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Universal vs separated banking with deposit insurance in a macro model *

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Abstract

A macroeconomic model is developed to analyse integration of retail and investment banks with and without deposit insurance. Benefits flow from elimination of double marginalization and insurance premia which retail banks otherwise charge investment banks. Deposit insurance increases average output, whether banks are universal or separated, and can be welfare improving as it counters monopoly distortion. However, when unfavourable shocks hit the economy, the size of government bailout is larger with integrated than with separated banks.

The welfare assessment of the structure of banks depends on the kinds of shock hitting the economy, the degree of competitiveness of the banking sectors as well as on the efficiency of government intervention (the excess burden of deposit insurance). Scenarios are sketched in which different banking structures are desirable.

JEL Classification: E13, E44; G11; G24; G28.

Keywords: Financial intermediation in DSGE models, separating commercial and investment banking, competition and risks, systematic and idiosyncratic risks, bailouts, deposit insurance and economic wedges.

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1 Introduction

Recent events in world financial markets have again demonstrated that maintaining financial and macroeconomic stability are not separate policy challenges, at least some of the time. As a result, the issue of macro-prudential regulation has moved centre-stage and seems set to become an evolving feature of macroeconomic policies. More specifically, policymakers are debating whether restrictions on so-called universal banking¹, as in the US Banking Act of 1933, should be reintroduced.² In the UK the Vickers Report proposed ‘ring-fencing’ retail activities from investment banking activities. Similar ideas have been put forward by Paul Volcker in the US and Erkki Liikanen in the EU.³

There are many considerations in assessing the desirability of such restrictions, but the Vickers Report concludes that in their absence the banking sector will be insufficiently capitalised; difficult to restructure when default occurs; prone to excessive risk-taking; and more vulnerable to a variety of financial shocks. The ring-fence is not the only policy proposal⁴, but it is fair to say that it forms a central recommendation and remains controversial in the UK and elsewhere.

In this paper a simple macroeconomic model is developed to begin studying separated versus universal banking structures and to enquire: What inefficiencies might attend current and proposed banking structures? How does deposit insurance affect banks’ optimizing strategies? Which shocks to the economy and financial system are likely to be most harmful across alternative structures? What macroeconomic policy trade-offs are likely to emerge and how might these interact with existing regulations, principally deposit protection?

In the model a final-goods sector is reliant for its production on the output of a stylized, monopolistically competitive investment banking sector. The investment banks, in order to hire labour, raise loans from a monopolistically competitive retail banking sector. Retail banks are funded by private agents’ deposits. Initially we set out the problems facing the

¹Universal banks combine both retail/utility banking activities with investment banking operations.

²The 1933 Act is popularly known as the Glass-Steagall Act. Amongst other things it separated retail banking from investment banking. That provision was effectively repealed in 1999 by the Gramm–Leach–Bliley Act.

³See The Final Report of the Independent Commission on Banking Chaired by Sir John Vickers, generally known as The Vickers Report: <http://bankingcommission.independent.gov.uk/>. The ring-fence, broadly speaking, would require a separately capitalised institution for retail, including SME, banking. The idea is to isolate these agents from financial market risk and provide a clear focus for any government bail out. The practical implementation of a ring-fence remains somewhat contentious.

⁴Other notable proposals are directed at changing pay structures, requiring more and better capital and liquidity, withdrawing hidden central bank subsidies, making banking more competitive, reconfiguring supervisory architecture domestically, and coordinating supervision across countries better, making banks draw up living wills and taxing certain types of financial transactions.

retail banks and the investment banks separately; we then ‘merge’ these institutions to model the implications of universal banking.

The model has three distinctive features. First, investment banks have projects (production functions) with random returns. They make hiring decisions before cost and demand conditions are known. In choosing the amount of labour to employ, they determine the likelihood of default. And for the reasons just noted, final goods producers and retail banks are impacted by the investment banks’ decisions.

Second, there is a rich menu of shocks. Investment banks are subject to idiosyncratic, bank-specific, shocks. They also face a shock that is common to all investment banks. Hence, depending on whether common or idiosyncratic shocks are dominant, the economy can be well-insured against, or vulnerable to, financial shocks. In addition, there is a common macroeconomic shock similar to a TFP or quality of (financial) capital shock, as in Gertler and Kiyotaki (2010).

Third, we assume that in the event of default by an investment bank, retail banks are covered by government-provided deposit protection so that they can meet their obligations to depositors. In a sense retail banks are bailed out. Investment banks have limited liability and they are allowed to continue trading next period.⁵ Thus depositors have no need to monitor individual retail banks and retail banks have no need to monitor individual investment banks. The banks understand that they will be bailed out and this distorts their optimization problems: It boosts the size of the investment bank sector and narrows spreads in the retail sector. There are no institution-specific costs associated with bankruptcy. However, the bail out is costly to agents as a whole since government action is distortive. Nevertheless, we show the bailout may still be welfare-enhancing.

The framework provides initial insights into a number of the questions posed above. First, deposit insurance (in effect bank bailouts) may be welfare-enhancing. That is because the monopoly wedges in the economy mean that output of the financial sector (and final goods) may be inefficiently low. Deposit insurance stabilizes interest rates inducing higher precautionary labour supply and an expansion in banks’ balance sheets; encouraging *more* risk-taking by banks may well be the optimal second-best policy, akin to light-touch regulation. Hence, if the monopoly distortion is sufficiently large, deposit insurance is welfare-improving regardless of the structure of banking. Second, a key trade-

⁵An alternative description of the environment is that banks go bust and are replaced next period by new banks such that market structure is identical period-to-period. Thus, we have deposit protection but no bank bailouts. The model is not rich enough to distinguish these alternative interpretations although the distinction may well be important in reality.

off that emerges is one between a higher cost of funding (i.e., double marginalization) and relatively low default when banks are separated, against more competitive pricing and larger government bailouts under universal banking. Double marginalization and the required credit spread are, in welfare terms, more costly distortions when the distortive impact of government intervention is low and monopoly distortions are high.⁶ When common shocks to the overall efficiency of the financial system dominate, universal banking may remain the preferred structure, but the judgement is finer; despite boosting the output of investment banks, final goods production and hence consumption, universal banking results in larger taxpayer bail-outs. The overall welfare assessment of the structure of banks depends on the kinds of shock hitting the economy, the degree of competitiveness of the investment and retail banking sectors as well as on the efficiency of government intervention. It is not difficult to come up with scenarios in the model in which separated banking may be preferable. One such scenario is when investment banking is competitive and common shocks are volatile.

1.1 Outline of paper

The rest of the paper is set out as follows. Section 2 describes the behavior of private agents and final goods producers. The investment banks and their interaction with retail banks are modeled in Section 3 together with credit spreads and the costs of the retail bank bail out. Section 4 solves out for the general equilibrium of the baseline model and sets out how different assumptions concerning banking structure and bailouts affects the baseline model. Building on some preliminary welfare analysis at the end of Section 4, Section 5 presents a detailed analysis of the wedges of inefficiency in the decentralised economy. It analyses how the wedges associated with monopoly power and the excess burden of government action may be attenuated or exacerbated by deposit insurance/bailouts. The social planning solution is also analyzed. We set out a simple numerical analysis of the model economy in Section 6 which clarifies some of the findings in Section 5. Section 7 summarizes and concludes. Appendices contain additional calculations, derivations and proofs referred to in the text.

2 Macroeconomic Framework

The basic set up of the model is as follows: The economy consists of continua of households, monopolistically competitive, risk-neutral banks and final goods producers. There is also a

⁶Boyd, Chang and Smith (1998) argue that a universal banking structure requires a larger FDIC involvement. See also Boot and Thakor (1997). To the best of our knowledge, the present paper is the first to provide welfare analysis of the benefits of universal versus separated banking.

government. Households consume the final goods, provide labour to the investment banks and deposit their savings in the retail banks. The retail banks, if separate from investment banks, lend to investment banks who use the funds to hire labour. The investment banks make their hiring and production decisions before they observe their productivity and the demand for their output. That output is an intermediate good; that is, an input to the production of the final good. Investment banks have differing rates of profitability because they face idiosyncratic shocks. Because of idiosyncratic and common shocks, the value of banks' assets are stochastic, and some of the banks may default. The role of government is to bail out the banks when necessary and possible. The components of the model are now described in more detail.

2.1 Households

There is a continuum of identical households in the economy who evaluate their utility using the following criterion:

$$E_0 \sum_{t=0}^{\infty} \beta^t (\log(C_t) - \lambda N_t). \quad (1)$$

E_t denotes the expectations operator at time t , β is the discount factor, C_t is consumption and $N_t = \int_j N_t(j) dj$ is labour, where $N_t(j)$ is the quantity of labour supplied to investment bank j . λ is a time-invariant preference parameter.

Consumption is defined over a basket of goods and indexed by i , $C_t \equiv \left[\int_0^1 c_t(i)^{\frac{\theta-1}{\theta}} di \right]^{\frac{\theta}{\theta-1}}$, where $\theta > 1$ is the elasticity of substitution. The price-level is $P_t = \left[\int_0^1 p_t(i)^{1-\theta} di \right]^{\frac{1}{1-\theta}}$. The demand for each good is given by

$$Y_t(i) = \left(\frac{p_t(i)}{P_t} \right)^{-\theta} Y_t^d, \quad (2)$$

where $p_t(i)$ is the nominal price of the final good produced by firm i and Y_t^d denotes aggregate demand. All investment banks pay the same real wage for the same labour. As a result, $w_t(i) = w_t$, $\forall i$. All households provide the same amount of labour to each investment bank.

The household's nominal flow budget constraint is

$$P_t C_t + D_t^h = R_{t-1}^h \Gamma_t D_{t-1}^h + W_t N_t + \Pi_t, \quad (3)$$

where D_t^h is the nominal value of deposits in retail banks at the start of date t . There are many retail banks and agents spread their risks across banks. Between date $t-1$ and the start of t deposit balances earn a nominal gross interest return of $\Gamma_t R_{t-1}^h$, where R_{t-1}^h is the gross interest each bank agrees to pay ex ante. However, the ex post return may be smaller if the bank's assets at the end of the period are lower than $R_{t-1}^h D_{t-1}^h$. In that case

banks will pay only proportion Γ_t^s of their obligations. If there is deposit insurance then Γ_t^g is provided by government. Therefore the proportion of the contracted return actually received by the depositors is $\Gamma_t = \Gamma_t^s + \Gamma_t^g$. If deposit insurance is not provided, $\Gamma_t^g = 0$. However, when deposit insurance is provided, there exists the possibility that profits may be so low in the economy that governments may not have the capacity to bail out in full the depository institutions. The Γ reflects these eventualities, hence it is stochastic and $\Gamma \leq 1$. The precise form of Γ will change with these various scenarios and will be derived below.

In period t , agents have to decide how much of their current wealth to place in retail banks, D_t^h , given W_t , the nominal wage in period t , the expected return on deposits and Π_t , the corporate profits remitted to the individual net of the cost of bailing out banks.

Necessary conditions for an optimum include:

$$C_t = \frac{W_t}{\lambda P_t}; \quad (4)$$

and

$$E_t \left\{ \Gamma_{t+1} R_t^h \frac{\beta C_t}{C_{t+1}} \frac{P_t}{P_{t+1}} \right\} = 1. \quad (5)$$

2.2 The final goods sector

The production of final goods depends on A_t , common to all producers, and a financial product, $X_t(i)$:

$$Y_t(i) = A_t X_t(i). \quad (6)$$

We think of $X_t(i)$ as a bundle of financial and consultancy services demanded by firm i . The assumption is that these financial services are necessary for production of the final output. The aggregate level of output will be of the same form as (6) so that another way to view the impact of financial services is that they help eradicate misallocation of resources across firms and hence increase aggregate output; see Restuccia and Rogerson (2008), Hsieh and Klenow (2009) and Greenwood, Sanchez and Wang (2010) and Pratap and Urrutia (2012). A_t may be thought of as an aggregate macro shock to output or as a utilization shock, reflecting factors such as the costs of using the financial system.

Firms set prices to maximize their profit

$$\max_{P_t(i) X_t(i)} \left(\left(\frac{P_t(i)}{P_t} \right) Y_t(i) - \frac{Q_t}{P_t} X_t(i) \right),$$

where Q_t/P_t is the real price of the output of the financial sector.

In a symmetric equilibrium $\frac{P_t(i)}{P_t} = 1$, and using the first order conditions for firms, it is straightforward to derive an aggregate real price, (7), and the aggregate demand for the

financial product (8):

$$\frac{Q_t}{P_t} = \frac{\theta - 1}{\theta} A_t; \quad (7)$$

$$X_t^d = Y_t/A_t. \quad (8)$$

2.3 Banks

There are potentially two types of banks in the model, investment banks and retail banks. The output of investment banks, as noted, comprises a bundle of financial goods and services demanded by the final goods producers. Investment banks may be separated from retail banks. In that case, investment banks finance their activities by borrowing funds from retail banks. The only role for retail banks is to collect deposits from households and channel funds to the investment banks. In this ‘interbank market’ they are monopolistic competitors. Where there is no distinction between investment and retail banks (i.e., there is universal banking) there is no role for such a loan market. The banking sectors are now set out.

3 Financial intermediation

3.1 The investment bank sector

Assume that investment and retail banks are separate. Agents deposit savings in retail banks. The retail banks bundle and sell these funds to an investment banking sector. The investment banks in the model need loans from retail banks to pay the wage bill ahead of selling their output to the final goods sector. However investment banks’ output is stochastic. If that output is below some value then these banks default, ending up with negative net assets. If these losses in aggregate are large, retail banks may not be able to repay depositors the rate R_t^h . The banks’ losses may be made good by the taxpayer. However, in any case, banks are allowed to continue trading in the next period. If output is high enough, profit is remitted to private agents.

The investment bank produces output at $t + 1$, $X_{t+1}(j)$, by employing labour at time t . Labour is homogeneous and is used with the following production technology to which all banks have access:

$$X_{t+1}^s(j) = \Omega_t \varepsilon_{t+1} e_{t+1}(j) N_t(j). \quad (9)$$

Here, $N_t(j)$ is the labour input employed by investment bank j , $\Omega_t > 0$ is the time t expected return common to all investment banks. ε_{t+1} is a shock that is also common to all banks and $e_{t+1}(j)$ is a j -specific shock. It is assumed that $e_{t+1}(j) \geq 0$, $\varepsilon_t \geq 0$, $E_t \varepsilon_{t+1} = 1$, and

$E_t e_{t+1}(j) = 1$ and $(e_{t+1}(j), \varepsilon_{t+1})$ are independently distributed. The cumulative distributions of ε_{t+1} and e_{t+1} are denoted by $F^\varepsilon(\varepsilon)$ and $F^e(e)$, are time-invariant and common to all banks.

At the start of period t the investment bank borrows the amount $B_t(j) = W_t N_t(j)$ from retail banks. In the next period the investment bank receives $Q_{t+1}(j) X_{t+1}(j)$, and pays $B_t(j) R_t^C$ to the retail bank, where $Q_{t+1}(j)$ denotes the price per unit $X_{t+1}(j)$, and R_t^C is the interest due on the loan.

The market for the output of the investment banking sector is assumed to be monopolistically competitive and the demand for output of bank j is

$$X_t^d(j) = \left(\frac{Q_t(j)}{Q_t} \right)^{-\eta} X_t^d. \quad (10)$$

The superscript d indicates ‘demand’, and $\eta > 1$ is the demand elasticity between the variety of products or the degree of competition⁷. The aggregate price next period, Q_{t+1} , and aggregate demand, X_{t+1} , are exogenous to the bank’s decision.

Clearly, the optimal strategy includes selling everything demanded ex-post so that in equilibrium,

$$X_{t+1}^d(j) = X_{t+1}^s(j) = X_{t+1}(j). \quad (11)$$

Combining (9), (10) and (11) shows that the ex-post price depends on the realization of common and specific banking shocks

$$Q_{t+1}(j) = Q_{t+1} \left(\Omega_t \varepsilon_{t+1} e_{t+1}(j) \frac{N_t(j)}{X_{t+1}^d} \right)^{-1/\eta}. \quad (12)$$

So, the bank’s assets at the end of period are

$$Q_{t+1}(j) X_{t+1}(j) = [\Omega_t \varepsilon_{t+1} e_{t+1}(j) N_t]^{1-1/\eta} X_{t+1}^{1/\eta} Q_{t+1}. \quad (13)$$

Expression (13) clearly shows that conditional expected profit depends not only on the productivity shocks, $\varepsilon_{t+1} e_{t+1}(j)$, but also on the state of the macroeconomic environment, represented by (X_{t+1}, Q_{t+1}) .

Ex-ante, the investment bank needs to decide on the level of borrowing/labour input. We suppose that investment banks have limited liability and act as though profit is bounded below at zero. So, assuming banks are risk-neutral, expected profit is

$$\begin{aligned} E_t \Pi_{t+1}(j) &= \max [E_t Q_{t+1}(j) X_{t+1}(j) - W_t N_t R_t^C, 0] \\ &= \max [E_t [\Omega_t \varepsilon_{t+1} e_{t+1}(j) N_t]^{1-1/\eta} X_{t+1}^{1/\eta} Q_{t+1} - W_t N_t R_t^C, 0]. \end{aligned} \quad (14)$$

The limited liability distortion means that banks will seek to maximize profits on a subset of states of nature. As a result, they will choose borrowing and a cut-off value for a

⁷The aggregate demand for financial intermediation is defined over a basket of services indexed by j , $X_t \equiv \left[\int_0^1 X_t(j)^{\frac{\eta-1}{\eta}} di \right]^{\frac{\eta}{\eta-1}}$, where $\eta > 1$ is the elasticity of substitution. The aggregate price index is $Q_t = \left[\int_0^1 Q_t(i)^{1-\eta} di \right]^{\frac{1}{1-\eta}}$.

composite of the shocks facing the bank below which default will occur. We now construct the bank's optimization problem in detail.

First, note that aggregate price and demand will depend on the realisation of both macro and banking shocks

$$Q_{t+1} = \frac{\theta - 1}{\theta} A_{t+1} P_{t+1}; \quad (15)$$

$$X_{t+1}^d = X_{t+1}^s, \quad (16)$$

where we assume that $A_t = A_{t-1}^\rho u_t$, $0 < \rho < 1$ and u is lognormally distributed with $E(u) = 1$. It is assumed that price stability is maintained in the final goods market. That makes P_{t+k} completely predictable, and for convenience we normalise it to 1.

