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EXTERNALITIES, MONOPOLY AND THE OBJECTIVE FUNCTION OF THE FIRM

David Kelsey and Frank Milne

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David Kelsey
Department of Economics
University of Exeter
and University of Birmingham, England.

Frank Milne
Department of Economics
Queens University, Canada.

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Abstract

This paper provides a theory of general equilibrium with externalities and/or monopoly. We assume that the firm's decisions are based on the preferences of shareholders and/or other stakeholders. Under these assumptions a firm will produce fewer negative externalities than the comparable profit maximising firm. In the absence of externalities, equilibrium with a monopoly will be Pareto efficient if the firm can price discriminate. The equilibrium can be implemented by a 2-part tariff.

Address for Correspondence David Kelsey, Department of Economics, The University of Birmingham, Edgbaston, B15 2TT, ENGLAND, (until 31st July 2004). Department of Economics, School of Business and Economics, University of Exeter, Exeter, Devon, EX4 4PU, ENGLAND, (after 1st August 2004).

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1 INTRODUCTION

1.1 Background

Traditionally it has been assumed that firms maximise profits. However in the presence of market distortions, it is not typically the case that owners will wish firms to maximise profits. The usual justification for profit maximisation is the Fisher Separation Theorem (see Milne (1974), Milne (1981)), which says that if there are no externalities, the firm has no market power and financial markets are complete, all shareholders will wish to maximise the value of the firm. This result does not apply if there are externalities between the firm and its shareholders. In this case, shareholders will not just care about the effect of firm's decisions on their wealth but will also care about the direct (externality) effects of the decisions upon their utility. For instance, a shareholder who lives near a factory with a smoking chimney, will want less production than the profit maximising level and less production than one who lives further away. Thus we see both disagreement between shareholders and deviations from profit maximisation.

Although we use pollution as an example of an externality, it is not the most important one. Another is the dislike that many people have from investing in firms, which behave in socially irresponsible ways, such as supporting repressive regimes or damaging the environment. Alternatively the externality could be interpreted as private benefits of control, perquisites (see Jensen & Meckling (1976)) or other services not captured by market variables. These are the externalities discussed most often in the corporate control literature. We suspect that these are important factors in proxy fights and takeover contests. Other examples are firm-specific investments provided by workers or managers (see section 5).

If there is imperfect competition, the Fisher Separation Theorem breaks down in two ways. Firstly, in general, there will be disagreement between different shareholders about the policy of the firm. Secondly, typically, no shareholder will wish to maximise profits. The Fisher Separation Theorem does not apply if there is imperfect competition, since in that case, a change in the firm's production plan will affect prices as well as shareholders' wealth. Profits are not well defined since there will be more than one price system in terms of which profits can be expressed. If the firm changes its production plan typically a shareholder's old budget set will not be a subset of the new one and no unambiguous comparisons can be made.

As argued above, in the presence of market distortions, shareholder unanimity cannot be guaranteed. However it is still the case that there are some decisions on which all members of the control group will agree. Firstly we show that, under some assumptions, for any plan which is not productively efficient, there will be some production plan which is unanimously preferred. Secondly, all members of the control group will agree that the firm should produce less/more than the profit-maximising level of negative/positive externalities. Thus conventional profit-maximising models may have overstated the size of the distortions due to externalities. Thirdly if the firm has monopoly power all will agree on the desirability of using a 2-part tariff.

1.2 Modelling Firm's Decisions

We consider an economy with externalities and/or monopoly. As we argue above, there is a no unambiguous justification for assuming profit maximisation when markets are distorted. However, it is not clear what the alternative should be. At present there is no widely accepted economic model of the internal decision-making of firms. To resolve this we propose a relatively general model. Despite the generality, our model is able to make a number of predictions concerning equilibrium behaviour. The firm is modelled as a collection of individuals, each of whom is maximising his/her utility. Decisions are made by a process of aggregating the preferences of a group of decision-makers within the firm.

One possibility, is to assume decisions are made by a majority vote of shareholders, see for instance Hart & Moore (1996) or Renstrom & Yalcin (2003). If the firms' choice is one-dimensional (e.g. price), it will be determined by the median shareholders' preference. However one can object to these models by arguing that, in practice, management have more influence than shareholders. To model this, we assume that decisions are made by a group of individuals, which we shall refer to as the *control group*. We do not make specific assumptions about the composition of the control group since our model does not require them. For example, the control group could consist of the shareholders and senior management.

At present there is no widely accepted theory of the internal structure of the firm (for recent surveys of the governance literature see Shleifer & Vishny (1997), Allen & Gale (2000) and Tirole (2001)). For this reason we use an abstract model. We make, what we believe to be the mild assumption, that the firm's procedures respect unanimous preferences within the control group. Such rules would include, inter alia, those which give a major role for management.

Another major point of this paper is to emphasize the connection between traditional public economics and the theory of the firm. We can think of the firm as an entity which provides local public goods, e.g. profits and private benefits of control to shareholders and/or employees (see Holmstrom (1999)). This establishes a connection between our model of a firm and the theory of a public project in an economy with symmetric information and real or pecuniary externalities.

Our model does not deal with asymmetric information or competing oligopolistic firms. To incorporate asymmetric information we would require a general equilibrium model with asymmetric information. Such models exist (see Prescott & Townsend (2000)) but address the issues in terms of competitive clubs. Related issues, which arise in the context of incomplete markets and oligopoly are discussed in Kelsey & Milne (1996) and Kelsey & Milne (2003).

1.3 Organisation of the Paper

This paper aims to provide a general framework to model the internal decision-making of firms. In section 2 we present a general equilibrium model with externalities and/or monopoly. We begin the characterisation of equilibrium by showing that it is independent of the choice of numeraire and that, under some conditions, it is productively efficient. It is difficult to get clear comparative statics or policy conclusions when there are multiple distortions. For this reason the two subsequent sections consider externalities without monopoly and monopoly without externalities. Section 3 shows a case where externalities are partially internalised within the control group leading to a Pareto improvement on the Walrasian equilibrium. Returning to the monopoly problem, by similar reasoning if the control group are consumers of the firm's products and the monopolist uses uniform pricing then the price will be below the usual monopoly price. Again the distortion is partially internalised. However this involves cutting price to nonmembers of the control group as well, hence some profit has been lost. The natural response is to practice price discrimination, which we study in section 4. We show that perfect price discrimination implies Pareto efficiency and that the firm's preferred pricing system can be implemented with two-part tariffs. In the conclusion we discuss another interpretation of our model, where the firm is a monopsonist or externalities flow between the firm and a supplier. In particular we consider hold-up problems within the firm. The appendix contains proofs of those results not proved in the text.

This paper is intended to be part of a larger research programme on the theory of the firm. The aim is that the general existence result can be used in more specific settings to derive policy conclusions. In a companion paper, Kelsey & Milne (2003), we consider the objective function of the firm in oligopolistic industries. We find similar results to those in the present paper. In addition we find that the constitution of the firm can influence the equilibrium in the product market. This implies that there will be an optimal constitution of the firm to suit the conditions it faces in product and input markets. Hence we are able to endogenise the objective function of the firm.

2 EQUILIBRIUM

In this section we consider a general equilibrium model with monopoly and externalities. We have chosen a relatively simple model to illustrate the issues, which arise from endogenising decision-making within firms. It has been adapted from Edlin, Epelbaum & Heller (1998) to suit our purposes. The model is not intended to be the most general model of imperfect competition. Instead it has been chosen to study the economic effects of the internal organisation of firms.

2.1 Model

There is a single firm with market power, firm 0, which we shall refer to as *the monopolist*. There is in addition a fringe of F competitive firms, $1 \leqslant f \leqslant F$.

2.1.1 Markets

The model has J goods. Goods 1 to $\bar{j}$, $(\bar{j} \geqslant 1)$ are competitive goods, while goods $\bar{j}+1$ to J are monopoly goods. Thus we can write a vector of goods as $x = \langle x_c, x_m \rangle$ to denote the competitive and monopoly goods separately. There are markets in all goods. There is no market in shares. Since there is no uncertainty, diversification is not a possible motive for trading shares. We shall use $p_m \in \mathbb{R}^{J-\bar{j}}$ and $p_c \in \mathbb{R}^{\bar{j}}$ to denote respectively the price vectors for monopoly goods and competitive goods. Let $p = \langle p_c, p_m \rangle$ denote the price vector. Let $P = \mathbb{R}^J_+$ be the space of all price vectors. We shall make the following assumption.

Assumption 2.1 *All economic agents including the monopolist are price-takers for competitive goods.*

This ensures that at least one market is undistorted. It is almost impossible to derive clear policy implications in economies in which every market is distorted.

2.1.2 Firms

We require firms to satisfy the following assumptions.

Assumption 2.2 *Firm f has production possibilities described by a production function $\phi^f : \mathbb{R}^J \to \mathbb{R}$, i.e. $Y^f = \left\{ y^f \in \mathbb{R}^J : \phi^f \left(y^f \right) \geqslant 0 \right\}$, for $0 \leqslant f \leqslant F$. Moreover,*

1. *the function ϕ^f is assumed to be continuous and concave;*

2. $\phi^f \left(\bar{y}_c, \lambda \hat{y}_m + (1-\lambda) \tilde{y}_m \right) > \lambda \phi^f \left(\bar{y}_c, \hat{y}_m \right) + (1-\lambda) \phi^f \left(\bar{y}_c, \tilde{y}_m \right);$

3. *the production set, Y^f, is bounded above and non-empty;*

4. $0 \in Y^f$.

