Economics Department Discussion Papers Series

ISSN 1473 – 3307

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Paper number 08/14

URL: http://business-school.exeter.ac.uk/economics/papers/

The Consequences of Zakat for Capital Accumulation

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October 28, 2008

Abstract

The payment of zakat by the owners of wealth is one of the five pillars of Islam. Many countries operate with no enforcement of the obligation to pay, making zakat a form of voluntary redistribution. We analyze how zakat affects capital accumulation in a model that explicitly recognizes the voluntary nature of zakat. The voluntary payment is modelled using both warm-glow and social custom frameworks. These are embedded within an overlapping generations model with heterogenous consumers and endogenous population growth. The results show that zakat can raise the capital-labor ratio when it is motivated by the warm-glow but welfare can be non-monotonic in the strength of the warm-glow. In the social custom model reduced participation can lead to a reduced capital-labor ratio as the rate of zakat is increased.

JEL classification codes: E62, H21, P51

Acknowledgements: Thanks are due to Nigar Hashimzade, seminar participants in Exeter, and to participants in the Singapore conference on Leadership, Altruism, and Social Organization.

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1 Introduction

Paying zakat is one of the five pillars of Islam stated in the Koran. The payment of zakat is a religious obligation and must be fulfilled by Muslims who wealth above the minimum threshold. Al-Qardawi (1997) argues that zakat is not just a form of worship but also that the receipt of the proceeds of zakat is a right of the poor. There are two types of zakat: zakat-al-fitr and zakat-al-mal. *Zakat-al-fitr* must be paid for charity at the end of Ramadhan (Zayas, 1960). We focus on *zakat-al-mal* which is the zakat on wealth. This is levied on different types of wealth at a benchmark rate of 2.5 percent. Zakat on wealth must be paid by Muslims who own wealth above the *nisab*, the minimum exemption limit, and must be paid annually. The goal of zakat is to redistribute wealth from the owners of nisab (the non-poor) to the non-owners of nisab (the poor).

The institution of zakat can be viewed as a system of voluntary redistribution supported by religious obligation. It represents an alternative institutional form to redistribution through statutory taxes and transfers. Interesting questions are therefore raised concerning the economic consequences of this institution.

There have been several studies of the economic effect of zakat (Siddiqi 1979, Rahman 2003, Kuran 2006). Most Islamic economists argue that zakat has a positive effect on the economy. It is generally concluded that the effect of the implementation of zakat is to increase aggregate demand, increase the capital stock, and raise economic growth. There are two separate arguments underlying these claims. The first observes that zakat is a transfer of wealth from the rich to the poor, so if the poor have a higher marginal propensity to consume than the rich then aggregate demand will rise. The second is the claim that zakat payers will increase their saving ratio in order to avoid the depletion of wealth due to the payment of zakat. Embedding these arguments within a static Keynesian model leads to the conclusion that the increase in saving will ultimately be matched by an increase in investment (Kahf 1980, Khan 1984). The level of output therefore rises in equilibrium. Some of the limitations of this reasoning are identified by Iqbal (1985) who observed that the consumptions effects of transfers of wealth from the rich to the poor could cancel unless a Keynesian aggregate consumption function was assumed.

There are two further significant limitations of this previous literature. First, the models used are static so provide a poor framework for addressing the effects of zakat on capital accumulation and growth. In order to see the overall effects of zakat on an economy's capital accumulation and consumption, it is necessary to consider how the economy is affected in the long run. Thus, we choose to analyze zakat within an overlapping generation model in order to derive long-run implications for capital accumulation, economic welfare and consumption. Second, the models do not incorporate the motive for paying zakat within the analysis of consumption and saving decisions. The economic representation of this motive has to be a key component of the analysis.

The motivation for the paying zakat is important because it is a religious obligation rather than a statutory obligation. In other words, zakat is not a tax but is instead a voluntary contribution to wealth redistribution. We consider

two different explanations for why such a voluntary contribution is made. The first is to model the reward from making the payment as a form of warm-glow (Andreoni 1990). The alternative is to model the religious obligation behind contributions to zakat as a social custom. We discuss the relative merits of these two approaches in more detail below. In our modelling we also assume that the growth rate of population is endogenous. In particular, we assume the growth rate of the poor population is dependent upon the income level of the poor. This assumption can be motivated by the argument that both fertility and the survival rates of infants being dependent on income. In any case, this mechanism provides an interesting feedback from zakat to the dynamic evolution of the economy.

The analysis explores the economic consequences of the institution of zakat employing standard economic models drawn from the literature on voluntary contirbution. The results demonstrate that the institution of zakat can increase the capital-labor ratio in the model that we analyze if contribution is explained by the warm-glow. The results for the social custom model are not so promising for the institution. As the rate of zakt is increased the model predicts a fall in the capital-labor ratio through decreased participation in the social custom.

Section 2 reviews the operation of zakat and the modelling of voluntary contributions. Section 3 analyzes the implications of zakat in a warm-glow framework. Section 4 studies the social custom model. The warm-glow and the social custom are combined in Section 5. Conclusions are given in Section 6.

2 Operation and Modelling of Zakat

Zakat is not purely an act of worship. Apart from its religious objectives zakat also has the socioeconomic objective of reducing inequality in the distribution of wealth between the rich and the poor (Ahmad,1980). The Koran permits a zakat payment to be made directly to recipients or through a zakat collection authority (Mannan, 1986). Both centralized (or obligatory) and decentralized (or voluntary) modes of zakat administration are permitted. Malaysia, Saudi Arabia, and Pakistan are among the countries where zakat is administered by the state (Kuran, 2006). In many other countries the state is not directly involved.

Zakat payments on wealth are made on an annual basis. The forms of wealth that are zakatable, and the ratios of zakat payment on each, were not fully specified in the Koran. The rates of zakat payment, time of payment, the nisab, and the method of organizing and administering zakat collection and distribution were adopted through the practice or sayings of the Prophet Muhammad. According to Shaik (1979, cited by Zaman 1980) a majority of Islamic jurists agree on the rates in Table 1 and the zakatable assets in Table 2.

Item	Rate
On all types of wealth	2.5%
Mines and treasure troves	20%
Produce from irrigated land	5%
Produce from unirrigated land	10%
Animals	1.5%-2.5%

Table 1: Rates of zakat

Types of wealth or asset	Nisab	Rate
Gold	85 grams of gold	2.5% of the value
Silver	595 grams of silver	2.5% of the value
Cash in hand, stocks, bonds, trading goods, or any other liquid asset	Amount equivalent to 85 grams of gold	2.5% of the value
Agricultural produce	653 kg per harvest	5% (irrigated land) 10% (non-irrigated land)
Product of mines	Any amount	20% of the value

Table 2: Types of zakatable assets, nisab, and rates

Islamic jurists have different opinions about which sources of wealth should be subject to zakat. Kahf (1997) noted three *fiqhi* views, or opinions, on zakat collection. The narrow opinion includes agricultural products, gold and silver (except for personal use), trading goods, short-term net returns, and cash in hand. This is the most agreed upon method of zakat calculation. The middle opinion includes all types of assets listed in the narrow opinion plus earnings on fixed assets, as well as wages, salaries, and professional incomes. The latter is controversial in relation to the zakatability of new forms of wealth. The third opinion includes items under the narrow and middle opinions plus fixed assets, which are assets used to generate income. This would include assets such as buildings, furniture, machinery, and containers. However, a majority of contemporary scholars believe business fixed assets should be exempt from zakat.

The distribution of the proceeds of zakat is clearly targeted to eight zakat recipients in the Koran. As mentioned in Surah 9 verses 58-60:

> "*A collected sadaqa is for the poor, the destitute, those who collect it, reconciling people's hearts, freeing slaves, those in debt, spending in the Way of Allah and wayfarers. . . .*"

Five of the eight eligible recipients of zakat proceeds are considered to fall under the poor category (Al-Abdin, 1997): the poor, the destitute, slaves, those in debt, and wayfarers.

The question whether zakat is a tax has been widely debated. Economists such as Kahf (1999), Kamali (1999), and Kuran (2006) have referred to zakat as a tax. Certainly, zakat shares with taxation the aim of achieving economic

and social objectives. Where they differ is that payment of tax is mandatory whereas the payment of zakat is a religious obligation. It is important to observe that there is limited evidence of state enforcement of zakat payment even where legislation permits this. Hence, in most countries zakat is voluntary and is made either directly to the poor or to a zakat collection authority. A further difference is that tax rates can be changed as part of fiscal policy whereas the rate of zakat is fixed through religious ruling. Finally, the recipients of the proceeds of zakat are specifically defined whereas hypothecation is rarely used for taxation. However, zakat shares with taxation the property that it results in the transfer of purchasing power from one economic agent to another. It should therefore be possible to explore the economic effects of zakat using the same tools as applied to taxation provided that the motives for giving zakat are adequately represented.

