Optimal Fiscal Policy in a Model of Firm Entry and Financial Frictions

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Optimal Fiscal Policy in a Model of Firm Entry and Financial Frictions[*]

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Abstract: This paper develops a general equilibrium model of firm entry and financial frictions. Movements in the volatility of firm-level shocks and aggregate productivity generate procyclical entry and a countercyclical firm default rate. We derive analytical results for optimal fiscal policy and show that the government faces two trade-offs. The first arises from a profit destruction and a consumer surplus effect when firm entry is endogenous. The second arises because financial frictions reduce firm entry and default is costly. We also study the optimal mix of taxes on labor-income and firm profits in a quantitative version of the model. We find that a countercyclical labor-income tax is always part of the optimal fiscal policy, whereas the cyclicality of the profit tax is sensitive to the source of aggregate fluctuations.

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1. Introduction

This paper studies optimal fiscal policy in a model of firm entry and financial frictions. We first develop a general equilibrium model with procyclical firm entry and a countercyclical firm default rate.[1] Next, we provide analytical expressions for optimal fiscal policy and show that the government faces two trade-offs. The first arises from a profit destruction and a consumer surplus effect when firm entry is endogenous. The second arises because financial frictions reduce firm entry and default is costly. We then study the optimal mix of taxes on labor-income and firm profits in a quantitative version of the model. We find that a countercyclical labor-income tax is always part of the optimal fiscal policy, whereas the cyclicality of the profit tax is sensitive to the source of aggregate fluctuations.

The general equilibrium model we develop features a continuum of firms subject to idiosyncratic demand shocks. Each firm produces a differentiated good and competes in a monopolistically competitive market. Firms face a technological constraint when transferring resources to households that provide labor services and factor payments are financed by working capital loans provided by financial intermediaries with an imperfect monitoring technology.[2] In this environment, firms that produce goods with a relatively low level of ex-post demand default and firm entry and exit are jointly determined. Financial frictions thus alter the macroeconomic effects of shocks to firm-level volatility and aggregate productivity through the response of firm entry.[3]

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[1]Lee and Mukoyama (2015) find that the firm entry rate is more cyclical than the exit rate (based on US manufacturing data). Giesecke *et al.* (2011) report that the value-weighted default rate for US non-financial firms rises in recessions. Historical default rates, such as those published by Moody's, and which use issuer-weighted default rates, show the same pattern. See Exhibit 5 in Ou *et al.* (2011).

[2]The financial frictions in our model differ from Bernanke *et al.* (1999) in that firms are monopoly suppliers of a differentiated good. Imperfect monitoring also interacts with the working capital constraint, similar to Jermann and Quadrini (2012).

[3]It is well-established that firm entry plays an important role in influencing aggregate fluctuations. For

Consider the implications of an increase in the volatility of firm-level shocks for the production decision of an individual firm.[4] When volatility rises, the firm attempts to take advantage of a potentially good realization of demand by hiring labor and expanding production. It is optimal for the firm to do this because the market is monopolistically competitive and there is uncertainty with respect to the price at which a firm can sell it's good.[5] Expanding production, however, amounts to committing to a greater level of borrowing, in advance, and increased borrowing requires each firm to generate more revenue to avoid default. As the volatility of firm-level shocks rises, so does the minimum (default-threshold) level of demand required for an individual firm to be able to repay its loan. Increasing firm-level volatility therefore leads to a rise in the rate of firm default.

New firms are created each period by paying a one-time cost and firms enter until their expected profit, conditional on not defaulting, is sufficient to cover the cost of entry. An increase in volatility affects the firm entry decision through two off-setting channels. A greater level of borrowing and an increased probability of default lead to a drop firm entry, whereas the possibility of increased profit, via a good realization of the demand, encourages entry.[6] The default channel generated by financial frictions dominates, and in equilibrium, an increase in the volatility of firm-level shocks leads to an economy with fewer, more indebted firms, alongside lower aggregate employment and a lower wage rate.

We begin by analyzing optimal fiscal policy when lump-sum taxation is available and show

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example, Gourio *et al.* (2016) show that reduced firm entry leads to persistent negative effects on GDP.

[4]Our analytical results focus on exogenous movements in the volatility of idiosyncratic (firm-level) shocks. Our approach is supported by the DSGE and VAR-based evidence in Christiano *et al.* (2014) and Caldara *et al.* (2016) respectively. Also see the discussion in Bloom *et al.* (2011).

[5]This is a direct consequence of uncertainty over the idiosyncratic level of demand. The effect of price uncertainty on factor inputs, for a competitive firm, is analysed in, for example, Sandmo (1971) and Hartman (1972). What matters for our analysis is that revenue (profit) is concave in prices.

[6]There is a parallel between firm entry in our model and the decision to serve a second market once an entry decision has been made. See Garetto and Fillat (2015).

that the government faces two trade-offs.[7] The first stems from changes in profit per-firm (a profit destruction effect) and in product variety (a consumer surplus effect) when firm entry is endogenous. This is the firm entry trade-off. Without financial frictions, firm entry should be subsidized because the profit destruction effect is relatively weaker than the consumer surplus effect.[8] The second trade-off is a direct consequence of financial frictions - the financial frictions trade-off. In this case, a subsidy should be used to mitigate the reduction in firm entry associated with financial frictions but taxation is required to offset for the resource implications of firm default. Overall, the strength of taxation is determined by the costs of firm default and firm creation.

The financial-frictions trade-off we identify is captured by a single statistic: the endogenous default-threshold level of demand. This also determines the aggregate default rate in the economy. We relate optimal fiscal policy to this statistic and show analytically how the taxation of firm profits should react to movements in the volatility of firm-level shocks. Our main result is that the taxation of profits should be pro-cyclical when default is sufficiently costly. An increase in the volatility of firm-level shocks causes a recession in which firm entry falls and the default-threshold level of demand and firm default rate rise. Raising taxes is optimal because, if the government subsidizes firms in an attempt to mitigate the impact of financial frictions on firm entry, the resource costs associated with default increase.[9]

We extend the baseline model to a dynamic setting and undertake a quantitative analysis with firm-level volatility and aggregate productivity shocks. In the calibrated version of the model, a positive one-time one-standard-deviation shock to the firm-level volatility generates

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[7]In the analytical version of our model we abstract from the public finance aspects of optimal fiscal policy we consider in the quantitative analysis.

[8]This result is an artifact of the specification of entry costs and the form of preferences. It is not central to the results we derive for optimal stabilization policy.

[9]In our model defaults cause deadweight losses to society. Glover (2016) shows that the average firm expects to lose just under half of its value in default.

a 0.19 percentage point rise in the default rate on impact. We compare the response of the default rate in this volatility-induced recession to a recession induced by a negative one-standard-deviation shock to aggregate productivity. We find that the initial increase in default is larger (at 0.59 percentage points) and is less persistent. The differential response of default across recessions is to due to the transmission mechanism of shocks. A persistent increase in volatility encourages firms to expand and take on more debt, whereas a persistent drop in productivity, only operates - in a quantitative sense - through it's unanticipated component.[10]

We then revisit the results on fiscal policy and study the optimal mix of taxes on labor-income and firm profits when the government issues state-contingent real debt. Consistent with the analytical results, we find that taxes on firm profits should rise in a recession if movements in volatility are the source of aggregate fluctuations. A one-standard deviation volatility shock generates a 1 percentage point increase in the the profit tax on impact. Conditioning on the size of the business cycle, productivity-induced recessions result in a similar-sized response of profit taxes, albeit in the opposite direction. By raising taxes in a volatility-induced recession it is possible to mitigate the future implications of higher default rates, whereas in a productivity-induced recession, only the initial rise in default matters. In both cases, we find taxes on labor-income are counter-cyclical, quantitatively sizable, although the initial response is short-lived.

Our results on tax policy are closely related to Chugh and Ghironi's (2015) study of optimal fiscal policy in a model of endogenous firm entry. They find that the Ramsey-optimal long-run tax on firm profits is identical to that when lump-sum taxation is available and taxes (on labor-income and firm profits) should not respond to aggregate productivity shocks when

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[10]Overall, we find that productivity (volatility) shocks generate relatively larger fluctuations in macroeconomic (financial) variables.

preferences are of Dixit-Stiglitz type.[11] The design of fiscal policy with firm entry has also been studied in environments with physical capital (Coto-Martinez *et al.*, 2007), long-run risk (Croce *et al.*, 2013), and oligopolistic competition (Colciago, 2016).[12] We contribute to this recent literature by allowing for financial frictions and focusing on the implications for short-run optimal stabilization policy arising from changes in firm-level volatility.

There is a related literature that has shown how the interaction of financial market frictions and changes in firm-level volatility play and important role in explaining aggregate fluctuations. For example, in Christiano *et al.* (2014) a widening in the distribution of productivity shocks increases the fraction of loan defaults, and in Gilchrist *et al.* (2014), financial frictions magnify shocks to firm-level volatility through movements in credit spreads. Arellano *et al.* (2012) argue that the majority of the decline in employment during the 2007-09 recession can be explained by an increase in the volatility of firm-level shocks. Our analytical and quantitative results imply that changes in the volatility of firm-level shocks not only help explain aggregate fluctuations but that the interaction of financial frictions and firm-level shocks play a role in shaping policy decisions.

Finally, our paper contributes the literature on firm entry and exit more broadly. Our approach is most similar to Bilbiie *et al.* (2012).[13] To their model of firm entry we allow for endogenous exit by incorporating ex-post firm-level heterogeneity, a working capital constraint, and financial frictions. A complementary approach to studying firm entry and

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[11]In Chugh and Ghironi (2015), the extent to which profits should be taxed (in the long-run and short-run) is discussed in the context of preference aggregation. We choose to work with a form of preferences that lead to constant long-run taxes to focus on the role of financial frictions.

[12]Lewis and Winkler (2015) also analyse tax policy with endogenous firm entry. They focus on the structure of demand and costs of firms entry.

[13]Bilbiie *et al.* (2012) assume monopolistic competition in the product market. Chatterjee and Cooper (1993) and Devereux *et al.* (1996) also develop general equilibrium models with procyclical firm entry and monopolistic competition.

exit, which amends Hopenhayn's (1992) model with ex-ante heterogeneous firms to allow for investment in physical capital and aggregate shocks, is developed by Clementi and Palazzo (2015). Our modelling choices - which imply a symmetric employment decision by firms in equilibrium - are driven by the desire to generate relatively simple policy implications. A general point, however, is that, in either setting, firm entry is a form of investment in which up-front costs incurred to start a business generate expected future profits.

The remainder of the paper is organized as follows. In section 2 we develop a general equilibrium model of firm entry and financial frictions. We show how shocks to firm-level volatility affect firm entry and the firm default rate and we derive analytical expressions for optimal fiscal policy when lump-sum taxation is available. In sections 3 and 4 we assess the quantitative impact of exogenous changes in firm-level volatility and aggregate productivity. In section 5 we revisit fiscal policy and study the optimal mix of taxes on labor-income and firm profits when the government issues state-contingent real debt. A final section concludes.

2. Static Model

In this section we develop a general equilibrium model of firm entry and financial frictions. We first discuss the setup of the model, specify the optimization problem for households and firms, and the definition of equilibrium. We then study optimal fiscal policy.