Let $y = \left\langle y^0, ..., y^F \right\rangle, y^f \in Y^f$, denote the economy's production vector and $Y = \left\{ \left\langle y^0, ..., y^F \right\rangle : y^f \in Y^f, 0 \leqslant f \leqslant F \right\}$ the economy's production set.

Assumption 2.3 (Free Disposal) *If* $z \in \sum_{f=0}^{F} Y^f$ *then* $-z^+ \in \sum_{f=0}^{F} Y^f$, *where* $z^+ = \langle \max\{0, z_0\}, ..., \max\{0, z_F\} \rangle$.

This says that any unwanted outputs can be disposed of at zero cost.

Assumption 2.4 *Firm* f *is a competitive firm for* $1 \leqslant f \leqslant F$. *These firms are price-takers for all goods. They neither produce monopoly goods nor use them as inputs,* $y = \langle y_c, y_m \rangle \in Y^f \Rightarrow y_m = 0$, *for* $1 \leqslant f \leqslant F$.

Firm 0 is a monopolist and is able to set the price for monopoly goods. However it is a price-taker in the market for competitive goods. One can motivate this by observing that, while some firms are big enough to set some prices, it is unlikely that any given firm would have sufficient market power to set prices for *all* goods. For instance, one would not expect Microsoft to consider the impact of its decisions on the price of paper since it is relatively small in that market.[1]

It would be possible to modify our model so that price taking behaviour for competitive goods could be derived rather than assumed. Consider an economy where the group of competitive goods are always desired by consumers and are produced by a competitive firm (or industry). Assume the firm(s) uses a linear technology, which uses the numeraire as an input.[2] As the commodities are always desired in positive amounts, they will be produced and their prices will be set equal to the constant marginal cost in terms of the numeraire. Thus these commodities' prices will be invariant to the monopolist's decision.

2.1.3 Consumers

There are H consumers $1 \leqslant h \leqslant H$. We assume that the consumer h has consumption vector x^h, which lies in a consumption set $X^h \subset \mathbb{R}^J \times Y$.

[1] For further discussion of this assumption see Edlin et al. (1998).

[2] This would require appropriate modification of Assumption 2.2.

Assumption 2.5 *For all* h, X^h, *is bounded below, non-empty, closed, convex and* $\mathbb{R}_+^J \times Y \subseteq X^h$, *where* $\mathbb{R}_+^J$ *denotes the non-negative orthant of* $\mathbb{R}^J$.

Assumption 2.6 *Consumer* h *has a utility function:* $u^h = u^h\left(x_c^h, x_m^h, y\right)$, *which is continuous in all arguments and weakly concave and increasing in* $\left\langle x_c^h, x_m^h \right\rangle$ *and strictly increasing in* x_c^h.

Note we allow for possible externalities between firms and consumers.

Assumption 2.7 *Individual* h *has endowments* ω_c^h *of competitive goods,* ω_m^h *of monopoly goods and* θ_h^f *of shares in firm* f, *where* $0 \leqslant \theta_h^f \leqslant 1$ *and* $\sum_{h=1}^H \theta_h^f = 1$. *We assume* $\omega_h \in \operatorname{int} X^h$, *where* $\omega_h = \left\langle \omega_c^h, \omega_m^h \right\rangle$.

Individual h has a budget constraint:

$$p_m.x_m^h + p_c.x_c^h \leqslant p_m.\omega_m^h + p_c.\omega_c^h + \sum_{f=0}^F \theta_h^f p.y^f. \tag{1}$$

This generates demand functions $x_m^h(p_c, p_m, y)$ and $x_c^h(p_c, p_m, y)$. Define the corresponding aggregate demand functions, $x_m(p_c, p_m, y) = \sum_{h=1}^H x_m^h(p_c, p_m, y)$ and $x_c(p_c, p_m, y) = \sum_{h=1}^H x_c^h(p_c, p_m, y)$.

Definition 2.1 *Define* $v^h(p, y) = \max_{x^h \in X^h} u^h\left(x^h, y\right)$, *subject to (1). The function* v^h *represents individual* h*'s induced preferences over the production plans and pricing decisions of the firms.*

2.1.4 Firms' Decisions

As already argued, it is not desirable to assume that the monopolist maximises profit. In addition, since there are externalities between the competitive firms and their shareholders, the Fisher Separation Theorem does not apply to them either. Instead of profit maximisation we assume that firm f can be represented as maximising a preference relation $\succcurlyeq^f$, defined on $P \times Y$. Hence, in general, firm's preferences may depend on the price vector and the output of all firms. This binary relation will arise from some process of aggregation of the preferences of the control group. In this section we describe the properties of this relation.

We assume that the decisions of firm f are made by a group of individuals $C^f \subset \{1, ..., H\}$, which we shall refer to as the *control group* of firm f. Our results do not depend crucially on the composition of the control group, hence we do not need to be more specific. We assume that the firm's preferences depend on the preferences of the control group $\langle v_h \rangle_{h \in C^f}$ and shareholdings $\theta_1^f, ..., \theta_H^f$. Note that we do not exclude the possibility that individuals, who are not shareholders (e.g. managers), are able to influence the firm's preferences. We shall not model the internal decision making of the control group explicitly but simply assume that whatever procedure is used, respects unanimous preferences.

Assumption 2.8 *For* $\tilde{f} \neq \hat{f}$, $C^{\tilde{f}} \cap C^{\hat{f}} = \emptyset$.

This says that there is no overlap between the control groups of different firms. We make this assumption to avoid issues of collusion, which are beyond the scope of the present paper.

Assumption 2.9 *Firm f's preferences satisfy the* Strong Pareto Principle *i.e.* $\forall h \in C^f, v_h \langle \hat{p}, \hat{y} \rangle \geqslant v_h \langle \tilde{p}, \tilde{y} \rangle$ *and* $\exists \hat{h} \in C^f, v_{\hat{h}} \langle \hat{p}, \hat{y} \rangle > v_{\hat{h}} \langle \tilde{p}, \tilde{y} \rangle \Rightarrow \langle \hat{p}, \hat{y} \rangle \succ^f \langle \tilde{p}, \tilde{y} \rangle$.

Equivalently, we are assuming there is costless Coasian bargaining within the control group.

Assumption 2.10 *The firm's strict preference relation,* $\succ^f$, *has open graph.*

This is a continuity assumption and is largely technical in nature. We do not need to assume completeness or transitivity of the firm's preferences. However to prove existence of equilibrium we need the following assumptions.

Assumption 2.11 *The firm's preferences satisfy* $\left\{y : \langle \tilde{p}, y \rangle \succ^f \langle \tilde{p}, \tilde{y} \rangle\right\}$ *is convex for* $0 \leqslant f \leqslant F$.

Note that we only assume convexity of preferences over goods for a given price vector. Preferences over prices are not necessarily convex. Competitive firms satisfy the above assumptions. They maximise their preferences taking all prices and any externalities produced by other firms as given. In contrast, firm 0 when supplying monopoly goods takes into account the effect of its decisions on the price of monopoly goods and any externalities produced by other firms. In addition we assume that the preferences of the monopolist are acyclic.

Definition 2.2 *A binary relation* $\succcurlyeq$ *on a set* X *is said to be* acyclic *if there do not exist* $x_1, ..., x_n \in X$ *such that* $x_i \succ x_{i+1}$ *for* $1 \leqslant i \leqslant n-1$ *and* $x_n \succ x_1$.

Assumption 2.12 *The preferences of the monopolist, firm 0,* $\succ^0$ *are acyclic.*

As is well known from the social choice literature, group preferences are likely to be incomplete or intransitive or both, (see for instance Sen (1970)). Because of this, we do not assume completeness and/or transitivity.[3] However we do assume acyclcity. This is a weaker assumption than transitivity, which enables us to avoid the Arrow Impossibility Theorem, see Sen (1977).[4] For some examples of decision procedures for firms, which satisfy our assumptions see Kelsey & Milne (1996).

2.2 Existence

Next we shall define and demonstrate existence of equilibrium. Although all trade takes place at a single moment of time, the model is formally sequential. First the monopolist, firm 0, chooses a vector of monopoly goods. Secondly there is trade in competitive goods. At the second stage all agents including the monopolist take prices as given. The vector of monopoly goods is treated as part of the shareholders' endowment. The second stage is a competitive equilibrium with non-standard preferences as in Shafer & Sonnenschein (1975). The monopolist chooses the initial vector of monopoly goods to achieve its most preferred equilibrium at the second stage.[5]

[3] Social choice problems may not be as great as they appear at first sight. Hansmann (1996) argues that the control groups of firms have relatively homogenous preferences. Hence the assumption of unrestricted domain, commonly used in social choice theory, may not hold in this context. This is true both of conventional investor-controlled firms and of various kinds of non-profit firms and cooperatives.

[4] The firm's problem is one of making a choice. In this respect it differs from the Arrow problem of making a social welfare judgement. Arrow required all social alternatives to be ranked. In contrast making a choice merely requires selecting a best element from a set of alternatives. The weaker condition of acyclicity is sufficient for the latter problem but not the former.

[5] A model with a similar sequential structure is used in Cornwall (1977).

First we take the output of monopoly goods as exogenous and define an equilibrium for the competitive sector of the economy.

