portfolio efficiency and risk-adjusted returns within the optimization framework.

7. CONSTRAINTS AND LIMITATIONS:

The Markowitz Portfolio Optimization model makes several assumptions about the behaviour of financial markets and the characteristics of assets that are included in a portfolio. These assumptions are:

• **Normal distribution of asset returns**: The Markowitz model assumes that asset returns are normally distributed, which means that they follow a bell-shaped curve. This assumption is important because it allows the model to use statistical methods to estimate the expected return and variance of assets.

• **Investors are risk-averse**: The Markowitz model assumes that investors are risk-averse, which means that they prefer portfolios that provide a higher expected return for a given level of risk or a lower level of risk for a given expected return.

• **Single period investment**: The Markowitz model assumes that the investment period is a single period, which means that it does not consider the effects of compounding over time.

• **No taxes, transaction costs, or other frictions**: The Markowitz model assumes that there are no taxes, transaction costs, or other frictions associated with buying or selling assets.

• **Stable correlation between assets**: The Markowitz model assumes that the correlation between assets is stable over time. This assumption can be problematic in volatile markets where correlations may change rapidly.

• **No short-selling restrictions**: The Markowitz model assumes that investors can sell assets short, which means they can borrow assets and sell them with the expectation of buying them back at a lower price in the future. This assumption can be problematic in practice because short-selling is not always allowed or feasible.

• **Expected returns are constant**: The Markowitz model assumes that expected returns are constant over time. This assumption may not hold true in practice, as asset returns can be influenced by a variety of factors, including macroeconomic conditions, company performance, and changes in investor sentiment.

8. EFFICIENT FRONTIER METHOD

The Efficient Frontier epitomizes optimal portfolio construction within modern portfolio theory (MPT), representing the delicate balance between risk and return inherent in investment decisions. Each point along this frontier signifies a specific portfolio mix, deemed efficient because it provides the maximum expected return for a given level of risk, or conversely, the minimum risk for a given expected return.

Visualized graphically, the Efficient Frontier illustrates the spectrum of risk-return trade-offs achievable by adjusting asset allocations. Typically sloping upwards, it denotes that higher returns come with increased risk. However, not all points on the Efficient Frontier are equally suitable for investors; the ideal portfolio choice depends on individual risk tolerance, financial objectives, and investment horizon.

Investors willing to embrace higher risk for potential higher returns may gravitate towards portfolios on the right end of the Efficient Frontier, where riskier assets dominate. Conversely, those with lower risk tolerance may favor portfolios on the left end, offering lower risk albeit lower expected returns.

The shape of the Efficient Frontier is molded by various factors, including expected returns, asset volatilities, and correlations. Diversification, achieved through spreading investments across assets with low correlations, plays a pivotal role in shaping the frontier, allowing investors to mitigate risk without sacrificing returns.

In essence, the Efficient Frontier serves as a guide for investors, aiding in the selection of portfolios that align with their risk preferences. By pinpointing portfolios offering the optimal risk-return balance, it empowers investors to craft strategies that best suit their financial goals.

9. Program:

1. Importing Libraries:

- Import the necessary Python libraries for data manipulation (`pandas`, `numpy`), fetching financial data (`yfinance`), date handling (`datetime`), plotting (`matplotlib.pyplot`), and portfolio optimization (`EfficientFrontier`, `risk_models`, `expected_returns`).

2. Defining Assets and Date Range:

- Define a list of assets (stocks) for the portfolio, such as MSFT (Microsoft), GOOG (Alphabet Inc.), AMZN (Amazon), and AAPL (Apple Inc.).

- Specify the start and end dates for fetching historical data, such as '2019-05-01' to the '2024-05-01' .

3. Downloading Data Using yfinance:

- Use the `yfinance` library to download historical daily closing prices of the assets from Yahoo Finance.

- Retrieve the closing prices for the defined assets within the specified date range.

4. Resampling Data to Monthly Frequency:

- Convert the daily data to monthly frequency by resampling and taking the last closing price of each month.

