Optimal Fiscal Feedback on Debt
in an Economy with Nominal Rigidities

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Abstract

We examine the impact of different degrees of fiscal feedback on debt in an economy with nominal rigidities where monetary policy is optimal. We look at the extent to which different degrees of fiscal feedback enhances or detracts from the ability of the monetary authorities to stabilise output and inflation. Using an objective function derived from utility, we find the optimal level of fiscal feedback to be small. There is a clear discontinuity in the behaviour of monetary policy and welfare either side of this optimal level. As the extent of fiscal feedback increases, optimal monetary policy becomes less active because fiscal feedback tends to deflate inflationary shocks. However this fiscal stabilisation is less efficient than monetary policy, and so welfare declines. In contrast, if fiscal feedback falls below some critical value optimal monetary policy becomes strongly passive, and this passive monetary policy leads to a sharp deterioration in welfare.

Key Words: Fiscal Policy, Feedback Rules, Debt, Macroeconomic Stabilisation
JEL Reference Number: E52, E61, E63, F41

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1 Introduction

With the occasional and notable exception, most governments now see one of their primary economic responsibilities as ensuring that the national debt stays within reasonable bounds. In some cases explicit targets for the debt to GDP ratio have been announced, with the implication that if debt deviates from this target, some form of 'fiscal feedback' via taxes or spending will operate. However it is also recognized that any attempt to control the debt stock, or the public sector deficit, too tightly may induce instability in other macroeconomic variables. In this paper we examine this trade-off.

We work with a standard New Keynesian model of a closed economy with Calvo pricing to examine how optimal monetary policy varies with the degree to which fiscal instruments respond to the state of government indebtedness, as measured by fiscal feedback. Traditionally it was thought that some minimum level of fiscal feedback was required for a stable and determinate economy. However, the Fiscal Theory of the Price Level literature[1] has argued that a determinate equilibrium may be possible when no feedback occurs, but where prices adjust to ensure the government's intertemporal budget constraint holds. If fiscal feedback is strong, such that fiscal adjustments ensure the government's budget is satisfied, monetary policy is 'active' in the sense that it focuses on the control of inflation; if the fiscal feedback is negligible, monetary policy becomes 'passive' in that it must also ensure fiscal solvency. This paper provides a systematic treatment of the nature of equilibrium over the range of fiscal feedback parameters.[2]

Our analysis forms a bridge between the literature on optimal monetary policy and the literature on joint monetary-fiscal optimization. The former work, as exemplified by Woodford (2003), examines optimal monetary responses to shocks where there are lump-sum taxes available to continuously satisfy the government budget constraint. The latter literature, and in particular Benigno and Woodford (2004), Schmitt-Grohe and Uribe (2004) and Schmitt-Grohe and Uribe (2007), have looked at joint optimization where there are no constraints on the movement of at least one fiscal instrument. However, neither of these approaches allow us to examine the implications that the (mis)conduct of fiscal policy can have on the operation of optimal monetary policy. Here we do this by computing optimal monetary policy *conditional* on fiscal actions in the form of simple debt-controlling rules. This description of fiscal policy appears to be a more realistic modelling of current institutional arrangements. It is generally the case that fiscal policy

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[1] See Leeper (1991), Woodford (1996), but also Buiter (2002) for a more critical view.

[2] The terms 'active' and 'passive' monetary policy are derived from Leeper (1991). Leeper (1991) also describes strong fiscal feedback as 'passive fiscal policy' and negligable fiscal feedback as 'active fiscal policy'.

is far less flexible than monetary policy, and partly as a result, the focus of policy makers seems to be on how quickly government indebtedness should be corrected, as the debates around the Stability and Growth Pact of the European Monetary Union illustrate.[3]

We provide a welfare ranking of policies based on a measure derived from consumer utility, and we also compute joint monetary-fiscal optimization as a benchmark. We find that active monetary policy combined with sufficient fiscal feedback clearly dominates a passive monetary policy when fiscal feedback is negligible. The optimal degree of fiscal feedback occurs when debt follows a path very close to a unit root process, mirroring the path of debt under joint monetary-fiscal optimization. When government spending is the fiscal instrument, this optimal degree of fiscal feedback produces a level of welfare that is only very slightly worse that joint monetary-fiscal optimization, suggesting that the costs of restricting fiscal policy to a simple debt feedback rule are negligible in this case.

However, we find that there are two cases where optimal monetary policy can take on passive features (in the sense that interest rates initially fall following a cost-push shock) even when there is significant fiscal feedback. The first is when the initial level of debt is high, so that changes in interest rates are a particularly effective way of managing debt. The second is where the fiscal feedback parameter is too large from a welfare perspective, such that this strong feedback stabilizes inflation as well as debt. As a result, monetary policy is no longer required to play an active stabilization role, but this substitution is inefficient in terms of social welfare.

We find that the strength of fiscal feedback affects the stability properties of the economy. When fiscal feedback is either completely absent or above a certain threshold optimal monetary policy is able to stabilize both inflation and debt. When fiscal feedback is non-zero but small, we find that optimal monetary policy chooses policies that allow debt to follow a mildly explosive path. This path for debt implies that the economy as a whole does not return to a stationary equilibrium after a temporary shock. Nevertheless, discounting implies that this mildly explosive behavior represents the optimal outcome for monetary policy.

Our initial results are derived assuming that government spending is the fiscal instrument in the feedback rule. We then examine a feedback rule that uses income taxes as an instrument. This produces similar results, although in this case feedback that is too strong (in welfare terms) has less of an impact on monetary policy and welfare. We also examine the robustness of our results to replacing 'infinitely lived' consumers with consumers who face a constant probability of

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[3]This does not imply fiscal policy makers are 'irrational', but may simply reflect overriding political economy concerns that are outside the scope of this paper.

death (and leave no bequests). In this case, joint monetary-fiscal optimization no longer implies a unit root process for debt. While other elements of our results remain unchanged, this analysis does highlight the intergenerational aspects of different degrees of fiscal feedback.

The paper is organized as follows. Section 2 outlines our core model, where consumers are 'infinitely lived'. In Section 3 we present the case of joint monetary-fiscal optimization, which provides a benchmark for our analysis. Section 4 presents our main results, where we use a feedback rule from debt to government spending. We discuss the method and form of the solution in Section 4.2, and then examine its properties in terms of monetary policy and welfare in Section 4.3. Section 5 examines the robustness of our results. We first consider using taxes, or taxes and spending, as fiscal instruments (Section 5.1), then vary the level of steady state debt (Section 5.2) and consider different shocks (Section 5.3). Finally we adapt our model to include consumers with finite lives, and look at intergenerational issues (Section 5.4). Section 6 concludes.

2 The Model

2.1 Consumers

Our model of the household sector is familiar from Woodford (2003). Our economy is inhabited by a large number of individuals, who specialize in the production of a differentiated good (indexed by z), and who spend $h(z)$ of effort in its production. They consume a basket of goods C, and derive utility from per capita government consumption G. Individuals' maximization problem is

$$\max_{\{C_v, h_v\}_{v=t}^\infty} \mathcal{E}_t \sum_{v=t}^\infty \beta^{v-t} \left[u(C_v, \xi_v) + f(G_v, \xi_v) - v(h_v(z), \xi_v) \right]. \tag{1}$$

Here ξ is a preference shock. The price of a differentiated good z is denoted by $p(z)$, and the aggregate price level is P. An individual chooses optimal consumption and work effort to maximize criterion (1) subject to the demand system and the flow budget constraint:

$$P_t C_t + \mathcal{E}_t \left(Q_{t,t+1} \mathcal{A}_{t+1} \right) \leq \mathcal{A}_t + (1 - \Upsilon_t) \left(w_t(z) h_t(z) + \Pi_t(z) \right) + T, \tag{2}$$

where $P_t C_t = \int_0^1 p(z) c(z) dz$ is nominal consumption, $\mathcal{A}_t$ are nominal financial assets of a household, Π_t is profit and T is a constant lump-sum tax/subsidy. Here w is the wage rate, and Υ_t is a tax rate on income. $Q_{t,t+1}$ is the stochastic discount factor which determines the price in period t to the individual of being able to carry a state-contingent amount $\mathcal{A}_{t+1}$ of wealth into period $t+1$. The riskless short term nominal interest rate i_t has the following representation in

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terms of the stochastic discount factor:

$$\mathcal{E}_t(Q_{t,t+1}) = \frac{1}{(1+i_t)}.$$

Each individual consumes the same basket of goods. Goods are aggregated into a Dixit and Stiglitz (1977) consumption index with the elasticity of substitution between any pair of goods given by $\epsilon_t > 1$ (which is a stochastic elasticity with mean ϵ^4), $C_t = \left[\int_0^1 c_t^{\frac{\epsilon_t-1}{\epsilon_t}}(z)dz\right]^{\frac{\epsilon_t}{\epsilon_t-1}}$.

We assume that the net present value of individual's future income is bounded. We also assume that the nominal interest rate is positive at all times. These assumptions rule out infinite consumption and allow us to replace the infinite sequence of flow budget constraints of the individual by a single intertemporal constraint,

$$\mathcal{E}_t \sum_{v=t}^{\infty} Q_{t,v} C_v P_v \leq \mathcal{A}_t + \mathcal{E}_t \sum_{v=t}^{\infty} Q_{t,v} \left\{ (1-\Upsilon_v)\left(w_v(z)h_v(z) + \Pi_v(z)\right)\right) + T \right\}. \tag{3}$$

The optimization requires that the household exhaust its intertemporal budget constraint and, in addition, the household's wealth accumulation must satisfy the no Ponzi game condition:

$$\lim_{s \to \infty} \mathcal{E}_t\left(Q_{t,s}\mathcal{A}_s\right) = 0. \tag{4}$$

We assume the specific functional form for the utility from consumption component, $u(C_v, \xi_t) = \frac{(C_v \xi_t)^{1-1/\sigma}}{1-1/\sigma}$. Household optimization leads to the following dynamic relationship for aggregate consumption:

$$C_t = \mathcal{E}_t\left(\left(\frac{1}{\beta}\frac{P_{t+1}}{P_t}Q_{t,t+1}\right)^{\sigma} C_{t+1}\right). \tag{5}$$

Additionally, aggregate (nominal) asset accumulation is given by

$$\mathcal{A}_{t+1} = (1+i_t)\left(\mathcal{A}_t + (1-\Upsilon_t)\left(W_t N_t + \Pi_t\right) - P_t C_t - T\right), \tag{6}$$

where W_t and N_t are aggregate wages and employment.

We linearize equation (5) around the steady state (here and everywhere below for each variable X_t with steady state value X, we use the notation $\hat{X}_t = \ln(X_t/X)$). Equation (5) leads to the following Euler equation (intertemporal IS curve):

$$\hat{C}_t = \mathcal{E}_t \hat{C}_{t+1} - \sigma\left(\hat{\imath}_t - \mathcal{E}_t \hat{\pi}_{t+1}\right) + \xi_{t+1} - \xi_t. \tag{7}$$

Inflation is $\pi_t = \frac{P_t}{P_{t-1}} - 1$ and we assume steady state inflation is zero.

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[4] We make this parameter stochastic to allow us to generate shocks to the mark-up of firms.

2.2 Price Setting

Price setting is based on Calvo contracting as set out in Woodford (2003). Each period agents recalculate their prices with fixed probability $1-\gamma$. If prices are not recalculated (with probability γ), they remain fixed. Following Woodford (2003) and allowing for government consumption terms in the utility function, we can derive the following Phillips curve for our economy[5]:

$$\hat{\pi}_t = \beta\mathcal{E}_t\hat{\pi}_{t+1} + \frac{(1-\gamma\beta)(1-\gamma)\psi}{\gamma(\psi+\epsilon)}\hat{s}_t, \tag{8}$$

where marginal cost is

$$\hat{s}_t = \frac{1}{\psi}\hat{Y}_t + \frac{1}{\sigma}\hat{C}_t + \frac{\tau}{(1-\tau)}\hat{\Upsilon}_t - \hat{\zeta}_t + \left(\frac{v_{y\xi}}{v_y} - \frac{u_{C\xi}}{u_C}\right)\hat{\xi}_t + \hat{\eta}_t.$$

The shock $\hat{\eta}_t$ is a mark-up shock and $\hat{\zeta}_t$ is a technology shock, as we assume the production function $y_t = Z_t h_t$, $Z_t = Z_{t-1}\zeta_t$, where ζ_t has a mean of unity. Here $\psi = v_y/v_{yy}y$ and τ is the steady state income tax rate.

Under flexible prices and in the steady state the real wage is always equal to the monopolistic mark-up $\mu_t = -(1-\epsilon_t)/\epsilon_t$. Optimization by consumers then implies:

$$\frac{\mu^w}{\mu_t} = \frac{v_y(y_t^n(z),\xi_t)}{Q_t\left(1-\hat{\Upsilon}_t^n\right)u_C(C_t^n,\xi_t)}, \tag{9}$$

where superscript n denotes natural levels (see Woodford (2003)), and μ^w is a steady state employment subsidy which we discuss below. Linearization of (9) yields

$$\hat{Y}_t^n\frac{1}{\psi} + \hat{C}_t^n\frac{1}{\sigma} + \frac{\tau}{(1-\tau)}\hat{\Upsilon}_t^n - \hat{\zeta}_t + \left(\frac{v_{y\xi}}{v_y} - \frac{u_{C\xi}}{u_C}\right)\hat{\xi}_t = 0. \tag{10}$$

2.3 Fiscal Constraint

The government buys goods (G_t), taxes income (with tax rate Υ_t), raises lump-sum taxes, pays an employment subsidy and issues nominal debt $\mathcal{B}_t$. The evolution of the nominal debt stock can be written as:

$$\mathcal{B}_{t+1} = (1+i_t)(\mathcal{B}_t + P_t G_t - \Upsilon_t P_t Y_t - T + \mu^w).$$

The employment subsidy (μ^w) and lump-sum taxes (T) are constant. This equation can be linearized as (defining $B_t = \mathcal{B}_t/P_{t-1}$ and denoting the steady state ratio of debt to output as χ) :

$$\chi\hat{B}_{t+1} = \chi\hat{\imath}_t + \frac{1}{\beta}\left(\chi\hat{B}_t - \chi\hat{\pi}_t + (1-\rho)\hat{G}_t - \tau\left(\hat{\Upsilon}_t + \hat{Y}_t\right)\right). \tag{11}$$

where $\rho = C/Y$ in steady state.

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[5]The derivation is identical to the one in Woodford (2003), amended by the introduction of mark-up shocks as in Beetsma and Jensen (2004a).

2.4   Aggregate Relationships

Output is distributed as wages and profits:

$$Y_t = W_t N_t + \Pi_t. \tag{12}$$

Government expenditures constitute part of demand, so the national income identity can be written as

$$Y_t = C_t + G_t, \tag{13}$$

and in steady state $G = (1 - \rho)Y$. The linearized national income identity is then:

$$\hat{Y}_t = (1 - \rho)\hat{G}_t + \rho\hat{C}_t. \tag{14}$$

2.5   Behavior of the Economy

We now write down the final system of equations for the 'law of motion' of the out-of-steady-state economy. We simplify notation by using lower case letters to denote 'gap' variables, where the gap is the difference between actual levels and natural levels i.e. $x_t = \hat{X}_t - \hat{X}_t^n$. The model consists of an intertemporal IS curve (15), the Phillips curve (16), national income identity (17), and an equation explaining the evolution of debt (18). We could use the linearized assets accumulation equation (6) instead of (18) as they are equivalent (equation (19)). The system is:

$$c_t = \mathcal{E}_t c_{t+1} - \sigma(i_t - \mathcal{E}_t \pi_{t+1}), \tag{15}$$

$$\pi_t = \beta \mathcal{E}_t \pi_{t+1} + \kappa \left(\frac{1}{\sigma} c_t + \frac{1}{\psi} y_t + \frac{\tau}{(1-\tau)} \tau_t + \hat{\eta}_t \right), \tag{16}$$

$$y_t = (1 - \rho)g_t + \rho c_t, \tag{17}$$

$$\tilde{b}_{t+1} = \chi i_t + \frac{1}{\beta} \left(\tilde{b}_t - \chi \pi_t + (1 - \rho) g_t - \tau (\tau_t + y_t) \right) + \hat{\delta}_t, \tag{18}$$

$$\tilde{a}_t = \tilde{b}_t, \tag{19}$$

where parameter $\kappa = \frac{(1-\gamma\beta)(1-\gamma)\psi}{\gamma(\psi+\epsilon)}$ and $\hat{\delta}_t = \frac{\chi(1-\beta)}{\beta}\hat{\xi}_t - \left(\frac{(1-\beta)}{\beta} + \frac{(1-\rho_q)}{\sigma} \right) \frac{\chi\psi\sigma}{(\sigma+\psi)}\hat{\zeta}_t$ is a composite shock. We denote $\tilde{b}_t = \chi\hat{B}_t$. Note that preference and technology shocks only appear in so far as they impact on debt, while cost push shocks matter through the Phillips curve.

It remains to specify policy. We do this in Section 3, where we also discuss some benchmark results of full optimization.

2.6 Calibration

We take the model's frequency to be quarterly. To achieve a steady state rate of interest of approximately 4%, we set the household discount rate β to 0.99. Output is normalized to one, and the ratio of government consumption to output, $1 - \rho$, is 0.25, which determines the relative preference for government spending in utility. The remaining parameters of the utility function are typical of those used in the literature, see e.g. Canzoneri et al. (2006). The elasticity of intertemporal substitution σ is taken as $1/1.5$, the Calvo parameter γ is set at 0.75 so as to imply average contracts of about a year, the elasticity of demand is taken as $\varepsilon = 7.0$ to achieve a 17% mark up, and the elasticity of labour demand is taken as $\psi = 1/3$.

We consider three values for the debt to GDP ratio. Our 'high' debt level corresponds to 60% of annual output, which is the level of debt in a number of European economies. For analytical purposes, it is useful to consider a 'low debt' case where the steady state debt is zero. (Although unusual, such cases are not unknown: New Zealand, Sweden and Ireland have net debt to GDP ratios close to zero.) Our base case for χ is the midpoint of 30% of annual output. We discuss these figures further in Section 5.2 below.

