



Counting Methods (Part 3-Pigeonhole principle)

Exercise

1. There are 400 students in a programming class. Show that at least 2 of them were born on the same day of a month.
2. Let $A = \{a_1, a_2, a_3, a_4, a_5, a_6, a_7\}$ be a set of seven integers. Show that if these numbers are divided by 6, then at least two of them must have the same remainder.
3. Let $A = \{1, 2, 3, 4, 5, 6, 7, 8\}$. Show that if you choose any five distinct members of A , then there will be two integers such that their sum is 9.
4. From the integers in the set $\{1, 2, 3, \dots, 19, 20\}$, what is the least number of integers that must be chosen so that at least one of them is divisible by 4?

① Pigeonholes = 365 days a year

Pigeon : 400 student ✓

1st form ✓

$$k = \left\lceil \frac{400}{365} \right\rceil = 1.095$$

$$= 2$$

This shows there are at least 2 student are born on the same day & month

② Let $X = \{a_1, a_2, a_3, a_4, a_5, a_6, a_7\}$
(pigeon) ✓

$Y = \{0, 1, 2, 3, 4, 5\}$ (pigeonholes)

$$|X| > |Y|$$

Hence at least two will have the same remainder

③ $A = \{1, 2, 3, 4, 5, 6, 7, 8\}$ (pigeon) ✗

$B = \{(1, 8), (2, 7), (3, 6), (4, 5)\}$ (pigeonholes)

$|pigeon| > |pigeonholes|$ 2nd form

A: 5 elements we chose

$$|A| > |B|$$

$$(4) \quad X = \{1, 2, \dots, 19, 20\}$$

$$Y = \{4, 8, 12, 16, 20\} \quad |Y| = 5$$

where Y is divisible by 4

$$20 - 5 = 15 \text{ (not divisible by 4)}$$

$$15 + 1 = 16 \text{ (least number of int}$$

that must be chosen so

$$|Y| = 5$$

that at least one of

$$|X| = 16$$

them are divisible by 4)

$$|X| > |Y|$$

