Economics Department
Discussion Papers Series

FIVE-EQUATION MACROECONOMICS A SIMPLE VIEW OF THE INTERACTIONS BETWEEN FISCAL POLICY AND MONETARY POLICY

Tatiana Kirsanova, Sven Jari Stehn and David Vines

Paper number 06/10

FIVE-EQUATION MACROECONOMICS
A SIMPLE VIEW OF THE INTERACTIONS BETWEEN FISCAL POLICY
AND MONETARY POLICY[*]

Tatiana Kirsanova

University of Exeter

Sven Jari Stehn

Brasenose College,

University of Oxford

David Vines

Economics Department and Balliol College, University of Oxford;

Australian National University; and CEPR

8 January 2006

Abstract

This paper studies the interactions of fiscal and monetary policy when they stabilise a single economy against shocks in a dynamic setting. We assume that fiscal and monetary policies both stabilise the economy only by causing changes to aggregate demand. Our findings are as follows. If the both policymakers are benevolent, then the best outcome is achieved when the fiscal authority allows monetary policy to perform nearly all of the burden of stabilising the economy. If the monetary authorities are benevolent, but the fiscal authorities have distorted objectives, then a Nash equilibrium will result in large welfare losses: unilateral efforts by each authority to stabilise the economy will result in a rapid accumulation of public debt. However, if the monetary authorities are benevolent and the fiscal authorities have distorted objectives, but there is a regime of fiscal leadership, then the outcome will be very nearly as good as it is in the regime in which both policymakers are benevolent.

Key words: Monetary Policy, Fiscal Policy, Macroeconomic Stabilisation
JEL codes: E52, E61, E63

1   INTRODUCTION

1.1   From Three-Equation Macroeconomics to a Five-Equation Setup

Thinking about macroeconomic policy has been transformed in the past ten years. Nearly all of us now analyse short-run macroeconomics using a simple *three-equation system*. This system contains an IS curve, a Phillips curve, and something like a Taylor-type rule for monetary policy. It was explained in a paper by Svensson (1997) and Ball (1999)[1], and was applied to the UK in a simple, clear, paper by Bean (1998). This setup contains no fiscal policy.

In this paper we introduce endogenous fiscal policy. We do this by adding a Taylor-type rule for fiscal policy, and also adding an equation which tracks the evolution of public debt. We show that one can use the resulting *five-equation system* to analyse the interactions of monetary policy and fiscal policy. For expositional purposes we set up our model in a backward-looking manner, which makes it particularly simple.[2] We believe that the three results established in the present paper will also be true in a "best-practice", fully micro-founded model, in which there are forward-looking elements.

In what follows we first consider the interaction of *simple rules* for both monetary policy and fiscal policy in this five-equation system. We then turn to investigate the games played between *optimising* monetary and fiscal policymakers.

Our whole investigation is driven by the following single, but fundamental, observation. The experience of best-practice Central-Bank independence teaches that we can safely assume that the monetary authority is farsighted and does not aim for excess output. By contrast, the lack of delegation of fiscal policy to an independent agent means that, as a result of political pressure, fiscal policymakers are likely to (i) discount the future too much, and/or (ii) aim for excess output.

This paper establishes that, under these circumstances, the following three things are true.

First, the best outcome is achieved if the fiscal authorities and monetary authorities are benevolent, and cooperate with each other in the setting of their macroeconomic policy instruments. We show that, in this case, the fiscal authorities will allow monetary policy to perform nearly all of the burden of stabilising the economy in the face of shocks.

Second, we show that very nearly exactly this same good outcome will occur if the fiscal authority is able to act as a Stackelberg leader, even if the fiscal authority discounts the future too much, and/or desires to aim for excess output. This is because, roughly speaking, fiscal leadership will mean that the fiscal authority will acknowledge that the monetary authority will be able to prevent its fiscal policy from having "stupid" effects. As a result it will decide not to be stupid, even if (i) and/or (ii) are true. Hughes Hallett (2005) concludes that for the UK, the evidence is

---

[1]   The classic paper is that by Clarida, Gali, and Gertler (1999) , which contains a forward-looking Phillips curve, unlike the model in present paper.

[2]   See Allsopp and Vines (2000), Carlin and Soskice (2005), Romer (2000), and Taylor (2000), for other user-friendly expositions of this setup.

supportive of this "Stackelberg interpretation of the Treasury's strategy". This is why we support Gordon Brown's fiscal policy framework for the UK.

Third, we show that if the fiscal authority plays Nash, then social welfare will be harmed, and possibly greatly harmed, if the fiscal authority discounts the future too much, and/or desires to aim for excess output. This is because, if the fiscal authority plays Nash, then it does not acknowledge that the monetary authority will be able to prevent its fiscal policy from having stupid effects. It will thus decide to play "stupid" if either it discounts the future or it aims for excess output. But the monetary authority will respond and will overturn its play. We show that, in response to an inflation shock, this can lead to monetary contraction, and then redoubled fiscal effects to expand; it will thus lead to fights between the fiscal authority and the monetary authority, fights which are possibly very costly. It is clearly the case that, in Europe at present, the twelve fiscal authorities cannot play the role of Stackelberg leader vis-à-vis the ECB. Hughes Hallett (2005) concludes that for the EMU, the evidence suggests "weak monetary leadership - or more likely, a straightforward non-cooperative game". That is why we are critical of the current arrangements for fiscal and monetary policy within EMU.

1.2   Plan of this paper

We proceed as follows. Section 2 sets out what we need to know about three-equation Taylor-rule macroeconomics. Section 3 shows how we can build on this, to create a five-equation system, and studies the effects of simple policy rules in such a system. We use this setup in Section 4 to study interactions between optimising fiscal and monetary authorities, and present simulations of policy games played between them. It is these simulations which establish our three key results.

Clearly, our aim in this paper is to discuss fiscal and monetary policy interactions, using dynamic models which build on three-equation Taylor-rule macroeconomics. By contrast with this, the existing literature on monetary and fiscal interactions concentrates almost entirely on static models. It is most helpful to describe those models *after* we have described the outcomes from our own dynamic models (rather than describing the work of others on fiscal-monetary interactions at the beginning of the paper, as most authors would do). This way we are able to compare the *outcomes* which can be obtained from studying these two, rather different, sorts of models. Section 5 presents this comparison of outcomes.

In that section we first compare our findings with those of a widely-cited paper by Dixit and Lambertini (2003). We show that this paper points directly towards the three results of our paper, although that is not how the authors themselves describe their findings. We also discuss an early paper by Blake and Weale (1998), and show that those authors anticipate our third result. In addition we mention papers by Adam and Billi (2005) and Hughes Hallet (2005). In both of these papers there is a discussion of the effects of public deficits on fiscal and monetary interactions, effects which are critical for our own results.

Section 6 summarises the findings of the paper.

2　THREE-EQUATION TAYLOR-RULE MACROECONOMICS

In this section of the paper we set out the three-equation Taylor-rule system - in which there is an IS curve, a Phillips curve, a Taylor rule for monetary policy, and no active fiscal policy. We also explain some ideas about dynamic control, since these will be useful in later sections of the paper. For the definition of variables, see Appendix A.

2.1　The Dynamic Three-Equation Model

The first equation is an IS curve, showing the evolution of the output gap (y_t), driven by the real interest rate r_t :

$$y_t = \kappa y_{t-1} - \sigma r_{t-1} + \varepsilon_t \tag{1}$$

where ε_t is a demand shock. As discussed by Woodford (2003), an equation like this can be obtained from the optimising behaviour of individuals who choose consumption, given a budget constraint.

The second equation is an accelerationist Phillips Curve. This describes the dynamics of inflation (π_t) in terms of past inflation and the output gap:

$$\pi_t = \pi_{t-1} + \omega y_{t-1} + \upsilon_t \tag{2}$$

where υ_t is an inflation shock.

In these two equations, the *real* interest rate is taken to be the instrument of monetary policy, and it affects output with a lag of one period. It then takes output another period to affect inflation. Following Bean (1998) we assume that there is inertia, or "persistence", in output[3] as well as in the inflation process.[4] Despite its simplicity this model has been widely used to understand the basic mechanisms of monetary policy. In what follows we will use the calibration discussed in Appendix A which leads to the following values for parameters: $\kappa = 0.5$, $\sigma = 1.0$ and $\omega = 0.1$. As discussed in the appendix, these parameters are conventional in the literature.

The dynamic structure of the model means that some aspects of policy must be thought about intertemporally. In particular, the accelerationist nature of the Phillips curve makes the control of

---

[3]　Bean included a lagged output term in the IS curve for empirical reasons. A micro-founded approach might motivate such a lagged term in the IS curve by appealing to habit persistence in consumption.

[4]　We shall, see below that the coefficient of unity on lagged inflation in the (accelerationist) Phillips curve gives that equation a unit-root process, which is vital in what follows. It is possible that the IS curve could be written in the same way, if consumption were assumed to follow a forward-looking Euler equation. But that amendment would not affect the analysis which follows in the present paper.

inflation shocks (υ_t) a dynamic problem of sacrifice. There is a trade-off between the pain of a recession in the present and the benefit of disinflation in the future. But the pain is temporary: the equilibrium level of output is independent of the position adopted along this trade-off.

By contrast, in this system, other aspects of policy do not need to be thought about intertemporally. The control of demand shocks (ε_t) is not a dynamic problem. Once the policymaker sees the effects of a demand shock, and identifies that they are indeed the consequences of a demand shock, then the policy-maker should cut the interest rate in order to remove the effects of, say, a negative demand shock on the economy. In the control of such demand shocks, there is not a dynamic trade off between current sacrifice and future benefit, although, it does take one period to remove such shocks.

2.2   A "Satisfactory" Taylor Rule in the Three Equation Model

Taylor (1993) famously demonstrated that actual US monetary policy could be well described by a simple rule that relates the real interest rate[5] to inflation and to output, with parameters θ_π and θ_y respectively:

$$r_t = \theta_\pi \pi_t + \theta_y y_t \tag{3}$$

The first term in the Taylor rule shows that if inflation rises, then the real interest rate will be raised to weaken demand, which will reduce inflation. The second term shows that the real interest rate is raised if output rises.

We now describe what happens if a Taylor rule of the form (3) is applied to the system consisting of equations (1) and (2), in response to impulse shocks to inflation and to demand, using the parameters $\theta_\pi = 1.1$ and $\theta_y = 0.6$

We can see that, because of the persistence of inflation, it takes time to get inflation down after a shock, during which time interest rates must remain above base. It also takes some time to remove the effects of a demand shock, because of the lag in the effect of the interest rate in the IS curve and the persistence of output, and also because a transient output shock causes inflation, which then takes time to remove.

---

[5]   Taylor stated his rule as a rule for the *nominal* interest rate, but it is helpful to translate it into a rule for the *real i*nterest rate. This is possible because the real interest rate is here *defined* as the nominal interest rate minus the current inflation rate, which the monetary authority is able to observe.
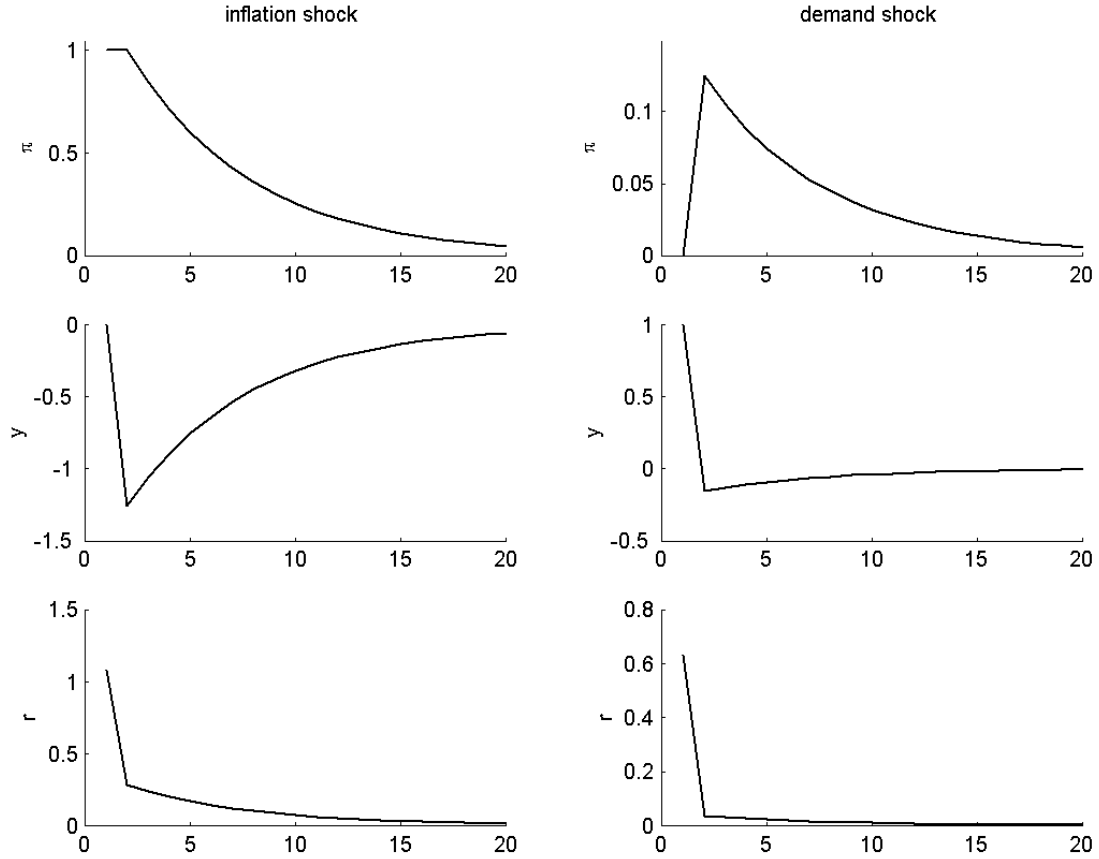
Figure 1: Impulse Responses

Different values of θ_π and θ_y give different outcomes. If $\theta_y = 0$, then we clearly require $\theta_\pi > 0$ for stability[6]. But there may be problems if θ_π is large. If inflation is above target at t=0, then a large value for θ_π will cause a large interest rate increase and so a large reduction in output, but, because of the lag in the IS curve, not until t=1. The output lag in the Phillips curve means that this will not cause a reduction in inflation until t=2. This lagged response means that inflation may overshoots below target, and then converge in a cyclical manner. That is, technically, the eigenvalues of the system under control of such rule can become complex[7]. A positive value for θ_y can prevent such overshooting, and so can ensure an outcome like that shown in Figure 1. Roughly speaking, this extra term is helpful because output is a predictor of future inflation; it causes the policymaker to lower the real interest rate when output has fallen, without waiting for the reduction in inflation which the fall in output will cause. [8] Figure 2 shows – to the right of the dotted line – combinations of θ_π and θ_y such that inflation control is both stable and non-cyclical. Figure 2 also shows that the value of $\theta_\pi = 1.1$, which was used to compute the

---

[6] The principle "if there is a sustained increase in inflation by k%, then the nominal interest rate should rise by more than k% (*i.e.* the real rate of interest should rise)" has become known as Taylor principle. A similar interest-rate rule was proposed long ago by Wicksell (1907).

[7] It is possible to show formally that cycles can arise, even without output persistence, *i.e.* even if $\kappa = 0$. It is also possible to show that, with $\kappa > 0$, the system can even become unstable, for values of θ_π larger than those shown in this figure.

[8] Allsopp and Vines (2000) discuss this issue in detail and provide a diagrammatic analysis.

outcomes in Figure 1, would produce cyclical outcomes unless θ_y is greater than approximately 0.3.

