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Complementarities and Macroeconomics: Poisson

Games

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Abstract

In many situations in macroeconomics strategic complementarities arise, and agents face a coordination problem. An important issue, from both a theoretical and a policy perspective, is equilibrium uniqueness. We contribute to this literature by focusing on the macroeconomic aspect of the problem: the number of potential innovators, speculators e.t.c. is large. In particular, we follow Myerson (1998, 2000) that in large games "a more realistic model should admit some uncertainty about the number of players in the game". In more detail, we model the coordination problem as a Poisson game, and investigate the conditions under which unique equilibrium selection is obtained.

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1 Introduction

In many situations in macroeconomics strategic complementarities arise: individual payoffs from taking a certain action are non-decreasing in the number of agents who adopt the same strategy. Examples include technological spillovers and innovation, currency crises, and others. Cooper (1999) is an excellent recent treatment of complementarities in macroeconomic environments.⁰

In these environments, agents are called to coordinate their actions, and a very important issue, from both a theoretical and a policy perspective, is whether beliefs are indeterminate and non-cooperative equilibrium in pure strategies is unique. The global games literature emphasise asymmetric information, that arises by means of idiosyncratic noisy signals about economic fundamentals, to obtain a unique equilibrium.¹ Morris and Shin (2001) provide an overview of this strand of research. Herrendorf et. al (2000), Burdzy et. al. (2001) and Frankel and Pauzner (2000) exploit heterogeneity of agents to the same effect.² Also, Lee and Mason (2002), study the interaction of heterogeneity, uncertainty about economic fundamentals and degree of complementarities.³

⁰I am indebted to G. Bulkley, E. Cannon, D. Demery, N. Duck, K. J. Van Garderen, I. Jewitt, C. Leaver, H. Polemarchakis and seminar participants at Bristol and Exeter for comments and lengthy discussions on earlier work this paper draws heavily upon. This paper also draws heavily on research under financial support from the European Commission under contract ERBFMBICT950108, which is acknowledged with thanks. The usual disclaimer applies.

¹In this approach, agents' private types are correlated. This approach, by utilising insights in Schelling (1980), shows how introducing a small number of dominant strategy types into a coordination game can select a unique Bayesian Nash equilibrium. See also Carlsson and van Damme (1993).

²A related work is the investigation in Baliga and Sjostrom (2004) of the implications for equilibrium selection, in an arms race game, of cheap talk when private types are independent. In an arms race the payoff from building up weapons is *decreasing* in the number of agents who adopt the same strategy, while the payoff from not acquiring new weapons is *increasing* in the number of agents who do not acquire new weapons.

³A related work is also Mason and Valentinyi (2003). There, the focus is on the interaction

Given the above work, the common view is that, in order to escape a prediction of indeterminacy of equilibria, one needs to have a sufficiently large degree of heterogeneity and/or of asymmetric information. However, as we show here, this may not be true in macroeconomic contexts. In particular, this paper complements the received literature by focusing on the 'largeness' of the situations in question. In more detail, our starting point is that in macroeconomic environments the number of potential innovators, speculators, e.t.c. is, by definition, very large. The standard assumption that every player takes every other player's behaviour as given and known when contemplating her best response therefore seems somewhat implausible. For instance, in large societies, it may be prohibitively expensive to collect the necessary information for who all the stakeholders are. As Myerson notes, in large games "it is unrealistic to assume that every player knows all the other players in the game; instead, a more realistic model should admit some uncertainty about the number of players in the game" (Myerson (2000) pp.7). We contribute to the literature on macroeconomics and complementarities by following Myerson's suggestion and allowing for population uncertainty in investigating the conditions under which unique equilibrium selection is obtained. Specifically, given the convenient properties associated with the Poisson distribution (see Myerson (1998)), we model the coordination problem as a Poisson game.

As a complementary justification for this modelling choice, suppose that the identity of every stakeholder is indeed common knowledge but also that binding individual orders for new technology, or for short-sales of a currency, must arrive with the inventor, or central bank, by a given time. Standard theory suggests that each agent will decide on her action by taking the number of orders at the collector's disposal as given. However, potential stakeholders may be ill, postmen may be on strike or computer networks may be down; in short, accidents will happen, albeit with a small, but strictly positive, probability. As of incomplete information and heterogeneities in a large class of games.

a result, in a large environment, stakeholders should actually view the number of players *in* the coordination game as a Poisson random variable (PRV).

