Economics Department
Discussion Papers Series
ISSN 1473 – 3307

Bank Equity and Macroprudential Policy

Keqing Liu

Paper number 15/03

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Keqing Liu[†]

University of Exeter

March 10, 2015

Abstract

We investigate a new macroprudential policy in a DSGE model with financial frictions. As Gertler, Kiyotaki and Queralto (2012), we propose to subsidize bank equities. However, our tax rate is different from their policy. The tax rate in our macroprudential policy is proportional to capital ratio gap while it is proportional to the shadow price of bank deposit in Gertler et al. (2012). Our policy has two advantages: Firstly, because bank's balance sheet structure is observable target for central bank, our policy is more applicable for practical policy design. Secondly, our policy makes individual banks choose to raise more capital. While it tightens the moral hazard constraint, the policy could raise the future value of investment and it shows the modified policy is welfare dominant.

Keywords: Macroprudential policy, Bank equity, Capital ratio, DSGE model

JEL classification: C61, E61, G28

1 Introduction

The financial crisis in 2009 has highlighted needs for macroprudential policy, which should focus on financial stability. It should help decrease the external loss in terms of real output for the whole economy caused by the financial distress. In Basel III, policy makers call for macroprudential policies like higher bank capital requirements and sufficient capital buffers. Designing macroprudential policy becomes the most important task for macroeconomists. For that purpose, Gertler and Kiyotaki (2012) constructed a DSGE model with financial frictions.

The financial shock is modelled as exogenous deterioration of bank assets, and the banks make decisions on their balance sheet structure before the financial shock comes. In a decentralised economy, they choose a capital ratio lower than the socially efficient level. Gertler, Kiyotaki and Queralto(2012) proposed a macroprudential policy which increase the capital ratio. They propose to subsidize bank equities. This paper shows how their policy can be improved by a changing the design of the subsidy rate. Our macroprudential policy choose the level of the subsidy ratio directly according to bank's capital ratio. The result shows that the new macroprudential policy can help raise the capital ratio to a higher level and it is welfare improving.

Low capital ratio on banks' balance sheet is an important factor of excessive systemic risk and financial instability, and it is crucial to explain why banks choose such a more risky balance sheet. Some literature, i.g., Lorenzoni (2008), Bianchi (2009), Korinek (2011), Stein (2011), shows that with financial frictions and credit constraints, banks tend to issue excessive short term debts and undervalue the importance of raising bank capital. This overborrowing feature makes commercial banks vulnerable to risks, because short term debts would be affected by credit crunch or 'fire sales' effect. There is a fundamental failure of the financial market that financial intermediaries issue too much short-term debts and leave the financial system excessively vulnerable to financial crisis. By following Gertler et al. (2012), An agent problem is embedded in the bank sector as financial frictions and the feature of individual banks in the model is consistent with the literature.

Because of this feature of commercial banks, literature for the macroprudential policy instrument focuses on regulations of bank capital. In Kashyap and Stein (2004), Dewatripont and Tirole (2012), bank regulations like time-varying capital requirement and countercyclical capital buffers is analyzed. Some researchers call for substantially higher capital ratio than the market determined ratio, like Admati et. al. (2010) and Hanson et al. (2011), saying that bank equity is not so costly. However, these instruments have a procyclicality feature. Perotti and Suarez (2009) propose liquidity risk charges to change the incentive of banks on the issuance of bank equities and short term debts, which could overcome the procyclicality of macroprudential policy. In Gertler et al. (2012), they introduce a macroprudential policy a in DSGE model to overcome the time consistency problem of the government bailout during the financial crisis. The macroprudential policy is designed to respond to the shadow cost of the deposit. This policy has a nice property: it induces bankers to choose higher capital ratio at the steady state, so it is actually working as capital ratio requirement. The policy is shown to be welfare improving. Based on their research, we try to find possible improvement of their policy because of two reasons: Firstly, for practical reasons, their policy target can not be easily observed in the economy, and policy makers may find difficulty in measuring it. Secondly, the macroprudential policy that targets at the shadow cost of the bank deposit haven't been shown to be optimal on social welfare improvement.

So could we find a macroprudential policy which has a better performance on mitigating the consequece of financial shocks and improving social welfare? To answer this question, we develop a new macroprudential policy in a macroeconomic model. The basic framework is similar as Gertler and Karadi (2011) and Gertler, Kiyotaki and Queralto (2012). In the model, banks have alternatives to issue either short term debts or bank equities. An agent problem between banks and bank managers is the source of financial frictions. The advantage of this model is that it is possible to explain why banks choose a low capital ratio before a financial crisis. Briefly, the moral hazard problem alters bank's balance sheet decision and household's saving choice, and it is shown to lower the capital ratio on bank's balance sheet.

In this paper, we provide an improved version of macroprudential policy which dominates the previous one in Gertler et al. (2012). Instead of designing the policy on the basis of the shadow price of the net worth, we target at the gap between bank's capital ratio and central bank's capital ratio anchor. One obvious advantage of this policy is that bank's capital ratio is a observable target for the central bank, and it reveals the commercial banks' ability against financial risks. In our policy, the central bank announces a benchmark capital ratio anchor and take proportional liquidity risk charges if individual banks choose not to reach the capital ratio anchor. The larger deviation to the capital ratio anchor is, the higher ratio of the charge will be on short term debts. The policy could offset externalities in the bank sector caused by the moral hazard problem. To get rid of the policy distortion on banks' balance sheet, central bank uses the tax the get from the deposit to subsidize banks issuing bank equities, so the policy can change banks' balance sheet only by changing the future value of bank equities and deposits. We compare the performance of new macroprudential policy with the Gertler et al.'s policy and find the results as follows: Firstly, with the implementation of the new macroprudential policy, the volatility of risky return and the credit spread is decreased more than the benchmark policy. Smaller volatility of return helps decrease the financial risk from banks' perception, so banks are more willing to lend fund to final goods producing firms. The decrease of the credit spread minishes the wedge in financial market caused by the moral hazard problem and help raise the total bank asset at the steady state. It helps stimulate the investment and real output production. Secondly, we can show that our macroprudential policy could encourage banks to keep a higher capital ratio before the financial crisis, because it raises the future value of the bank equity, and part of the reason is the reduction of volatility on risky return. Although the higher capital ratio reveals higher competition in the financial market, it tightens the moral hazard constraint and the negative effect is costly. Thirdly, the policy also raises the shadow price of bank assets, so the bank total lending increases. Meanwhile, the bank manager find it more profitable to keep bank assets in the bank, it relaxes the moral hazard constraint. In general, if the overall taxation ratio of the policy stays fixed, our new policy is shown to increase the excess value and decrease

the volatility of risky return more, and change banks' balance sheet structure to higher capital ratio. We use the consumption equivalence at the risky steady state to compare the policy performance, and the result shows that the social welfare is improved by the implementation of the new macroprudential policy, considering a range of parameters.

The outline of the paper is as follows. In Section 2 we introduce the model. In Section 3 we provide the simulation results and compare performances of two different macroprudential policies, and Section 4 concludes. We provided detailed derivation of the model and proofs in Appendix.

2 The Model

The model we use is a variant of Gertler et al. (2012). It is a standard DSGE model with financial frictions. In this section we describe each sector in the model: households, goods producers, capital producers and government. The summary of system of equations is listed in Appendix 5.1.

Before we begin, we introduce how a financial crisis is modeled. Following Gertler et al. (2012), the financial crisis is modeled as an exogenous shock on aggregate capital stock. As will become clear, the deterioration of the capital stock will be reflected on the asset side of individual banks. It is a simple way to introduce asset price variation. The aggregate capital stock suffers a negative capital quality shock when a financial crisis happens.

$$K_{t+1} = \psi_{t+1} S_t,$$

where K_{t+1} is the aggregate capital stock at the beginning of time period $t + 1$, S_{t+1} is the capital after physical production and capital accumulation at the end of period t, ψ_{t+1} is the capital quality shock from time t to $t + 1$. The shock ψ_{t+1} is an i.i.d. process, with an unconditional mean of unity. It is a trigger of large deterioration of capital and this is how to model a financial crisis.

2.1 Households

A representative household has a continuum of members with unit 1. There are f fraction of workers and $1 - f$ fraction of bankers in the households. Bankers work as bank managers and they can make decisions on bank's balance sheet. They transfer dividends back to households when they quit the bank sector. Workers supply labour to physical production firms to gain wages. Because there are a fraction of bankers quit the bank sector to become workers each period, we assume that some workers go into the bank sector with initial wealth to become bankers. So the fraction of bankers and workers keeps constant.

The utility function follows Guvenen (2009) and Greenwood, Hercowitz and Huffman (1988) and it allows for habit formation. The reason why we prefer

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this form of preference is that it produces labour volatility with little cost of complexity and improves the quantitative performance of the model. The representative household has the expected discounted objective function as follows.

$$\mathbb{E}_t \sum_{\tau=t}^{\infty} \beta^{\tau-t} U\left(C_\tau, L_\tau\right) = \mathbb{E}_t \sum_{\tau=t}^{\infty} \beta^{\tau-t} \frac{1}{1-\gamma} \left(C_\tau - hC_{\tau-1} - \frac{\chi}{1+\varphi} L_\tau^{1+\varphi} \right)^{1-\gamma}.$$

(1)

where $\mathbb{E}_t(\cdot)$ denote the expectation conditional on the information set at time t, β is discount factor, γ is risk aversion, h stands for the habit parameter, χ is the weight parameter of labour and φ is the Inverse Frisch elasticity of labour supply.

Note that households do not acquire capital nor provide funds directly to goods producers or capital producers. They supply fund only to banks. Households have two different assets to choose, the risk free short term debts (it is also called deposit, D_t) or the time-contingent bank equities (e_t). Households can also turn to riskless government bonds which is perfect substitutes to the short term debts.

Banks can choose the value of one unit of the bank equity to be any divisible size, so it is possible to normalize one unit of bank equity to correspond to one unit of the bank asset. Similarly, non-financial firms can do normalization of firm securities as well. Denote Z_t as the return of the bank asset from time $t-1$ to t. q_t is the price of the bank equity at time period t. The payoff of one unit of bank equity at time t will be $(Z_t + (1-\delta)q_t)\psi_t$. Goods producers are assume to be lack of fund and they always need to borrow from banks. As will become clear, banks lend money to the goods producers and firms use the fund to buy capital. So banks' asset side of balance sheet corresponds to the return of capital in non-financial firms. Plus the banks get all the gross profit earned by goods producers, their payoff accounts for the physical depreciation δ and the capital quality shock ψ_t.

Assume that the representative household maximizes his or her expected discounted utility function by choosing consumption (C_t), labour supply (L_t), quantities of riskless short term debts (D_t) and bank equities (e_t) at time t, subject to the budget constraint:

$$C_t + D_t + q_t e_t = W_t L_t + \Pi_t - T_t + R_t D_{t-1} + (Z_t + (1-\delta)q_t)\psi_t e_{t-1},$$

(2)

where W_t is the wage of workers, Π_t is household's net profit from the ownership of capital producing firms and dividends from banks, T_t is lump-sum tax, R_t is risk free return from time $t-1$ to t and the following notation $\Lambda_{t,t+1}$ is the stochastic discount factor from time period t to $t+1$.