Since Ω and ε are common to all banks, the aggregate supply of loans is given by

$$X_{t+1}^s = \bar{N} \Omega_t \varepsilon_{t+1} \left[\int_0^\infty [e]^{\frac{\eta-1}{\eta}} dF^e(e) \right]^{\frac{\eta}{\eta-1}} = \bar{N} \Omega_t \varepsilon_{t+1} \Delta, \quad (17)$$

where \bar{N} is the average number of employees at the other investment banks and $\Delta \equiv \left[\int_0^\infty [e]^{\frac{\eta-1}{\eta}} dF^e(e) \right]^{\frac{\eta}{\eta-1}}$ is the aggregate of idiosyncratic shocks across investment banks. There is no strategic interaction amongst the banks and \bar{N} is treated as parametric by each bank. So, combining (15) and (17) means that (14) can be written, omitting time subscripts, as

$$\Pi|_{u\varepsilon e^{1-1/\eta}} = N \max \left[\Omega u \varepsilon e^{1-1/\eta} \frac{(\theta-1)}{\theta} A_t^\rho \left(\frac{\bar{N} \Delta}{N} \right)^{1/\eta} - W_t R_t^C, 0 \right]. \quad (18)$$

That expression is positive if and only if

$$u \varepsilon e^{1-1/\eta} > \varepsilon^D, \quad (19)$$

where

$$\varepsilon^D = \Lambda \left(\frac{\bar{N}}{N} \right)^{-1/\eta}, \quad (20)$$

and for purposes later on it is convenient to define

$$\Lambda = \frac{\theta}{\theta-1} \frac{W_t R_t^C}{\Omega_t A_t^\rho} (\Delta)^{-1/\eta}. \quad (21)$$

Formula (20) represents an ex-ante planned default threshold chosen by an individual bank taking macroeconomic factors, W_t , R_t^C , \bar{N}_t and A_t , as given. However, the ex-post default rate depends on the realisation of the product of shocks, $s := u \varepsilon e^{1-1/\eta}$ where s is a random variable with lognormal density $f^s(s)$. If $s > \varepsilon^D$, then the bank will realise positive profits, otherwise profit are, in effect, zero. Hence, the complete investment banking problem can be written as:

$$\max_{N, \varepsilon^D} W_t R_t^C N_t \int_{\varepsilon^D}^{+\infty} \left[\left(\frac{s}{\varepsilon^D} \right) - 1 \right] f^s(s) ds; \quad (22)$$

$$\text{s.t. } \varepsilon^D - \Lambda \left(\frac{\bar{N}}{N} \right)^{1/\eta} = 0. \quad (23)$$

Denoting by μ the Lagrange multiplier on (23), it follows that the first order conditions for an optimum are,

$$\frac{\partial L}{\partial(N/\bar{N})} = \int_{\varepsilon^D}^{+\infty} \left[\frac{s}{\varepsilon^D} - 1 \right] f^s(s) ds - \frac{1}{\eta} \mu \Lambda \left[\frac{N}{\bar{N}} \right]^{\frac{1-\eta}{\eta}} = 0,$$

and

$$\frac{\partial L}{\partial \varepsilon^D} = \frac{1}{\varepsilon^D} \frac{N}{\bar{N}} (1 - \eta) \int_{\varepsilon^D}^{+\infty} \left(\frac{s}{\varepsilon^D} - \frac{\eta}{\eta - 1} \right) f^s(s) ds = 0, \quad (24)$$

where we have used that

$$\mu = \eta \frac{N}{\bar{N}} (\varepsilon^D)^{-1} \int_{\varepsilon^D}^{+\infty} \left[\frac{s}{\varepsilon^D} - 1 \right] f^s(s) ds. \quad (25)$$

In a symmetric steady state $\frac{\bar{N}}{N} = 1$, so that (23) implies

$$\varepsilon^D = \Lambda. \quad (26)$$

Thus, combining (24) and (26) we get

$$\int_{\Lambda}^{+\infty} \left[\frac{s}{\Lambda} - \frac{\eta}{\eta - 1} \right] f^s(s) ds = 0. \quad (27)$$

If the distribution of s is given, then Λ is just a constant which solves this integral equation. However, there may exist no, or many, solutions to this equation. In the appendix (Section 8.1) we derive sufficient conditions for a solution to exist. It is shown that a lognormal distribution satisfies those conditions and provides a unique Λ as a solution. Moreover, under a lognormal distribution the second order conditions for an optimum are satisfied. Importantly, one observes that this equation has implications for the relationship between the degree of competition and equilibrium default to which we turn presently.

Given Λ one may use (21) to compute the equilibrium wage

$$W_t = \frac{\theta - 1}{\theta} \Omega_t A_t^\rho \frac{\Lambda}{R_t^C} (\Delta)^{1/\eta}. \quad (28)$$

This relation implies that for a given loan rate, R^C , the equilibrium wage increases in: labour productivity, ΩA_t^ρ , the degree of competition in the final goods sector, θ , and the tolerance to risk, Λ , of the investment banks. Or, to state this last observation differently, banks which are prepared to take on higher risks will offer higher wages.

Finally, one can compute the default threshold, conditional on the realisation of common shock, by combining (19) and (26),

$$e^D = \left(\frac{\Lambda}{u\varepsilon} \right)^{\frac{\eta}{\eta-1}}. \quad (29)$$

Every bank with an idiosyncratic shock lower than e^D will be in default while those for whom $e \geq e^D$ will be able to meet their commitments. From (29) it follows that the conditional probability of default depends only on the unexpected macro shock u and the realisation of the common shock to the investment banking sector, ε . At an optimum, it does not depend on the expected gross return to the sector, Ω , or the current state of the macroeconomy, A_t .

3.2 Effect of limited liability

In the benchmark model set out above banks act as if they have limited liability: They maximize expected profit conditional that profit is weakly positive. It would be interesting to see what happens when they have a different objective. For example, one might introduce a capital requirement, a target for own funds. That way the bank needs to incorporate the possibility of some losses into its optimal program. Here we briefly consider the ‘extreme’ case when the bank has to confront all losses and so maximizes expected profit over all possible state of nature. In this case the bank’s objective is

$$\Pi(N) = E_t \left(N \Omega s_{t+1} \frac{(\theta - 1)}{\theta} A_t^\rho \left(\frac{\bar{N} \Delta}{N} \right)^{1/\eta} - W_t R_t^C N \right), \quad (30)$$

which is equivalent to

$$\max_N W R^C N \int_0^{+\infty} \left[\left(\frac{s}{\Lambda_0} \right) \left(\frac{\bar{N}}{N} \right)^{1/\eta} - 1 \right] f^s(s) ds. \quad (31)$$

As before Λ_0 does not depend on any individual decision since

$$\Lambda_0 = \frac{\theta}{\theta - 1} \frac{W_t R^C}{\Omega_t A_t^\rho} (\Delta)^{-1/\eta}. \quad (32)$$

However, in equilibrium, Λ_0 solves the first order condition to (31), which is

$$\int_0^{+\infty} \left[\frac{s}{\Lambda_0} - \frac{\eta}{\eta - 1} \right] f^s(s) ds = 0. \quad (33)$$

The solution to (33) exists and is unique for any s with finite expectation. Moreover, Λ_0 is smaller than Λ as defined in (27).

Proposition 1 *If Λ exists, then $\Lambda > \Lambda_0$.⁸*

To analyse the economic implication of Lemma 1 note that Λ determines the demand for labour such that a larger Λ is associated with a higher demand for labour. Therefore, ceteris paribus, limiting bank liability increases the demand for labour and thus implies higher output in the economy as a whole. We return to these issues below.

⁸The proof is in the appendix.

3.3 The retail banking and credit spread

There is a continuum of risk-neutral, retail banks indexed by i . Banks pay a state dependent, contractual interest rate on deposits of R_t^h , if possible. That deposit rate will be common across banks and need not be indexed by i . In the loans market banks are monopolistic competitors and set loan rates, $R_t^c(i)$. So, following Aksoy et al. (2009), banks face the following demand for loans

$$B_t^c(i) = \left(\frac{R_t^c(i)}{R_t^C} \right)^{-\delta} B_t^C. \quad (34)$$

Here $B_t^c(i)$ is bank i 's lending, R_t^C is a measure of the average interest rate on loans, $R_t^C = \left[\int_0^1 R_t^c(i)^{1+\delta} di \right]^{\frac{1}{1+\delta}}$, B_t^C is aggregate demand for loans, $B_t^C = \left[\int_0^1 B_t^c(i)^{\frac{\delta-1}{\delta}} di \right]^{\frac{\delta}{\delta-1}}$, and $\delta > 1$ is the elasticity of substitution between loans. The objective of each bank, therefore, is to maximize expected profits by choosing the rate charged on lending. If all borrowers remain solvent, the retail bank will earn nominal return R_t^C per unit loaned. In the case of default, the assets of the borrower are repossessed pro rata by the retail bank.

Retail banks maximize expected profit, $E_t \Psi_{t+1}$, given the demand for loans, (34), and knowing that their liabilities are limited:

$$E_t \Psi_{t+1}(R_t^c(i)) = E_t \max(R_t^c(i) B_t^c(i) - R_t^h B_t^c(i), 0). \quad (35)$$

Although the profits of individual retail banks are in effect bounded below by zero, the net worth of the retail bank sector is ultimately determined by outturns in the investment banking sector. In some states, an investment bank may not be able to repay its loan in full. One may characterize as follows that portion of the loan which can be repaid by the investment bank. At period $t+1$ every investment bank j has given liability, $W_t R_t^C N_t$, whilst its assets are stochastic and equal to $\Omega u \varepsilon e(j)^{1-1/\eta} \frac{(\theta-1)}{\theta} A_t^\rho N_t$. Thus, the assets to liabilities ratio can be written as

$$\frac{\Omega u \varepsilon e^{1-1/\eta} \frac{(\theta-1)}{\theta} A_t^\rho \Delta^{1/\eta}}{W R_t^C} = \frac{u \varepsilon e^{1-1/\eta}}{\Lambda}.$$

Therefore the borrower is in default if $\frac{u \varepsilon e^{1-1/\eta}}{\Lambda} < 1$, and the gross return generated by the borrower in default will be $\frac{u \varepsilon e^{1-1/\eta}}{\Lambda} R_t^c(i)$. Let $\Gamma_{u \varepsilon e}(u \varepsilon e)$ be the ratio of actual to contractual return conditional on a particular realisation of the shocks, that is

$$\Gamma_{u \varepsilon e}(u \varepsilon e) = \min \left(\frac{u \varepsilon e^{1-1/\eta}}{\Lambda}, 1 \right).$$

After averaging over all possible idiosyncratic shocks, one obtains the expected ratio of actual to contractual return conditional on the realisation of macro and systemic shocks, $u \varepsilon$:

$$\Gamma_{u \varepsilon}(u \varepsilon) = u \varepsilon \int_0^{\left(\frac{\Lambda}{u \varepsilon} \right)^{\frac{\eta}{\eta-1}}} \frac{e^{1-1/\eta}}{\Lambda} f^e(e) de + 1 - F^e \left(\left(\frac{\Lambda}{u \varepsilon} \right)^{\frac{\eta}{\eta-1}} \right). \quad (36)$$

In formula (36) the first term is expected revenue from repossession of the assets of the investment banks which are in default. The remaining term is the expected revenue from

non-defaulting banks. It is useful to establish some basic properties of the function $\Gamma_{u\varepsilon}(u\varepsilon)$. We do this in:

Proposition 2 $\Gamma_{u\varepsilon}(u\varepsilon)$ is an increasing and concave function. Therefore more favourable systemic or macro shocks bring average returns closer to the contractual return. Moreover $\Gamma_{u\varepsilon}(0) = 0$ and $\lim_{x \rightarrow \infty} \Gamma_{u\varepsilon}(x) = 1$.

Proof. See Appendix ■

The profit of the retail bank conditional on the realisation of the aggregate shocks will be

$$\Psi_{t+1}(R_t^c(i), u\varepsilon) = \max\left(\frac{R_t^c(i)}{R_t^h} \Gamma_{u\varepsilon}(u\varepsilon) - 1, 0\right) B_t^c(i) R_t^h. \quad (37)$$

Using the demand for loans (34) one writes

$$\Psi_{t+1}(R_t^c(i), u\varepsilon) = \max\left[\frac{R_t^c(i)}{R_t^h} \Gamma_{u\varepsilon}(u\varepsilon) - 1, 0\right] \left(\frac{R_t^c(i)}{R_t^h}\right)^{-\delta} B_t^C R_t^h. \quad (38)$$

For any $R_t^c(i)$ there is a value of common shock $u\varepsilon = y$ below which retail banks will default on their obligations. Therefore, the expected profit maximization problem (35) can be written as

$$\begin{aligned} \max_{\frac{R_t^c(i)}{R_t^h}, y} E\Psi_{t+1} &= \left[\int_y^{+\infty} \left(\frac{R_t^c(i)}{R_t^h} \Gamma_{u\varepsilon}(x) - 1\right) f_{u\varepsilon}(x) dx \right] \left(\frac{R_t^c(i)}{R_t^h}\right)^{-\delta} B_t^C R_t^h \\ \text{s.t. } \frac{R_t^c(i)}{R_t^h} \Gamma_{u\varepsilon}(y) &= 1. \end{aligned} \quad (39)$$

The first order conditions imply that in a symmetric equilibrium the retail banks default threshold y is defined as

$$\int_y^{+\infty} \left(\frac{\Gamma_{u\varepsilon}(x)}{\Gamma_{u\varepsilon}(y)} - \frac{\delta}{\delta - 1}\right) f_{u\varepsilon}(x) dx = 0, \quad (40)$$

and the spread R_t^C/R_t^h in equilibrium is

$$\frac{R_t^C}{R_t^h} = \frac{1}{\Gamma_{u\varepsilon}(y)}. \quad (41)$$

Proposition 3 There exists a solution to (40) which also satisfies the second order conditions.

Proof. See the appendix. ■

It is interesting to note that the wedge (credit spread) imposed by separated banking is larger than the familiar monopolistic wedge:

Proposition 4 The mark up charged by retail banks is greater than the standard monopoly pricing wedge.

Proof. From (40) and (41) it follows that

$$\frac{R_t^C}{R_t^h} = 1/\Gamma_{u\varepsilon}(y) = \frac{\delta}{\delta-1} \frac{[1 - F_{u\varepsilon}(y)]}{\int_y^{+\infty} \Gamma_{u\varepsilon}(x) f_{u\varepsilon}(x) dx} > \frac{\delta}{\delta-1}. \quad (42)$$

■

Since $\Gamma_{u\varepsilon}(x) \leq 1$, it follows that $\int_y^{+\infty} \Gamma_{u\varepsilon}(x) f_{u\varepsilon}(x) dx < [1 - F_{u\varepsilon}(y)]$. The ratio $\mu^R = \frac{[1 - F_{u\varepsilon}(y)]}{\int_y^{+\infty} \Gamma_{u\varepsilon}(x) f_{u\varepsilon}(x) dx} > 1$ represents the contribution of risk to the mark up (which is not related to market concentration). If returns are certain ($\Gamma_{u\varepsilon}(x) = 1$), it follows that $\mu^R = 1$ and only monopoly power would cause inefficiency. Therefore, expression (42) shows that uncertainty in the return on retail lending makes the wedge in the banking sector larger than it otherwise would be.

Furthermore, one can show that the spread declines with competition in retail banking:

Proposition 5 *The credit spread, $sp := \frac{R_t^C}{R_t^h}$, declines with competition in the retail banking sector, $\frac{\partial(sp)}{\partial\delta} < 0$.*

Proof. See the appendix. ■

3.4 Competition and risk taking

The above model can be used to look at the interaction between risk, default and the intensity of competition. Competition in the investment banking sector is reflected in η and risk is measured as the probability of default, $F(e^D)$. A key property of the model is that the default rate increases with competition. Specifically, it appears that for any particular realisation of common shocks, $u\varepsilon$, the conditional default threshold e^D and the conditional probability of default $F^e(e^D)$ increase in η . In the absence of common shocks it is possible to prove these statements analytically. However, the presence of common shocks complicates things somewhat. Briefly, consider the following function

$$g_1(\Lambda, \eta) := \int_{\Lambda}^{+\infty} \left[s - \Lambda \frac{\eta}{\eta - 1} \right] f^s(s) ds = 0.$$

Thus, $(d\Lambda/d\eta) = -(\partial g_1/\partial\eta) / (\partial g_1/\partial\Lambda)$. From the second order conditions to problem (22)-(23), it follows that $\frac{\partial g_1(\Lambda, \eta)}{\partial\Lambda} < 0$. However, absent additional restrictions, it is less straightforward to show that $\frac{\partial g_1(\Lambda, \eta)}{\partial\eta} > 0$ even for the case of the lognormal distribution.⁹

⁹In all of the numerical simulations reported later, and others besides, we found that default and the degree of competition in the investment banking sector are indeed positively related.

Moreover, although we have not established it formally, simulation results show that the retail banking default threshold y depends positively on competition in investment banking, η . That implies a lower degree of stability in retail banking but also lower spread when investment banking is more competitive.

The intuition as to why increased competition might raise the riskiness of the investment banking sector seems straightforward: The lower mark-up shrinks the cushion of excess profits that absorbs the impact of low e draws. That basic result seems, in spirit, consistent with arguments already in the literature that competition in financial markets may promote risk taking (see for example, Hellmann et al. (2000), Bolt and Tieman (2004), Repullo (2004) and Allen and Gale (2004)). The real issue, of course, is whether increased competition is welfare enhancing. For example, some microeconomic models suggest that competition not only increases risk but may also improve entrepreneurs' access to credit (Bolt and Tieman, 2004) and reduce the loan rate (Boyd and De Nicoló, 2005, Damjanovic, 2013). Thus higher risk may be positively correlated with higher investment, and therefore may promote production and welfare. That positive relation is also documented in some recent empirical work (see Claessence and Laeven, 2005). This is exactly what is observed in our model since employment, and therefore the output of the financial sector, is positively related to the expected default threshold.

