1.整个方法分2部分：向前的轨迹生成，即计算车头朝前从车位开出到路边的轨迹；向后的轨迹跟踪，即自车沿着生成的轨迹倒车入车位。本文中车的运动都是倒车。
2.轨迹也分为2部分：locating segment是圆弧，规划车开出车位；entering segment是Bezier曲线，规划车向前到start point。
3.适用于平行停入和垂直停入场景。有仿真，也有实车测试。
4.可以输出连续的steering angle，车一次操作，无需停顿后打方向盘。
5.匀速运动，仿真中0.1m/s，测试中没提。
6.和自行车模型的不同在于考虑了车辆重心的slip angle。
7.计算圆弧时，同样以前方碰撞和侧方碰撞作为边界条件。
8.没提动静态障碍物避障。
9.没提圆弧的结束点怎么确定？Bezier曲线方程又怎么确定？switch point又依据什么来确定？

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A Trajectory Planning Method Based on Forward Path Generation and Backward Tracking Algorithm for Automatic Parking Systems

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***Abstract*—This paper proposes a planning method based on forward path generation and backward tracking algorithm for Automatic Parking Systems, especially suitable for backward parking situations. The algorithm is based on the steering property that backward moving trajectory coincides with the forward moving trajectory for the identical steering angle. The basic path planning is divided into two segments: a collision-free locating segment and an entering segment that considers the continuous steering angles for connecting the two paths. MATLAB simulations were conducted, along with experiments involving parallel and perpendicular situations.**

I. Introduction

Parking is a very difficult task even for a skilled driver because of limited space, incoming vehicles, and fixed and moving obstacles such as pedestrians. Therefore, the development of parking assistant systems and autonomous parking systems have become important issues. Furthermore, an autonomous vehicle must perform numerous tasks to safely park the vehicle in a narrow space, including precise environment detection and parking maneuvers. Generally, autonomous parking systems consist of three steps: detection of the free parking space, path planning, and controlling the steering input and velocity of a vehicle, including accelerating or braking to track the desired trajectory [1-2].

The detection of the free parking space involves sensing and calculating the surrounding environment before the path generation to park the vehicle. Various vision methods have been proposed to find free parking spaces. Some have recognized a vacant space between adjacent vehicles using the three-dimensional (3D) structure of a parking lot [3-6]. This is the most widely used approach since adjacent vehicles can be easily recognized by various range-finding sensors [3-4]. Some detect a parking space based on markings on the road surface [5]. A parking space can also be recognized using a bird's eye-view edge image [6]. However, in this paper, we assume that the surrounding spatial information about parking spaces has already been acquired. Thus, we limit the research to the path planning and path tracking.

The Ackerman steering model, which uses the geometric relationship of the vehicle model, is the one of the most popular path tracking methods. However, it may cause a steering angle error when a turning radius is not sufficiently larger than the longitudinal length of the vehicle, because of the Ackerman's approximation in the model [7-9].

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Path planning can also be used for parking in a narrow space. Some studies on automatic parking have used a double circular approach, two-step approach, etc [10-15].

A double circular trajectory, by connecting two circular parts together, is often used for parallel and perpendicular parking [12-13]. However, there is an abrupt change of rotation at the rendezvous of two circular path because the direction of two circles is opposite, as shown in figure 1.

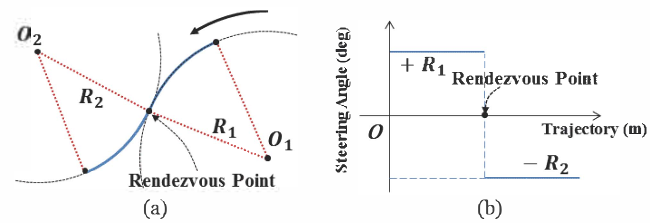

Fig. 1 An illustration of abrupt steering angle change at the rendezvous of two circles in a case of parallel parking maneuver.