Definition 2.3 *An equilibrium* $\left\langle x^*, y_c^{*0}, y^{*-0}, p^* | y_m^0 \right\rangle$ relative to a vector of monopoly goods, y_m^0, *consists of an allocation* x^*, *a vector of production plans for competitive firms* y^{*-0}, *a vector of competitive goods,* y_c^{*0} *for the monopolist and a price vector* p^*, *such that:*

1. $\sum_{h=1}^{H} x_h^* \leqslant \sum_{h=1}^{H} \omega_h + \left\langle y_c^{*0}, y_m^0 \right\rangle + \sum_{f=1}^{F} y^{*f}$;

2. x_i^* *maximises* $u_i\left(x_i, y^0, y^{*-0}\right)$, *subject to* $p.x_i \leqslant p.\omega_h + \theta_h^0 p^*.y^0 + \sum_{f=1}^{F} \theta_h^f p^*.y^{*f}$;

3. *there does not exist* $\hat{y}^f \in Y^f$ *such that* $\left\langle \hat{y}^f, y^{*-f}, p^* \right\rangle \succ^f \left\langle y^{*f}, y^{*-f}, p^* \right\rangle$, *for* $1 \leqslant f \leqslant F$;[6]

4. *there does not exist* $\hat{y}_c$ *such that* $\left(\hat{y}_c^0, y_m^{*0}\right) \in Y^0$ *and* $\left\langle \left(\hat{y}_c^0, y_m^0\right), y^{*-0}, p^* \right\rangle \succ^f \left\langle \left(y_c^{*0}, y_m^0\right), y^{*-0}, p^* \right\rangle$.

The vector of monopoly goods, y_m^0, is taken as given and the consumers, competitive firms and even the monopolist trade competitive goods taking prices as given. This makes precise the sense in which the monopolist is a price taker for competitive goods. For any given y_m^0, there may be one equilibrium, many equilibria or none.

Definition 2.4 *A* managerial equilibrium $\langle x^*, y^*, p^* \rangle$ *consists of an allocation,* x^*, *a production plan for each firm* y^*, *and a price vector* p^*, *such that:*

1. $\left\langle x^*, y^{*-0}, p^* | y_m^{*0} \right\rangle$ *is an equilibrium relative to* y^{*0};

2. *there does not exist* $\langle \tilde{x}, \tilde{y}, \tilde{p} \rangle$ *such that:*

   (a) $\left\langle \tilde{x}, \tilde{y}^{-0}, \tilde{p} | \tilde{y}_m^0 \right\rangle$ *is an equilibrium relative to* $\tilde{y}_m^0$,

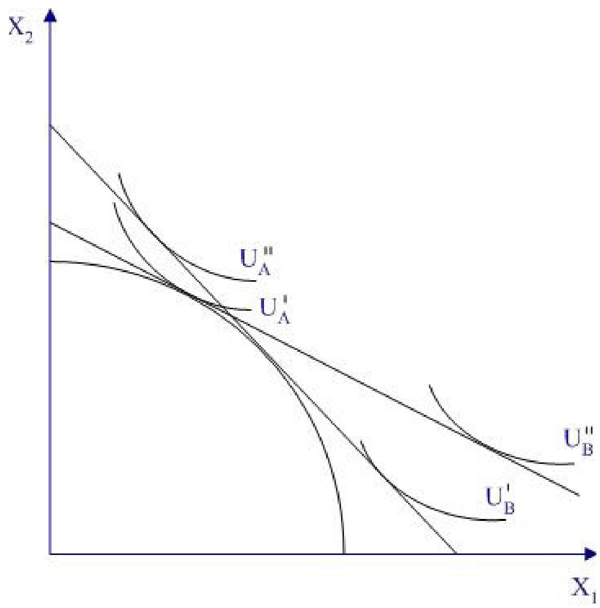
   (b) $\left\langle \left(\tilde{y}_c^0, \tilde{y}_m^0\right), \tilde{y}^{-0}, \tilde{p} \right\rangle \succ^0 \left\langle \left(y_c^{*0}, y_m^{*0}\right), y^{*-0}, p^* \right\rangle$.

If y_m^0 gives rise to multiple equilibria, we assume that the monopolist can choose its preferred equilibrium. In our opinion this is a reasonable way to model a monopolist

[6] As usual, y^{*-f} denotes the production vector of all firms other than f.

with power to set prices. The concept of profit maximisation is not well-defined as the Fisher Separation Theorem fails (for more detail see Milne (1981)). The following example indicates the problem.

Example 1 *Consider an economy with two consumers and two commodities. Each has the non-negative orthant in* $\mathbb{R}^2$ *as a consumption set and owns half the total endowment and production set. The production set is closed and convex. Thus both consumers have identical budget sets given any choice of* y*. However, since they have different preferences, they can have different rankings of the production vectors with price making by the monopolist, (see figure 1).*



Theorem 2.1 *Provided consumers satisfy Assumptions 2.5, 2.6 and 2.7, the monopolist satisfies 2.12 and all firms satisfy Assumptions 2.2, 2.3, 2.8, 2.9, 2.10 and 2.11, a managerial equilibrium exists.*

2.3 Choice of Numeraire

If firms maximise profit and there is imperfect competition, the real equilibrium will depend on the choice of numeraire or more generally the price normalisation rule, see for instance Böhm (1994). The intuition is clear: in a pure exchange economy, if one individual was given an objective, which depended on the numeraire, then changes in the numeraire could change the real equilibrium. A similar problem arises in an economy with production, if the firm's objective is to maximise profits in terms of the numeraire.

This problem does not arise in our model, since production decisions are based on utility maximisation by individuals. Hence the firm has a real objective. For instance, suppose that decisions of the firm are made by a majority vote of shareholders. Each one will have preferences which only depend on real consumption, hence the firm's decisions and consequently the equilibrium will be independent of the numeraire. Below we show that with our definition, equilibrium is independent of the numeraire.

Proposition 2.1 *The set of managerial equilibria does not depend on the choice of numeraire.*

This result follows from the sequential structure of our model. The second stage is a competitive equilibrium with non-standard preferences and hence is independent of the numeraire for the usual reasons. At the first stage the monopolist chooses his/her production plan. As explained above this decision depends only on real variables and hence is also independent of the numeraire.

2.4 Productive Efficiency

Here we show that monopoly is productively efficient even if it does not necessarily maximise profit. To do this we need to assume that there are no externalities.

Assumption 2.13 *There are no externalities, i.e.* $u^h\left(x_c^h, x_m^h, y\right) = u^h\left(x_c^h, x_m^h, 0\right)$ *for all* $y \in Y$ *and* $1 \leqslant h \leqslant H$.

Proposition 2.2 *Under Assumptions 2.1, 2.2, 2.6, 2.9, and 2.13 the equilibrium will be productively efficient.*

Proposition 2.2, shows that although the Fisher Separation Theorem does not apply, all shareholders will approve a change, which reduces costs while leaving output unchanged. Consider a point which is productively inefficient. Then the monopolist could directly supply output of competitive goods to shareholders in proportion to their shareholdings. Since the original position is productively inefficient, for a small increase, this is possible while leaving the firm's other net trades unchanged. This implies that the firm's profits are unchanged. As shareholders are price-takers for competitive goods this will be perceived as making them all better off.

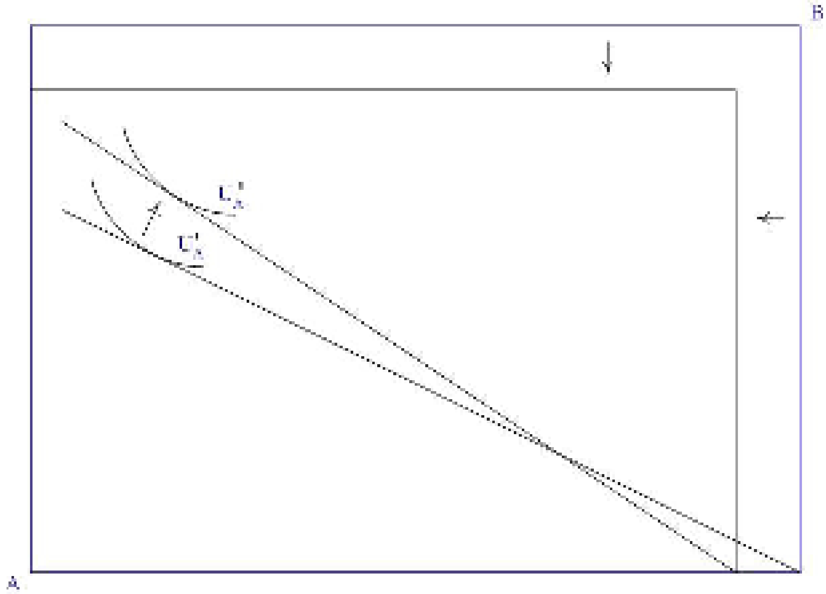
The following example shows that if the firm is not a price-taker in at least one input market, then an owner-manager may choose to be productively inefficient.

Example 2 ***Inefficient and Efficient Monopoly*** *Consider an economy with two consumers A and B, who have utility functions over two commodities, $u_i(x_{1i}, x_{2i})$, where $i = A, B$. Assume that consumer A has an endowment of an input L and wholly owns a production technology, where the input produces commodity 1 via a neoclassical production function $f(\ell)$. Consumer B has an endowment of commodity 2.*

We can construct an Edgeworth Box, where the height is commodity 2 and the length is commodity 1. By varying the amount of ℓ that consumer A puts into the firm she can alter the dimensions of the box. Assume that consumer A's firm is a monopoly supplier of commodity 1. It is possible that by reducing her input of ℓ and freely disposing of the remainder she can make herself better off, if relative prices move sufficiently in her favour, see figure 2.[7]

Alternatively, consider A's firm to be a perfect price discriminator, where A sets a non-linear price schedule that curves around the indifference curve through B's endowment. Assuming that A supplies inputs $\ell = L$, then we have a standard result, that the allocation is Pareto efficient. All the gains from trade are obtained by A. Consumer B is indifferent between trading or merely consuming his endowment of commodity 2. Clearly, in this case A will not reduce her input below L because that will reduce her welfare. Price discrimination is considered in more detail in section 4, where we show more generally that it can lead to efficient outcomes. The reason for the inefficiency of the first case is induced by the monopolistic distortion in prices, which in turn implies inefficiency in input supply.