According to Chapra (1992), the payment of zakat should be voluntary but with the state able to enforce payment if it wishes to do so. Since there are many countries in which zakat is not enforced but the level of payment is significant, it is necessary to consider explanations of such voluntary giving. In doing this we are seeking an economic representation of the religious motives that drive the payment of zakat. Alexander *et al.* (1997) suggested that people give because they expect gratitude and because of religious commitment. The two representations we consider, the warm-glow and social custom models, can be seen as formalizations of these explanations.

Andreoni (1989, 1990) motivated giving through the derivation of internal satisfaction; this satisfaction has become known as the warm-glow. In the warm-glow model the amount given is an argument of the utility function alongside other consumption goods and is chosen as part of the utility maximization process. In our context the warm-glow can be viewed as the pleasure derived from fulfillment of religious duties.

As zakat is a religious obligation, we would expect failure to meet this to result in a utility loss. This motivates our second way of modelling zakat. The social custom model of developed in Myles and Naylor (1996) captures the idea that there is a loss of welfare if an accepted form of behavior is not followed. In our model the accepted form of behavior is to meet religious obligations and provide zakat. Hence, an owner of nisab who does not give zakat loses utility from the social custom but can take advantage of higher disposable income.

3 A Warm-Glow Model of Zakat

This section considers the consequences for capital accumulation of zakat explained as an act of warm-glow giving. The model of the warm-glow is embedded within an overlapping generations model with endogenous population growth. It is assumed that there are two types of consumer in the economy: the rich who give to zakat and the poor who receive zakat. The poor are characterized as having a limited quantity of labor (in efficiency units) to supply, so qualify for zakat on the basis of a low income level. The rich supply a larger quantity

of labor than the poor and accumulate wealth on which zakat is paid.

3.1 Consumption and Production

Each consumer lives for two periods and works only in the first period of life. Labor is supplied inelastically. In the second period of life each consumer is retired. Labor is measured in efficiency units. A rich consumer is able to supply a greater number of efficiency units of labor than a poor consumer. The labor income of a rich consumer is divided between consumption and saving. In the second period of life savings are divided between consumption and the payment of zakat. A poor consumer receives labor income in the first period of life and a payment of zakat in the second period. The poor do not save, so all their income is consumed in the period in which it is received.

Consider a rich consumer born in period t. We denote consumption in the first period of life by $x^r_{t,t}$, consumption in the second period by $x^r_{t,t+1}$, and the payment of zakat, which is made in the second period of life, by z_{t+1}. The utility function is

$$U^r = U\left(x^r_{t,t}, x^r_{t,t+1}, z_{t+1}\right). \tag{1}$$

The appearance of z_{t+1} in the utility function captures the warm-glow derived from giving to zakat. The intertemporal budget constraint of the consumer is

$$w_t \ell^r = x^r_{t,t} + \frac{x^r_{t,t+1}}{1+r_{t+1}} + \frac{z_{t+1}}{1+r_{t+1}}, \tag{2}$$

where w_t is the wage rate, r_{t+1} the return on saving, and ℓ^r labor supply. For the remainder of this section it is assumed that units are such that $\ell^r = 1$. The maximization of utility generates the demand functions

$$x^r_{t,t} = x^r_{t,t}\left(w_t, r_{t+1}\right), \tag{3}$$

$$x^r_{t,t+1} = x^r_{t,t+1}\left(w_t, r_{t+1}\right), \tag{4}$$

and a zakat contribution of

$$z_{t+1} = z_{t+1}\left(w_t, r_{t+1}\right). \tag{5}$$

A poor consumer born in period t works in the first period of life and earns labor income $\ell^p w_t$. This labor income is assumed to be insufficient to finance saving; hence, consumption in the first period of life is

$$x^p_{t,t} = \ell^p w_t. \tag{6}$$

In the second period of life each poor consumer receives a payment of zakat, ζ_{t+1}. Consumption in the second period of life is

$$x^p_{t,t+1} = \zeta_{t+1}. \tag{7}$$

At the beginning of each period a new generation consisting of poor and rich consumers is born. The number of rich consumers born at time t is N^r_t and the

number of poor consumers born at t is N_t^p. The population growth rate of the rich is denoted n^r and the population growth of the poor at time t is n_t^p. The value of n^r is exogenous and constant, but n_t^p is endogenous. The dynamics of population for the two groups are given by

$$N_{t+1}^r = (1 + n^r) N_t^r, \tag{8}$$

and

$$N_{t+1}^p = (1 + n_t^p) N_t^p. \tag{9}$$

The growth rate of the poor, n_t^p, is determined by the total income of the poor. As described above, this can be motivated by the fertility and survival rates being related to income. The total income of the poor is labor income plus the amount of zakat received. Since zakat is distributed evenly among the poor it can be assumed that

$$n_t^p = \left(\frac{1}{v}\right)\left(\zeta_{t+1} + \ell^p w_t\right) - 1, \tag{10}$$

where the factor v can be given the interpretation of the income required to ensure that the survival rate is just high enough to keep the number of poor constant. By construction, $\zeta_{t+1} = z_{t+1} N_t^r / N_t^p$.

The firms in the economy are assumed to be competitive and all produce with the same constant returns to scale production. This permits the productive sector to be modelled using a representative firm. Total output in each period is composed of the sum of undepreciated capital and new output produced at the end of each production process. This output is allocated between re-investment and consumption. Denote the production function for (net) output by $F(K_t, L_t)$, where K_t is the capital stock in period t and L_t is aggregate labor supply. The representative firm chooses its use of capital and labour to maximize profit, π_t, where

$$\pi_t = F(K_t, L_t) - w_t L_t - r_t K_t. \tag{11}$$

Firms employ capital up to the point at which the marginal product F_K is equal to r_t, the cost of employing capital and employ labor up to the point at which the marginal product of labour F_L is equal to wage, w_t. Defining the per capita variables $y_t = \frac{Y_t}{L_t}$ and $k_t = \frac{K_t}{L_t}$, it follows that

$$y_t = f(k_t), \tag{12}$$

where $f(k_t) \equiv F\left(\frac{K_t}{L_t}, 1\right)$,

$$f(k_t) - k_t f'(k_t) = w_t, \tag{13}$$

and

$$f'(k_t) = r_t. \tag{14}$$

The production and consumption sectors are linked by the fact that at any time t the level of (gross) output must be equal to the sum of consumption and saving, s_t. Hence,

$$F(K_t, L_t) + K_t = (1 + n_t^r) x_{t,t}^r + \left(1 + n_{t-1}^r\right) x_{t-1,t}^r + (1 + n_t^r) s_t + (1 + n_t^p) x_{t,t}^p + \left(1 + n_{t-1}^p\right) x_{t-1,t}^p. \quad (15)$$

3.2 Equilibrium

The equilibrium of an overlapping generations economy can be interpreted either as the time path of the endogenous variables or as the steady state in which all per capita variables are constant. We find it worthwhile to consider both of these concepts in what follows. For both the time path and the steady state, the savings of the young constitute the capital stock for the following period, implying that the equilibrium condition for capital market can be expressed as the equality of total savings and the following period's capital stock. The time path of the endogenous variables is described by a pair of non-linear difference equations that determines the transition from one period to the next of the capital stock and the population of poor consumers. The steady state is obtained by determining the capital-labor ratio that is the fixed point of the non-linear difference equations.

3.2.1 Dynamic Evolution

Equilibrium in the capital market is achieved when capital demand in $t + 1$ is equal to saving in t. Since it is only the rich that save this equilibrium condition can be written

$$K_{t+1} = N_t^r \left[w_t - x_{t,t}^r\right]. \quad (16)$$

Recalling that $\ell^r = 1$, the total quantity of labor in period $t + 1$ is

$$L_{t+1} = N_{t+1}^r + N_{t+1}^p \ell^p. \quad (17)$$

Combining (16) and (17) the equilibrium can be expressed in term of the capital-labor ratio as

$$k_{t+1} = \frac{N_t^r \left[w_t - x_{t,t}^r\right]}{N_{t+1}^r + N_{t+1}^p \ell^p}. \quad (18)$$

Using the growth rate of the poor population defined by (10)

$$k_{t+1} = \frac{w_t - x_{t,t}^r}{1 + n^r + \frac{\ell^p}{v}\left[z_{t+1} + \frac{N_t^p}{N_t^r}\ell^p w_t\right]}. \quad (19)$$

Now define the proportion of poor to rich at time t by $\rho_t \equiv \frac{N_t^p}{N_t^r}$. Using the demand functions (3-4) and the determination of factor prices (13-14), the capital

market equilibrium condition can given the final form

$$k_{t+1} = \frac{f(k_t) - k_t f'(k_t) - x^r_{t,t}(f'(k_{t+1}), f(k_t) - k_t f'(k_t))}{1 + n^r + \frac{\ell^p}{v}\left[z_{t+1}(f'(k_{t+1}), f(k_t) - k_t f'(k_t)) + \rho_t \ell^p \left[f(k_t) - k_t f'(k_t)\right]\right]}. \tag{20}$$

Equation (20) is the non-linear difference equation that updates the capital stock from t to $t+1$. The significant difference to the standard overlapping generations model is that this equation also involves the endogenous variable ρ_t. A second non-linear difference equation now has to be obtained to describe the time path of this variable.

