Strategies and Spacing Requirements for Lane Changing and Merging in Automated Highway Systems

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***Abstract*—In automated highway systems (AHS) vehicles are expected to operate using their own sensing and control systems by interacting with other vehicles and the infrastructure in a way that guarantees safety, stability, and high capacity. One of the crucial vehicle maneuvers in an AHS environment that could affect safety and capacity is that of merging and lane changing. Such a vehicle maneuver would influence the operation of the vehicles in the vicinity and may have to alter their speed and position coordinates in order to accommodate the maneuver of the merging or lane changing vehicle in a way that safety is guaranteed and the disturbance in the traffic flow is minimized. In this paper, we analyze the problem of collision-free merging and lane changing. We examine various alternative scenaria for merging and lane changing and we present an algorithm for calculating the minimum safety spacing for lane changing (MSSLC); that is, we calculate the intervehicle spacing that the vehicles should maintain during a merging or lane changing maneuver so that in the case where one of the vehicles executes an emergency braking manuever, the rest of the vehicles have enough time and space to stop without any collision taking place. The calculation of the MSSLCs for merging or lane changing maneuvers is more complicated than the calculation of the *minimum safety spacing* for longitudinal vehicle following since, in the former case we have to take into account the particular lane-changing policy of the merging vehicle as well as the effect of combined lateral/longitudinal motion during the lane changing maneuver. The braking profiles of the vehicles involved in an emergency scenario during a lane changing maneuver depends on the particular AHS operational concept, i.e., on the degree of communication between the vehicles and between the vehicles and the infrastructure. We consider six different AHS operational concepts; for each concept we consider possible emergency braking profiles and we investigate the effect of the particular operational concept on the MSSLC.**

***Index Terms*—Automated highway systems (AHSs), lane changing, safety, spacing requirements.**

I. Introduction

URBAN highways in many major cities are usually congested and this makes driving difficult and raises the possibility of accidents, especially during merging and lane changing maneuvers. Human drivers engage in information gathering and decision making about driving conditions to determine if and when the conditions are favorable for a lane change. When they decide that the lane change can be successfully completed, they use their signals to notify other vehicles of their intent. Errors might result because the driver failed to collect critical information or failed to provide a signal or the other drivers failed to notice and take corrective action—for example, to provide additional spacing when needed. One of the promises of automated highway systems (AHS) is to increase the safety level of driving in highways and especially the safety of maneuvers like merging and lane changing by using advanced sensing and control systems to replace the inaccurate human actions. Therefore, AHS vehicles and/or infrastructure should have built-in intelligence that allows them to calculate the spacing requirements for safe merging and lane changing.

Vehicles need to maintain a certain "safety distance" between them, in order to be able to slow down or stop without collision when the leading vehicle performs a slow down or stopping maneuver. When another vehicle wants to merge in between, a spacing equal to the sum of the required safety distance between itself and the merging vehicle and the safety distance required between the merging vehicle and the leading vehicle, plus the length of the merging vehicle has to be created. There are several approaches to estimate the required spacing. If the following vehicle has no information about the vehicle class and braking capabilities of the merging vehicle, it has to make worst-case assumptions to allow for a large safety margin. Otherwise, extensive communications will be required between the vehicles involved so that each one can be informed of the vehicle class and braking capabilities of the others. In the latter case, the requirement for a large allowance for a safety margin can be significantly reduced, in effect allowing just enough space for the merging vehicle so that the spacing between them immediately after the merge is equal to the minimum safety distance calculated for longitudinal vehicle following [3].

The relative speed of the vehicle that intends to merge relative to the speed of the vehicles in the destination lane just prior to the merging is of great importance. The speed can be very different before the merging but it has to be matched after the merge. The speed before the merge is likely not to be the same because the merging vehicle might be accelerating from a ramp or it might be constrained by the speed of the traffic flow in the lane it occupies before merging into the new lane. This imposes additional constraints about the timing of the maneuver

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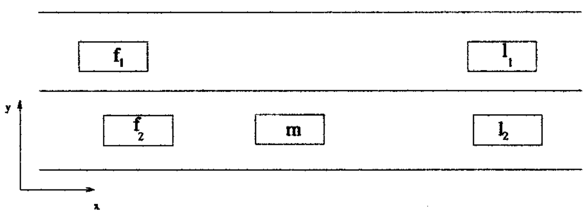

Fig. 1. Vehicle about to make a lane change.

and mostly in the amount of additional safety distance that will be required.