To summarise, a Taylor rule, with coefficients corresponding to a point to the right of the dotted line in Figure 2, will control our simple model economy in what we will call a *satisfactory* manner. By this we simply mean that behaviour of the system under control is both stable and non-cyclical.
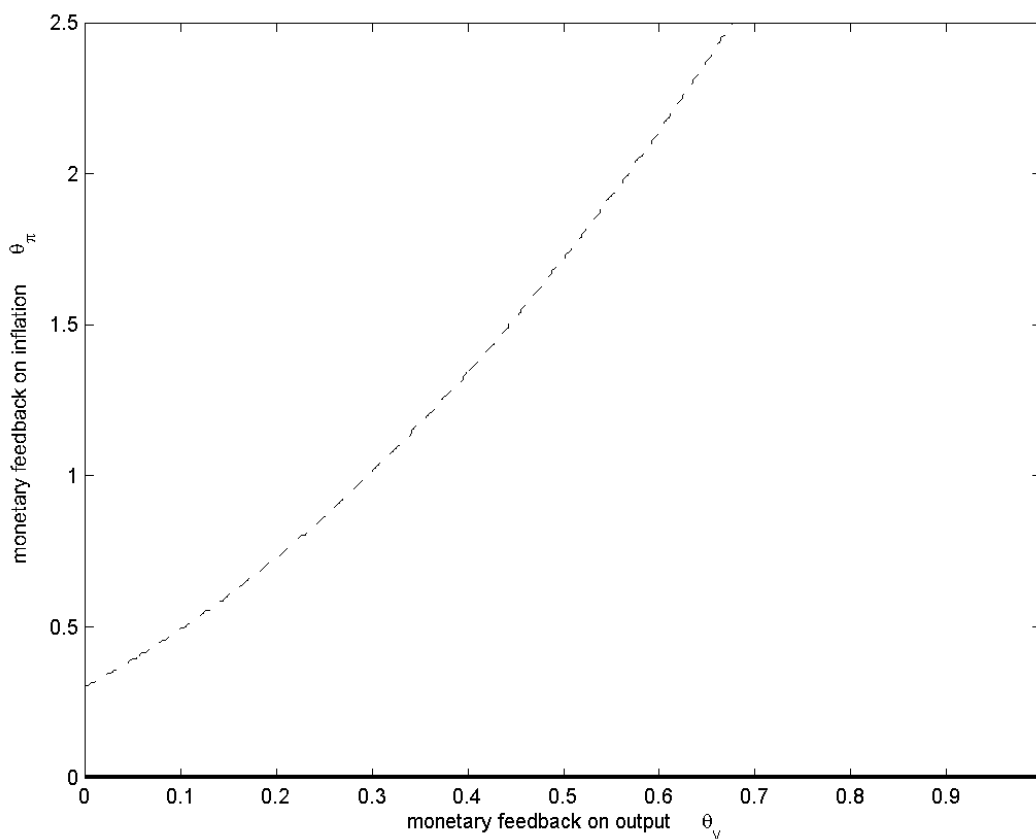

Figure 2: Cyclicality boundary for Monetary Policy

2.3 Optimal Monetary Policy in the Three Equation Model

But how would we control this simple economy in an *optimal* manner?

Let the preferences of the monetary policymaker be[9].

$$L = \mathcal{E}_o \frac{1}{2} \sum_{t=0}^{\infty} \beta^t \left(\pi_t^2 + \alpha \left(y_t - \overline{y} \right)^2 \right) \tag{4}$$

---

[9] This simple loss function is conventional. A formula similar to this, but not identical, can be derived from the utility function of a representative agent, reflecting the fact that, deep down, inflation matters because it distorts consumption and labour supply decisions across time, and so has quite complicated effects on the agent's utility. See Woodford (2003), Steinsson (2003); Leith and Wren-Lewis (2005) discuss this issue in some detail.

where $\mathcal{E}_o$ denotes expectations conditional on information available at time zero. Every period, the loss function penalises deviations of inflation from its target (here zero for simplicity), and of output from its target, $\bar{y}$, where $\bar{y}$ denotes the extent to which the output target is in excess of its potential (natural) level. The parameter α denotes the relative weight given to deviations of output from target; we use the conventional value of 0.5. (See for example Aizenman and Frenkel, 1985.) Future losses are discounted at the rate β. When there is no excessive discounting, and no aiming for excess output, β=0.99 and $\bar{y}$ =0.

Optimal monetary policy can be found by minimising loss function (4), subject to equations (1) and (2). Notice that the model represented by equations (1) and (2) contains two, and only two, lagged states, namely π_{t-1} and y_{t-1} . Using a standard theorem in control theory (see Chow, 1975), the optimal controller for such a system will be a feedback on both of these lagged states. But, because of the impact-lag of monetary policy, this is exactly what is achieved by the Taylor rule. There are thus values of θ_π and θ_y such that the Taylor rule (3), which has these parameters, is exactly the solution to the minimisation of (4), subject to (1) and (2). That Taylor rule will describe optimal monetary policy. The parameters θ_π and θ_y for the optimal rule will clearly depend on the parameters of the model, κ, σ, and ω. They will also depend on the parameter α which describes the preferences of the policymaker.[10] The values for θ_π and θ_y which were used to compute the outcomes shown in Figure 1 are in fact the optimal ones, for the values of κ, σ, ω, α and β that we have chosen. Bean (1998) demonstrates that θ_π and θ_y will both be higher, the lower is α. Despite this, Bean also shows, strikingly, that the variability of inflation and variability of output, and so the welfare outcomes, will not be greatly influenced by α. [11]

The optimal solution will deliver *fast* convergence to equilibrium (so that the terms in the infinite sum (4) are rapidly damped). Formally, the speed of convergence can be described by the absolute value of the biggest stable eigenvalue of the system under control. However, the optimal solution cannot converge *too* fast, as this might require too much sacrifice of output in initial periods after the beginning of stabilisation. The weight α in the loss function (4) defines the preferences of the policymaker on this matter. The optimal solution is also likely to have no cycles, because cycles increase the variability of economic variables, which would be penalised by the loss function (4). It is of course possible, that a *fast*-converging solution could nevertheless also have complex eigenvalues, and so exhibit rapidly converging, but cyclical, behaviour. But that turns out not to be the case in the present, simple, model. For this model the optimal controller is a member of the (large) set of what we have called "satisfactory" controllers. We can see, in Figure 2, that our optimal values of θ_π = 1.1 and θ_y = 0.6 correspond to a point to the right of cyclicality boundary.

---

[10]   Bean (1998) shows how to compute this optimal control rule analytically for the case in which *β=1*, and Henry *et. al.* (2006) show how to do this when β is less than unity.
[11]   See Henry *et. al* (2006) for a discussion of the reasons for this.

2.4 Solving the Problems of an Excess Output Target and of Discounting

Finally, we describe two important difficulties which must be dealt with, if there is to be a good monetary policy.

First, it is necessary that monetary policy solves the problem of inflation bias, a problem, which stems from having an excessive output target, $\bar{y}$, and which was first described by Kydland and Prescott (1977) and Barro and Gordon (1983). In the backward-looking model being used here, if inflation is initially low, then, if $\bar{y}>0$, the policymaker has an incentive to increase output above zero, and will do this until inflation has risen to such a high level that y>0 is no longer attractive. That incentive thus causes an inflation bias[12].

Second, it is necessary that the policy-maker does not discount the future excessively, i.e. it is necessary that β be close to, or equal to, unity. For any given α, the more the policymaker discounts the future (i.e. the smaller is β) the more she will seek to postpone the recession which is necessary to deal with an inflation shock. That is because such delay will cause the recession to happen at a time in the future when she cares less about it. As a result, although discounting will not cause a permanent inflation bias, it will cause disinflation to happen too slowly. [13]

The first-best solution to these problems is to delegate monetary policy-making to a "responsible" central banker, who will not target an excessive level of output and who will not discount the future.[14] Both John Vickers in the UK and Alan Blinder in the US have argued that the success of the MPC in the UK, and of the Federal Reserve Board in the US, is due to the fact that the monetary policy-makers in the UK and the US are of this kind.[15] (See Vickers, 1998, and Blinder, 1997, 1998.) In the UK, the MPC process has ensured that members of the committee are of the kind who do not target excess output or discount the future, partly because they have a professional reputation to defend, and partly because an inflation forecast is published along with the minutes of MPC meetings, in which there is a clear description of the policies which are required to get inflation back onto target, and of how and when they will be adopted. It is important for the feasibility of this "solution" that the time scale for monetary policy, in which inflation is brought under control at an optimal speed, is likely to be short. Practical experience suggests that we are talking about periods in the range one to three years. (See Figure 1, in which inflation is halved within 8 quarters of the occurrence of an inflation shock.)

---

[12] Barro and Gordon (1983) originally presented this story in a forward-looking but static model. See Clarida, Gali and Gertler (1999) for an alternative exposition.

[13] The literature has focussed on the "excess-output-target" problem, and very little has been written about the problem caused by excessive discounting by monetary policy-makers. But for such an argument, see Henry et al (2006).

[14] In a setup in which, unlike in (2), the Phillips curve is partly or wholly forward-looking, it will be important to be able to *trust* that the central banker does not target excess output and does not discount the future. That trust will enable the banker to be able to *promise* not to do either of these things, and the private sector will be able to know that there is not a *time-inconsistency* problem, in which the central banker promises to do one thing and then does another. That will make the economy easier to control. Because the model in this paper is backward-looking, these (important) issues cannot arise here.

[15] And that they are *trusted* to be of this kind.

It is possible that such a responsible central banker cannot be found i.e. it is possible that the act of delegation of monetary policy to an independent central bank is not, in itself, sufficient to solve the inflation-bias problem. The literature has attempted to find various mechanisms that can try to achieve second-best or third-best outcomes in these circumstances. These include appointing a Rogoff (1985) Conservative Central Banker, who has more inflation aversion than society (*i.e.* a lower α). An alternative, analysed by Svensson (1997), is to give the central bank an inflation target which somehow exactly offsets this inflation bias. It appears that the design of the European Central Bank was strongly influenced by the belief that this issue would remain a difficulty within EMU.

3   FIVE-EQUATION MACROECONOMICS-WITH-FISCAL-POLICY

3.1   The Dynamic Five-Equation Model

We now add fiscal policy to the model, by adding a description of the behaviour of the fiscal policy authority, and also an equation showing the evolution of public debt. For a discussion of how variables are defined see, again, Appendix A. The model presented here is analysed in more detail in Stehn (2006).

The first equation is, as before, a dynamic IS curve:

$$y_t = \kappa y_{t-1} - \sigma r_{t-1} + \phi b_t + \delta g_t + \varepsilon_t \tag{5}$$

where, as above, ε_t is a demand shock. As in equation (1), monetary policy sets the interest rate, which affects output with a lag. "Fiscal policy" will be taken to mean changes in government expenditure, g_t, not changes in tax rates. [16] If individuals have finite lives, then both government spending and the level of government debt will matter in the IS curve. (See Blanchard, 1985, and Yaari, 1965.) We can see that an increase in government spending, g, then has two effects. Firstly, it raises output directly via aggregate demand, with multiplier δ, which we set at 1.1. Secondly, this expenditure will lead it to issue extra public debt and – with Blanchard-Yaari consumers - a fraction of this will be treated as net wealth. As a result consumption, and thus output, will rise further, by an amount denoted by the parameter φ, which we set at 0.01, for reasons discussed in Appendix A. In order to avoid giving fiscal policy a "stealth advantage" in our investigations, we will assume that fiscal spending decisions need to be taken one period in advance of their actual implementation. We will therefore have a one-period "implementation lag" in the operation of fiscal policy, and a one-period "effect lag", as described above, in the operation of monetary policy.

The second equation is, as before, a standard accelerationist Phillips curve:

---

[16]   We could treat the tax rate, *τ*, as the fiscal instrument, instead of spending. We study spending because we want to study the effect of fiscal policy on demand. If the tax rate was the instrucment then this would have a direct influence on inflation that would complicate the analysis. We discuss this alternative approach in Section 5.

$$\pi_t = \pi_{t-1} + \omega y_{t-1} + \upsilon_t \tag{6}$$

Notice that in our Five Equation Model (i) both fiscal policy and monetary policy affect the IS curve, and (ii) neither policy influences inflation, other than through an indirect effect via output. This means that, in the control of inflation and output, the two instruments are perfect substitutes. That feature will be important in what follows.

We also need an equation which describes the accumulation of public debt. For simplicity we log-linearise the debt equation around the steady states of debt, b_0, set at 0.6, and the equilibrium interest rate, r_0, which we set at 0.011. [17] The real stock of debt at the beginning of this period (b_t) depends on the stock of debt at the beginning of last period, b_{t-1}, plus the flows that occur between t-1 and t, in the following way:

$$b_t = (1 + r_0) b_{t-1} + r_{t-1} b_0 + g_{t-1} - \tau y_{t-1} + \eta_t \tag{7}$$

where η_t is a debt shock.

The relevant flows consist of real interest payments, government spending and revenues. Tax revenues are assumed to vary with output, in a way which gives rise to 'automatic stabilisers', at a constant tax rate τ. Notice that we would return to the Three Equation Model if and only if (i) government expenditure was exogenous, so that we could include any changes in government spending in the (exogenous) demand shock, ε_t, and (ii) we could impose Ricardian Equivalence, by setting φ=0, and (iii) there were no other effects of debt accumulation. That last requirement would effectively mean that endogenous accumulation of debt did not induce changes in government expenditure or the interest rate, so as, for example, to avoid fiscal insolvency.

The Five Equation Model is completed by adding two equations showing the behaviour of monetary policy and fiscal policy to the three equations (5), (6) and (7).

3.2   Simple Policy Rules

How can we achieve *satisfactory* policy when fiscal policy and monetary policy follow simple rules, like the Taylor rule for monetary policy which we described in Section 2?

A Satisfactory Fiscal Rule when the Monetary Authorities follow a Taylor Rule

In Section 2, we learned that a satisfactory monetary policy for our simple model could be specified in the form of a Taylor rule, and that an *optimal* monetary policy for this model would also take the form of a Taylor rule, with particular (optimised) coefficients. Suppose the monetary

---

[17]   See Appendix A. Note that since the model is quarterly, this corresponds to an annual real interest rate of just over four percent.

authorities followed such a Taylor rule.[18] What would a fiscal policy rule need to look like, in these circumstances, in order to be *satisfactory*?

Let us assume that the fiscal authorities choose a simple rule of (8) form.

$$g_t = -\phi y_{t-1} - \mu b_{t-1} \tag{8}$$

In this simple setup, fiscal policy feeds back on the level of debt and also helps the monetary authority to stabilise output, leaving the monetary authority to stabilise inflation.

