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Transportation Research Part B 40 (2006) 251–264

TRANSPORTATION RESEARCH PART B

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Lane-changing in traffic streams

Jorge A. Laval [a], Carlos F. Daganzo [b,*]

[a] Institute of Transportation Studies, University of California, Berkeley, CA 94720, United States
[b] Department of Civil and Environmental Engineering, Transportation Group, University of California, Berkeley, CA, United States

Received 18 November 2004; received in revised form 7 April 2005; accepted 11 April 2005

Abstract

It is postulated that lane-changing vehicles create voids in traffic streams, and that these voids reduce flow. This mechanism is described with a model that tracks lane changers precisely, as particles endowed with realistic mechanical properties. The model has four easy-to-measure parameters and reproduces without re-calibration two bottleneck phenomena previously thought to be unrelated: (i) the drop in the discharge rate of freeway bottlenecks when congestion begins, and (ii) the relation between the speed of a moving bottleneck and its capacity.

1. Introduction

Freeway lane-changing has received increased scientific attention during the last decade. Research has produced both qualitative conjectures (Brackstone et al., 1998; Chang and Kao, 1991; Chowdhury et al., 1997; Wei et al., 2000; Greenberg et al., 2003) and empirical evidence (Mauch and Cassidy, 2002; Ahn, 2005; Cassidy and Rudjanakanoknad, 2005; Cassidy and Bertini, 1999), but a quantitative understanding of its impacts on traffic flow remains elusive.

* Corresponding author. Tel.: +1 510 642 3853; fax: +1 510 643 3955.
E-mail address: daganzo@ce.berkeley.edu (C.F. Daganzo).

0191-2615/$ - see front matter
doi:10.1016/j.trb.2005.04.003

This paper attempts to fill this gap. It considers freeway sections away from diverges, where the main incentive for drivers to change lanes is increasing their speed.[1] The main thesis is that a lane-changing vehicle acts as a moving bottleneck on its destination lane while accelerating to the speed prevailing on the lane, and that the ensuing disruption can trigger other lane changes. The freeway is therefore modeled as a set of interacting streams linked by the lane changes. The proposed model only needs one more parameter than the simplest traffic flow models (which require three) and explains several puzzling phenomena without re-calibration.

Existing traffic flow models do not address lane-changing phenomena properly. Extensions to kinematic wave (KW) theory (Lighthill and Whitham, 1955; Richards, 1956) with lane-changing (Munjal and Pipes, 1971; Munjal et al., 1971; Michalopoulos et al., 1984; Daganzo, 1997; Daganzo et al., 1997; Daganzo, 2002a,b) are inadequate because they treat lane-changing vehicles as a fluid that can accelerate instantaneously, and therefore do not sufficiently hinder following vehicles. Some microscopic simulation models consider realistic accelerations, but they have not yet been used successfully to model lane changes. Recent experience with these models (Buisson and Wagner, 2004) indicates that lane-changing greatly increases the complexity of the model specification and estimation process.

To overcome these problems, a hybrid approach was developed in Laval and Daganzo (2003). It combines the best features of microscopic and macroscopic models: the parsimony of the KW model (for the traffic stream) with the accuracy of microscopic models (for slower vehicles). Slow vehicles are treated in this reference as moving bottlenecks in a single KW stream, as in the KW theory of moving bottlenecks (Newell, 1993, 1998; Lebacque et al., 1998; Muñoz and Daganzo, 2002). Unfortunately, this requires as an input the maximum possible passing rates, which can only be guessed approximately.

This drawback is overcome in this paper by modeling each lane as a separate KW stream interrupted by lane-changing particles that completely block traffic; i.e., that allow no passing on the lane they occupy. The incremental-transfer (IT) principle for multilane KW problems (Daganzo et al., 1997), coupled with a one-parameter model for lane-changing demand, is used to predict the flow transfers between neighboring lanes. The appendix describes the constrained-motion (CM) model of Laval and Daganzo (2003), which can capture a vehicle's ability to accelerate after changing lanes recognizing both mechanical resistance and the restrictions imposed by downstream traffic. This CM model is used to generate particle trajectories. Section 2, describes the model and Section 3 tests it empirically.

2. The model

The model is presented in two parts: Section 2.1 for the multilane KW module, and Section 2.2 for the lane-changing particles.

[1] Generalizations to more complex geometries are easy to develop with the proposed modeling framework, but they require at least one more parameter to describe driver behavior. They will not be described here because they have not yet been tested.