Following Ireland (2004) and Canzoneri et al. (2006), the preference shock is calibrated as an AR(1) process $\hat{\xi}_t = \rho_\xi \hat{\xi}_{t-1} + \varepsilon_{\xi t}$ with $\rho_\xi = 0.9$ and $\sigma\left(\varepsilon_{\xi t}\right) = 0.03$. We calibrate the productivity shock as an AR(1) process $\hat{\zeta}_t = \rho_\zeta \hat{\zeta}_{t-1} + \varepsilon_{qt}$ with $\rho_\zeta = 0.9$ and $\sigma\left(\varepsilon_{\zeta t}\right) = 0.0075$. This is broadly in line with the values used in Canzoneri et al. (2006) $(\rho_\zeta, \sigma\left(\varepsilon_{\zeta t}\right)) = (0.92, 0.0090))$, Ireland (2004) $(\rho_\zeta, \sigma\left(\varepsilon_{\zeta t}\right)) = (1.00, 0.0109))$ and those used in Schmitt-Grohe and Uribe (2007) $(\rho_\zeta, \sigma\left(\varepsilon_{\zeta t}\right)) = (0.86, 0.0064))$. [6] Among these three studies, only Ireland (2004) uses a cost-push shock, which is AR(1) with a standard deviation of 0.0044. Smets and Wouters (2003) reports an i.i.d. cost push shock with a much smaller standard deviation in the model with inflation persistence, while Rudebusch (2002) estimates a standard deviation of 0.01 for an i.i.d. cost push shock. In the analysis below, we calibrate the standard deviation of an i.i.d. cost-push shock as 0.005. In our base line case this generates a standard deviation for inflation of 0.0038 that is the same order of magnitude as empirical data in developed countries over the last couple of decades.

3 Joint Optimisation as a Benchmark Case

The primary aim of this paper is to study the effect of a fiscal policy on monetary policy decisions. As a benchmark case we first compute a fully optimal policy i.e. joint monetary-fiscal optimiza-

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[6]Schmitt-Grohe and Uribe (2007) do not consider either taste shocks or cost-push shocks.

tion. We assume that both authorities jointly set their instruments $\{i_t, g_t, \tau_t\}$ to maximize the aggregate utility function:

$$\max_{\{i_s\}_{s=t}^{\infty}} \frac{1}{2} \mathcal{E}_t \sum_{s=t}^{\infty} \beta^{s-t} \left[u(C_s) + f(G_s) - \int_0^1 v(h_s(z)) dz \right]. \tag{20}$$

We show in Appendix A.2 that problem (20) implies the following optimisation problem:

$$\min_{\{i_s\}_{s=t}^{\infty}} \frac{1}{2} \mathcal{E}_t \sum_{s=t}^{\infty} \beta^{s-t} \left[a_\pi \pi_s^2 + a_c c_s^2 + a_g g_s^2 + a_y y_s^2 \right] + \mathcal{O}(3), \tag{21}$$

where $\mathcal{O}(3)$ collects terms of higher than second order and terms independent of policy, and all a_i are positive. This quadratic approximation to social welfare is obtained assuming that there is a constant production subsidy $\mu^w = T$ that eliminates the distortion caused by monopolistic competition and income taxes in steady state. (This approach follows Woodford (2003). Sutherland (2002) and Benigno and Woodford (2004) use an alternative way of eliminating first order terms from welfare, while Schmitt-Grohe and Uribe (2004) do not use a linear quadratic framework, but instead adopt a Ramsey approach.) We assume that the authorities have sufficient credibility to commit to the time inconsistent plan, so it can implement the first best time inconsistent solution.

Note that expression (21) contains a quadratic term in government spending, g. This term enters the welfare expression because it is assumed in (1) that households derive utility from the consumption of public goods, and that the level of government spending in steady state reflects this. However, if we instead assumed that government spending was pure waste, but the government still used g as a policy instrument, changes in g would still influence social welfare through the national income identity, but it would not constitute an independent source of welfare loss.

In order to solve for the fully optimal policy we specify system matrices and use MATLAB code by Söderlind (1999). The procedure is straightforward and non-innovative, so we only discuss the results, which are summarized in Table 1.

Table 1 shows that cost-push shocks have a much greater impact on social welfare than preference or productivity shocks when policy is optimal. This is a well known result, and arises because mark up shocks change the relationship between inflation and output, both of which are key policy objectives. In contrast, the impact of preference or productivity shocks only matter through their impact on debt: if lump sum taxes were available as a stabilization policy instrument, monetary policy could fully offset these shocks. Although lump sum taxes are constant in our case, the behavior of debt discussed below means that shocks to debt have

Shocks applied	Welfare loss of fully optimal monetary and fiscal policy if fiscal instruments are:		
	spending	taxes	spending and taxes
cost-push	2.054	0.230	0.220
taste	2.53×10^{-5}	1.25×10^{-6}	1.19×10^{-6}
productivity	4.49×10^{-5}	2.22×10^{-6}	2.12×10^{-6}
all shocks	2.054	0.230	0.220

Table 1: Welfare implications of shocks, measured as percent of steady state consumption under monetary-fiscal optimisation.

little impact on welfare, see also Leith and Wren-Lewis (2007). Our analysis below is based on applying the full menu of shocks, but as cost-push shocks are likely to dominate we also consider a case where there are only preference and productivity shocks (Section 5.3).

Figure 1 shows the path of key variables following a unit cost-push shock. The most straightforward case is where only government spending is available as an instrument. Here monetary policy responds to the increase in inflation generated by the cost-push shock, raising interest rates in both the initial and subsequent periods. Higher interest rates raise the level of debt, which increases gradually but eventually stabilizes at a new higher level. This unit root process for steady state debt, which is noted for joint monetary-fiscal optimization in Benigno and Woodford (2004) and Schmitt-Grohe and Uribe (2004), is an extension of a result from the tax smoothing literature (Barro (1979)). As the welfare function is convex and there is discounting, it is preferable to move fiscal instruments by a small amount permanently to service a new higher level of debt, rather than change them by a large amount on a temporary basis to return debt to its initial level. Note, however, that the cut in government spending is larger in the first period of the simulation than subsequent periods, so there is a very modest attempt to reduce the size of the long run increase in debt. The reasons for this, and its implications for policy under discretion, are discussed in Leith and Wren-Lewis (2007). When income taxes are available as a fiscal instrument, then there is an initial attempt to directly offset to cost-push shock by cutting taxes. Lower income taxes increase the incentive to work, which directly reduces the inflationary consequences of the cost-push shock.

4   Fiscal stabilisation of debt

4.1   Simple Feedback Rules

A number of authors have used simple fiscal rules where a fiscal instrument responds to deviations in debt from some reference value (for example Schmitt-Grohe and Uribe (2007), Canzoneri et al. (2006) among others). In the first part of our analysis we focus on the use of government spending as an instrument, but we broaden it to include income taxes subsequently. Empirical estimates of fiscal policy reaction functions have tended to focus on cyclical behavior rather than debt feedback (see Favero and Monacelli (2005), Taylor (2000), Auerbach (2002) for example), but the evidence suggests that both government spending and taxes do move to stabilize debt (see Muscatelli et al. (2004) for example). Our initial focus on spending is expositionally convenient, for reasons that become clear when we consider taxes in Section 5.1. We postulate that out-of-steady-state government expenditure G_t is related to out-of-steady-state debt according to the following simple feedback rule:

$$G_t - G_t^n = -\lambda \left(B_t - B \right),\tag{22}$$

Log-linearisation of this rules yields

$$g_t = -\frac{\lambda}{(1-\rho)}\tilde{b}_t,\tag{23}$$

Simple mechanistic rules for fiscal policy more accurately reflect institutional rigidities in fiscal policymaking than full optimization, where the latter would imply that fiscal instruments would immediately respond in an optimal fashion to contemporaneous shocks. Of course there are a variety of potential simple rules, but as our focus in this paper is on how debt stabilization affects optimal monetary policy, the specification above seems appropriate. In addition, we note that debt is the only state variable in our model. Furthermore, as we show below, when government spending is the instrument this rule comes very close to reproducing the outcome that would occur under full optimization, and so more complicated rules appear unnecessary in this case.

In what follows we shall explore the implications of different values of the fiscal feedback parameter λ. We are interested in two key questions. First, can we distinguish clearly between two policy 'regimes', as suggested by the Fiscal Theory of the Price Level and the results in Leeper (1991) and Leith and Wren-Lewis (2000)? If we can, how does welfare and optimal monetary policy compare between regimes? Second, what is the optimal degree of fiscal feedback on debt, and what are the implications for welfare and monetary policy of departing from this optimum?

4.2 The Solution

In order to solve the model we use the method of Lagrange multipliers. Unlike the case of joint optimization in Section 3, an analysis of first order conditions is crucial to explain certain dynamic properties of the solution.

The central bank chooses the nominal interest rate to minimize social loss (21) subject to the evolution of the economy. It is instructive to simplify the dynamic system (15)-(19) that describes the evolution of out-of-steady-state economy, as observed by the monetary policymaker. We substitute equations (17) and (52) into (16), (15) and (18), leaving only three dynamic equations for c_s, π_s and $\tilde{b}_s$:

$$
\begin{aligned}
c_s &= \mathcal{E}_t c_{s+1} - \sigma(i_s - \mathcal{E}_t \pi_{s+1}), \\
\pi_s &= \beta \mathcal{E}_t \pi_{s+1} + \kappa \left(\frac{1}{\sigma} + \frac{\rho}{\psi} \right) c_s - \kappa \frac{\lambda}{\psi} \tilde{b}_s + \kappa \hat{\eta}_s, \\
\tilde{b}_{s+1} &= \chi i_s + \frac{1}{\beta} \left((1 - (1 - \tau)\lambda) \tilde{b}_s - \chi \pi_s - \tau \rho c_s \right) + \hat{\delta}_s.
\end{aligned}
$$

Let the period at which optimization is taking place be period t. We construct the Lagrangian

$$
\mathcal{L} = \mathcal{E}_t \sum_{s=t}^{\infty} H_s,
$$

where each term H_s has the following form

$$
\begin{aligned}
H_s &= \frac{1}{2} \beta^{s-t} \left[a_\pi \pi_s^2 + a_c c_s^2 + a_g \left(\frac{\lambda}{(1-\rho)} \right)^2 \tilde{b}_s^2 + a_y \left(-\lambda \tilde{b}_s + \rho c_s \right)^2 \right] \\
&\quad + \beta^{s-t} L_{s+1}^c \left(\mathcal{E}_t c_{s+1} - \sigma(i_s - \mathcal{E}_t \pi_{s+1}) - c_s \right) \\
&\quad + \beta^{s-t} L_{s+1}^\pi \left(\beta \mathcal{E}_t \pi_{s+1} + \kappa \left(\frac{1}{\sigma} + \frac{\rho}{\psi} \right) c_s - \kappa \frac{\lambda}{\psi} \tilde{b}_s + \kappa \hat{\eta}_s - \pi_s \right) \\
&\quad + \beta^{s-t} L_{s+1}^b \left(\chi i_s + \frac{1}{\beta} \left((1 - (1 - \tau)\lambda) \tilde{b}_s - \chi \pi_s - \tau \rho c_s \right) + \hat{\delta}_s - \tilde{b}_{s+1} \right).
\end{aligned}
$$

In order to minimize the loss function in (21), we differentiate the Lagrangian, $\mathcal{L}$, with respect

to L^c, L^π, L^b, π, c, b and i. The first order conditions for optimality are:

$$\frac{\partial \mathcal{L}}{\partial \pi_s} = 0 = \beta^{s-t} a_\pi \pi_s + \sigma \beta^{s-t-1} L^c_s + \beta^{s-t+1} L^\pi_s - \beta^{s-t} L^\pi_{s+1} - \beta^{s-t-1} \chi L^b_{s+1}, \tag{24}$$

$$\frac{\partial \mathcal{L}}{\partial c_s} = 0 = \beta^{s-t} a_c c_s + \beta^{s-t} a_y \rho \left(-\lambda \tilde{b}_s + \rho c_s \right) + \beta^{s-t-1} L^c_s - \beta^{s-t} L^c_{s+1} \tag{25}$$

$$+ \kappa \left(\frac{1}{\sigma} + \frac{\rho}{\psi} \right) \beta^{s-t} L^\pi_{s+1} - \frac{\tau \rho}{\beta} \beta^{s-t} L^b_{s+1},$$

$$\frac{\partial \mathcal{L}}{\partial \tilde{b}_s} = 0 = \beta^{s-t} a_g \left(\frac{\lambda}{(1-\rho)} \right)^2 \tilde{b}_s - \lambda \beta^{s-t} a_y \left(-\lambda \tilde{b}_s + \rho c_s \right) \tag{26}$$

$$- \beta^{s-t} \kappa \frac{\lambda}{\psi} \chi L^\pi_{s+1} + (1 - (1-\tau)\lambda) \beta^{s-t-1} L^b_{s+1} - \beta^{s-t-1} L^b_s,$$

$$\frac{\partial \mathcal{L}}{\partial i_s} = 0 = -\sigma \beta^{s-t} L^c_{s+1} + \beta^{s-t} \chi L^b_{s+1}, \tag{27}$$

$$\frac{\partial \mathcal{L}}{\partial L^c_{s+1}} = 0 = \mathcal{E}_t c_{s+1} - \sigma(i_s - \pi_{s+1}) - c_s, \tag{28}$$

$$\frac{\partial \mathcal{L}}{\partial L^\pi_{s+1}} = 0 = \beta \mathcal{E}_t \pi_{s+1} + \kappa \left(\frac{1}{\sigma} + \frac{\rho}{\psi} \right) c_s - \kappa \frac{\lambda}{\psi} \tilde{b}_s + \kappa \hat{\eta}_s - \pi_s, \tag{29}$$

$$\frac{\partial \mathcal{L}}{\partial L^b_{s+1}} = 0 = \chi i_s + \frac{1}{\beta} \left((1 - (1-\tau)\lambda) \tilde{b}_s - \chi \pi_s - \tau \rho c_s \right) + \hat{\delta}_s - \tilde{b}_{s+1}, \tag{30}$$

along with initial conditions $L^c_t = L^\pi_t = 0, \tilde{b}_t = \bar{b}$ (see Currie and Levine (1993)).[7]

The linear difference system (24)-(30) is closed with a dynamic process for the two exogenous shocks $\hat{\eta}_{st}$ and $\hat{\delta}_t$ and this makes it to be of ninth order. It should be solved subject to nine boundary conditions. We know five initial values: (i) initial values for predetermined endogenous variables (debt in our case); (ii) Pontryagin's maximum principle requires setting to zero initial conditions for those Lagrange multipliers which are associated with dynamic constraints on non-predetermined variables (see e.g. Currie and Levine (1993)), L^π and L^c in our case; (iii) and initial values of shocks $\hat{\eta}$ and $\hat{\delta}$. We need to define an appropriate transversality conditions to close the system. Welfare will be finite if variables grow slower than at a rate of $1/\sqrt{\beta}$, $\lim_{t\to\infty} \beta^{t/2} x_t = 0$ for any variable x_t. Moreover, by imposing this transversality condition we guarantee that this solution to the system (24)-(30) is unique if it exists.[8]

The system (24)-(30) plus dynamic processes for the shocks can be written in a matrix form $\Omega z_{t+1} = \Psi z_t$ where $z_t = (\hat{\eta}_t, \hat{\nu}_t, \tilde{b}_t, \nu^\pi_t, \nu^c_t, i_s, \nu^b_t, \pi_t, c_t)'$, and $\nu^k_t = \beta^{s-t} L^k_t$, $k \in \{\pi, b, c\}$. This linear

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[7]In all the analysis below we take $\bar{b} = 0$ i.e. we assume we start from a position in which debt is at its steady state.

[8]Solutions to the dynamic system (24)-(30) would always exist if we imposed $\lim_{t\to\infty} \beta^{t/2} x_t < \infty$. However, in this case there can be multiple solutions with the asymptotic growth rate $1/\sqrt{\beta}$. Such solutions would not lead to finite welfare, but would still solve the system (24)-(30).

system has nine generalized eigenvalues[9], which are functions of the fiscal feedback parameter λ. The dynamics of the system can be fully described in terms of these eigenvalues. Moreover, the speed of convergence of the economy towards the steady state is determined by the largest eigenvalue of this system among those that are less than $1/\sqrt{\beta}$. This implies that system (24)-(30) can have solutions that exhibit a (moderate) explosion. We now examine if this is indeed the case.[10]

To do this, it is instructive to calculate values of λ that can generate an eigenvalue $\Phi(\lambda) = 1$. We show in Appendix C that there are two solutions to the problem where some $\Phi(\lambda) = 1$ in the area $0 \leq \lambda < \infty$. They are $\lambda = 0$ and $\lambda = \lambda^*$ where

$$\lambda^* = \frac{1 - \beta}{1 - \frac{\tau \psi}{\psi + \sigma \rho}}. \tag{31}$$

(The economic interpretation of λ^* is discussed in the next section.) More specifically, at these two points for λ we have the following structure of the generalized eigenvalues of system (24)-(30).

1. If $\lambda = 0$ then the nine generalized eigenvalues to this problem are:

$$\Phi_1 = \Phi_2 = \Phi_3 = 0,$$

$$\Phi_4 = \frac{1}{2} \left(\frac{\left((\beta + 1) + \frac{\kappa \epsilon}{\rho} \left(\frac{1}{\sigma} + \frac{\rho}{\psi} \right) \right)}{\beta} - \sqrt{\left(\frac{\left((\beta + 1) + \frac{\kappa \epsilon}{\rho} \left(\frac{1}{\sigma} + \frac{\rho}{\psi} \right) \right)}{\beta} \right)^2 - \frac{4}{\beta}} \right) < 1,$$

$$\Phi_5 = 1, \ \Phi_6 = \frac{1}{\beta},$$

$$\Phi_7 = \frac{1}{2} \left(\frac{\left((\beta + 1) + \frac{\kappa \epsilon}{\rho} \left(\frac{1}{\sigma} + \frac{\rho}{\psi} \right) \right)}{\beta} + \sqrt{\left(\frac{\left((\beta + 1) + \frac{\kappa \epsilon}{\rho} \left(\frac{1}{\sigma} + \frac{\rho}{\psi} \right) \right)}{\beta} \right)^2 - \frac{4}{\beta}} \right) > \frac{1}{\beta},$$

$$\Phi_8 = \Phi_9 = \infty > \frac{1}{\beta}.$$

There are five generalized eigenvalues that are strictly less than $1/\sqrt{\beta}$, and the biggest of them is equal to one.

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[9]Since Ω is singular, we deal with *generalised* eigenvalues, i.e. solutions Φ of the equation $\det(\Psi - \Phi\Omega) = 0$. See Klein (2000) on solutions of such systems. In what follows we will often omit the word 'generalised', but we keep the meaning.

[10]Note that we work with a linearised model and an infinite time horizon, so equations only remain valid in a neighbourhood of the steady state. Although *at some point* a moderate explosion would exceed these boundaries, the stability analysis is unaffected.

2. If $\lambda = \lambda^*$, then the eigenvalues are

$$\begin{aligned}
\Phi_1 &= \Phi_2 = \Phi_3 = 0, \\
\Phi_4 &= \text{solution of quadratic equation} < 1, \\
\Phi_5 &= 1, \ \Phi_6 = \frac{1}{\beta}, \\
\Phi_7 &= \text{solution of quadratic equation} > \frac{1}{\beta}, \\
\Phi_8 &= \Phi_9 = \infty > \frac{1}{\beta}.
\end{aligned}$$

There are five generalized eigenvalues that are strictly less than $1/\sqrt{\beta}$, and the biggest of them is equal to one.