For convenience, we define marginal utility to consume $U_{C,t}$ and stochastic discount factor $\Lambda_{t,\tau}$ as

$$U_{C,t} \equiv J_t^{-\gamma} - \beta h J_{t+1}^{-\gamma},$$

(3)

$$J_t = C_t - hC_{t-1} - \frac{\chi}{1+\varphi}L_t^{1+\varphi},$$

$$\Lambda_{t,\tau} \equiv \beta^{\tau-t}\frac{U_{C,\tau}}{U_{C,t}}. \tag{4}$$

We can derive Euler equation, labour supply by solving representative household's optimization problem.

$$R_{t+1}\mathbb{E}_t\left(\Lambda_{t,t+1}\right) = 1, \tag{5}$$

$$\mathbb{E}_t\left(\Lambda_{t,t+1}R_{e,t+1}\right) = 1, \tag{6}$$

$$\mathbb{E}_t U_{Ct} W_t = \chi J_t^{-\gamma} L_t^\varphi, \tag{7}$$

$$R_{e,t} = \frac{(Z_t + (1-\delta)q_t)\,\psi_t}{q_{t-1}}. \tag{8}$$

2.2   Goods Producers

The competitive goods producers buy capital (K_t) from capital producers and hire labour (L_t) from households to produce identical final output. We assume that manufacturers are always lack of fund. Also, they cannot borrow fund directly from households, so they have to borrow fund from banks to proceed physical production. Assume that banks have access to all the information of their borrowing firms. So firms borrow money from banks without any frictions. They commit to pay all the profit they earn to banks. They get fund from banks by issuing state-contingent firm securities with price Q_t to banks. As mentioned before, firms can normalize one unit of firm security to correspond to one unit of capital they use for production. In banks sector, banks could also choose the value per unit of bank equity, so we also normalize one unit of bank equity such that its claim corresponds to the future return of one unit of bank asset S_t, so as to relate it to profit per capital Z_t. After the normalization of both firm equities and bank equities, the price of capital can be equal to the price of firm securities, Q_t.

The firm produces output with a constant returns to scale Cobb-Douglas production function

$$Y_t = A_t K_t^\alpha L_t^{1-\alpha}, \tag{9}$$

where , $0 < \alpha < 1$ and A_t denotes aggregate productivity. In this paper, we focus on investigating the effect of capital quality shock ψ_t, which captured a financial crisis. We assume there is no technological shock in the model for

simplicity, so the technological parameter is a constant number, $A_t = 1$ for $t = 1, 2, \ldots$.

First order condition w.r.t. labour is

$$W_t = (1 - \alpha)\frac{Y_t}{L_t}.$$

We denote Z_t as return of bank equity, and it also stands for the profit per unit of capital. Here it can be expressed as

$$Z_t = \frac{Y_t - W_t L_t}{K_t} = a\frac{Y_t}{K_t} = a\frac{A_t L_t^{1-\alpha}}{K_t^{1-\alpha}} = aA_t\left(\frac{L_t}{K_t}\right)^{1-\alpha}.$$

Since all the profit of capital flows from firms to banks via the time-contingent firm equities, manufacturers have no profit left each period. The return of firm equity accounts for the return per unit of capital and physical depreciation, the price change of the capital, Q_t. So the return of firm equity can be written as

$$R_{k,t} = \frac{[Z_t + (1 - \delta)Q_t]\,\psi_t}{Q_{t-1}}. \tag{10}$$

2.3 Capital Producers

Capital producers use final output as input to produce new capital with flow variable adjustment costs. They sell capital to goods producers at the price Q_t. We assume that capital producers work as a independent sector and households own them. The profit they make flows back to households at the end of each period. Adjustment costs function $f(\cdot)$ is convex, and it has the property that $f(1) = f'(1) = 0$, $f''(x) > 0$ for any $x > 0$. The profit maximization problem of capital producers is to choose the scale of investment I_t to produce capital given the price of capital Q_t,subject to the adjustment costs. The objective function is

$$\max \mathbb{E}_t \sum_{\tau=t}^{\infty} \Lambda_{t,\tau} \left\{ Q_\tau I_\tau - I_\tau \left[1 + f(\frac{I_\tau}{I_{\tau-1}})\right] \right\}.$$

The concavity of the objective function ensures the interior solution can be found. The first order condition with respect to I_t shows that the marginal cost of capital production should be equal to the price of the capital, Q_t.

$$Q_t = 1 + f(\frac{I_t}{I_{t-1}}) + \frac{I_t}{I_{t-1}}f'(\frac{I_t}{I_{t-1}}) - \mathbb{E}_t\left[\Lambda_{t,t+1}\left(\frac{I_{t+1}}{I_t}\right)^2 f'(\frac{I_{t+1}}{I_t})\right] \tag{11}$$

In the simulation, we specify the adjustment costs to be quadratic: $f(\frac{I_\tau}{I_{\tau-1}}) = \Psi\left(\frac{I_\tau}{I_{\tau-1}} - 1\right)^2.$

2.4 Banks

In complete and competitive financial market, the return of bank equities, bank deposits and firm equities should be equal. They tend to be slightly different in our model because of the moral hazard problem in bank sector. In our model, the individual banks work as financial intermediaries and gain profit by the spread between the returns of lending and borrowing. They absorb deposits and issue bank equities to raise fund, and then lend the raised fund along with their retained capital to goods producers. Since the households cannot lend money directly to the firms, the flow of fund must go through the banks.

Each period they keep their retained profit in the bank as net worth. Individual banks face their flow constraints of balance sheet, where the total value of the loans they provide to non-financial firms should not exceed the total value of raised fund plus net worth they have at the current period.

$$Q_t s_{p,t} = n_t + q_t e_t + D_t. \tag{12}$$

The total value of the loans at period t should be the scale of firm securities times price of the it, Q_t. We denote $s_{p,t}$ as the amount of firm securities that banks buy from non-financial firms, e_t as the amount of bank equity that households buy from banks, n_t as the net worth the bank have, D_t is the bank deposit, q_t is the price of bank equities. As we mentioned before, we assume there is no financial friction between firms and banks. It indicates that all the profit of capital flows from firms to banks via the time-contingent firm equities. It also means that any factors which could affect the profit per capital in final output production would affect the asset side of the banks, too.

As shown later in the optimal decision of banks (Equation 19), the net worth is a crucial variable when they face the moral hazard constraint on raising fund. We have another two assumptions to ensure that banks cannot accumulate enough net worth to get rid of raising fund from households. Firstly, there is no way could new banker go into the sector with sufficient initial net wealth. Secondly, we assume that banks have a constant probability of quitting the bank sector each period. Therefore, banks have to issue equities and absorb deposits each period. Net worth of the bank at the beginning of time period t is equal to the profit of bank's investment in the last period. It is the gross payoff from the bank assets, net of the borrowing cost of bank equities and deposits. A low of motion of net worth n_t can be written as

$$n_t = R_{k,t} Q_{t-1} s_{p,t-1} - R_t D_{t-1} - R_{e,t} q_{t-1} e_{t-1}. \tag{13}$$

In the net worth accumulation function above, the return of bank equity $R_{e,t}$ is derived in households sector (Equation 8). The gross return of bank asset $R_{k,t}$ from $t-1$ to t is derived in goods producers sector (Equation 10).

The objective of individual banks is to accumulate the retained earning to get the maximal dividends when they quit the bank sector, facing with the net

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worth accumulation constraint described above. To be specific, their objective is to maximize the expected discounted value of the future terminal dividends considering all the possibility of quit at each period in the future. We denote bankers' probability of exiting the financial market as σ, the value function at the maximum can be expressed as

$$V_t = \max \mathbb{E}_t \left[\sum_{\tau=t+1}^{\infty} (1-\sigma)\sigma^{\tau-t-1}\Lambda_{t,\tau}n_\tau \right].$$ (14)

Before we add the moral hazard problem into the bank sector, we define two important ratios: the leverage ratio ϕ_t and the ratio of outside bank equity to the total bank assets, x_t. The former is defined as the ratio of total bank assets to net worth. It is an indicator how much the bank risk is leveraged. The latter is an crucial indicator of the moral hazard constraint. They are defined as follows.

$$\phi_t = \frac{Q_t s_{p,t}}{n_t},$$ (15)

$$x_t = \frac{q_t e_t}{Q_t s_{p,t}}.$$ (16)

We embed a moral hazard problem in the bank sector between shareholders and bank managers. It was first introduced in Gertler and Karadi (2010). After the bank obtain the fund from households, the bank manager who control the bank assets have incentive to transfer a fraction of the bank assets back to his family. Denote the proportion of diversion as Θ_t. After households recognize the possibility of bank manager's cheating, they limit the fund they provide to banks, and proportion of deposits and bank equities changes, too.

We assume the fraction of diversion from the total bank assets (Θ_t) depends on the current composition of bank's borrowing, e.g., x_t. Generally, it is more difficult to divert assets funded by short term deposits than outside equities. Because debts are the first priory and require banks to meet a certain value of payment with full responsibility. But dividend payments are tied to the performance of bank's assets, which is difficult to monitor by the outside bank equity holders.

So, the fraction of diversion is relevant to x_t, we specify it to be quadratic as follows, with $\varepsilon < 0$, $\kappa > 0$ (Equation 17). The intuition is: if households hold some bank equities, they somehow have more access to the information compared with the short term debt holders, therefore they have some advantages in monitoring the bank managers. Thus the bank managers will decrease the fraction of diversion if households choose to be a shareholder with small amount of bank equities. As households increase the fraction of bank equities they buy, however, the bank managers have larger incentive to cheat,

$$\Theta(x_t) = \theta \left(1 + \varepsilon x_t + \frac{\kappa}{2}x_t^2 \right).$$ (17)

The two-players game can be briefly described as follows: If the bank divert some fraction of assets back to households, the debts will default and bank will face bankruptcy, bank manager will get zero dividend. The creditors can only get the remaining fraction $1 - \Theta(x_t)$ of total asset as liquidation assets. Since households can recognize the bank manager's incentive to divert fund, they will restrict the amount of lending, such that bank manager's profit of cheating cannot exceed the dividends if they choose to keep the money in the bank. In this way, bank managers will not choose to cheat, but the banks will face an external funding constraint: the maximum value of the future dividends should be larger than the payoff of diversion at each period.

$$V_t \geq \Theta(x_t)Q_t s_t \tag{18}$$

By doing this, households can avoid bank managers' diversion behavior each period. The moral hazard constraint involves the value function V_t, which we don't know yet. So we should find the functional form of the value function first. In Gertler et al. (2012) they use guess and verify method to get the optimal solution, but here we can show a straightforward way to derive the specific form of the value function:

$$V_t = (\mu_{s,t} + x_t \mu_{e,t})Q_t s_t + v_t n_t, \tag{19}$$

where

$$v_t = \mathbb{E}_t \left(\Lambda_{t,t+1} \Omega_{t+1} \right) R_{t+1},$$

$$\mu_{s,t} = \mathbb{E}_t \left[\Lambda_{t,t+1} \Omega_{t+1} \left(R_{k,t+1} - R_{t+1} \right) \right],$$
$$\mu_{e,t} = \mathbb{E}_t \left[\Lambda_{t,t+1} \Omega_{t+1} \left(R_{t+1} - R_{e,t+1} \right) \right],$$
$$\Omega_{t+1} = (1 - \sigma) + \sigma \left[\phi_t \left(\mu_{s,t+1} + x_{t+1} \mu_{e,t+1} \right) + v_{t+1} \right].$$

There are more details of the moral hazard problem between bank managers and bank equity holders in Gertler and Karadi (2010). The detail of the proof is shown in Appendix 5.2.1.

