4.2 Methods

Two different methods are used for combining fixed-order calculations and EFT calculations.

4.2.1 Method implemented in `FeynHiggs`

The hybrid approach as implemented in **FeynHiggs** was orginally introduced in [19]. It was further developed in [20, 21]. Its main idea is to directly combine the result of the fixed-order calculation, namely the renormalized Higgs self-energies, with the resummed logarithms obtained in EFT approach. This is done by adding the EFT result to the Higgs self-energies. I.e., the renormalized hh self-energy of the full MSSM is replaced by

$$\hat{\Sigma}_{hh}^{\rm MSSM}(p^2) \to \hat{\Sigma}_{hh}^{\rm MSSM}(p^2) + \Delta\hat{\Sigma}_{hh}. \tag{2}$$

Here, $\hat{\Sigma}_{hh}^{\rm MSSM}$ is the renormalized MSSM self-energy calculated using the fixed-order calculation. The quantity $\Delta\hat{\Sigma}_{hh}$ contains the result of the EFT approach, namely the resummed logarithms, as well as subtraction terms. These ensure that logarithms already contained in $\Delta\hat{\Sigma}_{hh}$ are not counted twice,

$$\Delta\hat{\Sigma}_{hh} = -\left[2v^2\lambda(M_t)\right]_{\rm log} - \left[\hat{\Sigma}_{hh}^{\rm MSSM}(m_h^2)\right]_{\rm log}. \tag{3}$$

The sublabel "log" indicates that only logarithmic terms are taken into account. For the quantity $2v^2\lambda(M_t)$ which is obtained only in numerical form, this is achieved by subtracting all non-logarithmic terms which can be calculated easily in an iterative procedure.
In the course of this procedure, it is important to pay attention of the renormalization schemes used in different parts of the calculation. E.g., the fixed-order calculation of `FeynHiggs` by default uses the OS scheme for the renormalization of the stop sector. For the EFT calculation however, $\overline{\rm DR}$ input parameters are needed. Therefore, in this case a parameter conversion is necessary. Since we need only to reproduce the logarithms up to the order of fixed-calculation in the EFT calculation, it is sufficient to perform the conversion between the OS and the $\overline{\rm DR}$ scheme taking into account only one-loop large logarithmic contributions (see [20]). Only in the conversion of the stop mixing parameter such one-loop logarithms appear,

$$X_t^{\overline{\rm DR},\rm EFT} = X_t^{\rm OS}\left[1 + \left(\frac{\alpha_s}{\pi} - \frac{3\alpha_t}{16\pi}(1 - X_t^2/M_S^2)\right)\ln\frac{M_S^2}{M_t^2}\right], \tag{4}$$

where M_S is the stop mass scale. If the optional $\overline{\rm DR}$ renormalization of the fixed-order result of **FeynHiggs** is used (available from version `2.14.0` onward), such a conversion is not needed.

After combining the fixed order and the EFT result, the in this way logarithmically improved self-energy is then used to calculated the Higgs pole mass in the full model. I.e., in the limit of $M_A \gg M_Z$, we have to solve the equation

$$p^2 - m_h^2 + \hat{\Sigma}_{hh}^{\rm MSSM}(p^2) + \Delta\hat{\Sigma}_{hh} = 0. \tag{5}$$

The solution of this equation induces higher order corrections due to the momentum dependence of the self-energy. These higher order terms can contain large logarithms, which are already included in resummed form in the EFT correction. Therefore, it is important to solve the equation only up to the order at which the fixed-order self-energy is calculated (see [21] for more details).

This hybrid approach is so far restricted to the MSSM. To extend it to other models, the first step would be perform a fixed-order calculation and an EFT calculation for the extended model. Afterwards, new subtraction terms would have to be calculated. First steps into the direction of automatizing fixed-order calculations have been taken (see [HB: SARAH refs]). EFT calculations are however not automatized so far. Therefore, an automatization of the **FeynHiggs** hybrid approach is not feasible at the moment.