These two values of fiscal feedback, 0 and λ^*, split the set of possible feedbacks into the two open sets: $0 < \lambda < \lambda^*$ and $\lambda > \lambda^*$ We can also show numerically that for a relatively wide range of $0 \leq \lambda \leq \Lambda$, $\Lambda \gg \lambda^*$, the following holds:

(i) some eigenvalues remain either smaller than one or greater than $\frac{1}{\beta}$ for any $0 \leq \lambda \leq \Lambda$: $\|\Phi_8(\lambda)\| = \|\Phi_9(\lambda)\| = \infty$, $\|\Phi_1(\lambda)\| = \|\Phi_2(\lambda)\| = \|\Phi_3(\lambda)\| = 0$, $\|\Phi_4(\lambda)\| < 1, \|\Phi_5(\lambda)\| > 1/\beta$. None of these eigenvalues is the 'biggest stable eigenvalue' and so none of them will determine the rate of convergence of the economy to the steady state following a shock.

(ii) Eigenvalues $\Phi_5(\lambda), \Phi_6(\lambda)$ behave in the following way. First of all, they do not intersect: $\Phi_5(\lambda) < 1/\sqrt{\beta}$, and $\Phi_5(\lambda) > 1/\sqrt{\beta}$.[11] If $0 \leq \lambda \leq \lambda^*$, $\Phi_5(\lambda)$ increases from $\Phi_5(0) = 1$ up until it almost reaches $1/\sqrt{\beta}$ and then decreases to $\Phi_5(\lambda^*) = 1$.$\Phi_6(\lambda)$ decreases from $\Phi_6(0) = 1$ down until it almost reaches $1/\sqrt{\beta}$ and then increases back to $\Phi_6(\lambda^*) = 1/\beta$. For $\lambda > \lambda^*$, $\Phi_5(\lambda) < 1$ and $\Phi_6(\lambda) > 1/\beta$. $\Phi_5(\lambda)$ has a key role in determining the dynamic properties of the economy.

This dependence of eigenvalues on the value of fiscal feedback λ is shown schematically in Panel I in Figure 2, where eigenvalues are plotted against λ. Depending on the value of λ we can distinguish three cases.

1. *Strong fiscal feedback when $\lambda \geq \lambda^*$.* When $\lambda > \lambda^*$ we have five eigenvalues which are strictly less than one, and four explosive eigenvalues (which are strictly greater than $1/\beta > 1/\sqrt{\beta}$). Given five initial conditions and transversality conditions we obtain a unique solution. When t increases, all economic variables, $\tilde{b}_t, c_t, \pi_t$, once disturbed, necessarily converge to their steady state values. We say that in this case the solution and the steady state it converges

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[11]We show this analytically in Appendix C.

to are *asymptotically stable*. When $\lambda = \lambda^*$ then $\Phi_5 = 1$ and $\Phi_6 = 1/\beta$. The sixth eigenvalue is ruled out by the transversality conditions but Φ_5 is accepted by them. The optimal monetary policy thus generates unit-root dynamics of economic variables in response to shocks. We say that in this case the solution, and the steady state it does not diverge from, are *just stable*.

2. *Zero fiscal feedback when $\lambda = 0$.* If $\lambda = 0$ we have $\Phi_5 = 1$, $\Phi_6 = 1/\beta$. Again, transversality conditions classify Φ_5 as a just stable eigenvalue. We thus obtain a unique solution. We can check that in this case neither of the economic variables b_t, c_t, π_t nor instrument i_t will exhibit unit-root behavior, as the Lagrange multipliers have unit root dynamics.

3. *Weak fiscal feedback when $0 < \lambda < \lambda^*$.* For the intermediate range of parameter λ, $0 < \lambda < \lambda^*$, there are five eigenvalues that are less than $1/\sqrt{\beta}$, but one of them is greater than one. The model thus exhibits explosive behavior. This explosive behavior is modest, as variables grow at an asymptotic rate that is slower than the steady state rate of interest, $1/\beta$. The implied loss is finite.

These results suggest that we can define three different *regimes*, determined by the two threshold values of parameter λ. Two regimes, the zero fiscal feedback regime and the strong fiscal feedback regime, are regimes with stable solutions. The third regime, the weak fiscal feedback regime, generates a moderately explosive solution that delivers finite social loss. Although such behavior is theoretically possible, we are not aware of another example in the literature where it would be *optimal* for monetary policy to support moderate explosion. Although this distinction is made on the basis of dynamic stability properties, we shall see in the next section that there is a substantial difference among these regimes in the implied economic behavior of the policymaker.

4.3   Optimal Monetary Policy and Welfare

Panel II in Figure 2 presents the values of some key magnitudes as we change the degree of fiscal feedback λ. The first two columns are identical except for the scale of λ: the first column focuses on small positive values of fiscal feedback, whereas the second column gives results for a much broader range.

The top two rows report monetary policy responses to the cost-push shock and debt. It is well known (see Appendix B) that the unique solution for the optimal interest rate reaction function in linear-quadratic models can be presented in the form of a linear relationship:

$$i_t = \theta_\eta \hat{\eta}_t + \theta_\zeta \hat{\zeta}_t + \theta_b \tilde{b}_t + \vartheta_\pi L_t^\pi + \vartheta_c L_t^c, \tag{32}$$

with feedback coefficients θ on predetermined states and predetermined Lagrange multipliers. We plot two parameters from the implied reaction function for monetary policy (32): the feedback coefficient on the mark-up shock θ_η and on debt θ_b.[12] Solid lines indicate solutions that are asymptotically stable, while dotted lines are solutions that involve mild explosions (see the discussion in the previous section).

The bottom row of Panel II plots the social welfare loss expressed as a percent reduction in steady-state consumption. We discuss below the size of these welfare losses, and compare them to the results in the relevant literature.

4.4 Three policy regimes

Figure 2 and the analysis in Section 4.2 suggest that there are two regimes in which all processes are either asymptotically stable or unit root, and one in between which exhibits moderate explosive behavior. We discuss them in turn.

The first regime occurs when $\lambda = 0$. Here we have no feedback from debt to fiscal variables, so we might suppose that debt in this model would be unstable. However, the results show that monetary policy ensures the *asymptotic* stability of economic variables. It achieves this in two ways, both shown in Panel II in Figure 2. First, θ_b ensures that any positive movement of debt leads to a large fall in interest rates, which leads to correction through the government's budget constraint. Second, the reaction to a positive cost-push shock is also to reduce interest rates.

The negative feedback on debt is not surprising, given inaction by the fiscal authorities. The negative reaction to the cost-push shock is more interesting, and it raises the question of how monetary policy stabilizes inflation in this case. To understand what is going on, Panel I in Figure 3 plots impulse responses to the cost-push shock for two cases, both of which set $\lambda = 0$: optimal monetary policy (solid line), and for fixed nominal interest rates, which is a standard example of a passive monetary policy in the literature. The cost push shock raises inflation, and when interest rates are fixed this reduces debt. However, when inflation falls back, it returns to a small negative number, and from there gradually converges to zero. This reduction in inflation slowly increases debt, and allows it to return to its initial level. When monetary policy is optimal, then in the first period there is a large reduction in interest rates, as Panel II in Figure 2 also shows, but this is followed by an increase in interest rates. The key to why this path is optimal is that both consumption and inflation are forward looking. Higher future interest rates largely offset

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[12]The other ϑ parameters are less informative. ϑ_L represent the integral control part of the reaction function and therefore feedback on slow moving variables.

the impact of the immediate decline in interest rates on first period consumption, and thereafter consumption is below base. Inflation depends on current and future consumption, so inflation is lower in all periods as a result of this behavior in consumption. Of course debt depends on its own past value through the budget constraint. Thus, by cutting interest rates initially and raising them subsequently, monetary policy is able to both stabilize debt and moderate the initial increase in inflation.[13]

In contrast, where $\lambda \geq \lambda^*$, the reaction to the cost-push shock is generally to raise interest rates. The system is asymptotically stable for $\lambda > \lambda^*$ and exhibits a unit-root behavior for $\lambda = \lambda^*$. The contrast between behavior in the two regimes is illustrated in Panel II in Figure 3, which plots the impulse response for key economic variables following a cost push shock for two values of λ: $\lambda = 0$ as a dashed line, and $\lambda = \bar{\lambda} > \lambda^*$ as a solid line, where $\bar{\lambda}$ is only marginally higher than λ^* so the solution is asymptotically stable.

As we would expect from simply considering the government's budget constraint, the value λ^* that produces unit-root behavior in debt will be a function of the steady state real interest rate. However it is also a function of the tax rate, as the reduced form debt equation in Section 4.2 shows.[14] A reduction in government spending will reduce debt directly, but it also reduces income and therefore income taxes, which raises debt. Thus λ^* has to be greater than the steady state real interest rate to just stabilize debt.

These two regimes are separated by a region $0 < \lambda < \lambda^*$ where the system is not asymptotically stable, but has a 'moderately' explosive solution. In this case fully optimal monetary policy chooses not to ensure the convergence of the economy back to the steady state. As we have seen above, there is a clear conflict for monetary policy in both stabilizing debt and stabilizing inflation when fiscal feedback fails to stabilize debt. As a result, monetary policy pushes close to the boundary one of these objectives, which is to allow debt to mildly explode in a manner that keeps the welfare loss finite and allows policy to reduce the initial impact on inflation.

There is a clear parallel between our results using optimal monetary policy and the active and passive regimes described by Leeper (1991) and Leith and Wren-Lewis (2000). In their case a passive monetary policy is defined as a negative response of real interest rates when inflation is

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[13]Although optimal policy produces larger deviations from the inflation target after the initial period than under the fixed interest rate policy, the convexity of the welfare function implies that the impact of this on welfare is more than offset by the impact of the reduction in inflation in the initial period.

[14]Leith and Wren-Lewis (2000) use a determinate condition (which is a necessary but not sufficient condition for a determinate or stable solution) to calculate analytically a value of fiscal feedback of a similar order of magnitude that divides their two policy regimes. They can do this because they use a Taylor rule to describe monetary policy, rather than calculate optimum monetary policy.

above target in a Taylor rule, whereas in our case it corresponds to a substantial fall in real interest rates following a positive cost-push shock. When there is no fiscal feedback, fiscal instruments do nothing to prevent a debt interest spiral. To avoid an explosive solution for debt, monetary rather than fiscal policy must stabilize the government's debt stock, along the lines discussed above. However, unlike the literature cited above, our alternative regime does not always involve an active monetary policy, a point which we return to when we look at alternative steady state values of debt.

4.5   Fiscal feedback and welfare

As Panel II in Figure 2 shows, there is a non-trivial difference in the levels of welfare attained in the two asymptotically stable regimes, which at its greatest is around 0.25% of steady state consumption. While the papers cited above, and the Fiscal Theory of the Price Level more generally, have shown that a lack of fiscal feedback does not necessary lead to model instability, it is clear from our results that weak or zero fiscal feedback seriously damages the ability of monetary policy to reduce the social costs of macroeconomic shocks. These results are in contrast with those obtained by Schmitt-Grohe and Uribe (2007) where the difference does not exceed 0.05% of steady state consumption. This contrast reflects our inclusion of cost-push shocks, which in models of this kind are the most problematic for monetary policy. Taste/technology shocks are much less important because, with optimal monetary policy, they only influence our economy through their impact on debt, as we showed in Section 2.5, and the gain from eliminating these effects is small.

The optimal value of λ (which we denote as $\bar{\lambda}$, and which was used to plot dashed lines in Panel II in Figure 3) is very close to the lowest possible value that sustains this regime, λ^*. As we noted above, at $\lambda = \lambda^*$, one of the system's eigenvalues is exactly unity, and this corresponds with a unit root process for debt. At the optimal value of lambda ($\lambda = \bar{\lambda}$), therefore, debt is *almost* a unit root process, but will eventually return to its original steady state value. We saw in Section 3 that joint monetary-fiscal optimization would imply a unit root process for debt. However, we noted from Figure 1 that under fully optimal fiscal policy the debt implications of the shock are not completely accommodated: there is an attempt in the first period to reduce spending and thereby moderate the eventual increase in debt. This initial path for government spending cannot be replicated under our simple feedback rule, because spending is tied to debt. Although this short term difference is small in quantitative terms, it helps explain why the optimal level of fiscal feedback ($\bar{\lambda}$) is very slightly above that required for a unit root process. The optimal value of fiscal feedback, although it does not produce a unit root process, is extremely close to one:

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debt is substantially above its original level even after 500 years. (This is shown as $p = 0.0$ case in Figure 6, Panel II.) The value $\lambda = \bar{\lambda}$ implies that for every \$100 that debt is above its steady state level, government spending is reduced by \$1.25 a quarter.

For $\lambda > \bar{\lambda}$, Panel II in Figure 2 shows that the welfare loss steadily increases, although even when adjustment becomes large (a value of $\lambda = 0.3$ implies that government spending falls by \$30 each quarter for every \$100 that debt is above steady state), the loss is never as great as in the case of $\lambda = 0$. However, the increase in loss does demonstrate the macroeconomic costs involved in attempting to correct debt too quickly when government spending is the fiscal instrument.[15] While a policy that set λ a little above $\bar{\lambda}$ would have little cost, setting a much larger value for λ would incur significant costs.

Increasing λ beyond its optimal value does have a noticeable impact on optimal monetary policy: the response of interest rates to the cost push shock initially falls as λ increases, and becomes negative for a time. One reason for this is as follows. For large λ, fiscal policy helps stabilize the impact of a cost push shock. The shock raises debt (see above), which with large λ implies a substantial decline in government spending. This deflates the economy, implying less of a need for real interest rates to rise. However, this form of feedback is less efficient at demand stabilization than monetary policy, as the values for welfare show. Although both fiscal and monetary policy act directly on demand (through public and private consumption respectively), fiscal policy only acts when debt changes, whereas optimal monetary policy can respond directly to inflationary shocks, and is therefore more efficient. These results also suggest that we cannot characterize this policy regime as *always* involving an active monetary policy, a point that is reinforced (and explained) when we look at higher initial debt levels. However, monetary policy is always considerably more active than when there is minimal fiscal feedback.

Finally, we compare welfare under joint monetary-fiscal optimization with welfare when fiscal feedback is optimal. We argued above that, given current institutional arrangements, fiscal feedback represents a more realistic view of fiscal policy setting than a fully optimal fiscal policy, but it is interesting to note what the costs of this are. The difference between welfare in the two policies can be observed by comparing Table 1, (when government spending is the only instrument) with welfare when fiscal feedback is at the optimal $\bar{\lambda}$. This amounts to only 0.002% of steady state consumption. In this case, therefore, there is only a small cost in restricting fiscal policy to respond to debt alone.

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[15]The costs of larger λ 'come from' the quadratic term in g in social welfare. If we artificially delete this term, the loss function after λ^* would be flat. However, as we noted above, even if all government spending was pure waste, g would still influence social welfare.

5 Generality of the Results

5.1 Tax rate as an instrument

Our choice of government spending rather than the income tax rate as the dependent variable in the fiscal feedback rule was essentially arbitrary. One argument in favour of using taxes rather than spending is that the latter is less flexible, and some components of spending may be effectively exogenous. If we used income taxes instead, then the fiscal feedback rule would become

$$(\Upsilon_t - \Upsilon_t^n) Y = \omega (B_t - B), \qquad (33)$$

and its log-linearised version

$$\tau_t = \frac{\omega}{\tau} \tilde{b}_t. \qquad (34)$$

It can be shown (see Appendix C) that if we put $\omega \neq 0$ but impose $\lambda = 0$ then the system of first order conditions is structurally very similar to the one where we used g_t as an instrument. We can easily obtain similar analytical results: (i) if $\omega = 0$ then there is an asymptotically stable regime with passive monetary policy, (ii) if $\omega \geq \omega^*$ where

$$\omega^* = \frac{(1 - \beta)}{\left(1 - \frac{\tau \psi \sigma \rho}{(1-\tau)(\psi + \sigma \rho)}\right)}, \qquad (35)$$

then there is an asymptotically stable regime with active monetary policy and (iii) if $0 < \omega < \omega^*$ then there is a moderately explosive regime.

Panel I in Figure 4 repeats Panel II in Figure 2 for varying ω but keeping $\lambda = 0$; i.e. fiscal feedback involves income taxes, and not government spending. Although the pattern is broadly the same as in Figure 2, there are three notable differences. First, at the optimal feedback parameter, welfare is slightly better than with feedback on government spending. Second, this optimal value is just below, rather than just above, the value of feedback associated with a unit root debt process (ω^*).[16] Third, as ω increases beyond ω^*, the welfare cost of the cost-push shock increases at a much more gradual rate than when government spending was the fiscal instrument. The reason for this is that government spending impacts directly on demand, whereas taxes work

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[16]Recall that the fully optimal path of taxes following a cost-push shock involved a large initial cut, which was then reversed. Clearly a simple feedback rule on debt cannot replicate this, but feedback that is just below ω^* makes a partial attempt. In contrast, when government spending was the fiscal instrument, its optimum path overshot its long run level level, so $\bar{\lambda} > \lambda^*$ in that case.

through consumption and labour supply. The income effect will be smoothed by consumers, so strong feedback on debt will interfere less with monetary policy.

What if we used both government spending and taxes? If we vary (λ, ω) over the domain $((\lambda, \omega) \in [0, \Lambda] \times [0, \Omega], \Lambda \gg \lambda^*, \Omega \gg \omega^*)$ then we can plot the value of losses as a function of λ and ω. Panel II in Figure 4 plots level contours in the non-explosive area. The minimum loss is achieved with a mixed policy, but where the feedback on government spending is small. As a result, this policy is similar to the tax only policy discussed above.

5.2 Varying the steady state level of debt

We look at two alternative steady state levels of debt, zero and doubling the base case to 60% of GDP.[17] The latter still involves levels that are well below those in many industrialized countries. However our model is not complex enough to distinguish between debt of different maturities, and so we may be overestimating the impact of changes in short term rates on debt interest payments. As a result, a conservative choice of steady state debt levels seems appropriate. The key parameters that determine the debt accumulation process are ρ, τ and χ. However, not all of them are independent. If we take the share of government spending to output $(1 - \rho)$ as given, then there is a relationship that links the steady state level of the tax rate τ with the steady state level of debt to output ratio, χ :

$$\tau = (1 - \beta) \chi + 1 - \rho. \tag{36}$$

This relationship either determines τ for given χ, or determines χ given τ. In what follows we assume that χ and ρ determine τ. The higher the level of debt the higher the steady state level of taxes. Higher taxes widen the area over which optimal monetary policy produces outcomes that are not asymptotically stable, as formulae (31) and (35) show. (See the discussion of λ^* in Section 4.3.)