As labour demand positively depends on Λ , increased competition will result in higher production and lower prices for financial services, which in turn will result in higher consumption and welfare. However, it is worth recalling that deposit protection/bank bailouts are assumed. That implies, following Kareken and Wallace (1978), that risk taking may be excessive in the decentralised equilibrium. However, be that as it may, there is also a sense that risk-taking may be too low from an optimal policy perspective. That is because the monopolistic (and separated) structure of the banking sector is (are) also distorting outcomes relative to the first-best. Encouraging banks to be more risky—loosely speaking, light-touch regulation—may actually be welfare-enhancing, *ceteris paribus*. We analyse this issue fully in Sections 5 and 6.

3.5 Limited liability and the interest spread

A natural question is: What difference does the bailout of retail banks make to the interest rate spread? One consequence of the bailout is that it limits bank liability, so the bank maximizes expected profit conditional that profit is positive. It does not care about states in which losses are made, for in that case the government will bail out the bank and this is equivalent to a zero profit outcome for the bank. Now we consider the case where the

government somehow convinces banks to maximize unconditional expected profit and the bank acts as though it faces unlimited liability. In that case one may calculate the interest spread as follows (where the superscript "UL" stands for "unlimited liability"). The profit maximization problem (39) is now:

$$\max_{R_t^c(i)} E\Psi_{t+1}^{UL} = \left[\int_0^{+\infty} \left(\frac{R_t^c(i)}{R_t^h} \Gamma_{u\varepsilon}(x) - 1 \right) f_{u\varepsilon}(x) dx \right] \left(\frac{R_t^c(i)}{R_t^c} \right)^{-\delta} B_t^C R_t^h \quad (43)$$

and the associated first order condition in a symmetric equilibrium is

$$\int_0^{+\infty} \left(\frac{R_t^C}{R_t^h} \Gamma_{u\varepsilon}(x) - \frac{\delta}{\delta - 1} \right) f_{u\varepsilon}(x) dx = 0.$$

Further, define the constant

$$\Gamma = \int_0^{+\infty} \Gamma_{u\varepsilon}(x) f_{u\varepsilon}(x) dx.$$

It follows immediately that the spread in the absence of bailouts is

$$\text{sp}^{UL} = \frac{\delta}{\delta - 1} / \Gamma. \quad (44)$$

It seems intuitive that the spread ought to be smaller when losses are bounded. That is because, absent the zero lower bound on profits, the bank will have to set interest rates in order to maintain profitability over a larger range of shocks. That is accomplished via a higher spread. Indeed, that intuition is correct and it is the case that

$$\text{sp}^{UL} = \frac{\delta}{\delta - 1} / \Gamma > 1 / \Gamma_{u\varepsilon}(y) = \text{sp}$$

or

Proposition 6 *When banks expect no bound on losses, the spread is larger than when their losses are limited to zero.*

Proof. *See the appendix.* ■

3.6 Government intervention

It is now possible to characterize in more detail the government's behavior. In this stylized model government's only function is to raise funds for deposit insurance. The size of the government bail out is denoted G . It is assumed that government intervention is costly. Such costs, denoted here by $g(G_t)$, are generally associated with monitoring costs and distortive taxation. For tractability, we assume this g function is linear in G . It is then the case that

$$Y_t = C_t + gG_t, \quad g \geq 0. \quad (45)$$

Now we will compute the size of government intervention. First note that the actual (or average) return, R_t^{ca} per unit borrowed by investment banks, will depend on the realisation of the macro environment,

$$R_t^{ca}(u\varepsilon) = R^C \Gamma_{u\varepsilon}(u\varepsilon). \quad (46)$$

Government intervention will occur if $R_t^{ca}(u\varepsilon) < R^h$, that is, when $u\varepsilon < y$. The size of the government bailout, G , depends on a number of considerations as follows. To begin with, if the banking firm is separated (indicated by the S superscript as before), then for one dollar of deposits, agents will receive *from the bank* the interest rate R_t^h times $\Gamma_{u\varepsilon}^{SN}(u_{t+1}\varepsilon_{t+1})$, where

$$\Gamma_{u\varepsilon}^{SN}(u_{t+1}\varepsilon_{t+1}) = \min\left(\frac{\Gamma_{u\varepsilon}(u\varepsilon)}{\Gamma_{u\varepsilon}(y)}, 1\right). \quad (47)$$

Function $\Gamma_{u\varepsilon}^{SN}(u_{t+1}\varepsilon_{t+1})$ represents the proportion of contracted deposits which depositors will obtain from retail banks in the case where deposits are not insured by the government. Superscript “ SN ” indicates “*Separated banking with No insurance*”.

It also follows that for any dollar of bank liabilities, the required bailout or deposit insurance is $1 - \Gamma_{u\varepsilon}^{SN}(u\varepsilon) \geq 0$. However, the economy-wide budget constraint (45) imposes a natural restriction on the size of G_t . For example, in the event that total national output is required to repay households under deposit insurance, the return on deposits is the only source of funds for consumption, and equation (45) becomes $Y_t = (1 + g)G_t$. And whilst it is a rare event in our model, it may even happen that total insurance exceeds fiscal capacity. Therefore the size of government intervention is restricted by the size of GDP , $G_t \leq Y_t/(1 + g)$. In sum then, we assume that government guarantees the following compensation to the public

$$G_{t+1}^{SI} = N_t W_t R_t^h \min\left(\frac{Y_{t+1}}{N_t W_t R_t^h} \frac{1}{1 + g}; (1 - \Gamma_{u\varepsilon}^{SN}(u_{t+1}\varepsilon_{t+1}))\right), \quad (48)$$

where the first term after the min operator reflects the fact that the required bailout might exceed fiscal capacity. In that case, the government is unable to fulfil its desire to see returned R_t^h per dollar deposited. Using the expressions for aggregate output, financial services, wages and interest spread, that is (6), (17), (28) and (41), it follows that

$$G_{t+1}^{SI} = N_t W_t R_t^h \min\left(\frac{\theta}{\theta - 1} \frac{u_{t+1}\varepsilon_{t+1}}{\Gamma_{u\varepsilon}(y)\Lambda(\Delta)^{1/\eta-1}} \frac{1}{1 + g}; (1 - \Gamma_{u\varepsilon}^{SN}(u_{t+1}\varepsilon_{t+1}))\right). \quad (49)$$

Next, define the ratio of government insurance to deposits as

$$\Gamma_{u\varepsilon}^{gS}(u_{t+1}\varepsilon_{t+1}) = \min\left(\frac{\theta}{\theta - 1} \frac{u_{t+1}\varepsilon_{t+1}}{\Gamma_{u\varepsilon}(y)\Lambda(\Delta)^{1/\eta-1}} \frac{1}{1 + g}; (1 - \Gamma_{u\varepsilon}^{SN}(u_{t+1}\varepsilon_{t+1}))\right).$$

Finally, the return to depositors equals the sum received from the banks and from the government: $\Gamma_{u\varepsilon}^{SI} = (\Gamma_{u\varepsilon}^{gS}(u_{t+1}\varepsilon_{t+1}) + \Gamma_{u\varepsilon}^{SN}(u_{t+1}\varepsilon_{t+1}))^{10}$ so that the consumption Euler equation takes the following form:

$$E_t \left\{ \frac{\beta C_t}{C_{t+1}} R_t^h \Gamma_{u\varepsilon}^{SI} \right\} = 1.$$

¹⁰That is: $\Gamma_{u\varepsilon}^{gS}(u_{t+1}\varepsilon_{t+1}) + \Gamma_{u\varepsilon}^{SN}(u_{t+1}\varepsilon_{t+1}) = \min\left(\frac{\theta}{\theta - 1} \frac{u_{t+1}\varepsilon_{t+1}}{\Lambda(\Delta)^{1/\eta-1}} \frac{1}{1 + g} + \Gamma_{u\varepsilon}^{SN}(u_{t+1}\varepsilon_{t+1}); 1\right).$

It is useful to note that the likelihood and size of government support to the banking system declines in the credit spread. To see this more explicitly, consider the case where the government faces no fiscal capacity constraints. In that case one may write that $G_{t+1}^{SI} = W_t N_t R_t^h \max \left[\left(1 - \frac{R_t^c}{R_t^h} \Gamma_{u\varepsilon}(u\varepsilon) \right), 0 \right]$ and the negative effect of the spread is readily apparent.

4 Financial structure and economic outcome

Clearly, the precise form of the equilibrium relations of the model change depending on the nature of the shocks, whether or not there is universal banking and deposit insurance. We will contrast four variants of the model with different financial structures indexed by superscript $J = \{UI, UN, SI, SN\}$, where S stands for separated, U for universal, I indicates that the government provides deposit insurance and N means that no deposit insurance is provided. The variants of the model will differ along three key dimensions: (i) the return on deposits, which is reflected in the Euler equation; (ii) the size of government intervention and (iii) the credit spread.

Table 1 below summarizes the differences where, as before, $\Gamma_{u\varepsilon}^J$ is the proportion of the deposits liabilities paid by the banks, and $\Gamma_{u\varepsilon}^{gJ}$ is the proportion which is paid for by the government.

Table 1: Difference across the models

$$\text{sp}^J = \begin{cases} 1; & \text{for Universal banking, } J = UI, UN \\ 1/\Gamma_{u\varepsilon}(y); & \text{for Separated banking, } J = SI, SN \end{cases};$$

$$\Gamma_{u\varepsilon}^J(u\varepsilon) = \min(\Gamma_{u\varepsilon}(u\varepsilon) \text{sp}^J, 1);$$

$$\Gamma_{u\varepsilon}^{gJ}(u\varepsilon) = \begin{cases} \min\left(\frac{\theta}{\theta-1} \frac{u\varepsilon}{\Lambda(\Delta)^{1/\eta-1}} \frac{\text{sp}^J}{1+g}; (1 - \Gamma_{u\varepsilon}^J(u\varepsilon))\right) & \text{with insurance; } J = UI, SI \\ 0, & \text{without insurance; } J = UN, SN \end{cases}$$

The generic equations of the model are set out below as part of the definition of the decentralised equilibrium:

Definition 1 *A decentralised equilibrium with separated banking and bailouts is a set of plans, $\{C_{t+k}, Y_{t+k}, N_{t+k}, W_{t+k}, X_{t+k}, Q_{t+k}, R_{t+k}^c, R_{t+k}^h\}_{k=0}^\infty$, given initial conditions, $\{A_{t-1}, N_{t-1}, R_{t-1}^h, R_{t-1}^c, W_{t-1}\}$, and exogenous shocks, $\{u_{t+k}, \varepsilon_{t+k}, \Delta_{t+k}\}_{k=0}^\infty$, and satisfying*

conditions (50)-(59).

$$\frac{1}{\beta C_t} = E_t \left\{ [\Gamma_{u\varepsilon}^{gJ}(u_{t+1}\varepsilon_{t+1}) + \Gamma_{u\varepsilon}^J(u_{t+1}\varepsilon_{t+1})] R_t^h \frac{1}{C_{t+1}} \right\}; \quad (50)$$

$$G_{t+1} = N_t W_t R_t^h \Gamma_{u\varepsilon}^{gJ}(u_{t+1}\varepsilon_{t+1}); \quad (51)$$

$$\frac{R_t^C}{R_t^h} = sp^J; \quad (52)$$

$$C_t = \frac{W_t}{\lambda}; \quad (53)$$

$$W_t = \frac{\theta-1}{\theta} \Omega_t A_t^\rho \frac{\Lambda}{R^C} (\Delta)^{1/\eta}; \quad (54)$$

$$A_{t+1} = A_t^\rho u_{t+1}; \quad (55)$$

$$Q_t = A_t \frac{(\theta-1)}{\theta}; \quad (56)$$

$$Y_t = A_t X_t; \quad (57)$$

$$X_{t+1} = \Omega \Delta \varepsilon_{t+1} N_t; \quad (58)$$

$$C_t = Y_t - g(G_t). \quad (59)$$

As one can see, equations (53-59) are identical across the variants of the model, whereas differences are apparent in the Euler equation (50), the size of government intervention (51) and the credit spread (52).

The above bloc of equations can be used to derive tractable expressions for equilibrium consumption, labour and the deposit rate, as described presently, and these will be used in subsequent sections of the paper to analyze welfare under varying assumptions; with and without bailouts and under separated and universal banking. First, begin by rewriting some of the above equations as follows,

$$C_t = \frac{1}{\lambda} \frac{\theta-1}{\theta} \Omega_t A_t^\rho \frac{\Lambda}{R^C} (\Delta)^{1/\eta}; \quad (60)$$

$$Y_{t+1} = A_t^\rho u_{t+1} \Omega_t \Delta \varepsilon_{t+1} N_t; \quad (61)$$

$$G_{t+1}^{SI} = N_t W_t R_t^h \Gamma_{u\varepsilon}^{gJ}(u_{t+1}\varepsilon_{t+1}) = N_t \frac{\theta-1}{\theta} \Omega A_t^\rho \Lambda (\Delta)^{1/\eta} \frac{\Gamma_{u\varepsilon}^{gJ}(u_{t+1}\varepsilon_{t+1})}{sp^J}; \quad (62)$$

$$C_{t+1} = Y_{t+1} - gG_{t+1}; \quad (63)$$

$$R_t^C = sp^J R_t^h. \quad (64)$$

Combining (60) with (64) results in (65) and substitution of (61) and (62) into (63) gives (66)

$$R_t^h C_t = \frac{1}{\lambda} \frac{\theta-1}{\theta} \Omega_t A_t^\rho \Lambda (\Delta)^{1/\eta} \frac{1}{sp^J}; \quad (65)$$

$$C_{t+1} = A_t^\rho N_t \Omega_t \Delta \left[u_{t+1} \varepsilon_{t+1} - \tilde{g} \frac{\Gamma_{u\varepsilon}^{gJ}(u_{t+1}\varepsilon_{t+1})}{sp^J} \right]. \quad (66)$$

Where $\tilde{g} := g \frac{\theta-1}{\theta} (\Delta)^{1/\eta-1} \Lambda$. Combine the previous two equations with the consumption Euler equation (50) to obtain

$$N^J = \frac{\beta}{\lambda} \frac{\theta-1}{\theta} \Lambda (\Delta)^{1/\eta-1} \Upsilon^J, \quad (67)$$

where

$$\Upsilon^J = \int_0^{+\infty} \frac{[\Gamma_{u\varepsilon}^J(u_{t+1}\varepsilon_{t+1}) + \Gamma_{u\varepsilon}^J(u_{t+1}\varepsilon_{t+1})] / \text{sp}^J}{u_{t+1}\varepsilon_{t+1} - \tilde{g}\Gamma_{u\varepsilon}^J(u_{t+1}\varepsilon_{t+1}) / \text{sp}^J} dF_{u\varepsilon}. \quad (68)$$

These equations are of central importance in understanding employment, output and interest rates across the variants of the model.

4.1 Employment, output and interest rates

Clearly, N is proportional to Υ which differs across variants of the model. Table 2 reports these expressions which are straightforward to derive.

Table 2: Financial structure and Employment		
	Universal (U)	Separated (S)
I	$\Upsilon^{UI} = \int_0^{+\infty} \frac{\Gamma_{u\varepsilon}^G(u\varepsilon) + \Gamma_{u\varepsilon}(u\varepsilon) dF_{u\varepsilon}}{u\varepsilon - \tilde{g}\Gamma_{u\varepsilon}^G(u\varepsilon)}$	$\Upsilon^{SI} = \int_0^{+\infty} \frac{\Gamma_{u\varepsilon}(y) [\Gamma_{u\varepsilon}^{GS}(u\varepsilon) + \Gamma_{u\varepsilon}^S(u\varepsilon)] dF_{u\varepsilon}}{u\varepsilon - \tilde{g}\Gamma_{u\varepsilon}^{GS}(u\varepsilon) \Gamma_{u\varepsilon}(y)}$
N	$\Upsilon^{UN} = \int_0^{+\infty} \frac{\Gamma_{u\varepsilon}(u\varepsilon) dF_{u\varepsilon}}{u\varepsilon}$	$\Upsilon^{SN} = \int_0^{+\infty} \frac{\Gamma_{u\varepsilon}(y) \Gamma_{u\varepsilon}^S(u\varepsilon) dF_{u\varepsilon}}{u\varepsilon}$

The relationships in Table 2 will prove useful in establishing a number of properties concerning the link between equilibrium employment and the financial structure of the economy. First we look at employment and output and then briefly at interest rates. Note that production in the economy will have the same relation to Υ as does N as it positively depends on labour input (61). Moreover, given these relations, one can immediately establish the impact of deposit insurance on output and employment.

4.1.1 The effect of deposit insurance

There is a positive effect of deposit insurance on labour and output which we establish in the following proposition:

Proposition 7 *Equilibrium employment and output are larger in an economy with government deposit insurance.*

Proof. One can easily show that $\Upsilon^{UI} > \Upsilon^{UN}$. Note that $\Gamma_{u\varepsilon}^G(u\varepsilon) + \Gamma_{u\varepsilon}(u\varepsilon) > \Gamma_{u\varepsilon}(u\varepsilon)$ and $\frac{1}{u\varepsilon - \tilde{g}\Gamma_{u\varepsilon}^G(u\varepsilon)} > \frac{1}{u\varepsilon}$.

Similarly, $\Upsilon^{SI} > \Upsilon^{SN}$, as $[\Gamma_{u\varepsilon}^{GS}(u\varepsilon) + \Gamma_{u\varepsilon}^S(u\varepsilon)] > \Gamma_{u\varepsilon}^S(u\varepsilon)$ and $\frac{1}{u\varepsilon - \tilde{g}\Gamma_{u\varepsilon}^{GS}(u\varepsilon) \Gamma_{u\varepsilon}(y)} > \frac{1}{u\varepsilon}$. ■

Proposition 7 states that deposit insurance increases employment. It is important to note that deposit insurance impacts the equilibrium outturn of the economy in a number of ways.

In particular, there are two channels which we now briefly describe and which we analyse further in subsequent sections on welfare and efficiency of equilibrium. First, deposit insurance $\Gamma_{u\varepsilon}^{GS}(u\varepsilon)$ works to increase the supply of labour and deposits. That is because it stabilizes the return to savings; by implication the return to working is more certain. Second, from (66), g is seen to have a direct negative effect on C_{t+1} . Smaller expected future consumption causes an increase in savings and therefore supply of deposits. Moreover, given the cost of government intervention one has to work more to maintain a given level of consumption. These effects can be seen, respectively, in the numerator and denominator of (68).