At the end of bank sector's description, we need to mention that the value of Q_t and q_t are slightly different generally. Since banks can exclusively provide funds to the goods producers without any frictional costs, the price of one unit of capital (Q_t) should be a bit lower than the price of the bank equity (q_t). Another source of difference is from the moral hazard problem, which would alter the bank equity market condition compare to the frictionless case.

2.5 Net Worth, Assets and Resource Constraint

We mentioned in Section 2.1 that a fraction of workers go into the bank sector to become bank managers with a certain amount of initial wealth each period. We assume that the initial wealth they bring in is a fixed proportion of the total

bank asset at the end of the current period. Denote the initial wealth to be $n_{y,t}$ and the constant fraction to be ξ, so we have

$$n_{y,t} = \xi R_{k,t} Q_{t-1} s_{p,t-1}.$$

The net worth accumulation function can be modified as follows if we consider the dynamics of bankers and workers.

$$n_t = \sigma \left(R_{t,k} Q_{t-1} s_{p,t-1} - R_t D_{t-1} - R_{e,t} q_{t-1} e_{t-1} \right) + n_{y,t}, \qquad (20)$$

where σ is the probability of bankers surviving in the bank sector for one period.

The domestic final output is split into three parts: consumption (C), investment (I) and government purchase (G). Capital producers endure adjustment cost in the capital production procedure. So the resource constraint is

$$Y_t = C_t + \left[1 + f \left(\frac{I_t}{I_{t-1}} \right) \right] I_t + G_t. \qquad (21)$$

Non-financial firms produce the final output with a constant rate of physical depreciation, δ. The accumulation of aggregate capital stock is subject to

$$S_{t+1} = (1 - \delta) K_t + I_t \qquad (22)$$

where S_{t+1} is the aggregate capital stock 'in progress' at the end of time period t after the physical production. Capital 'in progress' is then transformed into the capital at the next period after the realization of the exogenous capital quality shock, ψ_{t+1}, from time period t to $t+1$.

2.6 Macroprudential Policy

In our model, individual banks tend to issue more short term debts with financial frictions and it makes themselves exposed to risks. Banks didn't consider the externality that holding too much short term debt will increase the systemic risk and make the financial system more vulnerable. During the financial crisis, credit spread increases sharply and it raises the cost of investment. The shrinkage of firms borrowing causes larger decrease of real output production. To solve the firms' borrowing problem, Gertler and Karadi (2010) propose a credit policy: the central bank lend money directly to the firm by buying large amount of firm equities. It is functioning as financial intermediaries during the financial crisis. By doing this, firm's borrowing keeps high and the consequences of the financial crisis on real economy can be mitigated.

Although the credit policy could maintain the firm's borrowing in a financial crisis, it has adverse effect on the bank sector. It is shown in Gertler et al. (2012) that the possible implementation of credit policy induces the individual banks to rely more on short term debts and take even more excessive risk. That is

because the banks have the perception that the central bank is going to intervene the credit market directly during the financial crisis. In this case, they choose to issue more short term debts than without credit policy. There is literature showing the similar time consistency problem of credit intervention. Diamond and Rajan (2009) mention that this credit intervention is not always feasible. And if it is feasible, it could encourage individuals banks to increase leverage or fund even more short term debts. In Farhi and Tirole (2009), affecting the financial market only by interest rate is not enough. They suggest that the focus of financial regulation should be on aggregate leverage and liquidity hoarding. Chari and Kehoe (2010) explore the time inconsistency and find out that the government intervention during the financial crisis has more severe time inconsistency problem.

To overcome these problems, A macroprudential policy is introduced in Gertler et al. (2012). The policy is aimed at encouraging individual banks to issue more bank equities instead of short term debts. By doing this, government offers banks a subsidy (τ_t^e) for each unit of bank equity they issue. To offset the overall taxation effect on bank's balance sheet, government finance the subsidy by taxing each unit of the private intermediated bank asset. Therefore central bank can only change the bank's balance sheet decision by changing the value of bank equities and short term debts in the future. So the banks' balance sheet has been modified into

$$\left(1 + \tau_t^k\right) Q_t s_{p,t} = N_t + \left(1 + \tau_t^e\right) q_t e_t + D_t, \tag{23}$$

where the taxation/subsidy ratio level corresponds to key variables in bank sector. In Gertler et al. (2012), they make it to respond to the shadow cost of bank deposit v_t,

$$\tau_t^e = \frac{\tau_1}{v_t}, \tag{24}$$

where τ_1 is a constant parameter chosen by the central bank. The price of the net worth reveals the cost of short term debts. So the higher the cost of the deposit is, the banks are unlikely to choose short term debts, so the smaller total subsidy ratio the central bank will apply to change bank's incentive. In this policy scheme, higher tax ratio causes the excess value of bank equity to increase, therefore induce banks to issue more bank equities than before. After the implementation of such macroprudential policy, the banks' capital ratio is increased. So it works as a capital ratio requirement. However, it has not been shown that this policy is not a optimal way to raise banks' capital ratio, so it might be possible that we find some improvement on policy design. Also, the shadow cost of deposit is not a perfect target for the central bank because they might face difficulty in practical implementation.

Based on their research, we introduce a new macroprudential policy. To what extent the central bank is going to intervene during the crisis depends on the design of macroprudential policy rule. Lots of literature on bank regulations suggests that macroprudential policy should focuses on regulations of

bank capital and capital requirements. So here we choose to make the subsidy ratio correspond directly to the capital ratio gap. The idea of the policy is from Perotti and Suarez (2009), Hanson et al. (2011). In this policy framework, central bank announces a benchmark capital ratio x^* for individual banks. Banks would be punished by higher charges on bank assets if they decide to have a lower ratio of bank equity to bank asset. This policy could help banks to keep higher capital ratio before the financial crisis. The motivation of the policy is that individual banks only consider how to maximize their future dividends pay off when they make balance sheet decision. They don't know holding to much short term debts makes them vulnerable to the financial shocks. Also, they don't know themselves that they could have been better hedged against risk if they choose to issue more bank equities.

In our model, capital ratio is an obvious indicator of bank's ability against risk. Higher capital ratio means the bank's risk to financial crisis is better hedged. That is because the return of bank equity is time-contingent. So it might change according to the performance of the bank's investment project, while the return of short term debt is constant and determined ex ante. We introduce the policy which targeted directly at the capital ratio,

$$\tau_t^e = \tau_0 \times (x^* - x_t). \tag{25}$$

In this macroprudential policy rule, x^* is the central bank's benchmark capital ratio anchor. It is exogenous and constant throughout the financial crisis in the model. τ_0 is the policy response parameter set by the central bank. The story is as follows: the central bank announces which level of capital ratio would be safe for individual banks to hedge the systemic risk, say, x^*, at the beginning. Central bank will make a asset transfer from banks asset to the bank equity if commercial banks doesn't reach such benchmark capital ratio anchor. They can also choose the taxation/subsidy rate parameter τ_0 in Equation (25), which means they could decide to what extent they are going to alter the bank's incentive of banks' equity issuance throughout the whole financial sector. In this policy, not only central bank could choose the taxation rate τ_0, but also the overall taxation τ_t^e depends on the current condition of bank's balance sheet. If the bank choose a risky balance sheet, the subsidy/taxation would be larger since the gap between the current capital ratio x_t and benchmark capital anchor x^* is higher. The good thing is, central bank could automatically change the degree of the intervention without changing the policy parameter when individual banks increase the capital ratio by about 10 percent during the financial crisis, shown in the simulation section. In order to offset the net effect of the policy on banks balance sheet decision, the following condition should be satisfied.

$$\tau_t^e = \frac{\tau_t^k}{x_t} \tag{26}$$

By changing the taxation scheme, the individual banks' optimal value func-

tion changes from Equation (19) to

$$V_t = \left[\left(\mu_{s,t} - \tau_t^k v_t \right) + \left(\mu_{e,t} + \tau_t^e v_t \right) x_t \right] Q_t s_t + v_t n_t. \qquad (27)$$

Also, the optimal choice of bank's capital ratio x_t is different from previous solution, Equation (**??**). It is slightly modified to

$$\left(\mu_{s,t} + \mu_{e,t} x_t \right) \left(\varepsilon + \kappa x_t \right) = \left(1 + \varepsilon x_t + \frac{\kappa}{2} x_t^2 \right) \left[\mu_{e,t} + \tau_0 v_t \left(x^* - x_t \right) \right]. \qquad (28)$$

The detailed derivation is shown in the Appendix.

There are some advantages of this policy rule compare to the macro prudential policy in Gertler et al. (2012). Firstly, focusing on individual bank's the capital ratio is a more direct and reasonable way because capital ratio is important information of bank's ability against risk. It makes sense to change the incentive of the banks more if they choose a low capital ratio on balance sheet. Secondly, central bank can determine two parameters to design the policy. They can change the policy by changing either τ and x^*, where the first parameter τ determine the ratio effect and the second one indicates level effect. The previous policy version just have one ratio parameter τ_1 and the sensitivity of deposit price level v_t is very small when we change different policy parameter. Thirdly, by comparing the performance of these two policies, our policy increases the social welfare more if we compare the consumption equivalence. In the simulation section, we evaluate baseline model without policy as a benchmark and compare both of the macroprudential policies with it to see how these macroprudential policies mitigate the negative effect of the financial shock.

3 Simulation

In this section we show numerical simulation results of the model. In Gertler et al., they show that the macroprudential policy which respond to the shadow cost of bank deposit is welfare improving. Here we choose the policy parameters which provides the best policy performances in both macroprudential policies repsectivelty and evaluate the performances of these two policies. The result shows that our macroprudential policy improves the overall social welfare more than the previous policy in these aspects: Firstly, our macroprudential policy help reduce volatility of risky returns and credit spread more to enhance the stability of the financial system. Secondly, new macroprudential policy is more sensitive to bank's balance sheet, and it provides higher capital ratio. Thirdly, the decrease of credit spread at the steady state reflects that our policy can cut down the wedge in the financial market and raise firm's borrowing. We compare the welfare of different policies using consumption equivalence at the risky steady state and show that the new macroprudential policy is more welfare improving. The details of the welfare description is shown in Appendix.

3.1 Parameters

We have 14 parameters in our baseline model. They are discount factor β, relative risk aversion parameter γ, the habit coefficient h, the weight coefficient of labour in utility function χ, the inverse Frisch elasticity of labour supply φ, the production function parameter α, physical depreciation rate δ, the elasticity of the price of capital to investment Ψ, banks' probability of survival σ, and some parameters like θ, ε, κ for moral hazard setting . Most of preference and technological parameter values are conventional in macroeconomic literature. The other parameters in bank sector is from Gertler et al. (2012). They are all provided in the table as follows.

