2017 IEEE Intelligent Vehicles Symposium (IV)
June 11-14, 2017, Redondo Beach, CA, USA

The Future of Parking: A Survey on Automated Valet Parking with an Outlook on High Density Parking

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***Abstract*— In the near future, humans will be relieved from parking. Major improvements in autonomous driving allow the realization of automated valet parking (AVP). It enables the vehicle to drive to a parking spot and park itself. This paper presents a review of the intelligent vehicles literature on AVP. An overview and analysis of the core components of AVP such as the platforms, sensor setups, maps, localization, perception, environment model, and motion planning is provided. Leveraging the potential of AVP, high density parking (HDP) is reviewed as a future research direction with the capability to either reduce the necessary space for parking by up to 50 % or increase the capacity of future parking facilities. Finally, a synthesized view discussing the remaining challenges in automated valet parking and the technological requirements for high density parking is given.**

I. INTRODUCTION

Parking is considered as one of the most challenging driving tasks, where the vehicle has to be moved backwards in narrow spaces with high probabilities of collision [1]. The complexity lies in the generation of a suitable parking maneuver, the simultaneous control of the vehicle's longitudinal and lateral motion, and the avoidance of obstacles in the environment. In addition, modern vehicle designs, safety requirements, and aerodynamics have a negative impact on the all round visibility of a vehicle. As a consequence, 40 % of accidents with physical loss or damage occur during parking or maneuvering [2]. Today's vehicles address this problem by providing fully automated parking functions for parallel and perpendicular parking. The parking task will even be further eased by automated valet parking (AVP) that allows passengers to leave the car in a drop-off zone, e.g. in front of a parking lot, while the car accomplishes the parking task on its own, see Figure 1.

High density parking (HDP) is a logical extension of AVP with the vision to increase the capacity of future parking facilities or decrease the massive land use necessary for parking. This can be accomplished in two ways: (1) taking an existing parking layout and packing the vehicles denser as no exit space for humans is necessary anymore, and (2) changing the parking layout and allowing shunting operations to park and unpark vehicles.

Given that vehicles are parked 96 % of their time [3], the need for HDP exists in many cities. For instance, a serious shortage of parking space can be seen in Beijing, where only 1.6 million parking spaces were available to 4.37 million

Fig. 1: Automated valet parking, where the car is left in a drop-off zone and the parking task is initiated with a mobile device.

registered cars in 2014 [4]. The result is illegal parking that decreases traffic flow and contributes to Beijing's heavy congestion and pollution. However, creating additional parking space is expensive and contributes heavily to land use. This problem can be seen in midtown Atlanta, Georgia, USA, where 21 % of the area is only dedicated to parking [5], [6].

In this regard, the **main contributions** of this paper are:

- Analysis and comparison of the core components of AVP. The focus lies on the platforms, sensor setups, maps, localization, perception, environment model, and motion planning developed in previous AVP research projects.
- Review of existing concepts in HDP and their impacts on parking.
- Synthesized view including a discussion on the gaps and remaining challenges of AVP and the future research directions in HDP.

The remainder is **organized** as follows. After an overview of representative contributions from international research groups to AVP, Section II presents and compares the core components of AVP in the intelligent vehicles literature. A review of the state of the art in HDP is presented in Section III. Remarks, perspectives and a discussion on the remaining challenges are finally given in Section IV.

II. AUTOMATED VALET PARKING

In 2007, the DARPA Urban Challenge (DUC) initiated numerous research projects that started to tackle the challenges of autonomous driving. Besides highway and urban driving, automated valet parking has always been of major interest in order to relieve the driver from parking. An overview of representative AVP research projects is displayed in Table I.