To derive these effects explicitly consider the Euler equation

$$1 = E_t \frac{u'(C_{t+1})}{u'(C_t)} \beta R_t^h \Gamma,$$

where the labour supply is $u'_c(C_t) = v'_n/W$, and v'_n is the marginal disutility from labour. In our model utility is separable in consumption and labour, and v'_n is constant so that

$$1 = \frac{\beta W R_t^h}{v'_n} E_t \Gamma u'_c(C_{t+1}).$$

If we know that C_{t+1} depends on N_t , and some parameter g , such that $C_{t+1} = C(N, g)$, where $C_N > 0, C_g < 0$ ¹¹ then we may write

$$v'_n(N) = \beta W R_t^h E_t \Gamma u'(C(N, g)).$$

It follows by the implicit function theorem that

$$\frac{dN}{dg} = - \frac{\beta W R_t^h E_t \Gamma u''(C(N, g)) C_g}{\beta W R_t^h E_t \Gamma u''(C(N, g)) C_N - v''(N)} > 0 \quad (69)$$

assuming that $v''(N) > 0$, $u''(C) < 0$, and that

$$\frac{dN}{d\Gamma} = - \frac{\beta W R_t^h E_t u'(C(N, g))}{\beta W R_t^h E_t \Gamma u''(C(N, g)) C_N - v''(N)} > 0. \quad (70)$$

It follows that labour supply increases in the certainty of the deposit return, Γ , and apparently, the inefficiency of government, g . The latter effect is due to a precautionary motive similar to that discussed in Kimball (1990).

4.1.2 Universal banking increases aggregate output

It is also interesting to observe that employment, and therefore output, is greater in the model with universal banking:

¹¹Those assumptions may be valid in our model, see formula (66). It would be valid if government were able to provide complete deposit insurance.

Proposition 8 *Universal banking results in higher employment in equilibrium than a separated banking system.*

Proof. One can easily show that $\Upsilon^{UN} > \Upsilon^{SN}$ since, by definition, $\Gamma_{u\varepsilon}^S(u\varepsilon) = \min\left(\frac{\Gamma_{u\varepsilon}(u\varepsilon)}{\Gamma_{u\varepsilon}(y)}, 1\right)$ and therefore $\Gamma_{u\varepsilon}(y)\Gamma_{u\varepsilon}^S(u\varepsilon) = \min(\Gamma_{u\varepsilon}(u\varepsilon), \Gamma_{u\varepsilon}(y)) \leq \Gamma_{u\varepsilon}(u\varepsilon)$. Similarly, $\Upsilon^{UI} > \Upsilon^{SI}$, which we show in the appendix. ■

It is not very surprising that vertical integration eliminates a credit spread and promotes production¹². Proposition 8 shows exactly this. However, it can only be welfare improving if production is below the optimal level because of some other distortions. Moreover, in section 5.4 we will see that when deposit insurance is provided, the cost of bailout is larger with a universal banking structure. Therefore it is possible that when the economy is hit by an adverse shock, consumption is smaller under universal banking due to the high cost of government intervention.

4.2 Interest rates

To compute R_t^C we use (65), (66) lagged one period and the expression for N^{SI} to yield

$$R_t^{CJ} = \frac{1}{\beta} \frac{\Omega_t}{\Omega_{t-1}} \frac{A_t^\rho}{A_{t-1}^\rho} \frac{1}{[u_t \varepsilon_t - \bar{g} \Gamma_{u\varepsilon}^{GS}(u_t \varepsilon_t) / \text{sp}^J]} \frac{1}{\Upsilon^J}. \quad (71)$$

When government is efficient, $g = 0$, the interest rate goes in the opposite direction to employment; bailouts reduce the deposit interest rate and separated banking results in lower interest rates.

For the case $g > 0$ things are less clear-cut and the impact of g may be non-monotonic. On the one hand, a high probability of large government intervention may increase the deposit rate as it is distortive. On the other hand, bailouts are a source of insurance and will tend to boost employment and consumption and depress the deposit rate. It appears that the second effect may often dominate. Appendix 8.9 solves out for the deposit rate under the model variants.

We also establish in the appendix the following propositions concerning the deposit rate:

Proposition 9 *Without bailouts the deposit rate is lower in a separated banking system. And*

Proposition 10 *With bailouts and with efficient government, $g = 0$, the deposit rate is lower in a separated banking system.*

¹²The other costs and benefits of vertical integration we will consider in section 5.4

4.3 Welfare analysis

In the previous section it was proved that in terms of production an economy with deposit insurance is larger than one without deposit insurance. However there are two main reasons why larger output does not necessarily imply higher welfare. First, the economy may have over-employment when the marginal benefit of consumption is smaller than marginal disutility from labour supply. Second, part of aggregate output is not consumed as it is lost in the process of funding government intervention (i.e., the excess burden of intervention).

One may compare utility across economies with universal banks and no deposit protection and with such insurance.

Welfare is given by

$$W^J = \log(C^J) - \lambda N^J.$$

Combining this with the present time version of (66) results in

$$W^J = \log N^J - \lambda N^J + \log \left[u_t \varepsilon_t - g \frac{\theta - 1}{\theta} \Lambda(\Delta)^{1/\eta - 1} \frac{\Gamma_{u\varepsilon}^g(u_t \varepsilon_t)}{\text{sp}^J} \right] + \log(A_t^\rho \Omega_t \Delta) \quad (72)$$

where the first term $\log N^J - \lambda N^J$ is associated with efficiency of the labour supply, and the second term is associated with shocks and inefficiency of government intervention. It is easy to see that welfare is a concave function with respect to labour and it achieves its maximum when $N^J = 1/\lambda$. From section 4.1 we know that the largest labour supply is in an economy with universal banking and deposit insurance, $J = UI$. So, if for that economy the labour supply is still below its efficient level, $N^{UI} < 1/\lambda$, and if inefficiency is not too high, then the welfare ranking is the same as the ranking of employment levels across models. The key finding is that:

Proposition 11 *If $N^{UI} < 1/\lambda$ then for any realisation of the shock*

i) universal banking welfare dominates separated banking without insurance, $W^{UN} > W^{SN}$.

Furthermore, if government is efficient, $g = 0$, then

ii) universal banking welfare dominates separated banking with insurance, $W^{UI} > W^{SI}$;

iii) insurance improves welfare for any banking structure, $W^{UI} > W^{UN}$ and $W^{SI} > W^{SN}$.

Proof. Note that when $g = 0$, welfare (72) is

$$W^J = \log N^J - \lambda N^J + \log u_t \varepsilon_t + \log(A_t^\rho \Omega_t \Delta);$$

and the term related to common shocks are the same across models, which implies that welfare is determined solely by labour efficiency. If the model with the largest labour supply is such that $N^{UI} < 1/\lambda$, then an increase in labour supply is welfare improving. The rest of the proof follows immediately from Propositions 7 and 8. ■

5 Efficiency, welfare and financial structure

In this section we pursue a more general analysis of the costs and benefits of separated versus vertically integrated financial systems. Then we look at the costs and benefits of deposit insurance. In general, overall economic efficiency and welfare depend on the efficiency of resource utilization. In the present model, labour is the main resource. In the following subsection we will analyse how particular financial structures affect the demand and supply of labour. We start with the social planner problem and compute the efficient allocation. Then we add in the wedges of inefficiency associated with decentralised decision making amongst the economic players in the model. Finally, we analyse how different financial structures either increase or decrease those wedges.

5.1 Social planner problem

A general version of the social planner's problem relevant to the present set-up is as follows:

$$U(N) = \max_N E_0 \sum_{t=0}^{\infty} \beta^t (u(C_t) - v(N_t)) \quad (73)$$

with the following constraints on the choice of labour

$$\begin{aligned} Y_{t+1} &= F(N_t, u_{t+1}) \\ C_{t+1} &= Y_{t+1}. \end{aligned}$$

The optimal choice is given by

$$\beta E_t u_c(F(N_t, u_{t+1})) F_N(N_t, u_{t+1}) = v'(N_t).$$

The left hand side of this expression reflects the marginal benefit of an extra unit of labour, and the right hand side reflects the cost. This is taken as the benchmark against which the decentralized outcomes under different scenarios for banking and bailouts are compared.

In our model the problem is $\max E_0 \sum_{t=0}^{\infty} \beta^t (\log(C_t) - \lambda N_t)$, subject to $C_t = A_t \Omega \Delta \varepsilon_t N_{t-1}$. The labour supply optimizing this objective is implied by $\beta E_t u_c(F(N_t, u_{t+1})) F_N(N_t, u_{t+1}) = v'(N_t)$ and is easily seen to be $N^* = \beta/\lambda$.

5.2 Decentralised structure of the economy

It will be useful to set out fully and carefully all the distortions attendant with the decentralised equilibrium. Recall that the model economy operates as follows: First households supply labour and deposits. Their behaviour determines wages and the return

on deposits via labour supply and the Euler equation. Retail banks take deposits and produce loans. Therefore the return on deposits, R^h , can be considered as the marginal cost facing retail banks whilst the return on credit, R^C , is the marginal benefit (profit). As for investment banks, their marginal cost is given by $R_t^C W_t$, and $Q_{t+1} \Omega \Delta \varepsilon_{t+1}$ is their marginal profit. The marginal costs for the final good producers is Q_{t+1} whilst the marginal benefit is A_{t+1} . It is therefore straightforward to compute the wedges between costs and benefits for retail banking, $\mu^C = R_t^C / R_t^h$; investment banking, $\mu^I = E_t Q_{t+1} \frac{dX_{t+1}}{dN_t} / R_t^C W_t$; and the production sector, $\mu^F = \frac{dY_{t+1}}{dX_{t+1}} / Q_{t+1}$. The product of these wedges is defined as¹³

$$\mu^C \times \mu^I \times \mu^F = \frac{E_t dY_{t+1} / dN_t}{R_t^h W_t}. \quad (74)$$

We will refer to that product as the "production wedge" and denote it $\mu^m := \mu^C \times \mu^I \times \mu^F$.

We formally define the "household wedge" as the residual after "production wedge" in the hypothetical case when government intervention is costless: $Y_{t+1} = C_{t+1}$. In that case the following equality should hold

$$\beta E_t u_c(F(N_t, u_{t+1})) F_N(N_t, u_{t+1}) = \mu^H \times (\mu^C \times \mu^I \times \mu^F) \times v'(N_t),$$

which can be written as

$$W_t R_t^h \beta E_t u'(C_{t+1}) \frac{d}{dN_t} C_{t+1} = \mu^H v'(N_t) E_t \frac{d}{dN_t} C_{t+1}. \quad (75)$$

The household wedge arises because households do not take into consideration the correlation between the return on savings and their labour supply decision. Recall that investment banks borrow to fund their labour demand so that in general the marginal product of labour and the marginal benefit from additional savings will be correlated. The implications of that correlation for the household wedge will be worked out below.

Finally, government intervention imposes an additional wedge of inefficiency as part of total output is lost, for example to costly administration of deposit insurance. Hence, $C_t = C(Y_t; G_t)$ where consumption increases with output but declines in government intervention. Formally, we define the government wedge μ^g as

$$\beta E_t u_c(F(N_t, u_{t+1})) F_N(N_t, u_{t+1}) = \mu^g \times \mu^H \times (\mu^C \times \mu^I \times \mu^F) \times v'(N_t). \quad (76)$$

Combining this equation with (74) gives

$$\beta E_t u_c(F(N_t, u_{t+1})) F_N(N_t, u_{t+1}) = \mu^g \frac{E_t dY_{t+1} / dN_t}{R_t^h W_t} \mu^H v'(N_t).$$

¹³In general one should include a "correlation markup", μ^{IF} , such that $\mu^{IF} = \frac{E_t Q_{t+1} \frac{dX_{t+1}}{dN_t} \frac{dY_{t+1}}{dX_{t+1}} / Q_{t+1}}{\mu^I E_t \mu^F}$. In our model μ^F is constant and uncorrelated with $Q_{t+1} \frac{dX_{t+1}}{dN_t}$, and so $\mu^{IF} = 1$.

Now use (75) to obtain

$$E_t u_c(F(N_t, u_{t+1})) F_N(N_t, u_{t+1}) = \mu^g E_t F(N_t, u_{t+1}) \frac{E_t u'(C_{t+1}) \frac{d}{dN_t} C_{t+1}}{E_t \frac{d}{dN_t} C_{t+1}}. \quad (77)$$

To summarize then: In the model economy there are potentially five wedges of inefficiency relative to the efficient allocation. We summarise these wedges in the following table.

Table 3. Economic wedges		
Notation	Agent	Definition
μ^C	Commercial bank	$R_t^c = \mu^C R_t^h$;
μ^I	Investment bank	$E_t Q_{t+1} \frac{dX_{t+1}}{dN_t} = \mu^I R_t^c W_t$;
μ^F	Final producer	$\frac{dY_{t+1}}{dX_{t+1}} = \mu^F Q_{t+1}$;
μ^H	Households	$W_t R_t^h \beta E_t u'(C_{t+1}) \frac{d}{dN_t} C_{t+1} = \mu^H v'(N_t) E_t \frac{d}{dN_t} C_{t+1}$;
μ^g	Government	$\mu^g = \frac{E_t [u'(C_{t+1}) F_N(N_t, u_{t+1})] E_t \frac{d}{dN_t} C_{t+1}}{E_t (F_N(N_t, u_{t+1})) E_t [u'(C_{t+1}) \frac{d}{dN_t} C_{t+1}]}$

Upon multiplying all the wedges, one obtains a measure of the total distortion

$$\mu = \mu^H \times \mu^C \times \mu^I \times \mu^F \times \mu^g,$$

so that the decentralised equilibrium in the economy is determined by

$$\beta E_t u_c(F(N_t, u_{t+1})) F_N(N_t, u_{t+1}) = \mu v'(N_t).$$

Clearly, the decentralised allocation coincides with the efficient (planner's) solution if and only if $\mu = 1$. First we look briefly at the monopolistic wedges and then we turn to the more interesting issue of how the financial structure affects each of the wedges.

5.3 Production wedges

The production process in the model may go through 3 vertical stages: up to 2 stages of financial intermediation and a final production stage. Therefore this ‘triple’ marginalization may result in a large wedge between the social marginal benefit and marginal disutility of labour

$$\beta E_t u_c(F(N_t, u_{t+1})) F_N(N_t, u_{t+1}) = \mu^m v'(N_t),$$

where μ^m is the triple monopolistic mark up,

$$\mu^m = \mu^C \times \mu^I \times \mu^F$$

As a result, the labour market is distorted and production is lower than its socially optimal level. In the example which we consider above

$$N_t = \frac{\beta}{\mu^m \lambda} < N^*. \quad (78)$$

The final good sector markup, $\mu^F = \frac{\theta}{\theta-1}$, does not depend on financial structure, so we will concentrate attention on the markups imposed by the financial intermediary sectors. Recall that μ^C and μ^I denote the markups imposed by retail and investment banks respectively. Then the total production markup will be $\mu^m = \mu^C \mu^I \frac{\theta}{\theta-1}$. And of course, scaling this composite term are the wedges associated with households, who provide deposits and labour, and government, which provides costly deposit insurance. So, the wedge of inefficiency associated with investment banking is

$$\mu^I = E_t Q_{t+1} \frac{dX_{t+1}}{dN_t} \frac{1}{R_t^C W_t},$$

which may be written as

$$\mu^I = \frac{\theta-1}{\theta} A_t^\rho \Omega \Delta \frac{1}{R_t^C W_t} = (\Delta)^{1-1/\eta} / \Lambda.$$

One concludes that the larger is the planned default threshold Λ , the smaller is the monopolistic wedge associated with investment banking. And from Proposition 1 it follows that when the banking industry operates with limited liability, the investment banks' mark up is lower compared to when they are required to absorb their losses. In the latter case $\mu^{I0} = (\Delta)^{1-1/\eta} / \Lambda_0 = \frac{\eta}{\eta-1}$. Therefore, when investment banks have unlimited liability, the associated wedge equals the simple monopolistic markup. As $\Lambda > \Lambda_0$, it follows that $\mu^I < \mu^{I0}$. The conclusion follows therefore that limitation of banks' liabilities reduces the investment banking mark up and *may* promote efficiency.

5.4 Cost and benefit of vertical integration

The desirability of universal banks versus separate investment and retail banking entities is the subject of much current debate. The above framework allows one to begin to examine some key underlying themes. To that end, consider the model economy where one retail bank is integrated with one investment bank. There is an obvious cost associated with this course of action since retail banks are less diversified¹⁴. Households on the other hand continue to diversify their investment portfolio by depositing in each universal bank. On the other hand, a benefit ought to flow from cheaper funding for investment banks, due to the eradication of the interest wedge between retail and investment banks, $\mu^C = R_t^C / R_t^h$. Assume that the integrated bank can now take deposits directly. As before, intermediation requires labour input. However, now the integrated bank cost of borrowing is equal to the deposit rate, R_t^h , and $\mu^C = 1$.

¹⁴The benefits from risk diversification were clearly shown in Greenwood and Jovanovic (1990). In our model the absence of risk diversification in the banking sector is partly offset by bailouts.

As shown in Proposition 4, due to uncertainty about return the spread is larger than the monopoly distortion among retail banks, $\delta/(\delta - 1)$. It was also established in Proposition 6 that banks' unlimited liability structure increases the spread and therefore the mark up. Thus, bailouts, by making banks less liable for their losses, can move the economy towards a more efficient allocation. And if government is perfectly efficient, so that there is no excess cost to operating the bailout scheme, $\mu^G = 1$, then elimination of the retail banks' margin is welfare improving. On the other hand, when government intervention is costly, μ^G may be higher under universal banking compared with separated banking. Moreover, even if labour supply is closer to its efficient level under universal banking, a sufficiently unfavorable shock may reduce drastically current consumption due to costly government intervention. In section 6 some examples are provided to illustrate these and other scenarios.