Table.1 Parameters

β	0.99	discount factor
γ	2	relative risk aversion parameter
h	0.75	habit formation coefficient
χ	0.25	weight coefficient of labour
φ	0.33	inverse Frisch elasticity of labour supply
α	0.33	share of capital
δ	0.0025	physical depreciation rate
Ψ	1	the elasticity of the price of capital to investment
σ	0.9685	bank probability of survival
ξ	0.00289	households transfer ratio to new banks
θ	0.264	moral hazard parameter
ε	-1.21	
κ	13.41	

The parameters in non-financial sectors we use are conventional. We choose each bank has 3.15 percent probability of dying out so that individual banks can survive about 8 years on average. In Gertler et al. (2012), the parameters for bank sector are chosen in order to capture the characteristics of financial intermediaries before the financial crisis. For example, at steady state, the spread between the firm stock return and the risk-free rate is set to be close to 1 percent. The leverage ratio ϕ is chosen to satisfy the balance sheet structure where the total bank asset to retaining capital plus bank equities is approximately 4. Bank choose to sell 40 percent of bank equity to households. The risky steady state values are shown in Table.2 below. The first column displays the risky steady state of some key variables in economy without any policy. The second and the third columns illustrate the steady states with two different macroprudential policies, respectively.

Table.2 Risky steady states			
	no policy	Gertler et al.	modified policy
Y	24.04	24.16	24.22
C	18.73	18.81	18.85
L	8.22	8.25	8.27
K	212.45	214.06	214.84
N	31.90	31.74	31.41
r (%)	0.9974	0.9873	0.9864
r_k (%)	1.2417	1.2271	1.2226
spread (%)	0.2443	0.2398	0.2362
x	0.1009	0.1594	0.1791
v	1.65	1.72	1.78
μ_e	0.04142	0.02984	0.02993
μ_s	0.2709	0.3112	0.3164
$\mu_s + x\mu_e$	0.2751	0.3160	0.3218
ϕ	6.67	6.73	6.82
optimal ratio (τ_1/τ_0)	0	0.0027	0.09
τ^e	0	0.0046	0.00021
τ^k	0	0.00073	0.000037
J	1.5720	1.5765	1.5787
$\mathcal{W}$	-80.26	-79.94	-79.76
$\Gamma(\%)$	0	0.168	0.269

3.2 Comparison Between Two Macroprudential Policies

We show the simulation results of two different macroprudential policy rules we discussed in Section 2.6. The central bank taxes the bank assets and use the tax to subsidize bank equities they issue like Equation (23). The subsidy ratio on each unit of bank equity respond to indexes in the bank sector. We use the macroprudential policy in Gertler et al. (2012) as a benchmark, they make the subsidy ratio respond to the shadow price of net worth v_t. The difference in our policy is, to what extent we are going to tax/subsidize depends on the current balance sheet structure, like Equation (25). The current bank's capital ratio is a reasonable indicator of the bank's current ability against risk and we can use it to determine the tax level. Briefly, both of these policies could help encourage banks to issue larger fraction of bank equities, and they help reduce the volatility of the risky return rate, so as to raise the shadow price of both bank equities and bank assets. Compared with the benchmark policy, the new macroprudential policy has opposite effects on bank sectors. On one hand, the increase of bank equities to bank assets ratio tightens the moral hazard constraint. In the simulation, we choose the parameter of κ and ε such that the fraction of diversion is increasing to x_t when capital ratio is higher than 10 percent. So, from the steady state of x_t in Table.2, higher steady state value of x with macroprudential policy increases the bank managers' fraction of

diversion from banks. On the other hand, the increase of shadow price of bank asset induce banks incentivizes the bank managers to keep bank asset in the bank sector in order to get more dividends in the future. We find the results as follows: although our new macroprudential policy increase the capital ratio to a higher level to make the moral hazard constraint worse, it help decrease the volatility of risky return rate more, increase the competitiveness of the financial market and have higher shadow price of bank assets to increase the total lending scale of banks. The net effect shows to be positive, so it improves the social welfare. We try different parameters of the model within a reasonable interval and show the result is robust.

In the simulation of the benchmark policy, we check different subsidy ratio parameters τ_1 within an interval, from 0 to 0.01^1, and pin down the value of τ_1 to be equal to 0.0027, such that the social welfare reaches the highest level within this policy framework. The implementation of the benchmark macroprudential policy induces banks to keep higher ratio of bank equities to bank assets and raises the shadow price of bank assets compared with the baseline model. The second column of Table.2 shows the steady state result of the benchmark policy with highest welfare in the simulation. At the steady state, the ratio of bank equities to total bank assets x increases by more than 50 percent after the implementation of the policy. That is why the policy can be regarded as central bank's capital requirement. Compared the second moment of variables with the baseline model, we find out that the volatility of risk free return R_t and risky return R_k are significantly lower. Together with the decrease of the credit spread by 1.9 percent at the steady state, the benchmark policy helps enhance the stability of financial system. Also, the steady state of risk free return R_t, risky return R_k and the credit spread is lower than that of in baseline model. In this respect, the benchmark macroprudential policy helps stimulate banks to increase the total bank assets. We know that if the financial market are complete and perfect, the credit spread is zero and interest rate should be the same. The credit spread in our model is caused by the moral hazard problem in financial sector. The benchmark policy narrows the interest rate spread, which means it cut down the wedge in the financial market. From this aspect, the policy could help raise the firms borrowing by removing the imperfection in the financial market. Together with the lower volatility of the return rate in the second moments, the shadow price of the bank assets, $\mu_s + x\mu_e$, is shown to be 14 percent higher than that of without policy. This also helps raise the total fund that banks lend to the firms, $Q_t K_t$. As a result, it is shown from the stochastic steady state table that the representative households lifetime utility is about 0.39 percent larger in this scenario The consumption equivalence is 0.17 percent if the second moments is included. In general, the benchmark macroprudential

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[1]When τ^c is larger than 0.01, the welfare decreases significantly because the moral hazard constraint is quadratic and the cost of the policy increases sharply. It is the similar case how we choose τ_0 in our new macroprudential policy.

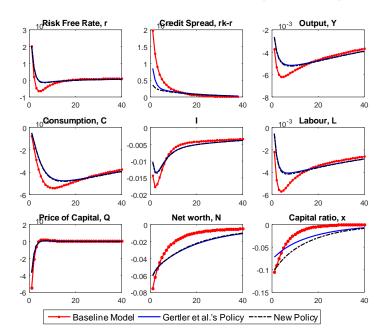policy could help improve the social welfare.

Figure.1 Baseline model v.s. two Macroprudential policies

We run simulations for the modified macroprudential policy. We assume that the central bank sets the benchmark capital ratio anchor for individual banks to be 20%[2]. and the taxation/subsidy ratio parameter τ_0 is set to be optimal ($\tau_0 = 0.09$), such that the households' welfare is improved to the highest level within this policy framework. The third column of Table.2 illustrates the steady state of key variables in economy with that policy. From the simulation result, the consumption equivalence is about 0.27 percent, about 40 percent higher than the benchmark macroprudential policy in Gertler et al.. To be specific, The higher steady state values of capital and labour supply is key factor of higher level of output, consumption and welfare. In bank sector, the capital ratio is 17.91% with new macroprudential policy, and it is higher than the benchmark policy. From Equation (17), we know that if the capital ratio is higher than 10 percent, the fraction of diversion is increasing w.r.t. capital ratio with a increasing speed. So in this case, the banks suffer moral hazard constraint more than that of the benchmark policy. Banks rely less on the short term debts and the profit that banks make from spread of return is smaller. Therefore, the steady state of net worth N_t is lower. It is not good for banks, but it somehow reveals the information that the competitiveness of the financial market is enhanced more with smaller return spread and bank's profit. Because in the financial market without any financial frictions, the return spread will be zero and the profit of the banks will be zero, too. The most important thing

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[2] We do different simulations when benchmark capital ratio anchor of the central bank goes from 15% to 25%, and the results are consistent.

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is, our policy have more stabilization effect on the bank's scale of lending and firm's borrowing during the financial crisis with the new policy. Because of the lower quarterly return spread of 0.236% at steady state, the total value of bank asset in financial intermediaries and the value of bank asset is higher than the benchmark policy. The higher level of capital stock makes the economy perform better. Although higher capital ratio tightens the moral hazard constraint, the reduction of volatility in risky returns and the decrease of the deadweight loss in the financial market make the economy reach a higher welfare.

Figure.1 shows the impulsive response functions after a negative 'capital quality' shock with two different macroprudential policies. The baseline model's response function without any policy is also plotted on the same graphs as comparison. When the economy is hit by a financial shock, the credit spread suddenly increases by 20 basis points quarterly without policy. The benchmark macroprudential policy could mitigate the suddenly credit spread to 10 basis points, while the macroprudential policy on capital ratio performs even better. It could stabilize the credit spread to only 5 basis points rise-up. So the macroprudential policy on capital ratio has the best stabilization effect on the credit spread. We can see the same result by checking the second moment of the risky returns. It is an important factor to explain why our new macroprudential policy has the best performance. With the reduction of the return spread at the steady state, more stable and competitive financial market can the economy have, and greater stabilization of the real variables could be achieved because of the banks transmission mechanism to the real economy. That confirms our analysis of the financial market in Section 2.6 before. Bank capital to asset ratio is higher than baseline model case at the steady state, but the recovery speed of capital ratio and banks' new worth is slower. Because the time-contingent bank equities could mitigate the loss of banks caused by the financial shock, it decreases when a negative shock comes. The banks are better hedged with policy because of higher capital ratio, but the lower bank equity return after the shock induces households to turn to saver asset like short term debts. It reduces the bank's profit at the same time. That is the reason why we can see that capital ratio x_t goes down, and the recovery of the ratio is also slower after the financial shock. It can be seen in the Figure.1 that both financial sector variables (e.g., interest rate r) and production sectors variables (e.g., output Y, consumption C) have lower variances. Generally it tells us that the economy is more stabilized.

The macroprudential policy we propose causes higher capital ratio than that of in benchmark policy. It is good for stability of the financial system, but costly for the moral hazard problem. The decrease of interest rate spread shows the policy narrows the imperfection of the financial market. The stimulation shows that our policy's positive effect on the financial market overwhelms the negative effect caused by the moral hazard problem, thus it turns out to be a higher level of bank's total lending scale in economy than Gertler et al.'s policy. In the next section, we try to find other reasons why our macroprudential policy performs

better on improving the social welfare by considering the policy performances with different tax ratio parameters.

3.3 Explaining the Advantages of the Modified Policy

In this final section, we compare these two macroprudential policies to see why the macroprudential policy on capital ratio has a better performance. We extract the difference in the system of equations, and try to analyze the differences between the two policies in a certain range of tax ratio parameters τ_0/τ_1. Since the difference comes from the policy rule and the banks optimal choice on balance sheet with each policy, so if we rearrange Equation (28), we could get the expression as follows.

$$\mu_{s,t}\left(\varepsilon + \kappa x_t\right) = \tau_0 v_t \left(x^* - x_t\right)\left(1 + \varepsilon x_t + \frac{\kappa}{2}x_t^2\right) + \mu_{e,t}\left(1 - \frac{\kappa}{2}x_t^2\right). \qquad (29)$$

We extract the optimal decision of banks with the macroprudential policy in Gertler et al. (2012) in a similar way, it can be simplified as

$$\mu_{s,t}\left(\varepsilon + \kappa x_t\right) = \tau_1 \left(1 + \varepsilon x_t + \frac{\kappa}{2}x_t^2\right) + \mu_{e,t}\left(1 - \frac{\kappa}{2}x_t^2\right). \qquad (30)$$

We can see that the only difference between these two macroprudential policies are the overall tax ratio part $\tau_0 v_t\left(x^* - x_t\right)$ and τ_1 on the right hand side.