2.1. The multilane KW module

A continuum multilane extension of the KW model for a highway with $n = 2$ lanes was first presented in Munjal and Pipes (1971), see also Munjal et al. (1971); Michalopoulos et al. (1984). For $n > 2$, the conservation equation for a single lane, ℓ, is

$$\frac{\partial k_\ell}{\partial t} + \frac{\partial q_\ell}{\partial x} = \Phi_\ell, \quad \ell = 1, \ldots, n, \tag{1}$$

where $k_\ell(t,x)$ and $q_\ell(t,x)$ give the density and flow on ℓ at the time-space point (t,x). The inhomogeneous term Φ_ℓ is the *net lane-changing rate onto* lane ℓ, in units of veh/time–distance. It was postulated in Michalopoulos et al. (1984) that Φ_ℓ was a linear function of the set of k_ℓ's, but the idea was not applied successfully at the time because effective numerical methods (Daganzo, 1993, 2005b; Lebacque, 1996; Daganzo et al., 1997) had not yet been developed. The next two subsections show how the ideas in Munjal and Pipes (1971); Munjal et al. (1971) and Michalopoulos et al., 1984) can be refined and then implemented in discrete time.

2.1.1. The continuum formulation

Define the vector $k(t,x) \doteq [k_1(t,x), \ldots, k_n(t,x)]$ and assume that the one-directional *lane-changing rate* from lane ℓ to lane ℓ' (with $\ell \neq \ell'$) is a function, $\Phi_{\ell\ell'}$, of k, t and x. The net lane-changing rates are related to the one-directional rates by

$$\Phi_\ell = \sum_{\ell' \neq \ell} \Phi_{\ell'\ell} - \Phi_{\ell\ell'}. \tag{2}$$

The proposed model specifies the $\Phi_{\ell\ell'}$ instead of the Φ_ℓ and does not require linearity. The $\Phi_{\ell\ell'}$ must realistically represent the competition between drivers' desires for changing lanes, and the available space capacity in the target lane. To strike a balance between these two factors, we first specify three sets of functions of (k,t,x) defining: (i) a desired lane-changing rate from ℓ to ℓ' (i.e., a demand for lane-changing in units of veh/time–distance) $L_{\ell\ell'}, \ell \neq \ell'$, (ii) a desired set of through flows on ℓ, T_ℓ, (in units of veh/time) and (iii) the available capacity on lane ℓ, μ_ℓ (in units of veh/time). Formally,

$$L_{\ell\ell'} = L_{\ell\ell'}(k,t,x), \tag{3a}$$

$$T_\ell = T_\ell(k,t,x), \tag{3b}$$

$$\mu_\ell = \mu_\ell(k_\ell,t,x). \tag{3c}$$

A competition mechanism, $\mathscr{F}$, then determines the actual one-directional lane-changing rates $\Phi_{\ell\ell'}$ and through flows q_ℓ from (3), i.e.

$$(\Phi_{\ell-1,\ell}, q_\ell, \Phi_{\ell+1,\ell}) \doteq \mathscr{F}(L_{\ell-1,\ell}, T_\ell, L_{\ell+1,\ell}, \mu_\ell). \tag{4}$$

The demand functions L and T of (3a) and (3b) are obtained by disaggregating with a choice model the sending (or demand) function of KW theory. The capacity function μ_ℓ is the receiving (or supply) function of KW theory (Daganzo, 1993, 1994; Lebacque, 1996). The transformation $\mathscr{F}$ should reflect sensible priority rules, which depend upon the nature of the lane-changing maneuvers (discretional or mandatory). We will describe $\mathscr{F}$ explicitly in the next section.

2.1.2. The discrete-time formulation

It is assumed here that the fundamental diagram (FD) of each lane is triangular with free-flow speed u, wave speed $-w$ and jam density κ. (This accounts for three of the four model parameters.) All lanes are partitioned into small cells of length Δx and time is discretized into steps of duration Δt; see Fig. 1. The following three-dimensional grid is used ($t_j \doteq j\Delta t, x_i \doteq i\Delta x, \ell$). For numerical stability it is assumed that

$$\Delta x \doteq u\Delta t. \tag{5}$$

Indices i and j will be used to denote the calculated values of a variable at a discrete point (t_j, x_i); e.g., $k^j_{i\ell}$ will denote the discrete approximation of $k_\ell(t_j, x_i)$. In the discrete world, the conservation equation becomes

$$\frac{k^{j+1}_{i\ell} - k^j_{i\ell}}{\Delta t} + \frac{q^j_{i\ell} - q^j_{i-1,\ell}}{\Delta x} = \sum_{\ell' \neq \ell} \Phi^j_{i-1,\ell'\ell} - \Phi^j_{i\ell\ell'}, \quad \forall \ell. \tag{6}$$