The time path of the number of poor consumers relative to the number of rich consumers is described by

$$\frac{N^p_{t+1}}{N^r_{t+1}} = \frac{[1 + n^p_t]}{[1 + n^r]} \frac{N^p_t}{N^r_t}. \tag{21}$$

Using (10), (3-4), and (13-14), this can be written

$$\rho_{t+1} = \frac{\ell^p \left[f(k_t) - k_t f'(k_t)\right]}{v\left[1 + n^r\right]} \rho_t + \frac{z_{t+1}(f'(k_{t+1}), f(k_t) - k_t f'(k_t))}{v\left[1 + n^r\right]}. \tag{22}$$

The pair of equations (20) and (22) constitute the system of non-linear difference equations that determine the time paths of the endogenous variables k_t and ρ_t.

3.2.2 Steady State

The steady state is achieved when all per capita variables are constant. In addition, the growth rate of the poor population must be equal to the growth rate of the rich population. If this were not the case then one population group would eventually become insignificant in size relative to the other.

Using (22) the steady-state level of ρ satisfies

$$\rho = \frac{z(f'(k), f(k) - k f'(k))}{v\left[1 + n^r\right] - \ell^p \left[f(k) - k f'(k)\right]}. \tag{23}$$

From (10) it can be seen that this is the value of ρ that ensures $n^p = n^r$, so confirming that the two populations grow at the same rate in the steady state.

The steady-state capital-labor ratio satisfies the condition

$$w - x^r_1 = k\left[1 + n^r + \left[z + \rho \ell^p w\right] \frac{\ell^p}{v}\right], \tag{24}$$

where x^r_1 denotes the steady-state level of consumption of a rich consumer in the first period of life. Using the solution (23) for the steady-state value of ρ and exhibiting the functional dependence gives the steady-state condition for the capital labor ratio is

$$k = \frac{\left[f(k) - k f'(k) - x^r_1(f'(k), f(k) - k f'(k))\right]\left[v\left[1 + n^r\right] - \ell^p \left[f(k) - k f'(k)\right]\right]}{\left[1 + n^r\right]\left[v\left[1 + n^r\right] - \left[\left[f(k) - k f'(k)\right] - z(f'(k), f(k) - k f'(k))\right] \ell^p\right]}. \tag{25}$$

The efficiency of the steady-state allocation in an overlapping generations economy can be assessed by comparing it with the Golden rule capital-labor ratio. The Golden rule is obtained by finding the capital-output ratio that maximizes consumption per capita in the steady state. There are three features that distinguish our model from the standard case. First, the population growth rate is endogenous. Second, we have two types of consumer so need to take account of the distribution of consumption between types. Third, there is the transfer of zakat from the rich to the poor, and the value of this has to be determined as a component of the Golden rule.

To characterize the Golden rule it is necessary to consider the feasible set of consumption levels in the first and the second periods of life. The budget constraint of a rich consumer and the two steady-state conditions can be written as

$$w = x_1^r + \frac{x_2^r}{1+r} + \frac{z}{1+r}, \tag{26}$$

$$\rho = \frac{z}{v\left[1+n^r\right] - \ell^p w}, \tag{27}$$

and

$$w - x_1^r = k\left[1 + n^r + \left[z + \rho\ell^p w\right]\frac{\ell^p}{v}\right]. \tag{28}$$

Eliminating x_1^r and z from (26 - 28) gives

$$x_2^r = k\left[1+n^r\right]\left[1+r\right]\left[1+\rho\ell^p\right] - \rho\left[v(1+n^r) - \ell^p w\right]. \tag{29}$$

The aggregate level of consumption at time t must satisfy

$$\begin{aligned} N_t^p x_p^1 + N_t^r x_r^1 + N_{t-1}^p x_p^2 + N_{t-1}^r x_r^2 &= N_t^r\left[\rho w\ell^p + x_r^1 + \frac{z}{1+n^r} + \frac{x_r^2}{1+n^r}\right] \\ &= \left[1+\rho\ell^p\right] N_t^r\left[w + \left[r - n^r\right]k\right]. \end{aligned} \tag{30}$$

The Golden rule is chosen to maximize consumption per capita at time t. Using (30) consumption per capita, c_t, is

$$c_t = \frac{\left[1+\rho\ell^p\right] N_t^r\left[w + \left[r-n^r\right]k\right]}{N_t^p + N_t^r + N_{t-1}^p + N_{t-1}^r}. \tag{31}$$

Dividing the top and bottom of (31) by N_t^r and using the definition of population growth in the steady-state

$$c = \frac{1+\rho\ell^p}{1+\rho}\frac{1+n^r}{2+n^r}\left[w + \left[r-n^r\right]k\right]. \tag{32}$$

Hence, the Golden rule value of k solves

$$\max_{\{k\}} \frac{1+\rho\ell^p}{1+\rho}\frac{1+n^r}{2+n^r}\left[f(k) - kf'(k) + \left[f'(k) - n^r\right]k\right], \tag{33}$$

which has the standard Golden rule necessary condition

$$n^r = f'(k). \tag{34}$$

The Golden rule capital-labor ratio matches the marginal productivity of capital to the growth rate of the rich population (which is same as growth of poor population in the steady state).

Observe also that, for any value k, consumption per capita is maximized when $\rho = 0$. This is an obvious result. The poor are less productive than the rich so that it would be efficient to reduce the poor population. However, this does not occur because the institution of zakat ensures the growth of the poor population.

3.3 Simulation

A simulation analysis of the economy described above is now undertaken. The simulation will contrast the economy with the institution of zakat to economies that use standard forms of taxation to obtain redistribution. Both the time paths and the steady state will be studied.

The simulation is based on the utility function for the rich

$$U^r = \alpha \ln\left(x^r_{t,t}\right) + \beta \ln\left(x^r_{t,t+1}\right) + \gamma \ln\left(z_{t+1}\right), \tag{35}$$

the utility for the poor

$$U^p = \alpha \ln\left(x^p_{t,t}\right) + \beta \ln\left(x^p_{t,t+1}\right), \tag{36}$$

and the production function

$$f\left(k_t\right) = k_t^a. \tag{37}$$

With these functional forms the time path of capital is described by

$$k_{t+1} = \frac{\left[1-a\right]\left[\frac{\beta+\gamma}{\alpha+\beta+\gamma}\right]k_t^a}{\left[1+n^r\right] + \left[1-a\right]k_t^a\left[\frac{\gamma\left[1+ak_{t+1}^{a-1}\right]}{\alpha+\beta+\gamma} + \rho_t \ell^p\right]\frac{\ell^p}{\upsilon}}, \tag{38}$$

and the time path of the population proportion is

$$\rho_{t+1} = \frac{\ell^p\left[1-a\right]k_t^a}{\upsilon\left[1+n^r\right]}\rho_t + \frac{\gamma\left[1+ak_{t+1}^{a-1}\right]\left[1-a\right]k_t^a}{\left[\alpha+\beta+\gamma\right]\upsilon\left[1+n^r\right]}. \tag{39}$$

The basic parameter values used in the simulations are: $\alpha + \beta + \gamma = 0.75$, $a = 0.36$, $\ell^p = 0.1$, $n^r = 0.05$, and $\upsilon = 1$. The initial value of the capital-labor ratio is $k_0 = 0.01$ and the initial ratio of poor to rich is $\rho_0 = 0.05$.

The first step is to compare the growth path of the capital-labor ratio for different values of the preference for giving zakat, γ. The results are summarized in Figure 1. This figure is constructed under the assumption that $\alpha = \beta$, so as γ is increased α and β are reduced equally. It can be seen in the figure that the

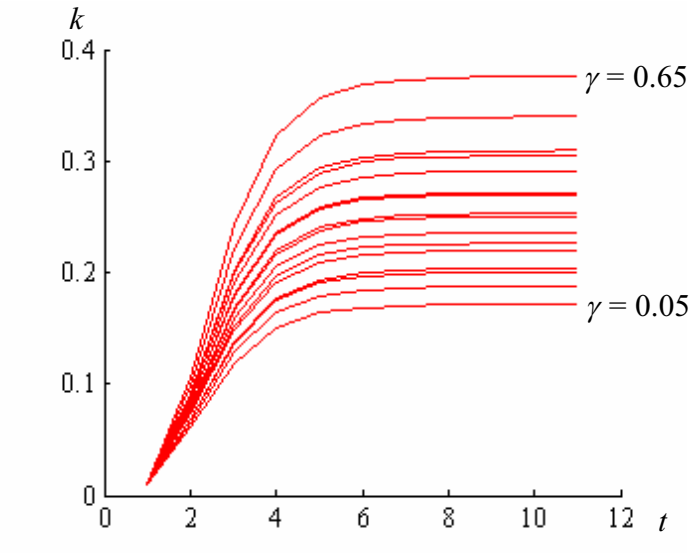

Figure 1: Capital-labor ratio: warm-glow effect

growth rate of the capital-labor ratio increases with γ. Moreover, the steady state level also increases with γ. Within the context of the model analyzed these results demonstrate that the institution of zakat can raise the growth rate of the capital-labor ratio.