After substituting in these two policy rules we will have a fully complete Five Equation Model. For simplicity we take θ_π and θ_y to have the optimal values discussed in Section 2. We can then investigate the behaviour of the system for different values and ϕ and μ. The result of this stability analysis is summarised in Figure 3.
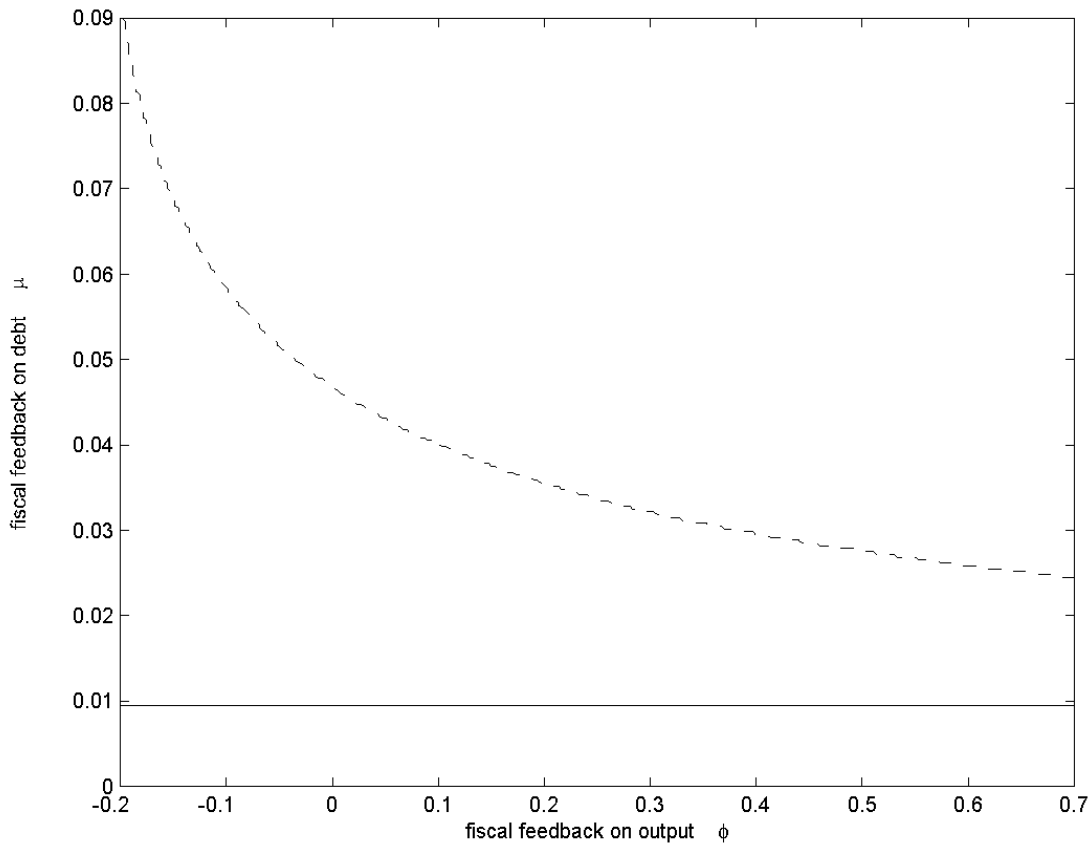

Figure 3: Stability and Cyclicality Boundaries for Fiscal Policy

---

[18] An attentive reader might think that the class of Taylor rules could no longer contain the optimal rule. "This is because," that reader might claim (using the argument of Chow (1975) which we appealed to in section 2.3) "with the lagged value of debt now appearing as an additional state variable, the system is now third order: thus any optimal monetary rule must be third order, feeding back on debt as well as inflation and output." But we will see below that the optimal coefficient, with which monetary policy should feed back on debt, is very close to *zero*. So the Taylor rule derived above in our analysis of the Three Equation system is a good approximation to the *optimal* policy rule for the model being analysed here.

To ensure stability, the economy must be *above* the solid line, which traces out the stability boundary for the fiscal rule. Below this line, the feedback coefficient on debt is too small, so that debt interest payments will cumulate without bound, leading to system instability.

The dashed line is a cyclicality boundary for the fiscal rule; it shows that cycles will result if μ is *above* this line. To understand how these cycles can arise, suppose for simplicity that $\phi = 0$, and consider a positive inflation shock at time t=0. This will raise the interest rate (via the Taylor rule). That causes output to fall at t=1 and so inflation will fall at t=2, as in the Three Equation system. But the interest rate increase will also cause debt to increase, at t=1, and the output fall will reduce tax revenues and make debt increase even more, at t=2. Because fiscal policy feeds back on this rising debt, there will be fiscal contraction and that will causes further falls in output at t=2 and t=3, and subsequently, and so inflation will go on falling at t=3 and t=4, and beyond then. If this feedback on debt is too strong, that will make inflation more likely to overshoot than it was in the Three Equation Model, and the economy will be more likely to exhibit cycles[19]. As we discussed above for the Three Equation Model, such cycles are likely to be welfare-reducing, since an adjustment to equilibrium, which avoided such cycles, would probably lead to smaller divergences of inflation and output from their target levels.[20] It is obvious that this tendency towards cycles is enhanced if there is also substantial fiscal feedback on output – that is why the dotted line in Figure 3 slopes downward. Even although, in that case, fiscal policy tries to moderate the effects on output of monetary control of inflation, it does so in such a way as to induce cumulative movements in debt which, if there is strong feedback on debt, will cause a cyclical outcome.

Figure 4 plots impulse responses to a unit inflation shock for two sets of fiscal feedback coefficients. The solid line plots the dynamic response of the economy is there is a small feedback on debt ($\mu = 0.03$), and if there is no fiscal feedback on output ($\phi = 0$). These parameters ensure stability but do not lead to cycles. The dashed line corresponds to the case of larger fiscal feedback ($\mu = 0. 15$) but still with no feedback on output; we can see that both output and inflation overshoot in this case. We could also illustrate the case with this larger fiscal feedback on debt and also with a large response to output (say with $\phi = 0.4$). As we might expect from Figure 3, a system with such a fiscal rule cycles even more severely.

---

[19]   We can put this point more formally as follows. Consider a Three Equation Taylor rule system with values of the Taylor rule coefficients such that it is "satisfactory", *i.e.* such that it does not cycle. Now add equations (7) and (8) to that system. Then, in general, a sufficiently large μ can be found such that the five-equation system will have two complex eigenvalues, and so will be cyclical.

[20]   The reason for these cycles in the present model is the persistence of inflation. Note, however, that even for entirely forward looking models, welfare is maximised for small fiscal feedback, see Kirsanova and Wren-Lewis (2005).
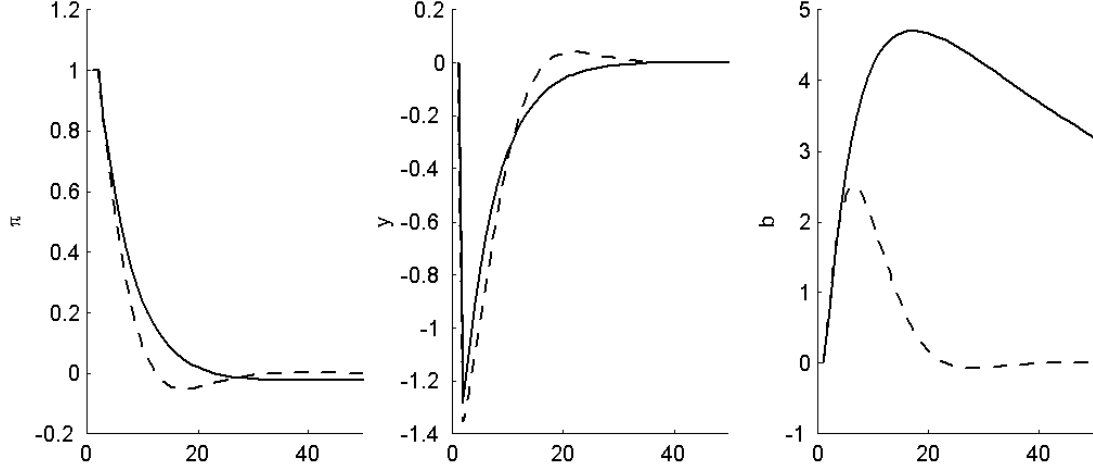
Figure 4: Impulse Responses to Inflation Shock

This analysis suggests that if we know that monetary policy stabilises the economy well, then a good fiscal policy should feed back on debt with a small coefficient, and with a small response to output, if any. This lack of response to output is sensible because output is well controlled by monetary policy alone. We will show, later in Section 4 , that a fiscal policy very like this is not only satisfactory, but also optimal.

Optimal Monetary Policy in Face of a Simple Fiscal Policy Rule

Let us describe what optimal monetary policy looks like when fiscal policy is constrained to follow a simple feedback on the level of debt, in the manner suggested in the previous paragraph:

$$g_t = -\mu b_{t-1} \qquad\qquad (8')$$

Notice that, in this case, the automatic stabilisers mean that debt accumulation will also depend negatively on the level of output, as shown in equation (7).

Now we substitute the fiscal rule (8') into the system and solve for optimal monetary policy, which minimises the social loss function, *L,* given in equation (4). We use the same value for α that we used above, α=0.5. Subject to this constraint, optimal monetary policy will take the form of a rule in which there is a feedback on the three lagged state variables, π_{t-1}, y_{t-1}, and b_{t-1} of the system [21]. These optimal feedback coefficients can be written as θ_π, θ_y θ_b and are plotted in Figure 5 as a function of the strength of fiscal feedback, μ.
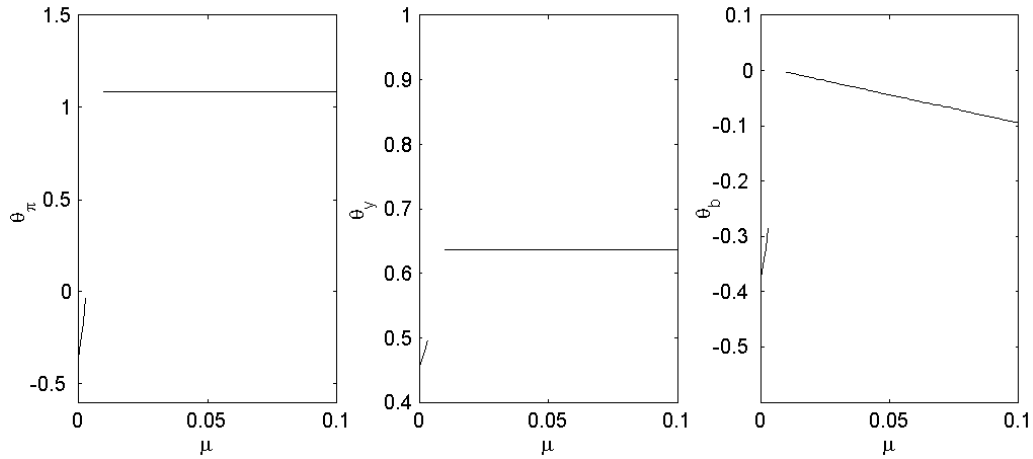
---

[21] See again Chow (1975)

Figure 5: Feedback Coefficients for Monetary Policy Rule

We can notice two things from Figure 5.

First, it is clear that as soon as the fiscal feedback on debt, μ, is high enough, the optimal monetary policy will be "conventional" [22]. That is, it will feed back on inflation and output, with coefficients which satisfy Taylor principle. For obvious reasons, Leeper (1991) used the word "active" to describe this system.

Second, we can see from the final panel of the figure that, when fiscal feedback on debt is small – i.e. with a coefficient just above the interest rate - then the optimal monetary feedback on debt will be negligibly different from zero and the optimal monetary policy rule in this Five Equation Model can be described by a Taylor rule with only two coefficients. These coefficients θ_π and θ_y are almost exactly equal to the coefficients which we found in Section 2, in the Three Equation system in which debt was ignored.

We can also see that, as the fiscal feedback on debt is tightened, *i.e.* as μ is increased, the optimal monetary feedback on debt will be made *negative*, but the values of θ_π and θ_y will be essentially unchanged. The reason for this is the following. The inflation shock raises debt (because monetary policy raises the interest rate), and if μ is large this would lead to a substantial decline in government spending, which would deflate the economy. So fiscal policy essentially helps stabilise the economy against an inflation shock. This means that there would be less of a need for real interest rates to rise in order to stabilise inflation. But this form of fiscal policy is less efficient at the stabilisation of demand than monetary policy: our calculations of L – not reported here - show that overall welfare is maximised if the fiscal feedback on debt μ is very small, *i.e.* with a coefficient just above the interest rate.

The "Fiscal Theory of the Price Level" in this Setup

---

[22]   Leith and Wren-Lewis (2000) show that this change in regime happens when μ is just above the interest rate.

15

Traditionally it was thought that some minimum level of fiscal feedback on debt was required for the economy to be stable. That is, it was thought that μ needed to be large enough to stabilize debt, in the way that we have assumed up until now. However, this view was challenged, most noticeably by Leeper (1991), who put forward what has become known as the "Fiscal Theory of the Price Level". This argued that a stable equilibrium may be possible without such fiscal feedback, if monetary policy becomes driven by the need to ensure the fulfilment of the government's intertemporal budget constraint. Figure 5 reveals that this is indeed true, in the setup being studied here.[23]

Figure 5 shows the nature of optimal monetary policy when μ is very small, so that fiscal feedback is *not* large enough to stabilise debt. In this case optimal monetary policy will *lower* the interest rate in response to an inflation shock. This regime was called "passive" by Leeper (1991), who was the first to discuss this kind of outcome. What we see is a monetary policy that does not satisfy Taylor principle, and so does not succeed in stabilising inflation around its target. Instead this "passive" monetary policy is devoted to ensuring that debt does not explode, which – given the appearance of debt in the IS curve – would cause the economy to explode. By behaving in a passive way – and not controlling inflation – monetary authority is able to ensure the solvency of the fiscal authority.[24] That is to say, in these circumstances, the optimising monetary authority will choose to abandon the control of inflation, and go instead for the control of debt.[25]

This discussion suggests that fiscal feedback on debt is important, and that its absence can force monetary authorities to take over the task of debt stabilisation, to ensure macroeconomic stability, at the expense of inflation control. McCallum and Nelson (2005) discuss this issue in detail. The results of our calculations of L – not reported here - show that such a "passive regime" has severely negative welfare consequences[26]. We do not discuss it further.

3.3   Interim Summary

We have learned a considerable amount from this investigation of simple rules.

(a)   If monetary policy follows an active stabilisation policy (*i.e.* follows a Taylor rule) then a satisfactory fiscal policy will require a feedback on debt as in (8') (i) with a positive parameter μ which is (ii) above the interest rate but small. And vice versa: we have also learned that if the

---

[23]   Some readers may be surprised at a discussion of the "fiscal theory of the price level" in the context of an entirely backward-looking model. But Leith *et.al.* (2003) show that it is not crucial that prices be able to jump so as to deflate nominal debt – they, too, discuss how this issue can be thought of in an entirely backward-looking setting.

[24]   Figure 5 shows that in this case fiscal policy will also feed back on output, but with a lower coefficient than before. This feedback means that, as tax receipts fall with falling output, the interest rate is lowered, so as to reduce interest payments.

[25]   It does so because doing this causes a sustained inflationary loss, which is *finite* in each period, but doing the opposite would (as we have seen) cause the economy to explode, and that would lead to loss which *tended towards infinity* in each period.

[26]   See Kirsanova and Wren Lewis (2005). That paper discusses the above issue in a microfounded model with forward-looking elements, which also allows a welfare analysis.

fiscal feedback on debt μ is above some threshold but small, then a satisfactory outcome will require an "active" monetary policy.

(b)   Also, to a first-order approximation, we can say that, in this setup (i) the *optimal* monetary policy will be *exactly* the same as the optimal Taylor rule obtained in Section 2, and (ii) the *optimal* fiscal feedback on debt will have a parameter just above the interest rate.

But there is more to learn.

4   OPTIMISING FISCAL-AND-MONETARY-POLICY GAMES

We now investigate what happens if *both* monetary policy makers, and fiscal policy makers, derive policy in an optimising manner. Once we do this, game theoretic issues emerge, such as the type of interaction between the policy-makers, and the timing of this interaction.

If we are to discuss optimising fiscal policy, then the social welfare function will need to be extended. Let us assume that agents derive utility from the provision of public goods purchased by government spending. In our simple linearised model we can let g_t denote the deviation of public spending from its desired level. We include in the social welfare function a penalty for non-zero values of g_t. If policymakers are benevolent, then this penalty should enter their objectives too.

