two_stream_scatter_mod

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This document describes two_stream_scatter_mod, a module that solves the two-stream equations with scattering for a semi-grey atmosphere (i.e. two 'grey bands' are treated: one representing shortwave radiation and the other longwave).

In their most generic form, the two-stream equations with scattering can be written (Pierrehumbert, 2010)

$$\frac{\mathrm{d}}{\mathrm{d}\tau^*}I_+ = -\gamma_1 I_+ + \gamma_2 I_- + \gamma_\mathrm{B}\pi B(T) + \gamma_+ L_\circledast \exp\left(-(\tau^*_\infty - \tau^*)/\cos\zeta\right) \quad (1)$$

$$\frac{\mathrm{d}}{\mathrm{d}\tau^*}I_- = \gamma_1 I_+ - \gamma_2 I_- - \gamma_\mathrm{B}\pi B(T) - \gamma_- L_\circledast \exp\left(-(\tau^*_\infty - \tau^*)/\cos\zeta\right) \quad (2)$$

$$\frac{\mathrm{d}}{\mathrm{d}\tau}I_{-,\mathrm{dir}} = \frac{L_\circledast}{\cos\zeta}I_{-,\mathrm{dir}} \quad (3)$$

where I_+ and I_- are upward and downward diffuse fluxes, and $I_{-,\mathrm{dir}}$ is a downward direct flux. τ is the optical depth, with $\tau^* = \tau/2$, and is defined so that $\tau = 0$ at the surface, and $\tau = \tau_\infty$ at the top-of-atmosphere. B is the Plank function, and $L_\circledast$ is the value of $I_{-,\mathrm{dir}}$ at τ_∞. ζ is the zenith angle. When calculating shortwave fluxes, B is zero. When calculating longwave fluxes, $L_\circledast$ is zero (and consequently all radiation is diffuse).

The optical depth is defined as

$$\frac{\mathrm{d}}{\mathrm{d}p}\tau = \frac{1}{g}\left(\chi_\mathrm{abs} + \chi_\mathrm{sca}\right)$$

where

$$\chi_\mathrm{sca} = a_\mathrm{sca} + b_\mathrm{sca}q + c_\mathrm{sca}\log(\mathrm{CO_2}/\mathrm{CO_{2,ref}})$$
$$\chi_\mathrm{abs} = a_\mathrm{abs} + b_\mathrm{abs}q + c_\mathrm{abs}\log(\mathrm{CO_2}/\mathrm{CO_{2,ref}}).$$

a, b and c are absorption/scattering coefficients (depending on subscript), and are parameters to be chosen by the user. We can define a single scattering albedo,

$$\omega_0 \equiv \frac{\chi_\mathrm{sca}}{\chi_\mathrm{abs} + \chi_\mathrm{sca}}$$

and then we have

$$\gamma_1 = \gamma\left(1 - \hat{g}\omega_0\right) + \gamma'\left(1 - \omega_0\right)$$
$$\gamma_2 = \gamma\left(1 - \hat{g}\omega_0\right) - \gamma'\left(1 - \omega_0\right)$$
$$\gamma_\mathrm{B} = 2\gamma'\left(1 - \omega_0\right)$$
$$\gamma_+ = \frac{1}{2}\omega_0 - \gamma\omega_0\hat{g}\cos\zeta$$
$$\gamma_- = \frac{1}{2}\omega_0 + \gamma\omega_0\hat{g}\cos\zeta$$

where γ=γ'=1 and $\hat{g} = \frac{3}{2}\tilde{g}$ when hemi-isotropic closure is used, with $\tilde{g}$ the asymmetry factor.

The model presented above is grey in the sense that B and $L_\circledast$ are integrated over all frequencies ν, and that a, b and c are independent of ν.

The diffuse equations (1 and 2) can be rewritten in vector form

$$\frac{\mathrm{d}}{\mathrm{d}\tau^*}\mathbf{V} - \mathbf{M}\left(\tau^*\right) \cdot \mathbf{V} = \mathbf{F}\left(\tau^*\right)$$

where V is a vector of the form $[I_{+,0}, I_{-,0}, I_{+,1}, I_{-,1}, I_{+,2}, I_{-,2}, \ldots]$ containing entries for I_- and I_+ at each model half level. Boundary conditions are imposed on the system through modification of the matrix $\mathbf{M}$. This equation can be written in the form $\mathbf{A}\mathbf{x} = \mathbf{b}$, with $\mathbf{A}$ a band diagonal matrix. This system is then solved via LU decomposition to obtain I_- and I_+ at each model half level using an algorithm obtained from: `http://jean-pierre.moreau.pagesperso-orange.fr/f_matrices.html`.

References

Pierrehumbert, R. T., 2010: *Principles of Planetary Climate*.