The effect of changing γ upon the evolution of the proportion of poor population is shown in Figure 2. As γ is increased the growth rate of the proportion of the poor in the population and the steady-state value increase. Not surprisingly, a stronger preference for giving zakat causes more redistribution and permits a larger population of poor consumers to be sustained.

The second set of simulations compare the growth path with zakat to the growth path with an income tax. The growth path for zakat is computed for the parameter values $\alpha = \beta = \gamma = 0.25$. For the income tax the warm glow is no longer relevant. To ensure a meaningful comparison the adjustment $\alpha' = \alpha + \frac{\gamma}{2}$, $\beta' = \beta + \frac{\gamma}{2}$ is adopted. The budget constraint for rich with income taxation becomes

$$\ell^r_{t,t}\left(1-\tau\right) = x^r_{t,t} + \frac{x^r_{t,t+1}}{1+r\left(1-\tau\right)}, \tag{40}$$

so both labor income and interest income are taxed at the rate τ. It is assumed that the poor do not pay tax, and that the tax revenue raised from the rich is used to fund a payment to the poor. This payment is received by the poor in the second period of life.

Figure 3 shows the growth of the capital stock with zakat (solid line) and growth with income taxation (dashed lines). The interesting feature is that the

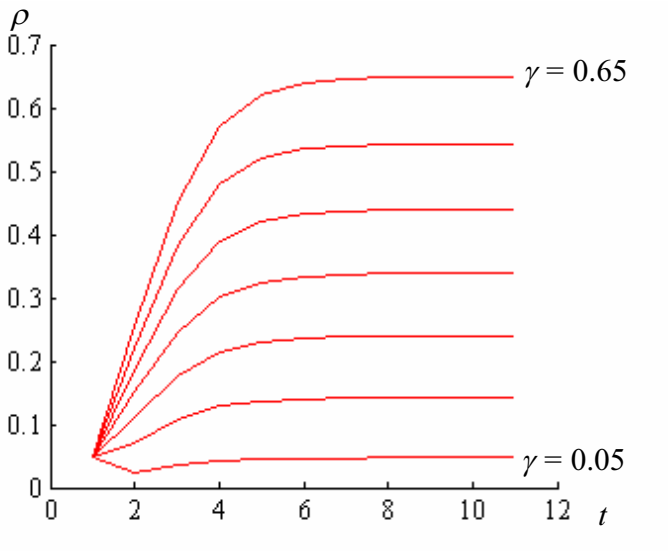

Figure 2: Population proportion: warm-glow effect

growth rate of the capital-labor ratio with zakat is faster than with no tax. Hence, saving to support the poor either through zakat or with a tax will raise the rate of capital accumulation and the steady state level of capital. It can also be seen that high income tax rates do eventually lead to a higher steady-state capital-labor ratio than for zakat. It should not be presumed that this is necessarily good for welfare since this model can suffer from excessive capital accumulation. Welfare will be investigated in the analysis of the steady state. As a general observation the growth path with zakat has the same form as with taxation so there is no fundamental difference in the performance of the economy between the institutions.

Figure 4 shows the growth of the proportion of poor to rich with zakat (solid line) and growth with income taxation (dashed lines). When there is no tax the growth rate of the poor is driven to zero since income is not sufficient to support a positive level. Again, high rates of income tax are able to support a larger poor population than zakat. The initial part of the growth path does have a different structure. The steady-state levels do not differ, in the sense that an income tax of (approx.) $\tau = 0.35$ results in the same capital-labor ratio and proportion of poor as the zakat economy. Contrasting the growth paths shows that zakat leads to a slower initial rate of growth than the income tax.

Figure 5 shows the growth path of the capital-labor ratio with zakat (solid line) and growth with a wealth tax (dashed lines). Zakat is often interpreted as

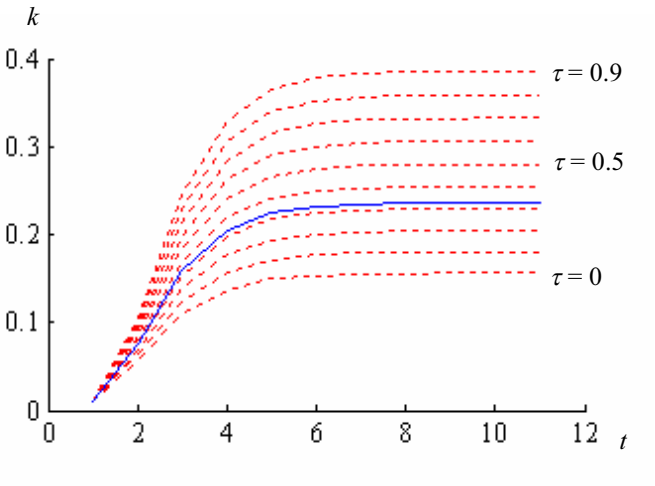

Figure 3: Capital-labor ratio: zakat and income taxation

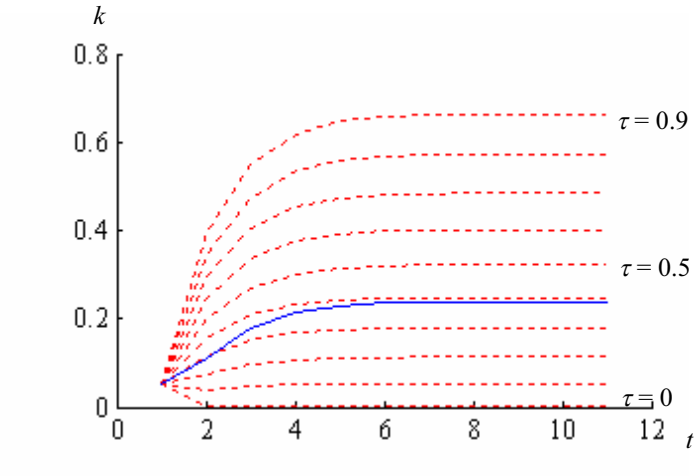

Figure 4: Population proportion of poor: zakat and income tax

a voluntary tax on wealth so this seems an appropriate comparison. The budget constraint for the wealth tax is

$$\ell^r_{t,t} = x^r_{t,t} + \frac{x^r_{t,t+1}}{(1 + r_{t+1})(1 - \tau)}. \tag{41}$$

The same adjustment to the values of α and β is used as for the income tax. The figure shows that zakat leads to a significantly higher capital-labor ratio than the wealth tax. In this case the wealth tax proves to have a detrimental effect upon capital accumulation. Figure 6 shows the growth of the proportion of poor to rich with zakat (solid) and growth with wealth taxation (dashed lines). It can be seen in the figure that for the parameter values chosen zakat is equivalent of at wealth tax of 60 percent.

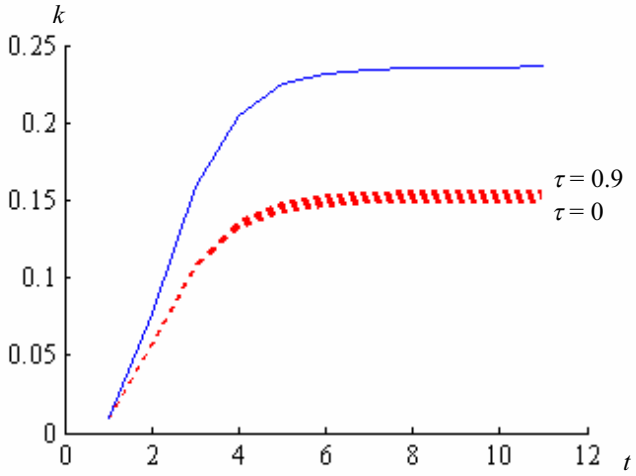

Figure 5: Capital-labor ratio: wealth tax and zakat

Figure 7 shows how the steady state values of k and ρ change as γ is increased. An increase in γ raises both the steady state capital-labor ratio and the proportion of the poor population. The positive relation between the steady-state capital-labor ratio and the strength of preference for giving zakat demonstrates that there can be a sense in which the institution of zakat leads to an increased capital stock.