[7] This is an example, in reverse, of the classical immiserising growth argument in international trade, see Dixit & Norman (1980), Ch.5.



Our result on productive efficiency could, in principle, be extended to economies with externalities. However care needs to be taken over the appropriate definition of productive efficiency. The externalities need to be taken into account when defining efficiency. Consider a situation with two firms, call them firm A and firm B. These firms are otherwise similar except that firm B produces twice as much pollution per unit of output compared to firm A. Intuitively, efficiency requires that firm B should not produce. This could be achieved by counting pollution as an input into the production process. However proceeding in this direction may create problems. In the present paper, each consumer potentially gets an externality from the use of every physical commodity by every firm. Hence we would need a new good for the effect of each output or input of each firm on every individual. A total of $J(F+1)H$ new goods. However with such a large number of goods, productive efficiency is a very weak criterion. It could be criticised as merely saying each process is an efficient way of producing itself. For further discussion of productive efficiency and externalities see Sen (1973).

3 EXTERNALITIES

This section presents a special case of the previous model, where there are externalities but no monopoly power. In this case we show a non-profit maximising firm produces less than the profit-maximising level of negative externalities. A similar result was proved in a partial equilibrium context by Roemer (1993), who showed that pollution would be reduced if a firms' decisions were made by majority voting. We extend this to general equilibrium and to any decision rule which respects unanimity. Our model is also different because it has multiple firms and variable labour supply. By similar reasoning we may show that a non-profit maximising firm will produce more positive externalities than a profit maximising firm.

3.1 Model

There are two traded goods, a consumption good y and labour L. In addition there is a negative externality, e.g. pollution z, which is not traded but enters into the utility and production functions. We shall normalise the price of y to 1. The price of labour is denoted by w. In order to focus on the effect of externalities, throughout this section we shall assume that all firms are price-takers.

Consumers

Assumption 3.1 *There are H consumers, $1 \leqslant h \leqslant H$. Individual h has utility function: $u^h = y^h - c^h\left(\ell^h\right) - D^h\left(\sum_{f=1}^F d^h\left(z^f\right)\right)$, where $c^{h\prime} > 0, c^{h\prime\prime} > 0$, $D^{h\prime} > 0, D^{h\prime\prime} > 0, d^{h\prime} > 0$, $d^{h\prime\prime} > 0$ and z^f denotes the level of externality produced by firm f. Consumer h has an endowment $\bar{\ell}^h$ of labour.*

This utility function has two familiar special cases. First where the externality is a pure public bad, $u^h = y^h - c^h\left(\ell^h\right) - D^h\left(\sum_{f=1}^F z^f\right)$. Secondly where utility is additively separable between the externalities produced by different firms, $u^h = y^h - c^h\left(\ell^h\right) - \sum_{f=1}^F d^h\left(z^f\right)$. Since utility is quasi-linear, the aggregate labour supply can be written in the form $L^S = L^S(w)$.

Firms There are F identical firms, which produce the consumption good from labour according to the production function $y = g(L, z)$, where g is C^2 strictly concave and increasing in both arguments. Firms are assumed to be price-takers on both input and output markets. We assume that all firms use the same decision rule. No assumptions are imposed on this rule other than that it respects unanimity. Individuals are not, however, assumed to be identical. Hence the firm faces a non-trivial collective choice problem. To preserve symmetry, we require that consumers all suffer the same disutility from the externalities produced by any given firm.

All individuals are assumed to have an equal number of shares in each firm. We assume that individuals do not coordinate their voting across different firms. Thus they cannot implement a collusive outcome by reducing output at all firms simultaneously.[8] We shall only consider symmetric equilibria.

Theorem 3.1 *For any decision rule which satisfies unanimity, in symmetric equilibrium, there will less than the profit-maximising level of negative externality.*

Although we have shown unambiguous results, the reader will have noticed that we required strong assumptions on preferences and production. This should not be surprising as we are dealing with an abstract second best setting, where apparently perverse comparative static results can occur. To see this more clearly, observe that our profit-maximising model above can be thought of as an economy, where there are no Lindahl prices for externalities. In contrast the non-profit maximising firm has marginal conditions that mimic Lindahl prices for externalities flowing to the control group. Thus our problem is comparing distorted and less distorted economies, neither of which are first best.

Consider a partial equilibrium world, where the output and input prices are held constant. If a firm is faced with additional Lindahl shadow prices for the externality from the control group, it will reduce output of a negative externality, for the usual

[8] This may be rigorously justified in a model with many types of consumer, where all consumers of a given type have the same preferences and endowment of goods. The distribution of shares over types is the same for all firms. However no individual owns shares in more than one firm, hence there is no possibility for coordinating voting between different firms.

revealed preference reasons. This intuitive result requires no feedback effects through prices induced from the general equilibrium conditions. It is these effects that can overturn the partial equilibrium intuition. However in general equilibrium strong assumptions are needed since it is difficult to get clear comparative static results when agents' actions are strategic substitutes. We have assumed that if one firm pollutes more this reduces the marginal benefit of polluting, hence externalities are strategic substitutes. This seems a natural assumption if the externality is pollution. The marginal damage of pollution is thought to be increasing in many environmental problems. However, while realistic, it is difficult to establish general comparative static results with this assumption. Comparative statics could be established with less restrictive assumptions if externalities were strategic complements, (see Milgrom & Roberts (1990), Milgrom & Shannon (1994)).

3.2 Expanding The Control Group

Our preceding observations on Second Best results, imply that it is difficult to make unqualified assertions about the welfare implications of expanding the control group. For example, in a related literature in incomplete asset markets it is well-known (see Hart (1975), Milne & Shefrin (1987)) that introducing more markets for asset trading can be welfare reducing.[9]

Therefore an increase in the control group, that moves the economy from one second best equilibrium to another, could, in principal, have any welfare result. Clearly if the original control group can choose to add or veto the addition of new members, they will only introduce members that enhance the welfare of both old and new members. Notice that our non-profit model allows for transfers, so that new members could compensate existing ones for the benefits of entry. In an abstract way this encapsulates the bargaining that occurs in takeovers and mergers, where side-payments and conditions are negotiated by shareholders, management and key employees.

One deficiency is that there is no obvious limit to the size of the control group. It could be possible to include all agents in an efficient allocation for the economy

[9] Other examples of counter-intuitive comparative statics occur in international trade, taxation etc. and are well known in the public economics literature (see Laffont (1988)).

and compensate potential losers. In short, the control group would be equivalent to some efficient, all inclusive planning agency. Clearly this is unrealistic as we have omitted any costs of bargaining within the firm or with potential new members of the control group. Thus we could allow for bargaining costs that rise with the size of the control group. This cost, limiting the size of the control group, is similar to crowding or congestion costs in the theory of clubs, where such costs limit the size and composition of clubs.[10]

4 PRICE DISCRIMINATION

In this section we consider a variant of the model of section 2, where there is monopoly power but no externalities. By similar reasoning to that used in the previous section, we may show that if a monopolist practices uniform pricing it will set a price below the profit-maximising level. This will happen if control group members are also consumers of the firm's output. Starting at the profit maximising level, a price reduction has a second order effect on profits but a first order effect on the consumer surplus. We shall not discuss this in detail since there is already a fairly large literature on the case where consumers wholly or partially control a uniform pricing monopolist. (See, for instance, Farrell (1985), Hart & Moore (1996), Kelsey & Milne (2003) and Renstrom & Yalcin (2003)).

When a uniform pricing monopolist reduces the price, those within the firm gain consumer surplus. However cutting price reduces the profits, which can be made from non-members of the control group. This suggests that the firm would like to practice price discrimination, selling at marginal cost to members of the control group, while charging outsiders a higher price. In practice, discounts for staff are common and discounts for shareholders are not unknown. Hence there is a case for investigating price discrimination with non-profit maximising firms. We focus on the extreme case

[10] There are obvious parallels with our theory of the firm and club theory, see Prescott & Townsend (2000) for an explicit connection in a general equilibrium model with asymmetric information. See Cornes & Sandler (1996), for a survey; and Conley & Wooders (2001) and Ellickson, Grodal, Scotchmer & Zame (1999) for recent formulations of endogenous clubs embedded in a private market system.

of perfect price discrimination. Our main results are that, the outcome will be Pareto optimal and can be implemented by two-part tariffs. This extends some results of Edlin et al. (1998) from profit maximisation to general objective functions for the firm.

4.1 Model

The model is similar to that of section 2, the main modifications being that we assume no externalities and allow the monopolist to price discriminate. For perfect price discrimination to be possible it is necessary that households should not be able to trade in the goods it produces. Hence we shall require that no individual has any endowment of monopoly goods in this section. Moreover individuals are not able to trade monopoly goods among themselves, hence there are no resale prices for these goods. This enables us to prove efficiency of the equilibrium. To apply calculus techniques, the utility function is C^2, the production function is C^1 and both functions satisfy appropriate Inada conditions.

4.1.1 Consumers

Consumers satisfy Assumption 2.6. Let R^h be the total amount which individual h pays for monopoly goods. Individual h's income, net of payment to the firm is $I^h = p_c.\omega^h - R^h$. To prove existence we need to make an additional assumption, which says that the firm is the only source of monopoly goods. This will be a maintained hypothesis throughout this section.

Assumption 4.1 *All individuals have zero endowment of monopoly goods,* $\omega_m^h = 0$, *for* $1 \leqslant h \leqslant H$.