The core components of an AVP system are visualized in Figure 2. Based on this diagram, Section II-A analyzes and compares the platforms and sensor setups deployed in previous contributions. An in-depth overview of the utilized offline maps and localization approaches is given

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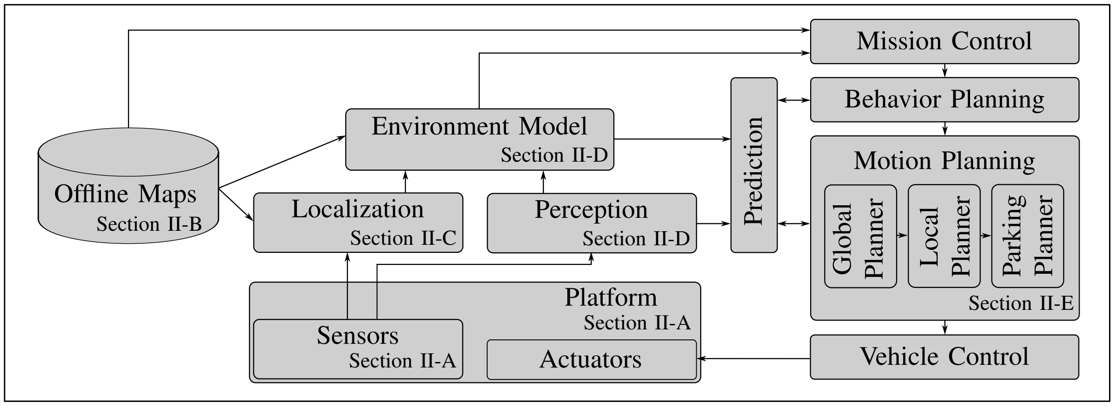

Fig. 2: Core components and their dependencies in automated valet parking, based on [7].

in Section II-B and II-C. The perception and environment model are reviewed in Section II-D, and a comparison of the different motion planning techniques is highlighted in Section II-E.

TABLE I: Representative contributions to AVP

Research Study	Description
Urmson et al. [8] and Montemerlo et al. [9], 2008	Carnegie Mellon's and Stanford's entry in the DUC. Proving the feasibility of autonomous driving and parking with onboard sensors only.
Kümmerle et al. [10], 2009	Autonomous driving and parking in a multi-level parking structure using onboard laser scanners, an inertial measurement unit (IMU) and odometry.
Jeevan et al. [11] and Stanek et al. [12], 2010	AVP with automotive grade sensors. GPS-denied localization with a camera and odometry.
Löper et al. [13], 2013	AVP as travel assistance. Information of onboard sensors is enhanced by a parking space occupancy detection camera.
Min and Choi [14], 2013	Infrastructure-based AVP solution, where an external server guides the vehicle to the parking spot.
Ibisch et al. [15], 2013 and Lenz et al. [16], 2014	Audi's approach towards autonomous driving in a parking garage. Infrastructure-based solution for localization and navigation of the vehicle.
Han and Choi [17], 2014 and Min and Choi [18], 2015	AVP using vision-only information to localize the vehicle on the parking lot.
Friedl, Hupka, and Tanzmeister [19], 2015	BMW's AVP system. Autonomous driving and parking with onboard sensors only in a GPS-denied parking garage.
Klemm et al. [20], 2016	AVP in the AutoPLES project. Realizing driving and parking in a GPS-denied parking garage.
Furgale et al. [21], 2013 and Schwesinger et al. [22], 2016	AVP in the V-Charge project. Using cameras and ultrasound sensors to navigate and park in GPS-denied parking structures.

A. Platforms and Sensor Setups

Recent approaches in AVP aim at reducing the number of required sensors to fulfill the driving task. Especially the use of already close-to-market sensors is favored. Table II compares Carnegie Mellon's platform [8] used in the DUC with the V-Charge test vehicle [21], [22].

TABLE II: Comparison of Carnegie Mellon's platform in the DUC [8] with the V-Charge test vehicle [21], [22]

Hardware	'07 Chevrolet Tahoe	VW e-Golf VII
#LIDARs	11 (front, rear, side)	-
#Radars	5 (front, rear, side)	-
#Stereo cameras	-	2 (front, rear)
#Mono cameras	2 (front)	4 fisheye cameras (surround)
#Ultrasound sensors	-	12 (front, rear)
GPS, IMU	Yes, Applanix with Omnistar Virtual Base Station corrections	Yes, standard GPS receiver
Odometer	Yes	Yes
Processing Unit	10 Core2Duo processors (2.16 GHz)	6 PCs