The two-step approach to a continuous curvature was proposed in the prior-arts [14-15]. In the first step, a collision-free path is generated without considering the kinematic restrictions on the movement of the car. Then, a collision-free path is approximated to a feasible path. This approach overcomes problem of abrupt angle change at the connecting point. However, the determination of the starting point for a parking maneuver is still difficult.

The purpose of this paper is to provide a method for an autonomous parking system that determines the starting point based on the forward path generation and a backward tracking algorithm. It can consider a minimum distance required to avoid collision with obstacles and space limit as well as other kinematic constraints in the path planning. Our goal is providing a path generation algorithm which can generate a parking path from a desired starting point and heading angle to the destination with referred heading angle in the parking lot. To implement proposed method, a backward tracking model of the lateral motion is developed using a bicycle kinematic model, which considers the slip angle from the vehicle's center of gravity. The proposed tracking model has the advantage of being able to express backward parking trajectory by a forward path of heading-out from parking lot to the desired position in the road. In addition, forward path planning is implemented by dividing the path into two segments: a locating segment and an entering segment. The arc in the locating segment and the Bézier curve [16] in the entering segment are connected together. The arc can consider the minimum distance between the front corner of a vehicle and a static obstacle using the geometric model. The proposed method can be applied to both parallel parking and perpendicular parking. The steering input angle can be

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extracted from these two paths using the backward tracking model. This paper is organized into five sections. Section II describes the outline of the proposed algorithm for an autonomous parking system. The path tracking method, including forward and backward tracking based on the geometric relationship, is described in section III. In section IV, the path planning is explained in detail. Simulation and experimental results are presented in section V. Finally, the conclusions and future work are provided in section VI.

II. Outline Of Proposed Parking Algorithm

We propose a parking algorithm that can determine the maneuvering zone for an autonomous parking system based on the forward path generation and backward tracking.

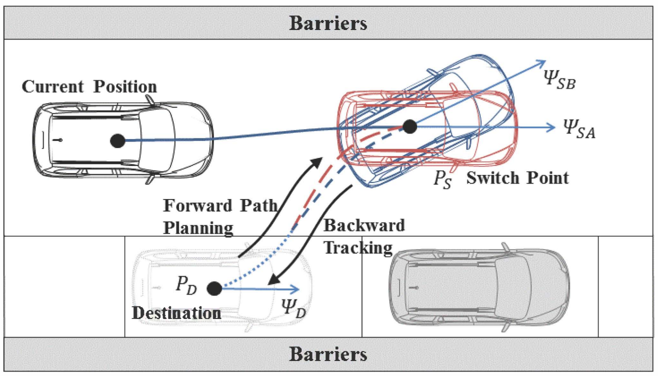

Fig. 2 An Illustration of the proposed path planning algorithm applied to parallel parking maneuver.

starting point 就是图中的switch point

A forward path is generated to escape from the destination parking point, heading-out to the starting point. First, the position and heading angle at the final destination are determined. Then, the switch point and heading angle of the parking maneuvers are determined, applying the forward path planning algorithm used in the steering control of autonomous vehicles [11]. If the parking space is limited, we must apply an algorithm to calculate the minimum turning radius [11-12]. The vehicle moves from the current position to the switch point (P_S) with heading angle Ψ_S and parks at the destination (P_D) with heading angle Ψ_D by backward tracking. Figure 2 shows an example of the proposed algorithm used for a parallel parking maneuver.

The forward path from the destination (P_D) to starting point (P_S) is equivalent to the backward tracking from the starting point (P_S) to destination (P_D). Furthermore, our path planning has the advantage of being able to designate the starting point with a certain heading angle considering obstacles and other kinematic constraints.

III. Kinematic Model For Steering Control

Many studies have dealt with the Ackerman steering model for path tracking in forward driving. If the front-wheel angle δ_f is small, and the front wheels drive the vehicle, the front wheel angle can be approximated as follows [9]:

$$\tan\delta_f \cong \frac{L_f + L_r}{R_{Acm}} \tag{1}$$

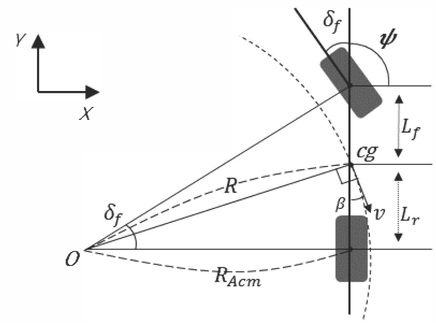

Fig. 3 Modified bicycle kinematic model for backward moving.