In this paper, we analyze the problem of collision-free merging and lane changing. We examine various alternative scenaria for merging and lane changing and we present an algorithm for calculating the *minimum safety spacing for lane changing* (MSSLC); that is, we calculate the spacing that the vehicles should have during a merging or lane changing maneuver so that in the case where one of the vehicles enters an emergency braking situation, the rest of the vehicles have enough time and space to stop without any collision taking place. The calculation of the MSSLCs for the merging or lane changing maneuver is more complicated than the calculation of the minimum safety spacing in the longitudinal case since, in the former case, we have to take into account the particular lane changing policy of the merging vehicle as well as the effect of combined lateral/longitudinal motion during the lane changing maneuver. The braking profiles of the vehicles involved in an emergency scenario during lane changing maneuver depend on the particular AHS operational concept, i.e., on the degree of communication between the vehicles and between the vehicles and the infrastructure. We consider six different AHS operational concepts; we present the possible emergency braking profiles of the vehicles for each operational concept and investigate the effects of the particular operational concept on the MSSLC.

Due to the similarities between the lane changing and the merging problem, we consider and analyze only the lane changing problem. The results of this work can be easily extended/modified for the case of merging. Similar studies on the evaluation of safety spacing were performed in [1] for the case of no deceleration or emergencies. As a result, the work in [1] leads to the calculation of safety spacing for lane changing under a constant speed vehicle movement. In our case, we calculate the intervehicle spacing requirements under possible emergency stopping situations during lane changing.

II. Lane-Changing Strategies

Consider the five vehicles shown in Fig. 1 that will be directly affected when one of them performs a lane change maneuver. The symbols ℓ_1, f_1, ℓ_2, m and f_2 stand for the leading vehicle in the destination lane, following vehicle in the destination lane, the leading vehicle in the originating lane, the vehicle that performs the lane-changing (which will be called thereafter as the merging vehicle), and the following vehicle in the originating lane, respectively. Each vehicle has length $L_i, i = \ell_1, f_1, \ell_2, m, f_2$. By assuming a two-dimensional (2-D) coordinate system as shown in Fig. 1, the vehicle's motion can be completely described by the 2-D vectors $x^{(i)}, v^{(i)}$, and $a^{(i)}, i = \ell_1, f_1, \ell_2, m, f_2$ of position, velocity, and acceleration, respectively. The position of each vehicle is measured with respect to the center of the front end of the vehicle, while the velocity and acceleration are measured with respect to the center of gravity of the vehicle. The first entry of the vectors $x^{(i)}, v^{(i)}, a^{(i)}$ denotes the longitudinal position, velocity, and acceleration, respectively, while the second entry stands for the lateral position, velocity, and acceleration, respectively. Finally, $c^{(i)}$ denotes the distance in the longitudinal direction between the center of the front end and the center of gravity of the ith vehicle.

Let d_{ij} denote the intervehicle distance in the longitudinal direction between the vehicles i and j, i.e.,

$$d_{ij} := x_1^{(i)} - x_1^{(j)} - L_i.$$

Obviously, during a lane changing operation, we are concerned about the longitudinal distances $d_{\ell_1 m}, d_{\ell_2 m}, d_{m f_1}, and\ d_{m f_2}$. If one of the above distances measured in the same lane (which will be called thereafter "intervehicle spacing") becomes nonpositive then a collision will occur. Moreover, following the work of [3] a lane-changing scenario must be such that it guarantees that no collision occurs if one of the vehicles enters an emergency braking manuever at any point in time before, during, and after the end of the lane changing maneuver. In other words, during the lane-changing maneuver, the intervehicle spacings $d_{\ell_1 m}, d_{\ell_2 m}, d_{m f_1}, d_{m f_2}$ must be large enough so that in the case where any of the five vehicles enters an emergency braking manuever, the other four vehicles have sufficient time and spacing to stop without any collision taking place.