Before embarking on our analysis, we feel that we should emphasise the following. First, Poisson games are not a special case of global games. As Myerson (1998) discusses both Bayesian games and Poisson games are subsumed by the general class of population uncertainty games. Second, as it will be made clear shortly after, our explanation of equilibrium uniqueness is analytically simpler than the existing explanations. Third, as we will see later on, our model offers different predictions to the ones in the received literature on complementarities and uniqueness.

The organisation of the paper is as follows. Next Section describes the model. Section 3 describes equilibrium. Finally, Section 4 discusses some applications and the last Section concludes.

2 The Model

 their action. Finally, assume that agents resolve any indifference in favour of the 'safe' strategy c=0.

Putting more structure into the model, define $\theta_H = \arg \max_{\theta} \{\theta \mid b(\theta, 1) \leq t\}$, after following the convention that if $b(\theta, 1) \leq t$ for any θ then $\theta_H \equiv +\infty$. Given the properties of the benefit function we have that $b(\theta, x^- + 1) > t$ for any level of fundamentals $\theta > \theta_H$ and any number of 'other' adoptions $x^- \geq 0$.

Define, 4 now, $\theta_L = \arg \max_{\theta} \{\theta \mid \lim_{x \to \infty} b(\theta, x) \leq t\}$, after following the conventions that if $\lim_{x \to \infty} b(\theta, x) \leq t$ for any θ then $\theta_L \equiv +\infty$, and if $\lim_{x \to \infty} b(\theta, x) > t$ for any θ then $\theta_L \equiv -\infty$. Given the properties of the benefit function we have that if θ_L is finite then $b(\theta, x^- + 1) \leq t$ for any level of fundamentals $\theta \leq \theta_L$ and any size of 'other' orders $x^- \geq 0$.

Note, from the properties of the benefit function, that $\theta_H > -\infty$ and $\theta_L \leq \theta_H$. Thus, if fundamentals are sufficiently high, i.e. $\theta > \theta_H$, equilibrium is unique: every player chooses the 'risky' action c = 1. If fundamentals are sufficiently low, i.e. $\theta \in (-\infty, \theta_L]$, then equilibrium is again unique: every player chooses the safe action c = 0. If, however, fundamentals are in an intermediate range, i.e. $\theta \in (\theta_L, \theta_H]$, then the incentive to choose the risky action c = 1 depends on the beliefs about the size of other orders x^- . To see an illustration of this, consider the benchmark case where the number of actual players in the game N is common knowledge, and define $\theta_L^N = \arg\max_{\theta} \{\theta \mid b(\theta, N) \leq t\}$. It follows directly that for any level of fundamentals $\theta \in (\theta_L^N, \theta_H]$ this benchmark game has two (pure-strategy) equilibria. One with $x^o = N$, and one with $x^o = 0$.

In what follows we assume instead, which is how we depart from other studies 4In a currency crises model high θ corresponds to 'bad' fundamentals, and to high devaluation if an attack succeeds. In a new technology adoption model, 'good' fundamentals correspond to high θ , and to high quality of invention (or high spillover effects).

⁵Superscript o denotes equilibrium variables.

 $^{^6}$ While, for $\theta \in (-\infty, \theta_L^N]$ the benchmark game has a unique equilibrium where every player chooses c=0

of strategic complementarities, that the number of actual players in the coordination game N is a PRV with mean n. We will refer to n as the population or game or group-size. Also, we focus hereafter to fundamentals $\theta \in (\theta_L, \theta_H]$.

3 Equilibrium

Introducing population uncertainty implies that players can no longer assign a strategy to other individual players, simply because they are not aware of who they all are. Instead, we describe strategic behaviour in terms of a distributional strategy (see Myerson (2000)). Such a strategy, τ_n , is defined as any probability distribution over the action set in a game of size n. That is $\tau_n(1)+\tau_n(0)=1$ and $\tau_n(c)\geq 0$. Note that τ_n can be interpreted as the 'beliefs' players choose to hold that a randomly sampled player will choose action c in a game of size n. As Myerson (1998) puts it "...going to a model of population uncertainty requires us specify a probability distribution over actions..., rather than for each individual player. In effect, population uncertainty forces us to treat players symmetrically in our game-theoretic analysis."

When players behave according to τ , the number of players of any type that choose any action c is a PRV with mean $n\tau(c)$ and hence the expected action profile in a game of size n is $n\tau \equiv \{n\tau(c)\}_{c\in C}$. To derive expected payoffs for each action, given $n\tau$, we make use of two special features of Poisson games. First, that the number of players choosing c is independent of the number of players who choose all other actions (see Myerson (1998)). Second, that any player in a Poisson game attaches the same probability that there are d individuals in the game with him with the probability that the external

⁷For notational convenience, we supress hereafter the dependence of the strategy τ on the size of the game n whenever there is no danger of confusion.