5.5 Household wedge

From definition (75), it is easy to see that the household wedge will be the only distortion in the absence of production and government wedges, and when the marginal cost of loans, $W_t R_t^h$, is equal to expected marginal revenue,

$$W_t R_t^h = E_t F'(N_t). \quad (79)$$

In the event that (79) obtains, then (75) is equivalent to

$$\beta E_t u_c(F(N_t, u_{t+1})) F_N(N_t, u_{t+1}) = \mu^H v'(N_t)$$

and μ^H is the only distortion associated with the market equilibrium. In fact, it is possible to compute the exact formula for the household wedge by combining (75) with the labour supply equation, $W_t u'(C_t) = v'(N_t)$ and the Euler equation, $\beta R_t^h E_t (\Gamma^J u'(C_{t+1})) = u'(C_t)$, yielding

$$\beta W_t R_t^h E_t (\Gamma^J u'(C_{t+1})) = v'(N_t).$$

It follows then that the household wedge may be written as

$$\mu^H = \frac{E_t [u'(C_{t+1}) F'(N_t)]}{E_t F'(N_t) E_t (\Gamma^J u'(C_{t+1}))}.$$

To understand what this expression implies for efficiency, it is convenient to decompose it into two 'sub-wedges', $\mu^H = \mu^{HN} \times \mu^{HD}$ where we define

$$\mu^{HN} := \frac{E_t [u'(C_{t+1}) F'(N_t)]}{E_t F'(N_t) E_t [u'(C_{t+1})]} \quad (80)$$

and

$$\mu^{HD} := \frac{E_t (u'(C_{t+1}))}{E_t (\Gamma^J u'(C_{t+1}))}. \quad (81)$$

5.5.1 Certain return on deposits

If deposits are completely safe ($\Gamma^J \equiv 1$), then $\mu^{HD} \equiv 1$ and $\mu^H \equiv \mu^{HN}$. It follows that μ^H is solely due to households not taking into account the correlation between their labour supply decisions and interest rates. It is straightforward to show that

$$\begin{aligned}\mu^{HN} &= 1 + \frac{\text{cov}[u'(C_{t+1})F'(N_t)]}{E_t F'(N_t) E_t [u'(C_{t+1})]} \\ &= 1 + \text{corr}[u'(C_{t+1})F'(N_t)] \text{cvar}[u'(C_{t+1})] \text{cvar}[F'(N_t)].\end{aligned}$$

Here $\text{cov}[\cdot]$ denotes covariance, $\text{corr}[\cdot]$ is the correlation coefficient and $\text{cvar}[\cdot]$ is the coefficient of variation. It is clear that $\mu^{HN} = 1$ when the marginal product of labour and the marginal utility of consumption are uncorrelated. However in the model both the marginal product of labour and future consumption increase with a common productivity shock. As $u'(\cdot)$ is a decreasing function, $\text{cov}[u'(C_{t+1})F'(N_t)] < 0$, and therefore $\mu^{HN} < 1$. Therefore, that opens up the possibility that the μ^{HN} sub-wedge may ameliorate the distortions to production caused by either monopolistic power or the uncertainty due to risk in the banking sector. We may pursue this point a little further. If one assumes that $F'(N_t) = a \frac{F(N_t)}{N_t}$ and that there is no excess burden associated with bailouts, ($g = 0$), then

$$F(N_t) = C_{t+1}, \tag{82}$$

and formula (80) can be written as $\mu^{HN} = \frac{E_t[u'(C_{t+1})C_{t+1}]}{E_t C_{t+1} E_t [u'(C_{t+1})]}$. In an appendix it is shown that

$$\mu^{HN} = 1 - s_u \frac{\text{var}(C_{t+1})}{(E_t C_{t+1})^2} \left(\frac{1}{1 + \sigma_3 s_u \frac{\text{var}(C_{t+1})}{(E_t C_{t+1})^2}} \right) + O3,$$

where s_u is the coefficient of relative risk aversion $s_u := -\frac{u''(E_t C_{t+1}) E_t C_{t+1}}{u'(E_t C_{t+1})}$, $\frac{\text{var}(C_{t+1})}{(E_t C_{t+1})^2}$ is the coefficient of variation ($\text{var}(\cdot)$ denotes variance) and is a good measure of uncertainty, and σ_3 is a half of the coefficient of relative prudence, $\sigma_3 := -\frac{1}{2} \frac{u'''(E_t C_{t+1}) E_t C_{t+1}}{u''(E_t C_{t+1})}$. Therefore, sub-wedge μ^{HN} declines in prudence, uncertainty and risk aversion and can offset the other positive wedges.

The positive relation between prudence and future consumption has been noted by Kimball (1990) and discussed with respect to the labour market in Flodén (2006). Our results are complementary in that a higher coefficient of prudence is shown to boost labour supply even when income is realised with a lag.

5.5.2 Uncertain return on deposits

The possibility of default by investment banks makes the return on deposit balances uncertain. In that case, $\mu^{HD} \neq 1$. This sub-wedge is related to the effect that uncertain

deposit rates have on current labour supply, as the proposition at the end of this section tries to make clear. Denote the actual return on balances by $R_t^h \Gamma(u_{t+1} \varepsilon_{t+1})$, where $\Gamma(u_{t+1} \varepsilon_{t+1})$ is an increasing function and $\Gamma(u_{t+1} \varepsilon_{t+1}) \leq 1$. Note that the denominator of μ^{HD} may be written as

$$E_t(\Gamma(u_{t+1} \varepsilon_{t+1}) u'(C_{t+1})) = E_t(\Gamma(u_{t+1} \varepsilon_{t+1}) E_t u'(C_{t+1})) + \text{cov}(\Gamma(u_{t+1} \varepsilon_{t+1}) u'(C_{t+1})).$$

As C_{t+1} and $\Gamma(u_{t+1} \varepsilon_{t+1})$ are positively related to the common shock $u_{t+1} \varepsilon_{t+1}$, they are necessarily positively correlated. On the other hand, the marginal utility of consumption, $u'(C_{t+1})$, is a decreasing function and therefore it is negatively correlated with $\Gamma(u_{t+1} \varepsilon_{t+1})$. Hence, $\text{cov}(\Gamma(u_{t+1} \varepsilon_{t+1}) u'(C_{t+1})) < 0$ and

$$1/\mu^{HD} = \frac{E_t(\Gamma^J u'(C_{t+1}))}{E_t(u'(C_{t+1}))} = E_t \Gamma(u_{t+1} \varepsilon_{t+1}) + \frac{\text{cov}(\Gamma(u_{t+1} \varepsilon_{t+1}) u'(C_{t+1}))}{E_t(u'(C_{t+1}))}. \quad (83)$$

One concludes that

$$\mu^{HD} = \frac{1}{E_t(\Gamma(u_{t+1} \varepsilon_{t+1})) [1 - \rho]} > 1$$

where $\rho = -\frac{\text{cov}(\Gamma(u_{t+1} \varepsilon_{t+1}) u'(C_{t+1}))}{E_t \Gamma(u_{t+1} \varepsilon_{t+1}) E_t u'(C_{t+1})} > 0$, and $\rho < 1$ because $\Gamma(u_{t+1} \varepsilon_{t+1})$ and $u'(C_{t+1})$ are non-negative numbers.¹⁵

Proposition 12 *Consider two deposit insurance schemes such that*

$$E_t \Gamma_1(u_{t+1} \varepsilon_{t+1}) u'(C_{t+1}) > E_t \Gamma_2(u_{t+1} \varepsilon_{t+1}) u'(C_{t+1}). \quad (84)$$

It follows that $\mu^{HD}(1) < \mu^{HD}(2)$.

Proof. This is an immediate consequence of (83). ■

One can show that when government is efficient, $g = 0$, deposit insurance satisfies property (84) and results in a lower household wedge. Indeed, property (84) is equivalent to $E_t[(\Gamma^1 - \Gamma^2) u'(C_{t+1})] > 0$. Let Γ^2 represent an economy without deposit insurance and Γ^1 an economy with insurance. Then $(\Gamma^1 - \Gamma^2)$ is the share of deposits paid by government. It is nonnegative and so $E_t[(\Gamma^1 - \Gamma^2) u'(C_{t+1})] > 0$. Therefore, one may conclude that deposit insurance works to induce a larger labour supply than otherwise would be the case and may improve welfare by offsetting the negative impact of monopolistic distortions. The effect is even greater when g is positive as government insurance reduces expected consumption and therefore increases marginal utility.

¹⁵That is, let X and Y be random positive variables, then $EX > 0$, $EY > 0$, and $EXY > 0$ so that

$$-\frac{\text{cov}(X, Y)}{EXEY} = 1 - \frac{EXY}{EXEY} < 1.$$

5.6 When deposit insurance is costly

Finally, we wish to examine the wedge associated with costly government intervention, μ^g , which is defined in (77) and can be presented as

$$\frac{E_t \left[u_c(Y_{t+1}) \frac{d}{dN_t} Y_{t+1} \right]}{\frac{d}{dN_t} Y_{t+1}} = \mu^g \frac{E_t u'(C_{t+1}) \frac{d}{dN_t} C_{t+1}}{E_t \frac{d}{dN_t} C_{t+1}}. \quad (85)$$

When there is an excess burden associated with the deposit insurance scheme, aggregate consumption is lower than aggregate output and equation (82) becomes

$$Y_{t+1} = F(N_t) = C_{t+1} + g_{t+1}.$$

where in our model $g_{t+1} = gG_{t+1}$. That in turn implies that

$$\frac{d}{dN_t} Y_{t+1} = F_N(N_t, u_{t+1}) = \frac{d}{dN_t} C_{t+1} + \frac{d}{dN_t} g_{t+1},$$

which we can use to compute the government wedge defined in (77) and prove the following proposition:

Proposition 13 *If the cost of government intervention, $\frac{dg_{t+1}}{dN_t}$, negatively correlates with consumption, then $\mu^g > 1$*

Proof. See appendix. ■

The implication of Proposition 13 is that the beneficial effects of deposit insurance/bailouts are reduced when government action is costly.

6 Relative welfare of Universal banking

The model that we have developed is capable of ranking welfare associated with universal and separated banking with or without deposit insurance/bank bailouts, in the face of different types of shocks. For instance, it has been demonstrated that labour supply is larger under universal banking. That is because the double marginalization problem is avoided, (boosting the demand for labour), and bailouts cause precautionary motives (increasing labour supply). Moreover, if $N^{UI} < 1/\lambda$ universal banking is also socially desirable. However, that conclusion is only one of a number possibilities. For instance, it may be that labour supply is too high under universal banking, $N^{UI} > 1/\lambda$, and that separated banking with no bailouts takes the economy closer to the optimum, on average. Or, it could be that separated banking with bailouts is closer to the efficient outcome compared with universal banking without bailouts.

Another perspective on comparing welfare across banking structures is to enquire how likely are policymakers to regret adopting universal versus separated banking. This is a potentially interesting question as policy in this area is likely to be held fixed for extended periods. That was case, for example, with the split between retail and investment banking in the US for much of the twentieth century. So it is interesting to ask what the model implies about *ex post* welfare under differing market structures for different realisations of the shocks. For example, it may be that a decision to permit or prohibit universal banking is welfare decreasing *ex post*, but welfare increasing on average. Specifically, if banks are broken up (universal banks are prohibited) at the start of the period and a good positive common shock is realized, then in principle one might have preferred in this state of nature to have had a universal banking structure. So, it seems that the average welfare comparison of different banking structures and the likelihood of regretting having a particular banking structure are both of interest. The welfare assessment of these various possibilities turns on certain key factors in the present model such as how volatile are common and idiosyncratic shocks; how competitive are retail and investment banking sectors; how generous is the deposit insurance scheme and how distortive is government intervention. We turn now to some examples briefly to illustrate these possibilities.

6.1 Comparative statics and relative benefit of separated banking

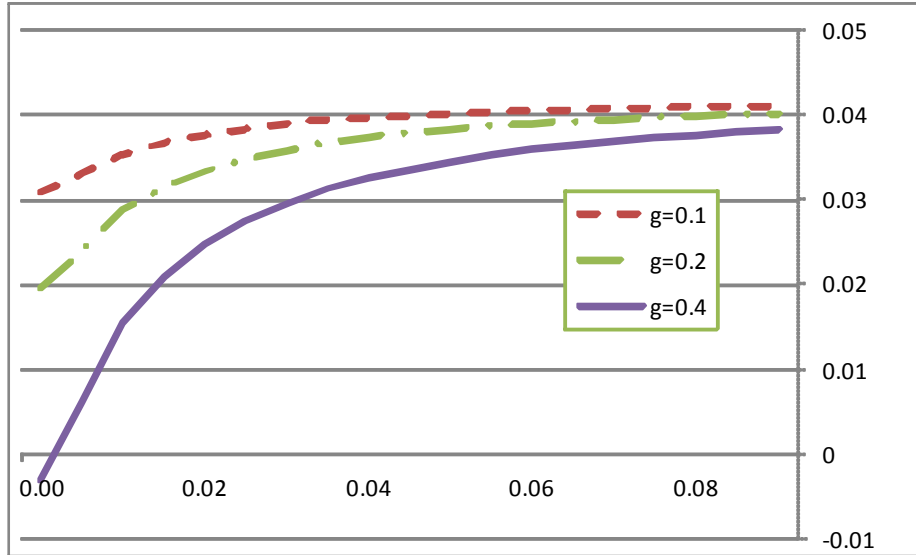
The model is as yet too simple to be the basis for any serious policy advice but may be useful in indicating how certain key attributes of the economic environment interact and sway judgement on banking structure. We take the following parameterization as our starting point: $\sigma_e = 0.1, \sigma_u = 0.1, \sigma_\varepsilon = 0.1, \eta = 4, \theta = 8, \delta = 10$. Under this parameterization the expected default rate of investment banks is 5%; $F_s(\Lambda) = 5\%$. That corresponds to an expected default rate for the retail banks of 0.4%, implying that retail banks rarely fail. In fact, it implies a government bailout once in every 250 years. In all the simulations full bailouts were possible and did not breach the fiscal limit. In the charts below, the x -axis displays the quantiles of the CDF of the shocks. So 0.04 means $\Pr(X < x) = 0.04$; that corresponds to a bad shock, drawn from the bottom 4% of the distribution. In the same way, $\Pr(X < x) = 0.5$ corresponds to a median value of the shock, and so on. The y -axis above the zero line indicates that welfare under universal banking is higher than under separated banking, and values beneath the zero line indicate the opposite.

Government efficiency and welfare: We investigated the desirability of universal banking for various degrees of inefficiency in terms of government action. The data are summarized in the following table and chart.

Table 4. Government efficiency

	$g = 0.1$	$g = 0.2$	$g = 0.4$
N^{UI}	0.6533	0.6535	0.6541
N^{SI}	0.5861	0.5861	0.5862
$E(W^{UI} - W^{SI})$	4.16%	4.14%	4.11%
$\Pr(W^{UI} < W^{SI})$	0	0	0.2%
ER^{hU}	1.39%	1.38%	1.36%
ER^{hS}	9%	17.7%	40%
ER^{CS}	21%	31%	56%

Figure 1: Welfare gain in benchmark economy



The conclusion from the simulation is that universal banking dominates separated banking. It is clear from the chart that for separated banking to be preferable, the composite shock has to be very bad indeed and government intervention highly distortive; in that case $E(W^{UI} - W^{SI}) < 0$ indicating that agents would require (a very small amount of) compensation on average to be indifferent between universal and separated banking. Indeed, the likelihood of universal banking being dominated, even under the most adverse of circumstances, is vanishingly small, $\Pr(W^{UI} - W^{SI} < 0) = 0.2\%$.

Idiosyncratic volatility: Intuition might suggest that separated banking is more attractive when investment banks are subject to higher idiosyncratic risk. In the appendix we provide a small amount of evidence that perhaps it is reasonable to increase σ_e above 0.1. We turn to that case next.

Table 5. Idiosyncratic volatility

$g = 0.2$	$\sigma_e = 0.1$	$\sigma_e = 0.2$	$\sigma_e = 0.3$
N^{UI}	0.65	0.68	0.73
N^{SI}	0.59	0.60	0.62
$E(W^{UI} - W^{SI})$	4.1%	4.2%	4.3%
$\Pr(W^{UI} < W^{SI})$	0	0	0
$F_s(\Lambda)$	5%	14%	30%
ER^{hU}	1.39%	1.35%	1.33%
ER^{hS}	18%	18.1%	18.3%
ER^{CS}	31%	32%	37%

In the above table, it is apparent that a more volatile economy results in higher employment along with a higher probability of default. These results are not surprising given the analysis earlier in the paper. In general in the model, volatility increases labour supply and output and that tends to push down on the interest rate. On the other hand, government intervention may be quite costly and that tends to reduce consumption. Overall the effect on interest rates is ambiguous. In this case, interest rates rise. Due to the higher level of employment when shocks are relatively benign, universal banking is preferable, but when shocks are relatively bad, universal banking is less attractive, as judged by the probability of default. Nevertheless, the probability that a universal banking sector would be suboptimal looking forward is zero, $\Pr(W^{UI} - W^{SI} < 0) = 0$. Alternatively, the average cost ex post to agents in a universal banking structure is still somewhat low, $E(W^{UI} - W^{SI})$.

Volatility of systemic risk: We now perform a similar experiment to the previous one only this time we drive up the volatility of the *systemic* shock, ε . Now the probability of a government bail out, $F_{u\varepsilon}(y)$, increases dramatically. As a result universal banking performs relatively less well. It is interesting to note that in equilibrium the deposit and lending rates actually decline in the volatility of the systemic shock.

Table 6. Systemic risk

$g = 0.2$	$\sigma_\varepsilon = 0.05$	$\sigma_\varepsilon = 0.1$	$\sigma_\varepsilon = 0.15$
N^{UI}	0.64	0.65	0.68
N^{SI}	0.58	0.59	0.60
$E(W^{UI} - W^{SI})$	4.2%	4.1%	4.0%
$\Pr(W^{UI} < W^{SI})$	0	0	0
$F_s(\Lambda)$	2.1%	5%	12%
ER^{hU}	2.2%	1.4%	0.07%
ER^{hS}	16.3%	17.4%	18%
ER^{CS}	31.2%	30.7%	30%

Competition in investment banking sector: We now consider the case of relatively competitive investment banking. This is an interesting case to consider because it implies that the investment banking sector is more likely to be a source of volatility for the rest of the economy, other things constant, as the cushion of profits for absorbing potential losses is lower. On the other hand, increased competition reduces the double marginalization problem, making separated banking more attractive, other things constant. Increases in the parameter η are used to simulate the effects of increased competition. The table below shows that increased competition tends to reduce the benefits of universal banking. However, it is interesting to note that it does not make a serious dent in the attraction of universal banking. The probability that an economy that adopted universal banking might be disappointed with its choice next period, $\Pr(W^{UI} - W^{SI} < 0)$, remains negligible.