Consumer h has utility function $u^h\left(x_c, x_m^h, y\right)$ defined over competitive and monopoly goods and possibly externalities from the firms. Individual h's budget constraint for competitive goods is $p_c.x_c^h \leqslant I^h$. The first order condition for the consumer's optimal choice of competitive goods is:

$$\frac{\partial u^h}{\partial x_{cj}} / \frac{\partial u^h}{\partial x_{ck}} = \frac{p_{cj}}{p_{ck}}. \tag{2}$$

The solution is individual h's demand function, denoted by $x_c^h\left(I^h, p_c, y\right)$.

Definition 4.1 *Define the indirect utility function of individual h by,* $V^h\left(x_m^h, R^h, p_c, y\right) = \max_{x_c} u^h\left(x_c, x_m^h, y\right)$ *such that* $p_c.x_c \leqslant p_c.\omega^h - R^h$.

By definition; $V^h\left(x_m^h, R^h, p_c, y\right) = u^h\left(x_c\left(I, p_c, y\right), x_m^h, y\right)$, hence $\frac{\partial V^h}{\partial I} = \sum \frac{\partial u^h}{\partial x_{cj}} \frac{\partial x_{cj}}{\partial I}$. Substituting from (2), $\frac{\partial V^h}{\partial I} = \frac{1}{p_{ck}} \frac{\partial u^h}{\partial x_{ck}} \sum p_{cj} \frac{\partial x_{cj}}{\partial I}$. By differentiating the budget constraint we obtain, $\sum p_{cj} \frac{\partial x_{cj}}{\partial I} = 1$. Hence:

$$p_{ck} \frac{\partial V^h}{\partial I} = \frac{\partial u^h}{\partial x_{ck}}. \tag{3}$$

Define $\bar{u}^h\left(p_c\right) = \max_{x_c} u^h\left(x_c, 0, y\right)$ such that $p_c.x_c \leqslant p_c.\omega^h$. Thus $\bar{u}^h\left(p_c\right)$ is the reservation utility, which consumer h can obtain if (s)he does not trade with the monopolist. Since we assume that the monopolist is a price-taker for competitive goods, $\bar{u}^h\left(p_c\right)$ can be taken as given by him/her.

4.1.2 Monopolist

The monopolist satisfies Assumption 2.1. We retain the assumption of symmetric information, hence there are no incentive compatibility problems. As usual, we can restrict attention to take it or leave it offers. The firm offers to supply individual h with a bundle x_m^h of monopoly goods in exchange for (gross) payment R^h. Since the monopolist implements unanimous preferences of the control group, the outcome can represented locally by maximising a weighted sum, $\sum_{h=1}^M \lambda^h u^h$, of their utilities for some non-negative weights λ^h. We may normalise the λ's by requiring $\sum_{h=1}^M \lambda^h \theta^h = 1$.[11] Hence, we may represent the firm's behaviour as the solution to the following optimisation problem. Choose $\langle R, x_m, y_c \rangle$ to maximise $\sum_{h=1}^M \lambda^h V^h\left(R^h, x_m^h, p_c\right)$ subject to the constraints:

$$\begin{aligned}
V^h\left(R^h, x_m^h, p_c\right) &= \bar{u}^h\left(p_c\right), \quad \text{for } M+1 \leqslant h \leqslant H; \\
\phi\left(\sum x_m^h, y_c\right) &= 0; \\
\sum_{h=1}^H R^h + p_c.y_c &= 0,
\end{aligned} \tag{4}$$

[11] This is only a local representation of the firm's preferences and does not imply that they are globally complete or transitive.

where $x_m = \left\langle x_m^1, ..., x_m^H \right\rangle$ and $R = \left\langle R^1, ..., R^H \right\rangle$. The first constraint says that non-members of the control group must achieve at least as much utility as they could obtain by not trading with the firm. The second restricts the firm to using feasible production plans. The final constraint is the firm's budget balance condition. The Lagrangian for the firm's optimisation problem is:

$$\begin{aligned}\mathcal{L} &= \sum_{h=1}^{M} \lambda^h V^h \left(R^h, x_m^h, p_c, y\right) + \sum_{h=M+1}^{H} \xi^h \left[V^h \left(R^h, x_m^h, p_c, y\right) - \bar{u}^h \left(p_c\right)\right] \\ &\quad + \mu\phi \left(\sum x_m^h, y_c\right) + \delta \left(\sum_{h=1}^{H} R^h + p_c.y_c\right). \end{aligned} \tag{5}$$

4.2 Equilibrium

Below we modify our definition of equilibrium to allow for price discrimination. As before we consider an economy which is formally sequential. First the monopolist chooses a profile of take it or leave it offers $\langle R, x_m \rangle$. Then all agents including the monopolist trade competitive goods taking prices as given.

Definition 4.2 *A equilibrium given $\langle R, x_m \rangle$ consists of allocations of competitive goods, a vector, y_c^*, of competitive goods and a price vector for competitive goods, $\langle x_c^*, y_c^*, y_m^*, p_c^* \rangle$, such that:*

1. $y_c^* + \sum_{h=1}^{H} x_c^{*h} = \sum_{h=1}^{H} \omega^h$;
2. $u^h \left(x_c^{*h}, x_m^h\right) \geqslant u^h \left(x_c^h, x_m^{*h}\right)$, *for all* x_c^h *such that* $p_c^*.x_c^h \leqslant p_c^*.\omega^h - R^{*h}$, *for* $1 \leqslant h \leqslant H$;
3. $u^h \left(x_c^{*h}, x_m^h\right) \geqslant \bar{u}^h \left(p_c\right)$ *for* $1 \leqslant h \leqslant H$.

Definition 4.3 *A PDM (price discriminating monopoly) equilibrium consists of a profile of take it or leave it offers, allocations of competitive goods, a production plan and a price vector for competitive goods, $\langle R^*, x_c^*, x_m^*, y_c^*, y_m^*, p_c^* \rangle$, such that:*

1. $\langle x_c^*, y_c^*, y_m^*, p_c^* \rangle$ *is an equilibrium given* $\langle R^*, x_m^* \rangle$;
2. $\langle R^*, x_m^*, y_c^*, y_m^* \rangle$ *solves the firm's optimisation problem (4);*

Theorem 4.1 *Given consumers satisfy Assumptions 2.5, 2.6, 2.7 and 4.1 and firms satisfy Assumptions 2.2, 2.3, 2.8, 2.9, 2.11 and 2.10, a PDM equilibrium exists.*

Proof. Given x_m and R, which satisfy the participation constraint, we can apply the same type of argument as Lemma A.3 to show there exists a pair $\langle R, x_m \rangle$, for which the competitive sector of the economy has an equilibrium. As before we may show that the set of equilibria contingent on $\langle R, x_m \rangle$ is closed. Since the set of attainable allocations is compact, we may assume that R and x are chosen from compact sets. Since the monopolist's feasible production set is compact and his/her objective is continuous, Lemma A.2 guarantees the existence of a maximum and thus a PDM equilibrium. ■

4.3 Efficiency

We shall now demonstrate that the equilibrium is efficient and can be implemented by a 2-part tariff, which consists of a personalised hook-up fee and a per unit price equal to marginal cost. Intuitively, total surplus can be maximised by setting price equal to marginal cost. Since surplus is maximised, the resulting equilibrium is efficient.

Definition 4.4 *We say that the firm uses* marginal cost pricing *if it sets a tariff,* $R^h = T^h\left(x_m^h\right) = t^h + p_m.x_m^h$, *where* $p_m = \left\langle p_{c1}\frac{\partial \phi}{\partial y_{m1}}/\frac{\partial \phi}{\partial y_1}, ..., p_{c1}\frac{\partial \phi}{\partial y_{mn}}/\frac{\partial \phi}{\partial y_1}\right\rangle$ *and* $T^h\left(x_m^h\right)$ *denotes the total amount consumer* h *pays for quantity* x_m^h.

To prove efficiency we need to assume that there are no externalities. Henceforth we shall suppress the dependence of u on y.

Theorem 4.2 *If Assumptions 2.1, 2.2, 2.6, 2.8, 2.9 and 2.13 are satisfied a PDM equilibrium is Pareto efficient and can be implemented by a 2-part tariff, in which the monopolist uses marginal cost pricing.*[12]

[12] This result does not hold if the firm is not a price-taker for competitive goods. The reasoning is the same as for a conventional monopolist. Starting at the efficient quantities, a small change in quantity will have a first order effect on prices in the competitive sector but only a second order effect on profits. Typically there will be a direction of change which will make all members of the control group better-off.

This result may have some applications for regulation. Regulated (or nationalised) firms are unlikely to maximise profit. It may be useful to know that their preferred pricing structure will consist of a two-part tariff. It is not surprising that the firm will wish to present outsiders with a 2-part tariff, since this pricing scheme is capable of extracting all their surplus. The firm also wishes to use a 2-part tariff with members of the control group. The reason is that, within the control group, it is desirable to allocate goods efficiently by using marginal cost pricing. Any redistribution between control group members can be achieved in a lump-sum manner by adjusting the hook-up fees. To clarify, for non-members of the control group the hook-up fee is equal to the total surplus. For the control group, the hook-up fee is not necessarily equal to total consumer surplus. Instead it is determined by a bargaining process or game within the control group.

An example of such bargaining would be partnerships in accounting or law firms where salaries and bonuses are determined by various formulae and bargaining in the group. Observe that such firms have partners (members of the control group) and non-partners. We have not modelled the determination of the control group. For further discussion see section 3.2.