We find the reason why the steady state of capital ratio x_t is higher in macroprudential policy than in benchmark policy at their optimal level repsectively. From the simulation result, the elasticity of capital ratio x_t w.r.t. tax ratio in our macroprudential policy is almost the same. That means as the tax ratio increases, the capital ratio will increase at the same speed in new macroprudential policy. To find the reason why the capital ratio is higher, we need to check the shadow price of the bank equity. From Equation (27) after we implement the policy, the price of bank asset $\mu_{s,t} + x_t\mu_{e,t}$ can be decomposed into two parts: the effect on bank equity $\mu_{e,t} + \tau_t^e v_t$ and the effect on bank deposit. As we choose different policy parameters, the value of bank equity is always larger with new macroprudential policy than with the benchmark policy. From Figure.2 below, it illustrates that the elasticity of it w.r.t. tax ratio is also significantly larger than Gertler et al.'s policy. We can also find how our macroprudential policy could affect the incentive of banks by checking the future value of bank equity without the policy effect, which is $\mu_{e,t}$. The results show that $\mu_{e,t}$ is decreasing faster in new policy. In the modified macroprudential policy, which means the new macroprudential policy has greater infulence on the bank equity. So the capital ratio in the new macroprudential policy is larger becaue it raises the futures price of bank equity faster than the benchmark policy.

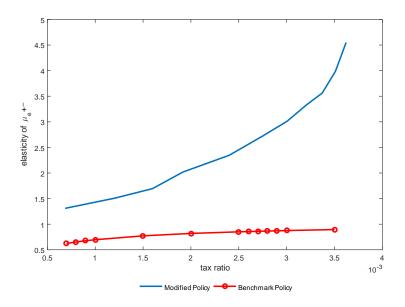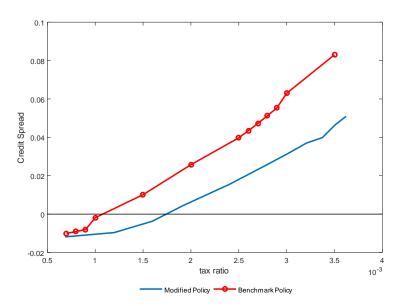Figure.2 Elasticity of $\mu_{e,t} + \tau_t^e v_t$ w.r.t. tax ratio

Figure.3 Elasticity of credit spread w.r.t. tax ratio

In the simulation result, we can see that both of two macroprudential policies alter bank's balance sheet in three different ways: Firstly, it decreases both the steady state and the volatility of the credit spread in the financial sector, hence raises bank's total lending and enhance the competitiveness of the financial market. Secondly, the increase of the bank capital to asset ratio x_t incentivizes the bank managers to cheat, so it tightens the moral hazard constraint (Equation 18) and lowers the welfare. Thirdly, as the capital ratio goes up with the

macroprudential policy, the future values of bank asset and net worth increase, which relaxes the moral hazard constraint at the same time. So bank managers are more willing to keep bank assets in banks for future dividend payments. All of these factors could affect the performance of the macroprudential policy on improving of the social welfare, and different policies have different sensitivities on these key variables as we change tax ratio parameters.

The overall tax ratio of the new macroprudential policy is 0.0036 at the optimal level, while it is 0.0027 in the benchmark policy. If we compare two different policies at its optimal level respectively, we can discover that the shadow price of the bank asset tomorrow $\mu_{s,t} + x_t\mu_{e,t}$ is 1.9 percent larger in the modified policy. The shadow price of the net worth v_t is larger as well. These results raise the value function Equation (27) at optimal value and slacken the moral hazard constraint. The credit spread is 1.7 percent smaller with modified policy, and it improves the social welfare because the deadweight loss caused by the financial market imperfection is smaller. The capital ratio is increased to 17.7 percent in new policy. It is higher than benchmark policy because the effect of taxation on the bank equity $\mu_{e,t} + \tau_t^e v_t$ is shown to be significantly larger. Higher capital ratio tightens the moral hazard constraint and it lowers the welfare. If we try to fix the moral hazard cost in two policies, we need to find the simulation result in benchmark policy when the tax ratio level is the same as the one of new macroprudential policy at the optimal level, 0.0036. Compared with these two cases, the future value of bank asset is even higher in benchmark policy. However, it doesn't help the welfare reach the level as new macroprudential policy, because the credit spread is 3.3 percent larger. The benchmark policy performs better in raising the bank's profit in the future, but worse in narrowing the wedge in financial market.

We check the elasticity of the key variables which might affect the performance of the policy w.r.t. tax ratio to see why these two policies could have different impact on social welfare. As shown in Figure.3, the credit spread goes down as we increase the tax parameter from zero, and then increases if the tax parameter is considerably large in both macroprudential policies. That tells us the implementation of macroprudential policy could help increase the competitiveness of the financial market when we choose the policy response to be small, but radical policy response would cause larger distortions on the financial market. In the simulation, results in Figure.3 show that the elasticity of credit spread w.r.t. tax ratio is always smaller in the modified policy. That tells us the new policy helps decrease the credit spread faster as the tax ratio increases from zero, and credit spread increases with a lower speed when tax ratio is high. That's why the simulation result shows that the new macroprudential policy performs better on reducing the credit spread at the steady state.

We know from Section 2.6 that both policies could raise the capital ratio by changing the future value of bank's lending and bank equity. As we mentioned before, we find out that as tax ratio goes up, the future value of bank asset is increasing, and so as the bank equity. So the capital ratio increases with

tax ratio parameter. Higher future price of bank asset and net worth means larger value function of banks, and it obviously raises the social welfare. The elasticities of them w.r.t. tax ratio decrease as the tax parameter increases, which means the benefit of the policy diminishes slowly. However, there is one opposite effect caused by the higher future price of bank equity: it raises the capital ratio and tightens the moral hazard constraint. The cost function is convex (Equation (17)) and the elasticity increases with capital ratio when x is larger than 10 percent. Because of two opposite effects on the social welfare, we cannot clarify if the new macroprudential policy is better in raising the capital ratio. From a dynamic point of view, as we increase the tax ratio, the benefit from raising the future price of bank asset together with the credit spread effect decreases and the cost of moral hazard problem increases. We find a level of tax ratio, such that the marginal benefit is equal to the marginal cost, to reach the optimal social welfare. But the two findings below ensures that our policy performs better: Firstly, both elasticities of shadow price of bank asset and bank equity are larger in new macroprudential policy. It means the capital ratio increases faster in the new policy than in the benchmark policy as tax ratio increases. Secondly, both of policies reach the optimal welfare when the elasticity of credit spread is positive. These two results ensure that the new macroprudential policy suffers lower excessive loss with smaller credit spread, given a fixed level of capital ratio.

In general, we find the reasons why our macroprudential policy performs better. Both of these two policies help improve the household's welfare by two ways: As the tax ratio increases, the credit spread goes down at the beginning and then increases as policy distortion goes up; The shadow price of bank asset increases with a diminishing rate. It raises bank's lending, but the moral hazard cost is quadratic, so it increases faster when we choose larger tax ratio parameters. Provided the property of these two policies in the simulation, it is possible to find some proper tax ratio values which improves the social welfare. Our policy can reach a better performance, because the elasticity of credit spread w.r.t. tax ratio is lower and the elasticity of shadow price of bank asset is higher. So it provides a more efficient way to vanish the imperfection in the financial market and raise the bank's total lending in the future.

4   Conclusion

In this paper, we provide an improved version of macroprudential policy in Gertler et al. (2012). The new macroprudential policy choose commercial bank's capital ratio as target, and it can incentivize the commercial banks to keep higher bank capital ratio before the financial crisis. Although the Gertler et al.'s policy could also work as a capital ratio requirement, the new macroprudential policy has two advantages. Firstly, this policy is easier to implement than the Gertler et al.'s policy. Banks' balance sheet structure is an observable

information for the central bank, while the shadow cost of the bank deposit cannot be obtained easily. Secondly, the macroprudential policy on capital ratio is welfare dominant to the benchmark macroprudential policy. We can show that the volatility of the risky return and the credit spread decreases more than the benchmark policy in the simulation, so it reduces the risk perception of the banks and raises the total lending through financial intermediaries. The steady state of the credit spread is also smaller. To some extent, it offsets the deadweight loss aroused by financial frictions in the financial market. The value of the bank equity in the future rises and the volatility on risky returns decreases, and the new macroprudential policy could encourage banks to keep a higher capital ratio. The higher capital ratio makes the moral hazard problem worse, but it also reveals the information that the financial market is more competitive than before. Moreover, the shadow price of bank assets increases, which slackens the moral hazard problem at the same time. It is because keeping bank assets in the bank to get future dividends is more profitable for bank managers than before. In general, the new policy increases the social welfare more because the overall positive effect we mentioned above exceeds the negative effect caused by the moral hazard problem. The result is robust within a range of parameters.

There are some further research about the interactions between macro prudential policy and monetary policy. Adrian and Shin (2010) explored the role of financial intermediaries in monetary economics. They find that it is important to consider the balance sheet quantities for monetary policy. Borio and Zhu (2012) argue that the changes of macroprudential regulation may alter the risk-taking channels and some prevailing macroeconomic models are not able to fully capture it. They believe that the bank capital regulations have an impact on the transmission mechanism of monetary policy, so it may reduce the effectiveness to monetary policy. However, in this paper, all terms are real and monetary policy cannot be considered. We can isolate the macroprudential policy effect to get rid of the interaction between these two policies. De Groot (2014) introduce nominal terms in the Gertler et al.'s framework and it might be interesting if we can investigate with macroprudential policies, how the monetary policy could be affected by the implementation of different macroprudential policies. That might be my further research topic.