TABLE I
Vehicle Variables And Parameters

Symbol	Definition
cg	Center of gravity of the vehicle
x, y	Coordinates of cg location
$L_f(L_r)$	Distance of front (rear) axle from cg
v	Velocity of vehicle
Ψ	Yaw angle
β	Slip angle with respect to the vehicle's longitudinal axis
δ_f	Front wheel angle
R_{Acm}	Turning radius of Ackerman steering model
R	Turning radius at cg

Eq. (1) is obtained from the approximation of the tangent function. Thus, it is valid when the turning radius R is sufficiently large compared to the longitudinal wheel base of the vehicle ($L_f + L_r$). When we are calculating the minimum turning radius under the condition of limited parking space, there may be an error when calculating the correct value of δ_f because the minimum turning radius is not sufficiently larger than the longitudinal length of the vehicle. In this case, the path tracking precision can decline because of the approximation of the front wheel angle if the vehicle has a long wheelbase or wide turning angle. To compensate for this weakness, the slip angle β from cg is considered in this work [17]. In addition, we have modified bicycle kinematic model by including backward driving mode, as shown in figure 3.

Assuming the rear wheels cannot be turned, the slip angle β can be obtained as a function of the steering angle δ_f as follows:

$$\tan\beta = \frac{L_f}{L_f + L_r}\tan\delta_f \tag{2}$$

The vehicle's turning radius R is taken from the bicycle kinematic model as follows:

$$R = \frac{L_f + L_r}{\tan\delta_f \cos\beta} = \frac{L_f}{\sin\beta} \tag{3}$$

Based on Eq. (2) and Eq. (3), the steering angle δ_f is calculated as a function of the turning radius R as follows:

$$\delta_f = \tan^{-1}\left\{\frac{L_f + L_r}{L_r}\cdot\tan\left(\sin^{-1}(L_r\cdot\frac{1}{R})\right)\right\} \tag{4}$$

Therefore, the steering angle input to track the path can be derived from the turning radius of the desired path.

The backward vehicle motion $(\dot{x}, \dot{y}, \dot{\Psi})$ is represented as a function of slip angle β as follows:

$$\dot{x} = \frac{dx}{dt} = -|V(t)|\, cos(\beta(t) + \Psi(t)) \tag{5}$$

$$\dot{y} = \frac{dy}{dt} = -|V(t)|\, sin(\beta(t) + \Psi(t)) \tag{6}$$

$$\dot{\Psi} = \frac{-|V(t)|}{L_r} sin\, \beta(t) \tag{7}$$

The trajectory of the vehicle from the initial point at t_0 to destination point t in backward moving situations $T_{backward\_x}(t)$, $T_{backward\_y}(t)$ is derived using integral forms of Eq. (5) and Eq. (6) as follows:

$$T_{backward\_x}(t) = \int_{t_0}^{t} -|V(\tau)|\, cos(\beta(\tau) + \Psi(\tau))\, d\tau \tag{8}$$

$$T_{backward\_y}(t) = \int_{t_0}^{t} -|V(\tau)|\, sin(\beta(\tau) + \Psi(\tau))\, d\tau \tag{9}$$

If an integration interval $[t_0, t]$ is reversed, it is equivalent to the trajectory of the vehicle during forward driving $T_{forward\_x}(t)$, $T_{forward\_y}(t)$ with interval $[t,\ t_0]$, as shown below:

$$T_{forward\_x}(t) = \int_{t}^{t_0} |V(\tau)|\, cos(\beta(\tau) + \Psi(\tau))\, d\tau \tag{10}$$

$$T_{forward\_y}(t) = \int_{t}^{t_0} |V(\tau)|\, sin(\beta(\tau) + \Psi(\tau))\, d\tau \tag{11}$$

Therefore, when the direction of the vehicle's velocity $V(t)$ and the values of the slip angles *β(t)* are reversed for the time interval, the backward trajectory from the destination to the starting point is equal to the forward trajectory from the starting point to the destination.