If the steady state level of debt is zero ($\chi = 0$) then the linearization in Section 2 should be changed as we cannot construct $\hat{B}_t$. As $\chi = 0$ then for small disturbances B_t itself will be 'small', so the correct version of the linearized budget constraint is

$$\tilde{b}_{t+1} = \frac{1}{\beta}(\tilde{b}_t + (1 - \rho)g_t - \tau y_t), \tag{37}$$

but where $\tilde{b}_t = B_t$. There are two differences between expressions (37) and (18). First, there are no first-order effects of interest rates and inflation on debt in (37) and, second, taste/productivity

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[17]For simplicity, we assume that government spending is the fiscal instrument.

shocks do not have first-order effect on debt if $B = 0$. Fiscal rule (52) remains the same, but the notation $\tilde{b}_t$ is recycled. We therefore need to solve the system (15), (16), (17), (37) and (52). Again, we can solve the system of first order conditions and obtain that there is a region of explosion when $0 < \lambda < \lambda^*$ where λ^* is given by the same formula (31) but where $\tau = 1 - \rho$ as follows from formula (36).

If $B \neq 0$ then monetary policy affects debt via two channels. The first channel is direct: a change in interest rates has a one-to-one effect on debt. The second channel is indirect: a change in interest rates influences price setting and consumption decisions, which impact on output and taxes. By putting $B = 0$ we eliminate the first channel but retain the ability to affect debt via the second channel.

Panel I in Figure 5 repeats Panel II in Figure 2 for the three levels of initial debt. The different policy regimes in terms of feedback parameters and welfare costs are evident in all cases, but there are some interesting differences. First, when fiscal feedback is zero, welfare losses are greatest when the steady state debt is also zero. This follows from the fact that in this case monetary policy is required to stabilize debt, but its ability to do so is severely weakened by the absence of a debt interest channel. The higher is debt, the less costly the absence of fiscal feedback is. Second, in the high debt case, the feedback parameter on the cost push shock remains negative, even when fiscal feedback becomes significant. This change in also reflected in the actual movement of interest rates following the cost-push shock, as Panel II in Figure 5 shows. Interest rates initially fall following a cost push shock, although they increase quite quickly thereafter. The consequences of a large initial stock of debt are therefore to delay the point at which interest rates rise. The reasons for this are very similar to our discussion of Figure 3 when there was no fiscal feedback. A key difference is that with no fiscal feedback monetary policy was forced to delay the increase in interest rates because it had to control debt, whereas in this case it is simply preferable to use monetary policy rather than fiscal feedback to control debt.

This result suggests that the link between optimal monetary policy and fiscal feedback on debt is rather more complex than a simple 'no feedback = passive monetary policy, significant feedback = active monetary policy' equation. If we compute a fully optimal fiscal policy when debt is high both the direct feedback of interest rates on the cost-push shock and the initial change in interest rates is negative. However, it remains the case that monetary policy is always more active/less passive when fiscal feedback is at or above its optimal level compared to when fiscal feedback is negligible. This remains true even if steady state levels are doubled yet again to over 120% of annual GDP.

5.3 Sensitivity to source of shocks

As we noted in Section 4.3, welfare losses are dominated by the impact of the cost-push shock. To what extent might our results therefore be specific to this shock? To examine this, we repeated the loss chart from the first two columns in Panel II in Figure 2 setting the cost push shock to zero. As the chart in the third column of the same panel shows, the basic characteristics of welfare as fiscal feedback varies are identical, although of course the size of the welfare losses are much smaller. The size of the welfare losses shown in this case are much more comparable to those in Schmitt-Grohe and Uribe (2007). This indicates that our basic results are robust to choice of shocks.

5.4 Blanchard-Yaari consumers and intergenerational effects

The results discussed so far assume that consumers are infinitely lived, so changes in government debt/personal wealth have no direct effect on the pattern of consumer spending over time. In this section we examine an alternative set up, where consumers have finite lives, using the framework due to Blanchard and Yaari (Blanchard (1985)). (Blanchard/Yaari consumers are also modelled in Leith and Wren-Lewis (2000), who examine issues of stability and monetary/fiscal policy interaction in a monetary union, as well as Smets and Wouters (2002) and Ganelli (2005)). With Blanchard/Yaari consumers, we now have a direct route whereby changes in government debt will influence changes in consumption, and we want to examine the extent to which the results described above continue to hold. Introducing Blanchard/Yaari consumers does, however, introduce costs in terms of complexity, which is why we do not examine them in the base case. Appendix D outlines the changes to the model when consumers have finite lives.

Panel I in Figure 6 repeats the analysis shown in Panel II in Figure 2 when government spending is the fiscal instrument. The broad pattern is the same, but there is one significant difference. The area where the economy is not asymptotically stable shifts slightly to the right. (We choose a deliberately high value for the probability of death for this figure so that this point is clear.) This is consistent with results in Leith and Wren-Lewis (2000), where the critical value of λ derived from the determinate stability condition is a positive function of p.[18] The economic

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[18] Our results go beyond those in Leith and Wren-Lewis (2000), who also consider Blanchard–Yaari consumers, in three respects. First, we show for negligible fiscal feedback that the optimal monetary policy is still passive (it responds negatively to inflation) even though it can also feedback directly from debt. Second, we show that the optimal monetary policy is strongly passive: the negative feedback on the cost push shock and inflation is very large. (This result is hinted at, but not established, in Leith and Wren-Lewis (2000).) Third, Panel I in Figure 6 shows that this passive monetary policy, while it stabilizes debt, has a clear welfare cost compared to the alternative regime with significant fiscal feedback.

reason for this is as follows. A cost push shock with an active monetary policy raises debt, and this has a positive impact on demand through consumption with Blanchard Yaari consumers. As a result, monetary policy will generate a larger increase in interest rates, which in turn requires a larger decrease in government spending to prevent a debt interest spiral. In fact, there is a natural neutrality result here. The net impact of debt on demand combines the positive wealth effect from Blanchard–Yaari consumers with the negative effect operating through fiscal feedback. It seems logical that if the former increases (because of larger p), then optimal λ should rise in a corresponding way, thereby neutralizing the overall impact of debt on demand.

We also compute the joint monetary-fiscal optimal policy when we have Blanchard Yaari consumers. Recall that with infinitely lived consumers, this policy implied a unit root process for debt, a result that is consistent with findings in Schmitt-Grohe and Uribe (2004) and Benigno and Woodford (2004). However, there has until now been no equivalent analysis in a model where consumers have finite lives and there are no bequests. We find that the random walk result does not hold in this case. Joint monetary-fiscal optimization produces a system where one of the eigenvalues is very close to one, but not equal to one. The reason for this is as follows. In a model with Blanchard Yaari consumers, the steady state real interest rate is no longer always equal to the rate of time preference, but instead is increasing in the steady state level of debt. A standard result from consumption smoothing is that if the real rate of interest differs from the rate of time preference we get 'tilting', and the same applies in this case to the path of public consumption chosen when the policy maker optimizes. This makes a pure random walk outcome suboptimal. However, as Panel II in Figure 6 shows, for realistic values of the probability of death the behavior of debt, both for a fully optimal policy and for optimal fiscal feedback, are very close to a unit root process, with less than half of any debt in excess of steady state eliminated after 250 years.

In this analysis, we have continued to assume that the monetary policy maker maximizes social welfare, which includes the welfare of unborn generations. (For a discussion of how this is done, see Appendix D.) However, this masks the potential for intergenerational conflict when fiscal feedback is modest. The right hand picture in Panel III in Figure 6 plots the difference between the *per period* social welfare loss for two values of λ: specifically $H_s(\lambda = 0.05) - H_s(\lambda = \lambda^*)$, where H_s is the per period loss at time s. In the first few periods the loss of feeding back on debt with large feedback outweighs the loss of the value of feedback that produces a unit root process in debt. After a number of periods, however, the (constant) loss from having unit root dynamics outweighs the loss of having strong feedback; in the latter case the economy is brought back to the steady state, so the loss will become zero. It is clear that from the point of view of future

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generations, a level of fiscal feedback that comes close to a unit root process for debt (i.e. the socially optimal $\bar{\lambda}$) is *not* preferred to one where fiscal feedback is more rapid. By implication, current generations will prefer a level of fiscal feedback that is less rapid than $\bar{\lambda}$.

We can confirm this by assuming that the monetary authority is 'captured' by currently living generations, and so maximizes an objective function which discounts per period social welfare at a rate equal to $\beta/(1+p)$, rather than β. In this case the monetary authority can allow debt to explode at a rate less than $\sqrt{(1+p)/\beta}$. An interesting result is that the optimal value (from the monetary policymaker's point of view) of fiscal feedback in this case becomes very small, at almost zero. In effect, current generations are able to disregard the debt problem completely, because it only affects future generations, and therefore prefer a value of fiscal feedback that allows them to maximize their impact on inflation. At this new optimal level of fiscal feedback debt explodes at a rate that is greater than $1/\sqrt{\beta}$, but which is still less than $\sqrt{(1+p)/\beta}$. We plot the policymaker's loss as a dashed line in the second picture in the Panel, indicating by a dotted line the area of explosion. Social welfare, plotted as a solid line, is only finite when the rate of explosion is smaller than $1/\sqrt{\beta}$, so the social loss becomes infinite in a neighborhood of the captured policymaker's best choice of fiscal feedback.

6   Conclusion

We have examined the impact of different degrees of fiscal feedback on debt in an economy with nominal inertia where monetary policy is optimal. Consumers are either infinitely lived, or of the Blanchard Yaari type. Our focus is on the extent to which different speeds of fiscal feedback on debt enhance or detract from the ability of the monetary authorities to stabilize output and inflation.

We use a welfare function derived from utility, and calculate joint fiscal-monetary optimization (i.e. a fully optimal fiscal policy with no constraints) as a benchmark. If consumers are infinitely lived we find that debt follows a unit root process, as found in Schmitt-Grohe and Uribe (2004) and Benigno and Woodford (2004). However, if consumers are of the Blanchard Yaari type, joint fiscal-monetary optimization no longer involves an exact unit root process, although it is close to it. If we then restrict fiscal policy to follow a simple feedback on debt, a formulation which seems closer to current institutional practice, we find the optimal level of fiscal feedback to be small. With this optimal degree of fiscal feedback, the behavior of debt is very close to a unit root process following shocks.

At low or moderate levels of initial debt we directly infer that optimal monetary policy is

active, in the sense that real interest rates initially rise following an increase in inflation, both for fully optimal fiscal policy and for the optimal level of fiscal feedback. In addition, we find that the costs of restricting fiscal policy to only respond to debt are small compared to a fully optimal fiscal policy, if government spending is the fiscal instrument. We also show that fiscal feedback using taxes rather than government spending is slightly preferable in welfare terms. If the initial debt stock is large so that changes in interest rates have a large impact on the government's budget constraint, then monetary policy may no longer be active under optimal fiscal feedback or joint fiscal-monetary optimization.

There is a discontinuity in the behavior of monetary policy and welfare either side of this optimal level of fiscal feedback. As the extent of fiscal feedback increases beyond the optimal level, optimal monetary policy becomes less active because fiscal feedback also tends to deflate inflationary shocks. However this fiscal stabilization is less efficient than monetary policy, and so welfare declines. In contrast, if fiscal feedback falls below the optimal level, then optimal monetary policy initially permits solutions that are mildly explosive, so that they are not asymptotically stable. In addition monetary policy becomes much more passive in nature, in the sense that interest rates initially fall sharply despite higher inflation. When fiscal feedback becomes zero optimal monetary policy remains passive, but the economy is asymptotically stable. This policy regime has strong similarities to the Fiscal Theory of the Price Level. We show that while this passive monetary policy may succeed in controlling debt, it leads to a sharp deterioration in welfare.

A   Steady State and Welfare

A.1   Government expenditures in steady state

The aggregate demand relationship (13) always holds along the dynamic path of the economy, which can be differentiated with respect to government expenditures in order to yield the following condition:

$$\frac{\partial Y_t}{\partial G_t} = \frac{\partial C_t}{\partial G_t} + 1 \tag{38}$$

Condition (9) also holds along the dynamic path of the economy. Its differentiation yields:

$$\frac{(1-\tau)}{\mu_t} u_{CC}(C_t)\frac{\partial C_t}{\partial G_t} = v_{yy}(Y_t)\frac{\partial Y_t}{\partial G_t} \tag{39}$$

Both conditions (38) and (39) hold in the steady state and can be solved for $\frac{\partial C_t}{\partial G_t}$ and $\frac{\partial Y_t}{\partial G_t}$:

$$\frac{\partial C_t}{\partial G_t} = -\frac{\rho\sigma}{(\psi + \rho\sigma)}, \quad \frac{\partial Y_t}{\partial G_t} = \frac{\psi}{(\psi + \rho\sigma)}$$

We assume that the steady state level of government expenditures is chosen to maximise the utility function (1) (subject to aggregate demand constraint and aggregate supply conditions), so that in the steady state[19]:

$$\frac{\partial}{\partial G}\left(u(C_s) + f(G) - v(Y_s)\right) = u_C(C)\frac{\partial C}{\partial G} + f_G(G) - v_y(Y)\frac{\partial Y}{\partial G} = 0 \tag{40}$$

From (38), (39) and (40) it follows that in equilibrium:

$$\frac{f_G}{u_C} = \frac{v_y}{u_C}\frac{\partial Y}{\partial G} - \frac{\partial C}{\partial G} = \frac{\psi(1-\tau)\mu^w/\mu + \rho\sigma}{(\psi + \rho\sigma)} \tag{41}$$

Note that for iso-elastic utility components

$$u(C_v) = \frac{C_v^{1-1/\sigma}}{1 - 1/\sigma}, \qquad f(G_v) = \zeta\frac{G_v^{1-1/\sigma}}{1 - 1/\sigma}$$

in the steady state:

$$\frac{f_G}{u_C} = \zeta\left(\frac{G}{C}\right)^{-1/\sigma} = \zeta\left(\frac{1-\rho}{\rho}\right)^{-1/\sigma} \tag{42}$$

A.2 Derivation of the Social Welfare Function

The derivation of the welfare metric is standard and for this model it is explained in detail in Kirsanova et al. (2006). The one-period (flow) welfare in (20) is $\mathcal{W}_t$:

$$\mathcal{W}_t = u(C_t) + f(G_t) - \int_0^1 v(y_t(z))dz$$

It can be linearised around the steady state

$$\mathcal{W}_t = Cu_C(C)\left(\hat{C}_t + \frac{1}{2}(1 - \frac{1}{\sigma})\hat{C}_t^2\right) + Gf_G(G)\left(\hat{G}_t + \frac{1}{2}(1 - \frac{1}{\sigma})\hat{G}_t^2\right)$$
$$- Yv_y(Y)\left(\hat{Y}_t + \frac{1}{2}(1 + \frac{1}{\psi})\hat{Y}_t^2 + \frac{1}{2}(\frac{1}{\psi} + \frac{1}{\epsilon})var_z\hat{y}_t(z)\right) \tag{43}$$

where we assumed $\sigma = -u_C/u_{CC}C = -f_G/f_{GG}G$, $\psi = -v_y/v_{yy}Y$.

A second-order approximation of aggregate demand (13) can be written as

$$\hat{C} = \frac{1}{\rho}\left(\hat{Y} - (1-\rho)\hat{G} - \rho\frac{1}{2}\hat{C}^2 - \frac{1}{2}(1-\rho)\hat{G}^2 + \frac{1}{2}\hat{Y}^2\right)$$

---

[19]Derivatives of constraints are equal to zero so we did not include them in the final expression.

so we can substitute consumption in (43) and obtain

$$\mathcal{W}_s = \rho u_C \left((1 - \frac{v_y}{u_C})\hat{Y}_s - (1 - \rho)(1 - \frac{f_G}{u_C})\hat{G}_s - \frac{\rho}{2\sigma}\hat{C}^2 - \frac{(1 - \rho)}{2}\left(1 + \frac{f_G}{u_C}\frac{(1 - \sigma)}{\sigma}\right)\hat{G}_s^2 \right.$$
$$\left. - \frac{1}{2}\left(\frac{v_y}{u_C}\frac{1 + \psi}{\psi} - 1\right)\hat{Y}_s^2 - \frac{1}{2}\frac{v_y}{u_C}\frac{\psi + \epsilon}{\psi\epsilon}var_z\hat{y}_s(z) \right)$$

To transform this equation into a more convenient form that does not include linear terms, we proceed as follows (see Beetsma and Jensen (2004b)). We have derived relationship (41) for f_G/u_C in the steady state. If the government removes monopolistic distortions *and* distortions from income taxation in the steady state using a subsidy[20]

$$\mu^w = \frac{\mu}{(1 - \tau)}, \tag{44}$$

then $f_G/u_C = 1$ and so the welfare function does not contain linear terms. The final formula for social welfare is

$$\mathcal{W}_s = -\rho u_C \left(\frac{\rho}{2\sigma}c_s^2 + \frac{(1 - \rho)}{2\sigma}g_s^2 + \frac{1}{2\psi}y_s^2 + \frac{1}{2}(\frac{1}{\psi} + \frac{1}{\epsilon})var_z\hat{y}_s(z) \right).$$

Woodford (2003) has shown that

$$\sum_{t=0}^{\infty} \beta^t var_z\hat{y}_s(z) = \sum_{t=0}^{\infty} \beta^t \frac{\gamma\epsilon^2}{(1 - \gamma\beta)(1 - \gamma)}\pi_t^2$$

so, using the conventional notation for gap variables, we get the final formula for the social welfare function:

$$\mathcal{W}_s = -\frac{\rho(\epsilon + \psi)\gamma\epsilon}{2\psi(1 - \gamma\beta)(1 - \gamma)}u_C \left(\frac{\kappa}{\epsilon}\left(\frac{\rho}{\sigma}c_t^2 + \frac{(1 - \rho)}{\sigma}g_t^2 + \frac{1}{\psi}y_t^2\right) + \pi_t^2 \right)$$

which is formula (21) in the main text.

