Inflation Bias with Dynamic Phillips Curves and Impatient Policy Makers

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Inflation Bias with Dynamic Phillips Curves and Impatient Policy Makers[*]

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Abstract

We generalise the analysis of inflation bias with dynamic Phillips curves in three respects. First, we examine the discretionary (time consistent) solution in cases where the Phillips curve has both a backward-looking and forward-looking component. Second, we show that the commitment (time inconsistent) solution does not normally involve zero inflation and output at its natural rate. Instead, with a purely forward-looking Phillips curve and positive discounting, it will involve a dynamic path for inflation in which steady state inflation is below its target. In this sense, we obtain negative inflation bias. Third, we show that the timeless perspective policy has the same steady state as the commitment case, but without any short-term output gains.

Key Words: Inflation Bias, Commitment, Discretion, Timeless Perspective Policy
JEL Reference Number: E52, E61, E63, F41

1 Introduction

The textbook model of inflation bias is based on a static Phillips curve. In the Barro-Gordon model (Barro and Gordon (1983)), discretionary (time consistent) policy produces positive inflation bias. If the authorities are able to commit in some way, and therefore implement a time inconsistent solution, then they can achieve at best target inflation, but with output at its natural rate.

However, most macroeconomic models involving price rigidity over the last ten or so years have involved dynamic Phillips curves, where current inflation depends on expected future inflation, and possibly also on the past level of inflation. Although there has been a good deal of analysis of inflation bias for these models (see Clarida et al. (1999), Woodford (2003), and Walsh (2003) for example), they have typically looked at a New Keynesian Phillips curve (where past inflation plays no role, and the NAIRU property does not hold), and assumed that policy makers use the same discount rate as the private sector.

There are clear reasons why we might expect policy makers to discount more heavily than the private sector. Where the monetary authority is an elected government, then the desire for reelection (and uncertainty about being reelected) may make policy makers shortsighted compared to the private sector (see Grilli et al. (1991) or Aidt et al. (2003) for example). Even in the case of an Independent Central Bank, Bank Governors (or members of a Monetary Policy Committee) are on fixed terms contracts that may encourage them to be relatively shortsighted, and these contracts generally do not contain provisions to prevent this happening.[1] Alternatively, agents in the economy may have heterogenous preferences, and policy makers may be partisan. The literature stemming from Barro and Gordon (1983) has often focused on possible differences between the preferences of the private sector and policy makers: e.g. Rogoff (1985), Walsh and Waller (1995) and Backus and Driffill (1985).

As a result, there seems to be a strong case for examining the implications of impatient (relative to the private sector) behaviour on the part of policy makers, in the context of a New Keynesian Phillips curve, which we do in this paper. In addition, we extend the results in Clarida et al. (1999) by looking at Phillips curves where both expected future and past inflation influence current inflation, but where the NAIRU property holds. We look at solutions assuming discretion, commitment and a timeless perspective. One new and interesting result is that the time inconsistent/commitment solution for impatient policy makers leads to steady state outcomes in

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[1]The forecast horizon of the Bank of England is between two and six years. Henry et al. (2006) discuss implications of short sighted behaviour for inflation and output targets.

which inflation is *below* its target level. (In fact, we show that if the Phillips curve has the NAIRU property, this will be true as long as the authorities use any discount factor less than one.)

In what follows, we use the term inflation bias to denote the difference between the steady state inflation rate and its target value. Although this usage is commonplace, some authors define inflation bias as a difference between steady state inflation rates under discretionary policy and under commitment, which will of course always be positive. This distinction only matters if the commitment policy does not deliver the inflation target in steady state, but this will be true for many of the cases we discuss below. We have no interest in arguing that one definition is better than another: it is simply more convenient for us to talk about inflation bias as we define it than to keep refering to differences between the steady state solution and the inflation target or between two endogenous steady state values.

In the tradition of Barro and Gordon (1983) and Clarida et al. (1999), we analyse inflation bias in a generic setting, rather than restricting ourselves to a particular microfounded set-up. Our analysis should be seen as complementary, rather than as an alternative, to fully specified general equilibrium models. We take the more eclectic approach here because the properties we demonstate are likely to apply to a wide range of particular models. The Phillips curve equations we use can be interpreted as linearised relationships around the flex price equilibrium.

The structure of this paper is as follows. We begin in Section 2 by briefly restating the familiar Barro-Gordon model as background to our analysis. This involves a Phillips curve in which current inflation depends on pre-set expectations of current inflation. The rest of the paper considers a dynamic Phillips curve. In Section 3 we introduce a general specification for the Phillips curve utilised by Christiano et al. (2005) among others, which has the NAIRU property but also allows for inflation inertia. We look at three solution concepts for this problem: time consistent, discretionary solutions (Section 3.1), time inconsistent, commitment solutions (Section 3.2) and Woodford's timeless perspective solution (Section 3.3). We show how the extent of inflation bias varies with discounting. We focus on the particular case of commitment policy with a completely forward looking Phillips curve, which either has the NAIRU property or has a New Keynesian formulation. Section 4 concludes.