The major difference between both platforms lies in the number of LIDARs and radars, and the inertial navigation system (INS) integrated in the vehicle. While the V-Charge test vehicle uses vision, ultrasound sensors, and a standard INS to accomplish the driving tasks, Carnegie Mellon's platform heavily relies on expensive LIDARs, radars, and a high performance INS. The reasons are threefold: (1) expenses and the appearance of the car played a minor role in the DUC focusing on the feasibility of autonomous driving and parking in complex scenarios, (2) experiences and advances in computer vision allow a realization of AVP core components like localization and perception with vision sensors only, and (3) the V-Charge test vehicle is optimized for driving in parking lots at low speed while the vehicles in the DUC had to master a broad set of driving tasks including on-road driving at medium speed.

B. Maps

The maps used for AVP can be divided into on- and offline maps. Online maps are constantly updated with new measurements from sensors and contain static obstacles and parking spot occupancies.

Offline maps for AVP can further be clustered into 3D geometric maps and semantic maps:

1) 3D Geometric Map: In particular for localization and collision avoidance, a 3D geometric map of the parking lot has to be recorded. Kümmerle et al. use a multi-level surface (MLS) map generated with GraphSLAM [23] from laser range data to represent the 3D structure of the environment [10]. The MLS map assigns the vertical extent of an object to its position in a 2D lattice.

Klemm et al. assign the geometric information obtained from laser data and a variant of offline GraphSLAM to several oriented signed distance (OSD) submaps [20]. A graph is used to connect these submaps to a global map. The data structure used stores the surface normal, the signed distance, and the occupancy of every obstacle. These information help to accelerate the collision check in the motion planning module.

A vision-only approach to generate the 3D map is implemented in [22] and [24]. Schwesinger et al. create the 3D map from images of the multi-camera system and a structure from motion (SfM) framework [22]. Chirca et al. use a visual EKF SLAM [25] to create the 3D map of the environment [24].

2) Semantic Map: Semantic maps contain additional information for localization, navigation, and driving. Typical information included in the semantic map are, for example, lanes and intersections, location of parking spaces, pick-up and drop-off zones, charging stations, sidewalks, and traffic signs [11], [12], [16], [13], [19], [20], [22]. An additional speed map is introduced in [22] to assign different reference speeds to certain areas in a parking lot.

C. Localization

A precise estimation of the vehicle's position and orientation is essential to avoid collisions and plan appropriate motions. Localization with GPS proved to be either inaccurate with position offsets of more than 1 m [9], or not available in certain environments like multi-level parking structures [10]. Therefore, LIDAR- and video-based localization methods are developed for AVP, which are analyzed in the following. The achieved accuracies are then summarized in Table III.

1) LIDAR-based Localization: Kümmerle et al. estimate the position of the car in a multi-storey parking structure using laser range measurements and a particle filter [26]. 3D scan-matching on a previously recorded MLS map is performed with the iterative closest points (ICP) algorithm [27]. The vehicle's orientation is obtained from an IMU. The authors do not provide any quantitative results.

A similar approach is shown in [20] for a multi-storey car park. The 2D position and heading of the vehicle are tracked using an extended Kalman filter (EKF) [28]. The prediction is made with information from the vehicle's odometry and the update with measurements from the laser range finder, the geometric map, and the fast PL-ICP algorithm [29]. Localization accuracies are not provided.

An infrastructure-based localization is described in [15]. Six 2D laser range finders are positioned at the ground of the parking garage parallel to the surface. The localization of the vehicle is conducted in three steps. First, measured data points are separated into localization-irrelevant background and localization-relevant active points. In a second step, a vehicle hypothesis is generated with a random sample consensus (RANSAC) algorithm. This hypothesis is finally tracked with an EKF.

In addition to the described approaches in AVP, recent trends in LIDAR-based localization for urban automated driving aim at increasing the robustness under changing and challenging environments. The interested reader is referred to the contributions in [30], [31], [32], [33].

2) Video-based Localization: The high sensor cost of LIDARs and advances in computer vision have created a high interest in vehicle localization with information from images only. Jeevan et al. use the front camera of the vehicle to detect artificial landmarks on the ground of a parking lot to estimate the vehicle's pose [11]. A Kalman filter is then used to track the pose of the vehicle.