5 CONCLUSION

The hypothesis of profit maximisation has been criticised both on empirical and theoretical grounds. Although we have found that there are a number of differences between profit maximising and non-profit maximising firms, we have also shown that some well-known results are independent of the objective of the firm. This suggests that many existing results on economics of firms and industries do not crucially depend on profit maximisation.

These arguments provide a possible rationale for controls on foreign ownership and may explain popular suspicion of foreign owned firms. If the control group of a foreign owned firm does not suffer externalities in the domestic economy, then such a firm would produce the profit-maximising level of externalities. By similar reasoning, monopolistic distortions would also be worse in a foreign owned firm. Hence there may be a case for regulating foreign owned firms more strictly. Even within a single country, there may be reasons for preferring relatively small locally owned firms to large national companies. Similarly our analysis of pollution problems, suggests that there may be advantages in having waste disposed of close to its place of production. This increases the chance that, those affected by negative externalities, will have some influence on the firm's decision.

So far the paper has emphasised the involvement of consumers in firms' decisions. But our theory is symmetric, so that we could assume the firm is a monopsonist in some input markets or that there are externalities flowing between the firm and a supplier of inputs. The most common examples are farm-owned marketing organisations or where the firm is owned by suppliers of a particular form of labour. So long as the firm acts competitively in all markets, except those for its own inputs, our arguments on productive efficiency continue to apply. Now let us turn to specific cases, where the supplier of the input can be influential in the decisions of the firm.

Assume that there are negative externalities between the firm and its suppliers. Then, as before, a non-profit maximising firm will produce less of such externalities. A special case of an externality arises from the hold-up problem. Assume that suppliers may make firm-specific investments, which are non-contractible, e.g. in human capital. Ex-post, the firm can appropriate these investments. This imposes a negative externality on the suppliers of inputs and hence reduces the incentive to provide firm specific investments.

With conventional firms there will be too little firm-specific human capital in equilibrium. However as already noted, a non-profit maximising firm will produce fewer negative externalities. Thus the hold-up problem will be reduced, and input suppliers will be more willing to supply firm-specific inputs, which brings about a Pareto improvement. It would be desirable to include all the suppliers so long as they are productive and add to the group's welfare. Observe that it is in the interest of the control group of the firm to include agents that suffer from the externality or benefit from the supply of firm specific human capital, given appropriate transfers.

This case relates directly to some recent papers on the theory of the firm (see

Hart & Moore (1990), Hart & Moore (1996) and Roberts & Steen (2000), where the initial members of the control group find it advantageous to include suppliers of firm specific human capital). But the general principles operate whether we are considering externalities or suppliers of firm-specific inputs.[13]

APPENDIX

A Existence and Efficiency

This appendix contains some technical results and proofs relating to the existence of equilibrium and productive efficiency.

Lemma A.1 *Let X be a finite set and let $\succcurlyeq$ be a reflexive and acyclic binary relation on X. Then there is a $\succcurlyeq$-maximal element of X.*

Proof. Choose $x_1 \in X$ arbitrarily. If there does not exist $x \in X$, such that $x \succ x_1$, the proof is complete. Otherwise choose $x_2 \in X$ such that $x_2 \succ x_1$. Define recursively x_{n+1} to be an element of X such that $x_{n+1} \succ x_n$ if such an element exists. Since X is finite and $\succcurlyeq$ is acyclic, this process must eventually terminate. The final point will be a $\succcurlyeq$-maximal element of X. ■

To prove existence we shall need the following result, which is a generalisation of the Weierstrass theorem.

Lemma A.2 *Let X be a topological space and let $\succcurlyeq$ be a reflexive and acyclic binary relation on X with open lower sections. Then if K is a non-empty compact subset of X, there exists a $\succcurlyeq$-maximal element of K.*

Proof. Suppose if possible that K contains no $\succcurlyeq$-maximal element. Since $\succcurlyeq$ has open lower sections, the sets $O_x = \{y : x \succ y\}, x \in K$, are an open cover of K. Hence there is a finite subcover, $O_{x_1}, ..., O_{x_m}$. Since $\succcurlyeq$ is acyclic the set $\{x_1, ..., x_m\}$ contains

[13] Notice that we assume that there exists an control mechanism that ensures efficient production rules. If for some reason we assume that such a mechanism cannot be used, then the control group will be constrained to use an inefficient (in the first best sense) mechanism. This is the central message of Bolton & Xu (1999), Hart & Moore (1990) and Roberts & Steen (2000).

a $\succcurlyeq$-maximal element. Without loss of generality assume this is x_1. Then it is not the case that $x_i \succ x_1$, for $1 \leqslant i \leqslant m$. But this implies that $x_1 \notin O_{x_i}$, for $1 \leqslant i \leqslant m$, which contradicts the fact that $O_{x_1}, ..., O_{x_m}$ is an open cover of K. The result follows. ■

Definition A.1 *Let $\mathcal{E}$ denote the set of ordered pairs $\langle y_m^0, p \rangle$, for which there exists $x \in X, y^{-0} \in Y^{-0}$, and y_c^0 such that $\left(y_c^0, y_m^0\right) \in Y^0$ and $\langle x, y_c^0, y^{-0}, p | y_m^0 \rangle$ is a competitive equilibrium relative to y_m^0.*

Lemma A.3 *Given that consumers satisfy Assumptions 2.5, 2.6 and 2.7, firms satisfy Assumptions 2.2, 2.3, 2.8, 2.9, 2.11 and 2.10, $\mathcal{E}$ is non-empty and closed.*

Proof. Consider the case where $y_m^0 = 0$. Then one can show an equilibrium exists by adapting the proof of Theorem 7.21 of Ellickson (1993). This result requires the production and consumption sets to be compact, however it may be adapted to our model as follows. Consider a sequence of truncated economies, where the production and consumption sets are bounded, such that the bounds tend to infinity as $n \to \infty$. Let $\langle x_n, y_{cn}^0, y_n^{-0}, p_n | y_{mn}^0 \rangle$ be the sequence of equilibria of the truncated economies. The attainable set is compact. By taking convergent subsequences, if necessary, we may assume that $x_n, y_{cn}^0, y_n^{-0}, p_n$ and y_{mn}^0 converge to limits $\bar{x}, \bar{y}_c^0, \bar{y}^{-0}, \bar{p}$ and $\bar{y}_m^0$ respectively.

We claim that $\langle \bar{x}, \bar{y}_c^0, \bar{y}^{-0}, \bar{p} | \bar{y}_m^0 \rangle$ is a competitive equilibrium relative to $\bar{y}_m^0$. Since the consumption and production sets are closed, $\bar{x}$ and $\bar{y}$ are feasible. As $\sum_{h=1}^H x_{nh} \leqslant \sum_{h=1}^H \omega_h + \left(y_{cn}^0, y_{mn}^0\right) + y_n^{-0}$, for all n, $\sum_{h=1}^H \bar{x}_h \leqslant \sum_{h=1}^H \omega_h + \left(\bar{y}_c^0, \bar{y}_m^0\right) + \bar{y}^{-0}$. Suppose, if possible, there exists $\breve{y}^f \in Y^f$ such that $\langle \bar{p}, \breve{y}^f, \bar{y}^{-f} \rangle \succ^f \langle \bar{p}, \bar{y}^f, \bar{y}^{-f} \rangle$, for $0 \leqslant f \leqslant F$. Since the graph of $\succ^f$ is open there exists $\epsilon > 0$ such that if $\|p - \bar{p}\| < \epsilon$, $\|\bar{y} - z\| < \epsilon$ and $\left\| \langle \breve{y}^f, \bar{y}^{-f} \rangle - w \right\| < \epsilon$ then $\langle p, w \rangle \succ^f \langle p, z \rangle$. For all sufficiently large n, $\|p_n - \bar{p}\| < \epsilon$, $\|y_n - \bar{y}\| < \epsilon$ and $\left\| \langle \breve{y}^f, \bar{y}^{-f} \rangle - \left\langle \breve{y}^f, y_n^{-f} \right\rangle \right\| < \epsilon$ hence $\left\langle p_n, \breve{y}^f, y_n^{-f} \right\rangle \succ^f \left\langle p_n, y_n^f, y_n^{-f} \right\rangle$. However this contradicts the fact that firm f is maximising its preferences in the equilibrium $\langle x_n, y_n, p_n \rangle$. A similar argument shows in $\langle \bar{x}, \bar{y}, \bar{p} \rangle$ consumers are maximising their preferences. It follows that $\langle \bar{x}, \bar{y}, \bar{p} \rangle \in \mathcal{E}$.

Now to demonstrate that $\mathcal{E}$ is closed. Let $\langle \tilde{y}_{mn}^0, \tilde{p}_n \rangle$ be a sequence of points from $\mathcal{E}$, which converges to a limit $\langle \tilde{y}_m^0, \tilde{p} \rangle$. Let $\tilde{y}_n$ and $\tilde{x}_n$ denote the corresponding vectors of equilibrium production and prices. By taking convergent subsequences, if necessary, we may assume that $\tilde{y}_n$ and $\tilde{x}_n$ converge to limits $\tilde{y}$ and $\tilde{x}$ respectively. By a similar argument to that above we may show that $\langle \tilde{x}, \tilde{y}_c^0, \tilde{y}^{-0}, \tilde{p} | \tilde{y}_m^0 \rangle$ is a competitive equilibrium relative to $\tilde{y}_m^0$, which establishes that $\mathcal{E}$ is closed. ■

Proof of Theorem 2.1 If we normalise prices to lie in the unit simplex, the set $\mathcal{E}$ is bounded and therefore compact. Proposition A.2 implies that $\succcurlyeq$ has a maximum over this set. It is easy to check that such a maximum is a managerial equilibrium. ■

Proof of Proposition 2.1 An equilibrium for a given level of output y^0 is a competitive equilibrium in a particular exchange economy. Since the set of competitive equilibria does not depend on the price normalisation, it follows that the set of competitive equilibria relative to a given output y^0 is also independent of it. Thus, for any given numeraire, the firm will have the same set of price-quantity combinations to choose from. Since the firm's preferences are defined over real variables, it will choose the same quantities. The result follows. ■

Proof of Proposition 2.2 The usual necessary condition for productive efficiency is that all firms are on their production frontiers and that all firms have equal marginal rates of transformation between any pair of goods. (Or that appropriate inequalities are satisfied at points where the production function is not differentiable.) Since the production sets are concave these conditions are also sufficient for productive efficiency.