The final analysis considers the welfare consequences of zakat. Define the per capita level of welfare by

$$W = \frac{N^r U^r + N^p U^p}{N^r + N^p}, \tag{42}$$

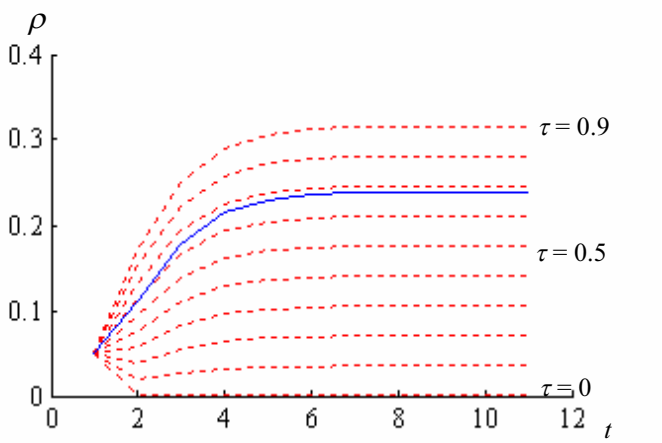

Figure 6: Growth of population proportion: wealth tax and zakat

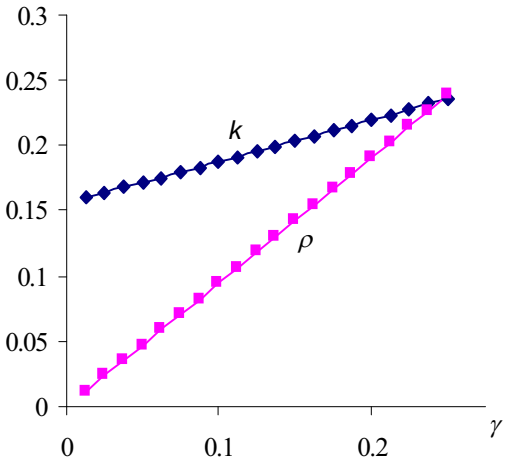

Figure 7: Capital and population in the steady state

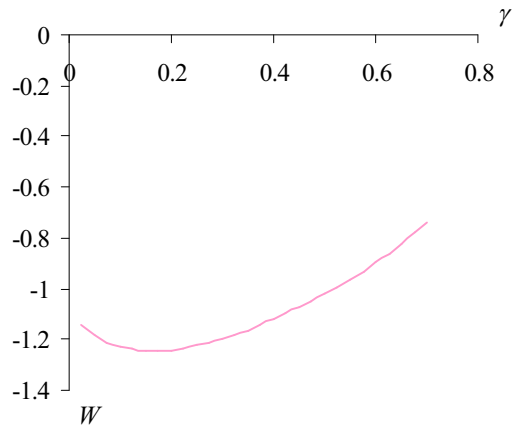

Figure 8: Welfare and incentive to give zakat

and observe that in a steady state this can be written as

$$W = \frac{1}{1+\rho}U^r + \frac{\rho}{1+\rho}U^p. \tag{43}$$

Figure 8 shows that relationship between zakat and per capita welfare. The U-shape is explained by the competing factors involved: as γ is increased the level of the steady-state capital-labor ratio rises but there is also an increased proportion of poor consumers. The effect of an intermediate value of γ is the existence of a significant proportion of poor consumers with a low level of welfare. The figure demonstrates the result that if the institution of zakat is poorly supported then making only a modest improvement to the extent of giving may reduce welfare.

These results have demonstrated that there can be a sense in which the institution of zakat increases the capital-labor ratio. A stronger preference for giving can raise both the rate of increase of the capital-labor ratio and its steady-state value. An equivalent outcome can be obtained using an income tax but an economy with zakat behaves differently to one with wealth taxation. The final point to stress is that the welfare effects are not so clear-cut. A strengthening of the institution of zakat increases both the capital-labor ratio and the proportion of the poor in the population. These two features have off-setting implications for welfare. For a utilitarian social welfare function the outcome is a non-monotonic relationship between the strength of the institution and the level of welfare.

4 Zakat as a Social Custom

A second explanation of voluntary giving is that it arises as the outcome of a social custom. The basic idea of a social custom is that those who do not act in the socially-approved manner suffer a utility loss relative to those who do act correctly (Myles and Naylor, 1996). If the population is heterogenous with respect to the size of this utility loss then those for which it is large will abide by the social custom whereas those for which it is small will not. This process partitions the population, with the point of partition endogenously determined in equilibrium.

This idea is now applied to zakat by assuming that the rich can choose whether or not to provide zakat. If they choose not to provide then they suffer a loss of utility as a consequence of the social custom. The choice of whether to provide is determined by comparing the utility obtained with no provision of zakat (higher consumption but a loss through the social custom) to the utility obtained when zakat is provided (lower consumption but no social custom loss). Whichever of these utilities is larger determines the chosen action.

Each rich consumer is characterized by a value of the utility loss from breaking the social custom. Denote the utility loss from breaking the social custom by χ. The distribution of χ across the population is given by $f(\chi)$, with strictly positive support on the interval $[\chi, \bar{\chi}]$. The cumulative distribution is given by $F(\chi)$.

Consider first the decision problem of a rich consumer who chooses to provide zakat. It is assumed that if a consumer chooses to provide zakat then the payment is made in accordance with the rules. If ϕ is the rate of zakat on wealth the optimization for a zakat payer is

$$\max_{\{x_{t,t}, x_{t,t+1}\}} U(x_{t,t}, x_{t,t+1}) \quad \text{s.t. } x_{t,t} + \frac{x_{t,t+1}}{[1+r_{t+1}][1-\phi]} = w_t. \tag{44}$$

Denote the maximum value function for the optimization in (44) by

$$V_z(w_t, r_{t+1}, \phi). \tag{45}$$

The decision problem of a rich consumer who does not provide zakat is

$$\max_{\{x_{t,t}, x_{t,t+1}\}} U(x_{t,t}, x_{t,t+1}) - \chi \quad \text{s.t. } x_{t,t} + \frac{x_{t,t+1}}{1+r_{t+1}} = w_t. \tag{46}$$

The maximum value function for the this optimization is denoted

$$V_0(w_t, r_{t+1}) - \chi. \tag{47}$$

Comparing the two maximum value functions, the consumer will provide zakat if

$$V_z(w_t, r_{t+1}, \phi) > V_0(w_t, r_{t+1}) - \chi, \tag{48}$$

but not provide zakat otherwise.

For given values of w_t and r_{t+1} there will be value χ^* at which

$$V_0\left(w_t, r_{t+1}\right) - \chi^* = V_z\left(w_t, r_{t+1}, \phi\right). \tag{49}$$

This value of χ^* partitions the population so that all those with $\chi < \chi^*$ do not pay zakat but all those with $\chi > \chi^*$ pay zakat. The value of χ^* is dependent upon the parameters facing the consumers, so

$$\chi^* = \chi\left(w_t, r_{t+1}, \phi\right). \tag{50}$$

The three types of consumer in the economy by p, d, and g; these are the poor, the rich that do not give, and the rich that give. The proportion of the rich that are of type g is $1 - F\left(\chi^*\right)$. Repeating the derivation that lead to (20) and (22) the time path of the capital-labor ratio is generated by

$$k_{t+1} = \frac{w_t - F\left(\chi^*\right) x_{t,t}^d + \left[1 - F\left(\chi^*\right)\right] x_{t,t}^g}{\left[1 + n^r\right] + \left[\left[1 - F\left(\chi^*\right)\right] z_{t+1} + \rho_t \ell^p w_t\right] \frac{\ell^p}{v}}, \tag{51}$$

and the time path of the population proportion is

$$\rho_{t+1} = \frac{\ell^p w_t}{v\left[1 + n^r\right]} \rho_t + \frac{\left[1 - F\left(\chi^*\right)\right] z_{t+1}}{v\left[1 + n^r\right]}, \tag{52}$$

with χ^* determined by (50), w_t by (13) and r_{t+1} by (14).

Figures 9 and 10 show the outcome of simulating the economy with a social custom. The simulation assumes a Cobb-Douglas utility function $U^i = x_{t,t}^i x_{t,t+1}^i$, $i = p, d, g$, with parameters $\alpha = \beta = 0.25$. The social custom utility loss, χ, is assumed to be uniformly distributed on the interval $[0, 2]$. Other parameter values are identical to those in the previous simulation.

Figure 9 shows the growth of the capital-labor ratio with the social custom. The economy is simulated for values of ϕ from 0.1 to 0.9 in steps of 0.1. The capital-labor ratio is decreased as the rate of zakat increases but, as the close bunching of the individual curves shows, changing the rate of zakat has little effect upon either the growth rate of the capital-labor ratio or the steady-state value.

Figure 10 shows the time-path of participation in the social custom. The effect of raising the rate of zakat is to reduce participation so that a higher value of ϕ results in a smaller proportion of the population providing zakat. It is this reduced participation that explains the fall in the capital-labor ratio.