In contrast to that, [22], [34], [35] and [36] use natural landmarks for point feature based localization (PFL). For instance, Schwesinger et al. match the detected landmarks from four monocular fisheye cameras with queried landmarks from an offline map [22], [34]. Finally, a refinement of an initial pose guess is conducted using a least-squares optimization.

A vision-only localization in the context of AVP is also realized in [17]. An EKF including a particle filter is implemented to track the vehicle's pose in two steps. First, the state of the vehicle is predicted with visual odometry. Secondly, the EKF is updated with the measured vehicle pose, which is estimated by a particle filter. The road marking features extracted from surround camera images, maps, and the vehicle's pose from the prediction step of the EKF are used to update the particle filter.

The precision and robustness of video-based localization methods are especially challenged at low illumination levels, where PFL frequently fails [37]. Such environments can often be found in multi-storey parking structures. To the authors' best knowledge, this problem has so far not been addressed in the context of AVP and only few publications exist for urban automated driving [37], [38].

TABLE III: Localization accuracies in AVP

	LIDAR-based		Video-based		
	[20], [26]	[15]	[11]	[22], [35]	[17]
Position error (mean±std) [cm]	-	11.5 ±5.4	20	10	13
Heading error (mean±std) [°]	-	1.7 ±1.16	2	-	1.21

D. Perception & Environment Model

The environment model with static and dynamic obstacles is a crucial input to the motion planner. Static obstacles are typically inserted in the already introduced online map. In [8], [9], [10], [12], [13], [15], a static obstacle map is generated with the information from laser range finders, see Figure 3. The standard Bayesian framework [39] can be used to update the map with new sensor measurements. Whether

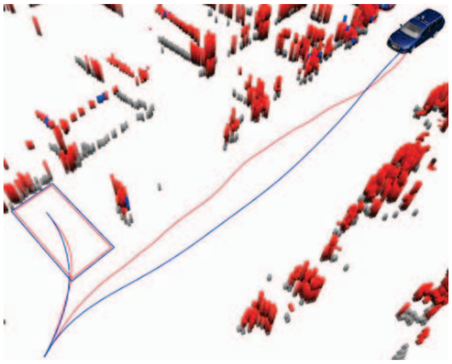

Fig. 3: Static obstacle map generated in [10] with a laser scanner.

or not a parking spot is occupied can then be detected by combining the obstacle map and the knowledge, where a parking lot is located from the semantic map [12].

A vision-only approach to detect static obstacles is realized in [22]. Here the fisheye cameras generate a depth map using stereo matching. The obstacles are then extracted by projecting the depth map into the 2D plane. An occupancy grid, which fuses the information from the cameras with the ultrasound sensors, is finally generated. The system detects most static obstacles except the arms of the barrier gate at the parking lot entrance [22].

In contrast to static obstacles, dynamic obstacles do not only need to be detected, but also associated with their velocity. The detection and tracking of a moving vehicle can, for example, be accomplished in three steps as shown in [8]. First, an algorithm extracts features like edges from laser scanner data. Secondly, the features are validated with radar data to assess whether they belong to a moving object. Thirdly, the valid features are associated with predicted object hypotheses. If the features can not be matched with an object hypothesis, a new object hypothesis will be initialized; otherwise, the state of the corresponding object is updated.

A similar approach is realized in [9], where two consecutive laser scans are compared to detect a moving object. If a moving object is detected, a particle filter, which tracks the object, will be initialized.

Detecting and tracking moving vehicles and pedestrians is realized in [22] with two different approaches. The first one uses the front stereo camera only. The features in the region of interest are extracted using aggregate channel features (ACF) and a soft-cascade classifier [40]. The second approach uses the monocular multi-cameras to detect vehicles and pedestrians with a Soft-Cascade+ACF classifier [41]. The tracking of the dynamic objects is conducted with an unscented Kalman filter [42].

Significant advances in deep learning, the availability of large data sets, and GPU-accelerated computing power have led to major improvements in perception. First successful applications in the context of urban automated driving show that these neural networks are able to conduct a semantic segmentation of an entire scene. The interested reader is referred to [43], [44].