This equation is ready for stepping through time, since there is only one term with time index $j+1$. At each iteration one calculates $L^j_{i\ell\ell'}$, $T^j_{i\ell}$ and $\mu^j_{i\ell}$ for every cell (i,ℓ) with (3) using the current densities k^j as arguments. One then computes the lane-changing rates and through flows $q^j_{i\ell}$, $q^j_{i-1,\ell}$, $\Phi^j_{i-1,\ell'\ell}$ and $\Phi^j_{i\ell\ell'}$ with the IT principle (4) and then evaluates $k^{j+1}_{i\ell}$ with (6). We now explain how to compute these rates with the IT principle.

2.1.2.1. Recipes for L, T and μ. As a first step we specify the arguments of the IT recipe: $L^j_{i\ell\ell'}$, $T^j_{i\ell}$ and $\mu^j_{i\ell}$. Recall that the sending function for triangular FDs gives the desired aggregate number of advancing moves in Δt; i.e.,

$$S^j_{i\ell} \doteq \Delta t \min\{uk^j_{i\ell}, Q\}. \tag{7}$$

Recipe for L: The desired number of lane-changing moves in Δt is given by

$$L^j_{i\ell\ell'}\Delta t\Delta x \doteq \pi^j_{i\ell\ell'}\Delta t S^j_{i\ell}, \quad \forall \ell, \forall\, \ell' \neq \ell, \tag{8}$$

where $\pi^j_{i\ell\ell'}$ is the fraction of choice-makers per unit time wishing to change from lane ℓ to lane ℓ'. We assume for maximum simplicity that the choice probability rate $\pi^j_{i\ell\ell'}$ is proportional to the

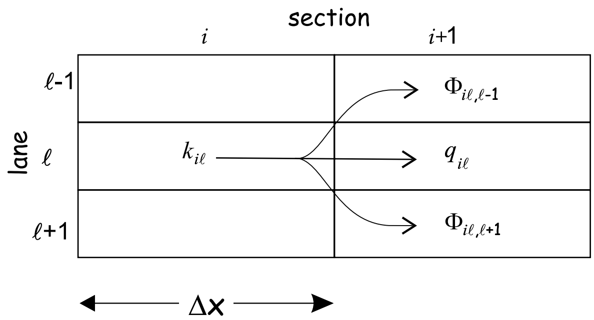

Fig. 1. Discretized freeway representation.

positive speed difference between lanes, $\Delta v^j_{i\ell\ell'} \doteq \max\{0, v^j_{i\ell'} - v^j_{i\ell}\}$, where $v^j_{i\ell}$ is the average speed on lane ℓ; i.e.,[2]

$$\pi^j_{i\ell\ell'} \doteq \frac{\Delta v^j_{i\ell\ell'}}{u\tau}, \quad \forall \ell,\ \forall \ell' \neq \ell, \tag{9}$$

where the parameter τ has units of time. Notice that τ^{-1} is an upper bound to $\pi^j_{i\ell\ell'}$; as such, τ can be interpreted as the time a driver takes to decide and execute a lane change when the origin lane is stopped and the target lane is freely flowing.

Recipe for T: It follows from the definition of $\pi^j_{i\ell\ell'}$ that the probability of staying in the same lane in the next Δt is $1 - \sum_{\ell' \neq \ell} \pi_{i\ell\ell'} \Delta t$, which will be in $(0,1]$ if we choose $\Delta t \ll \tau$. Thus, the desired number of through moves in the next Δt is

$$T^j_{i\ell} \Delta t \doteq \left(1 - \sum_{\ell' \neq \ell} \pi^j_{i\ell\ell'} \Delta t \right) S^j_{i\ell} \quad \forall \ell. \tag{10}$$

Recipe for μ: The available capacity is given by the receiving function for a triangular FD

$$\mu^j_{i\ell} \Delta t \doteq \Delta t \min\{w(\kappa - k^j_{i\ell}), Q\} \quad \forall \ell. \tag{11}$$