2 Barro-Gordon Model with Static Phillips Curve

We begin with the standard Barro-Gordon model (see Barro and Gordon (1983), and Blanchard and Fischer (1989) for a textbook exposition), partly because it is familiar, and partly to show the relationship between its results and those obtained with a dynamic Phillips curve. The authorities

minimise a quadratic discounted loss,

$$\min(\mathcal{L} = \frac{1}{2}\mathcal{E}_0 \sum_{t=0}^{\infty} \beta^t(\omega(\pi_t - \pi^*)^2 + (y_t - y^*)^2)) \tag{1}$$

with targets π^* and y^* are log-differences between targets and the steady state $(\Pi_t/\Pi_{t-1} = 1, Y_t = Y^n)$ around which the model is (implicitly) linearised. Parameter ω describes inflation aversion of the policymaker. The minimisation is subject to the Phillips curve in the form:

$$\pi_t = \pi_t^e + \phi y_t \tag{2}$$

where

$$\pi_t^e = \mathcal{E}_{t-1}\pi_t$$

namely, inflation expectations about the period t are set one period earlier, in period $t-1$ so they are predetermined in period t. These assumptions make the Phillips curve essentially static. The natural rate of output is zero, and we assume that the output target is above the natural rate: $y^* > 0$, which is the origin of the inflation bias problem. For example, the natural rate might be below the target rate because of distortions caused by monopolistic competition, as is now standard in the recent literature analysing monetary policy using welfare derived from agents' utility (see Benigno and Woodford (2004a) for example). The Hamiltonian for the optimisation problem can be written as[2]

$$\mathcal{H} = \mathcal{E}_0 \sum_{t=0}^{\infty} \left(\frac{1}{2}\beta^t(\omega(\pi_t - \pi^*)^2 + (y_t - y^*)^2) + \psi_t \left(\mathcal{E}_{t-1}\pi_t + \phi y_t - \pi_t \right) \right)$$

and the first order conditions for a time-consistent policy are

$$\frac{\partial H}{\partial \pi_t} = \beta^t \omega \left(\pi_t - \pi^* \right) - \psi_t = 0 \tag{3}$$

$$\frac{\partial \mathcal{H}}{\partial y_t} = \beta^t (y_t - y^*) + \phi \psi_t = 0 \tag{4}$$

$$\frac{\partial \mathcal{H}}{\partial \psi_t} = \mathcal{E}_{t-1}\pi_t + \phi y_t - \pi_t = 0 \tag{5}$$

This is a system of three *static* equations. Rational expectations imply that $\mathcal{E}_{t-1}\pi_t = \pi_t$, so the solution to this problem can be found as:

$$\pi = \pi^* + \frac{y^*}{\omega \phi} \tag{6}$$

$$y = 0 \tag{7}$$

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[2]We only consider the deterministic component as it greatly simplifies the algebra. This is without loss of generality because of certainty equivalence (see e.g. Anderson et al. (1996)).

Thus, as soon as the policymaker targets $y^* > 0$ there is a positive inflation bias, whose size is determined by the size of the output target, the degree of inflation aversion of the policymaker, and the size of the coefficient on demand pressure in the Phillips curve. A superior outcome in terms of welfare is possible if policy makers can commit to target inflation: actual inflation will be equal to target, but output will still be zero, i.e. at its natural rate.[3]

3   A Phillips Curve with Nominal Inertia

In this section we mainly examine a dynamic Phillips curve of the following form

$$\pi_t = \alpha \mathcal{E}_t \pi_{t+1} + (1 - \alpha)\pi_{t-1} + \phi y_t \tag{8}$$

where $0 \leq \alpha \leq 1$. This equation is a variant of the model set out in Calvo (1983). In Calvo model, monopolistically competitive firms reset their price with some fixed probability each period, and if prices are reset, the profit maximising price is calculated. In the variant above, those firms that do not reset their price based on profit maximisation instead index their price to lagged inflation (for a derivation, see Christiano et al. (2005)). This indexation means that current inflation depends on past inflation as well as expected future inflation. Svensson (2003) argues that the 'assumption that firms can index prices to average inflation between price adjustment opportunities has the advantage that the long-run Phillips curve becomes vertical rather than positively sloped', or in other words that the Phillips curve retains a NAIRU property.[4] This is in contrast to the New Keynesian Phillips curve, which is based on the original Calvo formulation, which has the form

$$\pi_t = \gamma \mathcal{E}_t \pi_{t+1} + \phi y_t \tag{9}$$

where γ is a private sector's discount factor. (See Woodford (2003), for example, for a derivation.) As McCallum (2002) has stressed, this New Keynesian Phillips Curve (NKPC) is inconsistent with the NAIRU hypothesis. With a NKPC, there is a (very steep) long run trade-off between output and inflation; in steady state $\pi = \frac{\phi}{(1-\gamma)}y$. Given the importance of the NKPC in the literature,

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[3]We study the *level bias* in a linear framework, although of course the economy will never come back to the point of linearisation. Any linearisation preserves the dynamic properties in a neighbourhood of that point, and these properties are the subject of our study. In what follows we will term the steady state displacement of the *linear system* as the 'steady state' although, of course, the true steady state is the explicit solution to the (non-available to us) non-linear system. The difference between them should be small, allowing us to work with this linear system (see Benigno and Woodford (2004b) on the analysis of permanent displacement in LQ models).