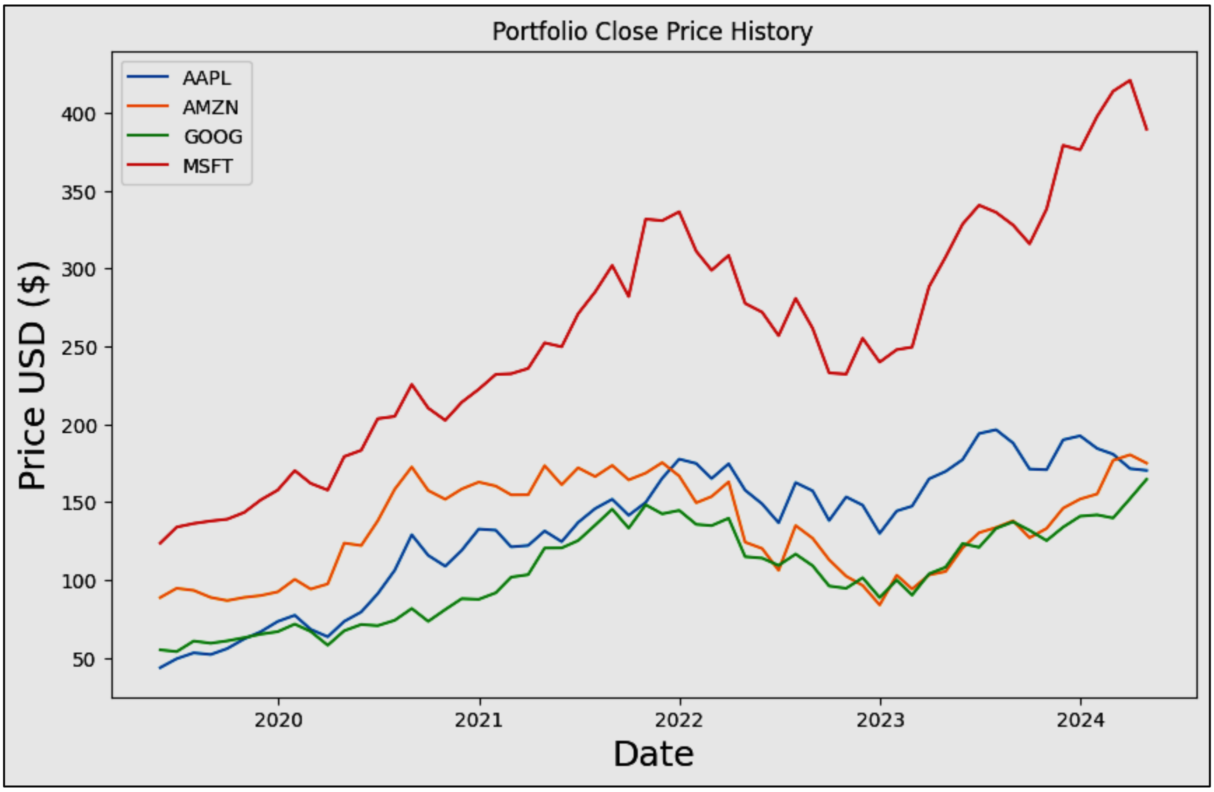
- This step ensures uniformity and simplifies further analysis.

5. Printing and Plotting Data:

- Print the first few rows of the monthly data to inspect and verify the downloaded data.

- Plot the adjusted close price history of the stocks to visualize their performance over time.

- This step helps in understanding the data and identifying any trends or patterns.



6. Calculating Monthly Returns:

- Calculate the monthly returns of each asset by computing the percentage change from one month to the next.

7. Calculating Annualized Covariance Matrix:

- Compute the covariance matrix of the monthly returns to assess the relationships between different assets.

- Multiply the covariance matrix by 12 to annualize it, assuming there are 12 months in a year.

Ticker	AAPL	AMZN	GOOG	MSFT
Ticker				
AAPL	0.087622	0.066494	0.042651	0.045562
AMZN	0.066494	0.106729	0.053796	0.048667
GOOG	0.042651	0.053796	0.073583	0.037936
MSFT	0.045562	0.048667	0.037936	0.048076

8. Defining Initial Weights and Calculate Portfolio Metrics:

- Define initial weights for the portfolio, often equal weights for simplicity (e.g., [0.25, 0.25, 0.25, 0.25]).

