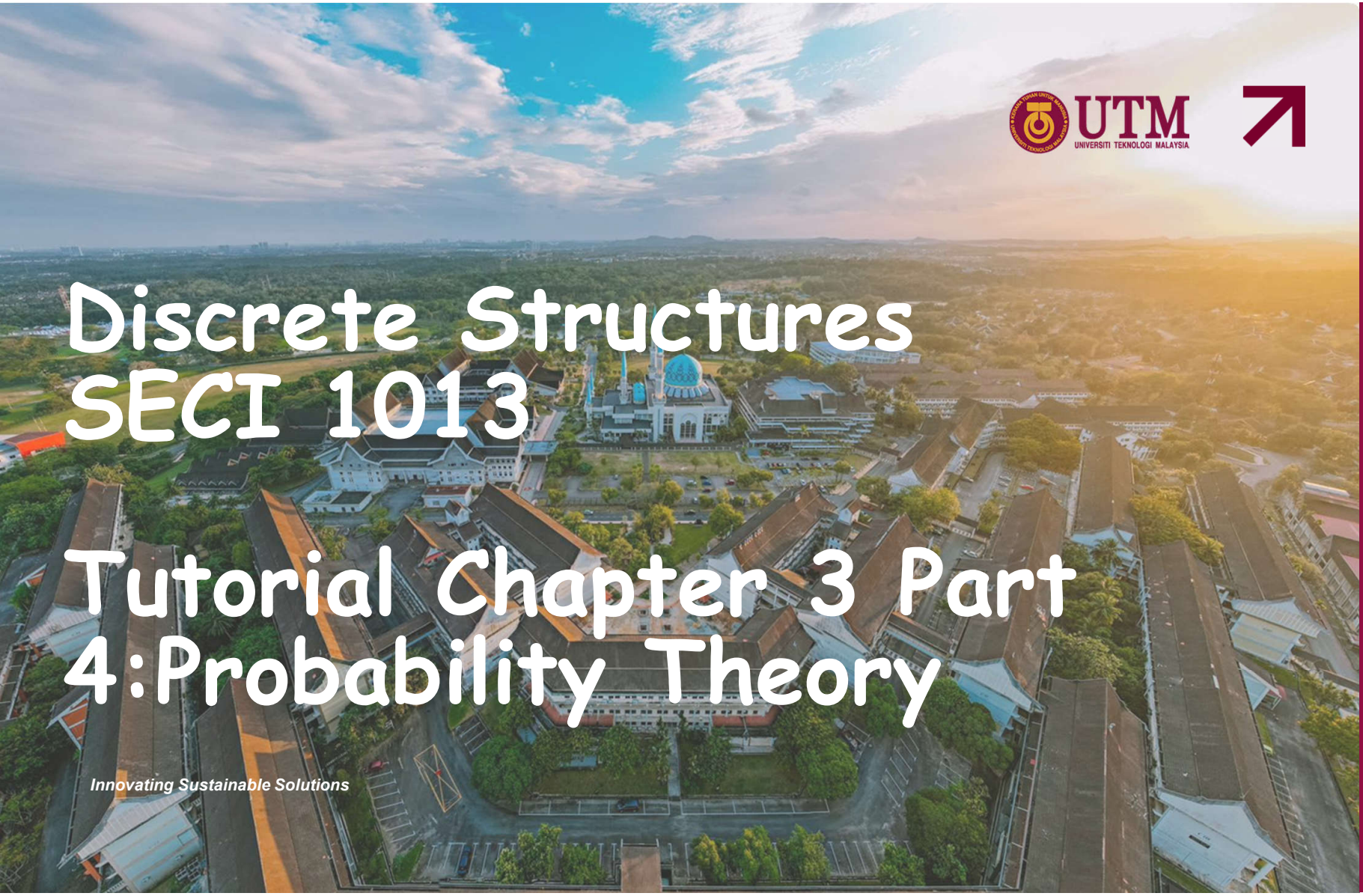

Discrete Structures SECI 1013

Tutorial Chapter 3 Part 4:Probability Theory

Innovating Sustainable Solutions

Theory of Computer Science

2025

Exercise #1

1 red = 4 pt, 1 blue = 2 pt

A bag contains 3 red balls and 3 blue balls. Each red ball is valued as 4 and blue ball as 2.

