PH4

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November 11, 2015

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1 Simple Harmonic Motion

In simple harmonic motion, acceleration is proportional to displacement and opposite in sign;

$$x = x_m \cos(\omega t + \phi)$$
$$\frac{\partial x}{\partial t} = -\omega x_m \sin(\omega t + \phi)$$
$$\frac{\partial^2 x}{\partial t^2} = -\omega^2 x_m \cos(\omega t + \phi)$$

An example of simple harmonic motion is a mass on a spring.

$$T = 2\pi\sqrt{\frac{m}{k}}$$

- m is mass
- k is spring constant

Pendula aren't but if they are long and the angle is small they pretty much are.

$$T = 2\pi\sqrt{\frac{L}{g}}$$

- L is length

1.1 Uniform Circular Motion

The projection of a point moving in uniform circular motion on a diameter of the circle in which the motion occurs executes SHM.

1.2 Relationships between Velocity, Acceleration & Displacement

1.3 Damped Oscillations

The energy of an oscillating system is the only factor directly related to the amplitude of the oscillation. If energy is removed from the system over time, such as by friction, the amplitude of the oscillation decreases over time. This is called **damped harmonic motion**. In damped harmonic motion, frequency and period stay the same, as they are not related to amplitude. In most cases, the damping is caused by an external force which does work in the opposite direction to velocity. Damped systems obey the following;

$$x(t) = x_m e^{-\frac{bt}{2m}} \cos\left(t\sqrt{\left(1 - \frac{b}{2m}\right)^2} + \phi\right)$$

1.4 Driven Oscillations

If energy is put into an oscillating system over time, the system is undergoing **driven oscillation**. Every system has a **natural frequecy** ω_d at which it oscillates if the oscillation is driven at this natural frequency, the driving **resonates** with the oscillation, and things get a bit out of hand.

$$x_m = \frac{F_0}{\sqrt{m^2\left(\omega^2 - \omega_d^2\right)^2 + b^2\omega_d^2}}$$

2 Momentum