As there are no externalities, the Fisher separation theorem can be applied to the competitive firms. Unanimity implies that these firms will maximise profit. The Fisher separation theorem can also be applied to the monopolist's trades at the second stage. Thus the second stage is a standard Walrasian equilibrium with profit-maximising firms. For the usual reasons all firms will set their marginal rate of transformation equal to the price ratio. (Again these are replaced by the appropriate inequalities at points where the production function is not differentiable.) Thus the first order conditions for productive efficiency are satisfied for competitive goods. Moreover the competitive firms will be producing on their production frontiers.

It remains to demonstrate that the monopolist operates on his/her production frontier. Let $\hat{y}^0 = \left\langle \hat{y}^0_c, \hat{y}^0_m \right\rangle$ be the equilibrium output of firm 0. Suppose, if possible, that there exists $\tilde{y}^0_c$ such that $\phi^f \left\langle \tilde{y}^0_c, \hat{y}^0_m \right\rangle \geqslant 0$ and $\tilde{y}^0_c < \hat{y}^0_c$. By Assumption 2.6, the equilibrium prices of competitive goods are strictly positive. Then if $\hat{p}$ denotes the equilibrium price vector and $\tilde{y}^0 = \left\langle \tilde{y}^0_c, \hat{y}^0_m \right\rangle, \hat{p}.\left(\tilde{y}^0 - \hat{y}^0\right) > 0$. By assumption, agents are price-takers for competitive goods. The two production plans $\hat{y}^0$ and $\tilde{y}^0$ only differ in the components referring to competitive goods. Hence if the surplus of $\hat{p}.\left(\tilde{y}^0 - \hat{y}^0\right)$ is divided among all members of the control group so that each receives a positive amount, the change will be perceived as giving all of them a larger budget set. Thus they will all be better off and consequently such a change will be approved by any decision rule which satisfies the Pareto principle. Thus we may conclude that there does not exist $\tilde{y}^0_c$ such that $\phi^f \left\langle \tilde{y}^0_c, \hat{y}^0_m \right\rangle \geqslant 0$ and $\tilde{y}^0_c < \hat{y}^0_c$.

Let $\hat{y}^0$ denote the equilibrium output of firm 0. Suppose if possible, that there exists $\tilde{y}^0$ such that $\phi^0\left(\tilde{y}^0\right) \geqslant 0$ and $\tilde{y}^0 > \hat{y}^0$. From above we must have $\tilde{y}^0_c = \hat{y}^0_c$. Consider $y^0_\lambda = (1-\lambda)\,\tilde{y}^0 + \lambda \hat{y}^0$. By Assumption 2.2, $\phi^0\left(y^0_\lambda\right) > 0$, for $0 < \lambda < 1$. By continuity $\phi^0\left(y^0_\lambda - \varepsilon e_1\right) > 0$, for some $\varepsilon > 0$. However if λ is sufficiently close to 1 and ε is sufficiently close to 0, $y^0_\lambda - \varepsilon e_1 > \hat{y}^0$ and $y^0_\lambda - \varepsilon e_1$ has a smaller component 1 than $\hat{y}^0$. This contradicts the previous paragraph and hence the result follows. ■

B Externality Model

This appendix contains the proof of the comparative statics result for our externality model. In symmetric equilibrium with profit-maximising firms the following conditions are satisfied:

$$g_L\left(\hat{L}, \hat{z}\right) = \hat{w}, \tag{6}$$

$$g_z\left(\hat{L}, \hat{z}\right) = 0, \tag{7}$$

$$F\hat{L} = L^S\left(\hat{w}\right), \tag{8}$$

where g_L denotes $\frac{\partial g}{\partial L}$ etc. Equations (6) and (7) are respectively the first order conditions for profit maximising choice of pollution and labour input, while equation (8) is the labour-market equilibrium condition.

As explained in section 4.1.2, we may represent the non-profit maximising firm's choice of inputs as maximising a weighted sum of utilities of control group members. Hence it may be characterised by the solution to the following optimisation problem:

$$\max \sum_{h=1}^{M} \lambda^h u^h = \sum_{h=1}^{M} \lambda^h \theta^h \left[g\left(L^f, z^f\right) - wL^f \right] - \sum_{h=1}^{M} \lambda^h D^h \left(\sum_{j=1}^{F} d^h\left(z^j\right) \right), \tag{9}$$

subject to $L^f \geqslant 0, z^f \geqslant 0.$[14]

Proof of Theorem 3.1 Consider the following problem,

$$\max_{L,z} \left\{ g\left(L^f, z^f\right) - wL^f - \alpha \sum_{h=1}^{M} \lambda^h D^h \left(\sum_{j=1}^{F} d^h\left(z^j\right) \right) \right\}. \tag{10}$$

If $\alpha = 0$, the solution to (10) gives the profit maximising values of L^f and z^f, while if $\alpha = 1$ this is the non-profit maximising firm's optimisation problem.

The first order conditions for (10) are:

$$g_L\left(L^f, z^f\right) = w, \tag{11}$$

$$g_z\left(L^f, z^f\right) = \alpha \sum_{h=1}^{M} \lambda^h D^{h\prime} \left(\sum_{j=1}^{F} d^h\left(z^j\right) \right) d^{h\prime}\left(z^f\right). \tag{12}$$

The Hessian of this problem is $H_\pi = \begin{pmatrix} g_{LL} & g_{Lz} \\ g_{Lz} & g_{zz} - \alpha\phi \end{pmatrix}$, where

$\phi\left(z^f\right) = \sum_{h=1}^{M} \lambda^h D^{h\prime\prime} \left(\sum_{j=1}^{F} d^h\left(z^j\right) \right) d^{h\prime}\left(z^f\right)^2 + \sum_{h=1}^{M} \lambda^h D^{h\prime} \left(\sum_{j=1}^{F} d^h\left(z^j\right) \right) d^{h\prime\prime}\left(z^f\right).$

The second order condition is that H_π must be negative semi definite at the optimum, which implies that its determinant must be positive, hence

$$g_{LL}\left(g_{zz} - \alpha\phi\right) - \left(g_{Lz}\right)^2 \geqslant 0. \tag{13}$$

We shall look for a symmetric equilibrium, where $L^f = L(\alpha)$, $z^f = z(\alpha)$ for $1 \leqslant f \leqslant F$. The conditions for such an equilibrium are:

$$g_L(L, z) = w, \tag{14}$$

$$g_z(L, z) = \alpha\psi(z), \tag{15}$$

$$FL = L^S(w), \tag{16}$$

[14] We do not need to consider corner solutions where $L^f = z^f = 0$, since in this case, it is trivially true that the firm produces less pollution than the profit maximising level.

where $\psi(z) = \sum_{h=1}^{M} \lambda_h D_h' (F d_h(z)) d_h'(z)$. Let the symmetric solution be $L(\alpha), z(\alpha)$, $w(\alpha)$. Substituting (14) into (16),

$$L^S (g_L (L, z)) = FL(\alpha). \tag{17}$$

Differentiating (15) and (17) with respect to α, we obtain:

$$\begin{aligned} g_{Lz} L'(\alpha) + g_{zz} z'(\alpha) &= \alpha\psi'(z) z'(\alpha) + \psi(z), \\ L^{S\prime} (g_L (L, z)) \left[g_{LL} L'(\alpha) + g_{Lz} z'(\alpha) \right] &= FL'(\alpha). \end{aligned}$$

Solving $L'(\alpha) = \frac{-L^{S\prime} g_{Lz} z'(\alpha)}{L^{S\prime} g_{LL} - F}$. Substituting $-\frac{L^{S\prime} g_{Lz}^2 z'(\alpha)}{L^{S\prime} g_{LL} - F} + g_{zz} z'(\alpha) = \alpha\psi'(z) z'(\alpha) + \psi(z)$. Hence $z'(\alpha) = \frac{\psi(z)\left[L^{S\prime} g_{LL} - F\right]}{L^{S\prime}\left[(g_{zz} - \alpha\psi'(z)) g_{LL} - (g_{Lz})^2\right] - \left[g_{zz} - \alpha\psi'(z)\right]}$. Note that from our assumptions on the derivatives of D, d and g we have,
$\psi'(z) = F \sum_{h=1}^{M} \lambda^h D^{h\prime\prime} \left(F d^h(z)\right) d^{h\prime}(z)^2 + \sum_{h=1}^{M} \lambda^h D^{h\prime} \left(F d^h(z)\right) d^{h\prime\prime} > 0$, $\psi(z) > 0$, $L^{S\prime} g_{LL} - F < 0$ and $g_{zz} - \alpha\psi'(z) < 0$.