IV. Forward Path Planning

1) Planning of minimum escape path

To park the vehicle in a limited parking space without back-and-forth movement switching during turning maneuvers, we need to compute the minimum distance d between the front corner of the vehicle and static obstacles. Blackburn derived an equation for the geometric relation using the outside of a front wheel as a reference point [18].

$$d \leq \sqrt{(L + K_f)^2 + (r^2 - L^2)^2 - \left(\sqrt{r^2 - L^2} - b\right)^2} - L - K_f \tag{12}$$

where r is the turning radius of the car's outside front wheel, $L(= L_r + L_f)$ is the wheel base length, K_f is the distance between the corner of the car and the front wheel, and b is the height of a front obstacle.

However, the accuracy is reduced by the approximation of the front wheel angle if the vehicle has a long wheel base or wide turning angle. To solve the shortcoming we have considered the position of cg instead of outside of a front wheel as a reference point, and we have derived both the minimum distance and the turning radius to steer the vehicle.

A) Parallel parking

To express the movement of the vehicle's cg, we will derive the distance d between the front corner of the car, point A, and a static obstacle with height of b, as shown in figure 4. To avoid a collision with b, A point of the car must travel along inside part of the circular path.

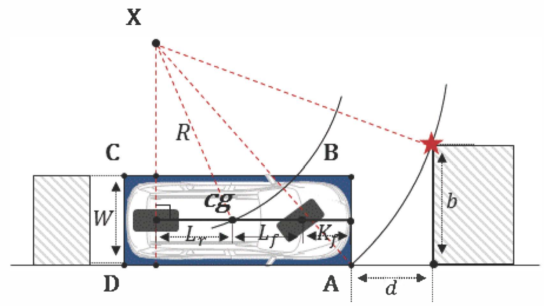

Fig. 4 An illustration of circular geometric model of cg reference for parallel parking.

We can measure the following distances: the distance between the rear (front) axle and cg $L_r(L_f)$, the distance between the corner of the car and a front wheel K_f, the height of a front obstacle b, and the width of the car W.

The value of d is derived as a parameter of minimum turning radius, R;

$$d \leq \sqrt{(L + K_f)^2 + \left(\sqrt{R^2 - L_r^2} + \frac{W}{2}\right)^2 - \left(\sqrt{R^2 - L_r{}^2} + \frac{W}{2} - b\right)^2} - L - K_f \tag{13}$$

Eq. (13) can be rearranged to calculate the turning radius R as follows:

$$R = \sqrt{L_r^2 + \left(\frac{d(d + 2L + 2K_f) - b \cdot W + b^2}{2b}\right)^2} \tag{14}$$

Therefore, if d and b are detected using range-senors, we can obtain the turning radius avoiding collisions in parallel parking situations for a rerestricted parking space.

B) Perpendicular parking

In the case of perpendicular parking, we must consider front as well as side collision possibility. If the vehicle turns to the left to escape the parking space, there is the possibility of the first collision between the side of the vehicle and a parked car. Another collision could be occured with a barrier and the left front end of thevehicle. To avoid these collisions, the vehicle must move straight forward first, and then turns to the left or right with a safe turning radius. To satisfy this situation, the boundary of the turning radius for the first collision R_{fc} and the boundary of the turning radius for the second collision R_{sc} will be derived as functions of the step forward distance s.

We can measure the vehicle's constants as mentioned in the previous section and sense the distance to the next parking space b_1, the length of the obstacle to the side b_2, and the distance to the front obstacle b_3.