We write the loss function, L_i, for the monetary authority ($i = m$) and for the fiscal authority ($i = f$) as follows:

$$L_i = \mathcal{E}_o \frac{1}{2} \sum_{t=0}^{\infty} \beta_i^t \left(\pi_t^2 + \alpha_i \left(y_t - \overline{y}_i \right)^2 + \gamma_i g_t^2 \right) \tag{9}$$

where α_i and γ_j describe the weights placed on output variability (relative to the policymaker's target level of output), and on public spending variability, by each of the policymakers, relative to inflation variability.

Note that the level of debt does not enter the loss function. The reason for this is intuitively clear from the stability analysis of our model which we presented in the previous section. We have learned that if the fiscal policy feeds back on debt with a large coefficient, then that is likely to be welfare-reducing. This is because the economy will exhibit cycles, and so the volatility of inflation and output, which *are* important for household welfare, will be higher[27]. But if the policy authorities had debt target in their objectives, then a policymaker would *have to* feed back on debt with a coefficient, proportional to the size of the penalty on debt in the objective function. That

---

[27]   These cycles could be removed if fiscal policy simultaneously fed back on inflation with a large coefficient. But in this case, there would be too much output sacrifice, following an inflation shock, and that would worsen the outcome for a policymaker with a positive value for α in loss function (9).

would compromise the stability of key macroeconomic variables[28] and lead to a loss of household welfare. We thus do not include debt in the social loss function. There may be separate political-economy reasons for stabilising debt, but we abstract from them in this paper.[29]

We will compare three different regimes, which result from three different forms of interaction between the policy authorities, in the face of shocks.

(i)   First, we take as a benchmark case the outcome which occurs if the fiscal and monetary policy-makers cooperate with each other in pursuit of a common objective, which is also the objective of the household sector.

(ii)   Second, we study the equilibrium which emerges when the fiscal authority moves first, as a Stackelberg leader, anticipating the response from the monetary authority. This is what we believe happens in the UK. In the light of our discussion in Section 2.4, we assume that the monetary policymaker is benevolent. We will examine what happens in this regime when the fiscal authority is benevolent like the monetary authority, and we also examine what happens when the fiscal authority is not benevolent.

(iii)   Third, we study the equilibrium when monetary and fiscal policy set their instruments simultaneously in a Nash game. This is what might happen within EMU. We again assume that the monetary authority is benevolent. We examine what happens in this setup when the fiscal authority is benevolent, and when it is not.

4.1   Benchmark Regime: Cooperation between Benevolent Policymakers

A benevolent policymaker has the objective function

$$L_b = \mathcal{E}_o \frac{1}{2} \sum_{t=0}^{\infty} \beta_b^t \left(\pi_t^2 + \alpha_b y_t^2 + \gamma_b g_t^2 \right) \tag{9'}$$

Such a policymaker does not aim for excess output (i.e. $\bar{y}=0$), and she discounts the future at a rate β, which is the social rate of discount, assumed to be $\beta=0.99$. The weights α and γ are supposed to reflect household preferences (see Woodford 2003). This is why we call such policymaker benevolent.

The outcome with cooperation between benevolent policymakers is displayed in Figure 6a. For a discussion of how all of the results displayed in Figures 6a to 6c are computed see Appendix B.

The optimal policy is almost *exactly* the same as that which was obtained in Section 3 for monetary policy, in the case in which the fiscal authority was feeding back very weakly on debt. This means that the optimal monetary rule, which emerges from the coordinated game with the

---

[28]   Unless there was signficant fiscal feedback on inflation. But that – as explained in the previous footnote - is undesirable.

[29]   See Stehn (2006) for an analysis of a setup in which debt is included in the objective function.

fiscal authority, is a conventional Taylor rule, and the optimal values of θ_π and θ_y are very nearly the same as those shown used to produce Figure 1. The optimal fiscal policy is to feed back onto debt in a way to make it only slightly more stable than a random walk, *i.e.* to feed back with a very small coefficient. That is, the optimal value of μ is just above the interest rate, and the optimal value for ϕ is very nearly zero. There is also a small fiscal feedback on inflation..

This means that the stabilisation of the economy is essentially carried out by monetary policy. The reason is partly due to the perfect substitutability between fiscal policy and monetary policy in the control of output and inflation (*i.e.* the fact that fiscal policy enters the IS curve in *exactly* the same way as monetary policy, and neither policy enters the Phillips curve). Consider the case of an inflation shock. After such a shock, anything that fiscal policy can do to reduce inflation, monetary policy can do, for exactly the same cost in terms of lost output. But changes in fiscal policy have been assumed to be costly (with penalty γ), whereas changes in monetary policy have been assumed not to be costly. Thus - obviously - the task of stabilisation should be carried out by monetary policy, since "anything that fiscal policy can do, monetary policy can do in a way which is cheaper". This is why fiscal policy *optimally* feeds back on inflation and debt with *very small* coefficients. Of course, debt has to be stabilised, even although debt is not directly in the social welfare function. That is because variability of debt disturbs output, via the consumption function. Since monetary policy can influence debt accumulation, one might have thought that it should deviate from stabilising inflation-and-output in order to help stabilise debt, especially since monetary and fiscal instruments are *not* perfect in stabilising debt. But this task of debt stabilisation would cause the monetary policy to be passive, i.e. to give up efforts to stabilise inflation and output. That would be welfare-reducing. (See Section 3.2 for the relevant intuition.) So the monetary policy-maker will only react to debt with a negligible coefficient and very nearly all of the stablisation of debt is done by fiscal policy. And, since fiscal stabilisation of debt is costly, the fiscal authority finds it optimal to exploit debt accumulation more-or-less up to the limit, and keeps the level of debt only just stationary.
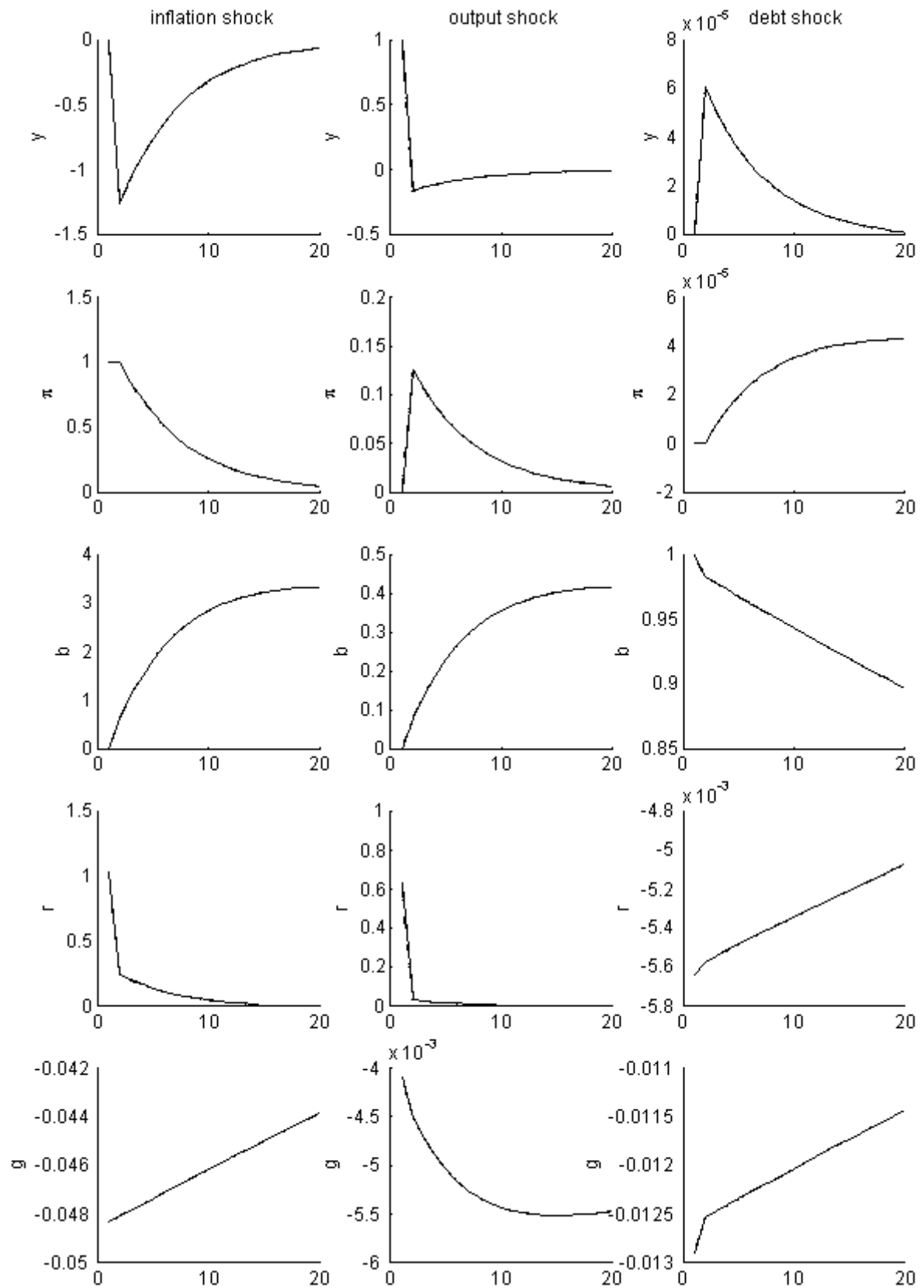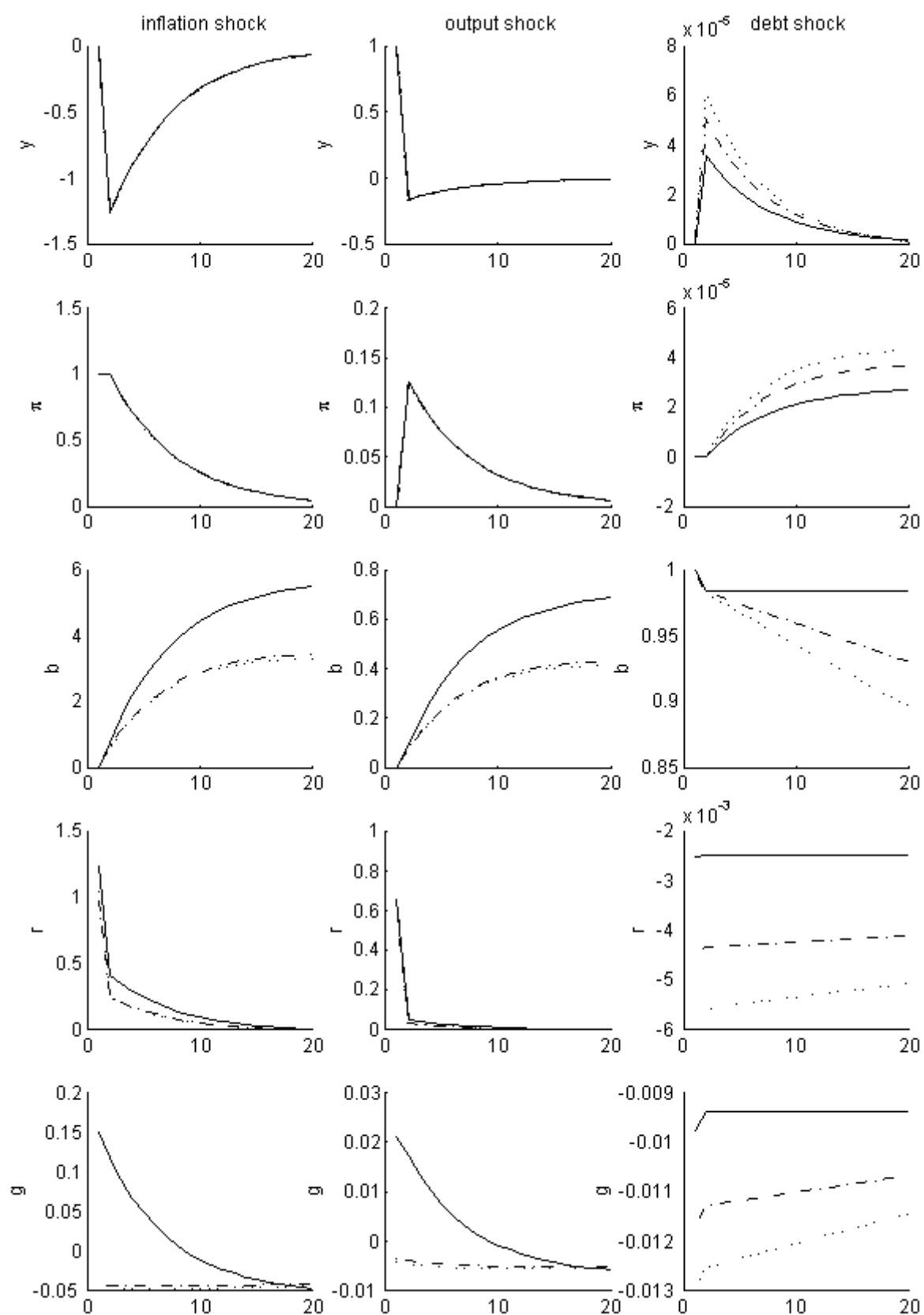
Figure 6a: Cooperation between Benevolent Authorities

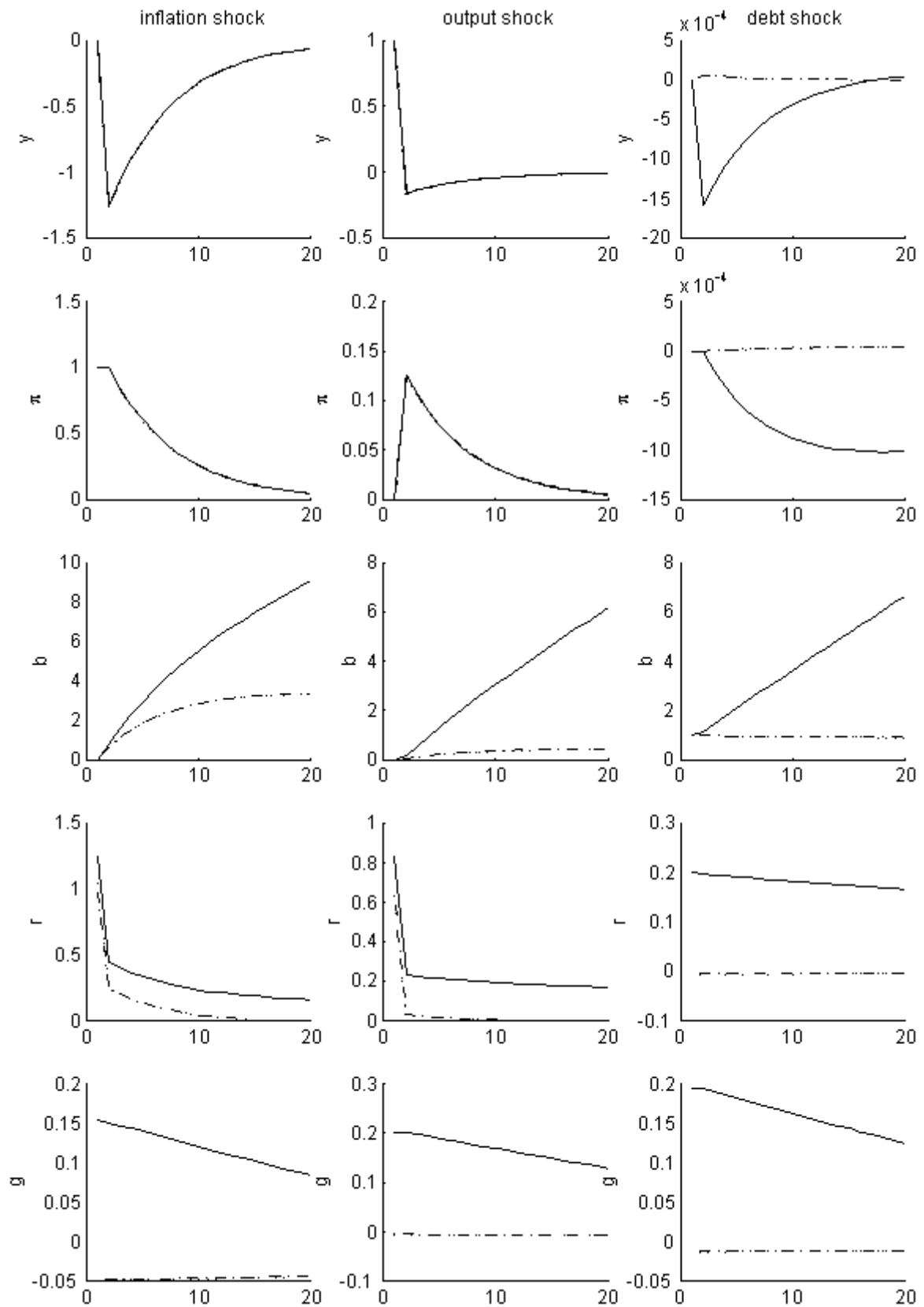**Figure 6b: The Fiscal Authority Discounts the Future**

Figure 6c: The Fiscal Authority has an Excessive Output Target

4.2   A Nash Game between the Fiscal and Monetary Policy-Makers

It is helpful to study next the equilibrium which emerges when the monetary and fiscal policymakers set their instruments simultaneously, in a Nash game. In the light of our discussion in Section 2.4, we assume that the monetary policymaker is benevolent. We examine what happens in this setup when the fiscal authority is benevolent like the monetary policymaker. But we also examine what happens if, for political reasons, the fiscal policymaker is not benevolent. That is, we examine what happens if the fiscal authority either discounts the future or aims for excess output.