This analysis of zakat as a social custom shows that participation in the provision of zakat changes with the rate of zakat to leave the capital-labor ratio barely affected. In this formulation there is little difference in the path of capital accumulation between an economy with the institution of zakat and one without. An increased rate of zakat discourages participation despite the social custom..

5 A Combined Model

The final analysis of zakat combines the warm-glow and the social custom models. The basic assumption is that the social custom utility is obtained by providing the minimum required level of zakat. But, since the amount of zakat

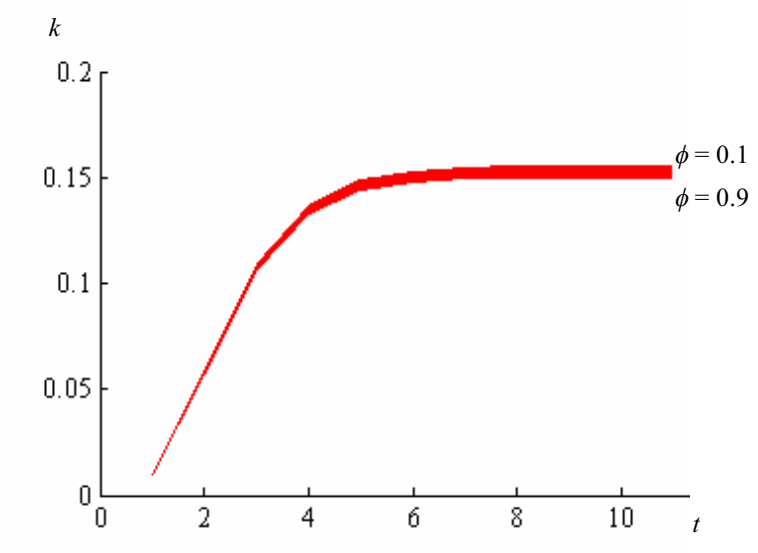

Figure 9: Capital-labor ratio: social custom

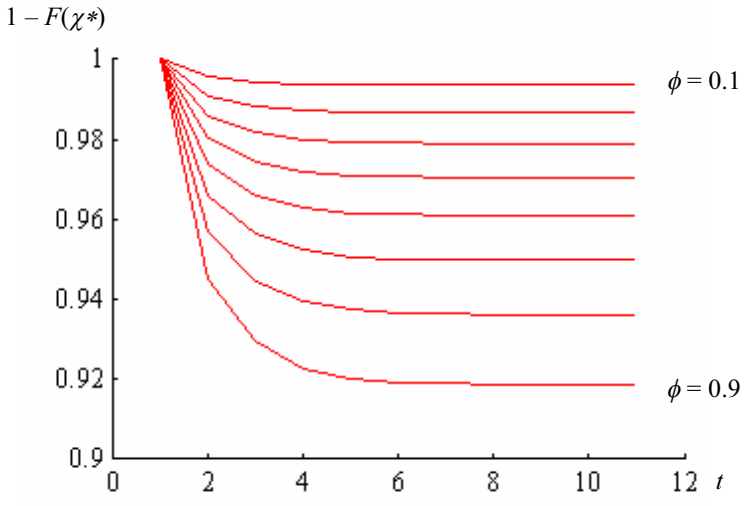

Figure 10: Participation in social custom

provided is voluntary, a warm-glow is obtained by paying in excess of the minimum level.

5.1 Economy

The economy contains poor and rich consumers. The description of the poor remains as in the previous sections. Rich consumers differ in the value of the social custom utility and the quantity of labor they can provide. Hence, each consumer is characterized by a pair $\{\ell^r, \chi\}$. The distribution of ℓ^r and χ is described by the function $f(\ell^r, \chi)$ which has strictly positive support on the set $\left[\underline{\ell^r}, \overline{\ell^r}\right] \times \left[\underline{\chi}, \overline{\chi}\right]$.

The utility of a rich consumer who does not provide zakat is

$$U^d = U\left(x^d_{t,t}, x^d_{t,t+1}\right) - \chi. \tag{53}$$

The value function corresponding to this utility is defined by

$$V_0(w_t, r_{t+1}; \ell^r) - \chi \equiv \max_{\{x^d_{t,t}, x^d_{t,t+1}\}} \left\{ \begin{array}{c} U\left(x^d_{t,t}, x^d_{t,t+1}\right) - \chi \\ \text{s.t. } x^d_{t,t} + \frac{x^d_{t,t+1}}{1+r_{t+1}} = w_t \ell^r \end{array} \right\} \tag{54}$$

The utility for a consumer that does provide zakat utility includes a warm glow from providing in excess of the minimum

$$U^g = U\left(x^g_{t,t}, x^g_{t,t+1}\right) + \Gamma\left(\xi_{t+1}\right), \tag{55}$$

where $z_{t+1} = \xi_{t+1} + \phi(1+r)s_t$. It is assumed that $\Gamma(0) = 0$ and $\Gamma'\left(\xi_{t+1}\right) > 0$. The optimization to determine the provision of zakat is

$$\max_{\{x^g_{t,t}, x^g_{t,t+1}, z_{t+1}\}} \left\{ \begin{array}{c} U\left(x^g_{t,t}, x^g_{t,t+1}\right) + \Gamma\left(\xi_{t+1}\right), \\ \text{s.t. } x^g_{t,t} + \frac{x^g_{t,t+1} + \xi_{t+1}}{[1+r_{t+1}][1-\phi]} = w_t \ell^r, \\ \xi_{t+1} \geq 0. \end{array} \right\} \tag{56}$$

Define $V_z(w_t, r_{t+1}, \phi; \ell^r)$ as the value function for the optimization in (56).

If $U\left(x^g_{t,t}, x^g_{t,t+1}\right)$ and $\Gamma\left(\xi_{t+1}\right)$ are concave then ξ_{t+1} is increasing in $w_t \ell^r$ whenever it is strictly positive. Consequently, there is a value $\hat{\ell}^r$ such that if $\ell^r \leq \hat{\ell}^r$ then $z_{t+1} = \phi(1+r_{t+1})s_t$ and if $\ell^r > \hat{\ell}^r$ then $z_{t+1} > \phi(1+r_{t+1})s_t$. Rich consumers with labor supply above $\hat{\ell}^r$ provide zakat above the minimum (if they choose to provide at all) and benefit from the warm-glow. The value of $\hat{\ell}^r$ is independent of χ, so the attainment of the warm-glow is separate from the decision on whether to provide.

A consumer $\{\ell^r, \chi\}$ will provide zakat if

$$V_z(w_t, r_{t+1}, \phi; \ell^r) > V_0(w_t, r_{t+1}; \ell^r) - \chi. \tag{57}$$

If this inequality is satisfied the level of z_{t+1} is determined by the optimization program (56). The right-hand side of (57) is linear in χ and the left-hand side independent of χ, so there exists a function $\chi = \chi(\ell^r)$ defined implicitly by

$$V_z(w_t, r_{t+1}, \phi; \ell^r) = V_0(w_t, r_{t+1}; \ell^r) - \chi, \tag{58}$$

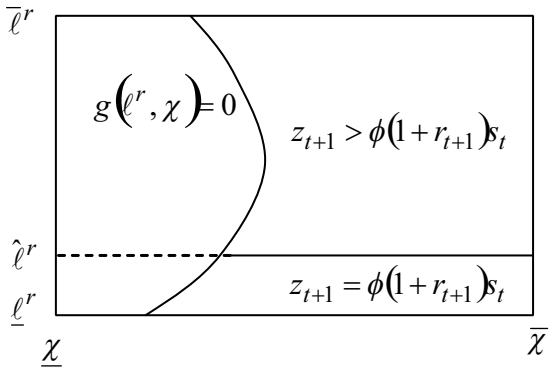

Figure 11: Division of population: warm glow and social custom

such that a rich consumer with labor supply ℓ^r will provide zakat if $\chi > \chi(\ell^r)$ but will not provide otherwise. Notice that for given χ there need not be a unique solution in ℓ^r to (58): the equation involves the intersections of two concave functions so that it is possible for them to intersect multiple times.

To give the analysis content it is assumed that $\underline{\ell}^r < \widehat{\ell}^r$. This implies that consumers with labor supply $\ell^r \in \left[\underline{\ell}^r, \widehat{\ell}^r\right]$ and $\chi > \chi(\ell^r)$ will provide zakat equal to $\phi(1 + r_{t+1})s_t$ and will obtain no warm-glow. Those with labor supply $\ell^r \in \left(\widehat{\ell}^r, \ell^r\right]$ and $\chi > \chi(\ell^r)$ will provide zakat to a level $z_{t+1} > \phi(1 + r_{t+1})s_t$. These results are summarized in Figure 11 that describes the separation of the population. The key point of the figure is that there need not be monotonicity in the relation between χ and ℓ^r.