Table 7. Competition

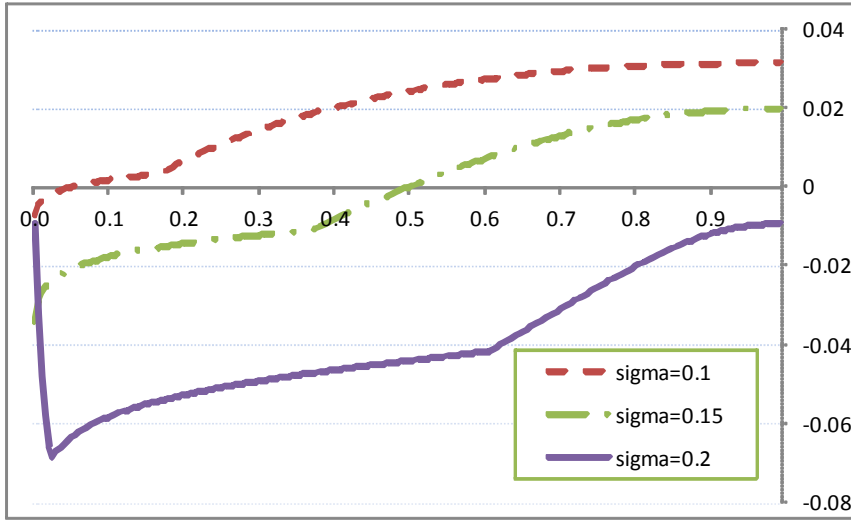
	$\eta = 4$	$\eta = 6$	$\eta = 8$
N^{UI}	0.65	0.77	0.87
N^{SI}	0.59	0.68	0.74
$E(W^{UI} - W^{SI})$	4.1%	3.1%	2.0%
$\Pr(W^{UI} < W^{SI})$	0	0	5.1%
$F_s(\Lambda)$	5%	27%	53%
$F_{u\varepsilon}(y)$	0.5%	5.7%	17%
ER^{hU}	1.4%	1.3%	1.2%
ER^{hS}	17.4%	19.8%	21%
ER^{CS}	30.7%	35.3%	40%

Competitive and volatile banking: The final case we consider is one where the competition in retail banking sector is higher than in the base case, $\delta = 10$, and investment banking is also competitive (as in the previous simulation, $\eta = 8$). It turns out that in this case we can have a situation where the economy is too large on average, $N^{UI} > 1$, and universal banking is welfare inferior. In general the welfare ranking depends on the volatility of the common shock; higher volatility increases the probability and the size of government intervention and reduces the relative advantage of the universal banking structure.

Table 8. High competition and government efficiency.

	$\sigma_\varepsilon = 0.1$	$\sigma_\varepsilon = 0.15$	$\sigma_\varepsilon = 0.2$
N^{UI}	0.87	0.97	1.15
N^{SI}	0.74	0.81	0.94
$E(W^{UI} - W^{SI})$	1.9%	-0.09%	-3.8%
$\Pr(W^{UI} < W^{SI})$	5%	59%	100%
$F_s(\Lambda)$	53%	69%	83%
$F_{u\varepsilon}(y)$	17%	37%	61%
ER^{hU}	1.1%	-0.3%	-2.2%
ER^{hS}	20%	18%	12%
ER^{CS}	39%	38%	33%

Figure 2. Welfare gain
Competitive and volatile investment banking



Now the welfare implications of sticking with universal banking appear unattractive as it is highly likely that $\Pr(W^{IN} - W^{SI}) < 0$. It would take very favourable shocks to the economy to result in universal banking dominating separated banking, ex post and, on average, it will not: $E(W^{UI} - W^{SI}) < 0$. And even if government intervention is relatively inefficient that conclusion is not overturned. It appears that the combination of bailouts, volatility and a reduction in the price markup in investment banking tilts the welfare comparison decisively in favour of separated banking.

7 Conclusion

According to the Vickers Report separating retail from investment banking is central to recapitalizing the banking sector, effecting orderly resolution of failed banks, aligning banks' risk-taking with the social good and insulating the economy better from financial shocks. No single model will capture all relevant factors for analysing the desirability of separating

retail from investment banking.¹⁶ However, the present model has set out a promising, tractable framework for clarifying aggregative dimensions to that issue.

In our model aligning banks' overall behaviour with the social good turns on a key trade-off; the eradication of a double marginalization problem (including a risk premium) in the financial sector, versus larger government bail-outs. That basic trade-off occurs whether shocks to the investment banks are idiosyncratic or common. However, bailouts per se may be welfare enhancing in the model as they ameliorate various underlying distortions. The degree of distortion of government action is important however. When such intervention was efficient, our model suggests that universal banking is preferable; double marginalization is more costly than government bail-outs. When government intervention is distortive then the interplay with shock variability may mean that separated banks are desirable.

An important qualification to that conclusion is that in practice, and unlike in our model, much of the growth in bank leverage is not due to rising retail deposits but interbank lending. That channel is not present in the model as it stands and so introducing it remains an important extension.

The Vickers Report seeks to encourage a more competitive banking industry and a more stable one. These two aims may sometimes be in conflict, as in our model. That is why in practice the report also recommends stricter risk asset ratio requirements and a limit on leverage. Our model is capable of being extended to include retained funds. That extension may permit a calculation of optimal capital requirements, and will enrich the assessment of which banking structure may be preferable.

There are a number of other extensions that seem important. We have assumed away institution-specific bankruptcy cost and so ignored the costs of restructuring in the event of default. Perhaps one may interpret g as reflecting these costs but it seems necessary to be much clearer on this issue before drawing policy conclusions. Second, whilst our assumption of comprehensive bank bailouts seems well motivated from a practical perspective, and could be welfare enhancing in the model, we have not analyzed optimal bailouts. Third, it is important to spell out more explicitly the different banking activities that are undertaken at different types of institutions. Being able to explain why certain structures evolve in the first place must be central to any policy that seeks to dismantle or support those structures. These are doubtless only a few of many extensions that are desirable.

¹⁶For a start, it is not even agreed what "separated" means. To give just one example: The ring-fence proposed in the Vickers report may be watered down by the Conservative-led coalition government in the UK on the basis that the risky activities have still been "separated" from the legitimate retail activities. In short, there is a lack of agreement as to what constitutes the dividing line between different kinds of banks and products.

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8 Appendix

8.1 Existence, uniqueness and the second order conditions for the investment banking problem

In the text the following equation was derived for a choice of Λ to characterize optimal behaviour in the investment banking sector,

$$\int_{\Lambda}^{+\infty} \left[\frac{\eta-1}{\eta} \frac{s}{\Lambda} - 1 \right] f^s(s) ds = 0. \quad (86)$$

First, it is established that no such solution need exist, and then that there may be many solutions. As an example for which no solution exists, consider the distribution

$$\begin{aligned} F(s) &= 1 - s^{-a}; \quad s > 1; \\ F(s) &= 0; \quad s < 1, \end{aligned}$$

where $a > 1$. Then the pdf is

$$\begin{aligned} f(s) &= as^{-a-1}; \quad s > 1; \\ f(s) &= 0; \quad s < 1. \end{aligned}$$

It follows then that (86) becomes

$$\int_{\Lambda}^{+\infty} \left[\frac{\eta-1}{\eta} \frac{as^{-a}}{\Lambda} - as^{-a-1} \right] ds \quad (87)$$

$$= \Lambda^{-a} \left(\frac{\eta-1}{\eta} \frac{a}{a-1} - 1 \right). \quad (88)$$

That expression is always positive if $a > \eta$, and a solution to (86) does not exist. On the other hand, if $a = \eta$, any Λ is a solution to (86).

8.2 Existence

To establish general conditions for existence and uniqueness of a solution, it is clear that one needs additional structure on the distribution function.

Definition 2 We call the number A the supremum of the domain of the pdf f , if $\forall x, x < A$. It follows that $F(x) < 1$, and $\lim_{x \rightarrow A} F(x) = 1$. For the lognormal distribution $A = +\infty$.

Definition 3 For any cdf $F(x)$ with positive domain, we define the "inverse log hazard function"

$$h_{il}(x) = \frac{(1 - F(x))}{xf(x)}. \quad (89)$$

We call it this since $h_{il}(x) = 1/h_l(x)$ where $h_l(x) = \frac{xf(x)}{(1-F(x))}$ is the hazard function for $y = \ln(x)$. Indeed, $F(X) = \Pr(x < X) = \Pr(y < \ln X) = F_y(\ln X)$. One may also compute the relation between the pdf x and pdf y .

$$f(X) = \frac{f_y(\ln X)}{X}. \quad (90)$$

It follows therefore that

$$h_l(x) = \frac{xf(x)}{(1-F(x))} = \frac{f_y(\ln X)}{1-F_y(\ln X)}.$$

To prove existence we will need the following assumption concerning the distribution:

Assumption A1:

$$\lim_{x \rightarrow A} h_{il}(x) = \frac{(1 - F(x))}{xf(x)} = 0.$$

Proposition 14 There exists a solution to (86) if the inverse log hazard rate converges to zero at the supremum of the domain,

$$\int_{\Lambda}^A \left[\frac{s}{\Lambda} - \frac{\eta}{\eta-1} \right] f(s) ds = 0. \quad (91)$$

Proof. Consider the function

$$g_{30}(x) := \frac{\int_x^A \left[s - x \frac{\eta}{\eta-1} \right] f(s) ds}{x(1-F(x))} = \frac{\int_x^A s f(s) ds}{(1-F(x))x} - \frac{\eta}{\eta-1}.$$

It is easy to see that $\lim_{x \rightarrow 0} g_{30}(x) = \lim_{x \rightarrow 0} \frac{Es}{x} = +\infty > 0$. That implies that there exists a $c > 0$, such that for any $x \leq c$, $g_{30}(x) > 0$. We now wish to prove that

$$\lim_{x \rightarrow A} g_{30}(x) < 0.$$

Hence, we compute

$$\lim_{x \rightarrow A} g_{31}(x) = \frac{\int_x^A s f(s) ds}{(1-F(x))x}.$$

Since both the numerator and the denominator converge to 0 and are differentiable, one may establish if L'Hôpital's rule can be applied. Thus,

$$\lim_{x \rightarrow A} g_{31}(x) = \frac{\int_x^A s f(s) ds}{(1-F(x))x} = \frac{xf(x)}{xf(x) - (1-F(x))} = \frac{1}{1 - \frac{(1-F(x))}{xf(x)}}.$$

And if $\lim_{x \rightarrow A} \frac{(1-F(x))}{xf(x)} = 0$ the limit exists and $\lim_{x \rightarrow A} g_{31}(x) = 1$. Thus

$$\lim_{x \rightarrow A} g_{30}(x) = \lim_{x \rightarrow A} g_{31}(x) - \frac{\eta}{\eta-1} = -\frac{1}{\eta-1}. \quad (92)$$

By the definition of limit, one concludes that there exist an x^* such that for any $x \in [x^*, A)$, $g_{31}(x) < -\frac{0.5}{\eta-1}$. ■

Corollary 15 *If Assumption A1 is true, and x is the largest solution to $g_{30}(x) = 0$ then $\forall x_1 > x$ we have that $g_{30}(x_1) \leq 0$.*

Proof. We will give a proof by contradiction. Assume that there is a solution, x , such that $g_{30}(x) = 0$ and there exists $x_1 > x$, such that $g_{30}(x_1) > 0$. However, since Assumption A1 holds, formula (92) obtains, and there is a solution x_2 such that $g_{30}(x_2) = 0$ and $x_2 > x_1 > x$. Therefore x is not the largest solution, and we have a contradiction. ■

From corollary 15 one also concludes that if x is the largest solution, $g'_{30}(x) \leq 0$ and function $g_{30}(x)$ cannot change sign from negative to positive at x .

8.2.1 Uniqueness and the second order conditions

Now, we may formulate a sufficient condition for uniqueness of the solution to $g_{30}(x) = 0$.

Assumption A2: The inverse log hazard rate, $\frac{(1-F(x_2))}{x_2 f(x_2)}$, is a strictly decreasing function.

Corollary 16 *If distribution F satisfies Assumptions A1 and A2, then function $g_{30}(x) = 0$ changes sign only once from positive to negative.*

Proof. We will give a proof by contradiction. Assume that there is a solution, x , such that $g_{30}(x) = 0$; and that $g_{30}(x) = 0$ changes sign only once from negative to positive. Then 1) $g'_{30}(x) \geq 0$ and therefore

$$g'_{30}(x) = \frac{-x f(x) [(1-F(x))x] - [(1-F(x)) - x f(x)] \int_x^A s f(s) ds}{[(1-F(x))x]^2} \geq 0. \quad (93)$$

As x is a solution, we can rewrite (93)

$$g'_{30}(x) = \frac{x f(x)}{(1-F(x))x} \left(\frac{1}{\eta-1} - \frac{(1-F(x))}{x f(x)} \right) \geq 0.$$

From Corollary 15 we know that there exist $x_2 > x$, such that $g_{30}(x_2) = 0$, and $g'_{30}(x_2) \leq 0$. That implies that

$$\frac{1}{\eta-1} - \frac{(1-F(x_2))}{x_2 f(x_2)} \leq 0 \leq \frac{1}{\eta-1} - \frac{(1-F(x))}{x f(x)};$$

or that

$$\frac{(1-F(x))}{x f(x)} \leq \frac{(1-F(x_2))}{x_2 f(x_2)},$$

which contradicts Assumption A2. ■

Function $g_{30}(x)$ is continuous and Corollary 16 implies that it changes sign only once from positive to negative. Assumption A2 implies that there can be only one solution with $g'_{30}(x) = 0$ therefore we can claim that there are no more than 2 solutions but only one of them corresponds to the situation when $g_{30}(x)$ changes sign from positive to negative. That solution will also satisfy the second order conditions of the initial problem.

Proposition 17 *If the distribution satisfies Assumptions A1 and A2, there is a unique solution x to $g_{30}(x) = 0$ at which function $g_{30}(x)$ changes sign from positive to negative. Only at this solution are both the first and the second order conditions satisfied.*

8.2.2 Lognormal distribution

It remains now to verify that the lognormal distribution satisfies assumptions A1 and A2. A1 asserts that

$$\lim_{x \rightarrow \infty} \frac{(1-F_y(x))}{f_y(x)} = 0. \quad (94)$$

Applying L'Hôpital's rule, it follows that

$$\lim_{x \rightarrow \infty} \frac{(1 - F_y(x))}{f_y(x)} = \lim_{x \rightarrow \infty} -\frac{f_y(x)}{f'_y(x)} = \lim_{x \rightarrow \infty} \left[-\frac{d}{dx} \ln(f_y(x)) \right]^{-1}.$$

Hence, one needs to verify that for the normal distribution it is the case that

$$\lim_{x \rightarrow \infty} \left[-\frac{d}{dx} \ln(f_y(x)) \right]^{-1} = 0. \quad (95)$$

And so,

$$\begin{aligned} f_y(x) &= \frac{1}{\sigma\sqrt{2\pi}} e^{-(x-\mu)^2/2\sigma^2}; \\ -\ln f_y(x) &= (x-\mu)^2/2\sigma^2 + \ln \sigma\sqrt{2\pi}; \\ \left[-\frac{d}{dx} \ln(f_y(x)) \right]^{-1} &= \sigma^2/(x-\mu); \\ \lim_{x \rightarrow \infty} \left[-\frac{d}{dx} \ln(f_y(x)) \right]^{-1} &= 0. \end{aligned}$$

So condition (95) is true for the lognormal function and A1 is satisfied.

Moreover, Thomas (1971) shows that the normal distribution has an increasing hazard rate. Therefore its inverse hazard rate is a decreasing function and assumption A2 is satisfied. It follows that for the lognormal distribution the solution exists. Moreover, the solution is unique and the second order conditions are satisfied.

8.3 Proof of Proposition 2

Proposition 2 $\Gamma_{u\varepsilon}(u\varepsilon)$ is an increasing and concave function. Therefore more favourable systemic or macro shocks bring average returns closer to the contractual return. Moreover $\Gamma_{u\varepsilon}(0) = 0$ and $\lim_{x \rightarrow \infty} \Gamma_{u\varepsilon}(x) = 1$.