2.1.2.2. IT principle. We now show how the IT principle in Daganzo et al. (1997) transforms the values of $L^j_{i\ell\ell'}$, $T^j_{i\ell}$ and $\mu^j_{i\ell}$ for every cell into the actual lane-changing rates and through flows. The IT recipe allocates differentials of flow to the desired target cell (i,ℓ) on a first-come-first-served basis. (We now drop the subindexes i and j for clarity). When total demand $T_\ell + \sum_{\ell \neq \ell'} \Delta x L_{\ell'\ell}$ is less than the available capacity μ_ℓ all the demands are fulfilled and able to advance to the target cell; otherwise the IT recipe prorates that available capacity to the different origin lanes according to their demands. It has been shown in Leclercq (2004) that if γ_ℓ represents the fraction of the demand able to advance the IT result reduces to

$$\gamma_\ell \doteq \min\left\{ 1, \frac{\mu_\ell}{T_\ell + \sum_{\ell \neq \ell'} \Delta x L_{\ell'\ell}} \right\}, \tag{12}$$

and the transfers to the target lane ℓ are

$$\Phi_{\ell'\ell} = \gamma_\ell L_{\ell'\ell} \quad \forall \ell' \neq \ell, \tag{13a}$$

$$q_\ell = \gamma_\ell T_\ell. \tag{13b}$$

Similar formulae have been proposed in Lebacque and Lesort (1999) for intersection modeling.

The quantities in (13) can be understood physically by taking the limit $\Delta t, \Delta x \to 0$. For example, consider a steady-state case where total demand exceeds available capacity. Combining (13) and (12) we find

$$\Phi_{\ell'\ell} = \frac{\mu_\ell}{T_\ell + \sum_{\ell \neq \ell'} \Delta x L_{\ell'\ell}} L_{\ell'\ell}. \tag{14}$$

[2] We propose using Eddie's generalized space-mean speed on a look-ahead section downstream of the current position during an evaluation period immediately preceding the current instant. Our tests used negligible evaluation periods and a look-ahead section comparable with the vehicle spacing.

Using (8) and (10) in (14) and after manipulation one obtains

$$\lim_{\Delta x\to 0}\Phi_{\ell'\ell} = \frac{\mu_\ell \pi_{\ell'\ell} S_{\ell'}}{u S_\ell}. \tag{15}$$

This shows that the $\Phi_{\ell'\ell}$'s tend to finite values as the mesh size tends to zero. Similarly, it can be shown that $q_\ell \to \mu_\ell$. In fact, manipulation of (5)–(13) reveals that the discrete model expresses a relationship between meaningful physical variables in the limit of $\Delta t, \Delta x \to 0$. Thus, Δt and Δx are not parameters of the model; they should be simply chosen as small as possible. This is reasonable; it was shown in Laval (2003) that model (5)–(13) is numerically stable and converges to a continuum solution in the dynamic case as the lattice is refined.

This completes the formulation of the discrete algorithm. More details can be found in Laval (2004). Note that Eqs. (5)–(13) have only introduced one additional parameter, τ. The model, however, is not yet sufficiently realistic. As its predecessors (Munjal and Pipes, 1971; Munjal et al., 1971; Michalopoulos et al., 1984), it ignores that the disruption caused by a lane change depends on the initial speed of the lane-changing vehicle and on its ability to accelerate. As a result, it underestimates the disruption caused by the lane changes. The following subsection shows how to remedy this problem.

2.2. Discrete lane-changing particles

The basic idea consists in quantizing the lane-changing rates from model (5)–(13) to generate discrete particles, and then treating them with the method in Daganzo and Laval (2005a) as temporary blockages that move with bounded accelerations. This is possible because the blockages have a known (zero) passing rate and trajectories that can be determined endogenously with the CM model of vehicle dynamics. In the CM model, particles move with maximum acceleration, but are constrained by their own power and the speed of traffic ahead (Laval and Daganzo, 2003). A distinguishing feature of the method is that particles are tracked with very high resolution in continuous space (no jumping).

To quantize the process we can simply evaluate the cumulative number of lane changes from (i,ℓ) to $(i+1,\ell')$ by time t_j, $\eta^j_{i\ell\ell'} \doteq \sum_{j'\leqslant j}\Phi^{j'}_{i\ell\ell'}\Delta t\Delta x$, and then use the "floor" function $\lfloor \eta^j_{i\ell\ell'} \rfloor$ to generate integer jumps. In our tests, we added a degree of randomness of people's choices, by generating particles as outcomes of Poisson variables with mean $\eta^j_{i\ell\ell'} - \eta^{j-1}_{i\ell\ell'}$, but this did not change the macroscopic results.