(a)   Benevolent Fiscal Policy Makers

The result is obviously the same as in the Benchmark Case. This is simply because when the players have identical objectives – as they would in this case – there can be no externalities between them. Consider a particular example of an inflation shock. In the cooperative solution, the monetary *and* fiscal authorities choose their efforts such that the marginal gain in inflation and output variability is equal to the marginal loss from the volatility of the fiscal instrument. Suppose now that the fiscal authorities were to attempt to play a more active part in stabilising an inflation shock than they do in the cooperative regime, so they decide to have an additional fiscal contraction. This will improve the inflation outcome, but at expense of the higher variability of the fiscal instrument. As their total loss will rise, the total cost of monetary authorities will rise too. The monetary authorities will have to step in and expand, in order to offset the effects of unwanted fiscal contraction on inflation and output, thus making sure that the fiscal authorities cannot achieve anything by contraction and so preventing them from undertaking such efforts. By a similar process of reasoning, *any* hypothetical outcome different from that presented in Section 4.1 can be shown to be suboptimal. When the objectives are the same, the cooperative and the Nash outcomes coincide.

(b)   The Fiscal Policymaker Discounts the Future

Suppose that the fiscal authority has a shorter planning horizon than the monetary authority ($\beta_F < \beta_M$), that is, suppose that she is impatient and so would choose to postpone losses to the future rather than accept them now. We assume that β_M remains at 0.99, but that β_F falls to 0.98, which is a *very small* reduction.

In the face of an inflation shock, the impatient fiscal authority will now have a desire to delay the required recession, and will attempt to do so when playing a Nash game. But the patient monetary authority will react, by raising the interest rate, so as to continue to fight inflation. As we discussed above in Section 4.1, the monetary authorities could *completely* offset any effect that the fiscal policy has on inflation and output. This desire to offset the fiscal effects on demand and

inflation will lead the monetary policy to a decision to raise the interest rate so as to *very nearly exactly* offset the intervention caused by the fiscal authority.

The outcome is the one shown in Figure 6b with a solid line. We there show that the monetary authorities offset the fiscal effect on output and inflation, but at the expense of higher interest rates. Because inflation is controlled via demand, every effort to increase demand by the fiscal authority will be offset by the monetary authority – all that the fiscal authority gets from higher government spending is higher interest rates. As a result inflation and output will be almost exactly the same as in the cooperative regime of benevolent policymakers. This outcome clearly yields lower welfare than what would happen in that cooperative case, because it leads to a higher, possibly a much higher, variance for government spending.

This outcome results in a higher accumulation of debt. Notice that this happens even although the degree of impatience of the fiscal authority is only trivially greater than that of the monetary authority *i.e.* even although the value of β is only trivially lower for the fiscal authority than for the monetary aughoritiy. We have shown that, if we were to further increase the impatience of the fiscal authorities, debt accumulation becomes an explosive process. This is because the debt accumulation process is already a *near random walk* process in the cooperative regime of benevolent policymakers. Thus it does not require very large increases in government spending, or vary large increases in interest rates, in order to reach a threshold, beyond which debt becomes explosive. That is why the plots for government expenditure, and for the interest rate, are not very different, in Figure 6b from what they were in the cooperative regime depicted in Figure 6a by a dotted line.[30]

This is a significant result. It shows how vulnerable the economy can be to even small differences of objectives, when the fiscal and monetary authorities play Nash against each other. This is because the perfect substitution of the two instruments in their effect on demand and thus, the ability of one authority to offset the effects of the other, will in effect lead to an unstable civil war between them. In this war, each authority will indeed offset each other's actions, but with a possibly catastrophic effect on debt.[31]

We can extend this analysis to a discussion of the case to the situation in which the fiscal authority has a lower relative weight on inflation than the monetary authority. In that case, in the Nash regime, the fiscal authority will attempt to counteract the contraction caused by the monetary authority, in the face of an inflation shock. In fact the outcome is identical to what happens when the fiscal authority discounts more heavily than the monetary authority. This is because caring relatively more about output is equivalent to discounting: a policymaker who cares more about output will delay any necessary reduction in output in exactly the same way as would be done by

---

[30] If the level of debt appeared in the authorities' loss function then that would decrease the sensitivity of the equilibrium to the discount factor: the threshold for β below which debt became explosive would be lower. But this would result in a higher fiscal feedback on debt, and in a higher volatility of inflation and output, than was optimal. If the debt target does not reflect *household* preferences, as it is likely to be the case, then the policymakers are not benevolent anymore, and a minimisation of *their* costs would not necessarily minimise household welfare.

[31] Of course this "perfect substitutability" property is an oversimplification. We discuss the possible structure of a more general model below in Section 5, when we discuss the model by Dixit and Lambertini (2003).

an impatient policy-maker.[32] Notice that the point being discussed here is *not* in an argument in favour of a conservative central banker. It is, instead, an argument that a fiscal authority will cause damage if she is inadequately conservative, if she plays Nash with a benevolent central bank.

(c)   Fiscal Policymaker has an Excessive Output Target

Suppose instead that the fiscal authorities face the incentive to target output above potential ($\bar{y} > 0$).

The outcome in the Nash regime is now much worse than it was in the previous case. Here higher government spending and higher interest rates are long-lasting. The fiscal authority expands to achieve its excessive target, whilst the central bank contracts to eliminate the consequent inflation. As a result, debt becomes a unit root process for *any* $\bar{y} > 0$, see Figure 6c, solid line. The cooperative regime is plotted there with a dotted line.

4.3   Fiscal Policymaker is a Stackelberg Leader

We now study the equilibrium which emerges when the fiscal authority moves first, as a Stackelberg leader, anticipating the response from the monetary authority. In the light of our discussion in Section 2.4, we assume that the monetary policymaker is benevolent. We first examine what happens in this setup when the fiscal authority is benevolent too. But we also examine what happens in this setup when, for political reasons, the fiscal policymaker is not benevolent.

(a)   Benevolent Fiscal Policymaker

Again the result is obviously the same as in the Benchmark Case. This is again because the game is one in which both policymakers are aiming for the same objective. Any hypothetical outcome different from that presented in Section 4.1 can be shown to be suboptimal to the fiscal policymaker, by a similar process of reasoning to that used in Section 4.2., and so we converge on the outcome discussed in Section 4.1.

(b)   Fiscal Policymaker Discounts the Future

Suppose that the fiscal authority has a shorter planning horizon than the monetary authority so that $\beta_F < \beta_M$. In the face of an inflation shock, an impatient fiscal authority would have a desire to delay the required recession. But a fiscal authority who is also a Stackelberg leader will also know that the patient monetary authority, who is a Stackelberg follower, will react so as to

---

[32]   Henry *et al* (2006) show that there is an exact equivalence result between the effects of a higher degree of discounting and an increased preference for output.

overturn the effects of such a fiscal action, in order to fight inflation in its (supposedly) benevolent manner. This means that the effects of the expansionary fiscal intervention on inflation and output would be overturned, in a similar way to what happens in the Nash regime (which we described above). But now, acting as a Stackelberg leader, the fiscal authority would know that this would happen. Therefore, there would be nothing to gain as a result of doing this. Since acting in this way, for no perceived benefit, would be costly, the fiscal authority would not do it.[33] This is also shown in Figure 6b with a dash-dotted line.

We can of course extend this case to the situation in which the fiscal authority has a lower relative weight on inflation than the monetary authority. Then, even although the fiscal policymaker would like to dampen the contraction caused by the monetary authority, in the face of an inflation shock, she will in fact *not* act in this way. This is because, just as in the discussion presented in the previous paragraph, she knows that any desirable (to her) effects which this (costly) action would have would be countermanded by the benevolent monetary authority.

(c)   Fiscal Policymaker has an Excessive Output Target

Instead suppose that the fiscal authorities face the incentive to target output above potential ($\bar{y} > 0$).

Here again the outcome in the Stackelberg regime is almost as good as in the benchmark cooperative case. The fiscal policymaker would like to set a high level of government spending and get a high level of output. But she knows that she will not, in these circumstances, get a high level of output, but just high interest rates. So the fiscal policymaker will not do this. This is shown in Figure 6c, by a dash-dotted line.

4.4   Fiscal Regimes: Gordon Brown versus EMU

These results show what is required for a good fiscal policy, if the world is adequately described by our model. We can summarise our findings as follows.

(a)   If the regime is one in which the fiscal authority is able to act as a Stackelberg leader then her preferences are relatively unimportant. Good monetary policy will ensure good outcomes.

(b)   If the regime is instead one in which the fiscal authority plays Nash then her preferences can be vitally important. If the preferences of the fiscal authority are such that she does any of: discounting the future, overvaluing output, or aiming for excess output, then the outcomes can be very bad indeed. Current institutional structures in OECD countries – with fiscal policy in the hands of regularly elected politicians – makes it both possible and likely that fiscal preferences will have some or all of these features.

---

[33]   There is a minimal threshold for β_F , below which the debt explodes. But this threshold is much lower than under Nash.

In a single country setting, like the UK, it is reasonable to argue that the fiscal authority is able to act as the Stackelberg leader. We therefore expect the social costs of a fiscal policymaker that targets excess output and/or discounts strongly to be small: "the Treasury will be tamed by the Bank of England". Note that such support for the UK policy framework framework does *not* depend on the oft-stated view that in the UK fiscal policy is "not really active" in the control of the economy, and that the UK Treasury deliberately *constrains* its policy, in a way which *prevents* it from helping the monetary authority to stabilise the economy, in a way which it might choose to do, if it were not so constrained. We have shown, instead, that the fiscal authority, if it is able to act as a Stackelberg leader, will *choose* to let the monetary authority do nearly all of the macroeconomic stabilisation of the economy, and that it will choose to do this even if it (i) over-discounts the future and (ii) would like to see output at an excessive level. Our support for Gordon Brown's strategy comes *not* from the UK's fiscal rules. It comes from the fact that, in the UK, the fiscal authority is able to play the role of Stackelberg leader.

Within EMU, by contrast, there is one central bank but many fiscal authorities. Clearly, these fiscal authorities cannot act as a Stackelberg leader unless they can co-ordinate amongst themselves. But that is obviously impossible within EMU: even a much less ambitious attempt to simply restrain deficits, in the form of the Stability and Growth Pact, was ineffective. EMU is thus forced to an outcome in which there cannot be Stackelberg leadership by the fiscal authorities. Our results suggest that there will be social welfare costs, possibly very large ones, whenever, within Europe, national fiscal policy interests differ from those of the ECB. The fact that fiscal decisions are in the hands of elected politicians suggests that these differences of interest exist, and that they may be large. This thus suggests that fiscal and monetary interactions within Europe may lead to low quality outcomes.

Note that the argument of the previous two paragraphs has significant institutional implications. It means that within EMU there may be a need for Fiscal Policy Councils, to prevent the bad effects of having fiscal authorities with distorted objectives. But, by contrast, this may not be true for the UK. In the UK Stackelberg leadership by the fiscal authority may, on its own, be enough to prevent such bad outcomes from emerging, even if the fiscal authorities are not benevolent.

Note that the argument in this paper about fiscal difficulties within EMU is based on the difficulty of stabilising shocks which are *symmetric* across Europe. It is quite different from the normal argument about problems within EMU, which is based on the difficulty of dealing with *asymmetric* shocks which occur separately in different countries. (See Allsopp and Vines, 1998.)

5   COMPARISON WITH OTHER RECENT WORK

We now compare our findings with those of a paper by Dixit and Lambertini (2003). It is useful to compare our work with that paper in some detail because it has been widely cited, and influential. In this section we will also relate our work to an earlier paper by Blake and Weale (1998), and to recent papers by Adam and Billi (2005) and Hughes Hallet (2005).

5.1   Dixit and Lambertini (2000, 2003)

The authors consider a stylised static model of a closed economy with two policymakers, in which output is suboptimally low, because of the monopoly power of firms, in the manner of Dixit and Stiglitz (1977). The model can be summarised by means of two reduced form equations. There is an aggregate demand function, in which output depends on the fiscal instrument, on unexpected inflation (because, say, an unexpected increase in inflation lowers the real interest rate), and on shocks. There is also an equation for inflation, which depends on any fiscal intervention, on monetary policy, and shocks. Monetary policy controls the money supply: an increase in the money stock raises inflation directly. The fiscal instrument affects both aggregate demand and inflation. In any time period, outcomes depend on this model structure, on the expected rate of inflation, on shocks, and on policy preferences. The model is static. This means, in particular, that policies are unconstrained by the requirement to keep public debt under control.

This paper is important from the methodological point of view. The model is well microfounded, and is suitable for analysing a range of policy scenarios. It is also a *static* model, and, as a result, the policy games played using it can be solved analytically, unlike in our paper.

However the model is so general that it can generate a whole assortment of the rankings of the possible strategic equilibria. For example, the authors show that, using reasonable values for the model's coefficients, Nash-equilibrium outcomes can be worse than those when there is fiscal leadership, as in our paper. But for other, also reasonable, values of these same coefficients the reverse is true[34]

In what follows, and in Appendix C, we show how, by restricting their setup, again in reasonable ways , we can get a much simpler version of their model which does not have these ambiguities. This stripped-down model also has the advantage that it is – in effect- a static simplification of our own model. Comparing the results which come from this Dixit-Lambertini-type model (DLT) enables us to make our own results much clearer. Our results have had to be obtained numerically, since they are so complex: they concern the *dynamic* outcomes of a intertemporally-optimsing policy-game, which is played in response to shocks. We will see that the DLT model produces *static* results, which can be derived analytically. These – we will show - are static simplifications of the dynamic numerical results obtained from our model. Doing all of this therefore helps us to underpin our own numerical results with analytical results. And it is especially useful since these analytical results come from a model derived from that in the Dixit and Lambertini paper, which is so well known.

The model of Dixit and Lambertini is different from our own model in five ways.