- Calculate metrics such as portfolio variance, volatility (standard deviation), and expected annual return based on these weights.

9. Printing Portfolio Performance Metrics:

- Print the calculated performance metrics of the portfolio, including expected annual return, volatility, and variance.

- This step provides insights into the risk and return characteristics of the initial portfolio.

10. Calculating Expected Returns and Covariance Matrix:

- Use historical data to estimate the expected returns and covariance matrix of the assets.

- Calculate the mean historical returns and sample covariance matrix of the monthly returns using appropriate functions from the `expected_returns` and `risk_models` modules.

11. Defining Risk Range and Risk-Free Rate:

- Define a risk range within which the portfolio's volatility should fall to meet specific risk preferences.

- Specify a risk-free rate, representing the return on a risk-free investment (e.g., government bonds), which is used in the Sharpe Ratio calculation.

12. Optimizing for Maximum Sharpe Ratio:

- Use the `EfficientFrontier` class to optimize the portfolio for the maximum Sharpe Ratio.

- Consider the specified risk-free rate in the optimization process to account for risk-adjusted returns.

13. Calculating and Print Portfolio Performance Metrics:

- Calculate the expected return, volatility, and Sharpe Ratio of the optimized portfolio.

- Print these metrics to assess the performance of the optimized portfolio and compare them with the initial portfolio.

14. Adjusting Portfolio to Meet Risk Constraints:

- Check if the portfolio's volatility falls within the specified risk range.

- If not, adjust the portfolio weights to ensure that the volatility remains within the desired range while maximizing the Sharpe Ratio.

15. Plotting the Efficient Frontier:

- Define a function to plot the efficient frontier, which represents the set of optimal portfolios that offer the highest expected return for a given level of risk.

- Highlight the risk range on the plot for visual reference to understand how the optimized portfolio aligns with the specified risk preferences.