⁸Without population uncertainty this property could not be satisfied since the total number of players who choose a certain action must be equal to the known N, see Myerson (1998) pp.9.

game theorist would attach on the event that there are d individuals in the whole game. This 'environmental equivalence' property implies that "from the perspective of any player..., the number of other players (not including himself) who choose action c is also an independent Poisson random variable with the same mean $n\tau(c)$ ", Myerson (1998), pp 16. Accordingly, the expected net gain of the typical agent from choosing c = 1 is

$$B(n\tau, \theta, t) \equiv -t + Eb(\theta, x^{-} + 1) \equiv -t + \sum_{x^{-}=0}^{\infty} f(x^{-} \mid n\tau(1))b(\theta, x^{-} + 1), \quad (1)$$

with f being the Poisson distribution with mean $n\tau(1)$, after following the convention that $f(0 \mid 0) = 1$.

In essence, then, each player resolves her decision by formulating 'beliefs' τ over the likely behaviour of the typical player in the game and then calculating which action maximises her expected utility given the resulting expected action profile. Following Myerson (2000), we establish an equilibrium if "all the probability of choosing action c comes from types for whom c is an optimal action, when everyone else is expected to behave according to this distributional strategy" (Myerson (2000), pp.11).

We then have directly that $\tau^o(1)=0$ is an equilibrium, as $B(0,\theta,t)=b(\theta,1)-t\leq 0$ for any $\theta\leq \theta_H$. Regardless of the group-size, if the typical agent expects that no other player in the game plays the risky action, he is certain that the size of 'other' orders is zero, i.e. $f(0\mid 0)=1$, and thereby finds it optimal to play safe as well. Anticipating, on the other hand, that every other player in the game chooses the risky action, implies that the representative agent's expected benefit from playing the risky action as well depends on her beliefs about the actual game-size $f(x^-\mid n)$ and the extend of complementarities, i.e. how $b(\theta,x)$ increases with x. In particular, $\tau^o(1)=1$ is an equilibrium if and only if $B(n,\theta,t)>0$, that is if and only if $\sum_{x^-=0}^{\infty}f(x^-\mid n)b(\theta,x^-+1)>t$. We have shown:

Proposition 1 In a Poisson Contribution Game of expected size n, with level

of fundamentals $\theta \in (\theta_L, \theta_H]$ and costs t, every player in the game playing the safe action is an equilibrium, i.e. $\tau^o(1) = 0$. This is the unique (pure-strategy) equilibrium if and only if $B(n, \theta, t) \leq 0$.

The condition $B(n; \theta, t) \leq 0$ can be used to investigate the interaction of game-size n, costs t and level of fundamentals $\theta \in (\theta_L, \theta_H]$, and the implications for uniqueness of equilibrium. In particular, we have in a straightforward manner that:

Proposition 2 In a Poisson Contribution Game of expected size n, with level of fundamentals $\theta \in (\theta_L, \theta_H]$ and costs t suppose that the net benefit from the risky action B() is increasing with game-size. Then every player in the game playing the safe action is the unique equilibrium, if group-size is sufficiently low, given t.

Proof. Directly after recalling $B(0, \theta, t) = b(\theta, 1) - t \le 0$ and defining the appropriate threshold level $n^* \ge 0$ by $B(n^*, \theta, t) = 0$.

In the next section we look into some examples that fit the above framework.

4 Examples

4.1 Currency Crises

We start with a model of currency crises. In this model potential speculators have a choice between not questioning a currency peg or borrowing one unit of the currency in question and selling it in the market for foreign currencies. Here, t is the riskless gross interest rate plus any transaction costs faced by the speculators.