Suppose now that at the time-instant $t = 0$ the merging vehicle starts performing the lane change maneuver. There are many alternative lane changing policies for the merging vehicle m and the best policy depends on many factors such as relative speed between the originating and destination lane, the spacing between the merging vehicle and the leading vehicle ℓ_2, etc. Despite the differences of the alternative lane changing policies they all can be described as follows: The merging vehicle starts adjusting its longitudinal velocity (by decelerating or accelerating) to make the spacing $d_{\ell_2 m}$ large enough; then it starts adjusting its longitudinal velocity (again by decelerating or accelerating) in order to make its longitudinal velocity equal to the velocity of the destination lane. Let us suppose that the time needed for the merging vehicle to adjust its longitudinal position and velocity is equal to $t_{\rm long}$. Regarding now the lateral motion of the merging vehicle at a certain time-instant $t_{\rm lat} \geq 0$ the merging vehicle starts developing a lateral acceleration in order to enter the destination lane. The lateral adjustment of the merging vehicle's motion may start at the same time as the adjustment of its longitudinal motion (in this case $t_{\rm lat} = 0$), it may start right after the adjustment of its longitudinal motion is completed (in this case $t_{\rm lat} = t_{\rm long}$) or it may start any time after the merging vehicle has initiated adjustment of its longitudinal motion (in this case $0 < t_{\rm lat} < t_{\rm long}$). The time-instants $t_{\rm long}, t_{\rm lat}$ as well as the profiles of the longitudinal and lateral accelerations specify the particular lane changing policy. In the next two subsections, we describe the possible profiles of the longitudinal and lateral accelerations of the merging vehicle.

A. The Longitudinal Acceleration Model for the Merging Vehicle

The profile of the longitudinal acceleration of the merging vehicle mainly depends on the relative velocity between the originating and the destination lane. When the vehicles in the destination lane move faster than in the originating one, then the merging vehicle must first decelerate in order to make its spacing with the leading vehicle ℓ_2 large enough for the lane changing maneuver, and then it must accelerate in order to adjust its velocity with the vehicle velocity in the destination lane. On the other hand, in the case where the vehicles in the destination lane move slower than in the originating one, then the merging vehicle must first decelerate in order to make its spacing with the leading vehicle ℓ_2 large enough for the lane changing maneuver, and then it may continue decelerating until its velocity becomes equal to that of the vehicles in the destination lane.

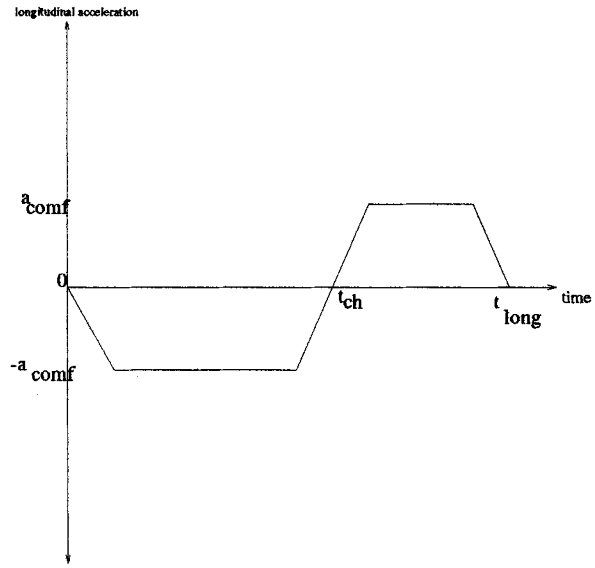

Fig. 2. Longitudinal acceleration profile in the case where the vehicle speed in destination lane is higher than that in the originating one.

Let V_d and V_o denote the velocity of the destination and originating lane, respectively. Let us examine the acceleration profiles of the merging vehicle in the case where $V_d > V_o$ (i.e., in the case where the vehicle speed in the destination lane is higher than that in the originating one) and $V_d \leq V_o$ (i.e., in the case where the vehicle speed in the destination lane is lower than that in the originating one).

- ***Case I:*** $V_d > V_o$. In this case, the merging vehicle initially decelerates in order to create enough spacing in the originating lane for a safe and collision-free lane changing maneuver. As soon as a sufficient spacing has been created it starts accelerating in order to match its velocity with the velocity V_d in the destination lane. We consider a simple model for the longitudinal acceleration profile of the merging vehicle. In particular, we assume that the merging vehicle's acceleration initially decreases linearly with respect to time until it reaches a limit $-a_{\rm comf}$, where $a_{\rm comf}$ is appropriately chosen to maintain safety and comfort of the passengers in the vehicle. Then, the acceleration remains constant and equal to $-a_{\rm comf}$ until a sufficient spacing has been created in the originating lane and then it switches from decelerating to accelerating. In particular, the acceleration starts increasing linearly until it reaches the positive acceleration limit $a_{\rm comf}$. The acceleration remains constant and equal to $a_{\rm comf}$ until the merging vehicle's velocity becomes equal to V_d. Fig. 2 shows the longitudinal acceleration profile for this case.