The aggregate values of demand and provision of zakat are computed by integrating over the population taking into account the critical values that separate the different behavior regions. To do this it is necessary to define the function $C(\ell^r)$ by

$$C(\ell^r) = \min\left\{\max\left\{\chi(\ell^r), \underline{\chi}\right\}, \overline{\chi}\right\}. \tag{59}$$

Using (59) the total amount of zakat provided is determined as

$$Z_{t+1} = N_t^r \int_{\underline{\ell}^r}^{\bar{\ell}^r} \int_{C(\ell^r)}^{\overline{\chi}} z_{t+1} f d\chi d\ell^r. \tag{60}$$

The aggregate consumption levels in the two periods of life are

$$X_{t,t}^r = N_t^r \left[\int_{\underline{\ell}^r}^{\bar{\ell}^r} \int_{\underline{\chi}}^{C(\ell^r)} x_{t,t}^d f d\chi d\ell^r + \int_{\underline{\ell}^r}^{\bar{\ell}^r} \int_{C(\ell^r)}^{\overline{\chi}} x_{t,t}^g f d\chi d\ell^r\right], \tag{61}$$

and

$$X^r_{t,t+1} = N^r_t \left[\int_{\underline{\ell}^r}^{\bar{\ell}^r} \int_{\underline{\chi}}^{C(\ell^r)} x^d_{t,t+1} f d\chi d\ell^r + \int_{\underline{\ell}^r}^{\bar{\ell}^r} \int_{C(\ell^r)}^{\overline{\chi}} x^g_{t,t+1} f d\chi d\ell^r \right], \quad (62)$$

Aggregate labor income is

$$W_t = N^r_t w_t \int_{\underline{\ell}^r}^{\bar{\ell}^r} \int_{\underline{\chi}}^{\overline{\chi}} \ell^r f d\chi d\ell^r, \quad (63)$$

and aggregate labor supply of the rich

$$L^r_t = N^r_t \int_{\underline{\ell}^r}^{\bar{\ell}^r} \int_{\underline{\chi}}^{\overline{\chi}} \ell^r f d\chi d\ell^r. \quad (64)$$

The dyanamics of the capital-labor ratio and the population proportion can now be derived. The capital stock is determined by

$$K_{t+1} = \left[W_t - X^r_{t,t} \right], \quad (65)$$

so that the capital-labor ratio is

$$k_{t+1} = \frac{W_t - X^r_{t,t}}{L^r_{t+1} + L^p_{t+1}}, \quad (66)$$

where L^p_{t+1} is the aggregate labor supply of the poor. Using the definition of the aggregates and the growth rate of the poor population this becomes

$$k_{t+1} = \frac{w_t \int_{\underline{\ell}^r}^{\bar{\ell}^r} \int_{\underline{\chi}}^{\overline{\chi}} \ell^r f d\chi d\ell^r - \int_{\underline{\ell}^r}^{\bar{\ell}^r} \int_{\underline{\chi}}^{C(\ell^r)} x^d_{t,t} f d\chi d\ell^r - \int_{\underline{\ell}^r}^{\bar{\ell}^r} \int_{C(\ell^r)}^{\overline{\chi}} x^g_{t,t} f d\chi d\ell^r}{[1+n^r] \int_{\underline{\ell}^r}^{\bar{\ell}^r} \int_{\underline{\chi}}^{\overline{\chi}} \ell^r f d\chi d\ell^r + \left[\int_{\underline{\ell}^r}^{\bar{\ell}^r} \int_{C(\ell^r)}^{\overline{\chi}} z_{t+1} f d\chi d\ell^r + \rho_t \ell^p w_t \right] \frac{\ell^p}{v}}. \quad (67)$$

The dynamics of the population are given by

$$\rho_{t+1} = \frac{\ell^p w_t}{v\left[1+n^r\right]} \rho_t + \frac{\int_{\underline{\ell}^r}^{\bar{\ell}^r} \int_{C(\ell^r)}^{\overline{\chi}} z_{t+1} f d\chi d\ell^r}{v\left[1+n^r\right]}. \quad (68)$$

5.2 Simulation

The simulation of the economy summarized in (67) and (68) is based on the quasi-linear utility function for rich consumers

$$U^r_t = \left[x^g_{t,t}\right]^\alpha \left[x^g_{t,t+1}\right]^\beta + \gamma \xi_{t+1}. \quad (69)$$

With this utility it follows that $\xi_{t+1} > 0$ when the Lagrange multiplier, λ_t, for the optimization with $\xi_{t+1} = 0$ is less than $\gamma\left(1+r_{t+1}\right)\left(1-\phi\right)$. This holds for $\ell^r > \hat{\ell}^r$ where

$$\hat{\ell}^r = \frac{\alpha+\beta}{w_t} \left(\frac{\alpha^\alpha \beta^\beta}{\gamma\left(1+r_{t+1}\right)^{1-\beta}\left(1-\phi\right)^{1-\beta}} \right)^{1/1-\alpha-\beta}. \quad (70)$$

The value function can be solved as

$$V_z^t = \begin{cases} \frac{\alpha^\alpha \beta^\beta (w_t \ell^r)^{\alpha+\beta} (1+r_{t+1})^\beta (1-\phi)^\beta}{(\alpha+\beta)^{\alpha+\beta}}, & \ell^r \leq \hat{\ell}^r, \\ \frac{\alpha^\alpha \beta^\beta \left(w_t \hat{\ell}^r\right)^{\alpha+\beta} (1+r_{t+1})^\beta (1-\phi)^\beta}{(\alpha+\beta)^{\alpha+\beta}} & \\ +\gamma w_t \left(1+r_{t+1}\right)(1-\phi)\left[\ell^r - \hat{\ell}^r\right], & \hat{\ell}^r < \ell^r. \end{cases} \tag{71}$$

The value function from not providing zakat is given by

$$V_0^t = \frac{\alpha^\alpha \beta^\beta \left(w_t \ell^r\right)^{\alpha+\beta} \left(1+r_{t+1}\right)^\beta}{(\alpha+\beta)^{\alpha+\beta}}. \tag{72}$$

From these value functions it follows that

$$\chi(\ell^r) = \begin{cases} \frac{\alpha^\alpha \beta^\beta (w_t \ell^r)^{\alpha+\beta} (1+r_{t+1})^\beta \left[1-[1-\phi]^\beta\right]}{[\alpha+\beta]^{\alpha+\beta}}, & \ell^r \leq \hat{\ell}^r, \\ \frac{\alpha^\alpha \beta^\beta (1+r_{t+1})^\beta \left[(w_t \ell^r)^{\alpha+\beta} - \left(w_t \hat{\ell}^r\right)^{\alpha+\beta} [1-\phi]^\beta\right]}{[\alpha+\beta]^{\alpha+\beta}} & \\ -\gamma w_t \left[\ell^r - \hat{\ell}^r\right][1+r_{t+1}][1-\phi], & \hat{\ell}^r < \ell^r, \end{cases} . \tag{73}$$

The simulation of this economy considers the effect of changing the rate of zakat. The parameter values remain as in last section. In addition, the parameter on the warm glow is $\gamma = 0.15$, the minimum and maximum values of labor supply are $\underline{\ell}^r = 1$ and $\overline{\ell}^r = 10$, and the upper and lower values of the social custom are $\underline{\chi} = 0$, $\overline{\chi} = 0.5$.

Figure 12 shows the time path of the capital-labor ratio for four values of the rate for zakat. The important observation is that the capital-labor ratio is reduced as the rate of zakat increases. This conforms with the outcome in the economy with just a social custom. Therefore, the institution of zakat reduces the steady-state capital-labor ratio in this economy compared to the economy without zakat.

Figure 13 shows the time path of the mean value of the $\chi(\ell^r)$ over the interval $\left[\ell^r, \bar{\ell}^r\right]$. The mean value, χ_t^m, increases as the rate of zakat increases which shows there is less participation in the provision of zakat. This is the explanation for the capital-labor ratio falling as the rate of zakat increases.

The final simulation considers the effect of increasing the utility obtained from the warm-glow upon the capital-labor ratio. Figure 14 shows that the rate of growth of the capital-labor ratio, and the steady-state value, increase with γ. Therefore, increased utility from the warm-glow raises the capital-labor ratio. This conforms to the result obtained from the economy with only the warm-glow.