2.1. Setup

The economy is populated by a measure $n_t > 0$ of firms and a measure one of households and financial intermediaries. Each firm has a linear production technology and supplies a differentiated good with an idiosyncratic level of demand, $\varepsilon \geq 0$. New firms are created each period by paying a one-time entry cost. Households consume a basket of goods and supply labor inelastically. Financial intermediaries hold deposits from households and issue

intra-period working capital loans to firms.[14] Each firm repays it's working capital loan to a financial intermediary if it has sufficient revenue to do so. Firms default if their revenue is insufficient to repay the loan. When a firm defaults, the intermediary repossesses the assets of the firm, subject to a cost of receivership.[15]

The timing of the model is as follows. At the beginning of the period new firms are created and households place deposits with financial intermediaries. Firms then make an employment decision and sign a contract with a financial intermediary to cover their working capital requirements. Production takes place and idiosyncratic demand (and revenue) is realized. Firms with a sufficiently high level of demand, $\varepsilon \in [\varepsilon_t^\star, \infty)$, sell their goods to households. Firms with a low level of demand, $\varepsilon \in [0, \varepsilon_t^\star)$, default. Households receive net-of-tax profits from production, interest payments on deposits, and a lump-sum transfer from the government. At the end of the period all non-defaulting firms exit.

Households Each household has a constant elasticity of substitution utility function,

$$C_t = \left\{ \int_{i \in \Omega} [\varepsilon \times c_t(i)]^\theta \, di \right\}^{1/\theta} \tag{1}$$

where $c_t(i)$ is the consumption of good $i \in \Omega$ and $1/(1-\theta) > 1$ is the elasticity of substitution. The integration over the probability space Ω is $n_t \int dG(\varepsilon)$ and $G(\varepsilon)$ is the cumulative distribution function of idiosyncratic demand shocks.[16] The standard deviation of firm-level demand is denoted σ_t and we refer to unanticipated changes in this variable as firm-level volatility shocks. Households are endowed with one unit of labor that they supply inelastically and they receive wages, w_t, in units of consumption. Households also own an equal

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[14]The technological restriction we place on the transfer of resources from firms to households is similar to Neumeyer and Perri (2005).

[15]Our formulation is equivalent to all firms selling their production and the financial intermediary bearing the burden of unpaid loans.

[16]Similar to Bernard *et al.* (2011) the firm-level shock reflects product attributes or product appeal. Midrigan (2011) refers to this shock as a quality shock.

share of firms.

Firms Each firm produces a differentiated good with technology,

$$y_t(i) = l_t(i) \tag{2}$$

where $y_t(i)$ is the output and $l_t(i)$ is the employment level of firm i. Firms use working capital to finance their labor requirements. Working capital requires a loan, at gross rate $r_t \geq 1$, equal to $w_t l_t(i)$.[17] The profit of firm i, with demand level ε, is written as,

$$\pi_t(i,\varepsilon) = p_t(i,\varepsilon) y_t(i) - w_t r_t l_t(i) \tag{3}$$

where $p_t(i,\varepsilon)$ is the price of good i in units of consumption, $p_t(i,\varepsilon) y_t(i)$ is firm revenue, and $w_t r_t l_t(i)$ is the debt of firm i. Throughout the analysis we assume firms operate under limited liability and act as though profit is bounded from below at zero. This implies a threshold level of demand, $\varepsilon_t^\star$, determines the mass of firms unable to meet their debt obligations ex-post. This default-threshold level of demand is defined as $\varepsilon_t^\star \equiv \inf\{\varepsilon : \pi_t(i,\varepsilon) > 0\}$ and the probability of default is $G(\varepsilon_t^\star) = \int_0^{\varepsilon_t^\star} dG(\varepsilon)$.

There is an unbounded mass of potential entrants. The creation of a new firm is subject to a one-time entry cost. Firms enter until conditional expected profits, $\pi(\varepsilon_t^\star) \equiv \int_{\varepsilon_t^\star}^\infty \pi_t(i,\varepsilon) dG(\varepsilon)$; that is, expected profit conditional on not defaulting, net of a profit tax, $\tau_t < 1$, is equal to the cost of entry. The free entry condition reads,

$$(1 - \tau_t)\pi(\varepsilon_t^\star) = f_e \tag{4}$$

where the cost of entry, $f_e > 0$, is specified in units of output.[18] All non-defaulting firms

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[17]The interest rate on loans, r_t, is strictly greater than the interest rate on deposits. The deposit rate is exogenous in this version of the model.

[18]Firms face entry costs before starting production (for example, see Restuccia and Rogerson (2008)). As emphasized by Djankov *et al.* (2002), entry costs not only reflect the time and effort of the entrepreneur, but also bureaucratic and transactions costs required for setting up a business. Higher taxes reduce entry rates. Da Rin *et al.* (2008) present evidence on taxes an entry rates in European countries.

exit the market each period.

Financial Intermediaries Each financial intermediary receives deposits from households and issues working capital loans to firms. When a firm defaults, a financial intermediary repossesses the firms assets, subject to a cost of receivership, $\kappa > 0$. The expected assets of a financial intermediary are the revenue from the repayment of loans, the assets from liquidated firms, less the cost of receivership. Financial intermediaries are competitive and earn zero profit, which leads to,

$$\left[\int_{\varepsilon_t^\star}^{\infty} dG\left(\varepsilon\right) + \int_0^{\varepsilon_t^\star} \left(\frac{\varepsilon}{\varepsilon_t^\star} \right)^\theta dG\left(\varepsilon\right) \right] r_t - r_t^d = \kappa \left[G\left(\varepsilon_t^\star\right) / w_t l_t \right] \tag{5}$$

where $\int_{\varepsilon_t^\star}^{\infty} dG\left(\varepsilon\right)$ is the survival probability of a firm and $\int_0^{\varepsilon_t^\star} \left(\varepsilon/\varepsilon_t^\star\right)^\theta dG\left(\varepsilon_t\right)$ is the ratio of assets-to-loans of defaulting firms. The liabilities of financial intermediaries are given by $r_t^d w_t l_t$, where r_t^d is the interest rate on deposits. Equation (5) defines the interest rate on working capital loans.

2.2. Optimization

Each household chooses the consumption level, $c_t\left(i\right)$, to minimize the cost of acquiring C_t, taking prices and income as given. This leads to a standard downward-sloped demand curve for each good,

$$c_t\left(i\right) = \left[\frac{p_t\left(i, \varepsilon\right)}{\varepsilon^\theta} \right]^{-1/(1-\theta)} Y_t \tag{6}$$

where Y_t is aggregate output.

Each firm chooses an employment level, $l_t\left(i\right)$, subject to demand and technological constraints, given by equations (2) and (6), market clearing, $c_t\left(i\right) = y_t\left(i\right)$. Proposition 1 characterizes the optimal employment decision of firm i. To economize on notation, in what follows we drop the i index.

Proposition 1 Profit maximization implies firm-level employment, $l\left(\varepsilon_t^\star\right)$, and the default-threshold level of demand, $\varepsilon_t^\star$, are determined by the following conditions.

$$\left[\frac{\Delta(0)}{l\left(\varepsilon_t^\star\right)^{1/\theta}}\right]^{1-\theta}\left(\varepsilon_t^\star\right)^\theta = w_t r_t \quad \text{and} \quad \theta\int_{\varepsilon_t^\star}^\infty \varepsilon_t^\theta dG(\varepsilon_t) = \left[1 - G\left(\varepsilon_t^\star\right)\right]\left(\varepsilon_t^\star\right)^\theta \tag{7}$$

where $\Delta\left(0\right) \equiv \left[\int_0^\infty \varepsilon^\theta dG\left(\varepsilon\right)\right]^{1/\theta}$.

Proof See Appendix A. ∎

The first equation in Proposition 1 determines firm-level employment as a function of the marginal cost of production - the wage rate multiplied by the interest rate on loans - $w_t r_t$.[19] One implication of this condition is that, all else equal, a higher default-threshold level of demand, $\varepsilon_t^\star$, or an increased probability of default, $G\left(\varepsilon_t^\star\right)$, is associated with greater employment. This is because an increase in employment, at the firm-level, requires more debt, with working capital loans equal to $w_t r_t l\left(\varepsilon_t^\star\right)$. In turn, to avoid default, a more indebted firm needs to generate more revenue, and this implies a higher default-threshold level of demand.

The second expression in Proposition 1 determines the default-threshold level of demand. Products with demand $\varepsilon_t \in [0, \varepsilon_t^\star)$ are taken into receivership and products with demand $\varepsilon_t \in [\varepsilon_t^\star, \infty)$ are sold directly to consumers. An important property of this equation is that it uniquely determines the default-threshold. Thus, whilst $\varepsilon_t^\star$ is directly affected by movements in the volatility of firm-level shocks, it is unaffected by the firms optimal employment decision.[20] This simplifying property allows us to characterize analytically the relation-

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[19]In this version of the model, because of the nature of the contract with financial intermediaries, the interest rate at which loans are re-paid is independent of aggregate uncertainty. We relax this assumption in section 3.

[20]We can re-introduce the dependence of the default-threshold level of demand on firm size by allowing for overhead costs. This point is relevant for the quantitative analysis of section 4.

ship between firm-level volatility, the default-threshold, and the probability of default. We summarize the result in Proposition 2.

Proposition 2 When idiosyncratic shocks have a log normal distribution and the default-threshold level of demand is defined by $\theta \int_{\varepsilon^\star}^\infty \varepsilon_t^\theta dG(\varepsilon_t) = [1 - G(\varepsilon_t^\star)] (\varepsilon_t^\star)^\theta$, the default threshold and the probability of default increase with the volatility of firm-level shocks for $G(\varepsilon_t^\star) < 1/2$.

Proof See Appendix A. ∎

An increase in the volatility of firm-level shocks is equivalent to an increase in the standard deviation of $\log(\varepsilon)$. For an individual firm, all else equal, a higher default-threshold level of demand raises the probability of default, and is consistent with an increase in firm-size. Proposition 2 shows that the default-threshold and probability of default also increase with the volatility of firm-level shocks. At this point it is useful to define $\Delta(\varepsilon_t^\star) \equiv \left[\frac{1}{1-G(\varepsilon_t^\star)} \int_{\varepsilon_t^\star}^\infty \varepsilon^\theta dG(\varepsilon) \right]^{1/\theta}$ such that the default-threshold level of demand is implicitly determined by $[\varepsilon_t^\star / \Delta(\varepsilon_t^\star)]^\theta = \theta$. This new variable captures the relative dispersion of demand across non-defaulting firms, is a measure of conditional expected revenue, and acts as an endogenous revenue shifter. As the default-threshold rises, so does conditional expected revenue.[21]

An increase in firm-level volatility leads to a higher default-threshold level of demand and also raises conditional expected revenue. This explains why firms increase their demand for labor when there is greater underlying volatility. With monopolistic competition (such that profits are concave in prices) and flexibility in the factor input (the distribution of demand is known before employment decisions are made) firms expand to take advantage of a potentially good realization of demand which is reflected in a positive shift in conditional

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[21]Formally, we can show, $\frac{\partial [\Delta(\varepsilon_t^\star)]^\theta}{\partial \varepsilon_t^\star} = \frac{dG(\varepsilon_t^\star)}{[1-G(\varepsilon_t^\star)]^2} \int_{\varepsilon_t^\star}^\infty \left[\varepsilon^\theta - (\varepsilon_t^\star)^\theta \right] dG(\varepsilon) > 0$ where $\varepsilon_t^\star < \varepsilon$ and $\Delta(\varepsilon_t^\star) > \Delta(0)$.

expected revenue. This change in factor demand under increased uncertainty - in our case via demand shocks - is consistent with an Oi-Hartman-Abel effect (Bloom, 2014). However, because an increase in conditional expected revenue is also associated with a rising default-threshold, as volatility rises, firms are increasingly unlikely to generate sufficient revenue to repay their working capital loans. This is why the increase in volatility also implies an increase in the proportion of firms that default.