B Optimal Commitment Plan

The fully optimal, time inconsistent (TI) policy requires the minimisation of the constrained loss function:

$$H = \min_{\{U_s\}_{s=t}^{\infty}} \mathcal{E}_t \sum_{s=t}^{\infty} H_s$$

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[20] financed by lump-sum taxation, $\mu^w = T$.

where every term of H_s has the form:

$$H_s = \frac{1}{2}\rho^{s-t}(Y_s'\mathsf{Q}_{11}Y_s + 2Y_s'\mathsf{Q}_{12}X_s + X_s'\mathsf{Q}_{22}X_s + 2Y_s'\mathsf{U}_1U_s + 2X_s'\mathsf{U}_2U_s + U_s'\mathsf{R}U_s)$$
$$+ \lambda_{s+1}^{n\prime}(\mathsf{A}_{11}Y_s + \mathsf{A}_{12}X_s + \mathsf{B}_1U_s - Y_{s+1}) + \lambda_{s+1}^{p\prime}(\mathsf{A}_{21}Y_s + \mathsf{A}_{22}X_s + \mathsf{B}_2U_s - X_{s+1})$$

where λ^n is a vector of non-predetermined Lagrange multipliers (those associated with the predetermined variables Y) and λ^p is a vector of predetermined Lagrange multipliers (those associated with the non-predetermined variables X). To derive the first order conditions we differentiate the constrained loss function with respect to X, Y, U, and λ to obtain the following system (we also used $L_s = \beta^{-s}\lambda_s$ to simplify notation):

$$\frac{\partial H}{\partial X_s} : \mathsf{Q}_{22}X_s + \mathsf{Q}_{21}Y_s + \mathsf{U}_2U_s + \beta\mathsf{A}_{12}'L_{s+1}^n + \beta\mathsf{A}_{22}'L_{s+1}^p - L_s^p = 0 \tag{45}$$

$$\frac{\partial H}{\partial Y_s} : \mathsf{Q}_{12}X_s + \mathsf{Q}_{11}Y_s + \mathsf{U}_1U_s + \beta\mathsf{A}_{11}'L_{s+1}^n + \beta\mathsf{A}_{21}'L_{s+1}^p - L_s^n = 0 \tag{46}$$

$$\frac{\partial H}{\partial U_s} : \mathsf{U}_1'Y_s + \mathsf{U}_2'X_s + \mathsf{R}U_s + \mathsf{B}_1'L_{s+1}^n + \mathsf{B}_2'L_{s+1}^p = 0 \tag{47}$$

$$\frac{\partial H}{\partial \lambda_{s+1}^{y\prime}} : \mathsf{A}_{11}Y_s + \mathsf{A}_{12}X_s + \mathsf{B}_1U_s - Y_{s+1} = 0 \tag{48}$$

$$\frac{\partial H}{\partial \lambda_{s+1}^{x\prime}} : \mathsf{A}_{21}Y_s + \mathsf{A}_{22}X_s + \mathsf{B}_2U_s - X_{s+1} = 0 \tag{49}$$

This system must be solved using initial conditions for all predetermined variables (Y_0 and L_0^p) and terminal conditions (transversality conditions) for all non-predetermined variables (X, L^n and U). We observe Y_0, and the Pontryagin maximum principle requires that the initial conditions for the predetermined Lagrange multipliers should be set to zero,[21] $L_0^p = 0$. For a unique solution, the system (45)–(49) should have as many explosive eigenvalues (i.e. absolute values outside the unit circle) as the number of non-predetermined variables.

Finally, the *unique* solution to the system can be written in a form:

$$\begin{bmatrix} Y_{s+1} \\ L_{s+1}^p \end{bmatrix} = Z_{11}S_{11}^{-1}T_{11}Z_{11}^{-1}\begin{bmatrix} Y_s \\ L_s^p \end{bmatrix}, \quad \begin{bmatrix} U_s \\ X_s \\ L_s^n \end{bmatrix} = Z_{21}Z_{11}^{-1}\begin{bmatrix} Y_s \\ L_s^p \end{bmatrix} \tag{50}$$

where matrices Z, S and T are obtained by solving a particular generalised eigenvalue problem, see e.g. Söderlind (1999).

The requirement to set initial conditions $L_0^x = 0$ highlights the problem of time-inconsistency associated with the fully optimal solution. As soon as optimisation is done at time t and μ_t^x

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[21] For a very clear explanation see Currie and Levine (1993).

is set to zero, this implies a certain time path for $\{L_s^p\}_{s=t}^\infty$ such that L_s^p is not necessarily zero for any $s > t$. It immediately follows that given an option to re-optimise at any time $s > t$, the policymaker will choose to re-set L_s^p to zero, reneging on the previously optimal plan. The optimal plan at time t is inconsistent from the perspective of any other time $s > t$.

Denote

$$T = Z_{11} S_{11}^{-1} T_{11} Z_{11}^{-1}, \quad N = -Z_{21} Z_{11}^{-1}$$

so we can partition N conformally with U, X and L_s^n and write $L_s^n = -N_y Y_s - N_\mu L_s^p$. It was shown in Currie and Levine (1993) that the minimum welfare loss for the stochastic problem under commitment is given by

$$W_t = -\frac{1}{2} tr(N_y(Y(t)Y'(t) + \frac{\beta}{1-\beta} V_{11})) \tag{51}$$

where $V_{11} = cov(\delta, \delta)$.

Note that from formula (50) it follows that $U_t = \theta_y Y_t + \vartheta_L L_t^p$ so we can write the optimal reaction function as the feedback rule (32) in the main text, where $L_t^p = \{L_t^\pi, L_t^c\}$ are Lagrange multipliers, associated with constraints on inflation and consumption. Lagrange multiplies themselves can be presented as discounted linear combination of *past* values of π and c. Thus, all right hand side variables in (32) are predetermined (see Currie and Levine (1993)). This representation of the optimal policy is useful in judging whether policy is active or passive: we can look at the sign (and size) of θ–coefficients, as they will determine the reaction of the interest rate to shocks in the short run. ϑ–coefficients are set on predetermined Lagrange multipliers, which move relatively slowly in the short run, as they are integrals of past variables.

C   General Solution to Optimisation problem when Government uses Spending and Taxes

Suppose fiscal policy can use both fiscal instruments:

$$g_t = -\frac{\lambda}{(1-\rho)}\tilde{b}_t, \tag{52}$$

$$\tau_t = \frac{\omega}{\tau}\tilde{b}_t. \tag{53}$$

then substitution of these rulse into system (15)-(19) yields:

$$c_s = \mathcal{E}_t c_{s+1} - \sigma(i_s - \pi_{s+1})$$

$$\pi_s = \beta\mathcal{E}_t\pi_{s+1} + \kappa\left(\frac{1}{\sigma} + \frac{\rho}{\psi}\right)c_s - \kappa\left(\frac{\lambda}{\psi} - \frac{\omega}{(1-\tau)}\right)\tilde{b}_s + \kappa\hat{\mu}_s$$

$$\tilde{b}_{s+1} = \chi i_s + \frac{1}{\beta}\left((1 - ((1-\tau)\lambda + \omega))\tilde{b}_s - \chi\pi_s - \tau\rho c_s\right) + \hat{\zeta}_t$$

and the social welfare function becomes:

$$\min_{\{i_s\}_{s=t}^{\infty}} \frac{1}{2}\mathcal{E}_t\sum_{s=t}^{\infty}\beta^{s-t}\left[\pi_s^2 + a_c c_s^2 + a_g\left(\frac{\lambda}{(1-\rho)}\tilde{b}_s\right)^2 + a_y\left(-\lambda\tilde{b}_s + \rho c_s\right)^2\right] + \mathcal{O}(3) \tag{54}$$

We form Lagrangian as in Section 4.2, which has the following flow terms:

$$\begin{aligned}
H_s = &\frac{1}{2}\beta^{s-t}\left[\pi_s^2 + a_c c_s^2 + \frac{a_g\lambda^2}{(1-\rho)^2}\tilde{b}_s^2 + a_y\left(-\lambda\tilde{b}_s + \rho c_s\right)^2\right]\\
&+L_{s+1}^c\left(\mathcal{E}_t c_{s+1} - \sigma(i_s - \pi_{s+1}) - c_s\right)\\
&+L_{s+1}^\pi\left(\beta\mathcal{E}_t\pi_{s+1} + \kappa\left(\frac{1}{\sigma} + \frac{\rho}{\psi}\right)c_s - \kappa\left(\frac{\lambda}{\psi} - \frac{\omega}{(1-\tau)}\right)\tilde{b}_s + \kappa\hat{\mu}_s - \pi_s\right)\\
&+L_{s+1}^b\left(\chi i_s + \frac{1}{\beta}\left((1 - ((1-\tau)\lambda + \omega))\tilde{b}_s - \chi\pi_s - \tau\rho c_s\right) + \hat{\zeta}_t - \tilde{b}_{s+1}\right)
\end{aligned}$$

In order to minimise the loss function, we differentiate the Lagrangian with respect to L^c, L^π, L^b, π, c, b and i. The first order conditions for optimality are:

$$\frac{\partial w}{\partial \pi_s} = 0 = \beta^{s-t}a_\pi\pi_s + \sigma L_s^c + \beta L_s^\pi - L_{s+1}^\pi - \frac{\chi}{\beta}L_{s+1}^b \tag{55}$$

$$\frac{\partial w}{\partial c_s} = 0 = \beta^{s-t}\left(a_c c_s + a_y\rho\left(\rho c_s - \lambda\tilde{b}_s\right)\right) + L_s^c - L_{s+1}^c + \kappa\left(\frac{1}{\sigma} + \frac{\rho}{\psi}\right)L_{s+1}^\pi - \frac{\tau\rho}{\beta}L_{s+1}^b \tag{56}$$

$$\frac{\partial w}{\partial b_s} = 0 = \beta^{s-t}\left(\frac{a_g\lambda^2}{(1-\rho)^2}\tilde{b}_s - \lambda a_y\left(\rho c_s - \lambda\tilde{b}_s\right)\right) - \kappa\left(\frac{\lambda}{\psi} - \frac{\omega}{(1-\tau)}\right)L_{s+1}^\pi$$

$$+ (1 - ((1-\tau)\lambda + \omega))\frac{1}{\beta}L_{s+1}^b - L_s^b \tag{57}$$

$$\frac{\partial w}{\partial i_s} = 0 = -\sigma L_{s+1}^c + \chi L_{s+1}^b \tag{58}$$

$$\frac{\partial w}{\partial L_{s+1}^c} = 0 = \mathcal{E}_s c_{s+1} - \sigma(i_s - \pi_{s+1}) - c_s \tag{59}$$

$$\frac{\partial w}{\partial L_{s+1}^\pi} = 0 = \beta\mathcal{E}_s\pi_{s+1} + \kappa\left(\frac{1}{\sigma} + \frac{\rho}{\psi}\right)c_s - \kappa\left(\frac{\lambda}{\psi} - \frac{\omega}{(1-\tau)}\right)\tilde{b}_s + \kappa\hat{\mu}_s - \pi_s \tag{60}$$

$$\frac{\partial w}{\partial L_{s+1}^b} = 0 = \chi i_s + \frac{1}{\beta}\left((1 - ((1-\tau)\lambda + \omega))\tilde{b}_s - \chi\pi_s - \tau\rho c_s\right) + \hat{\zeta}_s - \tilde{b}_{s+1} \tag{61}$$

The system can be written in a general matrix form $Sz_{t+1} = Qz_t$ as follows. (We have substituted out $i_t = \frac{1}{\sigma}c_{t+1} + \pi_{t+1} - \frac{1}{\sigma}c_t$ and denoted $\nu_t^\pi = \beta^{t-s}L_t^\pi, \nu_t^c = \beta^{t-s}L_t^c, \nu_t^b = \beta^{t-s}L_t^b$.)

$$
\begin{bmatrix}
1 & 0 & 0 & 0 & -\frac{\chi}{\sigma} & -\chi \\
0 & \vartheta & 0 & \chi & 0 & 0 \\
0 & 0 & -\sigma & \chi & 0 & 0 \\
0 & -\kappa\beta\left(\frac{\lambda}{\psi} - \frac{\omega}{(1-\tau)}\right) & 0 & (1-((1-\tau)\lambda+\omega)) & 0 & 0 \\
0 & -\beta\kappa\left(\frac{1}{\sigma} + \frac{\rho}{\psi}\right) & \vartheta & \tau\rho & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & \beta
\end{bmatrix}
\begin{bmatrix}
\tilde{b}_{t+1} \\
\nu_{t+1}^\pi \\
\nu_{t+1}^c \\
\nu_{t+1}^b \\
c_{t+1} \\
\pi_{t+1}
\end{bmatrix}
$$

$$
=
\begin{bmatrix}
\frac{1}{\beta}(1-((1-\tau)\lambda+\omega)) & 0 & 0 & 0 & -\left(\frac{\chi}{\sigma} + \frac{\tau\rho}{\beta}\right) & -\frac{\chi}{\beta} \\
0 & \beta & \sigma & 0 & 0 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 \\
-\left(\frac{a_g}{(1-\rho)^2} + a_y\right)\lambda^2 & 0 & 0 & 1 & \rho\lambda a_y & 0 \\
-a_y\rho\lambda & 0 & 1 & 0 & \left(a_c + a_y\rho^2\right) & 0 \\
\kappa\left(\frac{\lambda}{\psi} - \frac{\omega}{(1-\tau)}\right) & 0 & 0 & 0 & -\kappa\left(\frac{1}{\sigma} + \frac{\rho}{\psi}\right) & 1
\end{bmatrix}
\begin{bmatrix}
\tilde{b}_t \\
\nu_t^\pi \\
\nu_t^c \\
\nu_t^b \\
c_t \\
\pi_t
\end{bmatrix}
$$

All finite generalised eigenvalues can be found as a solutions of a matrix equation

$$\det(Q - \Phi S) = 0$$

We now find which values of parameters λ and ω deliver the 'boundary stability case', i.e. $\Phi = 1$ or $\Phi = \frac{1}{\beta}$

1. $\Phi = 1$

$$
Q-S =
\begin{bmatrix}
-1 + \frac{(1-\omega-\lambda(1-\tau))}{\beta} & 0 & 0 & 0 & -\frac{\tau\rho}{\beta} & \left(1 - \frac{1}{\beta}\right)\chi \\
0 & 0 & \sigma & -\chi & 0 & 1 \\
0 & 0 & \sigma & -\chi & 0 & 0 \\
-\lambda^2\left(a_y + \frac{a_g}{(1-\rho)^2}\right) & \kappa\beta\left(\frac{\lambda}{\psi} - \frac{\omega}{1-\tau}\right) & 0 & \omega + \lambda(1-\tau) & \lambda\rho a_y & 0 \\
-\lambda\rho a_y & \kappa\beta\left(\frac{1}{\sigma} + \frac{\rho}{\psi}\right) & 1-\beta & -\tau\rho & a_c + \rho^2 a_y & 0 \\
\kappa\left(\frac{\lambda}{\psi} - \frac{\omega}{1-\tau}\right) & 0 & 0 & 0 & -\kappa\left(\frac{1}{\sigma} + \frac{\rho}{\psi}\right) & 1-\beta
\end{bmatrix}
$$

Its determinant (up to constant multiplier) is

$\det(Q-S) = ((\omega((1-\tau)\psi + \rho\sigma(1-\tau(1+\psi))) + (1-\beta)\chi\psi) + \lambda(1-\tau)(((1-\tau)\psi + \sigma\rho) - \chi(1-\beta))))$
$\times\left((1-\beta)(1-\tau)(\psi + \sigma\rho) - \omega((1-\tau)\psi + \rho\sigma(1-\tau(1+\psi))) - \lambda\left(\psi(1-\tau)^2 + \rho\sigma(1-\tau)\right)\right)$

either

(i). $\omega((1-\tau)\psi + \rho\sigma(1-\tau(1+\psi))) + (1-\beta)\chi\psi) + \lambda(1-\tau)(((1-\tau)\psi + \sigma\rho) - \chi(1-\beta)) = 0$

or

(ii). $(1-\beta)(1-\tau)(\psi + \sigma\rho) - \omega((1-\tau)\psi + \rho\sigma(1-\tau(1+\psi))) - \lambda\left(\psi(1-\tau)^2 + \rho\sigma(1-\tau)\right) = 0$

2. $\Phi = \frac{1}{\beta}$

$$Q - \frac{1}{\beta}S = \begin{bmatrix} -\frac{1}{\beta} + \frac{(1-\omega-\lambda(1-\tau))}{\beta} & 0 & 0 & 0 & \frac{(1-\beta)\chi - \tau\rho\sigma}{\beta\sigma} & 0 \\ 0 & \beta - 1 & \sigma & -\frac{\chi}{\beta} & 0 & 1 \\ 0 & 0 & \frac{\sigma}{\beta} & -\frac{\chi}{\beta} & 0 & 0 \\ -\lambda^2\left(a_y + \frac{a_g}{(1-\rho)^2}\right) & \kappa\left(\frac{\lambda}{\psi} - \frac{\omega}{1-\tau}\right) & 0 & \frac{(\omega+\lambda(1-\tau)-1)}{\beta} + 1 & \lambda\rho a_y & 0 \\ -\lambda\rho a_y & \kappa\left(\frac{1}{\sigma} + \frac{\rho}{\psi}\right) & 0 & -\frac{\tau\rho}{\beta} & a_c + \rho^2 a_y & 0 \\ \kappa\left(\frac{\lambda}{\psi} - \frac{\omega}{1-\tau}\right) & 0 & 0 & 0 & -\kappa\left(\frac{1}{\sigma} + \frac{1}{\psi}\rho\right) & 0 \end{bmatrix}$$

Its determinant (up to constant multiplier) is

$$\det(Q-S) = \left(\omega\left((1-\tau)\psi + \rho\sigma\left(1 - \tau(1+\psi)\right)\right) + (1-\beta)\chi\psi\right) + \lambda(1-\tau)\left(((1-\tau)\psi + \sigma\rho) - \chi(1-\beta)\right)\right)$$
$$\times \left((1-\beta)(1-\tau)(\psi + \sigma\rho) - \omega\left((1-\tau)\psi + \rho\sigma\left(1 - \tau(1+\psi)\right)\right) - \lambda\left(\psi(1-\tau)^2 + \rho\sigma(1-\tau)\right)\right)$$

and we end up with *the same* pair of equations:

(i). $\omega\left((1-\tau)\psi + \rho\sigma\left(1 - \tau(1+\psi)\right)\right) + (1-\beta)\chi\psi) + \lambda(1-\tau)\left(((1-\tau)\psi + \sigma\rho) - \chi(1-\beta)\right) = 0$

or

(ii). $(1-\beta)(1-\tau)(\psi + \sigma\rho) - \omega\left((1-\tau)\psi + \rho\sigma\left(1 - \tau(1+\psi)\right)\right) - \lambda\left(\psi(1-\tau)^2 + \rho\sigma(1-\tau)\right) = 0$

Note that none of the expressions for stability boundaries depends on parameters of utility function; they only depend on the structural parameters of the economy.

These results suggest that (we used $\tau + \rho - 1 = (1-\beta)\chi$ to substitute out χ)

(i) If

$$\lambda = -\omega \frac{\left((\sigma+\psi) - \sigma\tau(1+\psi)\right)\rho}{\left((1-\tau)(\psi+1) - (1-\sigma)\rho\right)(1-\tau)}$$

then there is at least one generalised eigenvalue that is equal to one, and at least one generalised eigenvalue that is equal to $\frac{1}{\beta}$. This straight line passes through $(\lambda, \omega) = (0,0)$ point and for a wide range of reasonable parameters is negatively sloped.

(ii) If

$$\lambda = \frac{(1-\beta)}{\left(1 - \tau\frac{\psi}{(\psi+\sigma\rho)}\right)} - \omega\frac{\left(1 - \frac{\tau}{(1-\tau)}\frac{\rho\sigma\psi}{(\psi+\sigma\rho)}\right)}{\left(1 - \tau\frac{\psi}{(\psi+\sigma\rho)}\right)}$$

then there is at least one generalised eigenvalue that is equal to one, and at least one generalised eigenvalue that is equal to $\frac{1}{\beta}$. This straight line passes through $(\lambda, \omega) = \left(\frac{(1-\beta)}{\left(1 - \tau\frac{\psi}{(\psi+\sigma\rho)}\right)}, 0\right)$ and $\left(0, \frac{(1-\beta)}{\left(1 - \frac{\tau}{(1-\tau)}\frac{\rho\sigma\psi}{(\psi+\sigma\rho)}\right)}\right)$ point and for a wide range of reasonable parameters is negatively sloped.