Proof.

$$\frac{d\Gamma_{u\varepsilon}(u\varepsilon)}{du\varepsilon} = \left(\frac{\Lambda}{u\varepsilon} \right)^{\frac{\eta}{\eta-1}} \int_0^{\frac{e^{1-1/\eta}}{\Lambda}} \frac{e^{1-1/\eta}}{\Lambda} f^e(e) de > 0. \quad (96)$$

$$\frac{d^2\Gamma_{u\varepsilon}(u\varepsilon)}{d(u\varepsilon)^2} = -\frac{\eta}{\eta-1} \frac{1}{(u\varepsilon)^2} f^e \left[\left(\frac{\Lambda}{u\varepsilon} \right)^{\frac{\eta}{\eta-1}} \right] \left(\frac{\Lambda}{u\varepsilon} \right)^{\frac{\eta}{\eta-1}} < 0 \quad (97)$$

We apply L'Hôpital's rule to prove the last statement in the proposition. ■

8.4 Proof of Proposition 5

Proposition 5 The credit spread, sp , declines with competition in the retail banking sector,

$$\frac{\partial(sp)}{\partial\delta} < 0.$$

Proof. Consider the following function

$$g_{15}(y, \delta) = \int_y^{+\infty} \left(\frac{\Gamma_{u\varepsilon}(x)}{\Gamma_{u\varepsilon}(y)} - \frac{\delta}{\delta-1} \right) f_{u\varepsilon}(x) dx = 0. \quad (98)$$

It is easy to see that $\frac{\partial g_{15}(y, \delta)}{\partial \delta} = \frac{1}{(\delta-1)^2} \int_y^{+\infty} f_{u\varepsilon}(x) dx > 0$. Moreover, note that (39) can be written as

$$\max_y g_{20}(y) = \left[\int_y^{+\infty} \left(\frac{\Gamma_{u\varepsilon}(x)}{\Gamma_{u\varepsilon}(y)} - 1 \right) f_{u\varepsilon}(x) dx \right] \left(\frac{1}{\Gamma_{u\varepsilon}(y)} \right)^{-\delta}. \quad (99)$$

Thus, the first order condition is

$$g'_{20}(y) = \frac{\Gamma'_{u\varepsilon}(y)}{\Gamma_{u\varepsilon}(y)} \left(\frac{1}{\Gamma_{u\varepsilon}(y)} \right)^{-\delta} (\delta-1) g_{15}(y, \delta) = 0. \quad (100)$$

The second order condition at the optimal point, where $g_{15}(y, \delta) = 0$, is

$$g''_{20}(y) = \frac{\Gamma'_{u\varepsilon}(y)}{\Gamma_{u\varepsilon}(y)} \left(\frac{1}{\Gamma_{u\varepsilon}(y)} \right)^{-\delta} \frac{\partial g_{15}(y, \delta)}{\partial y}. \quad (101)$$

Since $\Gamma'_{u\varepsilon}(y) > 0$, the second order condition for a maximum implies that $\frac{\partial g_{15}(y, \delta)}{\partial y} < 0$. Therefore, the implicit function theorem yields

$$\frac{dy}{d\delta} = -\frac{\partial g_{15}(y, \delta)}{\partial \delta} / \frac{\partial g_{15}(y, \delta)}{\partial y} > 0.$$

And so,

$$\begin{aligned} \text{sp} &= \frac{R_t^c}{R_t^h} = \frac{1}{\Gamma_{u\varepsilon}(y)}; \\ \frac{d\text{sp}}{d\delta} &= -\frac{\Gamma'_{u\varepsilon}(y)}{\Gamma_{u\varepsilon}^2(y)} \frac{dy}{d\delta} < 0. \end{aligned}$$

■

8.5 Proof of Proposition 6

Proposition 6. *When banks have no bound on losses, the spread is larger than when their losses are limited.*

Proof. It is necessary and sufficient to show that

$$\Gamma < \frac{\delta}{\delta-1} \Gamma_{u\varepsilon}(y).$$

Note that y is defined as

$$\int_y^{+\infty} \left(\Gamma_{u\varepsilon}(x) - \frac{\delta}{\delta-1} \Gamma_{u\varepsilon}(y) \right) f_{u\varepsilon}(x) dx = 0. \quad (102)$$

Therefore

$$\begin{aligned}
\Gamma - \frac{\delta}{\delta-1}\Gamma_{u\varepsilon}(y) &= \int_y^{+\infty} \left[\Gamma_{u\varepsilon}(x) - \frac{\delta}{\delta-1}\Gamma_{u\varepsilon}(y) \right] f_{u\varepsilon}(x) dx + \int_0^y \left[\Gamma_{u\varepsilon}(x) - \frac{\delta}{\delta-1}\Gamma_{u\varepsilon}(y) \right] f_{u\varepsilon}(x) dx \\
&= \int_0^y \left[\Gamma_{u\varepsilon}(x) - \frac{\delta}{\delta-1}\Gamma_{u\varepsilon}(y) \right] f_{u\varepsilon}(x) dx.
\end{aligned} \tag{103}$$

By direct differentiation, one can show that that $\Gamma_{u\varepsilon}(x)$ is an increasing function:

$$\Gamma'_{u\varepsilon}(x) = \left(\frac{\Delta}{x}\right)^{\frac{\eta}{\eta-1}} \int_0^{\frac{e^{1-1/\eta}}{\Lambda}} \frac{e^{1-1/\eta}}{\Lambda} f^e(e) de > 0. \tag{104}$$

So one concludes that if $x < y$ then $\Gamma_{u\varepsilon}(x) < \Gamma_{u\varepsilon}(y)$ and therefore (103) is negative. That confirms that $\text{sp}^{UL} > \text{sp}$. ■

8.6 Proof that $\Lambda > \Lambda_0$, Lemma 1

Recall that

$$\int_{\Lambda}^{+\infty} \left[\frac{s}{\Lambda} - \frac{\eta}{\eta-1} \right] f^s(s) ds = 0;$$

and that Λ_0 is defined as

$$\int_0^{+\infty} \left[\frac{s}{\Lambda_0} - \frac{\eta}{\eta-1} \right] f^s(s) ds = 0.$$

This may be rewritten as

$$\Lambda_0 = \frac{(\eta-1)}{\eta} \int_0^{+\infty} s f^s(s) ds = \frac{\eta-1}{\eta} \Delta^{\frac{\eta-1}{\eta}}.$$

One may compare these two quantities as follows

$$\begin{aligned}
\int_{\Lambda}^{+\infty} s f^s(s) ds &= \frac{\eta}{\eta-1} \Lambda (1 - F^s(\Lambda)) \\
\Lambda_0 &= \frac{(\eta-1)}{\eta} \int_{\Lambda}^{+\infty} s f^s(s) ds + \frac{\eta-1}{\eta} \int_0^{\Lambda} s f^s(s) ds
\end{aligned}$$

$$\Lambda_0 = \Lambda (1 - F^s(\Lambda)) + \frac{\eta-1}{\eta} \int_0^{\Lambda} s f^s(s) ds = \Lambda + \frac{\eta-1}{\eta} \int_0^{\Lambda} (s - \Lambda) f^s(s) ds + \left(\frac{\eta-1}{\eta} - 1\right) F^s(\Lambda) \Lambda$$

That proves that $\Lambda > \Lambda_0$.

8.6.1 Existence of commercial bank default rate

In this appendix we prove Proposition 3. First we show that there is a solution to (40)

$$g_{12}(z) := \frac{\int_{-\infty}^{+\infty} \Gamma_{u\varepsilon}(x) f_{u\varepsilon}(x) dx}{(1 - F_{u\varepsilon}(z))} - \frac{\delta}{\delta - 1} \Gamma_{u\varepsilon}(z) = 0, \quad (105)$$

It is useful to establish some basic properties of the function $g_{12}(z)$. We do this in:

Lemma 18

$$\lim_{z \rightarrow \infty} g_{12}(z) = -\frac{1}{\delta - 1};$$

Proof. To prove the Lemma we apply L'Hôpital's rule:

$$\lim_{z \rightarrow \infty} \frac{\int_{-\infty}^{+\infty} \Gamma_{u\varepsilon}(x) f_{u\varepsilon}(x) dx}{(1 - F_{u\varepsilon}(z))} = \lim_{z \rightarrow \infty} \frac{\Gamma_{u\varepsilon}(z) f_{u\varepsilon}(z)}{f_{u\varepsilon}(z)} = \lim_{z \rightarrow \infty} \Gamma_{u\varepsilon}(z) = 1$$

■

Proposition 19 There exists a solution to (40)

Proof. It is easy to see that $g_{12}(0) \geq 0$. However from Lemma (18), there exists a z^* such that $\forall z > z^*$, $g_{12}(z) < -\frac{0.5}{\delta - 1}$. Moreover, as $g_{12}(z)$ is a continuous function, there is a solution at which $g_{12}(y) = 0$ and where $g_{12}(y)$ changes sign from positive to negative. ■

Corollary 20 If y is the biggest solution to $g_{30}(y) = 0$ then $\forall x_1 > y$ $g_{12}(x_1) \leq 0$.

Proof. We will give a proof by contradiction. Assume that there is a solution, y , such that $g_{12}(y) = 0$ and there exists $x_1 > y$, such that $g_{12}(x_1) > 0$. However, since A1 holds, formula (92) obtains, and there is a solution x_2 such that $g_{12}(x_2) = 0$ and $x_2 > x_1 > y$. Therefore y is not the largest solution. ■

8.7 The second order condition of the commercial bank problem

The initial problem is

$$\begin{aligned} \max_{R_t^c(i), y} E\Psi_{t+1} &= \left[\int_y^{+\infty} \left(\frac{R_t^c(i)}{R_t^h} \Gamma_{u\varepsilon}(x) - 1 \right) f_{u\varepsilon}(x) dx \right] \left(\frac{R_t^c(i)}{R_t^c} \right)^{-\delta} B_t^C R_t^h \\ \text{s.t.} \quad &: \frac{R_t^c(i)}{R_t^h} \Gamma_{u\varepsilon}(y) = 1. \end{aligned} \quad (106)$$

To proceed, we substitute in the constraint so that

$$\max_y E\Psi_{t+1}(y) = \left[\int_y^{+\infty} \left(\frac{\Gamma_{u\varepsilon}(x)}{\Gamma_{u\varepsilon}(y)} - 1 \right) f_{u\varepsilon}(x) dx \right] (\Gamma_{u\varepsilon}(y))^\delta B_t^C R_t^h \left(\frac{R_t^h}{R_t^c} \right)^{-\delta}. \quad (107)$$

The first order condition is

$$\frac{d}{dy} E\Psi_{t+1}(y) = \left[\int_y^{+\infty} ((\delta - 1) \Gamma_{u\varepsilon}(x) - \delta \Gamma_{u\varepsilon}(y)) f_{u\varepsilon}(x) dx \right] (\Gamma'_{u\varepsilon}(y)) (\Gamma_{u\varepsilon}(y))^{\delta-2} B_t^C R_t^h \left(\frac{R_t^h}{R_t^c} \right)^{-\delta} = 0 \quad (108)$$

or

$$\frac{d}{dy} E\Psi_{t+1}(y) = g_{12}(y) ((\delta - 1) (\Gamma'_{u\varepsilon}(y)) (\Gamma_{u\varepsilon}(y))^{\delta-2} B_t^C R_t^h \left(\frac{R_t^h}{R_t^c} \right)^{-\delta} (1 - F_{u\varepsilon}(y)))$$

and has the same sign as $g_{12}(y)$. The second order conditions are satisfied if and only if $g_{12}(y)$ changes sign from positive to negative. However, we have proved that such a y always exists.

8.8 Proof of Proposition 8 about employment under insurance/bailouts

Here we consider insured separated banking versus insured universal banking. This is an interesting comparison, with some real-world resonance. It will turn out that labour supply is unambiguously higher under universal banking, that is regardless of g and $u\varepsilon$. As to whether universal banking is still *preferable* to separated banking, that will depend on whether $N^{UI} \leq N^*$. To show that $N^{UI} > N^{SI}$, one needs to prove that $\Upsilon^{UI} > \Upsilon^{SI}$, where

$$\begin{aligned} \Upsilon^{UI} &= \int_0^{+\infty} \frac{\Gamma_{u\varepsilon}^G(u\varepsilon) + \Gamma_{u\varepsilon}(u\varepsilon)}{\left[u\varepsilon - g \frac{\theta-1}{\theta} (\Delta)^{1/\eta-1} \Lambda \Gamma_{u\varepsilon}^G(u\varepsilon) \right]} dF_{u\varepsilon} \\ \Upsilon^{SI} &= \int_0^{+\infty} \frac{\Gamma_{u\varepsilon}(y) [\Gamma_{u\varepsilon}^{GS}(u\varepsilon) + \Gamma_{u\varepsilon}^S(u\varepsilon)]}{u\varepsilon - g \frac{\theta-1}{\theta} \Lambda (\Delta)^{1/\eta-1} \Gamma_{u\varepsilon}^{GS}(u\varepsilon) \Gamma_{u\varepsilon}(y)} dF_{u\varepsilon}. \end{aligned}$$

As noted, for any realisation of the common shock it is the case that $\Upsilon^{UI} > \Upsilon^{SI}$. To prove that, we first show $\Gamma_{u\varepsilon}^D(u\varepsilon) > \Gamma_{u\varepsilon}(y) [\Gamma_{u\varepsilon}^{GS}(u\varepsilon) + \Gamma_{u\varepsilon}^S(u\varepsilon)]$ in Lemma 21; this turns out to be a statement about the interest wedge in the economy with separated banking, that is the effective deposit rate across different banking structures. Then in lemma 22, which is a statement about the size of bailouts across different banking structures, we prove that $\Gamma_{u\varepsilon}^G(u\varepsilon) > \Gamma_{u\varepsilon}^{GS}(u\varepsilon) \Gamma_{u\varepsilon}(y)$. That establishes that for any common shock $u\varepsilon$ and for any value of the government inefficiency, g ,

$$\frac{\Gamma_{u\varepsilon}^G(u\varepsilon) + \Gamma_{u\varepsilon}(u\varepsilon)}{\left[u\varepsilon - g \frac{\theta-1}{\theta} (\Delta)^{1/\eta-1} \Lambda \Gamma_{u\varepsilon}^G(u\varepsilon) \right]} > \frac{\Gamma_{u\varepsilon}(y) [\Gamma_{u\varepsilon}^{GS}(u\varepsilon) + \Gamma_{u\varepsilon}^S(u\varepsilon)]}{u\varepsilon - g \frac{\theta-1}{\theta} \Lambda (\Delta)^{1/\eta-1} \Gamma_{u\varepsilon}^{GS}(u\varepsilon) \Gamma_{u\varepsilon}(y)}$$

and therefore that $\Upsilon^{UI} > \Upsilon^{SI}$.

Lemma 21 *For any realisation $u\varepsilon$,*

$$\Gamma_{u\varepsilon}^G(u\varepsilon) + \Gamma_{u\varepsilon}(u\varepsilon) > \Gamma_{u\varepsilon}(y) [\Gamma_{u\varepsilon}^{GS}(u\varepsilon) + \Gamma_{u\varepsilon}^S(u\varepsilon)] \quad (109)$$

Proof. First recall the definitions

$$\Gamma_{u\varepsilon}^G(u\varepsilon) = \min \left(\frac{\theta}{\theta-1} \frac{u\varepsilon}{\Lambda(\Delta)^{1/\eta-1}} \frac{1}{1+g}; (1 - \Gamma_{u\varepsilon}(u\varepsilon)) \right).$$

Therefore

$$\Gamma_{u\varepsilon}^G(u\varepsilon) + \Gamma_{u\varepsilon}(u\varepsilon) = \min \left(\frac{\theta}{\theta-1} \frac{u\varepsilon}{\Lambda(\Delta)^{1/\eta-1}} \frac{1}{1+g} + \Gamma_{u\varepsilon}(u\varepsilon); 1 \right). \quad (110)$$

The other definitions are

$$\Gamma_{u\varepsilon}^{GS}(u\varepsilon) + \Gamma_{u\varepsilon}^S(u\varepsilon) = \min \left(\frac{\theta}{\theta-1} \frac{u\varepsilon}{\Gamma_{u\varepsilon}(y)\Lambda(\Delta)^{1/\eta-1}} \frac{1}{1+g} + \Gamma_{u\varepsilon}^S(u\varepsilon); 1 \right) \quad (111)$$

where

$$\Gamma_{u\varepsilon}^S(u\varepsilon) = \min \left(\frac{\Gamma_{u\varepsilon}(u\varepsilon)}{\Gamma_{u\varepsilon}(y)}, 1 \right). \quad (112)$$

Multiply (111) by $\Gamma_{u\varepsilon}(y)$ so that

$$\begin{aligned} & \Gamma_{u\varepsilon}(y) [\Gamma_{u\varepsilon}^{GS}(u\varepsilon) + \Gamma_{u\varepsilon}^S(u\varepsilon)] \\ &= \min \left(\frac{\theta}{\theta-1} \frac{u\varepsilon}{\Lambda(\Delta)^{1/\eta-1}} \frac{1}{1+g} + \min((\Gamma_{u\varepsilon}(u\varepsilon), \Gamma_{u\varepsilon}(y)); \Gamma_{u\varepsilon}(y)) \right). \end{aligned} \quad (113)$$

Now we need to compare (113) with (110). To do that it is necessary to compare universal and separated banking under different possible scenarios. The first case is when government plus universal banks are able to ensure that agents receive deposits plus interest at the nominal or contracted rate. It does not assume anything about default, i.e., the constellation of shocks:

Case 1: $\frac{\theta}{\theta-1} \frac{u\varepsilon}{\Lambda(\Delta)^{1/\eta-1}} \frac{1}{1+g} + \Gamma_{u\varepsilon}(u\varepsilon) > 1$, Then $\Gamma_{u\varepsilon}^G(u\varepsilon) + \Gamma_{u\varepsilon}(u\varepsilon) = 1$, and since $\Gamma_{u\varepsilon}(y) [\Gamma_{u\varepsilon}^{GS}(u\varepsilon) + \Gamma_{u\varepsilon}^S(u\varepsilon)] < \Gamma_{u\varepsilon}(y) < 1$, inequality (109) holds.

The next case examines what happens if the government reaches fiscal capacity under universal banking and where default has definitely occurred under separated banking (since $u\varepsilon < y$ has occurred).

Case 2: $u\varepsilon < y$; and $\frac{\theta}{\theta-1} \frac{u\varepsilon}{\Lambda(\Delta)^{1/\eta-1}} \frac{1}{1+g} + \Gamma_{u\varepsilon}(u\varepsilon) < 1$. Then from (110) $\Gamma_{u\varepsilon}^G(u\varepsilon) + \Gamma_{u\varepsilon}(u\varepsilon) = \frac{\theta}{\theta-1} \frac{u\varepsilon}{\Lambda(\Delta)^{1/\eta-1}} \frac{1}{1+g} + \Gamma_{u\varepsilon}(u\varepsilon)$; and from (113)

$$\begin{aligned} & \Gamma_{u\varepsilon}(y) [\Gamma_{u\varepsilon}^{GS}(u\varepsilon) + \Gamma_{u\varepsilon}^S(u\varepsilon)] \\ &= \min \left(\frac{\theta}{\theta-1} \frac{u\varepsilon}{\Lambda(\Delta)^{1/\eta-1}} \frac{1}{1+g} + \Gamma_{u\varepsilon}(u\varepsilon); \Gamma_{u\varepsilon}(y) \right) \end{aligned} \quad (114)$$

$$\leq \frac{\theta}{\theta-1} \frac{u\varepsilon}{\Lambda(\Delta)^{1/\eta-1}} \frac{1}{1+g} + \Gamma_{u\varepsilon}(u\varepsilon) = \Gamma_{u\varepsilon}^G(u\varepsilon) + \Gamma_{u\varepsilon}(u\varepsilon), \quad (115)$$

which again proves inequality (109) holds.