First, the authors' work is driven by a wish to analyse the problem of inflation bias. They consider how fiscal and monetary policies interact, not only in response to shocks, but also as a

---

[34]   There are some unambiguous conclusions in the paper, of course, but only concerning issues which are different from those which we are discussing here.

result of the fact that *both* policies are driven to do something about the fact that output is suboptimally low, as a result of the monopoly power of firms. Our paper does *not* study a setup like this, because we aim to study what we believe is current practice. We believe that it *is* appropriate to make an assumption of this kind for fiscal policymakers, who may well aim for excess output, above what the monopolistic economy has the capacity to sustain. But, as we explain in Section 3, we do *not* think that it is appropriate to assume this for monetary policy. Thus in our reformulation of the Dixit and Lambertini model we will also not assume this. All of our analysis in Section 4 concerns an economy in which inflation is only above target because of policy choices in response to shocks. Our reformulated Dixit andLambertini model will have this same property.

Second, in the published version of the paper (Dixit and Lambertini, 2003) the fiscal policy instrument is production subsidies. Higher subsidies counter the monopoly distortion and so raise output but they also *lower* prices directly; the overall effect on prices of increased subsidies is assumed to be negative, because the second effect is assumed to dominate the first. The results of the published version of the paper are fundamentally dependent on this idea that a fiscal expansion *lowers* the price level. Although this may be true in the short run, it cannot be true in the long run – since as we discuss in our paper, fiscal laxity leads to a continuing issue of public debt which will eventually lead to such large increase in expenditure as to *increase* the rate of inflation. It is impossible to accommodate both of these conflicting outcomes in a static model. One way out of that problem would be to locate the analysis in a dynamic model, like that in our paper.[35] Alternatively one could use government expenditure as the fiscal instrument, as in the authors do in their earlier working paper version of the paper (Dixit and Lambertini, 2000), and obtain a static model without this problem. We will recast their model in this way – which has the added advantage of making their work more easily comparable with our own, which also uses government expenditures as the fiscal policy instrument.

Third, the authors use the money supply rather then the interest rate as the policy instrument. This difference, which looks important, does not, in fact, matter at all. Their model contains a demand for money function which solves for the interest rate as a function of the money supply (and other things). Thus manipulation of the money supply in their model has the same effect as manipulating the interest rate in our model.

Fourth, in their model the inflation process is forward-looking. In the version of their model which we construct, this feature turns out not to offer a real advantage. That is because our version of their model is linear, and shocks are additive, so that "certainly equivalence" applies. This means that, since inflation expectations are formed before shocks are revealed, and since the level-bias is removed by monetary policy (in the way which we have described above), expected

---

[35] Weale *et.al.* (1989) study the problem examined here in a dynamic way, in a model in which a tax cut initially lowers inflation but also raises aggregate demand. In an accelerationist model of inflation, like that in this paper, this must eventually raise the rate of inflation.

inflation, prior to the realisation of shocks, will always be zero. Forward-lookingness thus does no important work.[36]

Fifth, and finally, their model is static. As a result, policies are unconstrained by the requirement to keep debt under control, so as to ensure fiscal solvency. As will be obvious from Sections 3 and 4 of this paper, we believe that this requirement necessarily imposes a dynamic structure on the interactions between fiscal and monetary policy, and that this dynamic process has an important influence on the outcomes from these interactions. One attempt to incorporate this feature in a static way in a linear version of the DLT model leads to the Hughes Hallett (2005) model, which we discuss later. Modelling these processes in an intertemporal, dynamic way would, in effect, turn the resulting model into our model.

In Appendix C we represent the Dixit and Lambertini model by a setup, which we have called the DLT model, in which (i) monetary policy has no level bias, (ii) government expenditure is used as the fiscal instrument, (iii) the monetary policy instrument is the real interest rate, (iv) the model is linear and shocks are additive, but (v) in which we stick to the authors' decision not to analyse debt. We obtain a model with a static IS curve and a static, expectations-augmented, Phillips curve. We show in Appendix C that the DLT model gives a set of static results, analogous to our dynamic results.

These findings from the DLT model correspond to an *exact* simplification of our dynamic results, *providin*g that we also impose the assumption on this model, which has been fundamental to our paper, that fiscal and monetary policy are perfect substitutes in the control of output and inflation. In the face of an inflation shock, cooperating benevolent authorities will cause a recession to emerge, and they will bring this about by monetary restraint, not fiscal restraint. The same will be true if the policy authorities play a game in which the fiscal authority is a Stackelberg leader, even if the fiscal authority has an excessive weight on output losses (which can be thought of as a static game equivalent to over-discounting the future, for reasons discussed in Section 4.) But if the fiscal and monetary authorities play a Nash game, then if the fiscal authority has an excessive weight on output losses, the outcome will be one of excessive monetary contraction, and excessive fiscal expansion. This will be welfare reducing. Under the assumption of perfect substitutability between instruments, the DLT model, in which assumptions (i)—(v) hold, delivers exactly the kinds of results that we describe in our paper.

We also show in Appendix C that the conflict between fiscal and monetary authorities, which occurs under Nash, and about which we have been so concerned in this paper, can be moderated if fiscal and monetary policy are not perfect substitutes in the control of inflation and output. In Section 4 of Appendix C we show what happens if this moderation is very significant. It remains a real question as to whether it is significant. We do not discuss that important question in this paper.

---

[36]   The original model of Dixit and Lambertini is non-linear, since coefficients are stochastic, and hence certainty equivalence does not apply (see Chow, 1975, pp 249, 290-291). Therefore expected inflation is *not* zero in their model.

5.2   Adam and Billi (2005)

One way to make the model dynamic, in the way which Dixit and Lambertini do not do, is presented by Adam and Billi (2005). This is a dynamic two period model, in which fiscal solvency *is* imposed. The purpose of their paper , again like that of Dixit and Lambertini and of our own paper, is to study how and why conflicts arise between fiscal and monetary authorities if they do not cooperate. As in our model, fiscal policy sets public consumption (*i.e.* government spending). And as in our model, this government spending enters positively into the representative agent's utility function.

Two important features of the paper should be noted.

First, both fiscal and monetary policies deliberately target a level of output above its potential level. This potential level is inefficiently low, due to the same monopolistic distortions as those studied in the Dixit and Lambertini paper. The analysis in this paper is entirely devoted to this excess-output-target problem. Indeed the authors do not consider the response of the economy to shocks, which is the key purpose of our paper, and of the Dixit and Lambertini paper.

Second, fiscal solvency is imposed by assuming that at all times, fiscal policy obeys a balanced-budget constraint. Thus, there is no debt in the economy and government spending is financed by distortionary labour taxes.

The two features, which we have just noted, drive many, if not all, of their outcomes. One of these, in particular, should be recorded. The fiscal authorities wish to increase output above its (suboptimally low) equilibrium level. That will, of itself, increase the rate of inflation. But because of the balanced budget requirement, the fiscal authority can only do this by increasing the level of (distortionary) taxes, thus driving *down further* the *equilibrium* level of output, and so increasing the inflationary problems which they themselves cause by aiming at the (too high) undistorted level of output. The monetary authority understands this. It realises that, because the fiscal authority dislikes inflation, it (the monetary authority) can moderate excessive spending by the fiscal authority, by itself creating an excessive level of inflation, one above its own target. If it does this then, at the margin, the fiscal authority will set a lower level of government spending, and will thus impose a smaller increase in (distortionary) taxes on the economy. This result follows directly from a willingness to believe that, at the margin, the monetary authority will wish to tolerate excessive inflation as a way of getting higher output. In our model, we have ruled out a study of this issue, for reasons explained in Section 3. We believe that, in the absence of shocks, the monetary authority will aim for the non-inflationary level of output, even if that level is forced to an unduly low level by the need of the fiscal authority to raise the taxes to support its expenditure (see Vickers 1998). We think that this is true of all independent central banks, including both the Bank of England and of the ECB.

This review does suggest to us, however, that there would be much gain from using the Adam and Billi two-period dynamic framework to study analytically – in a way which we have been unable to do - the problems of fiscal and monetary responses to shocks.

5.3   Hughes Hallet (2005), and Blake and Weale (1998)

One paper which it is important to mention is Hughes Hallett (2005). His model assumes, like ours, that inflation is controlled via output and that monetary and fiscal instruments are perfect substitutes in the control of output. Unlike Adam and Billi, he allows government spending to differ from tax revenues. As we discussed above, this model is essentially the DLT model, to which is added a static version of the budget constraint. However, unlike in our paper, the author puts some penalty on debt in the objective function.

That assumption has unwanted unwanted implications. The process of debt accumulation process is intrinsically dynamic. What, in the end, matters is enforcing the *intertemporal* solvency of the government. A static model with a penalty on debt is not, we believe, a good way of doing this, especially since we have shown that the optimal way to control debt is *slowly,* in order to avoid cycles. This kind of analysis requires an explicitly dynamic model.

Interestingly, in a much earlier paper, Blake and Weale (1998) consider a model which is much more like ours, and is similar to a dynamic version of the Hughes Hallett model. It has a static aggregate demand curve, a backward-looking accelerationst Phillips Curve and a debt accumulation equation. It is effectively our model with $\kappa = 0$. The fiscal instrument is tax, and the monetary instrument is the real interest rate. They do not consider games which maximise welfare functions like those in our games, but instead they "allocate" the responsibilities of policy to separate policy authorities. Thus the monetary authority stabilises inflation (subject to a cost of moving interest rates), and the fiscal authority stabilises debt (subject to a cost of moving taxes). They consider two regimes: a Nash game, and a second regime in which the two authorities jointly minimise the sum of the two objectives. They find that Nash can lead to much higher inflation and debt and interest rates, and lower taxation. What they show, very effectively, is that outcomes for the key macroeconomic variables can be very bad under Nash. This paper is a first step towards our own. There is no concept of "benevolent policymaker",or of social welfare, and their results can only be assessed using "common sense". Nevertheless the paper makes very clearly the general point which emerges from our work. Outcomes under Nash can be very bad, when the objectives of policymakers diverge.

6   CONCLUSIONS

In this paper we have studied the potential for fiscal policy to stabilise an economy, using a simple five equation model of a single economy. And we have shown how to use this model to study the interactions between fiscal policy and monetary policy.

We began with the now well-known setting of Three Equation Taylor-rule macroeconomics, which has been used to study the stabilisation of an economy by monetary policy alone. We used this setup to display what is meant by "good" monetary policy. In Section 3, we then extended this setup to a Five Equation system, and used that to describe what is meant by a "good" fiscal policy. While doing all this, we devoted much time to discussing the methodological issues of constructing good control policy in a dynamic economy.

In Section 4 we used this Five Equation Model to display the outcomes of optimal monetary and fiscal policies, when they are used cooperatively to stabilise the economy against shocks. We also considered non-cooperative outcomes when the authorities play Nash game against each other, or when the fiscal authorities lead in a Stackelberg game. We assumed that the monetary authorities were benevolent. But we studied what happens if the fiscal authorities are not benevolent, because they might have different objectives from those of society as a whole. We showed three things using numerical simulations of these games.

We first showed the outcome, if fiscal authorities and monetary authorities are benevolent, and cooperate with each other in the setting of their macroeconomic policy instruments. We show that, in this case, the response to an inflation shock will be one in which the fiscal authorities let monetary policy perform nearly all of the burden of stabilising the economy in the face of the shock. Debt will be controlled by fiscal policy, but only slowly.

Second, we showed that exactly this same good outcome will occur if the fiscal authority is able to act as a Stackelberg leader, even if the fiscal authority discounts the future too much, and/or desires to aim for excess output. We think that this is possibly what happens in the UK. That is why we support the current fiscal policy framework for the UK.

Third, we showed that if the fiscal authority plays against the monetary authority in a Nash game, then social welfare will be harmed, and possibly greatly harmed, if the fiscal authority discounts the future too much, and/or desires to aim for excess output. We think that this might happen within EMU. That is why we are very critical of the current institutional arrangements for fiscal and monetary policy within EMU.

In the last part of the paper we spent much time showing how our model can be related to the rest of the existing literature on the interactions between fiscal policy and monetary policy. To date, most of that literature considers static games played between the fiscal authorities and the monetary authorities (albeit repeated ones). We have therefore constructed a simple static model - the DLT model presented in Appendix C - which captures the key features of that static literature. In particular, this DLT model can be viewed as a simple stripped-down version of the model by Dixit and Lambertini (2003), which is widely-cited. We have used this DLT model to explain the key insights which come from the Dixit and Lambertini paper. We have also used it as a platform from which to view the other related papers. At the same time this DLT model is *also* a simple version of the dynamic model which we have analysed in the body of this paper. We have thus

also been able to use this model to get important analytical insight into the numerical simulation results which we have presented in the main body of the paper.

The results in this paper come from a model which is not explicitly micro-founded, and which agents are backward looking. We deliberately adopted such a model for expositional reasons. It would be possible to move to a model in which there are explicit microfoundations, in which agents are forward-looking, and in which there is a loss function derived from the underlying microfoundations. We believe that the three claims made in the paper would survive that transformation, unscathed.

REFERENCES

Adam, K. and R. Billi (2005). Monetary and Fiscal Interactions with Commitment and the Value of Monetary Conservatism. Mimeo, European Central Bank.

Aizenman, J. and J. Frenkel (1985). Optimal Wage Indexation, Foreign Exchange Intervention, and Monetary Policy. *The American Economic Review 75* (3), 402—423.

Allsopp, C. and D. Vines (1998). Macroeconomic policy after EMU: The assessment. *Oxford Review of Economic Policy* 14 (3), 1—23.

Allsopp, C. and D. Vines (2000). Macroeconomic policy: an assessment. *Oxford Review of Economic Policy* 16 (4), 1—32.

Ball, L. (1999). Policy rules for open economies. In J. Taylor (Ed.), *Monetary Policy Rules*. Chicago: University of Chicago Press.

Barro, R. and D. Gordon (1983a). A Positive Theory of Monetary Policy in a Natural Rate Model. *The Journal of Political Economy* 91 (4), 589—610.

Barro, R. and D. Gordon (1983b). Rules, discretion and reputation in a model of monetary policy. *Journal of Monetary Economics* 12, 101—121.

Bean, C. (1998). The new UK monetary arrangements: a view from the literature. *Economic Journal* 108 (451), 1795—1809.

Blake, A.P. and M. Weale (1998). Costs of separating budgetary policy from control of inflation: a neglected aspect of central bank independence. *Oxford Economic Papers 50*, 449—467.

Blanchard, O. (1985). Debt, deficits, and finite horizons. *Journal of Political Economy* 93 (2), 223—247.

Blanchard, O. and C. Kahn (1980). The solution of linear difference models under rational expectations. *Econometrica 48*, 1305—1311.

Blinder, A. (1997). What central bankers could learn from academics-and vice versa. *Journal of Economic Perspectives* 11, 3—19.

Blinder, A. (1998). *Central Banking in Theory and Practice*. Cambridge, MA: The MIT Press.

Calvo, G. (1983). Staggered prices in a utility maximising framework. *Journal of Monetary Economics* 12, 383—398.

Carlin, W. and D. Soskice (2005). The 3-equation New Keynesian Model: A Graphical Exposition. CEPR Discussion Paper No 4588.

Chow, G. (1975). *Analysis and Control of Dynamic Economic Systems*. London: John Wiley and Sons.

Clarida, R. H., J. Galí, and M. Gertler (1999). The science of monetary policy: A new Keynesian perspective. *Journal of Economic Literature* 37, 1661—1707.

Dixit, A. and L. Lambertini (2000). Fiscal Discretion Destroys Monetary Commitment, available from http://econweb.sscnet.ucla.edu/lambertini/papers/fiscdisc.pdf.

Dixit, A. and L. Lambertini (2003). Interactions of commitment and discretion in monetary and fiscal policies. *American Economic Review* 93, 1522—1542.