The complete hybrid model has good estimation and convergence properties. It is parsimonious since it only requires the relaxation time for lane-changing, τ, and the three usual KW parameters (free-flow speed, capacity and jam density), which are readily observed in the field. It continues to converge as $\Delta t \to 0$, since the introduction of discrete moving bottlenecks is akin to the specification of additional boundary conditions, as explained in Daganzo (2005a).

3. Empirical tests and discussion

All the numerical experiments assume that $\tau = 3$ s and the triangular FD on each lane has free-flow speed $u = 96.6$ km/h (60 mph), congested wave speed $w = -24$ km/h (-15 mph) and jam density per lane $\kappa = 93.2$ vpkpl (150 vpmpl). Additionally, all lane-changing vehicles are assumed to have the acceleration capabilities of a typical car. We used $\Delta t = 0.3$ s, but the results are insensitive to values in (0, 0.5) s.

3.1. Lane-drops

Recent experiments (Bertini and Leal, 2003) show a consistent reduction in discharge flow after the onset of congestion at bottlenecks caused by lane-drops. A similar reduction has been observed at merge bottlenecks (Cassidy and Bertini, 1999). This suggests that the drop in discharge rate may be caused by lane changers—since approaching mainline vehicles are forced away from the shoulder lane in both bottleneck configurations. We now explore this conjecture.

The proposed model was applied to a 0.5 km (0.31 miles) 3-lane freeway with a lane-drop at $x = 0.33$ km (0.2 miles); see Fig. 2. At $t = 0$ the freeway is empty and the input demand is held constant at 1.242 vph on the median and middle lanes (lanes 1 and 2, respectively) and 416 vph on the shoulder lane (lane 3). The cumulative count curves across all lanes, $N(t)$, produced by the proposed method for the four locations of Fig. 2 are shown in Fig. 3a. Curves are plotted on an oblique coordinate system with a "background flow" of 2630 vph. They show that flow drops from 2900 vph to 2630 vph around $t = 10$ min. This is a 9.3% drop. The magnitude of the drop varies across simulations and demand patterns but only slightly. The results are consistent with (Bertini and Leal, 2003).

Parts (b) and (c) of Fig. 3 show that the results of the simulation are also in agreement with the findings of ongoing finer-resolution empirical studies at merge bottlenecks (Cassidy and Rudjanakanoknad, 2005). These studies show that the reduction in bottleneck discharge rate occurs simultaneously with sharp increases in both, vehicular accumulation and lane-changing activity upstream of the bottleneck. Part (b) of the figure shows the lane-specific vehicle accumulation predicted by the model for $0.23\text{ km} \leqslant x \leqslant 0.33\text{ km}$; notice how the increase in accumulation (on all lanes) correlates nearly perfectly with the flow drop of part (a). Part (c) shows the cumulative number of lane changes upstream of the bottleneck on an oblique coordinate system with a background rate of 337 lane changes per hour. Notice how in this case too the lane-changing rate increases significantly in conjunction with the breakdown. Note too that the increase is roughly linear and that it stabilizes at a level of about 600 lane changes per hour. Cassidy and Rudjanakanoknad (2005) also found a linearly increasing rate and similar stabilization level.

All in all, the evidence strongly suggests that lane changes are the main cause of the drop in discharge rate, and that the proposed model can roughly capture the effect.

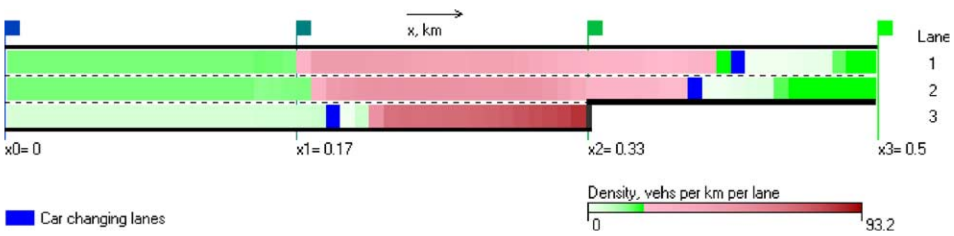

Fig. 2. Simulation of a lane-drop: configuration and snapshot at $t = 11$ min.