[4]See Ball and Mankiw (2001) for a discussion of the importance of the NAIRU property.

we also examine this formulation below.[5]

In the microeconomic derivation described in Christiano et al. (2005), the parameter α is related to the firms' discount factor, $\alpha = \frac{\gamma}{1+\gamma}$, and tends to one half as the discount factor tends to unity.[6] However, empirical estimates are far more wide ranging. Gali and Gertler (1999), Benigno and Lopez-Salido (2006) find a predominantly forward-looking specification, while Mehra (2004) finds an extremely backward-looking specification of the Phillips curve. Mankiw (2001) argues that stylised empirical facts are inconsistent with a predominantly forward-looking Phillips Curve. For this reason, we examine the model for any α. More generally, as the main points in this paper apply generically to most Phillips curves, we choose to work directly with (8), rather than restricting our analysis to one single formulation based on a particular microeconomic derivation.

The remainder of this section is organised as follows. We first look at the solution to the problem involved in minimising (1) subject to (8) under discretion i.e. the optimal time consistent policy. We then consider the first best commitment policy, which is time inconsistent. We then examine the solution under a 'timeless perspective' policy. Finally we replace (8) by (9), the New Keynesian Phillips curve.

The form of (1) is standard in the literature. One possibility would be to replace (1) by an objective function explicitly derived from private sector utility. However, this is not appropriate in our case for two reasons. First, we look at a number of Phillips curves (NAIRU/New Keynesian, forward, backward looking and hybrids), and therefore by implication a number of different specifications for private sector behaviour. Second, we focus on *non-benevolent* (short sighted) policy makers. As a result, it seems sensible to adopt the generic form of objective function used extensively in the literature.[7]

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[5] The NKPC does not allow for inflation inertia. Steinsson (2003) examines a model which extends the NKPC to include inflation inertia, by allowing some proportion of firms that do reset their price to index rather than set the profit maximising price. This results in an equation similar to (8), although with an additional term in lagged output. Our analysis could be applied to this formulation, but we would not gain any additional insights.

[6] This result is analogous to the standard New Keynesian Phillips curve based on the Calvo model, where the weight on expected inflation tends to one as the discount factor tends to unity. In both cases, the proportion of non-optimising firms does not influence the coefficients on inflation, but only the parameter on the forcing variable. The forcing variable will also be marginal costs rather than output, but we use output for expositional clarity.

[7] For example, Steinsson (2003) shows that a social welafre function based on his model involving backward looking price setters would involve the change as well as the level of inflation. However, there is no reason to believe that this modification would alter the main points made in this paper.

3.1 Solution under Discretion

The general solution to the problem 1, given the constraint (8), is complex to derive analytically and can be obtained numerically using the Oudiz and Sachs (1985) procedure.[8] We can illustrate the results in Figure 2. The upper panel of Figure 2 shows the steady state level of inflation bias for all values of α, and for several values of discounting[9]. The values for $\alpha = 0$ correspond to a purely backward looking Phillips curve, where the appendix shows that in steady state

$$\pi = \pi^* + \frac{(1-\beta)\, y^*}{\omega\phi} \tag{10}$$

$$y = 0 \tag{11}$$

There is an inflation bias, whose size depends on the authorities' rate of discounting, β. When the loss function is one-period, i.e. $\beta = 0$, then the inflation bias is identical to the one for the static model above (Section 2). If the authorities do not discount the future, i.e. $\beta = 1$, there is no inflation bias. The more policymakers discount the future, the larger is the inflation bias is. The reason why some discounting is required for inflation bias to be present is as follows. Without discounting, the (infinite) costs of the inflation bias would necessarily count for more in the welfare function than any short term welfare gains generated by higher output. Only with discounting are these inflation costs finite, and so only then is it possible for these short term output gains to justify positive inflation in the steady state. Note that, unless there is complete discounting, the amount of inflation bias in the steady state is always less than in the Barro-Gordon case, because the future inflation caused by targeting excess output causes policy to be less expansionary.[10]

When $\alpha = 1$, it is easy to show (Appendix B replicates results from many textbooks) that steady state inflation is independent of the level of discounting, and is given by

$$\pi = \pi^* + \frac{y^*}{\omega\phi} \tag{12}$$

$$y = 0 \tag{13}$$

This solution is identical to the one in the Barro-Gordon case. It is not dynamic, in the sense that inflation will immediately *jump* to this value and stay there, and output will always be at the natural rate. (This is the result derived by Clarida et al. (1999)). The intuition for this is

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[8]A popular implementation of the algorithm in MATLAB can be found in Söderlind (1999).

[9]Here and everywhere below we choose parameters, and in particular the discount rate, for presentational clarity and not necessarily realism.

[10]Levine (1988) effectively obtains this result, but does not draw on its significance. The issue is clearly discussed in Carlin and Soskice (2005).

as follows. In the static model, there must be no incentive for the authorities to trade-off more output for higher inflation. Only if inflation is positive, and equal to the level above, will this be the case. In the dynamic, discretionary case, the authorities should have no incentive to deviate from the optimal path. But this will only be true *in each period* along the entire dynamic path if inflation immediately jumps to its inflation bias level. As the incentive relates to the current period only there is no role for discounting.

Figure 1 shows the size of inflation bias for a full range of α and β in three dimensions, while the two dimesional Figure 2 selects some specific β.[11] In general inflation bias rises with α, but not always. At $\alpha = 0$ (a completely backward looking model), any one-off increase in output leads to a permanent increase in inflation, and so the incentive to undertake such an increase in output is offset at much lower levels of inflation bias if the discount rate (β) is reasonably large. As α increases towards 0.5, the initial impact of a one-off increase in output on inflation rises, and the persistence of this effect falls. Thus, if discounting is very high, inflation bias may fall as α increases.

