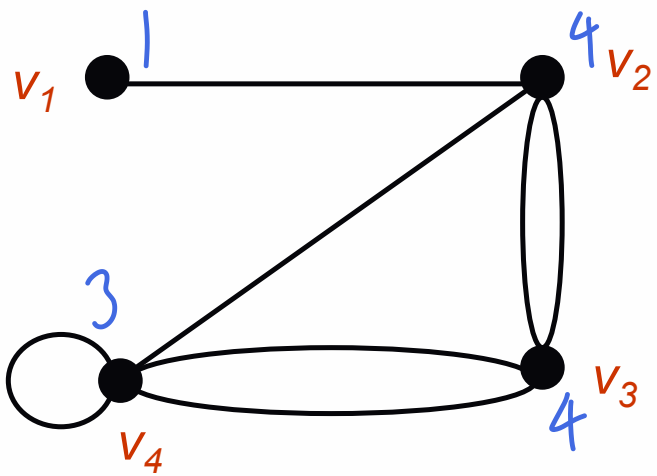
GRAPH THEORY

TUTORIAL

Definition and Key Concept

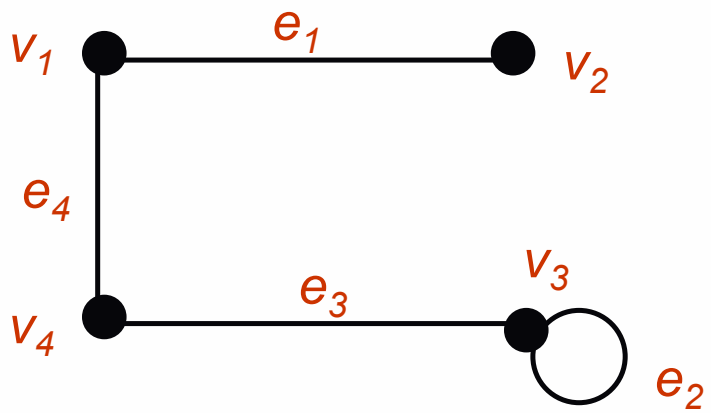
Exercise #1

- Find the degree of each vertex in the graph.

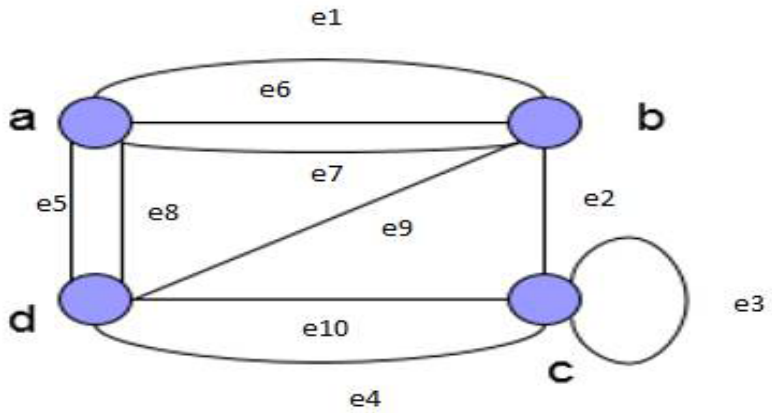


Exercise #2

- Find the adjacency matrix and the incidence matrix of the graph.