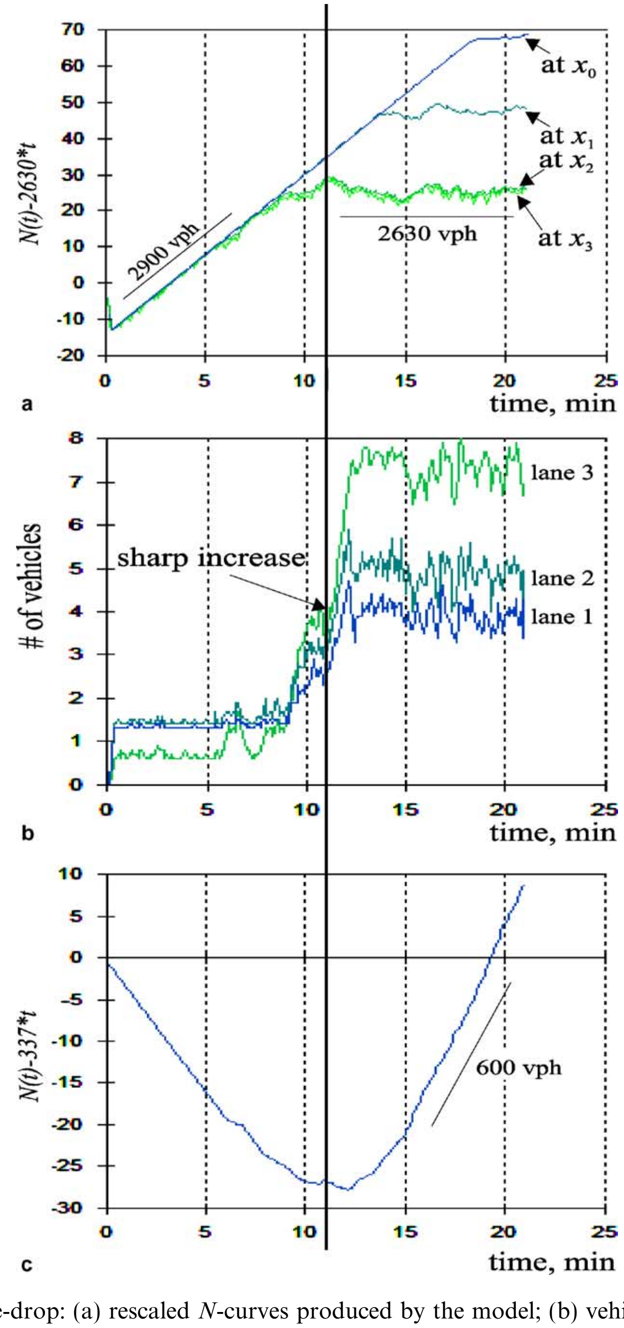

Fig. 3. Simulation of a lane-drop: (a) rescaled N-curves produced by the model; (b) vehicle accumulation on all lanes for 0.23 km $\leqslant x \leqslant$ 0.33 km; (c) cumulative number of lane changes upstream of the bottleneck.

3.2. Moving obstructions

Muñoz and Daganzo (2002) describes the results of several experiments where obstructions with a range of controlled speeds v were introduced in two freeways. It was found that there was a reproducible relation between the capacity of the moving bottleneck, Q_m (the queue discharge rate), and v, i.e.

$$Q_m = Q_m(v), \quad \text{with} \frac{\mathrm{d}Q_m}{\mathrm{d}v} > 0 \text{ for } v > 50 \text{ km/h}. \tag{16}$$

This phenomenon is still not well understood. Existing theories either assume that $\frac{\mathrm{d}Q_m}{\mathrm{d}v} = 0$ (see Newell, 1998; Lebacque et al., 1998; Giorgi et al., 2002), or introduce (16) as an exogenous boundary condition (Laval and Daganzo, 2003). By doing this, however, all the factors that could affect capacity are ignored.

To test the proposed theory we simulated the experiments in Muñoz and Daganzo (2002) for a broader set of conditions: $v \in \{0, \ldots, 80\}$ km/h and $n \in \{2,3,4\}$. Fig. 4 plots the results. It displays the (normalized) capacity, $\rho = Q_m(v)/Q_m(u)$, versus v. The match with empirical data (circles in the figure) is reasonably good, considering that no parameters were re-calibrated.

Curiously, the model predicts two regimes. Regime 1 ($v > 20$ km/h) where $\rho(v)$ increases, and regime 2 ($v < 20$ km/h) where $\rho(v)$ slightly decreases. Although Muñoz and Daganzo (2002) does not include experiments in regime 2, we note from Section 3.1 that the hybrid model predicts $\rho(0)$ reasonably well. Thus, the slight trend reversal is probably real.

