C4

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1 Parametric Curves

Parametric curves occur when x and y are defined in terms of a parameter, a parameter being a value that is the same in both the functions for x and for y. In a parametric curve C with parameter t;

$$C : x = f(t), y = g(t)$$

One differentiates a parametric eqution using the chain rule. If we were to differentiate y with respect to x by the chain rule, we would find

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{\mathrm{d}y}{\mathrm{d}t}\frac{\mathrm{d}t}{\mathrm{d}x} = \frac{f'(t)}{g'(t)}$$

Integration of parametric curves is done by integrating with respect to t. If one has to integrate with respect to something else, one can tranform it hence;

$$\int_{f(t)=a}^{b} g(t)\mathrm{d}f(t) = \int_{t=f^{-1}(a)}^{b} g(t)f'(t)\mathrm{d}t$$

This is then isomorphic to integration of non-parametric curves.

2 Implicit Differentiation

$$\frac{\mathrm{d}}{\mathrm{d}x}xy = \frac{\mathrm{d}y}{\mathrm{d}x}x + y$$
$$\frac{\mathrm{d}}{\mathrm{d}x}y^n = \frac{\mathrm{d}y}{\mathrm{d}x}ny^{n-1}$$

3 Forming Differential Equations

Differential equations describe scenarios in which;

$$\frac{\partial y}{\partial x} \propto y$$

Newton's law of cooling states that the rate of loss of temperature $-\frac{\partial \theta}{\partial t}$ is proportional to the the difference between the temperature θ of the body and the temperature θ_0 of its surroundings;

$$\frac{\partial \theta}{\partial t} = -k(\theta - \theta_0)$$

4 Integrating using Trigonometric Identites

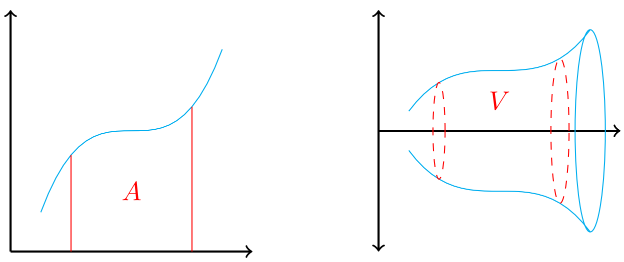
4.1 Examples

$$\int \tan^2 x \,\mathrm{d}x = \int \left(\sec^2 x - 1\right) \,\mathrm{d}x = \tan x - x + \mathit{const.}$$

$$\int \sin^2 x \, \mathrm{d}x = \frac{1}{2} \int [1 - \cos(2x)] \, \mathrm{d}x = \frac{1}{2}x - \frac{1}{4}\sin(2x) + \mathit{const.}$$

$$\int \sin(3x)\cos(3x) = \frac{1}{2} \int \sin(6x) \, \mathrm{d}x = -\frac{1}{12}\cos(6x) + \mathit{const.}$$

5 Volumes of Revolution



To calculate the volume if $y = f(x)$ has been revolved around the x-axis, we can model the volume of that as a cylinder with radius that varies with y.
πy^2 would be the 'surface area' of a given cross section. Therefore, the following would give you the volume between a point a and b:

$$\int_a^b \pi y^2 dx \quad \rightarrow \quad \pi \int_a^b y^2 dx$$