Exercise #3



- Find the adjacency matrix
- Find the incidence matrix

Exercise #4

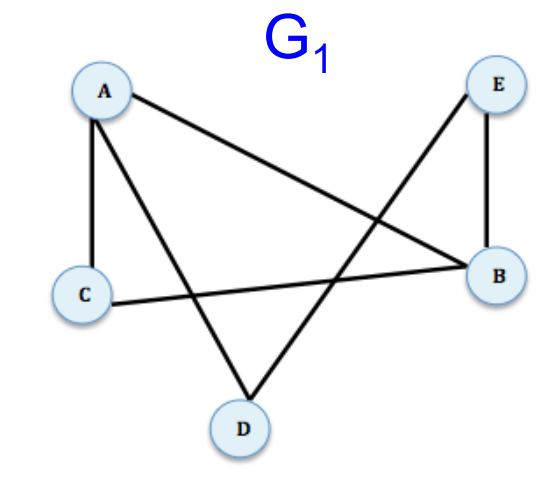
- A shuttle bus travels through eight residential colleges (C1, C2, C3, C4, C5, C6, C7, C8) to pick up students. The location of the colleges determines the bus route. There are two routes connecting C1 and C2. From C2, the shuttle can go to C3 through two routes, one route to C4 and one route to C5. There is only one route that connects C5, C6, and C7. In order to get to C7, the shuttle bus needs to pass through C6. C8 can only be access by a route through C7. During semester break; a shortcut route that connects C1, C4 and C7 can be used to shorten the journey.

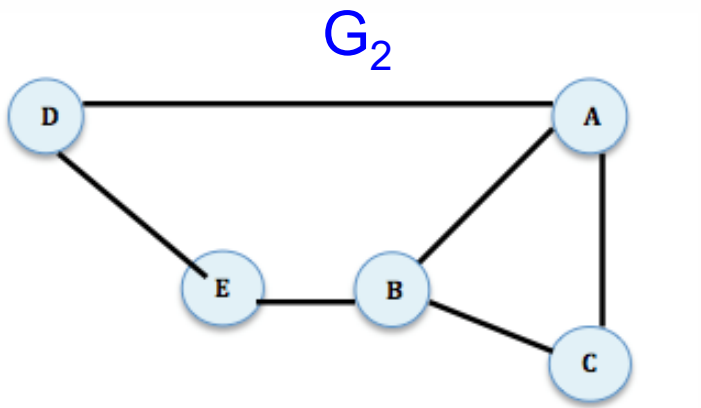
- Draw the graph.
- Is the graph is a simple graph? Justify
- List degree of each vertex

Isomorphisms

Exercise #5

Q: Show that the following two graphs are isomorphic.

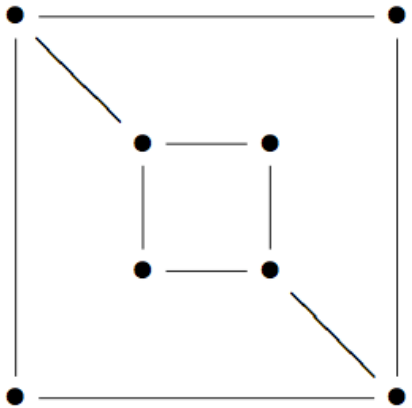




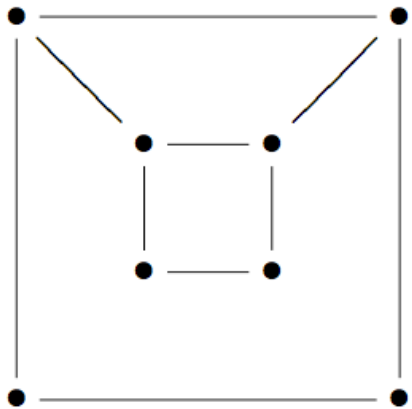
Exercise #6

Q: Is these two graphs are isomorphic?

G :



H :



Exercise 6

G:

	1	2	3	4	5	6	7	8
1	0	1	1	0	0	0	1	0
2	1	0	0	0	0	0	0	1
3	1	0	0	1	1	0	0	0
4	0	0	1	0	0	1	0	0
5	0	0	1	0	0	1	0	0
6	0	0	0	1	1	0	0	1
7	1	0	0	0	0	0	0	1
8	0	1	0	0	0	1	1	0

Since

H:

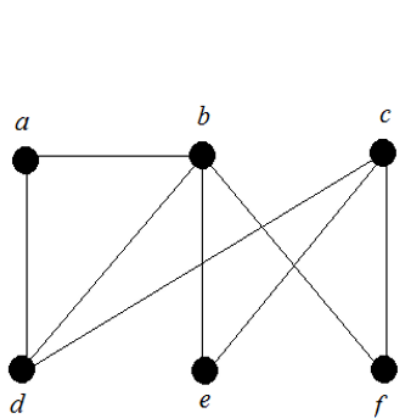
	1	2	3	4	5	6	7	8
1	0	1	1	0	0	0	1	0
2	1	0	0	1	0	0	0	1
3	1	0	0	1	1	0	0	0
4	0	1	1	0	0	1	0	0
5	0	0	1	0	0	1	0	0
6	0	0	0	1	1	0	0	1
7	1	0	0	0	0	0	0	1
8	0	1	0	0	0	1	1	0

$G \neq H$,

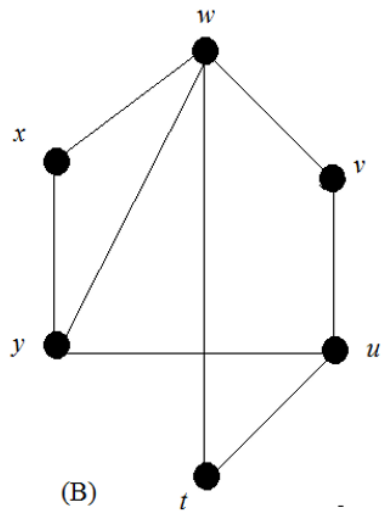
those two graph not isomorphic

Exercise #7

Determine whether the graphs in Figure 2 (A and B) are isomorphic. If the graphs are isomorphic, find their adjacency matrices; otherwise, give an invariant that the graphs do not share. (6 marks)



(A)

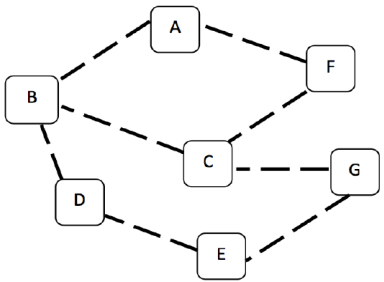


(B)

Euler/Hamiltonian Path, Circuits

Exercise #8

- A tourist plans to visit seven places of attractions in a country, namely A, B, C, D, E, F, and G. These places are connected with inter city railway links. The route across these places is

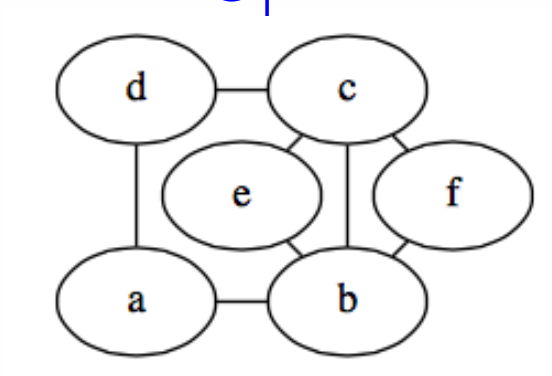


- Trace the journey of the tourist if he wanted to visit every place of attractions (A, B, C, D, E, F, G) exactly once starting and ending at E.
- Determine whether the tourist's journey in 2(d(i)) is an Euler circuit or Hamiltonian circuit
- If the inter city railway link from A to B is close due to maintenance work, plan the tourist's journey starting from A so he/she can still visit all the place of 5 attractions and ends at any of the places following the Euler trail rules

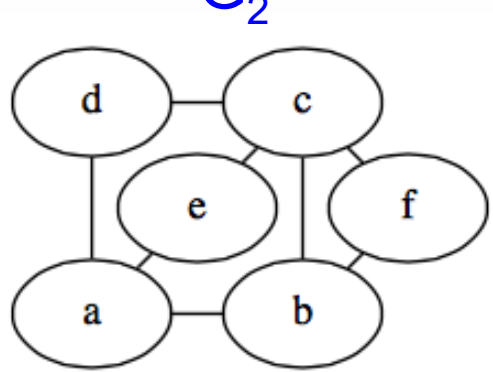
Exercise #9

Q: Which of the following graphs has Euler circuit?
Justify your answer.