2.3. Equilibrium

Labor is used for the production of goods and labor market equilibrium requires,

$$L = n\left(\varepsilon_t^\star\right) l\left(\varepsilon_t^\star\right) \tag{8}$$

where $L = 1$. Equation (8) implies that increased employment at the firm level, $l\left(\varepsilon_t^\star\right)$, translates directly into fewer operating firms, $n\left(\varepsilon_t^\star\right)$. Finally, the resource constraint of the economy is,

$$Y_t = C_t + f_e n\left(\varepsilon_t^\star\right) + \kappa\left[n\left(\varepsilon_t^\star\right) G\left(\varepsilon_t^\star\right)\right] \tag{9}$$

where $Y_t = \left[n\left(\varepsilon_t^\star\right)\right]^{(1-\theta)/\theta} \Delta(0)$ is aggregate output, $f_e n\left(\varepsilon_t^\star\right)$ represents investment (at the extensive margin), and $\kappa\left[n\left(\varepsilon_t^\star\right) G\left(\varepsilon_t^\star\right)\right]$ is the resource cost associated with firm defaults.

We now consider the impact of a change in the volatility of firm-level shocks onto aggregate variables. An immediate result is that there are two opposing effects of increased firm-level volatility on aggregate output. Expected revenue, $\Delta(0) = \left[\int_0^\infty \varepsilon^\theta dG\left(\varepsilon\right)\right]^{1/\theta}$, which increases with volatility, raises aggregate output, whereas the mass of operating firms, $n\left(\varepsilon_t^\star\right)$, falls, and this reduces output. The size of the fall in $n\left(\varepsilon_t^\star\right)$ is determined by the firm employment decision (see Proposition 1) and its strength onto output is greater the larger are the returns-to-variety (and the larger the monopolistic markup).[22] Overall, more volatility leads to lower output and from this point on we refer to this as a volatility-induced recession.

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[22] We focus on the case in which the returns-to-variety are less than unity. The returns-to-variety are

An increase in firm-level volatility raises the default-threshold level of demand and the firm default rate is countercyclical (by Proposition 2). A higher default rate is also associated with a rise in the economy-wide level of debt. This can be seen by summing over firm profits, given by equation (3), which implies, $\pi\left(\varepsilon_t^\star\right) = \left[\Delta(0)\right]^{1-\theta} \left[n\left(\varepsilon_t^\star\right)\right]^{1-1/\theta} l\left(\varepsilon_t^\star\right) \int_{\varepsilon_t^\star}^\infty \varepsilon^\theta dG\left(\varepsilon\right) - \left[1 - G\left(\varepsilon_t^\star\right)\right] wrl\left(\varepsilon_t^\star\right)$, and then using Proposition 1 and the free entry condition - equation (4). Doing so generates the following expression, $wl\left(\varepsilon_t^\star\right) r = \theta/\left(1-\theta\right)\left(1-\tau_t\right)\left[1 - G\left(\varepsilon_t^\star\right)\right]$. Since the overall mass of operating firms falls, and at the firm-level, employment rises, a volatility-induced recession is one in which there are fewer, larger firms, and a higher level of indebtedness.

Finally, we can explain the role of financial frictions on firm profits. Expected profit is given by $\pi_t = (1 - \theta)\left[Y_t/n\left(\varepsilon_t^\star\right)\right] > 0$ and this always rises in a volatility-induced recession. Moreover, this implies that the fall in the extensive margin of investment (firm entry) is larger than the fall in aggregate output. The easiest way to understand these points is to recall that the free entry condition requires conditional expected profit equal, $f_e/\left(1-\tau\right) = \pi\left(\varepsilon_t^\star\right) = D\left(\varepsilon_t^\star\right) \times \pi_t$, where $D\left(\varepsilon_t^\star\right) \equiv \left[1 - G\left(\varepsilon_t^\star\right)\right]\left[\Delta\left(\varepsilon_t^\star\right)/\Delta(0)\right]^\theta$ is an endogenous wedge generated by the presence of financial frictions. The financial frictions wedge falls as volatility rises.[23]

2.4. Fiscal Policy

Our model explains how changes in the volatility of firm-level shocks are consistent with procyclical firm entry and a countercyclical firm default rate. So far, however, we have not discussed the role of fiscal policy. In this section, we study optimal fiscal policy by

given by $(1-\theta)/\theta > 0$; the markup minus one, and there is increasing returns to an expansion in variety with the degree $1/\theta$. We make this restriction because firm entry costs are specified in units of output.

[23]Re-write the function $D\left(\varepsilon_t^\star\right)$ in the following way, $D\left(\varepsilon_t^\star\right) = \frac{\int_{\varepsilon_t^\star}^\infty \varepsilon^\theta dG(\varepsilon)}{\int_0^\infty \varepsilon^\theta dG(\varepsilon)} = 1 - \frac{\int_0^{\varepsilon^\star} \varepsilon^\theta dG(\varepsilon)}{\int_0^\infty \varepsilon^\theta dG(\varepsilon)} < 1$. The proof that $D'\left(\varepsilon_t^\star\right)$ is immediate. In the Appendix we also show formally that $D\left(\varepsilon_t^\star\right)$ falls with volatility for $G(\varepsilon_t^\star) < 1/2$.

considering the taxation of firm profits ($\tau_t < 1$) when a lump-sum tax balances the budget each period. Specifically, we assume the government budget constraint is,

$$T_t = \tau_t \left[n \left(\varepsilon_t^\star \right) \pi \left(\varepsilon_t^\star \right) \right] \tag{10}$$

where T_t is a lump-sum transfer to the household and the right-hand side of equation (10) is total government revenue.

An important result of this section is that the government faces two trade-offs when deciding on fiscal policy. The first is relatively standard and stems from a profit destruction effect and a consumer surplus effect when firm entry is endogenous.[24] The second trade-off stems from the reduction in firm entry generated by financial frictions and the resource implications of costly default. Despite the presence of financial frictions we can reduce the optimal fiscal policy problem of the government to one of choosing the mass of operating firms to maximize consumption. Specifically,

$$\max_{n(\varepsilon_t^\star)} C_t = \left[n \left(\varepsilon_t^\star \right) \right]^{(1-\theta)/\theta} \Delta(0) - n \left(\varepsilon_t^\star \right) \left[f_e + \kappa G \left(\varepsilon_t^\star \right) \right] \tag{11}$$

The solution to the optimization problem presented in equation (11) leads to the following proposition.

Proposition 3 The optimal profit tax is,

$$\tau_t = 1 - \frac{1}{\theta} \left[\frac{1}{1 + (\kappa/f_e) G \left(\varepsilon_t^\star \right)} \right] \frac{1}{D \left(\varepsilon_t^\star \right)} \tag{12}$$

where,

$$G \left(\varepsilon_t^\star \right) = \int_0^{\varepsilon_t^\star} dG \left(\varepsilon \right) \quad \text{and} \quad D \left(\varepsilon_t^\star \right) = \frac{\int_{\varepsilon_t^\star}^\infty \varepsilon^\theta dG \left(\varepsilon \right)}{\int_0^\infty \varepsilon^\theta dG \left(\varepsilon \right)} \tag{13}$$

and f_e and κ are positive constants and $\theta^{-1} > 1$.

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[24]This terminology is taken from Grossman and Helpman (1991) and is also used by Bilbiie *et al.* (2008).

Proof See Appendix A. ∎

It is immediate from Proposition 3 that without financial frictions; that is, when $G(\varepsilon_t^\star) \to 0$ and $D(\varepsilon_t^\star) \to 1$, optimal fiscal policy would be require a subsidy to firms equal to the monopolistic markup, $1/\theta$. A subsidy is required, in this case, because the returns-to-variety outweigh the reduction in profit per-firm implied by additional entry. This trade-off (which we call the firm entry trade-off) is relatively standard and is discussed in Bilbiie *et al.* (2008) and Chugh and Ghironi (2015). From the perspective of this analysis, an important property of equation (12), and the constant elasticity of substitution preferences we assume, is that, without financial frictions, the profit tax is independent of the business cycle and there is no role for short-run stabilization policy.[25]

With financial frictions the government faces a second trade-off when deciding on the rate at which to tax firm profits. The financial-frictions trade-off is captured by a single statistic: the default-threshold level of demand. Proposition 1 shows that this statistic is independent of the rest of the economy, and as such, we can start by thinking about how a change in $\varepsilon_t^\star$ affects optimal fiscal policy. An increase in $\varepsilon_t^\star$ implies a higher default rate, $G(\varepsilon_t^\star)$, which despite the fall in the mass of operating firms, $n(\varepsilon_t^\star)$, also increases the resource costs associated with firm default. As the default rate rises, it is therefore optimal to restrict entry, relative to the case without financial frictions, and the subsidy to firms falls.[26]

The second implication of a rising default-threshold is that the financial frictions wedge - captured by the variable $D(\varepsilon_t^\star)$ - falls with $\varepsilon_t^\star$. As discussed above, the financial frictions

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[25]Bilbiie *et al.* (2008) and Chugh and Ghironi (2015) both analyse the steady-state of a dynamic model without default. The exact specification of the profit tax depends entry costs and the form of preferences.

[26]We also note that since entry costs also matter for tax policy, any regulatory policy aimed at encouraging entry should be partly offset with taxation. It is well-documented that costs of firm entry vary across countries and that these have important macroeconomic implications (Poschke, 2010 and Barseghyan and DiCecio, 2011).

wedge relates conditional expected profit to expected profit, and as $D(\varepsilon_t^\star)$ falls; that is, as $\pi_t/\pi(\varepsilon_t^\star)$ rises, financial frictions have a stronger impact on profits and firm entry. In this case, greater financial frictions call for a subsidy to firms. Subsidizing firms, however, leads to an increase in entry, and magnifies the resource costs associated with default. This is precisely why the government faces a second trade-off when deciding on fiscal policy. Higher subsidies are required because firm entry is lower under financial frictions but taxation is possible when the resource costs of default are sufficiently large.

We also determine how fiscal policy reacts over the business cycle. To understand the response of the tax on firm profits, let $\kappa \to 0$, such that only the financial frictions wedge matters for the optimal policy decision (this special case has no implications for the response of output to a volatility shock). Define $x_t^\star \equiv [\ln(\varepsilon_t^\star) - \mu]/\sigma_t$, where σ_t is the measure of firm-level volatility and μ is the location parameter of the lognormal distribution. Following an exogenous change in the volatility of firm-level shocks, the optimal change in the profit tax is given by,

$$\widehat{\tau}_t = -\frac{\Phi'\left(-(x^\star - \theta\sigma)\right)}{\theta} \times [x^\star \widehat{x}_t^\star - \theta(\sigma\widehat{\sigma}_t)] \tag{14}$$

where a caret denotes the deviation of a variable from its long-run value (with $\widehat{\tau}_t \equiv \tau_t - \tau$) and $\Phi(x^\star)$ is the CDF of the normal distribution.[27] Since τ is negative when $\kappa \to 0$, the sign of the change in the profit tax that results from a change in firm-level volatility depends on the sign of the term in square brackets on the right-hand side of equation (14). In this term, $x_t^\star$ is negative and $\widehat{x}_t^\star$ is positive (since $\widehat{\varepsilon}_t^\star > 0$). Thus, for a given increase in volatility - such that $\widehat{\sigma}_t > 0$ - the subsidy to profits increases. However, as $\kappa > 0$ rises, so do the resource implications of default. Thus, when default is sufficiently costly, shocks to firm-level volatility are associated with pro-cyclical tax policy.

3. Dynamic Model

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[27]Specifically, $\Phi(x^\star) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{x^\star} \exp\left[-(x)^2/2\right] dx$ and $\Phi'\left(-(x^\star - \theta\sigma)\right) > 0$.