Henry, B., M. Satchi, and V. D. (2006). The effect of discounting on outcomes in inflation target regimes. *Economic Journal*. Forthcoming.

Hughes Hallett, A. (2005). In Praise of Fiscal Reastraint and Debt Rules: What the Euro Zone might do Now. CEPR DP 5043.

King, M. (1997). The inflation target five years on. *Bank of England Quarterly Bulletin* 37, 434—445.

Kirsanova, T. and S. Wren-Lewis (2005). Optimal Fiscal Feedback in an Economy with Inflation Persistence. Mimeo, University of Exeter. Available from www.ex.ac.uk/~tkirsano/papers.html

Kydland, F. E. and E. C. Prescott (1977). Rules rather than discretion: The inconsistency of optimal plans. *Journal of Political Economy 85* (3), 473—491.

Lambertini, L. and R. Rovelli (2004). Monetary and fiscal policy coordination and macroeconomic stabilization. A theoretical analysis. Mimeo, University of Bologna. Available from www.dse.unibo.it/rovelli/RR-Papers/emustacknew.pdf

Leeper, E. (1991). Equilibria under 'active' and 'passive' monetary and fiscal policies. *Journal of Monetary Economics* 27, 129—147.

Leith, C, Warren, P. and Wren-Lewis, S (2003), Fiscal Policy, Interest Rate Shocks and Prices, *Economic Modelling*, 20, 361-384

Leith, C. and S. Wren-Lewis (2000). Interactions between monetary and fiscal policy rules. *Economic Journal* 110, C93—C108.

McCallum, B. and E. Nelson (2005). Sense and nonsense in the fiscal theory of the price level. *Oxford Review of Economic Policy*. forthcoming.

Rogoff, K. (1985). The Optimal degree of Commitment to and Independent Monetary Target. *Quarterly Journal of Economics 100*, 1169—1189.

Romer, P. (2000). Macroeconomics without the LM curve. *Journal of Economic Perspectives* 14 (2), 149—157.

Söderlind, P. (1999). Solution and estimation of RE macromodels with optimal policy. *European Economic Review 43*, 813—823.

Stehn, S.J. (2006), "Optimal Fiscal Policy in a Simple Five-Equation Model", Forthcoming, Department of Economics Discussion Paper Series, Oxford University

Svensson, L. (1997). Optimal Inflation Targets, 'Conservative' Central Banks, and Linear Inflation Contracts. *The American Economic Review 87*, 98—114.

Taylor, J. (1995). The monetary transmission mechanism: An empirical framework. *Journal of Economic Perspectives* 9 (4), 11—26.

Taylor, J. (2000). Teaching macroeconomics at the principles level. *American Economic Review* 90 (2), 90—94.

Vickers, J. (1998). Inflation targeting in practice. *Bank of England Quarterly Bulletin* 38, 368—375.

Weale, M., A. Blake, N. Christodoloukais, D. Vines, and J. Meade (1989). *Macroeconomic Policy: Inflation, Wealth and the Exchange Rate*. London: Allen and Unwin.

Wicksell, K. (1907). The Influence of the Rate of Interest on Prices. *The Economic Journal 17*, 213--220.

Woodford, M. (2003). *Interest and Prices: Foundations of a Theory of Monetary Policy*. Princeton, NJ.: Princeton University Press.

Yaari, M. (1965). Uncertain lifetime, life insurance, and the theory of the consumer. *Review of Economic Studies* 32, 137—150.

APPENDIX A

In deriving our system of equations we have assumed that the behavioural consumption function is $C_t = (1-\tau)(a_1 Y_t + a_2 Y_{t-1}) - a_3(1+R_{t-1}) + a_4 B_t$ and the equation for aggregate demand is $Y_t = C_t + G_t$.

We substitute the consumption function into the equation for aggregate demand, and then log-linearise. Introducing new variables: $y_t = \ln\dfrac{Y_t}{Y}$, $g_t = \dfrac{G}{Y}\ln\dfrac{G_t}{G}$, $b_t = \dfrac{B}{Y}\ln\dfrac{B_t}{B}$, $1+r_t = (1+R_0)\ln\dfrac{1+R_t}{1+R_0}$, we obtain equation (5), and also equation (1) if government spending is ignored, where $\kappa = \dfrac{a_2(1-\tau)}{(1-a_1(1-\tau))}$, $\sigma = \dfrac{a_3}{(1-a_1(1-\tau))Y}$, $\phi = \dfrac{a_4}{(1-a_1(1-\tau))}$, and $\delta = \dfrac{1}{(1-a_1(1-\tau))}$.

The equation for accumulation of the real stock of public debt can be written as:

$$B_t = (1+R_{t-1})(B_{t-1} + G_{t-1} - \tau Y_{t-1})$$

Log-linearising this in a way similar to that used above, we obtain equation (8).

We calibrate $a_1 + a_2 = 0.7$ to be equal to the labour share in GDP (we take $a_1 = 0.5$), and we set $a_3 = 0.8$ which is less than one. The tax rate is set to 30%. For Blanchard-Yaari consumers we set the probability of death of 0.01 (that corresponds to average working life of 25 years) and so we calibrate $a_4 = 0.01$. The steady state real interest rate should be just above $1/\beta$ so we set $R_0 = 0.011$. The steady state level of debt is taken as 60% of GDP. This calibration leads to the parameter values reported in the main text.

APPENDIX B

This appendix explains the solution method used to compute the pictures shown in Figures 6a, 6b, and 6c.

Benevolent authorities and cooperation

When the authorities are benevolent and cooperate, the constrained loss function is

$$L^b = \mathcal{E}_o \frac{1}{2}\sum_{t=0}^{\infty}\left(\beta_b^t\left(\pi_t^2 + \alpha_b y_t^2 + \gamma_b g_t^2\right) + \lambda_t^y\left(\kappa y_{t-1} - \sigma r_{t-1} + \phi b_t + \delta g_t + \varepsilon_t - y_t\right)\right.$$
$$\left. + \lambda_t^\pi\left(\pi_{t-1} + \omega y_{t-1} + \upsilon_t - \pi_t\right) + \lambda_t^b\left((1+r_0)b_{t-1} + r_{t-1}B + g_{t-1} - \tau y_{t-1} + \eta_t - b_t\right)\right)$$

where the constraints include equations (5), (6) and (7).

In order to write the first-order-conditions (FOCs) , we need to differentiate this equation with respect to all of the instruments, g and r, and all state variables, π, y, b, λ_t^y, λ_t^π, and λ_t^b. We obtain the system of eight variables for eight unknowns, in which the state variables π, y and b are predetermined variables, and in which all instruments and Lagrange multipliers are jump variables. Note that the FOCs with respect to the Lagrange multipliers produce equations (5), (6) and (7). The system can be solved using the generalised Schur decomposition, as explained in Soderlind (1999).[37] As we minimise a quadratic loss function subject to linear system with additive shocks, our fully optimal solution is certainty equivalent, see Chow (1975). It means that the solution is the same as we would obtain if we replaced all stochastic variables (shocks in our case) with their mean values.

Nash equilibrium

When authorities play a Nash game, the monetary authorities' constrained loss function is:

$$L^{\mathrm{m}} = \mathcal{E}_o \frac{1}{2} \sum_{t=0}^{\infty} \left(\beta_{\mathrm{m}}^t \left(\pi_t^2 + \alpha_m y_t^2 + \gamma_m g_t^2 \right) + \lambda_t^{\mathrm{my}} \left(\kappa y_{t-1} - \sigma r_{t-1} + \phi b_t + \delta g_t + \varepsilon_t - y_t \right) \right.$$

$$+ \lambda_t^{\mathrm{m}\pi} \left(\pi_{t-1} + \omega y_{t-1} + \upsilon_t - \pi_t \right) + \lambda_t^{\mathrm{mb}} \left((1+r_0) b_{t-1} + r_{t-1} B + g_{t-1} - \tau y_{t-1} + \eta_t - b_t \right))$$

The first-order conditions for the monetary authority are seven equations which are obtained by differentiation of the L^{m} with respect to one instrument r, and all state variables, π, y, b, λ_t^{my}, $\lambda_t^{\mathrm{m}\pi}$, and λ_t^{mb}.

The fiscal authorities' constrained loss function is

$$L^{\mathrm{f}} = \mathcal{E}_o \frac{1}{2} \sum_{t=0}^{\infty} \left(\beta_{\mathrm{f}}^t \left(\pi_t^2 + \alpha_f y_t^2 + \gamma_f g_t^2 \right) + \lambda_t^{\mathrm{fy}} \left(\kappa y_{t-1} - \sigma r_{t-1} + \phi b_t + \delta g_t + \varepsilon_t - y_t \right) \right.$$

$$+ \lambda_t^{\mathrm{f}\pi} \left(\pi_{t-1} + \omega y_{t-1} + \upsilon_t - \pi_t \right) + \lambda_t^{\mathrm{fb}} \left((1+r_0) b_{t-1} + r_{t-1} B + g_{t-1} - \tau y_{t-1} + \eta_t - b_t \right))$$

The first-order conditions for the fiscal authority can be obtained by differentiation of L^{f} with respect to the instrument g, and all state variables, π, y, b, λ_t^{fy}, $\lambda_t^{\mathrm{f}\pi}$, and λ_t^{fb}. Note that, for both authorities, the FOCs with respect to Lagrange multipliers produce equations (5), (6) and (7).

To obtain the Nash solution we need to solve the two of systems of equations simultaneously. We obtain a final system of eleven equations, in which are included all seven FOCs for the monetary authorities and all seven of the FOCs for the fiscal authorities with respect to g, π, y and b. But since, for both authorities, the FOCs with respect to the Lagrange multipliers produce the same equations – *i.e.* equations (5), (6) and (7) - this gives eleven independent equations in eleven unknowns. Again, the resulting system can be solved using the with generalised Schur

---

[37]   This system has a singular left-hand side, so the method of Blanchard and Kahn (1980) is not suitable.

decomposition, knowing that π, y and b are predetermined variables and that all other variables are jump variables.

Stackelberg equilibrium with fiscal leadership

The optimisation problem for the monetary authorities is the same as for the Nash equilibrium, and we obtain seven first order conditions. The leader' however, takes into account the follower's reaction function, whereas the follower's reaction function is described by the first order conditions to the follower's optimisation problem. Hence, the loss function for the fiscal leader is the following:

$$L^{\mathrm{f}} = \mathcal{E}_o \frac{1}{2} \sum_{t=0}^{\infty} \left(\beta_{\mathrm{f}}^t \left(\pi_t^2 + \alpha_f y_t^2 + \gamma_f g_t^2 \right) + \lambda_t^{\mathrm{fy}} \left(\kappa y_{t-1} - \sigma r_{t-1} + \phi b_t + \delta g_t + \varepsilon_t - y_t \right) \right.$$

$$+ \lambda_t^{\mathrm{f}\pi} \left(\pi_{t-1} + \omega y_{t-1} + \upsilon_t - \pi_t \right) + \lambda_t^{\mathrm{fb}} \left((1 + r_0) b_{t-1} + r_{t-1} B + g_{t-1} - \tau y_{t-1} + \eta_t - b_t \right)$$

$$\left. + \upsilon_t^{\mathrm{f}\pi} \left(\frac{\partial L^{\mathrm{m}}}{\partial \pi_t} \right) + \upsilon_t^{\mathrm{fy}} \left(\frac{\partial L^{\mathrm{m}}}{\partial y_t} \right) + \upsilon_t^{\mathrm{fb}} \left(\frac{\partial L^{\mathrm{m}}}{\partial b_t} \right) + \upsilon_t^{\mathrm{fr}} \left(\frac{\partial L^{\mathrm{m}}}{\partial r_t} \right) \right)$$

where in the last line of the formula above we include, as additional constraints, the first order conditions of the follower with respect to all variables except the Lagrange multipliers[38]. We differentiate this constrained loss function with respect to *both* instruments, r and g, all predetermined state variables π, y and b, and all Lagrange multipliers, λ^f and υ^f. We obtain eleven equations, three of which repeat the FOCs of the follower's optimisation problem and not included in the final system. The final system consists of the first order conditions of the two players. It contains all seven FOCs of the follower's optimisation problem and eight (out of eleven) FOCs for the leader. We solve this system again using the generalised Schur decomposition, knowing that π, y and b are predetermined variables, that the Lagrange multipliers υ^f are predetermined variables too[39], and that all other variables are jump variables.

APPENDIX C

A STATIC DIXIT-LAMBERTINI-TYPE (DLT) MODEL

C.1 Model

Consider the following two-equation linear model:

$$y = -\sigma r + \delta g + \varepsilon$$
$$\pi = \pi^e + \omega y - \xi r + v$$

---

[38] The first order conditions with respect to Lagrange multipliers produce equations (5), (6) and (7) which are already included as constraints.

[39] As are those attached to constraints on jump variables. Pontryagin's Maximum Principle requires one to set initial conditions on predetermined Lagrange multipliers to zero.

This pair of equations has a slightly different form from that described in Section 5.1. But it effectively has the form that monetary and fiscal policy can each, separately, affect both output and inflation, which is the key feature of the system described by equations (1) and (2) in Dixit and Lambertini (2000).

Such a setup is a generalisation of our model in the following sense. If we set $\xi = 0$, so that fiscal and monetary policy only affect inflation via their effect on demand, and so are "perfect substitutes" in the control of output and inflation, then we get a model which is precisely a static simplification of our own model. (It is of course a simplification since it contains no debt equation.)

The objectives of the two policymakers can be written as

$$L_i = \frac{1}{2}\left(\pi^2 + \alpha_i\left(y - y_i\right)^2 + \gamma g^2\right)$$

where, as in the main text, $i = \{m, f, b\}$ stands for monetary, fiscal and benevolent.

Expected inflation π^e is exogenous (since expectations are formed before the shock draw), and the shocks ε and υ are also exogenous.

C.2   Optimal Outcomes in Three Regimes

It is straightforward to derive the first order conditions (FOCs) for each of the three regimes:

 (i) Nash equilibrium,
(ii) Fiscal-Leadership equilibrium,
(iii) the two policymakers with identical objectives jointly minimise the common loss function (Cooperation),

We can write down formulae for the reaction functions which emerge from each of these three sets of FOCs. Note that reaction functions can be either written in terms of variables (inflation and output), or in terms of instruments (real interest rate and government expenditure). We give both. These reaction functions are plotted in Figure 7. The left hand side panels plot reactions in inflation-output space and the right hand side panels plot them in instrument space.