E. Motion Planning

The motion planning module is responsible for generating a collision-free trajectory from a start to a goal configuration. While motion planning for autonomous vehicles is in general surveyed in [45] and [46], the focus of this section lies on the respective techniques for AVP including the following three stages: A **global planner** determines the route to the final destination by avoiding static obstacles. A **local planner** computes a trajectory that follows the global path and reacts on dynamic obstacles in the environment. Once the vehicle reaches the parking spot, a **parking planner** finally computes a trajectory into the parking bay. The three different stages are also visualized in Figure 2.

1) Global Planner: An overview of different global planners for AVP is given in Table IV.

TABLE IV: Overview of different global planners for AVP

Algorithm	Description
Dijkstra in [13]	Optimal graph search in a static road graph without heuristics.
A* in [16], [14], [19], [20], [21]	Optimal graph search in either a static road graph or a discretized state space using an admissible heuristic.
Hybrid A* in [9], [47]	Suboptimal graph search in a precomputed, discretized 4D state space using the maximum of two admissible heuristics.
Anytime D* in [8], [48]	Graph search in a precomputed 4D state lattice using the maximum of two admissible heuristics. The algorithm improves the initially suboptimal solution within the remaining sampling time.
RRT*-Connect in [49]	Sampling-based algorithm that randomly explores the environment and constructs a bidirectional path from the start to the goal pose and vice versa.

If a semantic map with a static road graph is available, the optimal route can be calculated either with Dijkstra's graph search [50] as shown in [13] or with an A* algorithm [51], see [16], [19], [20], [21]. The vertices in the static road graph represent start and end points of lanes, lane segments, or intersections and the edges the corresponding connections. Figure 4 visualizes such a graph of a parking garage.

The resulting global path neglects the kinematics of the vehicle and therefore, can not be executed by a vehicle with nonholonomic constraints. As a result, a smoothing step has to be conducted until the obtained path does not exceed the maximum curvature of the vehicle anymore. For example, this can be accomplished by a fourth degree polynomial as shown in [21].

In absence of a static road graph [8], [9], [14], a graph can be precomputed, where the vertices represent a 4D vehicle state (position, heading, driving direction) and the edges feasible transitions between the states. The costs assigned

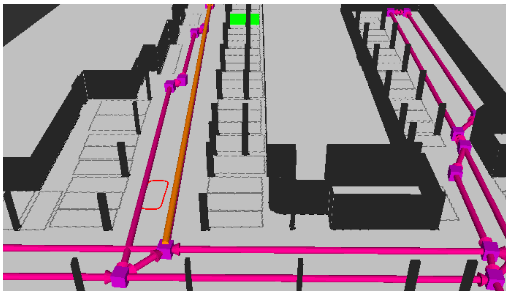

Fig. 4: Static road graph to determine the route (orange lane) from the start pose (red box on the ground) to the destination (green rectangle) [19].

to the vertices of the graph can, for instance, be based on an obstacle map, the current curvature of the path, the driving direction, and the distance to neighboring nodes [14].

In order to quickly find a feasible path from the start to the goal in such a graph, a variant of A* called Hybrid A* is used in [9], [47]. An Anytime D* algorithm [52] is implemented in [8], [48] that plans the vehicle's motion backwards from the goal to the starting point. The advantage of Anytime D* is that it quickly calculates an initial, suboptimal plan and improves the solution within the remaining time.

An alternative to a precomputed graph is a sampling-based approach. For instance, RRT*-Connect is presented in [49] that randomly explores the environment and computes simultaneously a bidirectional path from the initial configuration to the destination and vice versa. It converges asymptotically to the optimal solution [49].

Note that the approaches that do not rely on a static road graph [14], [49], [47], [48] can also be used for the local and the parking planner with little modifications.

2) Local Planner: All local planners share the idea of repetitively executing the local planner in order to react on changes in the environment. An overview of the different local planners for AVP is given in Table V.

Carnegie Mellon's team of the DUC used a model-predictive trajectory generator [54] to create the local path [8], [53]. A set of trajectories based on a second order spline with four degrees of freedom is optimized to satisfy the vehicle model and given constraints like a local goal point. The best trajectory is selected based on a metric including the distance to obstacles and the smoothness of the trajectory.