The two lines intersect at

$$\omega_0 = 1 - \beta - \frac{\rho(\psi + \sigma)}{\chi(\psi + 1)} < 0,$$

$$\lambda_0 = \frac{(1 - \sigma)\psi\rho}{(\psi + 1)\chi(1 - \tau)} + \frac{\sigma\rho}{\chi} > 0.$$

We plot the both lines in Figure 7. At point A there is no difference between passive and active policies. In other words, if we start at point like B (where the monetary policy reacts to cost push shock by raising interest rate, given that both tax and spending feed back on debt) and move along the line towards point A then the following happens. The reaction of the tax offsets the reaction of spending more and more. At points close to A monetary policy will have to react to debt (accommodate cost-push shocks) more and more.

We can start at point like C, where fiscal policy actually prevents debt stabilisation and so monetary policy stabilises debt. If we move towards A then the stabilising reaction of government spending on debt outweighs the destabilising reaction of taxes and monetary policy becomes more active. At point A monetary and fiscal policies neither passive nor active.

Note that we can move between B and C avoiding mild explosion. Although 'crossing this bridge' is costly, perhaps, it is of value to know that by changing fiscal parameters smoothly we can get out of the area of passive monetary policy without even mild instability.

D The Model with Blanchard Yaari consumers

This section gives an overview of changes to the model and the welfare metric. Further appendices E, F and G provide details of derivations. We need to make a number of changes to our model, described by equations (15)– (19). First, as consumers have a constant probability of death, p, the discount factor in formula (1) becomes $\beta/(1+p)$. Second, in the household budget constraint (2), the discount factor takes account of mortality, $\mathcal{E}_t(Q_{t,t+1}) = \frac{1}{(1+i_t)(1+p)}$. Third, these modifications and the fact that we now have an infinite number of living cohorts at each moment of time, results in a new system for aggregate variables. The first order conditions for individual consumption, and then aggregation of all such behavioural equations, leads to a pair of equations for aggregate consumption and for the average propensity to consume, instead of the single Euler equation (7):

$$\hat{C}_t = [\beta(1+i)]^{-\sigma}(\mathcal{E}_t\hat{C}_{t+1} + \frac{p\rho}{\Phi\theta}(\mathcal{E}_t\hat{A}_{t+1} - \mathcal{E}_t\hat{\pi}_{t+1} - \mathcal{E}_t\hat{\Phi}_{t+1})) - \sigma(\hat{\imath}_t - \mathcal{E}_t\hat{\pi}_{t+1}), \tag{62}$$

$$\frac{(1+p)(1+i)}{\beta^\sigma(1+i)^\sigma}\hat{\Phi}_t = \mathcal{E}_t\hat{\Phi}_{t+1} - (1 - \sigma)(\hat{\imath}_t - \mathcal{E}_t\hat{\pi}_{t+1}), \tag{63}$$

where $1/\Phi_t$ is average propensity to consume out of total resources, resources which consist of nominal financial wealth and human wealth. Equations (62) and (63) can be written in terms of gap variables. The resulting four equations should now be included in a system like that shown in equations (15)– (19), instead of equation (15).

To evaluate gains and losses we need a welfare metric. In the Blanchard-Yaari case, unlike in the infinitely-lived case, there is no obvious choice. Ideally total welfare should be evaluated using a social welfare function that aggregates across generations and weights the utility of every generation. It is not clear, however, how to treat future unborn generations. Calvo and Obstfeld (1988) discuss the importance of including unborn generation in the social welfare metric. If they are excluded, we introduce an additional source of time-inconsistency, as the policy which treats some particular generation differently will be necessarily time-inconsistent. However, straightforward aggregating of the utilities of unborn generations is not feasible for computational reasons. One way to overcome this difficulty is to suggest that the government uses a weighting scheme that makes the aggregate welfare of overlapping generations equivalent to the welfare of one infinitely long lived generation of consumers. A similar strategy was also adopted by Calvo and Obstfeld (1988). We therefore use formulae (21) to obtain our results.

E Derivation of Consumption Equation for Blanchard-Yaari consumers

E.1 Individual Relationships

To derive the first order conditions for the household's optimisation problem we write the Lagrangian as

$$L = \mathcal{E}_t \sum_{v=t}^{\infty} \left[\frac{\beta}{1+p} \right]^{v-t} [u(C_v^s, \xi_v) - v(h_v^s(z), \xi_v)]$$
$$- \lambda[\sum_{v=t}^{\infty} \mathcal{E}_t(Q_{t,v}^s P_v C_v^s) - \mathcal{A}_t^s - \sum_{v=t}^{\infty} \mathcal{E}_t(Q_{t,v}^s \int_0^1 (1-\tau)\left(w_v(z)h_v^s(z) + \Pi_v(z)\right) dz)]$$

when the first-order conditions are:

$$\frac{\partial L}{\partial h_v^s(z)} = -\left[\frac{\beta}{1+p}\right]^{v-t} v_h(h_v^s, \xi_v) + \lambda Q_{t,v}^s(1-\tau)w_v(z) = 0 \tag{64}$$

$$\frac{\partial L}{\partial C_v^s} = \left[\frac{\beta}{1+p}\right]^{v-t} u_C(C_v^s, \xi_v) - \lambda Q_{t,v}^s P_v = 0 \tag{65}$$

$$\frac{\partial L}{\partial \lambda} = \sum_{v=t}^{\infty} \mathcal{E}_t(Q_{t,v}^s P_v C_v^s) - \mathcal{A}_t^s - \sum_{v=t}^{\infty} \mathcal{E}_t\left(Q_{t,v}^s \int_0^1 (1-\tau)\left(w_v(z)h_v^s(z) + \Pi_v(z)\right) dz\right) = 0 \tag{66}$$

Divide the second FOC by itself and obtain:

$$\frac{\beta}{1+p}\frac{u_C(C_{v+1}^s, \xi_{v+1})}{u_C(C_v^s, \xi_v)}\frac{P_v}{P_{v+1}} = Q_{v,v+1}^s \tag{67}$$

For simplicity, we assume the particular utility function

$$u(C_v^s, \xi_v) = \frac{(C_v^s \xi_v)^{1-1/\sigma}}{1-1/\sigma}$$

so equation (67) now becomes:

$$C_v^s = \left[\frac{1+p}{\beta}\frac{P_{v+1}}{P_v}Q_{v,v+1}^s\right]^{\sigma} C_{v+1}^s \frac{\xi_{v+1}}{\xi_v} \tag{68}$$

Therefore

$$C_v^s = C_t^s \frac{C_v^s}{C_t^s} = C_t^s \prod_{k=0}^{v-t-1}\left(\frac{C_{t+k+1}^s}{C_{t+k}^s}\right) = C_t^s \prod_{k=0}^{v-1-t}\left[\frac{1+p}{\beta}\frac{P_{t+k+1}}{P_{t+k}}Q_{t+k,t+k+1}^s\right]^{-\sigma}\frac{\xi_{t+k}}{\xi_{t+k+1}}$$

$$P_v = P_t \frac{P_v}{P_t} = P_t^s \prod_{k=0}^{v-t-1}\left(\frac{P_{t+k+1}}{P_{t+k}}\right) = P_t \prod_{k=0}^{v-t-1}(1+\pi_{t+k+1})$$

We have for an *individual's* consumption and wealth from a generation born at time s :

$$P_t C_t^s + \sum_{v=t+1}^{\infty} Q_{t,v}^s P_v C_v^s = P_t C_t^s + P_t C_t^s \sum_{v=0}^{\infty} Q_{t,t+v+1}^s \frac{P_{t+1+v}}{P_t}\frac{C_{t+1+v}^s}{C_t^s}$$

$$= P_t C_t^s + P_t C_t^s \sum_{v=0}^{\infty} Q_{t,t+v+1}^s \frac{P_{t+1+v}}{P_t}\prod_{k=0}^{v}\left[\frac{1+p}{\beta}\frac{P_{t+k+1}}{P_{t+k}}Q_{t+k,t+k+1}^s\right]^{-\sigma}\frac{\xi_{t+k}}{\xi_{t+k+1}}$$

$$= P_t C_t^s + P_t C_t^s \sum_{v=0}^{\infty}\left(\frac{\beta}{1+p}\right)^{(v+1)\sigma}\frac{P_{t+1+v}}{P_t}\prod_{k=0}^{v}\left(\frac{P_{t+k+1}}{P_{t+k}}\right)^{-\sigma}(Q_{t+k,t+k+1}^s)^{1-\sigma}\frac{\xi_{t+k}}{\xi_{t+k+1}}$$

$$= P_t C_t^s + P_t C_t^s \sum_{v=1}^{\infty}\left(\frac{\beta}{1+p}\right)^{v\sigma}\prod_{k=0}^{v-1}\left(\frac{P_{t+k+1}}{P_{t+k}}\right)^{1-\sigma}(Q_{t+k,t+k+1}^s)^{1-\sigma}\frac{\xi_{t+k}}{\xi_{t+k+1}} = P_t C_t^s \Phi_t$$

36

where

$$\Phi_t = 1 + \sum_{v=1}^{\infty} \left(\frac{\beta}{1+p}\right)^{v\sigma} \prod_{k=0}^{v-1} \left(\frac{P_{t+k+1}}{P_{t+k}} Q_{t+k,t+k+1}^s\right)^{1-\sigma} \frac{\xi_{t+k}}{\xi_{t+k+1}} \tag{69}$$

$$= 1 + \left(\frac{\beta}{1+p}\right)^{\sigma} \left(\frac{P_{t+1}}{P_t} Q_{t,t+1}^s\right)^{1-\sigma} \frac{\xi_t}{\xi_{t+1}} \Phi_{t+1}$$

and from the last FOCs it follows that:

$$P_t C_t^s = \frac{1}{\Phi_t} \left(\mathcal{A}_t^s + \mathcal{H}_t^s\right) \tag{70}$$

where nominal human capital $\mathcal{H}_t^s$ is:

$$\mathcal{H}_t^s = \mathcal{E}_t \sum_{v=t}^{\infty} Q_{t,v}^s \left(\int_0^1 (1-\tau) \left(w_v(z) h_v^s(z) + \Pi_v(z)\right) dz + T_v^a\right)$$

E.2 Aggregate Relationships

We aggregate all relationships across all generations. The size of total population at time t is

$$\frac{p}{(1+p)} \sum_{s=-\infty}^{t} \left(\frac{1}{1+p}\right)^{t-s} = 1.$$

Therefore

$$C_t^a = \sum_{s=-\infty}^{t} \frac{p}{(1+p)} \left(\frac{1}{1+p}\right)^{t-s} C_t^s, \quad \mathcal{A}_t^a = \sum_{s=-\infty}^{t} \frac{p}{(1+p)} \left(\frac{1}{1+p}\right)^{t-s} \mathcal{A}_t^s$$

and we define aggregate nominal human capital as:

$$\mathcal{H}_t^a = \sum_{s=-\infty}^{t} \frac{p}{(1+p)} \left(\frac{1}{1+p}\right)^{t-s} \mathcal{E}_t \sum_{v=t}^{\infty} Q_{t,v}^s \left(\int_0^1 (1-\tau) \left(w_v(z) h_v^s(z) + \Pi_v^s(z)\right) dz + T_v^a\right)$$

$$= \mathcal{E}_t \sum_{v=t}^{\infty} Q_{t,v}^s \left((1-\tau) Y_v P_v + T_v^a\right)$$

where

$$Y_v P_v = \sum_{s=-\infty}^{t} \frac{p}{(1+p)} \left(\frac{1}{1+p}\right)^{t-s} \int_0^1 \left(w_v(z) h_v^s(z) + \Pi_v^s(z)\right) dz, \quad T_v^a = \sum_{s=-\infty}^{t} \frac{p}{(1+p)} \left(\frac{1}{1+p}\right)^{t-s} T_v^s$$

Aggregating relationship (70) yields:

$$\Phi_t P_t C_t^a = \mathcal{A}_t^a + \mathcal{H}_t^a.$$

We now derive a dynamic Euler equation for aggregate consumption. We note that

$$
\begin{aligned}
\mathcal{A}_{t+1}^a &= \frac{1}{Q_{t,t+1}^s}\frac{1}{1+p}\sum_{s=-\infty}^{t}\frac{p}{(1+p)}(\frac{1}{1+p})^{t-s}\left(\mathcal{A}_t^s - P_t C_t^s + \int_0^1 (1-\tau)w_t(z)h_t^s(z)dz + T_t^s\right) \\
&= \frac{1}{Q_{t,t+1}^s}\frac{1}{1+p}\left(\mathcal{A}_t^a + (1-\tau)P_t Y_t + T_t^a - P_t C_t^a\right) \\
\mathcal{H}_{t+1}^a &= \sum_{v=t+1}^{\infty} Q_{t+1,v}^s (1-\tau)Y_v P_v = \frac{Q_{t,t+1}^s}{Q_{t,t+1}^s}\sum_{v=t+1}^{\infty} R_{t+1,v}(1-\tau)Y_v P_v \\
&= \frac{1}{Q_{t,t+1}^s}\left(\sum_{v=t}^{\infty} Q_{t,v}^s(1-\tau)Y_v P_v - Q_{t,t}^s(1-\tau)Y_t P_t\right) = \frac{1}{Q_{t,t+1}^s}(\mathcal{H}_t^a - (1-\tau)Y_t P_t)
\end{aligned}
$$

Therefore

$$
\begin{aligned}
\Phi_{t+1}P_{t+1}C_{t+1}^a &= \mathcal{A}_{t+1}^a + \mathcal{H}_{t+1}^a = \mathcal{A}_{t+1}^a + \frac{1}{Q_{t,t+1}^s}(\mathcal{H}_t^a - (1-\tau)Y_t P_t) \\
&= \mathcal{A}_{t+1}^a + \frac{1}{Q_{t,t+1}^s}(\Phi_t P_t C_t^a - \mathcal{A}_t^a - (1-\tau)Y_t P_t) \\
&= \mathcal{A}_{t+1}^a + \frac{1}{Q_{t,t+1}^s}(\Phi_t P_t C_t^a - \mathcal{A}_{t+1}^a - P_t C_t^a) \\
&= -p\mathcal{A}_{t+1}^a + \frac{1}{Q_{t,t+1}^s}P_t C_t^a(\Phi_t - 1) \\
&= -p\mathcal{A}_{t+1}^a + \frac{1}{Q_{t,t+1}^s}P_t C_t^a\left(\frac{\beta}{1+p}\right)^{\sigma}\left(\frac{P_{t+1}}{P_t}Q_{t,t+1}^s\right)^{1-\sigma}\frac{\xi_t}{\xi_{t+1}}\Phi_{t+1}
\end{aligned}
$$

from where, taking expectations, we obtain

$$
C_t^a = \mathcal{E}_t\left(\left(\frac{1+p}{\beta}\frac{P_{t+1}}{P_t}Q_{t,t+1}^s\right)^{\sigma}\left(C_{t+1}^a + \frac{p\mathcal{A}_{t+1}^a}{P_{t+1}\Phi_{t+1}}\right)\frac{\xi_{t+1}}{\xi_t}\right) \tag{71}
$$

F   Fiscal policy and the Steady State

We assume that the government finances its deficit by bonds, and nominal bonds accumulate as:

$$
\mathcal{B}_{t+1} = (1+i_t)(\mathcal{B}_t + G_t P_t - \tau Y_t P_t) \tag{72}
$$

There is no physical capital in this model, so $\mathcal{A}_t = \mathcal{B}_t$. In a steady state with zero inflation and prices normalized to one, the following relationships will hold:

$$\mathcal{A} = (1+i)(\mathcal{A} + (1-\tau)Y - C) \tag{73}$$

$$\mathcal{B} = (1+i)(\mathcal{B} + G - \tau\overline{Y}) \tag{74}$$

$$\Phi C = \mathcal{A} + \mathcal{H} \tag{75}$$

$$\mathcal{H} = \frac{(1-\tau)(1+i)(1+p)}{(1+i)(1+p) - 1}Y \tag{76}$$

$$C = \rho Y \tag{77}$$

In order to obtain relationship (76) we computed steady state human capital as the net present value of steady state income, accounting for the mortality rate. Equations (73) and (74) are consistent with $Y = C + G$. We assume that steady state private consumption constitutes a share ρ of steady state income, so that steady state government consumption is $G = (1 - \rho)Y$.

A steady state value for Φ obtained from (69) is:

$$\Phi = \left(1 - \frac{[\beta(1+i)]^\sigma}{(1+p)(1+i)}\right)^{-1}$$

We substitute this into relationships (73)–(77), and simplify them to obtain:

$$\left(\rho - \left(1 - \frac{(\beta(1+i))^\sigma}{(1+p)(1+i)}\right)\frac{(1+i)(1+p)(1-\tau)}{(1+i)(1+p) - 1}\right)Y = \left(1 - \frac{(\beta(1+i))^\sigma}{(1+p)(1+i)}\right)\mathcal{A} \tag{78}$$

$$\mathcal{A} = \mathcal{B} = -\frac{(1+i)}{i}\left(1 - \tau - \rho\right)Y \tag{79}$$

Thus, in a steady state $\mathcal{A}$ ($= \mathcal{B}$) can be found from equation (79), if we know ρ and τ. Alternatively, if there is some steady state level for government debt in equilibrium, $\mathcal{A} = \mathcal{B} = BY$, equation (79) can be used to find the steady state level of the tax rate, which ensures this steady state level of debt, given the interest rate. Equation (78) is an equation for i, the steady state level of the interest rate. It has a unique solution[22] and in equilibrium $1 + i > 1/\beta$.

G   Derivation of Phillips Curve

From the first order conditions (64) and (65) it follows that:

$$\frac{v_h(h_s^i(z), \xi_s)}{(1-\tau)u_C(C_s^i, \xi_s)} = \frac{w_s(z)}{P_s} \tag{80}$$

---

[22]We checked it numerically for the wide range of parameters.

so the nominal wage is

$$w_t(z) = \frac{v_y(y_t^i(z), \xi_t)}{(1 - \tau)u_C(C_t^i, \xi_t)} P_t$$

Production possibilities are specified simply as

$$y_t^i(z) = h_t^i(z)$$

The cost of supplying a good is given as $Cost^i(z) = \frac{1}{\mu^w} w_s(z) h_s^i(z) = \frac{1}{\mu^w} w_s(z) y_t^i(z)$, where μ^w is a labour subsidy. We do not assume any other taxes and labour is the only cost to the firm.