The results of this section have been derived for an economy with a warm-glow and a social custom. An increase in the rate of zakat is met with a reduction in the participation in the social custom sufficient to reduce the capital-labor ratio. In contrast, an increase in the utility from the warm-glow raises the capital labor ratio. In results not reported here it has also been found that increasing the disutility from not paying zakat (formally, a right-shift in the interval $[\chi, \bar{\chi}]$)

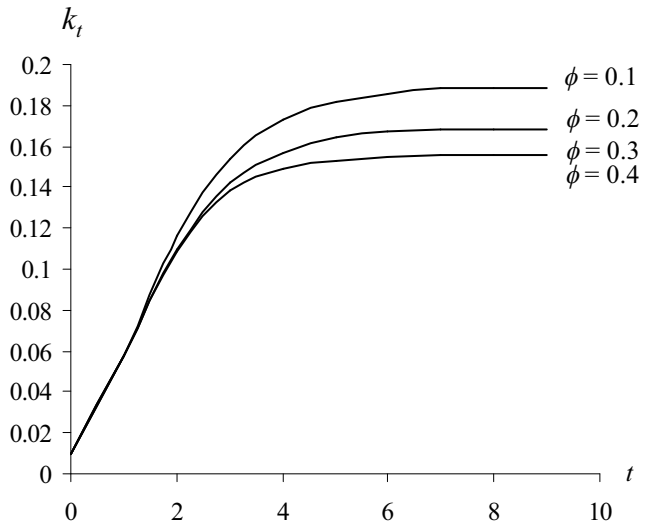

Figure 12: Time path of capital-labor ratio

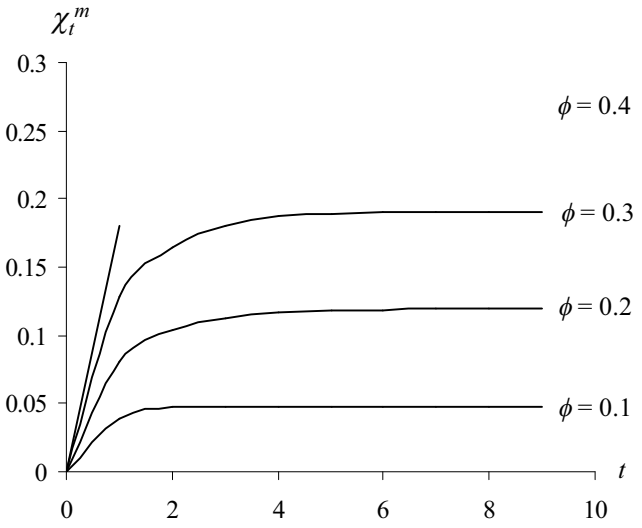

Figure 13: Time path of participation in zakat

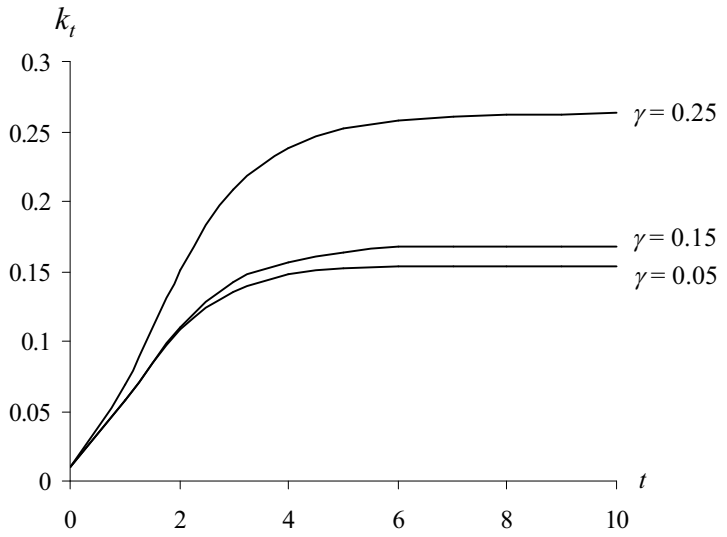

Figure 14: Capital-labor ratio and warm-glow

has very little effect on the capital-labor ratio. These results show that in an economy with the institution of zakat the most effective means of raising the capital-labor ratio is to enhance the warm-glow obtained from providing zakat.

6 Conclusions

The paper has considered the effects of the institution of zakat on capital accumulation. Zakat has been interpreted as a voluntary system of redistribution supported by the concept of religious obligation. It has been modeled as an alternative to redistribution financed by statutory taxation. Two alternative motivation for providing a voluntary payment of zakat have been analyzed. The warm-glow assumes that zakat gives a private utility return. The social custom assumes that there is a utility loss to not paying. These motives for giving zakat have been embedded within an overlapping generations model with heterogenous consumers and endogenous population growth.

With a warm glow the stronger is the preference for zakat the faster the capital-labor ratio grows and the higher it is in the steady state. This provides a sense in which the previous claims about zakat raising the capital stock are correct. The results from the social custom model are somewhat different. In this case the higher the rate of zakat the slower is the rate of growth of capital-labor ratio and the lower is the steady state level. Hence, the existence of a

social custom for providing zakat reduces the capital-labor ratio. These results carry over to a model that combines both the warm glow of giving and the social custom. A stronger preference for the warm-glow raises the capital-labor ratio but a higher rate of zakat reduces capital-labor ratio through reduced participation in the social custom.

The paper has explored two senses in which the question of a strengthening of the institution of zakat can be interpreted. The results show that increasing the perception of the warm-glow from providing zakat is more effective than raising the rate of zakat. In addition, we would like to claim that the analysis has demonstrated that the economic consequences of a religious obligation can be addressed by adapting existing economic models. The warm-glow and the social custom can explain voluntary contribution, and can be interpreted in terms that accord with the basic religious roots of the institution of zakat.

References

Ahmad, K. (1980) "Economic Development in an Islamic Framework", in Ahmad, K. (ed.), *Studies in an Islamic Economics*, Leicester: The Islamic Foundation.

Alexander, L., Coleman, R., Bennett, E., Dam, K. W., Delaney, H.R.J. and Dennis, K. O., (1997) *Giving better – giving smarter: The report of the National Commission on Philanthropy and Civic Renewal*, Washington: National Commission on Philanthropy and Civic Renewal.

Al-Abdin, A.Z. (1997) "Zakat and the alleviation of poverty in the Muslim world", *Hamdard Islammicus*, **20**, 65 – 82.

Al-Qardawi, Y. (1997) *Fiqh-Az-Zakat- A Comparative Study*, English Translation by Monzer Kahf, London:, Dar-Al Taqwa Ltd.

Andreoni, J. (1990) "Impure altruism and donations to public goods: A theory of warm-glow giving", *Economic Journal*, **100**, 464 – 477.

Chapra, M. U. (1992) *Islam and the Economic Challenge*, Leicester: The Islamic Foundation.

El-Ashkar, A. (1987) *The Islamic Business Enterprise*, London: Croom Helm.

Iqbal, M. (1985) "Zakat, moderation and aggregate consumption in an Islamic economy", *Journal of Research in Islamic Economics*, **3**, 45 – 61.

Kahf, M. (1980) "A Contribution to the Theory of Consumer Behaviour in an Islamic Society", in Ahmad, K (ed.), *Studies in Islamic Economics*, Leicester: The Islamic Foundation.

Kahf, M., (1997) *Economics of Zakat Book of Reading No.*2, Jeddah:, Islamic Development Bank.

Kahf, M. (1999) "The Performance of the Institution of Zakah in Theory and Practice", Presented at the International Conference on Islamic Economics towards the 21st century, Kuala Lumpur, 1999.

Kamali, M.H. (1999). Freedom Equity and Justice in Islam, Petaling Jaya, Ilmiah Publishers Sdn Bhd.

Khan, M.F. (1984) "Macro consumption function in an Islamic framework", *Journal of Research in Islamic Economics*, **1**, 1 – 24.

Kuran, T. (2006) *Islam and Mammon: The Economics Predicaments of Islamism*, Oxfordshire: Princeton University Press.

Mannan, M.A. (1986) *Islamic Economics: Theory and Practice*, London: Hodder and Stoughton.

Myles, G.D. and Naylor. R.A. (1996) "A model of tax evasion with group conformity and social customs", *European Journal of Political Economy*, **12**, 49 – 66.

Rahman,M. (2003), Zakat Calculation: Primarily Based on Fiqh-uz-Zakat by Yusuf-al-Qardawi, Leicester, UK, The Islamic Foundation.

Shaik, A.A. (1980) "Concept of Zakat: A Survey of Quranic Texts and their Explanations in *Shariah and Contemporary Economics*", in Zaman, M.R (ed.), *Some Aspects of the Economics of Zakat*, Indiana: American Trust Publications.

Siddiqi, S. A. (1979) "Zakat", in Sh Muhammad Ashraf (ed.), *Public Finance in Islam*, Lahore: Islamic Publications.

Zaman, R. (1980) "Policy Implications of Introducing Zakah into Bangladesh and into Saudi Arabia", in Zaman, M.R (ed.), *Some Aspects of the Economics of Zakat*, Indiana: American Trust Publications.

Zayas, F, G (1960) *The Law and Philosophy of Zakat*, Damascus: Al-Jadidah Press.