In contrast to the optimization-based approach in [8], Stanford's team also used its Hybrid A* global planner [47] to generate the local path in the DUC. Contrary to A*, Hybrid A* assigns a continuous vehicle state to every discrete configuration in the grid by predicting the vehicle's state with a given control input for a certain time. This modification of A* guarantees that the resulting path is drivable, but sacrifices the completeness of A* [47]. The developed algorithm is guided by the maximum value of the nonholonomic-without-obstacles heuristic and the holonomic-with-obstacles heuristic [47]. A conjugate-gradient descent smoother is then applied to the generated path. The execution of the algorithm takes 100 ms on one core of the Intel quad core computer installed in the vehicle [9].

Comparing the approaches in [8] and [9], it can be observed that dynamic obstacles are ignored by both planners as they were not part of the parking task in the DUC.

TABLE V: Overview of different local planners for AVP

Algorithm	Description
Model-predictive trajectory generator in [8], [53]	Optimizing the parameters of a second order spline to connect a local goal point with the current pose satisfying the vehicle constraints.
Hybrid A* in [9], [47]	Suboptimal graph search in a precomputed, discretized 4D state space using the maximum of the two admissible heuristics.
Terminal manifolds in [16]	Separate generation of the lateral and longitudinal trajectories using terminal manifolds and choosing the best combination that fulfills the kinematic and no-collision constraints.
Anytime Repairing A* in [19]	Graph search with an initial suboptimal solution in a precomputed 3D state lattice with kinematic compliant third order Bézier curves. The heuristic is equal to [8], [9]. The algorithm improves the solution within the remaining sampling time.
Sampling-based trajectory generator in [22]	Generating candidate motions along the reference path by forward simulating the vehicle dynamics including a vehicle controller with sampled control inputs.
Nonlinear optimization in [20]	Minimizing an objective function subject to nonlinear constraints that account for collision avoidance, the kinematics and dynamics of the vehicle.

Terminal manifolds [55] are used in [16] to generate optimal, collision-free trajectories in three steps. First, lateral and longitudinal trajectory sets are generated that neglect physical and collision-related constraints. Next, lateral and longitudinal trajectories are merged in every combination. Finally, the resulting trajectories are checked for physical and collision-related constraints, and the trajectory with the minimal cost is chosen. In contrast to the previously described local planners, this approach allows to check for collisions with dynamic objects making it more suitable for dynamic environments.

In the V-Charge project, candidate motions along the reference path are generated by forward simulating the vehicle dynamics including a vehicle controller [22]. A semantic map is used to predict other traffic participants 10 s ahead. Collisions with the other agents' predictions and the candidate motions are tested with a bounding volume hierarchy data structure [56]. The best collision-free trajectory is then chosen according to the distance to the reference path at the end of the trajectory [21], see Figure 5. Consequently, the resulting trajectory avoids collisions with dynamic objects making this planner suitable for dynamic environments.

A nonlinear constrained optimization problem [57] is solved in the AutoPLES project to generate the local trajectory [20]. The objective function penalizes acceleration, jerk, high yaw rates, and offsets to the middle of the lane center. The nonlinear constraints account for the kinematics and dynamics of the vehicle and geometric constraints in form of polygons for static and dynamic obstacles. Sequential quadratic programming [58] is used to solve the optimization problem.

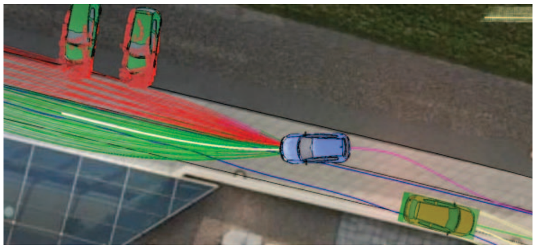

Fig. 5: Local planner developed in [40]. Red candidate motions visualize a possible collision with obstacles on the right side, green trajectories are collision-free.

3) Parking Planner: The parking planner is activated once the vehicle reaches a parking spot and generates a maneuver into the parking bay. The two most frequent approaches to plan a parking maneuver are: (1) use a predefined geometric set that connects the initial pose with the desired final pose, or (2) use graph-based methods to find the parking trajectory.