In this section we develop a dynamic version of the model in which firms are long-lived. We also generalize the financial contract between firms and financial intermediaries to account for productivity shocks.[28] The most important implication of this change is that the default threshold depends on the realization of aggregate productivity. The model thus generates a strong response of the credit interest rate to both volatility and productivity shocks. Finally, we endogenize the labor supply decision and the deposit interest rate which were fixed in the model of section 2 and allow the government to issue state-contingent real debt.

3.1. Long-Lived Firms, Endogenous Labor Supply, and Government Debt

We define the net worth of a firm as $z_t(\varepsilon, a_t) \equiv \max\left[(1 - \tau_t)\,\mathbb{E}_t\pi_t(\varepsilon, a_t), 0\right] + v_t(\varepsilon_t, a_t)$, where $v_t(\varepsilon, a_t)$ is the price of the firm at the end-of-period t, after the realization of uncertainty, and a_t is aggregate productivity. Under this formulation, once a firm defaults, its value is retained and sold by the firm in the following period. The instantaneous profit function is now written as,

$$\pi_t(\varepsilon, a_t) = \max\left\{ \varepsilon^\theta \times \left[n^{1/\theta}\Delta(0)\bar{l}_t\right]^{1-\theta} a_t l_t^\theta - w_t r_t(l_t + f_o), 0 \right\} \tag{15}$$

which is a generalization of equation (3) with production technology $y_t = a_t l_t$. In equation (15), the term $f_o > 0$ is a quasi-fixed overhead cost, and $\bar{l}_t$ is average firm-level employment, which is taken as given by the firm when maximizing net worth.[29] Throughout this section, due to presence of aggregate productivity shocks, and the specification of the financial contract, the default threshold and credit interest rate are such that, $\varepsilon_t^\star = \varepsilon^\star(a_t)$ and $r_t = r(a_t)$. To conserve on notation we suppress this index but it should be clear that both variables are implicit functions of aggregate productivity. We summarize the optimal employment choice in the following Proposition.

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[28]See sections 3.1 and 3.2 of Bernanke *et al.* (1999) for a discussion of a financial contract with aggregate risk and productivity shocks in the canonical model of the financial accelerator.

[29]In our specification the entire wage bill is borrowed in advance. Evidence for this assumption is presented in Lewis and Poilly (2012).

Proposition 4 Profit maximization implies the following optimal level of employment.

$$\mathbb{E}_t \int_{\varepsilon_t^\star}^\infty \left[\theta a_t \varepsilon^\theta - a_t (\varepsilon_t^\star)^\theta \left(\frac{l_t}{l_t + f_o} \right) \right] dG(\varepsilon) = 0 \tag{16}$$

for $\varepsilon > \varepsilon_t^\star$ where $\varepsilon_t^\star = \varepsilon^\star(a_t)$ is determined by $(\varepsilon_t^\star)^\theta \left[n_t^{1/\theta} \Delta(0) \right]^{1-\theta} a_t l_t - w_t r_t (l_t + f_o) = 0$ and $r_t = r(a_t)$ is determined by the zero-profit condition for financial intermediaries.

Proof See Appendix A. ∎

Household intertemporal utility is,

$$\mathbb{E}_0 \sum_{t=0}^\infty \beta^t u(C_t, L_t) \tag{17}$$

where L_t is total labor supply and $\beta \in (0, 1)$ is the subjective discount factor. Period utility, $u(\cdot)$, is a standard additively separable function. It is strictly increasing and strictly concave in C_t and strictly decreasing and strictly convex in L_t.

Households place deposits, d_t, with financial intermediaries, and hold shares, x_t, in firms. They also have access to a complete set of state-contingent government bonds, B_{t+1}^s. Households maximize their lifetime utility, subject to the following flow budget constraint,

$$d_t + \sum_s \frac{1}{r_{t+1}^s} B_{t+1}^s + C_t + \frac{n_{t+1}}{1-\delta} x_t v_t = r_{t-1}^d d_{t-1} + B_t^s + \left(1 - \tau_t^L\right) w_t L_t + n_t x_{t-1} z(\varepsilon_t^\star) \tag{18}$$

where r_t^d and r_{t+1}^s are the rates of return on deposits and bonds, respectively, τ_t^L is a tax on labor-income, and $z(\varepsilon_t^\star) = \int_{\varepsilon_t^\star}^\infty z(\varepsilon, a_t) dG(\varepsilon)$ is conditional expected net worth. Household decisions over deposits, bonds, and equity are characterized by standard consumption Euler equations and labor supply is determined by a standard labor-leisure trade-off which equates the ratio of the marginal utilities of consumption and leisure to the expected wage.

Each period there is an exogenous probability of firm exit equal to $\delta \in (0, 1)$. The free entry condition reads,

$$v_0 = \mathbb{E}_0 \sum_{t=1}^\infty M_{0,t} (1 - \tau_t) \pi(\varepsilon_t^\star) = f_e \tag{19}$$

In equation (19), the term $u_C(t)$ denotes the period t marginal utility from consumption and $M_{0,t} = [\beta(1-\delta)]^t u_C(0)/u_C(t)$ is a stochastic discount factor. As before, conditional expected profit in period t is denoted by $\pi(\varepsilon_t^\star)$. The law of motion for the mass of firms is,

$$n_{t+1} = (1-\delta)(n_t + n_{e,t}) \tag{20}$$

which reflects the assumption that there is a one-period lag between entry and production.

The government collects taxes on labor-income and firm profits and issues state-contingent real debt to finance an exogenous constant stream of government spending, $\mathfrak{G} > 0$. The flow government budget constraint is,

$$\tau_t^L w_t L_t + \tau_t n_t x_{t-1} \pi(\varepsilon_t^\star) + \sum_s \frac{1}{r_{t+1}^s} B_{t+1}^s = B_t + \mathfrak{G} \tag{21}$$

where $\tau_t^L w_t L_t + \tau_t n_t x_{t-1} \pi(\varepsilon_t^\star)$ is government income from taxation.

3.2. Model Summary

Table 1 presents the equations for the model economy,

===== **Table 1 Here** =====

where the $\varepsilon_t^\star$ index has been suppressed where possible. The term $u_L(t)$ denotes the period t marginal dis-utility from labor and the term $\eta_t \equiv \int_0^{\varepsilon_t^\star} \varepsilon^\theta dG(\varepsilon)$ reflects the ratio of assets-to-loans of defaulting firms. The conditions in Table 1 form a 12 variable system which solve the model for $\{C_t, Y_t, L_t, l_t, n_t, n_{e,t}, z_t, \pi_t\}$ and $\{w_t, r_{t-1}^d, r_t\}$, and $\{\varepsilon_t^\star\}$, with given tax policies, $\{\tau_t, \tau_t^L\}$, government expenditure, $\mathfrak{G} > 0$, and exogenous processes, $\{\sigma_t, a_t\}$.

4. Quantitative Analysis

In this section we undertake a quantitative analysis of the model developed in section 3.[30] We first outline the calibration of the steady-state and then compute impulse responses of endogenous variables for one-time shocks to firm-level volatility and aggregate productivity.

4.1. Parameterization and Calibration

This section discusses the parameterization and calibration of the steady-state of the model. We start with standard technology and preference parameters, followed by fiscal variables, and finally the distribution of firm-level shocks and fixed cost parameters. The discount rate is set at $\beta = 0.98$ and we adopt the following functional form for period utility,

$$u\left(C_t, L_t\right) = \ln C_t + \chi \frac{\left(1 - L_t\right)^{1-\upsilon} - 1}{1 - \upsilon} \tag{22}$$

The scale parameter $\chi > 0$ in equation (22) is set such that that households allocate 20 percent of their time to work in the steady-state. The Frisch elasticity of labor supply with respect to wages - here equal to $u_L/u_{LL}L > 0$ - is assumed to be 0.72, based on the empirical evidence in Heathcote *et al.* (2010). The elasticity of substitution between differentiated goods is set at 3.8. This value is taken from Bernard *et al.* (2003) and, without financial frictions, implies a markup of 35.7 percent. Finally, we set $\delta = 0.1$ to match an annual exit rate for firms of 10 percent.

Fiscal variables - government expenditure-to-output and labor-income and firm profit taxes - are set at $\left\{g, \tau^L, \tau\right\} = \{0.2, 0.2, 0.25\}$, respectively. When we consider optimal fiscal policy in section 4 we set the steady-state debt-to-output ratio at $b \equiv B/Y = 0.42$. The values for g and b are taken from Schmitt-Grohè and Uribe (2005) and the values for τ^L and τ are taken from Arseneau and Chugh (2010) and Gourio and Miao (2010), respectively.

We also assign a value to long-run firm-level volatility. Recall that the standard deviation of firm-level demand is denoted by σ_t and this captures firm-level volatility in the model.

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[30]In this section we also revert to the assumption that the government has a lump-sum transfer available to balance its budget.

Empirical estimates of firm-level volatility range from 0.09, used in Bachmann and Bayer (2012), to 0.23, used in Christiano *et al.* (2013). We use an intermediate value of $\sigma = 0.135$ based on Comin and Mulani (2006).[31] Given long-run micro volatility, we then calibrate overhead and default costs to match two features of the data. Normalizing the parameter governing entry costs to unity ($f_e = 1$), we set overhead costs parameter ($f_0 > 0$) such that the default rate is 1.0%. We base this calibration on historical value-weighted default rates of non-financials reported in Giesecke *et al.* (2011).[32] We then set the default cost parameter ($\kappa > 0$) to generate an average credit spread of 160 basis points which corresponds to the median of the BBB-Treasury spread as reported in Gilchrist *et al.* (2014).

Table 2 presents the parameters we use in our calibration.[33]

===== Table 2 Here =====

Our calibration has implications for variables other than those presented in Table 2. The average price-markup in our model is given by $[\theta\Delta(0)]^{-1}[\Delta(0)/\Delta(\varepsilon_t^\star)]^\theta$. Our calibration implies a price-markup of 21.7 percent once we account for financial frictions. Also, although the use of overhead labor in our economy is used to match default rates, it implies overhead costs account for 6.5 percent of total employment. Bartelsman *et al.* (2013) suggest that firms' use of overhead labor accounts for approximately 14 percent of total employment in U.S. manufacturing establishments. We can match this figure, but keeping default rates

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[31]Comin and Mulani (2006) use annual data on net sales from COMPUSTAT and find that since 1980 average (weighted) volatility is around 0.135. Arellano *et al.* (2012) - who also focus on demand shocks - use sales growth data and find 0.18.

[32]Giesecke *et al.* (2011) report that the mean (median) default rate for US nonfinancial is around 1.52% (0.54%). Over the long term, the also argue that credit spreads are twice as large as default losses, such that the average credit risk premium is 0.8%.

[33]When we perform sensitivity analysis we deviate from this baseline calibration and consider alternative values for σ, κ, and f_o.

below 1 percent requires lower firm-level volatility. Finally, business investment as a fraction of aggregate output across OECD countries is between 10 and 15 percent, and our calibration implies a value of 13.1 percent.

4.2. Exogenous Movements in Volatility and Productivity

In this section we consider the impact of a one-time one-standard-deviation shock to firm-level volatility and aggregate productivity. We focus the discussion on the response of financial variables - the default rate and credit interest rate spread - and the extent to which the interaction of financial frictions and the shock we consider effect the firm entry decision. Firm-level volatility and productivity are specified as independent AR(1) processes,

$$\eta_t = A_0 + A\eta_{t-1} + \omega_t \tag{23}$$

where $\eta_t = [\ln(\sigma_t), \ln(a_t)]^T$, $A_0 = [0.135, 1]^T$, and A is a 2×2 matrix describing the autoregressive component of the processes which we parameterize as $A = \begin{bmatrix} 0.83^4 & 0 \\ 0 & 0.93^4 \end{bmatrix}$. The term $\omega_t = [\omega_{\sigma,t}, \omega_{a,t}]^T$ is a vector of normally distributed, mean-zero shocks, where $Var(\omega_{a,t}) = 0.028^2$ and $Var(\omega_{a,t}) = 0.011^2$. The parameter values assigned to these processes are based on empirical estimates using US manufacturing data and are described in Chugh (2016).[34]

Figure 1 shows the response of key endogenous variables for a positive one standard-deviation shock to σ_t. The horizontal axis measures years and the vertical axis measures the percentage deviation from the steady-state, unless otherwise stated.