In Figure 3 we plot the optimal solution chosen by the policymaker when the existing state is $\pi = 0$ and $y = 0$. We plot the dynamic path of output and inflation for several values of alpha, with $\beta = 0.75$. In all cases output is above the natural rate in the short term, and as a result inflation rises towards the steady state. As we noted above, these paths are constant when $\alpha = 1$.

3.2 Solution under Commitment

We solve the model under commitment policy in Appendix A. We obtain the steady state values for inflation and output as well as equations describing the dynamics of the economy towards the steady state.

In the lower panel of Figure 2 we show how the steady state level of inflation bias varies with α for $\beta = 0.75$. If $\alpha = 0$, expectations play no role in the Phillips curve, and so discretionary and commitment solutions will be identical. (This is clearly true for any β.) Whereas inflation bias increases under discretion as α rises (i.e. as the importance of expected future inflation increases) unless discounting is very large, under commitment it falls. When $\alpha = \beta / (1 + \beta)$ the inflation bias becomes zero.[12] Thereafter inflation bias becomes *negative*.

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[11]Here we alter α, but leave ϕ unchanged. In microfounded derivations of the Phillips curve ϕ is likely to depend on α, but the exact relationship will vary depending on the particular derivation used. As the main points we make in this paper apply generically to Phillips curves, we do not want to select one particular derivation, so we keep ϕ constant in Figure 2. As a result, Figure 2 should be regarded as an illustration of the dependence of inflation bias on a particular parameter value, rather than its relationship to a behavioural characteristic of the economy.

[12]Note that if the policymaker's discount factor is equal to the one used by the private sector, then this value of

To understand this last result, we can focus on the special case where $\alpha = 1$, which is where there is no inflation inertia, and the Phillips Curve is purely forward looking. As the appendix shows formally, the steady state optimal solution for our dynamic model in the purely forward looking case is given by

$$\pi = \pi^* - \frac{(1 - \beta)}{\beta} \frac{y^*}{\omega\phi} \tag{14}$$

$$y = 0 \tag{15}$$

Output in steady state must be equal to its natural rate. However, as long as the discount factor β is below one, steady state inflation will be below target i.e. there is a negative inflation bias. It is apparent that this bias will depend upon β – the smaller is β the bigger it becomes. Only in the case of no discounting (i.e. $\beta = 1$) do we get the same solution as that in the static model, in which commitment ensures that there is no inflation bias. (Recall that in the static Barro-Gordon model, the optimal time inconsistent solution always involves zero inflation, and output at its natural rate.) Furthermore, the optimal solution is no longer constant, but follows the dynamic path discussed below.

The intuition for why there is no inflation bias when there is no discounting is identical to the case of the backward looking Phillips curve. The surprising result is that, with discounting, the steady state rate of inflation, for a forward-looking Phillips curve, is negative. The reason follows from a simple property of a forward-looking Phillips curve, which is that positive output is associated with *falling* inflation along any rational expectations saddle path. Figure 4 plots the optimal path towards the steady state for several values of α, including one. When $\alpha = 1$, output is above zero in the short term, but the cost of this is an inflation rate which is initially high and then falls gradually. Even though steady state inflation is not zero, the cost of this is offset by the short-term output gains, which are discounted by less. To see why steady state inflation has to be negative, imagine a hypothetical path in which steady state inflation was zero, as well as output. But this would mean that inflation would have to be higher in the short run than it is along the optimal path. Providing that there is any degree of discounting, the permanent gains from having inflation exactly on target in the steady state would count for less than the increase in inflation in the short run. (This is plotted in the third column of Figure 4.) This is why it will be optimal for the government to promise *negative* inflation in the future.

This optimum path is dynamically time inconsistent. After the initial period, the policy maker has an incentive to re-optimise. The dynamic path shown above would only occur if the

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α is the one implied by the microfoundations in Christiano, Evans and Eichenbaum (2005).

policy maker had some commitment mechanism that ensured it did not re-optimise. Whether the authorities could find a commitment mechanism that could sustain the dynamic path described above remains an open question. Nevertheless, it is important to recognise that this time inconsistent dynamic path is superior in welfare terms to an alternative path along which inflation is constant at its target level and output is always at its natural rate. More generally, we show in appendix that, for an arbitrary α in (8), the steady state inflation rate can be found as

$$\pi = \pi^* + \left(1 - \beta - \alpha \frac{(1 - \beta^2)}{\beta}\right) \frac{y^*}{\omega \phi}$$

which represents a linear dependance in α. Note that as ϕ enters in a multiplicative way and is always positive, then it has no impact on the point at which inflation bias becomes negative. Figure 1 shows graphically the level of inflation bias for a range of values for α and β.

3.3 Timeless Perspective Policy

Woodford (1999) introduced the 'timeless perspective' policy as an alternative optimisation solution: see Appendix C for brief discussion and derivation. This policy is derived from the dynamic first-order conditions for the fully optimal solution by simple change in *timing* of those variables that bring time-inconsistency. As a result, the steady state *levels* of the economy under a timeless perspective policy remain the same as the ones for the commitment policy, so there will continue to be negative inflation bias with discounting by policy makers and a forward looking, NAIRU Phillips curve. However, the dynamic path towards it is very different. As discussed in Woodford (2003) and derived in Appendix C, the timeless perspective policy generates sluggish adjustment of an instrument. For the entirely forward-looking Phillips curve, the level of output at any time t is completely determined by its level in the previous period $t - 1$. If we start with $y_0 = 0$, as we implicitly assumed in all examples above (but where this assumption did not play any role), then output will have to remain at zero for all time and inflation will jump immediately to its steady state level, which is negative. Thus, the losses from having inflation bias in the long run, are not compensated by having zero (or close to zero) inflation in at least one of the periods, and output closer to target in the short term.