i) What is probability of getting 4 balls with the total value is 10?

$$\frac{{}^{3}C_1 \times {}^{3}C_3}{{}^{6}C_4} = \frac{1}{5} = 0.2$$

ii) What is probability of getting total value of 6 with at most 3 balls?

$$\frac{{}^{3}C_1 \times {}^{3}C_1}{{}^{6}C_2} = 0.6\overline{3}$$

why 2 balls?

$$\frac{{}^{3}C_1 \times {}^{3}C_1}{{}^{6}C_2} + \frac{{}^{3}C_3}{{}^{6}C_3} = 0.6\overline{3}$$

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Exercise 2

$P(T) = \frac{1}{2}$

- An unprepared student who takes a 10-question true-false quiz and guesses at the answer to every question.

- i) What is the probability that the student answers every question correctly?

- ii) What is the probability that the student answers exactly 5 questions correctly?

i) $P(X=10) = {}^{10}C_{10}(0.5)^{10}(0.5)^{0}$

$= \frac{1}{1024}$

ii) $P(X=5) = {}^{10}C_{5}(0.5)^{5}(0.5)^{5}$

$= \frac{63}{256}$

Exercise #3

On New Year's Eve, the probability of a person having a car accident is 0.09. The probability of a person driving while intoxicated is 0.32 and probability of a person having a car accident while intoxicated is 0.15. What is the probability of a person driving while intoxicated or having a car accident?

$P(A \cap B) = 0.15$

let A = accident

B = driving w/ intox

$P(A) = 0.09$

$P(B) = 0.32$

$P(A \cup B) = 0.09 + 0.32 - 0.15$

$= 0.26$

Exercise #4

Mira is going to graduate from a computer science department in a university by the end of the semester. After being interviewed at two companies she likes, she assess that her probability of getting an offer from company A is 0.8, and her probability of getting an offer from company B is 0.6. If she believes that the probability that she will get offers from both companies is 0.5, what is the probability that she will get either from company A or company B (or both)? let A = comp A, B = comp B $P(A)=0.8$ $P(B)=0.6$

$P(A \cup B) = 0.8 + 0.6 - 0.5$ $P(A \cap B) = 0.5$

$= 0.9$

Exercise 5

- A balanced die is thrown 3 times and the resulting sequence of digits on the upper face is recorded.

- What is the probability of the event A that either all 3 digits are equal or none of then is 3 ?

$6 \times 6 \times 6 = 216$

Case 1: All 3 $= \frac{6}{216}$

Case 2: None 3 $= \frac{5 \times 5 \times 5}{216} = \frac{125}{216}$

$P(\text{Case 1} \cap \text{Case 2}) = \frac{5}{216}$

$P(A) = \frac{6}{216} + \frac{125}{216} - \frac{5}{216} = \frac{126}{216}$

Exercise #6

The probability that a doctor correctly diagnose a particular illness is 0.7. Given that the doctor makes an incorrect diagnosis, the probability that the patient files a lawsuit is 0.9. What is the probability that the doctor makes an incorrect diagnosis and the patient sues?

let D = correct diagnosis S = lawsuit

$P(D) = 0.7$ $P(D') = 0.3$ $P(S|D') = 0.9$

$$P(D' \cap S) = P(D') \times P(S|D')$$

$$= 0.3 \times 0.9$$

$$= 0.27$$

QUESTIONS RELATED TO BAYES RULE

Exercise #7

A department store has three branches that sells clothes. The customers can return the clothes if they bought the clothes in wrong sizes, the clothes have defects or if they simply change their mind. Suppose that out of all of the returned clothes from last month, **half are from branch A**, **3/10 from branch B** and **1/5 from branch C** (the details shown in Table 1).

Table 1: Data on returned cloths by branch.

	Branch A	Branch B	Branch C
Wrong size	3/5	1/3	3/8
Defects	1/10	1/2	1/4
Change mind	3/10	1/6	3/8

S
D
M

Exercise #7 (cont.)

i) What are the probabilities that the customers from branch A return the cloth because of they changed their mind? $P(M|A) = \frac{3}{10} = 0.3$

ii) If it was discovered that the customer return because of wrong size, what is the probability that he or she return it at branch C?

iii) If it was discovered that the customer return because of defects, what is the probability that he or she return it at branch B?

