

Technical University of Sofia

Informatics and Software Science - Optimization theory and big data analytics (2024/2025)

Project 1 - Topics of Algebra

Examiner: Prof. M. Tarulli

First Name:_____, Last Name:_____, Student No.:_____

Problem 1. Let be given the following real matrix:

$$A = \begin{pmatrix} 1 & -3 & -1 \\ 3 & 7 & 5 \\ 2 & -2 & 4 \end{pmatrix}$$

Then

- a) find its **LU** decomposition;
- b) explain rigorously what is a **LU** decomposition.

Problem 2. Let be given the following real matrix:

$$A = \begin{bmatrix} 5 & 1 & -1 \\ 2 & -3 & 4 \\ 3 & 2 & 1 \\ 7 & 4 & 2 \end{bmatrix}$$

Then

- a) find its **QR** decomposition and **LQ** decomposition;
- b) explain rigorously what is a **QR** decomposition and a **LQ** decomposition.

Problem 3. Let be given the following real matrix:

$$A = \begin{pmatrix} 2 & -1 & 1 \\ -1 & 5 & -1 \\ 1 & -1 & 2 \end{pmatrix}$$

Then

- a) prove by pivoting that A is positive defined;
- b) find its **Cholesky-Banachiewicz** and **LDL^T** decompositions.

Problem 4. Let be given the following real matrix:

$$A = \begin{bmatrix} 3 & 2 \\ 2 & 3 \\ 2 & -2 \end{bmatrix}$$

Then

- a) find its **SVD** decomposition;
- b) explain rigorously what is a **SVD** decomposition.