With a predominantly forward looking Phillips curve ($\alpha = 0.9$ for example) the optimal path is characterised by *rising* output (tending towards zero) and negative inflation, see Figure 5. This is a consequence of the new instrument equation for output, which requires that the difference in output is negatively related to the level of inflation. When inflation is negative, output must rise in the predominantly forward-looking Phillips curve.

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3.4 New Keynesian Phillips Curve

Until now, we have focused on Phillips curves that have the NAIRU property. Dynamic solutions involving the New Keynesian Phillips curve have been analysed extensively in Woodford (2003), but only for the case where the discount rate of the private sector and the policy maker are equal (i.e. the policy maker is benevolent).

We show in Appendix D.1, that in the case where the household discount factor is γ (see equation (9)), $\gamma \neq \beta$, we obtain the following expressions for the steady state output and inflation for a commitment policy:

$$
\pi = \pi^* + \frac{(\beta - \gamma)\left(y^* - \frac{(1-\gamma)}{\phi}\pi^*\right)}{\omega\phi\beta\left(1 + \frac{(\beta-\gamma)(1-\gamma)}{\omega\phi^2\beta}\right)}
$$

$$
y = \frac{\left(\pi^* + \frac{(\beta-\gamma)}{\omega\phi\beta}y^*\right)}{\left(\frac{\phi}{(1-\gamma)} + \frac{(\beta-\gamma)}{\omega\phi\beta}\right)}
$$

These results show that in the case where the authority's discount factor β is below the private sector's discount factor γ the equilibrium inflation is below its target *and* $y^* > 0$, unless the inflation target is very high.[13] In a sense our previous result with a NAIRU-consistent Phillips curve is a special case of this, where $\gamma = 1$. In contrast, if $\beta > \gamma$, (that is, if the authorities discount *less than* the private sector) we have positive steady state inflation. Only if the two discount rates are equal do we get target inflation in steady state, which is the case considered by Woodford (2003). If steady state inflation differs from target, then steady state output will also be non-zero even if the inflation target is zero, because the New Keynesian Phillips curve does not have the NAIRU property. In all cases the dynamics towards this new equilibrium is the same as that presented in Figures 3 and 4 for the NAIRU-consistent Phillips Curve.

Appendix B considers the case of the New Keynesian Phillips curve under discretion. This shows that inflation bias will be positive, and its size will be independent of the central bank's discount rate. In this qualitative sense, the results for the New Keynesian Phillips curve under discretion are the same as those derived for the NAIRU Phillips curve in section (3.1).

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[13] For this to hold, and for a realistic calibration of the Phillips curve, the inflation target should be of at least ten times higher than the output inefficiency, which is hardly realistic.

4   Conclusion

The received wisdom about inflation bias is based on an essentially static model, whereas the current literature works with Phillips curves which are dynamic in structure. We have examined both discretionary and commitment policies for Phillips curves with the NAIRU property that can be either backward looking, forward looking or some mixture of the two. We focus particularly on the impact of alternative discount rates used by policy makers, which has not been considered in the literature.

In the case of discretionary policy, we have generalised earlier results that were based on a purely forward looking model. If inflation has a backward-looking element, then the degree of bias caused by discretionary policy depends on both the rate of discount and the extent of forward-lookingness of the Phillips curve.
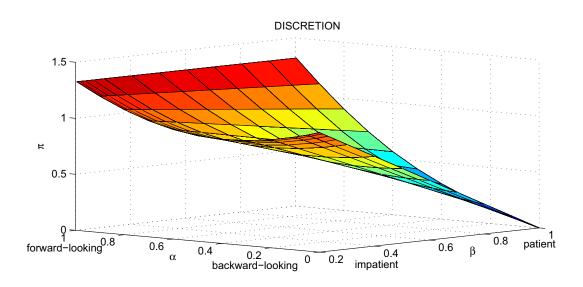
We have also examined the optimal time inconsistent policy. Whatever the level of discounting, the dynamic path towards the steady state will involve excess inflation and output above its natural rate. If policy makers discount, then steady state rate of inflation will in general not be equal to its target level, and if the Phillips curve is sufficiently forward-looking steady state inflation will be below target. In this sense, inflation bias can be negative.

The timeless perspective policy shares the same steady state as the commitment policy, so that inflation can be below target if there is discounting, providing that the Phillips curve is sufficiently forward-looking. However, if we start from a position in which output is at its natural rate, under the timeless perspective policy output stays at this level. That means that there will be the negative inflation bias, without any short run benefit of higher output.

These results for a NAIRU Phillips curve under commitment are also present for a New Keynesian Phillips curve, as long as the policy maker's discount rate exceeds that of the private sector. If the target inflation rate is moderate or low, then the steady state inflation rate will be below the target rate. As a consequence, steady state output will differ from the NAIRU, even if target inflation is zero. As with the case of a NAIRU Phillips curve, the dynamic path towards this steady state will be characterised by output above its steady state level and declining inflation.