Also, $b(\theta, x) \geq 0$ determines in a reduced form the size of the depreciation of the currency under scrutiny, as a proportion of the exchange rate during the peg, when fundamentals are θ and the size of the attack is x. In particular, for our purposes here, the benefit function is defined as follows. First, for any

 $\theta > \theta_H$ we have $b(\theta, 1) = \beta(\theta) > t$ with $\beta'(\theta) > 0$, while for any $\theta \le \theta_H$ we have $b(\theta, 1) = 0$. A single attack is not enough to cause the central bank to abandon the peg if the latter is sufficiently robust, i.e. if $\theta \leq \theta_H$. If, however, $\theta > \theta_H$ the peg is so fragile that even a single attack would induce the collapse of the fixed exchange rate regime. Furthermore, the depreciation is sufficient to cover the costs of attacking the peg. Note that here θ_H is independent of transaction costs t. Second, for any $\theta \in (-\infty, \theta_L]$ we have that $\lim_{x\to\infty} b(\theta, x) = 0$, while for any $\theta > \theta_L$ we have $\lim_{x\to\infty} b(\theta,x) = \beta(\theta) > t$. If $\theta \le \theta_L$ the peg is stable: no attack will cause its collapse. If instead $\theta > \theta_L$ an attack of infinite size will be too much for the central bank to contain and the peg will be abandoned. In addition, the depreciation is sufficient to cover the costs of short-selling the currency. Note that in this model θ_L is also independent of transaction costs $t.^9$ Third, for any $\theta \in (\theta_L, \theta_H]$ and $x \in (1, \infty)$ we have that $b(\theta, x)$ is a step function of the size of the attack x. In particular, $b(\theta, x) = 0$ if $1 < x < x^*(\theta)$ and $b(\theta, x) = \beta(\theta) > t$ if $x \ge x^*(\theta)$, with $x^{*'}(\theta) < 0$. If the attack on the peg is sufficiently high then the central bank abandons the fixed exchange rate regime and the currency under scrutiny depreciates by $\beta(\theta)$; otherwise the peg survives.

In this case, for any $\theta \in (\theta_L, \theta_H]$ we have $B(n, \theta, t) = -t + \beta(\theta) \sum_{x^- = x^*(\theta) - 1}^{\infty} f(x^- \mid n) = -t + \beta(\theta)[1 - F(x^*(\theta) - 2 \mid n)]$, where F() is the Poisson c.d.f. So, when fundamentals are in the 'grey' area $(\theta_L, \theta_H]$, survival of the peg is the unique equilibrium outcome if the total probability that an attack is successful, $1 - F(x^*(\theta) - 2 \mid n)$, is lower than or equal to the ratio of (maximum) benefits from the two actions $t/\beta(\theta) \equiv r$. Defining $G(y \mid n) \equiv \sum_{x^- = y}^{\infty} f(x^- \mid n)$ and $m \equiv x^*(\theta) - 1$ we thus have a unique equilibrium selection if $G(m \mid n) \leq r$. Not-

⁹An alternative model could have that a devaluation always occurs if the attack is very large, but for very good fundamentals the benefit cannot compensate for the transaction cost. That is, $\lim_{x\to\infty}b(\theta,x)=\beta(\theta)>0$ with $\beta'(\theta)>0$ and $\beta(\theta)\leq t$ for any $\theta\leq\theta_L$. In such a model, θ_L may depend on the level of transaction costs, and thereby be sensitive to policies like a Tobin tax.

ing that $G(m \mid n)$ is strictly increasing with the mean n (see theorem 33.2 in pp. 92 in Schmetterer, 1974), the latter condition can be re-written as $n \leq n^*(r, m)$, where $n^*(r, m)$ is the solution of r = G(m, n). Note also that $n^*(.,.)$ is strictly increasing with short-selling costs t, as the latter increase the ratio of benefits r and make the risky action less attractive. The critical group-size $n^*(.,.)$ is also strictly decreasing with θ , as the latter decreases the ratio of benefits r (due to $\beta' > 0$) and increases the fragility of the peg (due to $x^{*'} < 0$), and thereby makes the risky action more attractive. We clearly have that for any $\theta \in (\theta_L, \theta_H]$:

Corollary 1: In the currency crises model if the mean number of actual speculators is sufficiently low and/or transaction costs are sufficiently high, equilibrium is unique, no attack occurs and the peg survives.

Leaving this example note that here an increase in transaction costs from short-selling, by means, say, of the introduction of a Tobin tax, increases the ratio of benefits from not attacking and, hence, the critical group-size $n^*(.,.)$, and thereby increases the likelihood that no attack will occur. In Imposition of direct capital controls increases the robustness of the peg by increasing the minimum size of the attack necessary for the collapse of the regime, $x^*(\theta)$, for any level of fundamentals. Thus, this policy measure as well increases the critical game-size $n^*(.,.)$ and the likelihood that the peg will survive.