The final case looks at the situation where default has *not* occurred under separated banking, whilst agents do not receive their deposits plus contracted interest under universal banking. This case may occur when government intervention is sufficiently costly (g is large).

Case 3: $u\varepsilon > y$; and $\frac{\theta}{\theta-1} \frac{u\varepsilon}{\Lambda(\Delta)^{1/\eta-1}} \frac{1}{1+g} + \Gamma_{u\varepsilon}(u\varepsilon) < 1$. Then from (110) $\Gamma_{u\varepsilon}^G(u\varepsilon) + \Gamma_{u\varepsilon}(u\varepsilon) = \frac{\theta}{\theta-1} \frac{u\varepsilon}{\Lambda(\Delta)^{1/\eta-1}} \frac{1}{1+g} + \Gamma_{u\varepsilon}(u\varepsilon) > \Gamma_{u\varepsilon}(u\varepsilon) > \Gamma_{u\varepsilon}(y)$; but from (113)

$$\begin{aligned} & \Gamma_{u\varepsilon}(y) [\Gamma_{u\varepsilon}^{GS}(u\varepsilon) + \Gamma_{u\varepsilon}^S(u\varepsilon)] \\ &= \min \left(\frac{\theta}{\theta-1} \frac{u\varepsilon}{\Gamma_{u\varepsilon}(y) \Lambda(\Delta)^{1/\eta-1}} \frac{1}{1+g} + \Gamma_{u\varepsilon}(u\varepsilon); \Gamma_{u\varepsilon}(y) \right) \end{aligned} \quad (116)$$

$$\leq \Gamma_{u\varepsilon}(y) < \Gamma_{u\varepsilon}^D(u\varepsilon); \quad (117)$$

In order to complete the proof we need to show that $\Gamma_{u\varepsilon}^G(u\varepsilon) > \Gamma_{u\varepsilon}^{GS}(u\varepsilon) \Gamma_{u\varepsilon}(y)$. ■

Lemma 22 *For any realisation $u\varepsilon$,*

$$\Gamma_{u\varepsilon}^G(u\varepsilon) > \Gamma_{u\varepsilon}^{GS}(u\varepsilon) \Gamma_{u\varepsilon}(y) \quad (118)$$

Proof. We recall the definitions

$$\Gamma_{u\varepsilon}^G(u\varepsilon) = \min \left(\frac{\theta}{\theta-1} \frac{u\varepsilon}{\Lambda(\Delta)^{1/\eta-1}} \frac{1}{1+g}; (1 - \Gamma_{u\varepsilon}(u\varepsilon)) \right);$$

$$\Gamma_{u\varepsilon}^{GS}(u_{t+1}\varepsilon_{t+1}) = \min \left(\frac{\theta}{\theta-1} \frac{u\varepsilon}{\Gamma_{u\varepsilon}(y) \Lambda(\Delta)^{1/\eta-1}} \frac{1}{1+g}; \left(1 - \min \left[\frac{\Gamma_{u\varepsilon}(u\varepsilon)}{\Gamma_{u\varepsilon}(y)}, 1 \right] \right) \right);$$

And again, consider different cases:

Case 1: $u\varepsilon > y$, then $\Gamma_{u\varepsilon}^{GS}(u\varepsilon) = 0$ and inequality (118) holds.

Case 2: $u\varepsilon < y$, and $\frac{\theta}{\theta-1} \frac{u\varepsilon}{\Lambda(\Delta)^{1/\eta-1}} \frac{1}{1+g} < (1 - \Gamma_{u\varepsilon}(u\varepsilon))$; then $\Gamma_{u\varepsilon}^G(u\varepsilon) = \frac{\theta}{\theta-1} \frac{u\varepsilon}{\Lambda(\Delta)^{1/\eta-1}} \frac{1}{1+g}$; but $\Gamma_{u\varepsilon}(y) \Gamma_{u\varepsilon}^{GS}(u\varepsilon) < \frac{\theta}{\theta-1} \frac{u\varepsilon}{\Lambda(\Delta)^{1/\eta-1}} \frac{1}{1+g} = \Gamma_{u\varepsilon}^G(u\varepsilon)$.

Case 3: $u\varepsilon < y$, and $\frac{\theta}{\theta-1} \frac{u\varepsilon}{\Lambda(\Delta)^{1/\eta-1}} \frac{1}{1+g} > (1 - \Gamma_{u\varepsilon}(u\varepsilon))$; then $\Gamma_{u\varepsilon}^G(u\varepsilon) = (1 - \Gamma_{u\varepsilon}(u\varepsilon))$, and

$$\begin{aligned} \Gamma_{u\varepsilon}^{GS}(u\varepsilon) &= \min \left(\frac{\theta}{\theta-1} \frac{u\varepsilon}{\Gamma_{u\varepsilon}(y) \Lambda(\Delta)^{1/\eta-1}} \frac{1}{1+g}; \left(1 - \frac{\Gamma_{u\varepsilon}(u\varepsilon)}{\Gamma_{u\varepsilon}(y)} \right) \right) \leq \left(1 - \frac{\Gamma_{u\varepsilon}(u\varepsilon)}{\Gamma_{u\varepsilon}(y)} \right) \\ &< (1 - \Gamma_{u\varepsilon}(u\varepsilon)) = \Gamma_{u\varepsilon}^G(u\varepsilon) \end{aligned}$$

Thus it is proved that $\Gamma_{u\varepsilon}^G(u\varepsilon) > \Gamma_{u\varepsilon}^{GS}(u\varepsilon) \Gamma_{u\varepsilon}(y)$. ■

Lemma 23 *A model with universal banking and insurance gives higher employment in equilibrium than a separated banking system with insurance.*

Proof. It follow follows immediately from Lemmas 21 and 22 that $\Upsilon^{UI} > \Upsilon^{SI}$, and therefore that $N^{UI} > N^{SI}$. ■

8.9 Retail deposit rates

It can be shown from (65) that deposit interest rates R_t^{hXY} ($X = U, S$ and $Y = I, N$) under different scenarios are:

$$\begin{aligned} R_t^{hUN} &= \frac{1}{\beta} \frac{\Omega_t A_t^\rho}{\Omega_{t-1} A_{t-1}^\rho} \frac{1}{u_t \varepsilon_t} \frac{1}{\Upsilon^{UN}}; \\ R_t^{hUI} &= \frac{1}{\beta} \frac{\Omega_t A_t^\rho}{\Omega_{t-1} A_{t-1}^\rho} \frac{1}{u_t \varepsilon_t} \frac{1}{\left[1 - g \frac{\theta-1}{\theta} (\Delta)^{1/\eta-1} \Lambda \Gamma_{u\varepsilon}^G(u_t \varepsilon_t)/u_t \varepsilon_t\right]} \frac{1}{\Upsilon^{UI}}; \\ R_t^{hSN} &= \frac{1}{\beta} \frac{\Omega_t A_t^\rho}{\Omega_{t-1} A_{t-1}^\rho} \frac{1}{u_t \varepsilon_t} \frac{\Gamma_{u\varepsilon}(y)}{\Upsilon^{SN}}; \\ R_t^{hSI} &= \frac{1}{\beta} \frac{\Omega_t A_t^\rho}{\Omega_{t-1} A_{t-1}^\rho} \frac{1}{u_t \varepsilon_t} \frac{\Gamma_{u\varepsilon}(y)}{\Upsilon^{SI} \left[1 - g \frac{\theta-1}{\theta} \Lambda (\Delta)^{1/\eta-1} \Gamma_{u\varepsilon}(y) \Gamma_{u\varepsilon}^{GS}(u_t \varepsilon_t)/u_t \varepsilon_t\right]}. \end{aligned}$$

Now we can formulate some interesting facts about the deposit rate and financial structure.

Proposition 24 *Without bailouts the deposit rate is lower in a separated banking system,*

$$R_t^{hSN} < R_t^{hUN}$$

Proof. We need to show that

$$\Upsilon^{UN} < \frac{\Upsilon^{SN}}{\Gamma_{u\varepsilon}(y)}.$$

That follows since

$$\begin{aligned} \frac{\Upsilon^{SN}}{\Gamma_{u\varepsilon}(y)} &= \int_0^{+\infty} \frac{\min\left(\frac{\Gamma_{u\varepsilon}(u\varepsilon)}{\Gamma_{u\varepsilon}(y)}, 1\right)}{u\varepsilon} dF_{u\varepsilon} \\ \Upsilon^{UN} &= \int_0^{+\infty} \frac{\Gamma_{u\varepsilon}(u, \varepsilon)}{u\varepsilon} dF_{u\varepsilon} \end{aligned}$$

■

Proposition 25 *With bailouts and with efficient government, $g = 0$, the deposit rate is lower in a separated banking system, $R_t^{hSI} < R_t^{hUI}$.*

Proof. We need to show that

$$\Upsilon^{UI} < \frac{\Upsilon^{SI}}{\Gamma_{u\varepsilon}(y)}$$

To that end, note

$$\Upsilon^{UI} = \int_0^{+\infty} \frac{\min\left(\frac{\theta}{\theta-1} \frac{u\varepsilon}{\Lambda(\Delta)^{1/\eta-1}} + \Gamma_{u\varepsilon}(u\varepsilon); 1\right)}{u\varepsilon} dF_{u\varepsilon} \quad (119)$$

$$\frac{\Upsilon^{SI}}{\Gamma_{u\varepsilon}(y)} = \int_0^{+\infty} \frac{\min\left(\frac{\theta}{\theta-1} \frac{u\varepsilon}{\Lambda(\Delta)^{1/\eta-1}} + \min\left(\frac{\Gamma_{u\varepsilon}(u\varepsilon)}{\Gamma_{u\varepsilon}(y)}, 1\right); 1\right)}{u\varepsilon} dF_{u\varepsilon} \quad (120)$$

and hence it is true. ■

Proposition 26 *When government is efficient, $g = 0$, the interest rate goes in the opposite direction to employment; bailouts reduce the deposit interest rate; and separated banking results in lower interest rates.*

For the case $g > 0$ things are less clear-cut and the impact of g is non-monotonic. On the one hand, a high probability of large government intervention may increase the deposit rate as it is distortive. On the other hand, bailouts are a source of insurance and will tend to boost employment and consumption and depress the deposit rate.

8.10 Approximation to μ^{HN}

In this section we demonstrate how we derived the wedges associated with household behaviour. We begin with:

$$\mu^{HN} = \frac{E_t [u'(C_{t+1})C_{t+1}]}{E_t C_{t+1} E_t [u'(C_{t+1})]}.$$

Taking a second-order approximation of the numerator gives,

$$E_t [u'(C_{t+1})C_{t+1}] = u'(E_t C_{t+1})E_t C_{t+1} + u''(E_t C_{t+1}) \left(\frac{1}{2} \frac{u'''(E_t C_{t+1})E_t C_{t+1}}{u''(E_t C_{t+1})} + 1 \right) \text{var}(C_{t+1}) + O3.$$

One may rewrite this as follows

$$E_t [u'(C_{t+1})C_{t+1}] = u'(E_t C_{t+1})E_t C_{t+1} \left(1 + \frac{u''(E_t C_{t+1})E_t C_{t+1}}{u'(E_t C_{t+1})} \left(\frac{1}{2} \frac{u'''(E_t C_{t+1})E_t C_{t+1}}{u''(E_t C_{t+1})} + 1 \right) \frac{\text{var}(C_{t+1})}{(E_t C_{t+1})^2} \right).$$

Using these definitions

$$\begin{aligned} \sigma_3 &= -\frac{1}{2} \frac{u'''(E_t C_{t+1})E_t C_{t+1}}{u''(E_t C_{t+1})} \\ s_u &= -\frac{u''(E_t C_{t+1})E_t C_{t+1}}{u'(E_t C_{t+1})} \end{aligned}$$

gives

$$E_t [u'(C_{t+1})C_{t+1}] = u'(E_t C_{t+1})E_t C_{t+1} \left(1 - s_u (1 - \sigma_3) \frac{\text{var}(C_{t+1})}{(E_t C_{t+1})^2} \right).$$

It follows then that

$$\mu^{HN} = \frac{u'(E_t C_{t+1})}{E_t [u'(C_{t+1})]} E_t C_{t+1} \left(1 - s_u (1 - \sigma_3) \frac{\text{var}(C_{t+1})}{(E_t C_{t+1})^2} \right).$$

Note that the first term on the right hand side may be written as follows:

$$\frac{u'(E_t C_{t+1})}{E_t [u'(C_{t+1})]} = \frac{u'(E_t C_{t+1})}{[u'(E_t C_{t+1}) + \frac{1}{2}u'''(E_t C_{t+1}) + O3]} = \frac{1}{[1 + \sigma_3 s_u \frac{\text{var}(C_{t+1})}{(E_t C_{t+1})^2} + O3]}.$$

Simple substitution then verifies that the following approximation is indeed valid

$$\mu^{HN} = 1 - s_u \frac{\text{var}(C_{t+1})}{(E_t C_{t+1})^2} \left(\frac{1}{1 + \sigma_3 s_u \frac{\text{var}(C_{t+1})}{(E_t C_{t+1})^2}} \right) + O3.$$

8.11 Proof of Proposition 13

Proposition 13 *If government intervention, G_{t+1} , declines when consumption increases, then $\mu^g > 1$*

Proof. We use the fact that total output is either used for consumption or lost to society as a deadweight transaction cost associated with government activity:

$$\frac{d}{dN_t} Y_{t+1} = F_N(N_t, u_{t+1}) = \frac{dC_{t+1}}{dN_t} + \frac{dg_{t+1}}{dN_t},$$

where $g'(G_{t+1}) > 0$. We plug that expression into definition (77) and manipulate as follows:

$$\begin{aligned} \mu^g &= \frac{E_t[u'(C_{t+1})F_N(N_t, u_{t+1})] E_t \frac{dC_{t+1}}{dN_t}}{E_t(F_N(N_t, u_{t+1})) E_t \left[u'(C_{t+1}) \frac{dC_{t+1}}{dN_t} \right]} \\ &= \frac{E_t \left[u'(C_{t+1}) \left(\frac{dC_{t+1}}{dN_t} + \frac{dg_{t+1}}{dN_t} \right) \right] E_t \frac{d}{dN_t} C_{t+1}}{E_t \left(\frac{dC_{t+1}}{dN_t} + \frac{dg_{t+1}}{dN_t} \right) E_t \left[u'(C_{t+1}) \frac{d}{dN_t} C_{t+1} \right]} \\ &= \frac{E_t \left[u'(C_{t+1}) \left(\frac{dC_{t+1}}{dN_t} \right) \right] E_t \frac{d}{dN_t} C_{t+1} + E_t \left[u'(C_{t+1}) \left(\frac{dg_{t+1}}{dN_t} \right) \right] E_t \frac{dC_{t+1}}{dN_t}}{E_t \left(\frac{d}{dN_t} C_{t+1} \right) E_t \left[u'(C_{t+1}) \frac{dC_{t+1}}{dN_t} \right] + E_t \left(\frac{dg_{t+1}}{dN_t} \right) E_t \left[u'(C_{t+1}) \frac{dC_{t+1}}{dN_t} \right]}. \end{aligned}$$

By using the definition for covariance, one can write

$$\mu^g = \frac{E_t \left[u'(C_{t+1}) \frac{dC_{t+1}}{dN_t} \right] E_t \left(\frac{dC_{t+1}}{dN_t} \right) + E_t u'(C_{t+1}) E_t \left(\frac{dg_{t+1}}{dN_t} \right) E_t \frac{dC_{t+1}}{dN_t} + \text{cov}_t \left[u'(C_{t+1}) \frac{dg_{t+1}}{dN_t} \right] E_t \left(\frac{dC_{t+1}}{dN_t} \right)}{E_t \left[u'(C_{t+1}) \frac{dC_{t+1}}{dN_t} \right] E_t \left(\frac{dC_{t+1}}{dN_t} \right) + E_t u'(C_{t+1}) E_t \left(\frac{dg_{t+1}}{dN_t} \right) E_t \frac{dC_{t+1}}{dN_t} + E_t \left(\frac{dg_{t+1}}{dN_t} \right) \text{cov}_t \left[u'(C_{t+1}) \left[\frac{dC_{t+1}}{dN_t} \right] \right]}.$$

As government intervention declines when consumption increases and since marginal utility of consumption is a decreasing function, $\text{cov}_t \left[u'(C_{t+1}) \frac{dg_{t+1}}{dN_t} \right] > 0$, and $\text{cov}_t \left[u'(C_{t+1}) \frac{dC_{t+1}}{dN_t} \right] < 0$. It therefore follows that $\mu^g \geq 1$. ■

8.12 Aggregate equations and numerical analysis

To analyze the implications of the model for universal banking we compare welfare in the stochastic steady state of the model with and without universal banking. A key parameter is σ , the standard deviation of the shock to the investment banks. In order to get a rough guide on a reasonable value for this parameter we proceeded as follows. In the numerical analysis we employ the lognormal distribution $f(\varepsilon)$ with parameters σ and μ . Parameter μ is naturally chosen by normalization of the mean so that $E(\varepsilon) = 1$ implies $\mu = -\sigma^2/2$. The following data looks at the empirical volatility of the log returns to banking equity. The data, taken from yahoo finance, is the volatility of quarterly log returns.

Table UK	St. Dev	Time period
BARCLAYS	0.250	Q1 2003-Q2 2011
HSBC	0.106	Q3 2000-Q2 2011
Santander	0.170	Q3 2006-Q2 2011
RBS	0.343	Q1 2003-Q2 2011
Lloyds	0.200	Q3 2000-Q2 2011

Table	USA	St. Dev	Time period
Bank of America Corporation	0.208	Q3 1986-Q2 2011	
The Goldman Sachs Group	0.168	Q3 1999-Q2 2011	
JPMorgan Chase	0.166	Q1 1984-Q2 2011	
Morgan Stanley	0.170	Q2 1993-Q2 2011	
Credit Suisse Group	0.161	Q1 1999-Q2 2011	
Citigroup	0.190	Q1 1977-Q2 2011	

Hence $\sigma = 0.2$ appears a not unreasonable value.