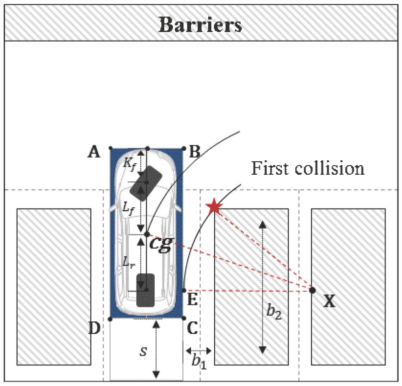

Fig. 5 First collision of circular geometric model for *cg* reference for vehicle during perpendicular parking.

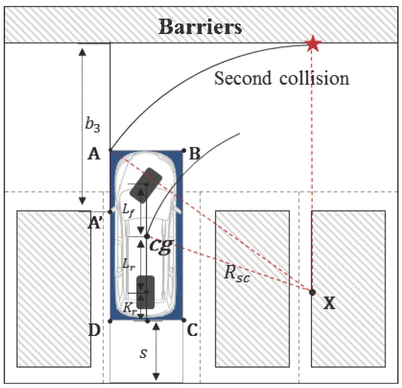

Fig. 6 Second collision of circular geometric model for *cg* reference for vehicle during perpendicular parking.

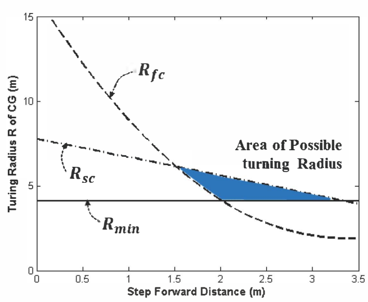

Fig. 7 Area of possible turning radius R during perpendicular parking.

The boundary value of turning radius R_{fc} and R_{sc} for the vehicle's cg can be simply derived each including the parameter of the step forward distance, s.

$$R_{fc} \geq \sqrt{\left(\frac{(s-b_2+K_r)^2+\ b_1^2+b_1\cdot W}{2b_1}\right)^2+L_r^2} \quad (15)$$

$$R_{sc} \leq \sqrt{\left(\sqrt{(s-b_3)(s-b_3-2K_f-2L)}-\frac{W}{2}\right)^2+L_r^2} \quad (16)$$

The area of possible turning radius R is shown in figure 7 as a function of step forward distance s. We list the vehicle's constants used for experiment in TABLE II, and set the default value of spatial parameters $b_1 = 0.4\,m$, $b_2 = 4.4\,m$, and $b_3 = 6\,m$ by reflecting regulation to specify dimension of the parking lot

2) *Path generation using Bezier curves*

A Bézier curve has been widely used as a parametric curve in path planning. This section will introduce the Bézier curve and its advantages. In a two-dimensional XY coordinate system, a Bézier curve of degree n can be represented using the following two equations with a parameter, τ [19]:

$$f_x(\tau) = \sum_{i=0}^{n}\binom{n}{i}(1-\tau)^{n-i}\,\tau^i P_{xi}\ (where\ 0 \leq \tau \leq 1) \quad (21)$$

$$f_y(\tau) = \sum_{i=0}^{n}\binom{n}{i}(1-\tau)^{n-i}\,\tau^i P_{yi}\ (where\ 0 \leq \tau \leq 1) \quad (22)$$

If three consecutive control points (P_{n-2}, P_{n-1}, P_n) are aligned in a straight line, zero curvature is obtained at the end point P_n. This is an advantage when designating a vehicle's heading angle in a lateral movement at the end point of desired path[16]. To obtain smooth input of the steering angles,, we need to connect the arc and Bézier curve.

3) *Forward path and backward tracking*

Dividing Eq. (6) by Eq. (5), the tangent of the vehicle motion is the sum of the vehicle heading angle and slip angle.

$$\frac{dy}{dx} = tan(\beta(t)+\Psi(t)) \quad (23)$$

Without reaching the expected heading angles for the two paths, as when reaching the sum of the vehicle's heading angle and the slip angle between the arc's end point and the Bézier curve's starting point, the two paths can be joined with smooth steering angles. The steering angles extracted from these two paths are reversed to park the vehicle for backward driving. Figures 8 and 9 show examples of the combined paths for parallel parking and perpendicular parking, respectively.