5 Appendix

5.1 Summary of models

5.1.1 System of equations (baseline model without policy)

$$Y_t = (\psi_t K_{t-1})^\alpha L_t^{1-\alpha} \tag{31}$$

$$Y_t = C_t + \left[1 + f\left(\frac{I_t}{I_{t-1}}\right)\right] I_t \tag{32}$$

$$K_t = (1-\delta)\psi_t K_{t-1} + I_{t-1} \tag{33}$$

$$R_{t+1}\mathbb{E}_t\left(\Lambda_{t,t+1}\right) = 1 \tag{34}$$

$$\mathbb{E}_t\left(\Lambda_{t,t+1}R_{e,t+1}\right) = 1 \tag{35}$$

$$j_t = \frac{J_t}{J_{t-1}} \tag{36}$$

$$J_t = C_t - hC_{t-1} - \frac{\chi}{1+\varphi}L_t^{1+\varphi} \tag{37}$$

$$(1-\alpha)\left[1 - \beta h\mathbb{E}_t\left(j_{t+1}^{-\gamma}\right)\right] Y_t = \chi L_t^{1+\varphi} \tag{38}$$

$$\Lambda_{t,t+1} = \beta\frac{\mathbb{E}_t\left(j_{t+1}^{-\gamma}\right) - \beta h\mathbb{E}_t\left(j_{t+1}^{-\gamma}j_{t+2}^{-\gamma}\right)}{1 - \beta h\mathbb{E}_t\left(j_{t+1}^{-\gamma}\right)} \tag{39}$$

$$R_{e,t} = \frac{\left[\alpha\left(\frac{L_t}{\psi_t K_{t-1}}\right)^{1-\alpha} + (1-\delta)q_t\right]\psi_t}{q_{t-1}} \tag{40}$$

$$R_{k,t} = \frac{\left[\alpha\left(\frac{L_t}{\psi_t K_{t-1}}\right)^{1-\alpha} + (1-\delta)Q_t\right]\psi_t}{Q_{t-1}} \tag{41}$$

$$Q_t = 1 + f(\frac{I_t}{I_{t-1}}) + \frac{I_t}{I_{t-1}}f'(\frac{I_t}{I_{t-1}}) - \mathbb{E}_t\left[\Lambda_{t,t+1}\left(\frac{I_{t+1}}{I_t}\right)^2 f'(\frac{I_{t+1}}{I_t})\right] \tag{42}$$

$$N_t = \sigma\left[(R_{k,t} - x_{t-1}R_{e,t} - R_t + x_{t-1}R_t)Q_{t-1}K_{t-1} + R_t N_{t-1}\right] + (1-\sigma)\xi R_{k,t}Q_{t-1}K_{t-1} \tag{43}$$

$$\Theta_t = \theta\left(1 + \varepsilon x_t + \frac{\kappa}{2}x_t^2\right) \tag{44}$$

$$N_t \phi_t = Q_t K_t \tag{45}$$

$$\phi_t = \frac{v_t}{\Theta_t - (\mu_{s,t} + x_t \mu_{e,t})} \tag{46}$$

$$v_t = \mathbb{E}_t(\Lambda_{t,t+1}\Omega_{t+1})R_{t+1} \tag{47}$$

$$\mu_{s,t} = \mathbb{E}_t \left[\Lambda_{t+1}\Omega_{t+1} \left(R_{k,t+1} - R_{t+1} \right) \right] \tag{48}$$

$$\mu_{e,t} = \mathbb{E}_t \left[\Lambda_{t+1}\Omega_{t+1} \left(R_{t+1} - R_{e,t+1} \right) \right] \tag{49}$$

$$\Omega_t = (1 - \sigma) + \sigma \left[(\mu_{s,t} + x_t \mu_{e,t})\phi_t + v_t \right] \tag{50}$$

$$\kappa \mu_{e,t} x_t^2 + 2\kappa \mu_{s,t} x_t + 2\varepsilon \mu_{s,t} - 2\mu_{e,t} = 0 \tag{51}$$

5.1.2 System of equations with macroprudential policy on x_t

$$Y_t = (\psi_t K_{t-1})^\alpha L_t^{1-\alpha} \tag{52}$$

$$Y_t = C_t + \left[1 + f\left(\frac{I_t}{I_{t-1}}\right) \right] I_t \tag{53}$$

$$K_t = (1 - \delta)\psi_t K_{t-1} + I_{t-1} \tag{54}$$

$$R_{t+1}\mathbb{E}_t \left(\Lambda_{t,t+1} \right) = 1 \tag{55}$$

$$\mathbb{E}_t \left(\Lambda_{t,t+1} R_{e,t+1} \right) = 1 \tag{56}$$

$$j_t = \frac{J_t}{J_{t-1}} \tag{57}$$

$$J_t = C_t - hC_{t-1} - \frac{\chi}{1+\varphi} L_t^{1+\varphi} \tag{58}$$

$$(1 - \alpha) \left[1 - \beta h \mathbb{E}_t \left(j_{t+1}^{-\gamma} \right) \right] Y_t = \chi L_t^{1+\varphi} \tag{59}$$

$$\mathbb{E}_t \Lambda_{t,t+1} = \beta \frac{\mathbb{E}_t \left(j_{t+1}^{-\gamma} \right) - \beta h \mathbb{E}_t \left(j_{t+1}^{-\gamma} j_{t+2}^{-\gamma} \right)}{1 - \beta h \mathbb{E}_t \left(j_{t+1}^{-\gamma} \right)} \tag{60}$$

$$R_{e,t} = \frac{\left[\alpha \left(\frac{L_t}{\psi_t K_{t-1}} \right)^{1-\alpha} + (1-\delta)q_t \right] \psi_t}{q_{t-1}} \tag{61}$$

$$R_{k,t} = \frac{\left[\alpha \left(\frac{L_t}{\psi_t K_{t-1}} \right)^{1-\alpha} + (1-\delta)Q_t \right] \psi_t}{Q_{t-1}} \tag{62}$$

$$Q_t = 1 + f(\frac{I_t}{I_{t-1}}) + \frac{I_t}{I_{t-1}} f'(\frac{I_t}{I_{t-1}}) - \mathbb{E}_t \left[\Lambda_{t,t+1} \left(\frac{I_{t+1}}{I_t} \right)^2 f'(\frac{I_{t+1}}{I_t}) \right] \tag{63}$$

$$N_t = \sigma \left[(R_{k,t} - x_{t-1}R_{e,t} - R_t + x_{t-1}R_t) Q_{t-1}K_{t-1} + R_t N_{t-1} \right] + (1-\sigma)\xi R_{k,t}Q_{t-1}K_{t-1} \tag{64}$$

$$\Theta_t = \theta \left(1 + \varepsilon x_t + \frac{\kappa}{2}x_t^2 \right) \tag{65}$$

$$N_t \phi_t = Q_t K_t \tag{66}$$

$$\phi_t = \frac{v_t}{\Theta_t - (\mu_{st} + x_t \mu_{et})} \tag{67}$$

$$v_t = \mathbb{E}_t(\Lambda_{t,t+1}\Omega_{t+1})R_{t+1} \tag{68}$$

$$\mu_{s,t} = \mathbb{E}_t \left[\Lambda_{t+1}\Omega_{t+1} \left(R_{k,t+1} - R_{t+1} \right) \right] \tag{69}$$

$$\mu_{e,t} = \mathbb{E}_t \left[\Lambda_{t+1}\Omega_{t+1} \left(R_{t+1} - R_{e,t+1} \right) \right] \tag{70}$$

$$\Omega_t = (1 - \sigma) + \sigma \left[(\mu_{s,t} + x_t \mu_{e,t})\phi_t + v_t \right] \tag{71}$$

$$\theta \left(\mu_{s,t} + \mu_{e,t}x_t \right) \left(\varepsilon + \kappa x_t \right) = \theta \left(1 + \varepsilon x_t + \frac{\kappa}{2}x_t^2 \right) \left[\mu_{e,t} + \tau_0 v_t \left(x^* - x_t \right) \right] \tag{72}$$

$$\left(1 + \tau_t^k \right) Q_t K_t = N_t + \left(1 + \tau_t^e \right) q_t e_t + D_t \tag{73}$$

$$x_t = \frac{q_t e_t}{Q_t K_t} \tag{74}$$

$$\tau_t^e = \tau_0 \left(x^* - x_t \right) \tag{75}$$

$$\tau_t^k = x_t \tau_t^e \tag{76}$$

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5.2 Optimization problem of households

The expected discounted utility function is

$$\mathbb{E}_t \sum_{\tau=t}^{\infty} \beta^{\tau-t} \frac{1}{1-\gamma} \left(C_\tau - hC_{\tau-1} - \frac{\chi}{1+\varphi} L_\tau^{1+\varphi} \right)^{1-\gamma}, \qquad (77)$$

subject to the household's constraint at period t

$$\Xi_t = C_t + D_{h,t} + q_t \bar{e}_t - W_t L_t - \Pi_t + T_t - R_t D_{h,t-1} - [Z_t + (1-\delta)q_t] \psi_t \bar{e}_{t-1} \le 0 \quad (78)$$

We solve it using Lagrangian. So Lagrangian can be constructed as

$$\mathcal{L} = \mathbb{E}_t \sum_{\tau=t}^{\infty} \beta^{\tau-t} \left\{ \frac{1}{1-\gamma} \left(C_\tau - hC_{\tau-1} - \frac{\chi}{1+\varphi} L_\tau^{1+\varphi} \right)^{1-\gamma} - \lambda_\tau \Xi_\tau \right\} \qquad (79)$$

Representative households choose variables C_t, L_t, $D_{h,t}$ and $\bar{e}_t$. First order conditions are listed as follows.

$$\frac{\partial \mathcal{L}}{\partial C_t} = \left(C_t - hC_{t-1} - \frac{\chi}{1+\varphi} L_t^{1+\varphi} \right)^{-\gamma} - \lambda_t - \beta h \mathbb{E}_t \left(C_{t+1} - hC_t - \frac{\chi}{1+\varphi} L_{t+1}^{1+\varphi} \right)^{-\gamma} = 0,$$

$$\frac{\partial \mathcal{L}}{\partial L_t} = -\chi \left(C_t - hC_{t-1} - \frac{\chi}{1+\varphi} L_t^{1+\varphi} \right)^{-\gamma} L_t^{\varphi} + \lambda_t W_t = 0,$$

$$\frac{\partial \mathcal{L}}{\partial D_{h,t}} = \mathbb{E}_t \left(-\lambda_t + \beta R_{t+1} \lambda_{t+1} \right) = 0,$$

$$\frac{\partial \mathcal{L}}{\partial e_t} = -\lambda_t q_t + \beta \mathbb{E}_t \left[\lambda_{t+1} \left(Z_{t+1} + (1-\delta)q_{t+1}\psi_{t+1} \right) \right] = 0.$$

By solving these equations, we can get Equation (5), Equation (6), Equation (7) and Equation (8) in Section 2.1.

5.2.1 Optimization problem of Bank Sector

We use technique of Bellman equations to solve the commercial bank's optimization problem because the value function shows up in the moral hazard constraint. Here we show two ways to derive the specific form of the value function.

Guess and Verify Method in Gertler et al. (2012): Firstly, we simplify the net worth accumulation constraint. We can actually simplify the net worth accumulation function (Equation 13) by plugging Equation (12) and Equation (16).

$$n_t = R_{k,t}Q_{t-1}s_{t-1} - R_t\left(Q_{t-1}s_{t-1} - n_{t-1} - q_{t-1}e_{t-1}\right) - R_{e,t}q_{t-1}e_{t-1}$$
$$n_t = R_{k,t}Q_{t-1}s_{t-1} - R_t\left(Q_{t-1}s_{t-1} - n_{t-1} - x_{t-1}Q_{t-1}s_{t-1}\right) - R_{e,t}x_{t-1}Q_{t-1}s_{t-1}$$
$$n_t = \left[R_{k,t} - R_t(1 - x_t) - R_{e,t}x_{t-1}\right]Q_{t-1}s_{t-1} + R_tn_{t-1} \tag{80}$$

Now we regard x_t, s_t as control variables and n_t as a state variable in this dynamic optimization problem. The Bellman equation can be written as follows.

$$V_{t-1}(x_{t-1}, s_{t-1}, n_{t-1}) = \mathbb{E}_{t-1}\Lambda_{t-1,t}\left\{(1-\sigma)n_t + \sigma\max_{s_t,x_t}\left[V_t(x_t, s_t, n_t)\right]\right\}. \tag{81}$$

The guess solution of value function is

$$V_t(x_t, s_t, n_t) = (\mu_{s,t} + x_t\mu_{e,t})Q_ts_t + v_tn_t. \tag{82}$$

The next step is to verify this specific form is the satisfy the Equation (81).