Our analysis has no direct implications for policy, because our focus is on the case where the policy makers discount rate differs from the private sector. However, it does have implications for the interpretation of historic evidence. Until now, periods in which inflation was persistently below an explicit or implicit inflation target would have been intrerpreted as evidence *against*

a policy maker having an output target above the natural rate. We have shown that there is another potential explanation, which is that below target inflation reflects the optimal outcome for a short-sighted policy maker with reputation and an overambitious output target.
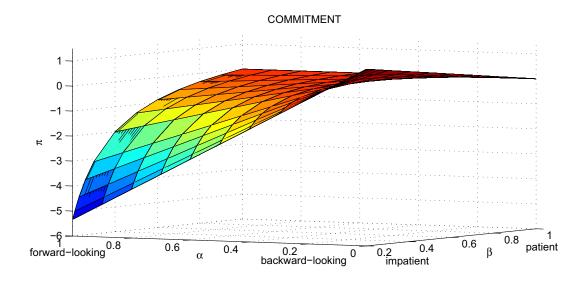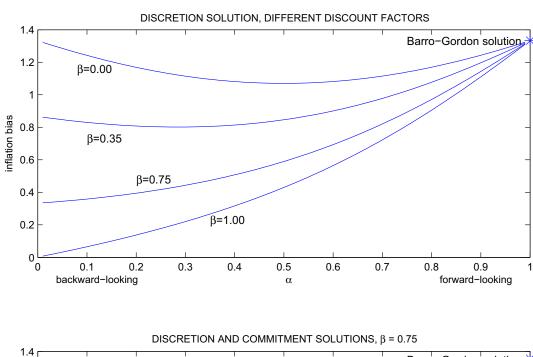
Figure 1: Inflation bias as a function of the household discount rate (β) and the degree of forward-lookingness of the Phillips curve (α).
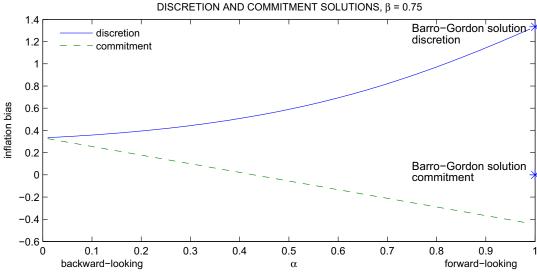
Figure 2: Relationship between the degree of forward-lookingness of the Phillips curve and the size of inflation bias for two policy regimes, discretion and commitment.
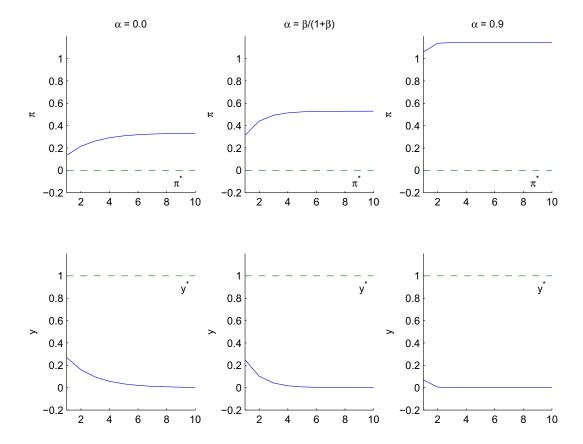
Figure 3: Dynamic path towards equilibrium under discretionary policy for $\beta = 0.75$ and different values of α.
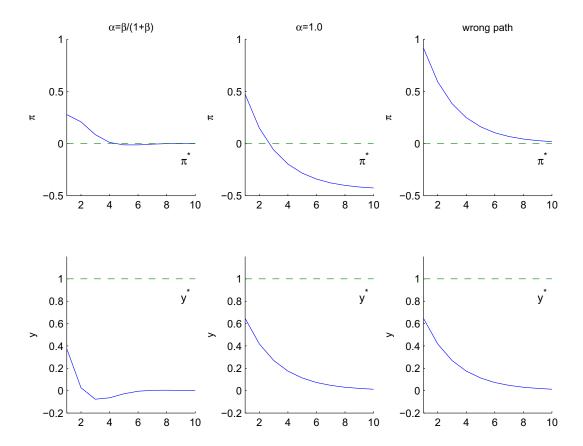
Figure 4: Dynamic path towards equilibrium under commitment policy for $\beta = 0.75$ and different values of α.
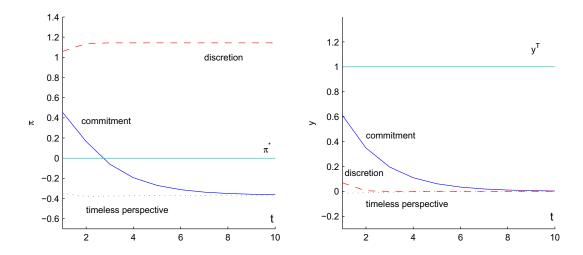
Figure 5: Dynamic path towards equilibrium under three policy regimes: commitment, discretion and timeless perspective, for $\alpha = 0.9$ and $\beta = 0.75$.