===== **Figure 1 Here** =====

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[34]Using annual manufacturing data, Chugh (2016) estimates innovations to firm-level volatility at 0.0276. We use this value because we want to take a conservative position on the role of volatility shocks. Chugh's long-run estimate of firm-level volatility is 0.156 which is close to the value we use.

A one-standard-deviation increase in firm-level volatility (a volatility-induced recession) leads to a 19 basis points rise in the firm default rate and a 11 basis points rise in the credit interest rate premium on impact. Increased volatility leads to a rise in the firm default rate in our economy because it is optimal for firms to expand and take advantage of potentially good realizations of demand. They can do so by hiring more labor but this requires that they take on more debt due to working capital constraints. In a more volatile economy, firms therefore need to generate a greater level of revenue to avoid default, and $\varepsilon_t^\star$ rises. The impulse responses also show that, from year 3 onwards, both the default rate and credit interest rate premium are lower than their pre-shock levels and afterwards slowly increase. This is a result of an interaction between the default-threshold level of demand and firm-level employment when there are overhead costs.[35]

The response of the mass of operating firms to a change in firm-level volatility is hump-shaped. Greater volatility reduces firm entry, and over time, less entry translates into a gradual reduction in the number of available goods. This effect peaks after 4 years by which time the initial rise in volatility has dropped by around 90 percent. Whilst the change in the mass of operating firms is relatively small (dropping by 0.2 percent) the transition back to the steady state is persistent. The model is relatively less successful at capturing labor market dynamics. Aggregate employment drops by less than 0.1 percent in response to an increase in volatility. Given the mechanism in the model this is perhaps not too surprising - recall that aggregate employment is $L_t = n_t \left(l_t + f \right)$ - because the only way for firms to take advantage of greater volatility is to expand production. Thus, a sufficiently large fall in the mass of operating firms is required for aggregate labor supply to fall.

In the calibrated economy, the long-run default rate is 1 percent, with an investment premium of 60 b.p., and the size of the innovations to volatility we consider are relatively

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[35]In section 2 we characterize the response of the default rate analytically assuming no overhead costs.

conservative.[36]   Despite this, the interaction of firm-level volatility shocks and financial frictions have important aggregate implications.   As a simple robustness check, in the first instance, we maintained the premium and reduced firm default to 0.4 percent (low default). We then maintained the default rate of 1 percent but reduced the premium to 30 b.p. (low premium).   In both cases, we kept long-run volatility at $\sigma = 0.135$.   In a second case, we set $\sigma = 0.1$ and used the baseline specification of 1 percent default with a 60 b.p. premium. Our main finding is that lowering the long-run volatility generates a larger response of firm entry, on impact, but firm-level employment is less persistent.

Figure 2 shows the response of key endogenous variables for a negative one standard-deviation shock to a_t.

===== **Figure 2 Here** =====

In a productivity-induced recession, we find the firm default rises by 59 basis points and the credit interest rate premium rises by 33 basis points on impact.   The initial response of the default rate and credit spread is broadly similar to that of a volatility-induced recession once we condition on the size of the business cycle.   The mechanism underlying the initial response of financial variables to the productivity shock is the following.   As firms become less productive, on average, some firms are unable to generate sufficient revenue, and these firms default.   This causes the default threshold (and the default probability) to rise.

Quantitatively, the increase in the default threshold depends in an important way on the cost of default, which is parameterized by $\kappa > 0$.   Consider the following date $t = 0$ partial equilibrium expression, derived using the zero profit condition for firms (the default-threshold

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[36]For example, Christiano *et al.* (2013) estimate innovations to be nearly twice as large.   Arellano *et al.* (2012) also assume a considerably larger value than we do.

equation) and for financial intermediaries (see equation (5)) which respectively determine $\varepsilon_0^\star = \varepsilon^\star(a_0)$ and $r_0 = r(a_0)$,

$$\widehat{\varepsilon}_0^\star = -\left[\frac{1}{\theta - \eta_{r,\varepsilon}(\kappa)}\right]\widehat{a}_0 \quad \text{where} \quad \eta_{r,\varepsilon}\left(\underset{-}{\kappa}\right) \tag{24}$$

for $\theta > \eta_{r,\varepsilon}(\kappa)$. The term $\eta_{r,\varepsilon}(\kappa)$ is decreasing in μ because whilst the default equation shows that $\varepsilon_0^\star$ declines with a_0 it increases with the credit interest rate, r_0. The elasticity of the interest rate with respect to default increases with the cost of default-threshold such that there is a feedback effect which does not occur with firm-level volatility shocks.

A second feature of productivity shocks is that, after the initial period, the default probability drops below it's pre-shock level whilst macroeconomic variables remain below their pre-shock level for many periods. This is because, after the initial period, the entire path of productivity is known, and lower productivity simply causes firms to cut back on production. Whilst the unexpected component of productivity has a strong effect on financial variables in the model, there is very little impact of future expected changes in productivity.[37]

5. Optimal Fiscal Policy

In this section we revisit the results on optimal fiscal policy. In section 2 the optimal tax on firm profits was characterized analytically under the assumption that lump-sum taxation was available to balance the budget. We showed that the government faced a firm entry trade-off and a financial frictions trade-off. In the quantitative version of the model there is an additional static distortion (an endogenous labor supply decision) and instrument (a tax on labor-income) and we assume the government issues state-contingent real debt. The policy choices - the optimal mix of taxes on labor-income and firm profits - are also therefore

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[37]This finding (i.e., procyclical default) is related to a well-known result in models of the financial accelerator with shocks to aggregate productivity. Despite the obvious differences, after periods $t = 1$, our default rate behaves in a similar way to the models of Calstrom and Fuerst (1997) and Gomes *et al.* (2003).

subject to a present value implementability constraint associated with the absence of lump-sum taxation.

In what follows the government solves the following reduced problem.

Definition 1 Given the exogenous processes $\{\sigma_t, a_t\}_{t=0}^{\infty}$ plans $\Omega_t \equiv \{n_{e,t}, n_{t+1}, C_t, L_t, l_t\}_{t=0}^{\infty}$ and $\{\varepsilon_t^{\star}\}_{t=0}^{\infty}$ represent the optimal allocation if they solve the following problem.

$$\max_{\{\Omega_t, \varepsilon_t^{\star}\}_{t=0}^{\infty}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \mathcal{U}(C_t, L_t, \xi) + \beta^t \lambda_{1,t} \left[Y_t - \mathfrak{G} - C_t - f_e n_{e,t} - \kappa n_t G\left(\varepsilon_t^{\star}\right) \right]$$
$$+ \beta^t \mathbb{E}_0 \lambda_{2,t} \left[L_t - n_t \left(l_t + f_o \right) \right] + \beta^t \lambda_{3,t} \left[(1 - \delta)\left(n_t + n_{e,t} \right) - n_{t+1} \right]$$
$$+ \beta^t \mathbb{E}_0 \lambda_{4,t} \left\{ \int_{\varepsilon_t^{\star}}^{\infty} \left[\theta a_t \varepsilon_t^{\theta} - a_t (\varepsilon_t^{\star})^{\theta} \left(\frac{l_t}{l_t + f_o} \right) \right] dG(\varepsilon_t) \right\} - \xi \mathcal{A} \tag{25}$$

where,

$$\mathcal{U}(C_t, L_t) \equiv u(C_t, L_t) + \xi \left[u_C(t)C_t + u_L(t)L_t \right] \tag{26}$$

and given,

$$\mathcal{A} \equiv u_C(0) \left[r_{-1}^d d_{-1} + b_0 + n_0 z\left(\varepsilon_0^{\star}\right) \right] \tag{27}$$

where $\{\lambda_{j,t}\}_{j=1}^4$ are lagrange multipliers associated with constraints, and ξ is a (constant) lagrange multiplier associated with the implementability constraint.

A detailed derivation of the reduced policy problem - as stated in equations (25)-(27) - is presented in Appendix B.[38] Here we focus of the substantive point that, relative to the case without default, the only additional constraint placed on the policy maker is the labor demand equation, as presented in Proposition 4. Given the structure of the problem in equations (25)-(27) there is a tight link between the default-threshold level of demand and

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[38]As in the standard Ramsey taxation problem, the government is assumed to commit, as of period zero, to time invariant policy functions for $t \geq 1$. Following Chugh and Ghironi (2016), we also assume that the schedule of state-contingent profit taxes is posted one period in advance.

the mass of operating firms. Once the government picks $\varepsilon_t^\star > 0$, for a given level of aggregate employment, L_t, the mass of operating firms is determined as a function of the underlying shocks. The remaining constraints - over resources, both in goods and labor markets, the law of motion for firms, and the present value implementability constraint - are the same as when financial frictions are absent.

We are interested in short-run stabilization policy for the set of shocks specified in section 4 - specifically, the optimal response of taxes in a recession.[39] Figure 3 plots the response of taxes on labor-income and firm profits to a one-time change in volatility and productivity.

===== **Figure 3 Here** =====

First consider the response of taxes when there is a volatility-induced recession (top panel). Following a one-standard deviation shock to firm-level volatility there is a 1 percentage points rise in profit taxation (implemented in periods $t \geq 1$) and 0.5 percentage points drop in labor-income taxation. We can explain the rise in the profit tax if we recall that firm entry falls and the default rate rises as the volatility of firm-level shocks increases. Reduced firm entry requires a subsidy to firms whereas an increase in the default rate requires taxation. Moreover, the response of the profit tax is consistent with the analytical section, which assumed lump-sum taxes were available. Given taxes on firm profits, we find that the labor-income tax should be counter-cyclical. The drop in the period $t = 0$ labor-income tax presented in Figure 3 is nearly 0.5 percentage points and is in anticipation of future expected changes in the profit tax. After period $t \geq 1$, however, the labor-income tax reverts to it's long-run level.

When a recession is generated by a productivity shock the response of taxation is markedly different (bottom panel). Most obvious is that the drop in profit taxes - at nearly 11

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[39]The impluse responses we report are based on an optimal stabilization policy and represent deviations from the steady-state of the model as calibrated in section 4.

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percentage points is large. However, if we condition on the size of the business cycle, this change in taxation is comparable to that which occurs when there are shocks to volatility. Intuitively, the explanation for the initial drop in the profit tax is that the government wants to reduce default costs. Because the profit tax is set in advance, and the government anticipates all future levels of productivity, it is optimal to encourage entry because, in periods $t \geq 1$, the default rate will be below its pre-shock level. We also find that the response of the labor-income tax is lagged one period when there are productivity shocks. The reason is that, whilst the productivity change has a large impact on the profit tax, this cannot be implemented until periods $t \geq 1$.

As with the analytical results, we can also ask what role default costs play in determining the optimal response of taxes. With volatility shocks the answer is straightforward. The lower the cost of default the less is the need to reduce subsidies to firms. With shocks to productivity, the relationship between default costs and fiscal policy is less clear. This is because, by lowering the cost of default, the sensitivity of the default rate to productivity shocks rises, as shown by equation (24). Overall, we find that with a lower default cost (that is, when there is a lower credit interest rate spread and long-run investment premium) the response of the profit tax is qualitatively unchanged, but quantitatively, the diminished is diminished.