We maximize this objective with the moral hazard constraint (Equation 18). The Lagrangian is constructed as

$$\mathcal{L} = (\mu_{s,t} + x_t\mu_{e,t})Q_ts_t + v_tn_t + \lambda_t\left[(\mu_{s,t} + x_t\mu_{e,t})Q_ts_t + v_tn_t - \theta\left(1 + \varepsilon x_t + \frac{\kappa}{2}x_t^2\right)Q_ts_t\right],$$
$$\mathcal{L} = (1 + \lambda_t)\left[(\mu_{s,t} + x_t\mu_{e,t})Q_ts_t + v_tn_t\right] - \lambda_t\theta\left(1 + \varepsilon x_t + \frac{\kappa}{2}x_t^2\right)Q_ts_t.$$

First order condition w.r.t. x_t and s_t:

$$\frac{\partial\mathcal{L}}{\partial x_t} = (1 + \lambda_t)\mu_{e,t}Q_ts_t - \lambda_t\theta\left(\varepsilon + \kappa x_t\right)Q_ts_t = 0$$
$$(1 + \lambda_t)\mu_{e,t} = \lambda_t\theta\left(\varepsilon + \kappa x_t\right)$$
$$(1 + \lambda_t) = \frac{\lambda_t\theta\left(\varepsilon + \kappa x_t\right)}{\mu_{e,t}}$$

$$\frac{\partial\mathcal{L}}{\partial s_t} = (1 + \lambda_t)(\mu_{s,t} + x_t\mu_{e,t})Q_t - \lambda_t\theta\left(1 + \varepsilon x_t + \frac{\kappa}{2}x_t^2\right)Q_t = 0$$
$$(1 + \lambda_t)(\mu_{s,t} + x_t\mu_{e,t}) = \lambda_t\theta\left(1 + \varepsilon x_t + \frac{\kappa}{2}x_t^2\right)$$

$$(\mu_{s,t} + x_t\mu_{e,t})Q_ts_t + v_tn_t = \theta\left(1 + \varepsilon x_t + \frac{\kappa}{2}x_t^2\right)Q_ts_t$$

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From the first two equations above, we express x_t as an implicit function of $(\mu_{s,t}, \mu_{e,t})$. The simplified version is as follows.

$$(\varepsilon + \kappa x_t)\left(\frac{\mu_{s,t}}{\mu_{e,t}} + x_t\right) = 1 + \varepsilon x_t + \frac{\kappa}{2}x_t^2$$

By substituting Equation (80) and Equation (82) into Bellman equation, it becomes.

$$
\begin{aligned}
(\mu_{s,t} + x_t\mu_{e,t})Q_t s_t + v_t n_t &= \mathbb{E}_t \Lambda_{t,t+1}\left\{(1-\sigma)n_{t+1} + \sigma\left[(\mu_{s,t+1} + x_{t+1}\mu_{e,t+1})\phi_{t+1}n_{t+1} + v_{t+1}n_{t+1}\right]\right\} \\
&= \mathbb{E}_t \Lambda_{t,t+1}\left\{(1-\sigma) + \sigma\left[(\mu_{s,t+1} + x_{t+1}\mu_{e,t+1})\phi_{t+1} + v_{t+1}\right]\right\}n_{t+1}.
\end{aligned}
$$

Denote $\Omega_{t+1} = (1-\sigma) + \sigma\left[(\mu_{s,t+1} + x_{t+1}\mu_{e,t+1})\phi_{t+1} + v_{t+1}\right]$, the Bellman equation becomes

$$(\mu_{s,t} + x_t\mu_{e,t})Q_t s_t + v_t n_t = \mathbb{E}_t \Lambda_{t,t+1}\Omega_{t+1}\left([R_{k,t+1} - R_{t+1}(1-x_{t+1}) - R_{e,t+1}x_t]Q_t s_t + R_{t+1}n_t\right) \tag{83}$$

So if we all of these variables $(v_t, \mu_{s,t}, \mu_{e,t}, \Omega_{t+1})$ as in Section 2.4, like

$$v_t = \mathbb{E}_t\left(\Lambda_{t,t+1}\Omega_{t+1}\right)R_{t+1},$$

$$\mu_{s,t} = \mathbb{E}_t\left[\Lambda_{t,t+1}\Omega_{t+1}\left(R_{k,t+1} - R_{t+1}\right)\right],$$

$$\mu_{e,t} = E_t\left[\Lambda_{t,t+1}\Omega_{t+1}\left(R_{t+1} - R_{e,t+1}\right)\right],$$

$$\Omega_{t+1} = (1-\sigma) + \sigma\left[\phi_t\left(\mu_{s,t+1} + x_{t+1}\mu_{e,t+1}\right) + v_{t+1}\right].$$

Compare both sides of Equation (83), they can be rearranged identical.

$$
\begin{aligned}
LHS &= \left\{\mathbb{E}_t\left[\Lambda_{t,t+1}\Omega_{t+1}\left(R_{k,t+1} - R_{t+1}\right)\right] + x_t\mathbb{E}_t\left[\Lambda_{t,t+1}\Omega_{t+1}\left(R_{t+1} - R_{e,t+1}\right)\right]\right\}Q_t s_t \\
&\quad + \mathbb{E}_t\left(\Lambda_{t,t+1}\Omega_{t+1}\right)R_{t+1}n_t \\
&= \mathbb{E}_t\left\{\Lambda_{t,t+1}\Omega_{t+1}\left[\left(R_{k,t+1} - R_{t+1}\right)Q_t s_t + x_t\left(R_{t+1} - R_{e,t+1}\right)Q_t s_t + R_{t+1}n_t\right]\right\} \\
&= \mathbb{E}_t\Lambda_{t,t+1}\Omega_{t+1}\left(\left[R_{k,t+1} - (1-x_t)R_{t+1} - x_t R_{e,t+1}\right]Q_t s_t + R_{t+1}n_t\right) \\
&= RHS
\end{aligned}
$$

So the Bellman equation can be satisfied for (n_t, s_t, x_t) at any time t with the specific form Equation (82). This method has a disadvantage that it might be difficult to get the correct functional form. We would show another method to derive the functional form without guessing the specific form in next section.

Derivation Method without guessing the form: Similarly, the objective function is

$$V_t = \mathbb{E}_t \left[\sum_{\tau=t+1}^{\infty} (1-\sigma)\sigma^{\tau-t-1} \Lambda_{t,\tau} n_\tau \right].$$

Firstly, we try to construct the Lagrangian without the moral hazard constraint.

$$\mathcal{L} = \mathbb{E}_t \sum_{\tau=t+1}^{\infty} \sigma^{\tau-t-1} \Lambda_{t,\tau} \left\{ (1-\sigma)n_\tau + \Omega_\tau g(n_\tau, n_{\tau-1}, x_{\tau-1}, s_{\tau-1}) \right\}.$$

where $g(n_t, n_{t-1}, x_{t-1}, s_{t-1}) = [R_{k,t} - R_t(1-x_t) - R_{e,t}x_{t-1}]Q_{t-1}s_{t-1} + R_t n_{t-1} - n_t = 0$. It can be rewritten as $\{[R_{k,t} - R_t(1-x_{t-1}) - R_{e,t}x_{t-1}]\phi_t + R_t\}n_{t-1} - n_t = 0$ by Plugging Equation (15). The reason as follows: we know that the net worth constraint of the bank is always binding, or else it will accumulate as much as assets as they can into infinity. We denote the maximum ratio of total assets to net worth at time t as ϕ_t, so equality always holds in Equation (15) as

$$\phi_t n_t = Q_t s_t.$$

The state variable here is net worth (n_t). We derive the first order condition w.r.t. n_{t+1}:

$$\frac{\mathcal{L}}{n_{t+1}} = (1-\sigma)\Lambda_{t,t+1} - \Lambda_{t,t+1}\Omega_{t+1} + \sigma\Lambda_{t,t+2}\Omega_{t+2} \left\{ [R_{k,t+2} - R_{t+2}(1-x_{t+1}) - R_{e,t+2}x_{t+1}]\phi_t + R_{t+2} \right\}$$
$$= 0.$$

It could be simplified as expression of Ω_{t+1}

$$\Omega_{t+1} = (1-\sigma)$$
$$+ \sigma\phi_t \frac{\Lambda_{t,t+2}}{\Lambda_{t,t+1}\Lambda_{t+1,t+2}} \left[\Lambda_{t+1,t+2}\Omega_{t+2}(R_{k,t+2} - R_{t+2}) + x_{t+1}\Lambda_{t+1,t+2}\Omega_{t+2}(R_{t+2} - R_{e,t+2}) \right]$$
$$+ \sigma \frac{\Lambda_{t,t+2}}{\Lambda_{t,t+1}\Lambda_{t+1,t+2}} \Lambda_{t+1,t+2}\Omega_{t+2}R_{t+2},$$

where $v_t = \mathbb{E}_t(\Lambda_{t,t+1}\Omega_{t+1})R_{t+1}$, $\mu_{s,t} = \mathbb{E}_t[\Lambda_{t,t+1}\Omega_{t+1}(R_{k,t+1} - R_{t+1})]$ and $\mu_{e,t} = \mathbb{E}_t[\Lambda_{t,t+1}\Omega_{t+1}(R_{t+1} - R_{e,t+1})]$, it is the same as the notation in Section 2.4.

$$\Omega_{t+1} = (1-\sigma) + \sigma\left[\phi_t\left(\mu_{s,t+1} + x_{t+1}\mu_{e,t+1}\right) + v_{t+1} \right]$$

From the Bellman equation, we can have

$$V_t = \mathbb{E}_t\left((1-\sigma)\Lambda_{t,t+1}n_{t+1} + \sigma\mathbb{E}_t\Lambda_{t,t+1}V_{t+1} \right) \tag{84}$$

Here we express the net worth accumulation function in a more simplified way. Denote the evaluation of net worth constraint as

$$n_{t+1} - \Gamma_t n_t = 0.$$

where
$$\Gamma_t = [R_{k,t+1} - R_{t+1}(1 - x_t) - R_{e,t+1}x_t]\phi_{t+1} + R_{t+1}$$

Assume that the price of net worth at time $t+1$ is Ω_{t+1} The value function today V_t should be equal to discounted total value of net worth tomorrow.