(ii) $P(C|S) = \frac{P(S|C) \times P(C)}{P(S)} = \frac{\frac{3}{8} \times \frac{1}{5}}{\left(\frac{3}{5} \times \frac{1}{2}\right) + \left(\frac{1}{5} \times \frac{3}{10}\right) + \left(\frac{3}{8} \times \frac{1}{5}\right)}$

$= \frac{3}{19} \approx 0.1579$

Exercise #8

(a) $P(\bar{M}) = |-0.51| = 0.49$

(b) (i) $P(R|M) = 0.095$

In Apple County, 51% of the adults are males. One adult is randomly selected for a survey involving credit card usage.

(a) Find the probability that the selected person is female.

(b) It is later learned that the selected survey subject was from a rural area. Also, 9.5% of males from a rural area, whereas 1.7% of female from a rural area.

(i) What is the probability of getting someone who is from a rural area, given that the person is a male?

(ii) What is the probability that the selected subject is a male, given that he comes from a rural area?

(iii) If the Apple County has 100,000 adult populations, find the number of females that come from an urban area.

(ii) $P(M|R) = \frac{P(R|M) \times P(M)}{P(R)}$

$= \frac{0.095 \times 0.51}{[0.095 \times 0.51 + 0.017 \times 0.49]}$

$= 0.8533$

Hint: Use the following notations

M = Male; $\bar{M}$ = female; R = from rural area; $\bar{R}$ = from an urban area

(iii) $\bar{M} = 100000(0.49) = 49000$

$P(\bar{R}|\bar{M}) = 1 - 0.017$

$= 0.983$

$\bar{R}|\bar{M} = 0.983 \times 49000$

$= 48\,167$

Let P: prison, G: guilty

Given: $P(P) = 0.45$; $P(G|P) = 0.4$; $P(G|P') = 0.55$

Exercise #9

In a study of pleas and prison sentences, it is found that 45% of the subjects studied were sent to prison. Among those sent to prison, 40% chose to plead guilty. Among those not sent to prison, 55% chose to plead guilty.

(i) $P(P') = 1 - 0.45 = 0.55$

i) If one of the study subjects is randomly selected, find the probability of getting someone who was not sent to prison.

ii) If a study subject is randomly selected and it is then found that the subject entered a guilty plea, find the probability that this person was sent to prison.

iii) If one of the study subjects is randomly selected, it is found that the subject is entered a guilty plea, find the probability that this person was not sent to prison.

iv) If a study subject is randomly selected find the probability of getting someone who was chose to plead guilty.

Exercise

A random sample of 200 adults are classified by gender and education level, as below

No	Education	Male	Female
1	Elementary	38	45
2	Secondary	28	50
3	College	22	17

If a person is picked at random from this group, find the probability that

a) The person is a male.
b) The person has elementary education among female.
c) The person is a male, given that the person has a secondary education.
d) The person does not have a college degree, given that the person is female.

a) $P(\text{Male}) = \frac{38+28+22}{200} = \frac{88}{200} = 0.44$

b) $P(\text{Elementary} \mid \text{Female}) = \frac{45}{45+50+17} = \frac{45}{112} = 0.4018$

c) $P(\text{Male} \mid \text{Secondary}) = \frac{28}{28+50} = \frac{28}{78} = 0.3590$

d) $P(\text{College}' \mid \text{Female}) = \frac{45+50}{112} = \frac{95}{112} = 0.8482$

Independent Events

Exercise

$P(\text{Football}) = \frac{360}{500} = 0.72$

$P(\text{Badminton}) = \frac{240}{500} = 0.48$

$P(\text{Basketball}) = \frac{200}{500} = 0.40$

University of DEF Sports Club surveyed a representative sample of students from year 1 to year 4 to find out what their three favorite sports were at university. They collected 500 surveys and found that football was chosen as a favorite sport by 360 students. Badminton was chosen as a favorite by 240 students. And basketball was the next top favorite with 200 votes. The club wants to know the probability of a student choosing football, badminton, and basketball as their favorite sports.

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Thank you!

Questions??