A   Commitment Solution

A fully-optimal (thus time-inconsistent) solution, can be found by using Pontryagin's Maximum Principle. We form the following Hamiltonian

$$\mathcal{H} = \mathcal{E}_0 \sum_{t=0}^{\infty} \left(\frac{1}{2} \beta^t (\omega(\pi_t - \pi^*)^2 + (y_t - y^*)^2) + \psi_t \left(\pi_t - \phi y_t - \alpha \pi_{t+1} - (1 - \alpha)\pi_{t-1} \right) \right)$$

Here ψ_{t+1} is a Lagrange multiplier.

The first order conditions are:

$$\frac{\partial \mathcal{H}}{\partial \pi_t} = \beta^t \omega(\pi_t - \pi^*) + \psi_t - \alpha \psi_{t-1} - (1 - \alpha)\psi_{t+1} = 0$$

$$\frac{\partial \mathcal{H}}{\partial y_t} = \beta^t (y_t - y^*) - \phi \psi_t = 0$$

$$\frac{\partial \mathcal{H}}{\partial \eta_t} = \pi_t - \phi y_t - \alpha \pi_{t+1} - (1 - \alpha)\pi_{t-1} = 0$$

This system collapses to the following equations, where we denoted $\beta^{-t}\psi_t = \mu_t$:

$$0 = \omega(\pi_t - \pi^*) + \mu_t - \frac{\alpha}{\beta}\mu_{t-1} - (1 - \alpha)\beta\mu_{t+1} = 0, \qquad t > 0 \tag{16}$$

$$0 = y_t - y^* - \phi\mu_t, \qquad t \geq 0 \tag{17}$$

$$0 = \pi_t - \phi y_t - \alpha \pi_{t+1} - (1 - \alpha)\pi_{t-1}, \qquad t \geq 0 \tag{18}$$

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The steady state values can be found as:

$$\pi = \pi^* + \left(1 - \beta - \alpha\frac{(1-\beta^2)}{\beta}\right)\frac{y^*}{\omega\phi} \tag{19}$$

$$\mu = -\frac{y^*}{\phi} \tag{20}$$

$$y = 0 \tag{21}$$

The dynamics towards equilibrium can be obtained by solving the system (16)-(18) numerically (using Currie and Levine (1993) formula) and imposing initial conditions $\pi_0 = \bar{\pi}, \mu_0 = 0$ and terminal conditions $\pi_\infty < \infty$, $\mu_\infty < \infty$. These four boundary conditions ensure a unique solution for this fourth order dynamic system.

B   Discretion

In general case, the discretionary solution can be obtained numerically. However, it is convenient to describe the two polar cases that can be solved analytically.

In the backward-looking case there are no expectational variables and the discretionary solution is the fully optimal solution. It can be obtained from the formulae above substituting $\alpha = 0$. The steady state values are

$$\pi = \pi^* + (1 - \beta)\frac{y^*}{\omega\phi}$$

$$\mu = -\frac{y^T}{\phi}$$

$$y = 0$$

In the forward looking case with $\alpha = 0$ we have no predetermined dynamic variables, so we can use the solution method described in Clarida et al. (1999) which explicitly treats all expectations variables as given. We form the following Hamiltonian

$$\mathcal{H} = \mathcal{E}_0\sum_{t=0}^{\infty}(\frac{1}{2}\beta^t(\omega(\pi_t - \pi^*)^2 + (y_t - y^*)^2) + \psi_{t+1}(\pi_t - \phi y_t - \pi_{t+1}))$$

Here ψ_{t+1} is a predetermined Lagrange multiplier (see Currie and Levine (1993)).

The first order conditions are:

$$\frac{\partial \mathcal{H}}{\partial \pi_t} = \beta^t \omega(\pi_t - \pi^*) + \psi_{t+1} = 0$$

$$\frac{\partial \mathcal{H}}{\partial y_t} = \beta^t(y_t - y^*) - \phi\psi_{t+1} = 0$$

$$\frac{\partial \mathcal{H}}{\partial \psi_{t+1}} = \pi_t - \pi_{t+1} - \phi y_t = 0$$

This system collapses to the following equations, where we denoted $\beta^{-t}\psi_t = \mu_t$:

$$0 = \omega(\pi_t - \pi^*) + \beta\mu_{t+1}, \qquad t > 0 \tag{22}$$

$$0 = y_t - y^* - \phi\beta\mu_{t+1}, \qquad\qquad t \geq 0 \tag{23}$$

$$0 = \pi_t - \pi_{t+1} - \phi y_t, \qquad\qquad t \geq 0 \tag{24}$$

The steady state values can be found as:

$$\pi_t = \pi^* + \frac{y^*}{\omega\phi} \tag{25}$$

$$\mu = -\frac{y^*}{\phi\beta} \tag{26}$$

$$y = 0 \tag{27}$$

and the economy will jump to the steady state immediately, as there are no predetermined state variables to delay the adjustment (See Clarida et al. (1999)).

C Timeless Perspective Policy

The required zeros in the initial conditions of the predetermined Lagrange multipliers for the commitment solution, and the implied time-inconsistency, can be explained using the observation that the system (16)-(18) is written for the time index $t = 1, 2, \ldots$ but equation (16) needs to be rewritten for $t = 0$ with $\mu_0 = 0$ while the rest of the system stays the same. We explicitly have one term less in (16) in the initial period. The private sector therefore observes that the policy maker in the initial period does something different from what it promises to do later on. Restarting the optimization at a later date yields the same first order conditions, an implied new policy in the first period and dynamic inconsistency.