Geometric parking planners are used in [11], [13], [14], and [18]. For instance, perpendicular and parallel parking are realized with circles of minimum turning radius and straight lines in [14]. In general, geometric parking planners are simple and computationally inexpensive, but less flexible.

A combination of a geometric and a graph-based parking planner is introduced in [22]. Three parking planners are executed one after another until a path is found [22]. The first tries to park the vehicle in one move with two straight lines and a circular arc. The second applies the previous strategy, but allows two switches in the driving direction of the vehicle. The third planner executes a Hybrid A* search. This hierarchical approach with increasing complexity allows to generate either a simple or a more advanced parking maneuver when necessary.

III. Automated High Density Parking

The necessity to build on the advances of AVP to park vehicles as efficiently as possible can, for instance, be seen in Los Angeles (LA) County, USA, where 10 million people are living. Chester et al. estimate a total number of 18.6 million parking spots in this area in 2010 [59]. As a result, 14 % of LA County's area is on average used for parking [59]. An overview of the coverage is displayed in Figure 6.

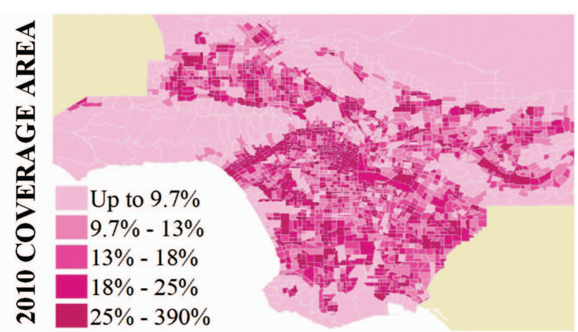

Fig. 6: Percentage of land used for parking in LA County [59]. Values greater than 100 % account for multi-level parking structures.

The reasons for the high number of parking spaces, especially in urbanized areas, are twofold. On the one hand, an increasing population with a high vehicle ownership rate increases the demand for parking space. On the other hand, most private and commercial buildings are required by law to fulfill peak parking demands [60].

To ease the parking shortage, international researchers have recently started to work on high density parking (HDP). The basic idea is to build on the opportunities of autonomous driving and AVP to increase the number of vehicles in a given area. An overview of the existing literature is given in Table VI.

TABLE VI: Representative contributions to HDP

Research Study	Description
Ferreira et al. [5], 2014	Vehicles are stacked in parallel rows. In order to access a vehicle, blocking vehicles are moved to another row. Density increases by 50 %.
Nunes et al. [61], 2014, d'Orey et al. [62], 2016	Build on top of [5]. Predictions/knowledge of a vehicle's exit time and different parking lot control strategies are used to reduce the distance traveled, number of shunting operations, and removal time.
Timpner et al. [63], 2015	Vehicles are parked in stacks perpendicular to the driveway. Density depends on the length of the stack, but increases by up to 33 %.

One of the first contributions to HDP is presented in [5], where vehicles are stacked in parallel rows. In order to exit a vehicle, blocking vehicles are moved to parallel rows.[1] It is shown that such a layout reduces the necessary space by 50 % compared to a conventional parking lot with equal capacity. Ferreira et al. show with real parking data that the average traveled distance per vehicle can even be reduced by 31.7 % compared to conventional parking. However, some vehicles are required to drive up to 246.3 % [5] longer distances due to the unlimited number of shunting operations.

Within this framework, the knowledge of a vehicle's exit time and intelligent parking lot control strategies can be used to optimize the results from [5]. The interested reader is referred to [61] and [62].

Another approach for HDP is shown in [63] by introducing parking stacks perpendicular to the driveway. The stacks allow to reduce the number of lanes on a parking lot while increasing the amount of parking spots.[2] The density raises with the length k of a stack [63]. For $k \to \infty$, it can be increased by 33 %. In contrast to [5], the maximum number of shunting operations between entering and leaving a stack is limited by $k - 1$.