The two-regime phenomenon is qualitatively explained by the spatial distribution of the lane changes. Fig. 5 shows the k-maps of one simulation for two bottleneck speeds, $v = 1$ and 32 km/h. Lane-changing locations are depicted as bold dots. Note from the white regions how lane changes reduce the flow downstream of the moving obstruction, as expected. The key observation here is that the closer a maneuver is to the bottleneck the bigger the void in front of the lane-changer. This is because lane-speed differences are greater close to the bottleneck. The reason for the capacity trend in regime 1 (Fig. 5b) is a direct consequence of this effect, because as v increases lane-speed differences drop and lane-changing becomes less and less disruptive.

Note now from Fig. 5a (regime 2) that there is a critical distance from the bottleneck beyond which lane-changing has no effect. Furthermore, for very low speeds a significant number of lane changes occur beyond this critical distance. The trend of regime 2 is then explained, because as

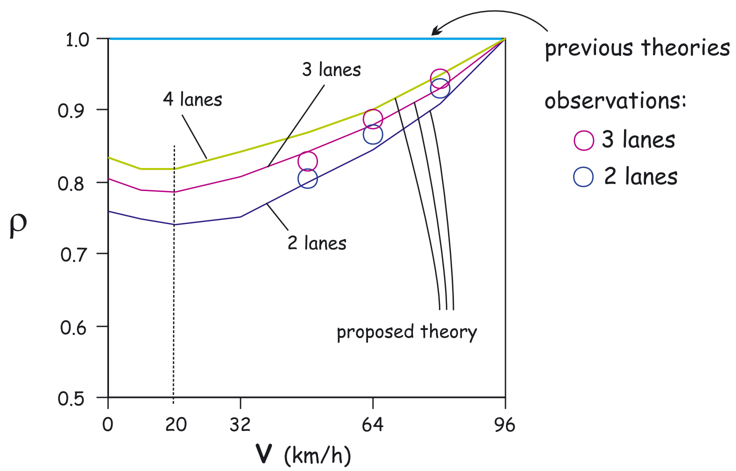

Fig. 4. Dimensionless bottleneck discharge rate, ρ, as a function of the slow vehicle speed v for $n \in \{2,3,4\}$ lanes.

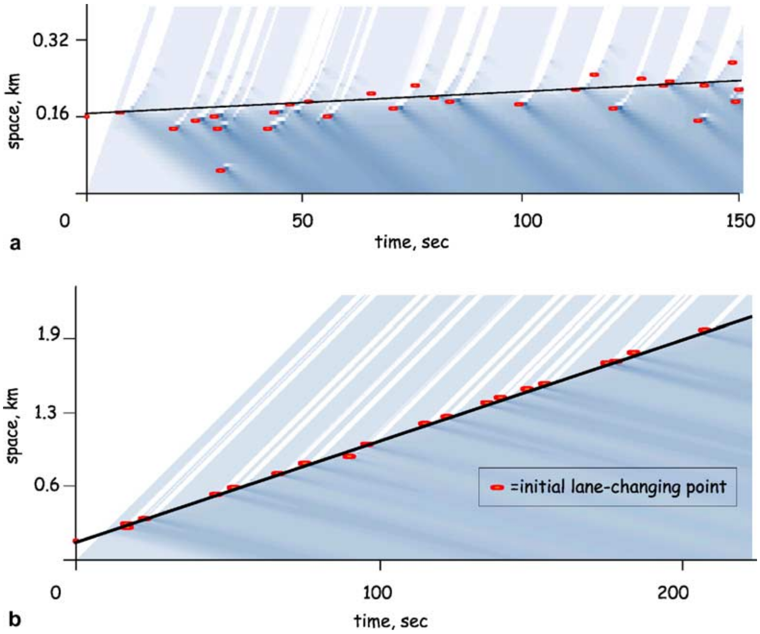

Fig. 5. k-maps for a moving obstruction traveling at (a) $v = 1$ kph and (b) $v = 50$ kph.