A potential resolution to this problem, suggested by Woodford (1999), is to design a policy such that the first order conditions and hence the optimal policy would *look the same* for the private sector for every period of time, including $t = 0$. Formally, it means that we eliminate the

predetermined Lagrange multipliers from the system and therefore have no associated problems with zero initial conditions. The resulting control rule would be time invariant.

We derive the timeless perspective policy by eliminating μ from the system (16)-(18). We use equation (17) to solve for predetermined $\mu_{t+1} = \frac{1}{\phi\beta}(y_t - y^*)$, and substitute μ into the remaining equations of the system. Note that when we substituted $\mu_t = \frac{1}{\phi\beta}(y_{t-1} - y^*)$ in equation (16) we assumed that relationship which is valid for predetermined μ for time $t + 1$, is also valid at time $t = 0$, so that it is *here* that we introduce the timeless perspective policy rule. We finally obtain the following dynamic system:

$$y_t = \beta(1-\alpha)y_{t+1} + \frac{\alpha}{\beta}y_{t-1} - \phi\omega(\pi_t - \pi^*) + \left(1 - \beta - \alpha\frac{1-\beta^2}{\beta}\right)y^* \tag{28}$$

$$\pi_t = \alpha\pi_{t+1} + (1-\alpha)\pi_{t-1} + \phi y_t \tag{29}$$

The steady state of this system is the same the one as for commitment, i.e. there is a steady state inflation bias (19).

It is apparent that timeless perspective policy introduces instrument persistence. It becomes important, therefore, to know the initial value for output, y_0. For all previously considered policies this information was not needed, as there were no constraints on the use of instrument. The dynamics towards equilibrium can be obtained by solving the system (28)-(29) numerically and imposing initial conditions $\pi_0 = \bar{\pi}, y_0 = \bar{y}$ and terminal conditions $\pi_\infty < \infty$, $y_\infty < \infty$. These four boundary conditions ensure a unique solution for this system of two equations of the second order.

D   New-Keynesian Phillips Curve

D.1   Commitment

We solve the problem of minimisation loss (1) subject to the New Keynesian Phillips Curve:

$$\pi_t = \gamma\pi_{t+1} + \phi y_t$$

The first order conditions (as above, or as in Woodford (2003), p. 472) are:

$$0 = \omega(\pi_t - \pi^*) + \beta\psi_{t+1} - \gamma\psi_t$$
$$0 = (y_t - y^*) - \beta\phi\psi_{t+1}$$
$$0 = \pi_t - \phi y_t - \gamma\pi_{t+1}$$

The steady state can be found as:

$$\pi = \pi^* + \frac{(\beta - \gamma)\phi}{((1-\gamma)(\beta-\gamma)+\omega\phi^2\beta)}\left(y^* - \frac{(1-\gamma)}{\phi}\pi^*\right)$$

$$y = \frac{(1-\gamma)}{((1-\gamma)(\beta-\gamma)+\omega\beta\phi^2)}\left(\phi\omega\beta\pi^* + (\beta-\gamma)y^*\right)$$

We can compute the derivative

$$\frac{\partial\pi}{\partial\gamma} = -\frac{\left((\gamma-\beta)+\phi^2\omega\right)y^* - (1-(\gamma-\beta))\omega\phi\pi^*}{\omega^2\phi^3\beta\left(1+\frac{(\beta-\gamma)(1-\gamma)}{\omega\phi^2\beta}\right)^2}$$

It is clear that if $\gamma > \beta$ *and* $y^* > 0$ then $\frac{\partial\pi}{\partial\gamma} < 0$ unless π^* is very high. As $\pi = \pi^*$ if $\gamma = \beta$ then bias becomes negative for $\gamma > \beta$.

D.2 Discretion

We solve the problem of minimisation loss (1) subject to the New Keynesian Phillips Curve:

$$\pi_t = \gamma\pi_{t+1} + \phi y_t$$

We form the following Hamiltonian

$$\mathcal{H} = \mathcal{E}_0 \sum_{t=0}^{\infty}(\frac{1}{2}\beta^t(\omega(\pi_t - \pi^*)^2 + (y_t - y^*)^2) + \psi_t(\pi_t - \gamma\pi_{t+1} - \phi y_t))$$

If there are no predetermined variables in the system, we can treat expectations as exogenous and the first order conditions are written as follows (Clarida et al. (1999)):

$$\frac{\partial}{\partial\pi_t} = \beta^t\omega(\pi_t - \pi^*) + \psi_t = 0$$

$$\frac{\partial}{\partial y_t} = \beta^t(y_t - y^*) - \phi\psi_t = 0$$

$$\frac{\partial}{\partial\psi_t} = \pi_t - \phi y_t - \gamma\pi_{t+1} = 0$$

We denote $\hat{\psi}_t = \beta^{-t}\psi_t$ and remove hats to recycle notation. We then find the steady state solution as:

$$\pi = \pi^* + \frac{\phi}{((1-\gamma)+\phi^2\omega)}\left(y^* - \frac{(1-\gamma)}{\phi}\pi^*\right)$$

$$y = \frac{(1-\gamma)}{((1-\gamma)+\phi^2\omega)}\left(\omega\phi\pi^* + y^*\right)$$

The steady state is defined by the private sector's discount factor only. The inflation bias is positive, unless π^* is unrealistically high.

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