

C3

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1 Partial Fractions

1.1 Splitting a Fraction with Two or More Linear Factors in the Denominator

$$\frac{x+3}{(x+2)(x+1)} = \frac{a}{x+2} + \frac{b}{x+1} = \frac{a(x+1) + b(x+2)}{(x+2)(x+1)}$$
$$\therefore x+3 = a(x+1) + b(x+2)$$

Here, two methods can be used; equating coefficients and substitution. Equating coefficients is rather self explanatory, and involves creating simultaneous equations from the fact that coefficients of different powers of x will be equal on both the left- and right-hand-side of the above equation, then solving for A and B;

$$\text{coefficients of } x: 1 = a + b$$

$$\text{constants: } 3 = a + 2b$$

The former rearranges to

$$b = 1 - a$$

Substituting this into the latter shows

$$3 = a + 2(1 - a) = 2 - a \therefore a = -1 \Rightarrow b = 2$$

Hence,

$$\frac{x+3}{(x+2)(x+1)} = \frac{2}{x+1} - \frac{1}{x+2}$$

Substitution involves substituting values for x which neglect one of the unknowns in our equation. In the above example, one would substitute the values -1 and -2 to neglect the terms containing a and b respectively;

$$x \rightarrow -1, b = 2$$

$$x \rightarrow -2, -a = 1 \therefore a = -1$$

And once again, we arrive at the same partial fractions,

$$\frac{x+3}{(x+2)(x+1)} = \frac{2}{x+1} - \frac{1}{x+2}$$

1.2 Splitting a Fraction with a Squared Linear Factor in the Denominator

$$\frac{a}{()^2} = \frac{a}{()} + \frac{b}{()^2} = \frac{a() + b}{()^2}$$
$$= a() + b$$

2 Periodic Functions

$$a \cos \theta + b \sin \theta = R \cos(\theta + \alpha) = R' \sin(\theta + \alpha')$$

$$a = R \cos \alpha = R' \sin \alpha, b = -R \sin \alpha = R' \cos \alpha$$

Example:

$$5 \cos 100\pi x + 10 \sin 100\pi x$$

$$5 = R \cos \alpha, 10 = -R \sin \alpha$$

$$-\frac{10}{5} = \tan \alpha$$

$$\alpha = \arctan -\frac{10}{5}$$