All producers of good z understand that sales depend on demand, which is a function of price. Intra-temporal consumption optimisation implies

$$y_t^a(z) = \left(\frac{p_t(z)}{P_t}\right)^{-\epsilon_t} Y_t^a$$

so with price flexibility a producer that wishes to maximise profit (in square brackets below) will choose a price at t that maximises

$$\max_{p_t(z)} \mathcal{E}_t \sum_{s=t}^{\infty} \gamma^{s-t} Q_{t,s} \left[p_t(z) y_s^i(z) - \frac{1}{\mu^w} w_s(z) y_s^i(z) \right]$$

$$\max_{p_t(z)} \mathcal{E}_t \sum_{s=t}^{\infty} \gamma^{s-t} Q_{t,s} \left[p_t(z) \sum_{i=-\infty}^{s} \frac{p}{(1+p)} \left(\frac{1}{1+p}\right)^{s-i} y_s^i(z) - \frac{w_s(z)}{\mu^w} \sum_{i=-\infty}^{s} \frac{p}{(1+p)} \left(\frac{1}{1+p}\right)^{s-i} y_s^i(z) \right]$$

$$\max_{p_t(z)} \mathcal{E}_t \sum_{s=t}^{\infty} \gamma^{s-t} Q_{t,s} \left[p_t(z) y_s^a(z) - \frac{w_s(z) y_s^a(z)}{\mu^w} \right] = \max_{p_t(z)} \mathcal{E}_t \sum_{s=t}^{\infty} \gamma^{s-t} Q_{t,s} P_s^\epsilon Y_s^a \left[p_t^{1-\epsilon}(z) - \frac{w_s(z) p_t^{-\epsilon}(z)}{\mu^w} \right]$$

This implies

$$\frac{\partial \text{Profit}^i}{\partial p_t(z)} = \mathcal{E}_t \sum_{s=t}^{\infty} \gamma^{s-t} Q_{t,s} Y_s P_s^{\epsilon_s} \left[p_t^{-\epsilon_s}(z)(1 - \epsilon_s) + \epsilon_s \frac{w_s(z) p_t^{-\epsilon_s-1}(z)}{\mu^w} \right] = 0$$

$$0 = \mathcal{E}_t \sum_{s=t}^{\infty} \gamma^{s-t} p_t^{-\epsilon_s-1}(z) Q_{t,s} Y_s \left(\frac{p_t(z)}{P_s}\right)^{-\epsilon_s} \left[(1-\tau) p_t(z) - \frac{\mu_s}{\mu^w} P_s \frac{v_y(y_s^i(z), \xi_s)}{u_C(C_s^i, \xi_s)} \right]$$

$$0 = \mathcal{E}_t \sum_{s=t}^{\infty} \gamma^{s-t} p_t^{-\epsilon_s-1}(z) Q_{t,s} Y_s \left(\frac{p_t(z)}{P_s}\right)^{-\epsilon_s} \left[(1-\tau) p_t(z) - \frac{\mu_s}{\mu^w} P_s \frac{v_y\left(\left(\frac{p_t(z)}{P_s}\right)^{-\epsilon_s} Y_s, \xi_s\right)}{u_C(C_s^i, \xi_s)} \right]$$

where we denote $\mu_s = -\frac{\epsilon_s}{(1-\epsilon_s)}$. The last equation is the equation for the optimal $p_t(z) = p_t^{\#}(z)$

In steady state this implies

$$0 = \sum_{s=t}^{\infty} (\gamma\beta)^{s-t} \left[(1-\tau) - \frac{\mu}{\mu^w} \frac{v_y(Y,1)}{u_C(C,1)} \right] = \left[(1-\tau) - \frac{\mu}{\mu^w} \frac{v_y(Y,1)}{u_C(C,1)} \right] \frac{1}{1-\gamma\beta} \tag{81}$$

Therefore:

$$\frac{v_y(Y,1)}{(1-\tau)u_C(C,1)} = \frac{w}{P} = \frac{\mu^w}{\mu}.$$

Using

$$P_s = 1 + \hat{P}_s$$
$$p_t(z) = 1 + \hat{p}_t(z)$$
$$\mu_t = \mu(1 + \hat{\mu}_t)$$

we can rewrite

$$\frac{v_y\left(\left(\frac{p_t^f(z)}{P_s}\right)^{-\epsilon_s} Y_s, \xi_s\right)}{u_C(C_s^i, \xi_s)} = \frac{v_y}{u_C} - \epsilon\frac{v_{yy}}{u_C}\left(\hat{p}_t^f(z) - \hat{P}_s\right) + \frac{v_{yy}Y}{u_C}\hat{Y}_s - \frac{v_y u_{CC}C}{u_C^2}\hat{C}_s^i + \left(\frac{v_{y\xi}}{u_C} - \frac{v_y u_{C\xi}}{u_C^2}\right)\hat{\xi}_s$$

$$= \frac{v_y}{u_C}\left(1 - \epsilon\frac{v_{yy}}{v_y}\left(\hat{p}_t^f(z) - \hat{P}_s\right) + \frac{v_{yy}Y}{v_y}\hat{Y}_s - \frac{u_{CC}C}{u_C}\hat{C}_s^i + \left(\frac{v_{y\xi}}{v_y} - \frac{u_{C\xi}}{u_C}\right)\hat{\xi}_s\right)$$

where the term with $\hat{\epsilon}_s$ is zero as it is multiplied by $\ln\frac{p^f(z)}{P} = 0$. Substitute everything in the term in square brackets in (81):

$$(1-\tau)p_t(z) - \frac{\mu_s}{\mu^w}P_s\frac{v_y\left(\left(\frac{p_t(z)}{P_s}\right)^{-\epsilon} Y_s, \xi_s\right)}{u_C(C_s^i, \xi_s)}$$

$$= (1-\tau)\left(1 + \hat{p}_t(z)\right) - \frac{\mu}{\mu^w}(1 + \hat{\mu}_s)\left(1 + \hat{P}_s\right)\frac{v_y}{u_C}\left(1 - \epsilon\frac{v_{yy}}{v_y}\left(\hat{p}_t^f(z) - \hat{P}_s\right)\right.$$

$$\left. + \frac{v_{yy}Y}{v_y}\hat{Y}_s - \frac{u_{CC}C}{u_C}\hat{C}_s^i + \frac{v_{y\xi}}{v_y}\hat{\xi}_s - \frac{u_{C\xi}}{u_C}\hat{\xi}_s\right)$$

$$= (1-\tau) - \frac{v_y}{u_C}\frac{\mu}{\mu^w} + (1-\tau)\hat{p}_t(z) - \frac{v_y}{u_C}\frac{\mu}{\mu^w}\left(\left(\hat{P}_s - \epsilon\frac{v_{yy}}{v_y}\left(\hat{p}_t^f(z) - \hat{P}_s\right)\right.\right.$$

$$\left.\left. + \frac{v_{yy}Y}{v_y}\hat{Y}_s - \frac{u_{CC}C}{u_C}\hat{C}_s^i + \frac{v_{y\xi}}{v_y}\hat{\xi}_s - \frac{u_{C\xi}}{u_C}\hat{\xi}_s + \hat{\mu}_s\right)\right)$$

$$= (1-\tau)\left(\hat{p}_t(z) - \hat{P}_s + \frac{\epsilon}{\psi}\left(\hat{p}_t^f(z) - \hat{P}_s\right) - \frac{1}{\psi}\hat{Y}_s - \frac{1}{\sigma}\hat{C}_s^i - \left(\frac{v_{y\xi}}{v_y} - \frac{u_{C\xi}}{u_C}\right)\hat{\xi}_s - \hat{\mu}_s\right)$$

In the last formula, we substituted $\frac{v_y}{u_C}\frac{\mu}{\mu^w} = (1-\tau)$.

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Knowing that $\hat{P}_s = \sum_{k=1}^{s-t} \pi_{t+k}$ we can simplify equation (81):

$$0 = (1-\tau)\mathcal{E}_t \sum_{s=t}^{\infty} \gamma^{s-t} \beta^{s-t} \left[\hat{p}_t^f(z) - \hat{P}_s + \frac{\epsilon}{\psi} \left(\hat{p}_t^f(z) - \hat{P}_s \right) - \frac{1}{\psi} \hat{Y}_s - \frac{1}{\sigma} \hat{C}_s^i - \left(\frac{v_{y\xi}}{v_y} - \frac{u_{C\xi}}{u_C} \right) \hat{\xi}_s - \hat{\mu}_s \right]$$

We solve this equation with respect to $\hat{p}_t^f(z)$, using $\sum_{s=t}^{\infty} (\gamma\beta)^{s-t} = \frac{1}{1-\gamma\beta}$ to obtain:

$$\hat{p}_t^f(z) = \frac{(1-\gamma\beta)}{\left(1+\frac{\epsilon}{\psi}\right)} \mathcal{E}_t \sum_{s=t}^{\infty} (\gamma\beta)^{s-t} \left(\left(1+\frac{\epsilon}{\psi}\right) \sum_{k=1}^{s-t} \pi_{t+k} + \frac{1}{\psi} \hat{Y}_s + \frac{1}{\sigma} \hat{C}_s^i + \left(\frac{v_{y\xi}}{v_y} - \frac{u_{C\xi}}{u_C} \right) \hat{\xi}_s + \hat{\mu}_s \right)$$

It is easy to check that $\sum_{s=t}^{\infty} (\gamma\beta)^{s-t} \sum_{k=1}^{s-t} \pi_{t+k} = \frac{1}{1-\gamma\beta} \sum_{k=1}^{\infty} (\gamma\beta)^k \pi_{t+k}$ so the last formula can be simplified to:

$$\hat{p}_t^f(z) = \mathcal{E}_t \sum_{k=1}^{\infty} (\gamma\beta)^k \pi_{t+k} + \frac{(1-\gamma\beta)}{\left(1+\frac{\epsilon}{\psi}\right)} \mathcal{E}_t \sum_{s=t}^{\infty} (\gamma\beta)^{s-t} \left(\frac{1}{\psi} \hat{Y}_s + \frac{1}{\sigma} \hat{C}_s^i + \left(\frac{v_{y\xi}}{v_y} - \frac{u_{C\xi}}{u_C} \right) \hat{\xi}_s + \hat{\mu}_s \right) \quad (82)$$

This formula is not the final Phillips curve, but Steinsson (2003) shows how it can be used to derive the final specification of the Phillips Curve, where we have rule-of-thumb price-setters, (8). Our derivation is identical to his, so will not be repeated here. Formula (82) demonstrates how mark-up shocks enter the Phillips curve. It also demonstrates that the *constant* wage income tax τ has no effect on the dynamic equations for log-deviations from the flexible price equilibrium (although it alters the equilibrium choice between consumption and leisure for the consumer).[23]

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[23]Steinsson (2003) assumes a stochastic tax rate, and shows that it enters the final specification of the Phillips curve similarly to the cost-push shock $\hat{\mu}$. He also suggests that this shock is relatively unimportant.