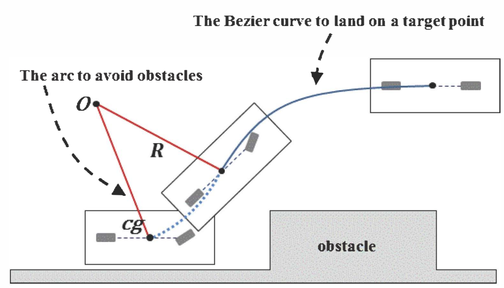

Fig. 8 An example of combined path for parallel parking.

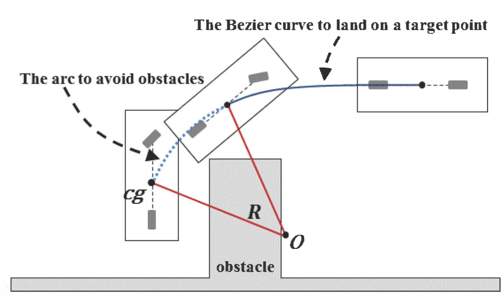

Fig. 9 An example of combined path for perpendicular parking.

V. Simulation, Experimental Results and Discussions

The backward steering angle is extracted to control the vehicle for parallel and perpendicular parking. To evaluate the proposed backward path planning and tracking model, MATLAB simulations were conducted.

The vehicle driving trajectory was represented by a white box that was scaled using the actual size of the test vehicle, a Sportage from Hyundai–Kia Motors. The *cg* of the vehicle was assumed to be in the middle of the wheel base. The velocity $V(t)$ is assumed to constant value 0.1 m/s. The vehicle parameters are listed in TABLE II.

TABLE II
Specification of the test vehicle.

Variable	Value
Overall length	4.32 (m)
Overall width	1.85 (m)
Wheel base	2.64 (m)
Maximum steering angle	34 (°)

TABLE III
National regulation of parking spacein Korea .

	Width	Length	Passing lane width
Parallel	2 (m)	6 (m)	3 (m)
Perpendicular	2.3 (m)	5 (m)	6 (m)

The basic environment for a parking space followed the parking lot enforcement regulations of South Korea for parallel parking and perpendicular parking, as listed in TABLE III [20].

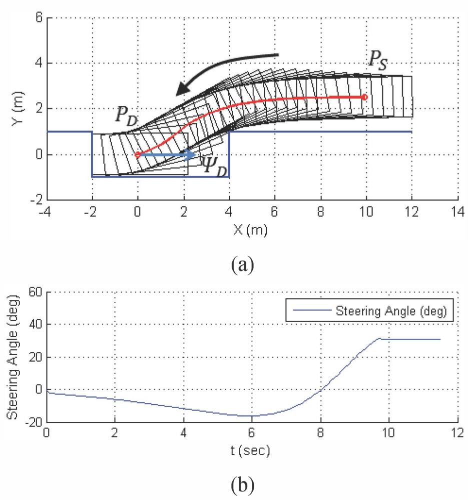

Fig. 10 MATLAB simulations of parallel parking. Generated path and simulated parking trajectory by backward tracking (a). Steering input angle of the backward parking (b).

1) Simulations

Figure 10 shows the simulated trajectory and the vehicle's heading and slip angle in a parallel parking situation. The steering angle has a relatively smooth transition at the joining point between the arc and the Bézier curve. This can be comfortable and suitable for passengers, without the need to stop while driving to turn the steering wheel at the joining point. Figure 11 shows the simulated trajectory and the vehicle's heading and slip angle in a perpendicular parking situation. A smooth transition is also shown for the steering angle in a perpendicular situation at the joining point between the arc and the Bézier curve. This can be comfortable and suitable for passengers, without the need to stop while driving to turn the steering wheel at the joining point.

