Techincal University - Sofia
Faculty of Applied Mathematics and Informatics

Project 1 - Topics of Algebra

Solution for version 4

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Project 1 - Topics of Algebra

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Problem 1. Let be given the following real matrix:

$$A = \begin{pmatrix} 1 & -3 & -1 \\ 3 & 7 & 5 \\ 2 & -2 & 4 \end{pmatrix}$$

Then

a) find its **LU** decomposition;

b) explain rigorously what is a **LU** decomposition.

Problem 2. Let be given the following real matrix:

$$A = \begin{bmatrix} 5 & 1 & -1 \\ 2 & -3 & 4 \\ 3 & 2 & 1 \\ 7 & 4 & 2 \end{bmatrix}$$

Then

a) find its **QR** decomposition and **LQ** decomposition;

b) explain rigorously what is a **QR** decomposition and a **LQ** decomposition.

Problem 3. Let be given the following real matrix:

$$A = \begin{pmatrix} 2 & -1 & 1 \\ -1 & 5 & -1 \\ 1 & -1 & 2 \end{pmatrix}$$

Then

a) prove by pivoting that A is positive defined;

b) find its **Cholesky-Banachiewicz** and **LDL**^{T} decompositions.

Problem 4. Let be given the following real matrix:

$$A = \begin{bmatrix} 3 & 2 \\ 2 & 3 \\ 2 & -2 \end{bmatrix}$$

Then

a) find its **SVD** decomposition;

b) explain rigorously what is a **SVD** decomposition.

1 Problem 1

$$A = \begin{pmatrix} 1 & -3 & -1 \\ 3 & 7 & 5 \\ 2 & -2 & 4 \end{pmatrix}$$

1.1 Solution for 1a

Lets start with L being the identity matrix

$$L = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

We perform the following operations on matrix A:

$$\left| \begin{array}{l} R_2 = R_2 - 3R_1 \\ R_3 = R_3 - 2R_1 \end{array} \right.$$

Where R_i is the ith row of the matrix.
We write the coefficient 3 in the matrix L at row 2 and column 1. The same can be done for the coefficient 2 for row 3 and column 1.

$$L = \begin{pmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ 2 & 0 & 1 \end{pmatrix} \qquad A = \begin{pmatrix} 1 & -3 & -1 \\ 0 & 16 & 8 \\ 0 & 4 & 6 \end{pmatrix}$$

We perform the following operation on matrix A

$$R_3 = -\frac{1}{4}R_2 + R_3$$

We write the coefficient $\frac{1}{4}$ in matrix L at row 3 and column 2.

$$L = \begin{pmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ 2 & \frac{1}{4} & 1 \end{pmatrix} \qquad A = \begin{pmatrix} 1 & -3 & -1 \\ 0 & 16 & 8 \\ 0 & 0 & 4 \end{pmatrix} = U$$

1.2 Solution for 1b

2 Problem 2

$$A = \begin{bmatrix} 5 & 1 & -1 \\ 2 & -3 & 4 \\ 3 & 2 & 1 \\ 7 & 4 & 2 \end{bmatrix}$$

2.1 Solution for 2a

2.2 Solution for 2b

3 Problem 3

3.1 Solution for 3a

3.2 Solution for 3b

4 Problem 4

4.1 Solution for 4a

4.2 Solution for 4b