Finally, we compare our results on optimal fiscal when there are productivity shocks to those reported in Chugh and Ghironi (2015) and Colciagio (2016) both of which adopt a similar mechanism for firm entry. Under the Dixit-Stiglitz type preferences we consider, taxes on labor-income and firm profits are invariant over the business cycle.[40] The reason is that the price-markup is constant along the business cycle which implies inefficiency wedges are also constant. The same point holds in our model without financial frictions. With financial

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[40]The type of tax-smoothing result discussed in Chugh and Ghironi (2015) is first presented in Chari *et al.* (1994) in the context of an RBC model.

frictions, however, the price-markup falls in a productivity-induced recession.[41] We cannot therefore map markups into tax responses without accounting for the resource implications of default costs and this is what creates the dual role of fiscal policy in our analysis.

6. Conclusion

This paper studies optimal fiscal policy in a general equilibrium model of firm entry and financial frictions. We provide analytical expressions for optimal fiscal policy and show that the government faces two trade-offs. The first arises from a profit destruction and a consumer surplus effect when firm entry is endogenous. The second arises because financial frictions reduce firm entry and default is costly. We also study the optimal mix of taxes on labor-income and firm profits in a quantitative version of our model with firm-level volatility and aggregate productivity shocks. We find a countercyclical labor-income tax is always part of the optimal fiscal policy but the cyclicality of the profit tax is sensitive to the source of aggregate fluctuations.

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[41]The average price markup with financial friction is equal to $[\theta/\Delta(0)]^{-1}[\Delta(0)/\Delta(\varepsilon_t^\star)]^\theta$. Without frictions this reduces to $[\theta/\Delta(0)]^{-1}$. The $\Delta(0)$ term remains in this expression due to heterogeneity in demand and is unaffected by productivity shocks.

Appendix A

In this Appendix we present proofs for Propositions 1-4.

Appendix A.1 (Proof of Proposition 1)

Firms maximize conditional expected profit, $\int_{\varepsilon_t^\star}^\infty \pi(i, \varepsilon_t) \, dG(\varepsilon_t)$, choosing employment level, $l_t(i)$, subject to technology, demand, and market clearing. In units of consumption, profit is given by, $\pi(i, \varepsilon_t) = p(i, \varepsilon_t) y_t(i) - w_t r_t l_t(i)$, where $y_t(i) = l_t(i) = Y_t \left[p(i, \varepsilon_t) / \varepsilon_t^\theta \right]^{-1/(1-\theta)}$ and $\varepsilon_t^\star = \inf \{ \varepsilon_t : \pi(i, \varepsilon_t) > 0 \}$. The unconstrained problem is,

$$\max_{l_t(i), \varepsilon_t^\star} \int_{\varepsilon_t^\star}^\infty \left\{ (\varepsilon_t)^\theta \left[l_t(i)/Y_t \right]^{\theta-1} l_t(i) - w_t r_t l_t(i) \right\} dG(\varepsilon) \tag{28}$$

with prices and $Y_t > 0$ given. The first order conditions implies,

$$\int_{\varepsilon_t^\star}^\infty \theta (\varepsilon_t)^\theta \left(l_t(i)/Y_t \right)^{\theta-1} dG(\varepsilon) - [1 - G(\varepsilon_t^\star)] w_t r_t = 0 \tag{29}$$

for all i. The threshold level of demand is determined by, $\pi(i, \varepsilon_t^\star) = 0$. We determine $\varepsilon_t^\star$ using the expression, $\pi(i, \varepsilon_t^\star) = (\varepsilon_t^\star)^\theta \left[l_t(i)/Y_t \right]^{\theta-1} l_t(i) - w_t r_t l_t(i) = 0$. Incorporating the above first-order condition, we then find, $\int_{\varepsilon_t^\star}^\infty \theta \varepsilon_t^\theta dG(\varepsilon_t) - [1 - G(\varepsilon_t^\star)] \varepsilon_t^{\star\theta} = 0$. The labor demand expression also pins-down the price of a good in our model for given costs of production. Using the demand curve, the average price is, $p(\varepsilon_t^\star) = w_t r_t \frac{1 - G(\varepsilon_t^\star)}{\theta} / \int_{\varepsilon_t^\star}^\infty \varepsilon_t^\theta dG(\varepsilon)$. Finally, we solve for employment using, $Y_t = l(\varepsilon_t^\star)^{(\theta-1)/\theta} \left[\int_0^\infty \varepsilon_t^\theta dG(\varepsilon) \right]$, which implies, $(\varepsilon_t^\star)^\theta \left[\int_0^\infty \varepsilon_t^\theta dG(\varepsilon) / l(\varepsilon_t^\star)^{1/\theta} \right]^{1-\theta} = w_t r_t$.

Appendix A.2 (Proof of Proposition 2)

We drop time subscripts and define the following function,

$$f(\varepsilon^\star, \sigma) \equiv \int_{\varepsilon^\star}^\infty \theta \varepsilon^\theta dG(\varepsilon, \sigma) - [1 - G(\varepsilon^\star, \sigma)] (\varepsilon^\star)^\theta = 0 \tag{30}$$

This implicitly determines $\varepsilon^\star$. Assume ε has a lognormal distribution with PDF, $g(\varepsilon, \sigma) = \frac{1}{\varepsilon\sigma\sqrt{2\pi}}\exp\left[\frac{-(\ln\sigma-\mu)^2}{2\sigma^2}\right]$, and define, $x \equiv (\ln\varepsilon - \mu)/\sigma \Leftrightarrow \varepsilon = \exp(x\sigma + \mu)$. The default-threshold level of demand is determined by,

$$f_1(x^\star, \sigma) = \frac{1}{\sqrt{2\pi}}\int_{x^\star}^\infty [\theta(\exp\theta\sigma x) - (\exp\theta\sigma x^\star)]\left\{\exp\left[\frac{-(x)^2}{2}\right]\right\}dx = 0$$

We express this condition in terms of a normal CDF, $\Phi(\varepsilon) \equiv \frac{1}{\sqrt{2\pi}}\int_{-\infty}^x \exp\left[-(x)^2/2\right]dx$, which gives,

$$f_1(x^\star, \sigma) = \theta\left[\exp\left((\sigma\theta)^2/2\right)\right]\Phi(-(x^\star - \theta\sigma))) - [\exp(\theta\sigma x^\star)]\Phi(-(x^\star)) \tag{31}$$

We prove Proposition 2 in following steps: (i), that $x^\star$ exists and is unique, (ii), that $\frac{\partial}{\partial x}f_1(x^\star, \sigma) < 0$, (iii), that $\frac{\partial}{\partial\sigma}f_1(x^\star, \sigma) > 0$ when $x^\star < 0$, and finally, (iv), that the financial frictions wedge, $D(\varepsilon^\star) = \int_{\varepsilon^\star}^\infty \varepsilon^\theta dG(\varepsilon) / \int_0^\infty \varepsilon^\theta dG(\varepsilon)$, falls with volatility.

Lemma 1 There exist and $x^\star$ which satisfies (31) and $\frac{\partial}{\partial x}f_1(x^\star, \sigma) < 0$.

Proof $\lim_{\varepsilon\to+\infty}\Phi(\varepsilon) = 1$ and $\lim_{\varepsilon\to-\infty}\exp(\varepsilon)\Phi(\varepsilon) = 0$ implies $\lim_{x^\star\to-\infty}f_1(x^\star, \sigma) = \theta\exp((\sigma\theta)^2/2) > 0$ and $\lim_{x^*\to+\infty}f_1(x^\star, \sigma) = -[\exp(\theta\sigma x^\star)] < 0$. Since $f_1(x^\star, \sigma)$ is a continuous function and it changes sign from positive to negative, a solution exists. We know $f_1(x, \sigma) > 0$ at some small x and $f_1(x, \sigma)$ crosses zero at least once. Let $x^\star$ be the first point where $f_1(x^\star, \sigma) = 0$. As $f_1(x^\star, \sigma)$ approaches the line from above, $\frac{\partial}{\partial x}f_1(x^\star, \sigma) \leq 0$.

To prove uniqueness, recall that the lognormal distribution has strictly decreasing hazard ratio (Thomas, 1971), and therefore,

$$f_2(x) = \int_x^\infty \exp\left[\frac{-(x)^2}{2}\right]\left(\exp\left[\frac{-(y)^2}{2}\right]\right)dy = \frac{\Phi(-x^\star)}{\Phi'(x^\star)} \tag{32}$$

is a strictly increasing function. In order to have multiple solutions there should be a $x^{\star\star} > x^\star$ such that $\frac{\partial}{\partial x}f_1(x^{\star\star}, \sigma) \geq 0$ and $f_1(x^{\star\star}, \sigma) = 0$. We will show that this is impossible. To prove our result we make use of the following lemma.

Lemma 2 If $\frac{\partial}{\partial x} f_1(x^\star, \sigma) \leq 0$, then for any $x^{\star\star}$ such that $x^{\star\star} > x^\star$, it is true that $\frac{\partial}{\partial x} f_1(x^{\star\star}, \sigma) < 0$.

Proof By contradiction. Assume there is an $x^{\star\star} > x^\star$ such that $\frac{\partial}{\partial x} f_1(x^{\star\star}, \sigma) \geq 0$. This implies,

$$\frac{\partial}{\partial x} f_1(x^{\star\star}, \sigma) = (1 - \theta) \frac{1}{\sqrt{2\pi}} [\exp \theta \sigma x^{\star\star}] \left(\exp \left[\frac{-(x^{\star\star})^2}{2} \right] \right) - \theta \sigma [\exp \theta \sigma x^{\star\star}] \Phi(-(x^{\star\star})) \geq 0$$

and,

$$\sqrt{2\pi} [\exp(-\theta \sigma x^{\star\star})] \left(\exp \left[\frac{(x^{\star\star})^2}{2} \right] \right) \times \frac{\partial}{\partial x} f_1(x^{\star\star}, \sigma) = (1 - \theta) - \theta \sigma f_2(x^{\star\star}) \geq 0 \tag{33}$$

However, $\frac{\partial}{\partial x} f_1(x^\star, \sigma) \leq 0$ implies $(1 - \theta) - \theta \sigma f_2(x^\star) \leq 0$. Combining this with the preceding expression we get $f_2(x^\star) \geq f_2(x^{\star\star})$. This contradicts the lemma of Thomas (1971) as presented in equation (32). Lemma 1 and Lemma 2 prove the existence and uniqueness of $x^\star$, the solution to (31) and that $\frac{\partial}{\partial x} f_1(x^\star, \sigma) < 0$.

Lemma 3 If $\Phi(x^\star) < 1/2$, then $x^\star$ increases with volatility and $D(\varepsilon^\star)$ falls with volatility.