10. CODE:

```
pip install yfinance matplotlib PyPortfolioOpt
import pandas as pd
import numpy as np
import yfinance as yf
from datetime import datetime
import matplotlib.pyplot as plt
from pypfopt.efficient_frontier import EfficientFrontier
from pypfopt import risk_models, expected_returns

# Define the list of assets and the date range
assets = ['AAPL', 'MSFT', 'GOOG', 'AMZN']
stockStartDate = '2019-05-01'
today = '2024-05-01'

# Download daily data using yfinance
df = yf.download(assets, start=stockStartDate, end=today)['Close']

# Resample to monthly frequency
df_monthly = df.resample('M').last()

# Print the DataFrame to check the results
print(df_monthly.head())

# Visually show the stock portfolio
title = 'Portfolio Close Price History'
# Get the stocks
my_stocks = df_monthly
# Create and plot the graph
plt.figure(figsize=(10, 6))
for c in my_stocks.columns.values:
    plt.plot(my_stocks[c], label=c)
plt.title(title)
plt.xlabel('Date', fontsize=18)
plt.ylabel('Price USD ($)', fontsize=18)
plt.legend(my_stocks.columns.values, loc='upper left')
plt.show()

# Calculate monthly returns
returns = df_monthly.pct_change().dropna()
print(returns.head())

# Create and show the annualized covariance matrix
cov_matrix_annual = returns.cov() * 12  # 12 months in a year
print(cov_matrix_annual)
```

```
# Define initial weights for the portfolio (equal allocation for
simplicity)
weights = np.array([0.25, 0.25, 0.25, 0.25])

# Calculate portfolio variance
port_variance = np.dot(weights.T, np.dot(cov_matrix_annual, weights))
print("Portfolio Variance:", port_variance)

# Calculate portfolio volatility
port_volatility = np.sqrt(port_variance)
print("Portfolio Volatility:", port_volatility)

# Calculate portfolio annual return
portfolio_simple_annual_return = np.sum(returns.mean() * weights) * 12
print("Portfolio Annual Return:", portfolio_simple_annual_return)

# Print the portfolio performance metrics
percent_var = str(round(port_variance, 4) * 100) + '%'
percent_vola = str(round(port_volatility, 4) * 100) + '%'
percent_ret = str(round(portfolio_simple_annual_return, 4) * 100) + '%'
print('Expected annual return: ' + percent_ret)
print('Annual volatility/ risk: ' + percent_vola)
print('Annual variance: ' + percent_var)

# Calculating the expected returns and the annualized sample covariance
matrix of asset returns
mu = expected_returns.mean_historical_return(df_monthly, frequency=12)
S = risk_models.sample_cov(df_monthly, frequency=12)

# Defining the risk range (e.g., annual volatility between 22% and 26%)
risk_min = 0.22
risk_max = 0.26

# Setting a risk-free rate of 4.502% (0.04502 annual risk-free rate)
risk_free_rate = 0.04502

# Optimization for maximum Sharpe ratio
ef = EfficientFrontier(mu, S)
weights = ef.max_sharpe(risk_free_rate=risk_free_rate)
cleaned_weights = ef.clean_weights()
print("Optimized Weights:", cleaned_weights)

# Calculating portfolio performance metrics with the specified risk-
free rate
expected_annual_return, annual_volatility, sharpe_ratio =
ef.portfolio_performance(risk_free_rate=risk_free_rate, verbose=True)

# Checking if the portfolio's volatility falls within the risk range
```

```
port_volatility =
ef.portfolio_performance(risk_free_rate=risk_free_rate)[1]

# Adjusting the portfolio to meet the risk range constraints
if port_volatility < risk_min or port_volatility > risk_max:
    target_risks = np.linspace(risk_min, risk_max, 100)
    best_sharpe = -np.inf
    best_weights = None
    for risk in target_risks:
        ef = EfficientFrontier(mu, S)
        ef.efficient_risk(risk)
        weights = ef.clean_weights()
        perf = ef.portfolio_performance(risk_free_rate=risk_free_rate)
        sharpe_ratio = perf[2]
        if sharpe_ratio > best_sharpe:
            best_sharpe = sharpe_ratio
            best_weights = weights

    cleaned_weights = best_weights
    ef.set_weights(best_weights)
    ef.portfolio_performance(risk_free_rate=risk_free_rate,
verbose=True)

print("Optimized Weights within Risk Range:", cleaned_weights)

# Manually plotting the Efficient Frontier with the risk range
def plot_efficient_frontier(mu, S, risk_min, risk_max, risk_free_rate):
    ef = EfficientFrontier(mu, S)
    max_return = ef._max_return()
    target_returns = np.linspace(mu.min(), max_return, 100)
    target_risks = []

    for ret in target_returns:
        ef = EfficientFrontier(mu, S)
        ef.efficient_return(ret)
        risk =
ef.portfolio_performance(risk_free_rate=risk_free_rate)[1]
        target_risks.append(risk)

    plt.figure(figsize=(10, 6))
    plt.plot(target_risks, target_returns, label='Efficient Frontier')
    plt.axvline(x=risk_min, color='r', linestyle='--', label='Risk
Min')
    plt.axvline(x=risk_max, color='g', linestyle='--', label='Risk
Max')
    plt.xlabel('Risk (Standard Deviation)')
    plt.ylabel('Return')
    plt.title('Efficient Frontier with Risk Range')
```

```python
    plt.legend()
    plt.show()

# Calling the function to plot the efficient frontier with risk range
plot_efficient_frontier(mu, S, risk_min, risk_max, risk_free_rate)
```

11.OBSERVATION:

This method effectively demonstrates how to optimize a stock portfolio to achieve the maximum Sharpe Ratio, considering a specified risk-free rate and risk range.

Optimized Weights: The code calculates the optimal allocation of weights for each stock (AAPL, MSFT, GOOG, and AMZN) to maximize the Sharpe Ratio.