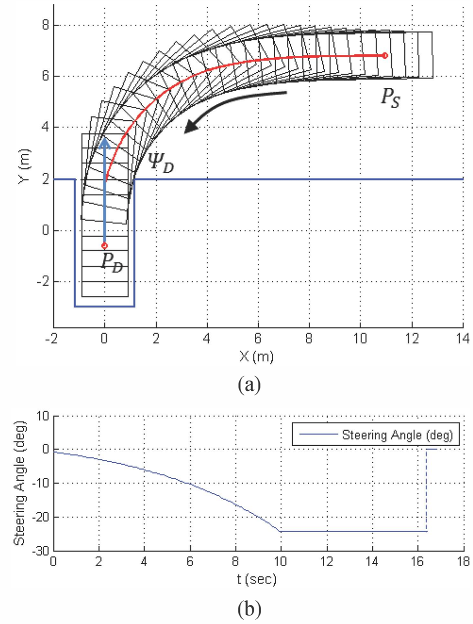

Fig. 11 MATLAB simulations of perpendicular parking. Generated path and simulated parking trajectory by backward tracking (a). Steering input angle of the backward parking (b).

2) Experiments

To evaluate the proposed parking algorithm, we performed experiments using the testvehicle of Sportage from Hyundai–Kia Motors. This vehiclehas a diesel engine, four-wheel drive, and motor-driven power steering (MDPS), which is an autonomous vehicle equipped with a controller and various range-senors. Vehicle's length and width are 4.45 m and 1.85 m with a maximum steering angle of 34°. DGPS sensors with resolution approximately 2 cm mounted on the roof rack was used to measure the heading angle, speed, and the position during the experimental parking maneuver. In the experiments, firstly forward path from destination in the parking lot to the current vehicle position was generated, then the vehicle moved backward direction to track the trajectory. Fig. 12 shows measured trajectory of the generated forward path and corresponding backward tracking path for a parallel parking maneuver.

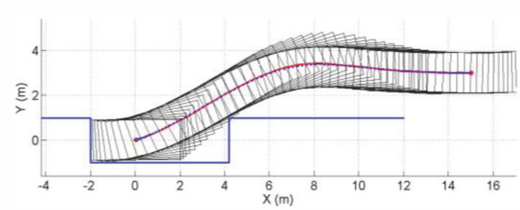

Fig. 12 Measured trace of generated forward path and corresponding backward tracking trajectoryfor parallel parking maneuver.

Fig. 13. Inside view of autonomous vehicle with main computer and motor-driven power steering.

Fig. 14 Snap shot pictures showing Experiment for parallel parking maneuver.

3) Discussions

The forward path generation by combining an arc to escape from the minimum space and a Bezier curve to move desired position with heading angle is proposed forautonomous parking applications. There are two segments divided into a locating segment and an entering segment. The locating path is an arc between the front corner of a vehicle and a obstacle, and is used to escape a parking space. The entering path is a Bezier curve from the ending point of the arc to a starting point, which has the advantage of continuous steering angles between the two paths. MATLAB simulations and experiments using the test vehicle were conducted to verify proposed forward path generation and backward tracking algorithm for automatic parking systems.

In the simulation and experiments, firstly, the forward path was generated to escape from the destination parking point, heading-out to the starting point. Secondly, an autonomous parking maneuver is performed by tracking the forward path backward from the starting point to the destination. The main advantage of the proposed method is that we can determine the maneuvering zone regardless of considering the current position of the vehicle.

In parallel parking, considering a minimum parking space width of 2 m and a length of 6 m, the minimum steering angle of the arc was calculated to be 31°. In perpendicular parking, to avoid a collision with a car to the side and the curb on the other side of the lane after moving forward 1.94 m, the minimum steering angle of the arc was found to be 24° with constraints that included a minimum aisle width of 6 m and parking space width of 2.3 m in simulations.

VI. CONCLUSIONS

We proposes a parking algorithm applicable to an autonomous parking system. The proposed method for an autonomous parking system determines the starting point based on the forward path generation and a backward tracking algorithm. The experimental results show that the proposed path planning is applicable to automated parking in a narrow space without back-and-forth movement switching.

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