Proof Using the definitions introduced above, $D(x^\star, \sigma) = 1 - \Phi(x^\star - \theta \sigma)$, and,

$$
\begin{aligned}
\frac{dD(x^\star, \sigma)}{d\sigma} &= \frac{\partial D(x^\star, \sigma)}{\partial \sigma} + \frac{\partial D(x^\star, \sigma)}{\partial x^\star} \frac{dx^\star}{d\sigma} \\
&= \theta \frac{1}{\sqrt{2\pi}} \exp \left[\frac{-(x^\star - \theta \sigma)^2}{2} \right] - \frac{1}{\sqrt{2\pi}} \exp \left[\frac{-(x^\star - \theta \sigma)^2}{2} \right] \frac{dx^\star}{d\sigma} \\
&= \frac{1}{\sqrt{2\pi}} \exp \left[\frac{-(x^\star - \theta \sigma)^2}{2} \right] \left(\theta - \frac{dx^\star}{d\sigma} \right)
\end{aligned}
$$

In order to determine the change in $D(x^\star, \sigma)$ when volatility increases, we require, $dx^\star/d\sigma < \theta$. We use definition of $x^\star$, $f_1(x^\star, \sigma) = 0$. From the implicit function theorem,

$$-\frac{\partial f_1(\cdot)}{\partial x} \left(\frac{dx^\star}{d\sigma} - \theta \right) = \frac{\partial f_1(\cdot)}{\partial \sigma} + \theta \frac{\partial f_1(\cdot)}{\partial x}$$

where,

$$f_1(x^\star, \sigma) = \theta \left[\exp\left((\sigma \theta)^2/2 \right) \right] \Phi(-(x^\star - \theta \sigma))) - [\exp(\theta \sigma x^\star)] \Phi(-(x^\star))$$

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which leads to,

$$\exp\left(-(\sigma\theta)^2/2\right)\frac{\partial f_1(\cdot)}{\partial x} = (1-\theta)\frac{1}{\sqrt{2\pi}}\exp\left[\frac{-(x^\star - \theta\sigma)^2}{2}\right] - \theta^2\sigma\Phi(-(x^\star - \theta\sigma))$$

and,

$$\exp\left(-(\sigma\theta)^2/2\right)\times\frac{\partial f_1(\cdot)}{\partial\sigma} = \theta^2(\sigma\theta - x^\star)\Phi(-(x^\star - \theta\sigma)) + \frac{\theta^2}{\sqrt{2\pi}}\exp\left[\frac{-(x^\star - \theta\sigma)^2}{2}\right] \tag{34}$$

This leads to,

$$\exp\left(-(\sigma\theta)^2/2\right)\left[\frac{\partial f_1(\cdot)}{\partial\sigma} + \theta\frac{\partial_1 f(\cdot)}{\partial x}\right] = \frac{1}{\sqrt{2\pi}}\exp\left[\frac{-(x^\star - \theta\sigma)^2}{2}\right] - x^\star\Phi(-(x^\star - \theta\sigma))$$

This expression is positive if $x^\star < 0$, which is equivalent to the probability of default being less than one half. As such, $D(\varepsilon^\star)$ falls with volatility under the same conditions as Proposition 2. Note that (34) implies $\frac{\partial f_1(\cdot)}{\partial\sigma} > 0$ when $x^\star < 0$. Therefore $\frac{dx^\star}{d\sigma} = \frac{\partial f_1(\cdot)}{\partial\sigma}/\left(-\frac{\partial f_1(\cdot)}{\partial x}\right) > 0$, and $x^\star$, and the probability of default $\Phi(x^\star) = G(\varepsilon^\star)$ increases with volatility.

Appendix A.3. (Proof of Proposition 3)

The policy problem is to choose τ_t to maximize consumption subject to the equilibrium conditions of the model. Where possible we suppress the index $\varepsilon_t^\star$ in what follows. First note that $\varepsilon_t^\star > 0$ is given and the labor demand equation is not a constraint faced by the policymaker. The policymaker chooses the following allocations and prices: $\{C_t, n_t, l_t, Y_t, w_t, r_t\}$. Since labor is inelastic, $l_t = 1/n_t$. We then replace consumption with the feasibility constraint $C_t = Y_t - n_t[f_e + \mu G(\varepsilon_t^\star)]$ and use the default threshold $w_t = \frac{1}{r_t}(\varepsilon_t^\star)^\theta[\Delta(0)]^{1-\theta}n_t^{(1-\theta)/\theta}$ to determine wages. The problem reduces to,

$$\max_{n_t, r_t} C_t = n_t^{(1-\theta)/\theta}\Delta(0) - n_t[f_e + \kappa G(\varepsilon_t^\star)]$$

$$+ \lambda_{1,t}\left\{(\varepsilon_t^\star)^\theta\left[\int_{\varepsilon_t^\star}^\infty dG(\varepsilon) + \int_0^{\varepsilon_t^\star}\left(\frac{\varepsilon}{\varepsilon_t^\star}\right)^\theta dG(\varepsilon) - \frac{r_t^d}{r_t}\right]\left[\frac{\Delta(0)}{n_t^{1/\theta}}\right]^{1-\theta} - \kappa n_t G(\varepsilon_t^\star)\right\}$$

It is immediate that $\lambda_{1,t} = 0$ by the choice of r_t. We discuss this reduced problem with $\lambda_{1,t} = 0$ in the text.

Appendix A.4. (Proof of Proposition 4)

We suppose that there is aggregate uncertainty. The instantaneous profit function of the firm is written as,

$$\pi_t\left(\varepsilon_t, a_t\right) = \varepsilon_t^\theta \times \left[n^{1/\theta}\Delta(0)\bar{l}_t\right]^{1-\theta} a_t\left[l_t(i)\right]^\theta - w_t r_t\left[l_t(i) + f_o\right] \tag{35}$$

where $\varepsilon_t^\star = \varepsilon^\star\left(a_t\right)$ and $r_t = r\left(a_t\right)$. For any particular a_t, we can compute $\pi_t\left(a_t\right)$, defined as $\pi_t\left(\varepsilon_t^\star, a_t\right) = \mathbb{E}_{\varepsilon_t}\pi_t\left(a_t\right)$, for all i. We write the constrained optimization problem of the firm as,

$$\max_{l_t(i)}\pi_t\left(a_t\right) = \int_{\varepsilon_t^\star}^\infty\left\{\varepsilon_t^\theta \times \left[\bar{l}_t n_t^{1/\theta}\Delta(0)\right]^{1-\theta} a_t l_t^\theta - w_t r_t\left(l_t + f_o\right)\right\}dG(\varepsilon) \tag{36}$$

subject to the threshold equation, $\pi_t\left(\varepsilon_t^\star\right) = 0$. Define the Hamiltonian as,

$$\mathcal{H}(l_t, \varepsilon_t^\star, r_t, a_t) = \pi_t\left(l_t, \varepsilon_t^\star, r_t, a_t\right) + \lambda_t \times f\left(l_t, \varepsilon_t^\star, r_t, a_t\right) \tag{37}$$

where $\lambda_t\left(\cdot\right)$ is the lagrange multiplier associated with the constraint. The Hamiltonian is at its maximum on average with respect to $\{r_t, \varepsilon_t^\star\}$ for any $\{a_t, l_t\}$, and so, $\int_{a_t}\frac{\partial}{\partial\varepsilon_t^\star}\mathcal{H}(t) = 0$ and $\int_{a_t}\frac{\partial}{\partial l_t}\mathcal{H}(t) = 0$. As $\frac{\partial}{\partial\varepsilon_t^\star}\pi\left(t\right) = 0$, the lagrange multiplier is zero and $\lambda\left(t\right) = 0$. Applying the equilibrium condition, $\bar{l}_t = l_t$, and we arrive at:

$$\frac{\partial H\left(t\right)}{\partial l_t} = \int_{\varepsilon_t^\star}^\infty\left\{\theta\varepsilon_t^\theta \times \left[n_t^{1/\theta}\Delta(0)\right]^{1-\theta} a_t\right\}dG(\varepsilon_t) - \left[1 - G\left(\varepsilon_t^\star\right)\right]w_t r_t = 0 \tag{38}$$

In order to proceed we need functional expressions for $\varepsilon^\star(a_t)$ and $r(a_t)$; the former is determined by the zero-profit default threshold condition and the latter by the zero-profit condition for financial intermediaries. Using the equation that determines $\varepsilon_t^\star$, we derive,

$$\mathbb{E}_t\int_{\varepsilon_t^\star}^\infty\left[\theta a_t\varepsilon_t^\theta - a_t(\varepsilon_t^\star)^\theta\left(\frac{l_t}{l_t + f_o}\right)\right]dG(\varepsilon) = 0 \tag{39}$$

We can easily relate this expression to $f\left(\varepsilon_t^\star, \sigma\right) = \theta\int_{\varepsilon_t^\star}^\infty\varepsilon^\theta dG(\varepsilon) - \left[1 - G\left(\varepsilon_t^\star\right)\right]\left(\varepsilon_t^\star\right)^\theta = 0$ as reported in Proposition 1 once we set $f_o = 0$ and eliminate aggregate productivity shocks.

Appendix B

In this Appendix we detail the Ramsey policy problem and derive the first-order conditions for optimal policy in the dynamic model.

Appendix B.1 (Derivation of the Present Value Constraint)

Recall that the household budget constraint - given by equation (18) in the text. We first multiply this constraint by the marginal utility of consumption, $u_C(t)$, impose the equilibrium condition $x_{t-1} = 1$, and integrate forward. We then use the labour supply and the dynamic Euler equations, $u_C(t) = \beta \mathbb{E}_t \left[u_C(t+1) r_t^j \right]$; $u_C(t) = \beta \mathbb{E}_t \left[u_C(t+1) r_t^d \right]$ and $v_t u_C(t) = \beta(1-\delta) \mathbb{E}_t \left[z(\varepsilon_{t+1}^\star) u_C(t+1) \right]$. Finally, we use dynamic equation for product creation, $n_t = (1-\delta)(n_{t-1} + n_{e,t-1})$, to write the present value constraint as,

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[u_C(t) C_t + u_L(t) L_t \right] = \mathcal{A} \tag{40}$$

where $\mathcal{A} \equiv u_C(0) \left[r_{-1}^d d_{-1} + b_0 + n_0 z(\varepsilon_0^\star) \right]$ is assumed exogenous.

Appendix B.2 (Definition of the Ramsey Problem)

Following Chugh and Ghironi (2015), the Ramsey policy maker picks τ_t^L and commits to pick τ_{t+1}^d in period t. The problem can be written as one of maximizing $\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t u(C_t, L_t)$, subject to all the conditions presented in Table 1, the present value constraint, given by equation (40), and a constraint that $\varepsilon_t^\star > 0$. Plans are made over $\{n_{e,t}, n_{t+1}, l_t, C_t, L_t, \pi(\varepsilon_t^\star)\}_{t=0}^{\infty}$, prices $\{w_t, r_t, r_{t-1}^d\}_{t=0}^{\infty}$, tax rates, $\{\tau_{t+1}^d, \tau_t^L\}_{t=0}^{\infty}$ and the default threshold, $\{\varepsilon_t^\star\}_{t=0}^{\infty}$. By choosing tax rates, however, the constraints on the labor-leisure and the Euler equation for shares (i.e., product creation) do not bind. Similarly, by picking wages and interest rates directly, the constraints on firm pricing, the zero profit condition for financial intermediaries, and the Euler equation for deposits do not bind. This allows us to re-write the reduced Ramsey policy problem as in the text where $\varepsilon_t^\star > 0$.