Since, $\phi(z) = \sum_{h=1}^{M} \lambda^h D^{h\prime\prime} \left(F d^h(z)\right) d^{h\prime}(z)^2 + \sum_{h=1}^{M} \lambda^h D^{h\prime} \left(F d^h(z)\right) d^{h\prime\prime}(z)$, in symmetric equilibrium $\psi'(z) > \phi(z)$. From the second order condition, $g_{LL}(g_{zz} - \alpha\phi) - (g_{Lz})^2 \geqslant 0$, hence $g_{LL}\left(g_{zz} - \alpha\psi'\right) - (g_{Lz})^2 \geqslant 0$. Therefore $z'(\alpha) < 0$. Letting α vary between 0 and 1, shows that, in the equilibrium, pollution is below the profit maximising level. ■[15]

C Price Discrimination

Proof of Theorem 4.2 Let $\langle x^*, y^*, p^*, R^* \rangle$ be a PDM-equilibrium. The first order conditions for the firm's optimisation problem are,

$$\begin{aligned} \lambda^h \frac{\partial V^h}{\partial x_{mj}^h} + \mu \frac{\partial \phi}{\partial y_{mj}} &= 0 \text{ for } 1 \leqslant h \leqslant H, \bar{j} + 1 \leqslant j \leqslant J; && (18) \\ -\lambda^h \frac{\partial V^h}{\partial I^h} + \delta &= 0 \text{ for } 1 \leqslant h \leqslant H; && (19) \\ \mu \frac{\partial \phi}{\partial y_{cj}} + \delta p_{cj} &= 0, \text{ for } 1 \leqslant j \leqslant \bar{j}. && (20) \end{aligned}$$

By the envelope theorem, $\frac{\partial V^h}{\partial x_{mj}^h} = \frac{\partial u^h}{\partial x_{mj}^h}$. From (3), (18), (19), and (20),

$$\frac{\partial u^h}{\partial x_{mj}^h} / \frac{\partial u^h}{\partial x_{c1}^h} = \frac{\partial u^k}{\partial x_{mj}^k} / \frac{\partial u^k}{\partial x_{c1}^k}, \text{ for } 1 \leqslant h, k \leqslant H, \bar{j} + 1 \leqslant j \leqslant J. \tag{21}$$

[15] We would like to thank Les Reinhorn for his comments on this proof.

From the consumer's first order condition,

$$\frac{\partial u^h}{\partial x^h_{cj}} / \frac{\partial u^h}{\partial x^h_{c1}} = \frac{\partial u^k}{\partial x^k_{cj}} / \frac{\partial u^k}{\partial x^h_{c1}}, \text{ for } 1 \leqslant h, k \leqslant H, 1 \leqslant j \leqslant \bar{j}. \tag{22}$$

From (19), $\frac{\mu}{p_{ck}} \frac{\partial \phi}{\partial y_{ck}} = \frac{\lambda^h}{p_{ck}} \frac{\partial u^h}{\partial x^h_{ck}}$. From (18), $\mu \frac{\partial \phi}{\partial y_{mj}} = -\lambda^h \frac{\partial u^h}{\partial x^h_{mj}}$. Dividing

$$\frac{\partial \phi}{\partial y_{mj}} / \frac{\partial \phi}{\partial y_{ck}} = -\frac{\partial u^h}{\partial x^h_{mj}} / \frac{\partial u^h}{\partial x_{ck}}, \tag{23}$$

which implies that the marginal rate of substitution is equal to the marginal rate of transformation. By (20),

$$\frac{\partial \phi}{\partial y_{cj}} / \frac{\partial \phi}{\partial y_{c\ell}} = \frac{p_{cj}}{p_{c\ell}}. \tag{24}$$

By concavity, (21), (22), (23) and (24) are sufficient conditions for Pareto optimality.

Implementation by 2-Part Tariffs Now assume that the firm offers consumers the 2-part tariff, $T^h\left(x^h_m\right) = t^h + p_m . x^h_m$, where $p_m = \left\langle p_{c1} \frac{\partial \phi}{\partial y_{m1}} / \frac{\partial \phi}{\partial y_{c1}}, ..., p_{c1} \frac{\partial \phi}{\partial y_{mn}} / \frac{\partial \phi}{\partial y_{c1}} \right\rangle$. The consumer's first-order condition is:

$$\frac{\partial u^h}{\partial x^h_{mj}} / \frac{\partial u^h}{\partial x_{ck}} = \frac{p_{mj}}{p_{ck}} = -\frac{p_{c1} \frac{\partial \phi}{\partial y_{mj}} / \frac{\partial \phi}{\partial y_{c1}}}{p_{ck}} = -\frac{\frac{\partial \phi}{\partial y_{mj}}}{\frac{\partial \phi}{\partial y_{ck}}}, \tag{25}$$

where the third equality follows from equation (20). Since (25) is equivalent to (23) the PDM equilibrium can be implemented by the 2-part tariff. ■

The above proof assumes that there are no competitive firms. However, it can be adapted to allow for the presence of competitive firms.

References

Allen, F. & Gale, D. (2000), Corporate governance and competition, *in* X. Vives, ed., 'Corporate Governance', CUP, Cambridge, UK, pp. 23–76.

Böhm, V. (1994), 'The foundations of the theory of monopolistic competition revisited', *Journal of Economic Theory* **63**, 1–24.

Bolton, P. & Xu, C. (1999), Ownership and managerial competition: Employee, customer or outside ownership, Working paper, Princeton University.

Conley, J. & Wooders, M. (2001), ‘Tiebout economies with differential genetic types and endogenously chosen crowding characteristics.’, *Journal of Economic Theory* **98**, 261–294.

Cornes, R. & Sandler, T. (1996), *The Theory of Externalities, Public Goods and Club Goods*, 2nd edn, Cambridge University Press, Cambridge, UK.

Cornwall, R. (1977), ‘The concept of general equilibrium in a market economy with imperfectly competitive producers’, *Metroeconomica* **29**, 55–72.

Dixit, A. & Norman, V. (1980), *Theory of International Trade*, CUP, Cambridge, UK.

Edlin, A., Epelbaum, M. & Heller, W. (1998), ‘Is perfect price discrimination really efficient?: Welfare and existence in general equilibrium’, *Econometrica* **66**, 897–922.

Ellickson, B. (1993), *Competitive Equilibrium: Theory and Applications*, CUP, New York.

Ellickson, B., Grodal, B., Scotchmer, S. & Zame, W. (1999), ‘Clubs and the market’, *Econometrica* **67**, 1185–1218.

Farrell, J. (1985), ‘Owner-consumers and efficiency’, *Economics Letters* **19**, 303–306.

Hansmann, H. (1996), *The Ownership of Enterprise*, Harvard University Press, Cambridge, Mass.

Hart, O. D. (1975), ‘On the optimality of equilibrium when the market structure is incomplete’, *Journal of Economic Theory* **11**, 418–443.

Hart, O. & Moore, J. (1990), ‘Property rights and the nature of the firm’, *Journal of Political Economy* **98**, 1119–1158.

Hart, O. & Moore, J. (1996), ‘The governance of exchanges: Members’ cooperatives versus outside ownership’, *Oxford Review of Economic Policy* **12**, 53–69.

Holmstrom, B. (1999), 'The firm as a subeconomy', *Journal of Law, Economics and Organization* **15**, 74–102.

Jensen, M. & Meckling, W. (1976), 'Theory of the firm: Managerial behavior, agency costs, and ownership structure', *Journal of Financial Economics* **3**, 305–360.

Kelsey, D. & Milne, F. (1996), 'The existence of equilibrium in incomplete markets and the objective of the firm', *Journal of Mathematical Economics* **25**, 229–245.

Kelsey, D. & Milne, F. (2003), 'Imperfect competition and corporate governance', *University of Birmingham, Discussion paper 03-01* .

Laffont, J. J. (1988), *Fundamentals of Public Economics*, MIT Press, Cambridge, Mass.

Milgrom, P. & Roberts, J. (1990), 'Rationalizability, learning, and equilibrium in games with strategic complementarities', *Econometrica* **58**, 1255–1277.

Milgrom, P. & Shannon, C. (1994), 'Monotone comparative statics', *Econometrica* **62**, 157–180.

Milne, F. (1974), 'Corporate investment and finance theory in general equilibrium', *Economic Record* pp. 511–533.

Milne, F. (1981), 'The firm's objective function as a collective choice problem', *Public Choice* **37**, 473–486.

Milne, F. & Shefrin, H. (1987), 'Information and securities: A note on Pareto dominance and the second best', *Journal of Economic Theory* **43**, 314–328.

Prescott, E. & Townsend, R. (2000), Firms as clubs in Walrasian markets with private information, Working paper.

Renstrom, T. & Yalcin, E. (2003), 'Endogenous firm objectives', *Journal of Public Economic Theory* **5**, 67–94.

Roberts, J. & Steen, E. V. (2000), Shareholder interests, human capital investment and corporate governance, Working paper, Stanford Business School.

Roemer, J. E. (1993), 'Would economic democracy decrease the amount of public bads?', *Scandinavian Journal of Economics* **95**, 227–238.

Sen, A. K. (1970), *Collective Choice and Social Welfare*, North Holland, Amsterdam.

Sen, A. K. (1973), The concept of efficiency, *in* M. Parkin & R. Nobay, eds, 'Contemporary Issues in Economics', Manchester University Press, pp. 196–210.

Sen, A. K. (1977), 'Social choice theory: A re-examination', *Econometrica* **45**, 217–245.

Shafer, W. & Sonnenschein, H. (1975), 'Equilibrium in abstract economies without ordered preferences', *Journal of Mathematical Economics* **2**, 345–348.

Shleifer, A. & Vishny, R. (1997), 'A survey of corporate governance', *Journal of Finance* **52**, 33–56.

Tirole, J. (2001), 'Corporate governance', *Econometrica* **69**, 1–35.