Nash equilibrium

    Variables :

$$MRF: \quad \pi = \frac{\sigma \alpha_m (y_m - y)}{\xi + \sigma \omega}$$

$$FRF: \quad \pi = \frac{\gamma (\sigma \pi^e + \sigma v - \xi \varepsilon) - \delta^2 \xi \alpha_f y_f}{\gamma \sigma - \delta^2 \xi \omega} + \frac{\gamma (\xi + \sigma \omega) + \delta^2 \xi \alpha_f}{\gamma \sigma - \delta^2 \xi \omega} y$$

    Instruments :

$$MRF: \quad g = \frac{\sigma^2 \alpha_m + (\xi + \sigma \omega)^2}{(\sigma \alpha_m + (\xi + \sigma \omega)\omega)\delta} r + \frac{\sigma \alpha_m (y_m - \varepsilon) - (\xi + \sigma \omega)(\omega \varepsilon + \pi^e + v)}{(\sigma \alpha_m + (\xi + \sigma \omega)\omega)\delta}$$

$$FRF: \quad g = \frac{((\alpha_f + \omega^2)\sigma + \xi \omega)\delta}{(\gamma + \delta^2 (\alpha_f + \omega^2))} r + \frac{\sigma \alpha_f (y_f - \varepsilon) - \delta \omega (\omega \varepsilon + \pi^e + v)}{(\gamma + \delta^2 (\alpha_f + \omega^2))}$$

Fiscal Leadership

    Variables :

$$MRF: \quad \pi = \frac{\sigma \alpha_m (y_m - y)}{\xi + \sigma \omega}$$

$$FRF: \quad \pi = \frac{((\xi + \sigma \omega)(\omega + 1) + \sigma \alpha_m)\gamma (\sigma \pi^e + \sigma v - \xi \varepsilon) - (\omega + 1)\delta^2 \xi^2 \alpha_f y_f}{\gamma \sigma (\xi + \sigma \omega)(\omega + 1) + (\delta^2 \xi^2 + \sigma^2 \gamma)\alpha_m}$$

$$+ \frac{\gamma (\omega + 1)(\xi + \sigma \omega)^2 + \delta^2 \xi^2 (\omega + 1)\alpha_f + \gamma \sigma (\xi + \sigma \omega)\alpha_m}{\gamma \sigma (\xi + \sigma \omega)(\omega + 1) + (\delta^2 \xi^2 + \sigma^2 \gamma)\alpha_m} y$$

    Instruments :

$$MRF: \quad g = \frac{\sigma^2 \alpha_m + (\xi + \sigma \omega)^2}{(\sigma \alpha_m + (\xi + \sigma \omega)\omega)\delta} r + \frac{\sigma \alpha_m (y_m - \varepsilon) - (\xi + \sigma \omega)(\omega \varepsilon + \pi^e + v)}{(\sigma \alpha_m + (\xi + \sigma \omega)\omega)\delta}$$

$$FRF: \quad g = \frac{(\alpha_m (\xi + \sigma \omega) - \sigma \alpha_f (\omega + 1))\xi \delta}{(\gamma (\xi + \sigma \omega) + \alpha_f \delta^2 \xi)(\omega + 1) + (\delta^2 \xi \omega - \sigma \gamma)\alpha_m} r$$

$$+ \frac{\delta \xi ((\omega + 1)\alpha_f (y_f - \varepsilon) + \alpha_m (\omega \varepsilon + \pi^e + v))}{(\gamma (\xi + \sigma \omega) + \alpha_f \delta^2 \xi)(\omega + 1) + (\delta^2 \xi \omega - \sigma \gamma)\alpha_m}$$

Cooperation

If the authorities cooperate, then the two instruments are chosen jointly from a single set of first order conditions. We have split this set into two subsets of first order conditions, in each of which the optimal outcome for one policy instrument is shown as a function of the value of the other policy instrument. This enables us to present the solutions for the two policy instruments as two reaction functions. Having such a representation is convenient for comparison of the results with those for the Nash equilibrium and for Fiscal Leadership.

Variables :

$$\text{MRF} : \quad \pi = \frac{\sigma \alpha_b \left(y_b - y \right)}{\xi + \sigma \omega}$$

$$\text{FRF} : \quad \pi = \frac{\gamma \left(\sigma \pi^e + \sigma v - \xi \varepsilon \right) - \delta^2 \xi \alpha_b y_b}{\gamma \sigma - \delta^2 \xi \omega} + \frac{\gamma \left(\xi + \sigma \omega \right) + \delta^2 \xi \alpha_b}{\gamma \sigma - \delta^2 \xi \omega} y$$

Instruments :

$$\text{MRF} : \quad g = \frac{\sigma^2 \alpha_b + \left(\xi + \sigma \omega \right)^2}{\left(\sigma \alpha_b + \left(\xi + \sigma \omega \right) \omega \right) \delta} r + \frac{\sigma \alpha_b \left(y_b - \varepsilon \right) - \left(\xi + \sigma \omega \right) \left(\omega \varepsilon + \pi^e + v \right)}{\left(\sigma \alpha_b + \left(\xi + \sigma \omega \right) \omega \right) \delta}$$

$$\text{FRF} : \quad g = \frac{\left(\left(\alpha_b + \omega^2 \right) \sigma + \xi \omega \right) \delta}{\left(\gamma + \delta^2 \left(\alpha_b + \omega^2 \right) \right)} r + \frac{\sigma \alpha_b \left(y_b - \varepsilon \right) - \delta \omega \left(\omega \varepsilon + \pi^e + v \right)}{\left(\gamma + \delta^2 \left(\alpha_b + \omega^2 \right) \right)}$$

The reaction functions plotted in Figure 7 are similar to those plotted in Dixit and Lambertini (2000). We plot these reaction functions for both the Nash-equilibrium case and the Fiscal-Leadership-equilibrium case, in both variable space and instrument space. The point where they intersect is either the Nash equilibrium, denoted by "N", or the Fiscal Leadership equilibrium, denoted by "FL". We also depict the cooperative equilibrium, which is denoted by "C".

C.3   A Special Case: $\xi = 0$

Note that if we put $\xi = 0$ then, as noted above, the model amounts to a static version of the model presented in this paper. In these circumstances we obtain the following solutions for the three regimes:

Cooperation

$$\pi = \frac{\alpha_b \left(\omega y_b + \pi^e + v \right)}{\alpha_b + \omega^2}, \quad y = \frac{\alpha_b y_b - \omega \left(\pi^e + v \right)}{\alpha_b + \omega^2}$$

$$r = \frac{-\alpha_b y_b + \left(\alpha_b + \omega^2 \right) \varepsilon + \omega \left(\pi^e + v \right)}{\sigma \left(\alpha_b + \omega \right)}, \quad g = 0$$

Nash

$$\pi = \frac{\alpha_m \left(\omega y_m + \pi^e + v \right)}{\alpha_m + \omega^2}, \quad y = \frac{\alpha_m y_m - \omega \left(\pi^e + v \right)}{\alpha_m + \omega^2}$$

$$r = \frac{\left(\dfrac{\delta^2}{\gamma} \alpha_f y_f + \varepsilon \right) \left(\alpha_m + \omega^2 \right) + \left(\dfrac{\delta^2}{\gamma} \left(\alpha_f + \omega^2 \right) + 1 \right) \alpha_m y_m + \left(\dfrac{\delta^2}{\gamma} \left(\alpha_f - \alpha_m \right) + 1 \right) \omega \left(\pi^e + v \right)}{\sigma \left(\alpha_m + \omega \right)},$$

$$g = \frac{\delta}{\gamma \left(\alpha_m + \omega \right)} \left(\left(\alpha_m + \omega^2 \right) \alpha_f y_f - \left(\alpha_f + \omega^2 \right) \alpha_m y_m + \left(\alpha_f - \alpha_m \right) \omega \left(\pi^e + v \right) \right)$$

Fiscal Leadership

$$\pi = \frac{\alpha_m \left(\omega y_m + \pi^e + v \right)}{\alpha_m + \omega^2}, \quad y = \frac{\alpha_m y_m - \omega \left(\pi^e + v \right)}{\alpha_m + \omega^2}$$

$$r = \frac{-\alpha_m y_m + \left(\alpha_m + \omega^2 \right) \varepsilon + \omega \left(\pi^e + v \right)}{\sigma \left(\alpha_m + \omega \right)}, \quad g=0$$

We plot the reaction functions and the resulting equilibria for these three regimes, in this special case, in the upper part of Figure 7. We can see three things.

(i)   In all three regimes the outcomes for inflation and output are the same. This is a consequence of the fact that the policy instruments are perfect substitutes in the control of output and inflation. (That this is the case can be verified from the formulae above, after noting that $\alpha_m = \alpha_b$ and $y_m = y_{b,.}$)

(ii)   In the Nash game, when the fiscal authorities have distorted objectives, they will wish for higher output than if they were benevolent. We can use the first row of Figure 7 to describe what then happens, by means of a virtual thought experiment. We start at point C in both boxes of the first row, which we would be at if the fiscal authorities were benevolent. A non-benevolent fiscal authority will increase *g,* above zero, even although this is costly, to reach its reaction function (which lies above the plotted region in the right-hand box). Monetary policy will then be tightened in order to fight the resulting inflation. But because the instruments are perfect substitutes this will return the economy right back to C in the left-hand box, but of course we will lie to the northeast of C in the right-hand box. The fiscal authorities will then expand in reply. This "civil war" between the two authorities will continue until the cost of raising *g* further becomes prohibitive. The two reaction functions intersect far to the right and above outside the plotted area in the right hand box. There is no debt in this model but if there was then this civil war would result in catastrophic consequences for debt, as in the model presented in the main body of this paper.

(iii)   Fiscal leadership will, however, replicate the cooperative equilibrium in the instrument space (providing that the monetary authorities are benevolent, as we assume). Thus, even if fiscal authorities are *not* benevolent, the outcome when there is fiscal leadership completely replicates the cooperative equilibrium which emerges when *both* policymakers are benevolent. This is because, when the fiscal authorities lead, they *know* that if they increase *g* in order to increase output, the monetary authority would then completely counteract their policy. They will therefore not do this.

These are essentially simple static versions of the results of the present paper, as reported in Section 4.

C.4   The more general case: ξ ≠ 0

These results do not hold in a model with $\xi \neq 0$. In this more general case the potential for conflict between the policy authorities is modified. This is fundamentally because when authorities are benevolent and cooperate, the interest rate can be raised to fight inflation, and there can then be fiscal expansion to offset the recession which these higher interest rates would create, without fully removing the effects on inflation of the higher interest rate. As a result, that both *r* and *g* will be raised more than when ξ=0, inflation will be *much* lower, and output will be *much* higher. The equilibrium, is denoted by the letter C in the lower parts of Figure 7. If we had debt in this model, then it would rise more than in the case of cooperation when ξ=0.

In the Nash game, when the fiscal authorities have distorted objectives, they can expand by more than they would if they were benevolent. They will thus create more inflation. Monetary policy will then contract more than if they were benevolent, in order to fight this resulting inflation. The fiscal authorities will expand in reply, and this continues until the cost of raising *g* further becomes prohibitive. But the battle between the policymakers will much less severe than when ξ = 0, because the higher inflation which the fiscal authorities cause is much more easily removed by the resulting increase in *r*. Overall, the outcomes of this Nash game are clearly much less bad than they would be when ξ=0. If we had debt in this model, then it would rise more than in the case of cooperation, but not nearly as much as in the Nash equilibrium when ξ=0.

When the fiscal authorities lead, they *know* that if they expand in order to fight the recession, then the monetary policy will contract in order to fight the resulting inflation. But, unlike in the case when ξ=0, they know that there is some "mileage" in their doing this, since the monetary authorities do not now need to fully remove the higher level of output in order to keep inflation low. As a result, the fiscal and monetary authorities will now fight each other, and we can see that they do this nearly as much as they would do in the Nash case. Again, if we had debt in this model, it would rise more than in the case of cooperation, and nearly as much as in the Nash equilibrium. Of course debt will now rise by *more* than it would in the fiscal leadership case when ξ=0.
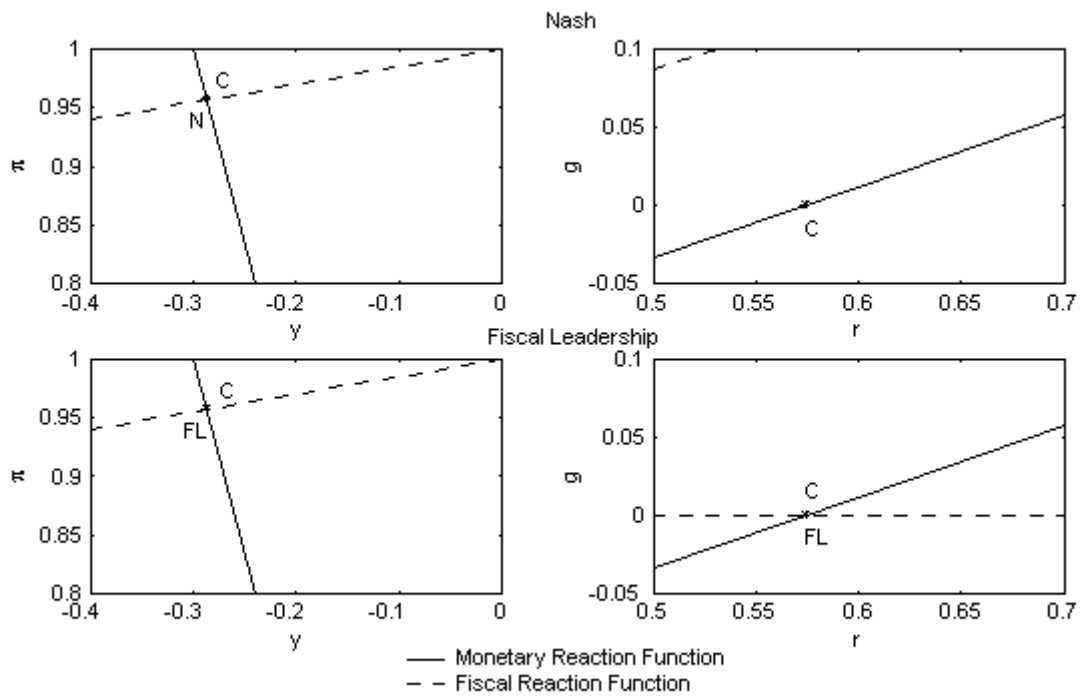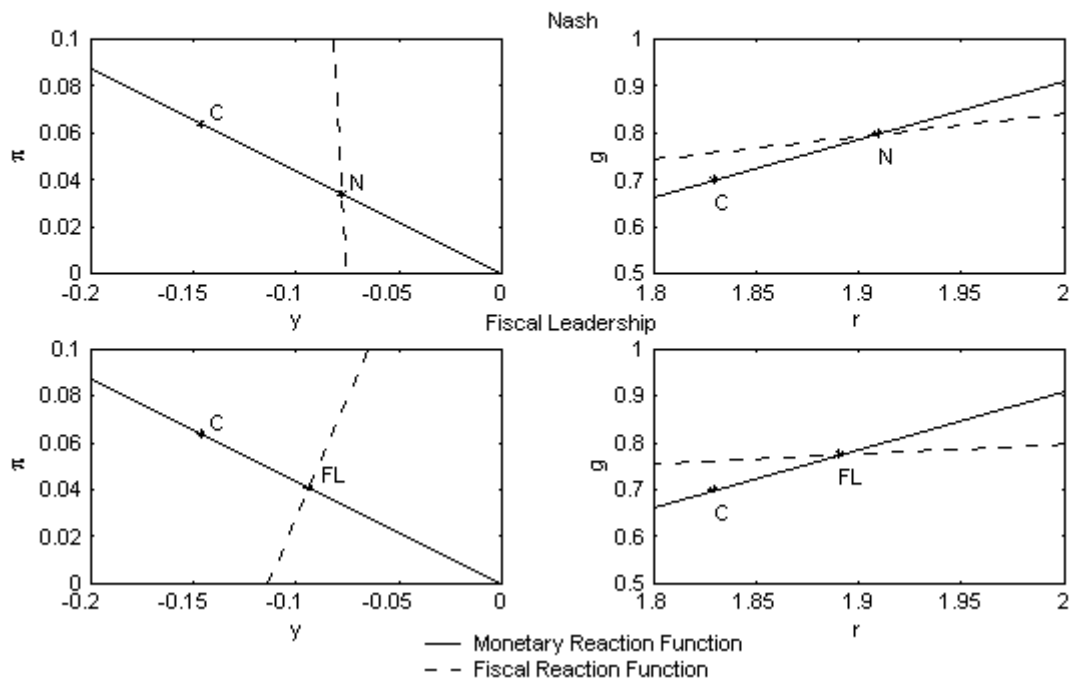
Figure 7: Monetary and Fiscal Reaction Functions for the Static Model