$$V_t = \mathbb{E}_t\Lambda_{t,t+1}\Omega_{t+1}n_{t+1}. \tag{85}$$

We will plug it into Bellman equation to get the exact form of Ω_{t+1}, and later we can prove Ω_{t+1} is the price of net worth. Plug Equation (85) into the simplified Bellman equation (84),

$$
\begin{aligned}
\mathbb{E}_t\Lambda_{t,t+1}\Omega_{t+1}n_{t+1} &= \mathbb{E}_t\left((1-\sigma)\Lambda_{t,t+1}n_{t+1} + \sigma\mathbb{E}_t\Lambda_{t,t+1}\left[E_{t+1}\Lambda_{t+1,t+2}\Omega_{t+2}n_{t+2}\right]\right) \\
&= \mathbb{E}_t\left((1-\sigma)\Lambda_{t,t+1}n_{t+1} + \sigma\mathbb{E}_t\Lambda_{t,t+1}\left[E_{t+1}\Lambda_{t+1,t+2}\Omega_{t+2}\Gamma_{t+1}n_{t+1}\right]\right) \\
&= \mathbb{E}_t\Lambda_{t,t+1}n_{t+1}\left[(1-\sigma) + \sigma\left(\Lambda_{t+1,t+2}\Omega_{t+2}\Gamma_{t+1}\right)\right]
\end{aligned}
$$

Since the equality of both sides, by comparing two sides of equation above after rearrangement, we can derive the exact form of Ω_{t+1}.

$$\Omega_{t+1} = (1-\sigma) + \sigma\Lambda_{t+1,t+2}\Omega_{t+2}\Gamma_{t+1} \tag{86}$$

Now let's prove it is the price of net worth at time period $t+1$.

$$\mathcal{L} = \mathbb{E}_t\left[\sum_{\tau=t+1}^{\infty}\sigma^{\tau-t-1}\Lambda_{t,\tau}\left[(1-\sigma)n_\tau - \Omega_\tau\left[n_\tau - \Gamma_{\tau-1}n_{\tau-1}\right]\right]\right]$$

Differentiate w.r.t. n_{t+1}, we could get

$$\mathbb{E}_t\Lambda_{t,t+1}(1-\sigma) - \mathbb{E}_t\Lambda_{t,t+1}\Omega_{t+1} + \sigma\mathbb{E}_t\Lambda_{t,t+2}\Omega_{t+2}\Gamma_{t+1} = 0,$$

which is the same as what we get before.

Finally, we substitute Equation (86) into Equation (85), the value function form is derived.

$$
\begin{aligned}
V_t &= \mathbb{E}_t\Lambda_{t,t+1}\Omega_{t+1}\Gamma_t n_t \\
&= \mathbb{E}_t\Lambda_{t,t+1}\Omega_{t+1}\left\{[R_{k,t+1} - R_{t+1}(1-x_t) - R_{e,t+1}x_t]\phi_t + R_{t+1}\right\}n_t \\
&= \mathbb{E}_t\Lambda_{t,t+1}\Omega_{t+1}R_{t+1}n_t + \mathbb{E}_t\Lambda_{t,t+1}\Omega_{t+1}(R_{k,t+1} - R_{t+1})\phi_t n_t \\
&\quad + \mathbb{E}_t\Lambda_{t,t+1}\Omega_{t+1}x_t(R_{t+1} - R_{e,t+1})\phi_t n_t,
\end{aligned}
$$

Compare it with the value function Equation (19).in the paper, they are the same.

5.3   Stochastic steady state

From We use a 'risky steady state' in our framework. It is because we believe that agents' perception of risk could alter their decision from the deterministic steady state today. They choose this different steady state when they expect a future shock, although the shock today turns out to be zero. This idea comes from other literature, i.e. Campbell (1994), Lettau (2003). We call it 'risky steady state' and its name comes from Coeurdacier et al. (2011). The method they use could be found in Lavan Mahadeva (2013). It is straightforward for small models, but not convenient for large models like DSGE models. The computational method we use is from de Groot (2013): they do a second-order approximation of the macroeconomic model around its deterministic steady state.

We want to find the risky steady state of our model, such that the second moment which generated by this level of steady state could lead us back to the same steady state. The only difference between the deterministic steady state and risky steady state is that we consider the second order (variance and covariance) on expected terms. In this way, the perception of risk will change the bank's balance sheet structure. To be specific, we use an iterated procedure to find the second moment and the risky steady state. Firstly, we log-linearize the whole model around the deterministic steady state without perception of risk. We use these steady states and non-linear dynamic system to find the second moment. of deterministic steady state. We introduce the second moment into our non-linear system of equations to derive a different steady state, so on so forth. We keep iterating and update the steady state until that, the steady state we adopt in dynamic system is exactly the same as that generated by second moment we derive from itself.

Generally, Our model could be described as n nonlinear system equations with n endogenous variables, where $\mathbf{y}_{t+1}^{+}$ is a vector of variables with future expectation, $\mathbf{y}_t$ denotes the current period variables and $\mathbf{y}_{t-1}^{-}$ is variables of the last period.

$$\mathbb{E}_t \left[\mathbf{f}(\mathbf{y}_{t+1}^{+}, \mathbf{y}_t, \mathbf{y}_{t-1}^{-}) \right] = \mathbf{0}$$

If we have ith equation with future expectation variables u_{t+1}, we try to approximate around the expected value of it, called $\mathbb{E}_t u_{t+1}$ to the second order. We would have

$$\Phi_i(\mathbb{E}_t \mathbf{u}_{t+1}) = \mathbf{f}_i \left(\mathbb{E}_t \mathbf{u}_{t+1} \right) + \mathbf{f}_i' \mathbb{E}_t (\mathbf{u}_{t+1} - \mathbb{E}_t \mathbf{u}_{t+1}) + \frac{1}{2!} \mathbb{E}_t \mathbf{f}_i'' \mathbb{E}_t (\mathbf{u}_{t+1} - \mathbb{E}_t \mathbf{u}_{t+1})^2 + o(\cdot) \simeq 0$$

$\mathbf{f}_i'(\cdot)$ and $\mathbf{f}_i''(\cdot)$ are first and second derivative of the function at value $\mathbb{E}_t \mathbf{u}_{t+1}$, respectively. We can figure out that $\mathbb{E}_t(\mathbf{u}_{t+1} - \mathbb{E}_t \mathbf{u}_{t+1})^2$ is not equal to zero and it is regarded as variance and covariance when we consider the second order approximation and it could be found as the second moment from the dynamic system around the risky steady state.

5.4 Welfare and Consumption Equivalence

We calculate the welfare $\mathcal{W}_t$ using the households lifetime utility from the current time period t to the infinity. So

$$\mathcal{W}_t = \mathbb{E}_t \sum_{\tau=t}^{\infty} \beta^{\tau-t} U\left(C_\tau, L_\tau\right) \tag{87}$$

We take a second order approximation of the utility function around the risky steady state. Since we do second order approximation around the steady state with future expectation terms, risk should be included. We evaluate the risk in the future using the second moment of consumption and labour supply. We show into details how the welfare is evaluated in Appendix.

We use the consumption equivalence in the baseline model compared with the policy regime. We denote Γ as the percentage increase of steady state value of consumption needed in baseline model to reach the same level the welfare in particular macroprudential policy. Suppose $\mathcal{W}^B$ and $\mathcal{W}^v$ are the welfare of baseline model and with particular macroprudential policy, respectively, Γ^v is the percentage of consumption increase such that

$$\mathcal{W}^v = \frac{U\left((1+\Gamma^v)C^B, L^B\right) + COV(C^B, L^B)}{1-\beta} \tag{88}$$

C^B and L^B denotes the risky steady state value of consumption and labour in baseline model without policy, $COV(C^B, L^B)$ includes the second order term at the steady state. We show the simulation result. the steady state welfare and the consumption equivalence for each scenarios in the last two rows of Table 1.

Here we provide the second order approximation of utility function around the risky steady state and we construct the consumption equivalence as Equation (88). The second order term $COV(C, L)$ of utility function of the Taylor expansion each period is calculated as follows.

$$
\begin{aligned}
COV(C, L) &= \frac{1}{2!}\mathbb{E}_t\left\{\left[\frac{\partial^2 U}{\partial C_{t+1}^2}\right]_{C,Y}(C_{t+1}-\mathbb{E}_t C_{t+1})^2\right\} \\
&\quad + \frac{1}{2!} \times 2 \times \mathbb{E}_t\left\{\left[\frac{\partial^2 U}{\partial C_{t+1}\partial L_{t+1}}\right]_{C,Y}(C_{t+1}-\mathbb{E}_t C_{t+1})(L_{t+1}-\mathbb{E}_t L_{t+1})\right\} \\
&\quad + \frac{1}{2!}\mathbb{E}_t\left\{\left[\frac{\partial^2 U}{\partial L_{t+1}^2}\right]_{C,Y}(L_{t+1}-\mathbb{E}_t L_{t+1})^2\right\} \\
&= \frac{1}{2}\left[\frac{\partial^2 U}{\partial C_{t+1}^2}\right]_{C,Y} var(C_{t+1}) + \left[\frac{\partial^2 U}{\partial C_{t+1}\partial L_{t+1}}\right]_{C,Y} cov(L_{t+1}, C_{t+1}) \\
&\quad + \frac{1}{2}\left[\frac{\partial^2 U}{\partial L_{t+1}^2}\right]_{C,Y} var(L_{t+1})
\end{aligned}
$$

$\left[\frac{\partial^2 U}{\partial C_{t+1}^2}\right]_{C,Y}$ denotes the second derivative of utility function in one period at the risky steady state and all the second derivatives are shown as follows.

$$\frac{\partial^2 U}{\partial C_t^2} = -\gamma \left(C_t - hC_{t-1} - \frac{\chi}{1+\varphi}L_t^{1+\varphi}\right)^{-\gamma-1} - h^2\gamma\beta \left(C_{t+1} - hC_t - \frac{\chi}{1+\varphi}L_{t+1}^{1+\varphi}\right)^{-\gamma-1}$$

$$\frac{\partial^2 U}{\partial C_t \partial L_t} = \chi\gamma \left(C_t - hC_{t-1} - \frac{\chi}{1+\varphi}L_t^{1+\varphi}\right)^{-\gamma-1} L_t^{\varphi}$$

$$\frac{\partial^2 U}{\partial L_t^2} = -\chi^2\gamma \left(C_t - hC_{t-1} - \frac{\chi}{1+\varphi}L_t^{1+\varphi}\right)^{-\gamma-1} L_t^{2\varphi} - \chi\varphi \left(C_t - hC_{t-1} - \frac{\chi}{1+\varphi}L_t^{1+\varphi}\right)^{-\gamma} L_t^{\varphi-1}$$

Welfare is expressed as the sum of expected discounted utility. We write it in a recursive form:

$$\mathcal{W}_t = U\left(C_t, L_t\right) + \beta\mathbb{E}_t\mathcal{W}_{t+1}$$

We expand the formula around the expected value of consumption and labour (the risky steady state) and use Law of Iterated Expectations, we could get the steady state value of welfare as

$$\begin{aligned} \mathcal{W} &= U\left(C, L\right) + \beta\left[U\left(C, L\right) + COV\left(C, L\right) + \beta\left(U\left(C, L\right) + COV\left(C, L\right) + ...\right)\right] \\ &= \frac{U(C, L) + \beta COV(C, L)}{1 - \beta} \end{aligned}$$

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