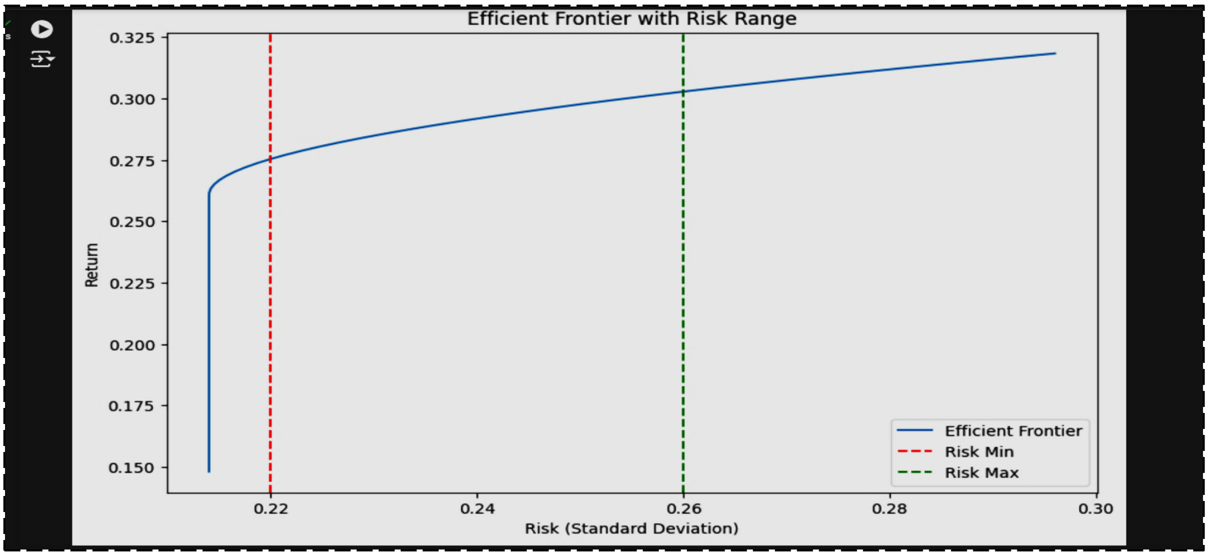
Portfolio Performance: The expected annual return, annual volatility, and Sharpe Ratio are computed and displayed for the optimized portfolio.

Risk Constraints: The code ensures that the portfolio's annualized volatility falls within a specified range. If the initial optimized portfolio does not meet this criterion, the code adjusts the weights to find the best Sharpe Ratio within this risk range.

Efficient Frontier: The code plots the efficient frontier, highlighting the portfolios with the highest return for each level of risk within the specified range.

OUTPUT OF THE PYTHON CODE :

```
Ticker          AAPL      AMZN      GOOG      MSFT
Date
2019-06-30  0.130519  0.066792 -0.020587  0.083118
2019-07-31  0.076394 -0.014179  0.125607  0.017244
2019-08-31 -0.020184 -0.048474 -0.023490  0.011668
2019-09-30  0.072962 -0.022733  0.026008  0.008487
2019-10-31  0.110684  0.023475  0.033724  0.031216
Ticker      AAPL      AMZN      GOOG      MSFT
Ticker
AAPL    0.087622  0.066494  0.042651  0.045562
AMZN    0.066494  0.106729  0.053796  0.048667
GOOG    0.042651  0.053796  0.073583  0.037936
MSFT    0.045562  0.048667  0.037936  0.048076
Portfolio Variance: 0.05663892103106575
Portfolio Volatility: 0.23798932965800326
Portfolio Annual Return: 0.2577040887562785
Expected annual return: 25.77%
Annual volatility/ risk: 23.799999999999997%
Annual variance: 5.66%
Optimized Weights: OrderedDict([('AAPL', 0.30362), ('AMZN', 0.0), ('GOOG', 0.11006), ('MSFT', 0.58632)])
Expected annual return: 27.8%
Annual volatility: 22.2%
Sharpe Ratio: 1.05
Optimized Weights within Risk Range: OrderedDict([('AAPL', 0.30362), ('AMZN', 0.0), ('GOOG', 0.11006), ('MSFT', 0.58632)])
```



12. CONCLUSION:

- The code and the Efficient Frontier graph together provide a comprehensive view of portfolio optimization.
- They help in understanding how to construct an optimal portfolio that maximizes returns for a given level of risk through diversification.
- By visualizing the Efficient Frontier, investors can make informed decisions about asset allocation, balancing their risk tolerance with expected returns, and achieving better long-term investment outcomes.

13. REFERENCES:

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