G_1

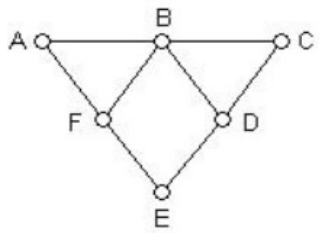


G_2

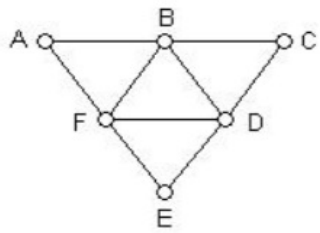


Exercise #10

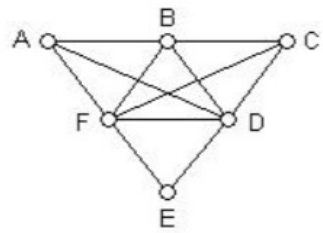
a) Determine whether the following graphs C1, C2 and C3 have an Euler circuit. Construct such circuit if exists. Otherwise, determine whether the graph has an Euler path and construct such a path if one exists.



Graph C1



Graph C2

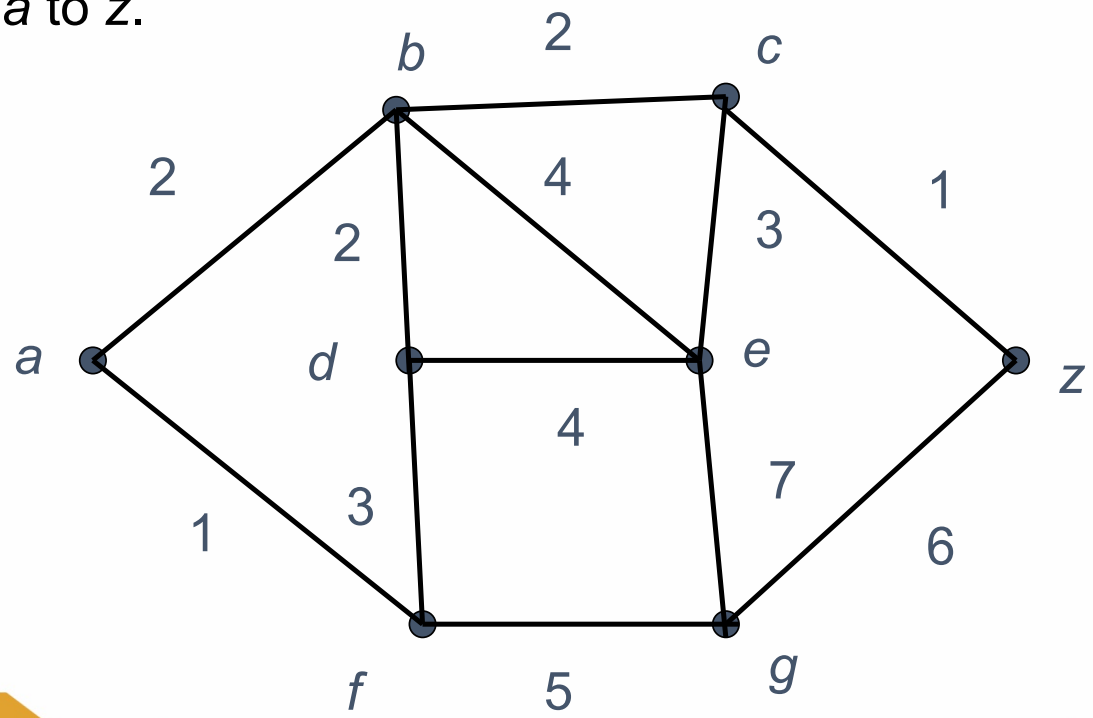


Graph C3

Dijkstra Algorithm

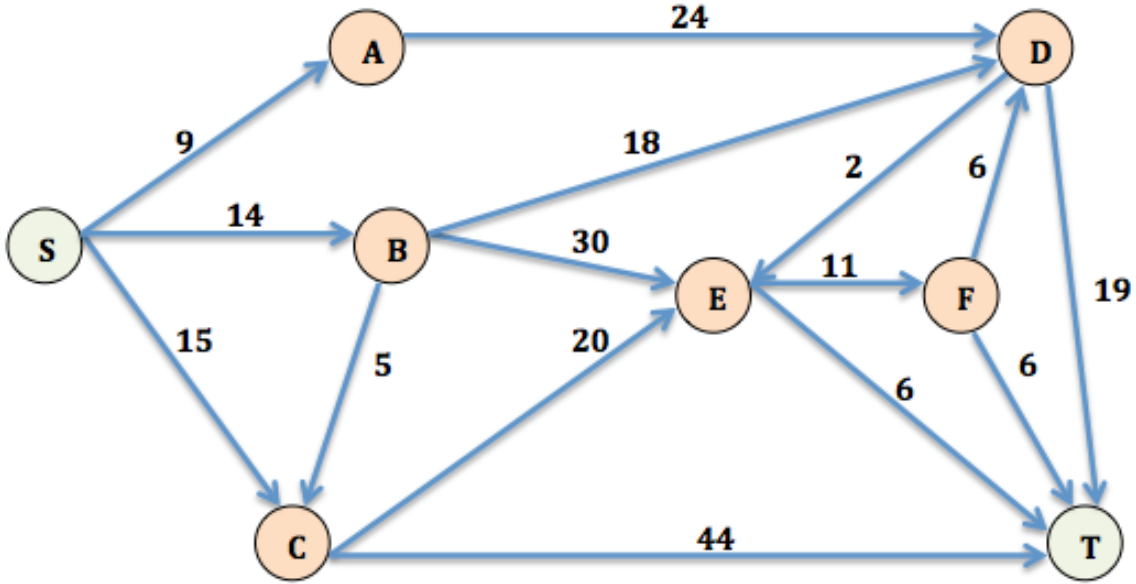