The present model of currency crises is very similar to the one in Morris and Shin (1998), where small differences in information determine the outcome of an incomplete information game. Yet, the main predictions about the kind of crises that can erupt are qualitatively different. In particular, the model in Morris and Shin (1998) features 'probing' attacks prior to the collapse of the peg, unless there is a sudden shift of sufficient size in fundamentals. In the latter 10Continuing from footnote 7, if θ_L is the lowest level of fundamentals for which an attack

¹⁰Continuing from footnote 7, if θ_L is the lowest level of fundaments for which an attack is not strictly profitable, i.e. $b(\theta_L) = t$, then the imposition of a Tobin tax would also result in an increase in θ_L and, so, in a reduction of the 'grey area' of fundamentals.

case, we have 'business as usual' prior to the launch of a successful attack on the peg. Here, instead, even if shifts in fundamentals are very small, the peg is not tested by the market until its collapse. This collapse, as in Morris and Shin (1998), takes place as soon as fundamentals cross a well-defined threshold (i.e. $\theta > \theta_H$).

4.2 Innovation and Positive Spillovers

We turn to a model of adoption of new technology with positive spillovers. In this model potential innovators have a choice between switching to a new technology or not. Here, t is the cost of innovating, of switching from the old to the new technology. Also, the benefit from innovating $b(\theta, x)$ is continuous and strictly increasing with the extend of innovation x and the quality of the new technology θ . In particular, for the purposes here, we assume that $b(\theta, x) = \gamma(\theta)x$ with $0 < \gamma(\theta) < t$ and $\gamma'(\theta) > 0$. That is, for individual sunk costs to be recouped enough innovations must take place. I addition, quality and the extend of innovation are complements. Note that in this model $\theta_H = +\infty$ and $\theta_L = -\infty$.

In this case, $B(n;\theta,t) = -t + \gamma(\theta) \sum_{x^-=0}^{\infty} f(x^- \mid n)(x^- + 1) = -t + \gamma(\theta)[n + 1]$. So, for any level of quality, survival of the old technology is the unique equilibrium outcome if the mean population-size n is lower than or equal to $[t - \gamma(\theta)]/\gamma(\theta)$. The critical upper bound on the game-size for unique equilibrium selection is again strictly increasing with costs t and strictly decreasing with fundamentals θ , and so we we have that for any $\theta \in \mathbf{R}$:

Corollary 2: In the innovation game if the mean number of actual innovators is sufficiently low and/or switching costs are sufficiently high and/or quality is sufficiently 'bad' (i.e. θ sufficiently low), equilibrium is unique, no innovation occurs and the old technology survives.

Leaving this example, we note that subsidies, which reduce the cost of in
11An alternative model of innovation with spillovers could be with $b(\theta, x) = \gamma(\theta) + \delta x$ with $\delta > 0$, $\gamma(\theta) + \delta < t$ and $\gamma' > 0$. In this case, the condition for uniqueness is $n \leq [t - \gamma(\theta) - \delta]/\delta$.

novating, make switching more likely as they reduce the critical game size n^* . The present model of innovations is very similar to the one in Lee and Mason (2002), where agents' preferences differ, and agents may also be faced with uncertainty about the fundamentals. Yet, the main characteristics of the equilibrium outcome, whenever uniqueness is ensured, are qualitatively different. In particular, the model in Lee and Mason (2002) features an intermediate area of fundamentals (or, under fundamental uncertainty, of common signals about the uncertain fundamentals) where the proportion of adoptions is greater than zero, less than one and increasing with the quality of fundamentals (or signals). Also, to the left of that area (i.e. for sufficiently 'bad' fundamentals) innovations do not take place, while to the right of that area (i.e. for sufficiently 'good' fundamentals) everyone innovates. Thus, the prediction is that, unless there is a sudden increase of a sufficient size in the quality of fundamentals, improvements in quality will be accompanied by small increases, if any, in the number of adoptions. Here, instead, adoptions are never partial. Specifically, our model predicts a sudden and abrupt adoption of new technologies by every agent as soon as fundamentals cross a well-defined threshold (i.e. $\theta > \theta_H$).

5 Conclusions

We investigated a coordination game under population uncertainty, and in particular when the number of players in the game is a Poisson random variable. This game may or may not be characterised by multiplicity of pure strategy equilibria. In fact, beliefs are not independent of fundamentals, a coordination problem does not arise, and unique equilibrium selection is obtained under certain conditions for fundamentals, mean population and transaction cost from innovating, short-selling e.t.c.

A very interesting line of research is to investigate the interaction of population uncertainty with heterogeneity, uncertainty about economic fundamentals and/or asymmetric information. Such research will expand our knowledge of complementarities in macroeconomics, enrich policy debates and provide empirical explorations with a wider range of theoretical modelling to draw upon.

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