$v \to 0$ more and more lane changes take place far upstream from the bottleneck where lane-speeds are similar. We recognize that these results could vary slightly with a more realistic lane-choice model, but they clearly illustrate the deleterious effect of lane changes near a bottleneck.[3]

3.3. Discussion

A four-parameter multilane hybrid model for traffic flow that recognizes the bounded accelerations of lane-changing vehicles has been introduced. The model appears to explain the reduction in flow observed after the onset of congestion at freeway lane-drops and the relationship between the speed of moving bottlenecks and their capacities. The ultimate cause for both phenomena appears to be the limited ability of lane changers to accelerate. (Additional simulations show that both effects disappear when the acceleration parameter of the model is increased.) The more detailed evidence also suggests that lane changes affect bottleneck behavior in ways that can be controlled to improve traffic flow. For example, since the spatial distribution of lane changes and the difference in lane speeds are found to be important determinants of bottleneck capacity, traffic managers may be able to increase capacity by forbidding lane changes and/or posting speed advisories at key locations upstream of bottlenecks; e.g., as in Daganzo et al. (2002).

We obtained virtually identical results after replacing the KW module with the cellular automata (CA) model in Daganzo (2004), using the same three macroscopic parameters. (This reference shows that the vehicle trajectories produced by the CA model in that reference and the KW

[3] A more realistic lane-choice model would recognize drivers' reluctance to change lanes with very high speed differences.

model with triangular FD produce the same vehicle trajectories to within a tolerance of a single jam spacing.) Thus, the predictions of the theory appear to be insensitive to the type of approximation (continuum/discrete) used for the traffic streams. Preliminary research also reveals that the model reproduces other phenomena quite well: the slow growth of instabilities in queued traffic, the emergence of synchronized FIFO traffic in space-time, and the lane-changing rates observed at merge bottlenecks during the "drop to capacity". This universality suggests that the proposed lane-changing theory should be also helpful in settings that require complex vehicle maneuvers, such as freeways with diverges and weaves, and surface streets with signalized intersections. The necessary extensions are easy to implement because any type of maneuver can be handled naturally within the proposed framework. The theory should of course be tested in complex environments once reliable data become available.

Acknowledgements

This research was partially funded by the International Center of Excellence on Future Urban Transport at the University of California, Berkeley, and by the University of California Transportation Center.

Appendix A. The constrained motion model

Let $x = \phi(t)$ be the position, x, of a moving bottleneck (MB) at time t. Its desired acceleration $a(v(t), x)$ is given by (A.4) in the next subsection. We use the following constrained motion rule for vehicles kinetics, were the MB's current speed, $v(t)$, is constrained both by its ability to accelerate and by traffic downstream, i.e.

$$v(t) = \min\{v_{\mathrm{dwn}}(t), v_{\mathrm{des}}(t)\}, \tag{A.1}$$

where $v_{\mathrm{dwn}}(t)$ is a numerical estimate in $[t, t+\Delta t)$ for the speed of the KW stream immediately downstream of the MB, and on the lane occupied by the MB. Notice that $v_{\mathrm{dwn}}(t)$ cannot exceed u. The $v_{\mathrm{des}}(t)$ is the "desired" MB speed, obtained with the relation

$$v_{\mathrm{des}}(t+\Delta t) \doteq v(t) + a(v(t), \phi(t))\Delta t. \tag{A.2}$$

The MB's position is then updated with

$$\phi(t+\Delta t) = \phi(t) + v(t)\Delta t. \tag{A.3}$$

To estimate the effects of the MB on the rest of the traffic stream and calculate $v_{\mathrm{dwn}}(t)$, we use the method in Daganzo and Laval (2005b). The overall procedure is made up of numerically stable components. As expected, it tends to the continuum solution as $\Delta t \to 0$ in all the numerical experiments in Laval and Daganzo (2003).

A.1. Free motion models

Free-motion models for the kinematics of an isolated vehicle (Gazizadeh et al., 1996; Sayers and Riley, 1996; Tong et al., 1998; Gillespie and Sayers, 1999; FHWA, 2000; Jiang et al., 2001;

Rakha et al., 2001) capture the physical limitations imposed by roadway geometry on the engine for typical driver behavior. Based on Newtonian mechanics, they give the "desired" acceleration of a vehicle, $a(t)$, as a function of its current speed $v(t)$, and the vehicle and road characteristics.

The free-motion model for cars incorporated in TWOPAS (FHWA, 2000) is used in this paper. It takes a linear form that depends on the grade at location x expressed as a decimal, $G(x)$, and on the car's maximum speed, $v_{\max}$, and zero-speed acceleration, a_0:

$$a = a_0(1 - v/v_{\max}) - gG, \tag{A.4}$$

where $g = 9.81$ m/s^2 is the acceleration of gravity. The car type used in the numerical demonstrations of Section 3 is a high performance car, defined with $v_{\max} = 155$ km/h and $a_0 = 4.3$ m/s^2.

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