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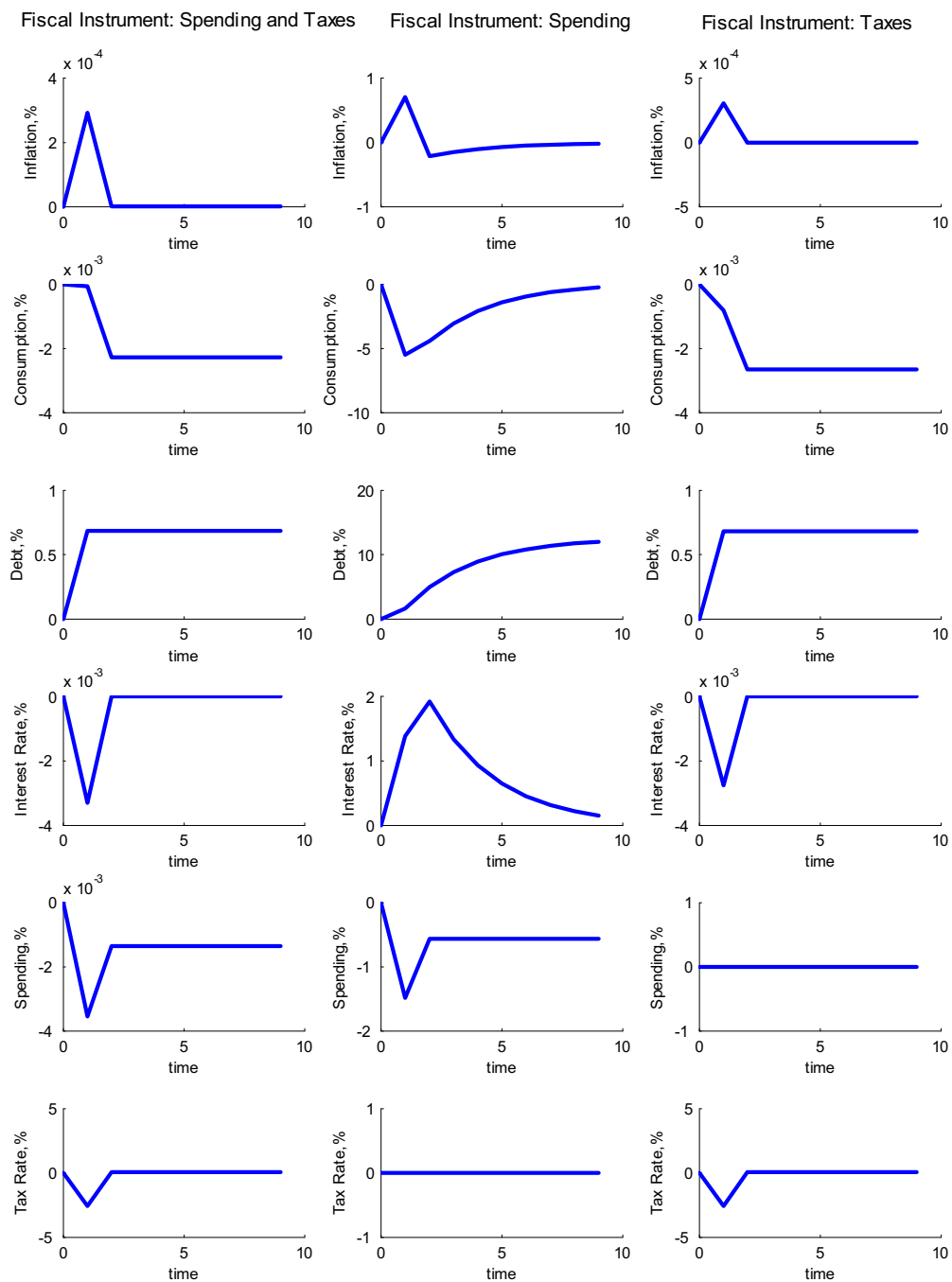
Figure 1: Responses to a unit cost-push shock under fully optimal monetary and fiscal policy using alternative fiscal instruments.
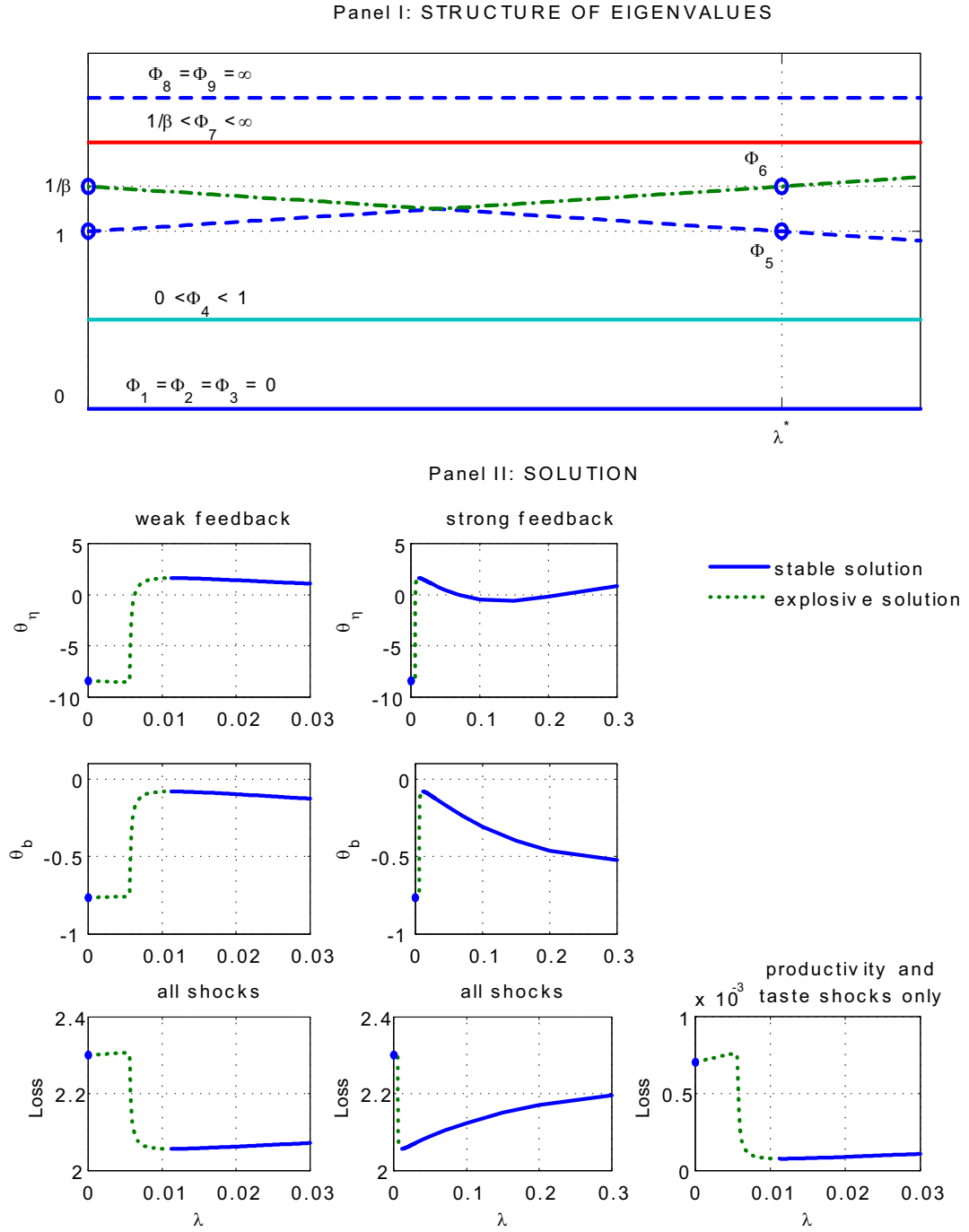
Figure 2: The structure of eigenvalues, coefficients of monetary policy reaction function and social welfare as a function of fiscal feedback. Fiscal policy uses government spending as an instrument.
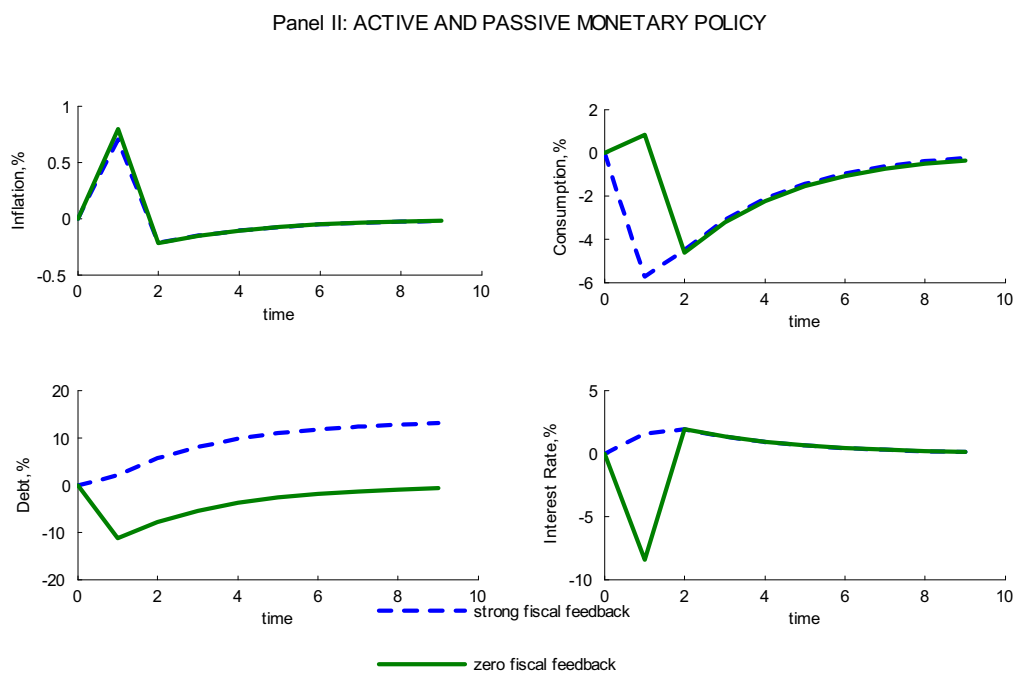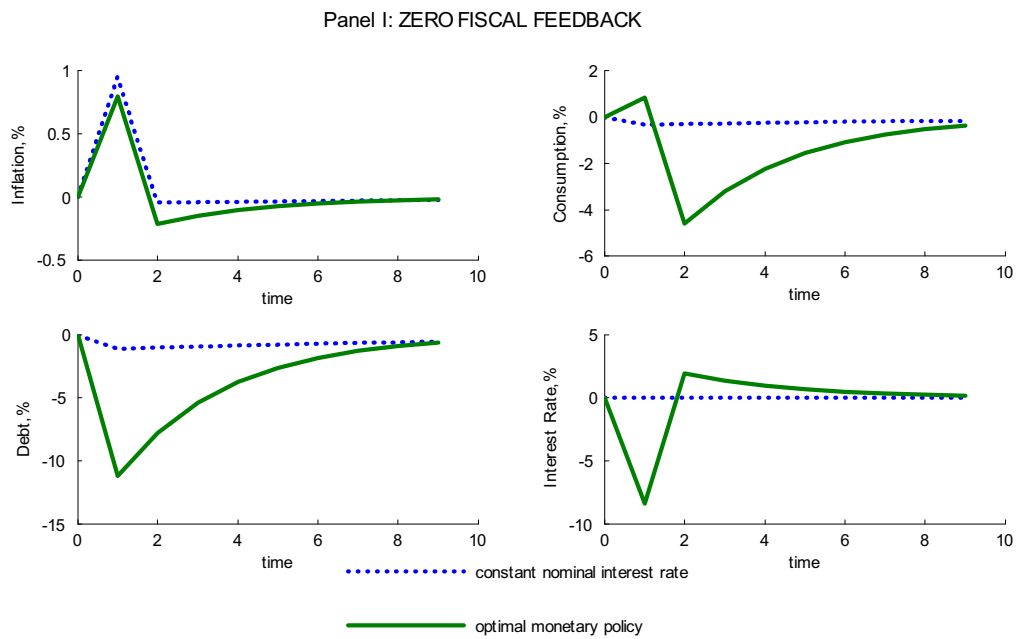
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Figure 3: Impulse responses to a unit cost-push shock, plotted for alternative feedback parameters. Fiscal policy uses government spending as an instrument.
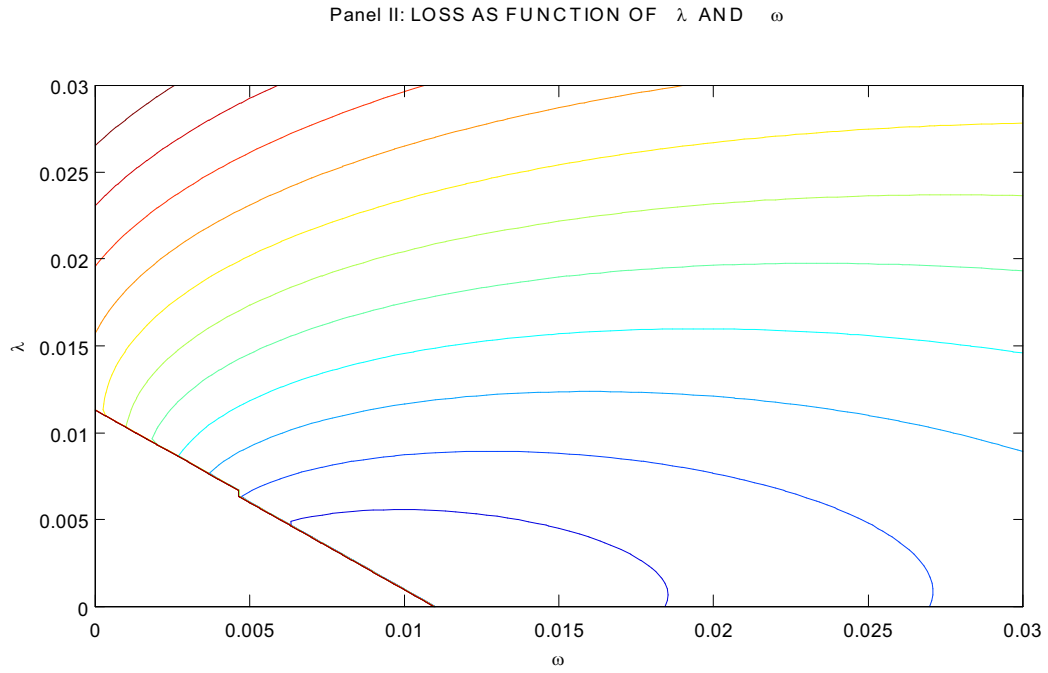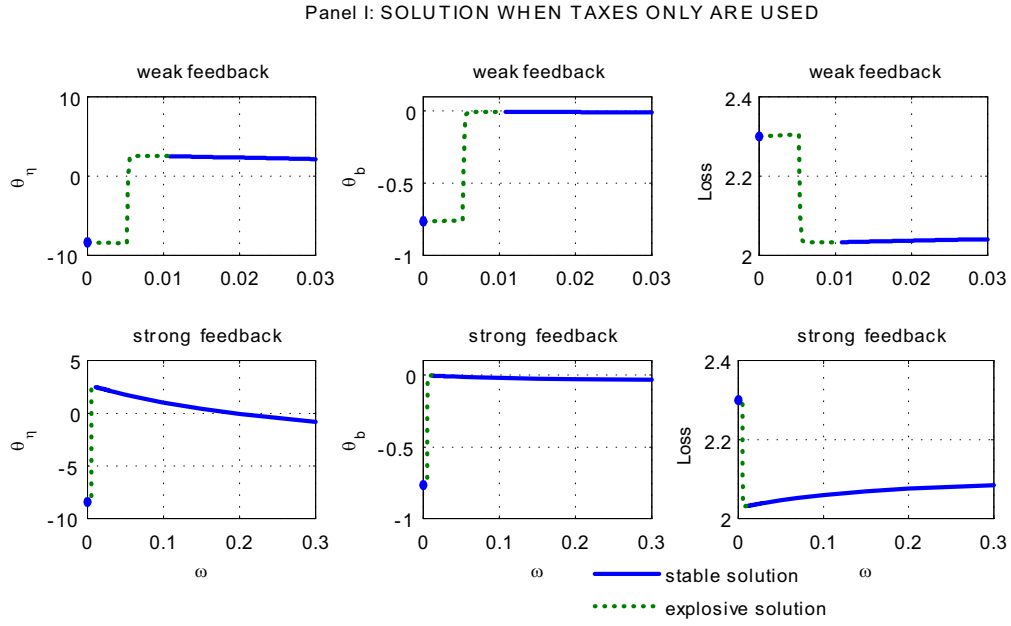
Figure 4: Coefficients of monetary policy reaction function and social welfare as a function of fiscal feedback. Fical policy uses taxes as an instrument in Panel I and uses taxes and spending in Panel II.

Panel I: MODEL SOLUTION FOR DIFFERENT VALUES OF B

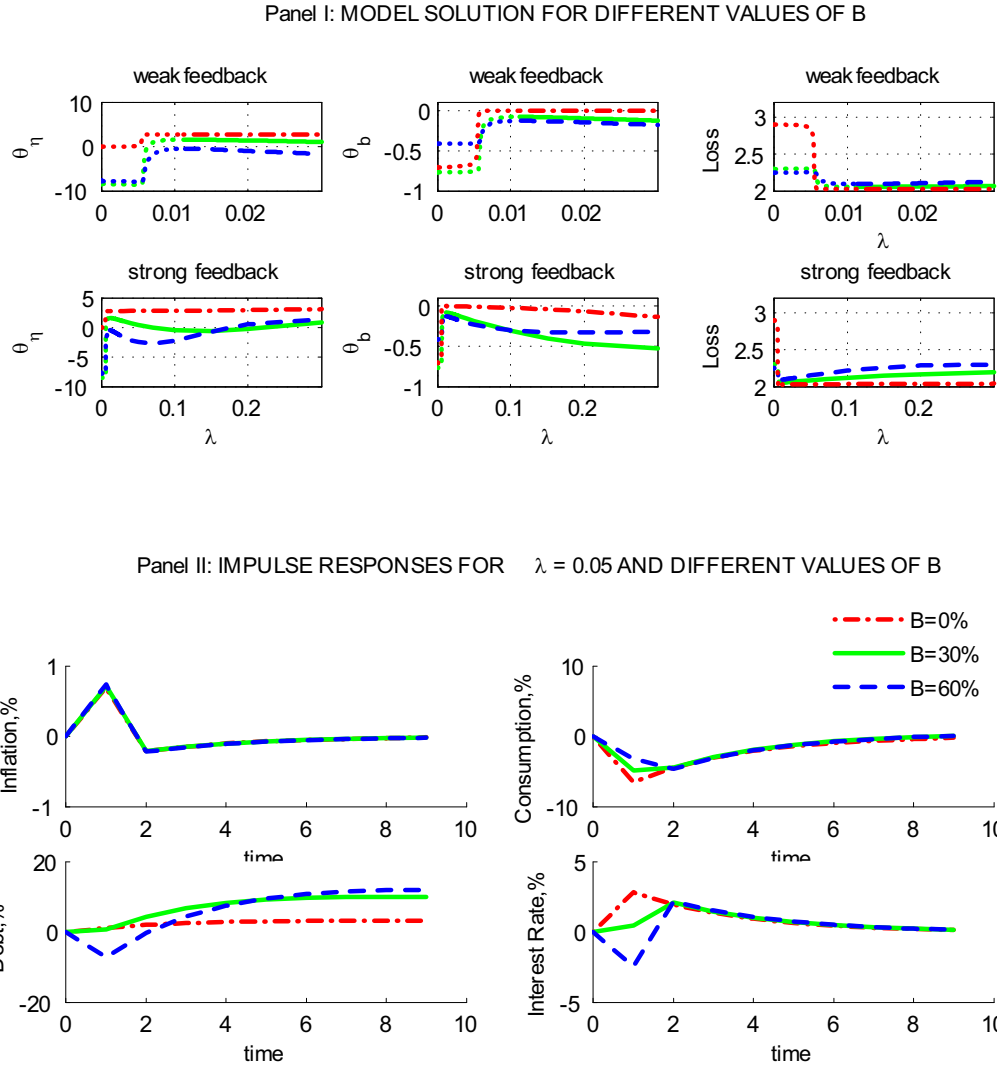Panel II: IMPULSE RESPONSES FOR $\lambda = 0.05$ AND DIFFERENT VALUES OF B

Figure 5: Coefficients of monetary policy reaction function and social welfare as a function of fiscal feedback plotted for alternative values of steady state debt (Panel I), and Impulse responses to a unit cost push shock (Panel II). Fiscal policy uses spending as an instrument.

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Panel I: WEAK FISCAL FEEDBACK, p=0.06

Panel II: IMPULSE RESPONSES TO A UNIT COST-PUSH SHOCK: DIFFERENT MORTALITY RATE, p
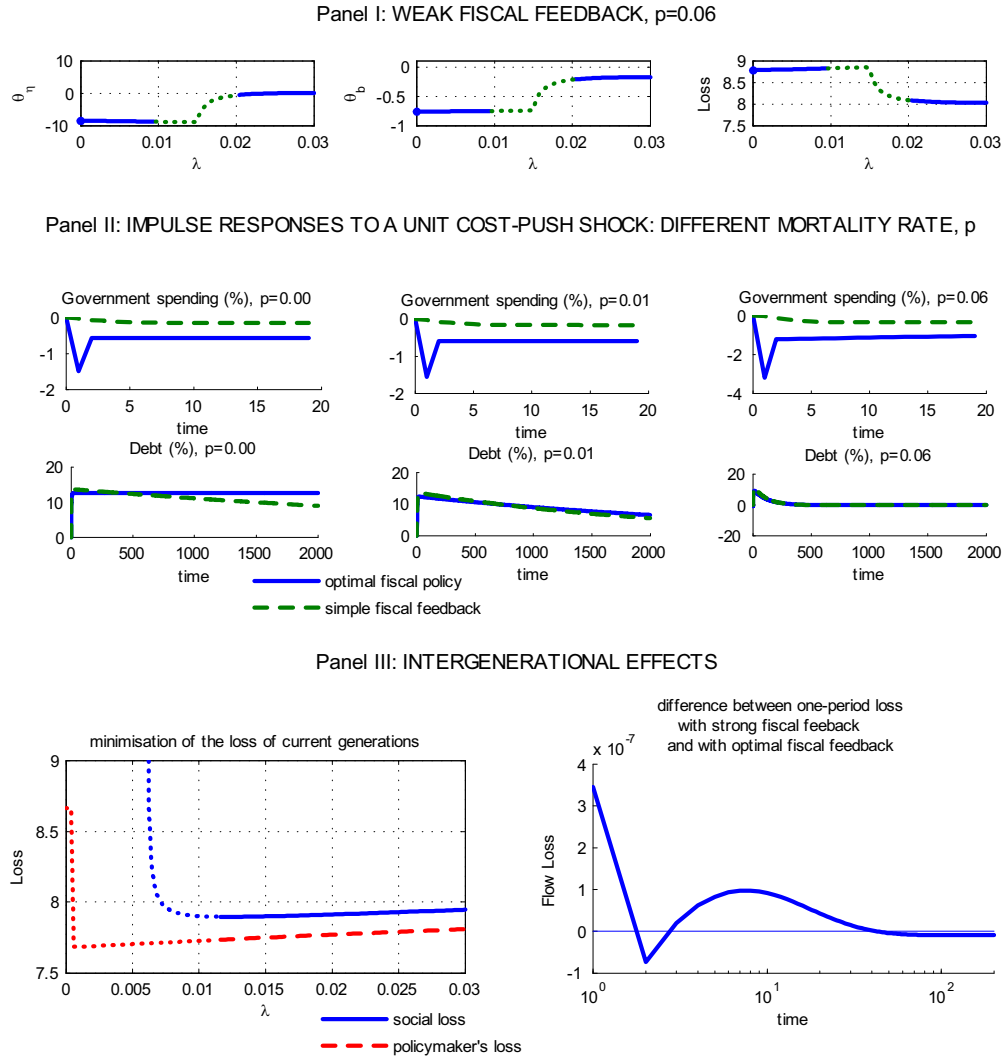
Panel III: INTERGENERATIONAL EFFECTS

Figure 6: Panel I and Panel II illustrate the effect of Blanchard-Yaari consumers on the monetary reaction function and social welfare, and impulse responses to a unit cost-push shock respectively. Panel III illustrates the effect of policymaker's discounting on the choice of optimal fiscal feedback.
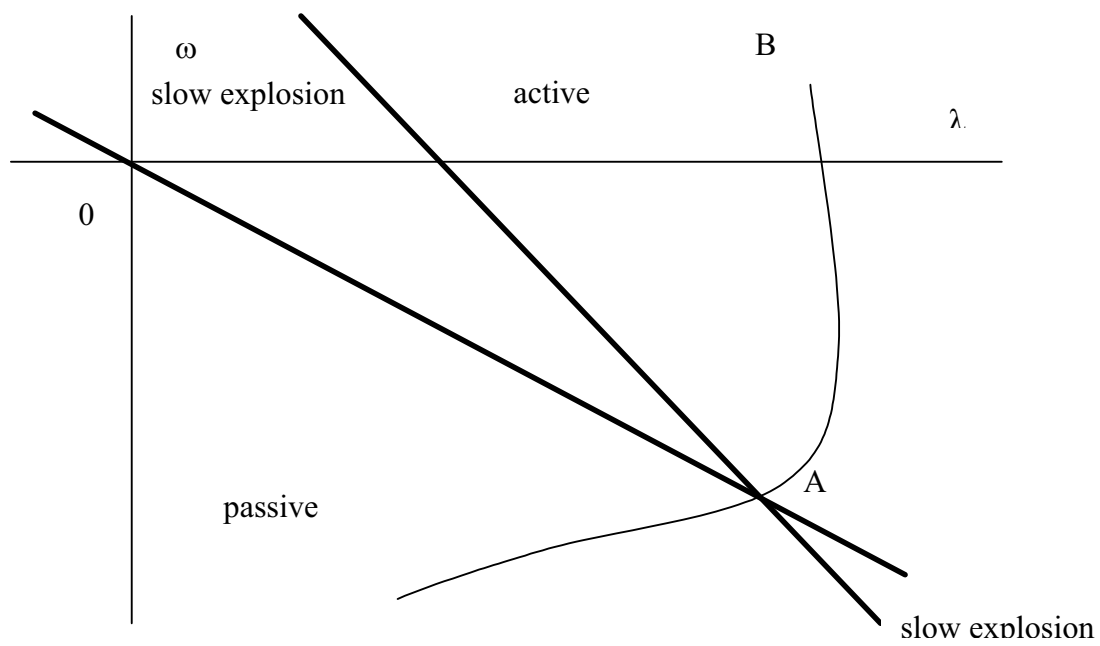
Figure 7: Dynamic properties of the economy under control