Exercise #11

Use Dijkstra's algorithm to find the length of a shortest path from *a* to *z*.



Exercise #12

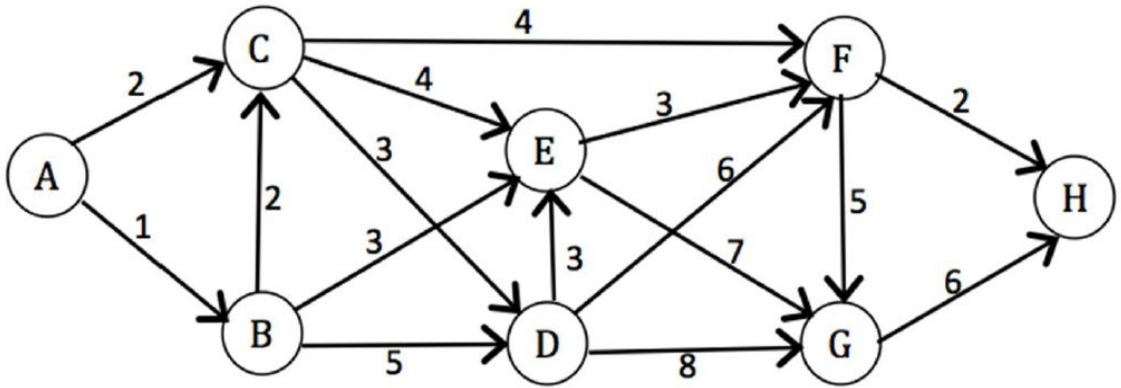
Q: Given a weighted digraph, find the shortest path from S to T, using Djik



Note: Weights are arbitrary numbers (i.e., not necessarily distances).

Exercise #13

The following network gives the distances in miles between pairs of cities A, B, …, and H.



Based on Dijkstra's algorithm, find the shortest path from city A to city H.