IV. Discussion

Automated valet parking has the potential of becoming one of the first level 5 [64] automated driving systems. This is mainly because the following three simplifications to the general self-driving car problem can be made: (1) the velocities are usually low, (2) the environment is closed and known in advance, and (3) the infrastructure can be equipped

[1] A video of Ferreira et al.'s simulation can be found under the following link: `https://youtu.be/cuNj2nAQGi0`.

[2] Timpner et al.'s simulation can be viewed under the following link: `https://youtu.be/pCzI-l8tsPY`.

with additional sensors. This leads to the crucial design choice of how to distribute the intelligence between the infrastructure and the vehicles. There are three possibilities, see Figure 7.

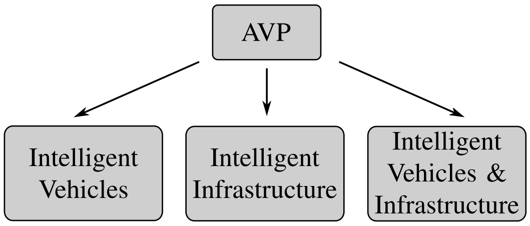

Fig. 7: Possible intelligence allocations in AVP.

The necessary intelligence for automated driving and parking can either be allocated to the vehicles without changing the infrastructure, to the infrastructure by only adding a remote control unit to the car, or to both for a mutual support. The advantages, disadvantages and remaining challenges of the different approaches are discussed below.

- **Intelligent vehicles**: The survey shows that a prototypical realization of AVP with already close-to-market sensors, such as ultrasound sensors and cameras is feasible. As vehicles are already becoming more intelligent, changes and investments in the infrastructure can be avoided. Apart from the question of the sensor setup, which we will discuss below, we see two major remaining challenges: (1) How can a robust and precise localization be guaranteed at all times in the context of a continuous changing environment of a parking lot? How do we deal with performance degradations, for example, of the localization module in the absence of an operator? (2) Once the vehicle is reinitialized after parking, how can it be guaranteed that occluded parts of the environment are obstacle-free, e.g. the tires, the underbody, parts below the bumpers, no low-hanging objects from the ceiling? A naive approach would be to never turn off the vehicle's perception during parking.
- **Intelligent infrastructure**: Only few contributions deal with the idea of integrating the discussed AVP core components in the infrastructure and sending the control commands to the vehicle. The advantages of this approach are a faster and wider availability to vehicles that can be controlled remotely. An appropriate interface for receiving the controls would be necessary among the manufacturers. The question concerning the sensor setup is in many aspects similar to the one above and is discussed below. Major disadvantages of this approach are the changes and the high investments in the infrastructure in order to integrate the sensors.
- **Intelligent vehicles & infrastructure**: To the authors' best knowledge, none of the AVP contributions deals with the idea of enhancing the perception and localization component of an autonomous vehicle with information from a sensor-equipped infrastructure. This would add a redundancy to the system that could increase the safety of the system. However, the necessity for high investments in the infrastructure remains.

Another key decision lies in the choice of the sensors, namely a vision-based vs. a LIDAR-based approach. The presented review shows that both sensors were used in the literature to tackle the AVP problem while a slight trend towards vision-only approaches can be observed. Especially in multi-storey parking structures, vision sensors are challenged by the difficult environment including low levels of illumination and shiny, smooth grounds causing reflections. LIDAR-based setups suffer from the problem of non reflective-surfaces and high cost. A hybrid approach using the strengths of several sensors and mutually verifying the incoming information might be a necessary concept to improve performance and robustness. Within this context, the potential of radar as a third sensor in AVP has to be evaluated.

So far, the research in HDP has focused on the introduction and evaluation of new parking layouts that leverage the opportunities of self-driving cars to park more vehicles in a given area. However, none of the existing literature analyzes how HDP can be integrated into already existing parking structures. In addition to that, the technological requirements for HDP have not been addressed yet. Arising questions in this regard are: Is it possible to park vehicles denser when average localization errors accumulate to 10 cm? Are the motion planning techniques capable of finding trajectories in cluttered environments, where it is not sufficient to approximate the shape of the robot and the obstacles with conservative bounding boxes? For that purpose, the requirements of localization, sensing, motion planning and actuator accuracies have to be determined and met in order to prove the feasibility of high density parking in real world applications.

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