Appendix B.3 (Optimality Conditions for the Ramsey Problem)

Consider the Ramsey problem defined in the text.

$$\max \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t u(C_t, L_t) + \beta^t \lambda_{1,t} \left\{ \rho_t \left[\Delta(0) \times a_t \right] l_t n_t - C_t - f_e n_{e,t} - \mathfrak{G} - \kappa n_t G\left(\varepsilon_t^\star\right) \right\}$$

$$+ \beta^t \lambda_{2,t} \left[L_t - n_t \left(l_t + f_o \right) \right] + \beta^t \lambda_{3,t} \left[(1 - \delta)\left(n_t + n_{e,t}\right) - n_{t+1} \right]$$

$$+ \beta^t \lambda_{4,t} \left\{ \theta \int_{\varepsilon_t^\star}^{\infty} \varepsilon^\theta dG\left(\varepsilon\right) a_t \left(l_t + f_o \right) - \left(\varepsilon_t^\star\right)^\theta \times a_t l_t \left[1 - G\left(\varepsilon_t^\star\right) \right] \right\}$$

$$+ \xi \left\{ \mathbf{A}_0 - \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[u_C(t) C_t + u_L(t) L_t \right] \right\}$$

Differentiating with respect to $\varepsilon_t^\star$, we find,

$$0 = \lambda_4 a_t \left\{ \left[(1 - \theta) l_t - \theta f_c \right] \left(\varepsilon_t^\star\right)^\theta g\left(\varepsilon_t^\star\right) - \theta l_t \left(\varepsilon_t^\star\right)^{\theta-1} \left[1 - G\left(\varepsilon_t^\star\right) \right] \right\} - \lambda_{1,t} \left[\kappa n_t g\left(\varepsilon_t^\star\right) \right] \qquad (41)$$

where $\lambda_{1,t}$ the lagrange multiplier associated with the resources (output) equation.[42] It is clear from this expression that $\lambda_{4,t} > 0$. The remaining first-order conditions (obtained from simply differentiating with respect to $\{n_{e,t}, n_{t+1}, l_t, C_t, L_t\}$) are,

$$\lambda_{1,t} f_e = (1 - \delta) \lambda_{3,t} \qquad (42)$$

$$\left(\frac{\beta}{\theta}\right) \mathbb{E}_t \left(\lambda_{1,t+1} \frac{Y_{t+1}}{n_{t+1}} \right) - \beta \mathbb{E}_t \left(\lambda_{2,t+1} \frac{L_{t+1}}{n_{t+1}} \right) = \lambda_{3,t} - \beta (1 - \delta) \mathbb{E}_t \lambda_{3,t+1} \qquad (43)$$

$$0 = \lambda_{1,t} \mathbb{E}_t \left(\frac{Y_t}{l_t} \right) - \lambda_{2,t} \mathbb{E}_t n_t + \lambda_{4,t} \frac{f_o}{(l_t + f_o)^2} \mathbb{E}_t \int_{\varepsilon^\star(a_t)}^{\infty} \left\{ a_t \left[\varepsilon^\star(a_t) \right]^\theta \right\} dG(\varepsilon) \qquad (44)$$

$$\lambda_{1,t} = u_C(t) \left\{ 1 + \xi \left[1 + \frac{u_{CC}(t) C_t}{u_C(t)} \right] \right\} \quad ; \quad -\lambda_{2,t} = u_L(t) \left\{ 1 + \xi \left[1 + \frac{u_{LL}(t) L_t}{u_L(t)} \right] \right\} \qquad (45)$$

where $\rho_t = n_t^{(1-\theta)/\theta}$. Without aggregate uncertainty, we have $\ldots + \lambda_{4,t} \left\{ \theta - \left[\frac{\varepsilon_t^\star}{\Delta(\varepsilon_t^\star)} \right]^\theta \right\}$ in equation (44).

---

[42]Note that, without uncertainty, we can show the final term in this expression is equal to $\lambda_{4,t} l_t \mathbb{E}_t \left[\frac{\varepsilon_t^\star}{\Delta(\varepsilon_t^\star)} \right]^\theta \theta (\Delta_{\varepsilon^\star} - 1)$, where $\Delta_{\varepsilon^\star} \equiv \varepsilon^\star \left[\Delta'(\varepsilon^\star) / \Delta(\varepsilon^\star) \right] < 1$.

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Table 1: Model Summary

Description	Equation
Labor Clearing and Production	$L_t = n_t \left(l_t + f_o \right) \quad \text{and} \quad Y_t = n_t^{1/\theta} \left[\Delta(0) \times a_t \right] l_t$
Resources	$Y_t - \mathfrak{G} = C_t + f_e n_{e,t} + \kappa n_t G \left(\varepsilon_t^\star \right)$
Labor demand	$\mathbb{E}_t \int_{\varepsilon_t^\star}^\infty \left[\theta a_t \varepsilon_t^\theta - a_t \left(\varepsilon_t^\star \right)^\theta \left(\frac{l_t}{l_t + f_o} \right) \right] dG(\varepsilon) = 0$
Labor supply	$w_t = -\dfrac{u_L(t)}{\left(1 - \tau_t^L \right) \mathbb{E}_t u_C(t)}$
Net worth	$z_t = \left(1 - \tau_t \right) D \left(\varepsilon_t^\star \right) \pi_t + v_t$
Expected profit	$\pi_t = n_t^{(1-\theta)/\theta} \left[\Delta(0) \times a_t \right] \left[(1 - \theta) l_t - \theta f_o \right]$
Mass of firms	$n_{t+1} = \left(1 - \delta \right) \left(n_t + n_{e,t} \right)$
Default threshold	$\left(\varepsilon_t^\star \right)^\theta \times \left[n_t^{1/\theta} \Delta(0) \right]^{1-\theta} a_t l_t - w_t r_t \left(l_t + f_o \right) = 0$
Financial intermediaries	$w_t \left(l_t + f_o \right) r_{t-1}^d + \kappa G \left(\varepsilon_t^\star \right) = \left[1 - G \left(\varepsilon_t^\star \right) + \frac{\eta_t}{\left(\varepsilon_t^\star \right)^\theta} \right] \left(l_t + f_o \right) w_t r_t$
Euler Equation (shares) and Entry	$f_e = \beta (1 - \delta) \mathbb{E}_t \left[\frac{u_C(t+1)}{u_C(t)} \right] z_{t+1}$
Euler Equation (deposits and bonds)	$1 = \beta \mathbb{E}_t \left[\frac{u_C(t+1)}{u_C(t)} \right] r_t^d$

Table 2: Exogenous Parameters and Calibration

Parameters Set Exogenously			
Statistic	Parameter	Value	Target/Source
Firm exit rate	δ	0.1	10%
Markup	θ	0.74	Bernard *et al.* (2003)
Discount factor	β	0.98	2% risk-free rate
Frisch elasticity	$v\left(\frac{1-L}{L}\right)$	0.72	Heathcote *et al.* (2010)
Sunk cost	f_e	1	Normalization

Calibrated Parameters				
Statistic	Parameter	Value	Target	Source
Volatility (long-run)	σ	0.135	-	Comin and Mulani (2006)
Interest rate spread	κ	2.84	160 b.p.	BBB-Treasury spread
Default rate	f_o	0.116	1%	Giesecke *et al.* (2011)
Hours worked	χ	1.341	20%	-

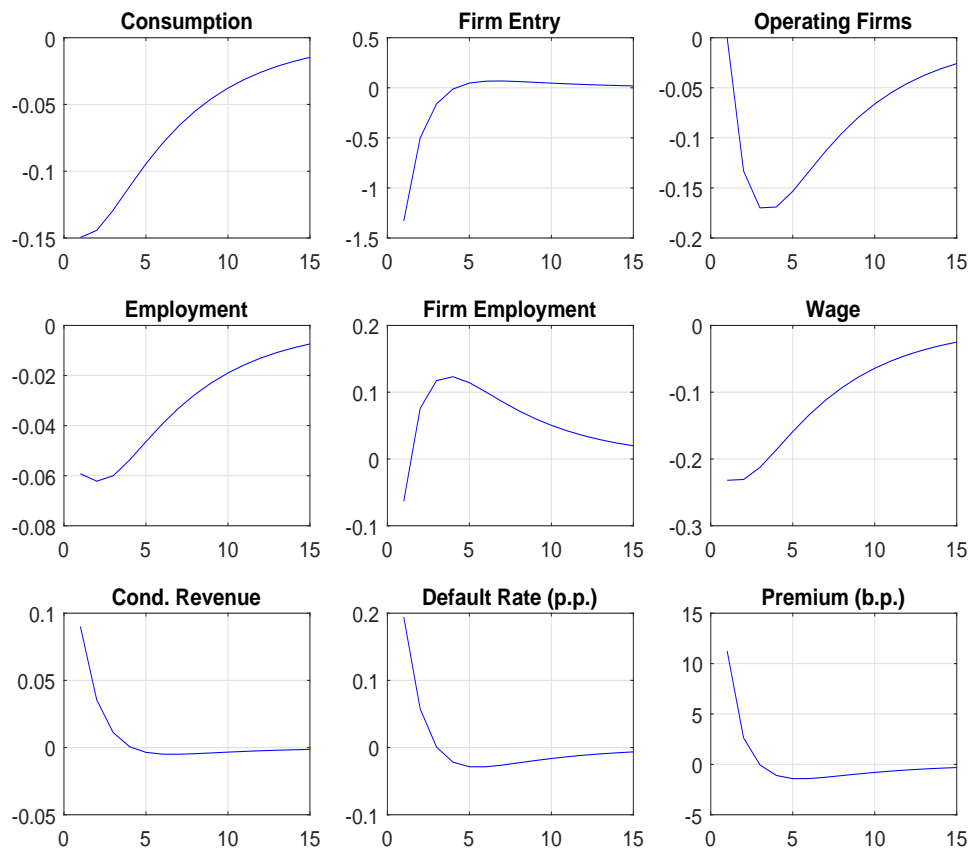Figure 1: Impulse Responses to a Volatility Shock

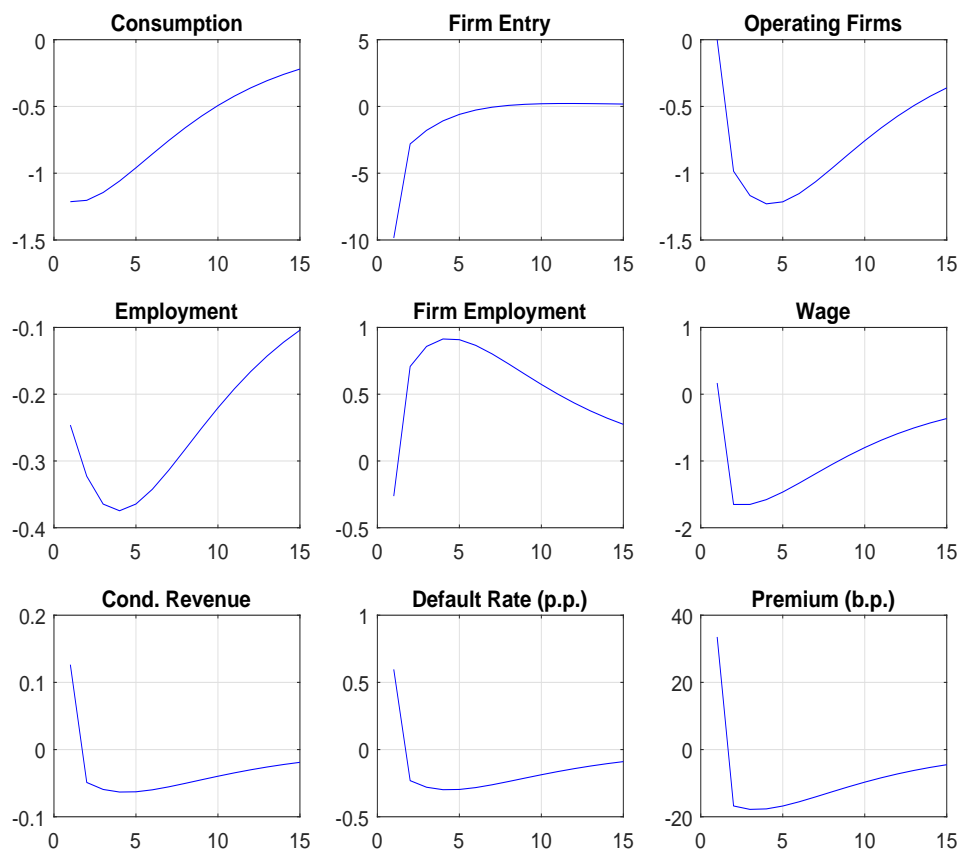Figure 2: Impulse Responses to a Productivity Shock

Figure 3: Tax Responses to a Volatility and Productivity Shock

Volatilty Shock

Legend:
- Labor Tax, τ_t^L
- Profit Tax, τ_t^π

Productivity Shock