  The constant $t_{\rm ch}$ in Fig. 2 denotes the time-instant at which the merging vehicle switches from decelerating to accelerating, while the constant $t_{\rm long}$ is such that the longitudinal velocity of the merging vehicle equals to the vehicle speed V_d in the destination lane.
- ***Case II:*** $V_d \leq V_o$. In this case, the merging vehicle decelerates in order to both create enough spacing in the originating lane for a safe and collision-free lane changing maneuver and match its velocity with the vehicle speed V_d in the destination lane. Similar to the previous case, we consider a simple model for the longitudinal acceleration profile of the merging vehicle. In particular, we assume that the merging vehicle's acceleration initially decreases linearly with respect to time until it reaches a limit $-a_{\rm comf}$, where $a_{\rm comf}$ is appropriately chosen to maintain safety and comfort of the passengers in the vehicle. Then, the acceleration remains constant and equal to $-a_{\rm comf}$ until both a sufficient spacing has been created in the originating lane and the merging vehicle's velocity becomes equal to V_d. When both the spacing in the originating lane guarantees a safe and collision-free lane changing maneuver and the velocity of the merging vehicle is equal to V_d, the acceleration is linearly increased to zero. The longitudinal acceleration profile, in the case where the vehicles in the destination lane move slower than in the originating one, is shown in Fig. 3.

Note that the acceleration profile for the case where $V_d > V_o$ is a general one since we can get the acceleration profile for the case where $V_d \leq V_o$ by setting $t_{\rm ch} = t_{\rm long}$.

B. The Lateral Acceleration Model for the Merging Vehicle

In our analysis, we use a somewhat simplified model of the lane change maneuver trajectory, which assumes a sinusoidal pattern of lateral acceleration. As a first-order approximation, the acceleration of a vehicle during normal lane change maneuvers can be modeled as a sine function of time [2], [5]. The variable parameters of this model are time and distance. The model is symmetric with respect to the direction of lane change, therefore, the direction of the change is not a factor. Merging, exiting and weaving have all similar motions in terms of kinematic equations and the same model can be applied for all these cases. According to this model, the lateral acceleration is given by

$$a_{\rm lat} = \begin{cases} \frac{2\pi d_l}{t_{\rm LC}^2} \sin\left(\frac{2\pi}{t_{\rm LC}}(t - t_{\rm lat})\right), & \text{if } t \in [t_{\rm lat}, t_{\rm LC} + t_{\rm lat}] \\ 0, & \text{otherwise} \end{cases} \tag{2.1}$$

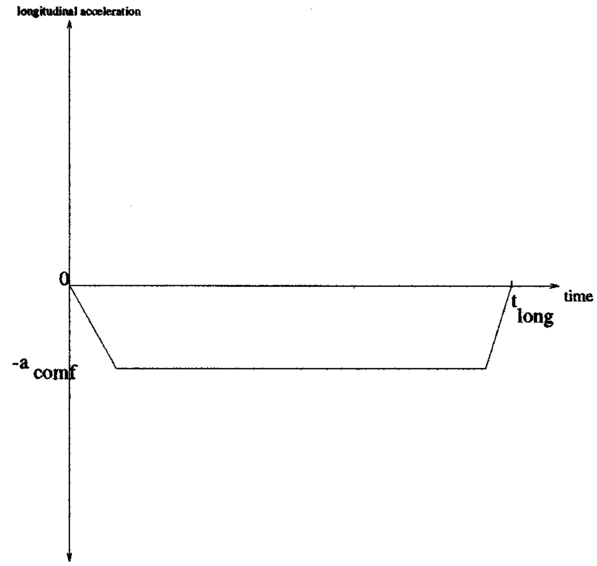

Fig. 3. Longitudinal acceleration profile in the case where vehicles in the destination lane move slower than in the originating one.

where

- a_{lat} instantaneous lateral acceleration;
- t_{LC} total time to complete the lane change;
- t elapsed time;
- d_I lateral intended lane change distance.

Therefore, the lateral peak acceleration A is

$$A = \frac{2\pi d_I}{t_{\mathrm{LC}}^2} \tag{2.2}$$

and the lane change frequency is

$$\omega = \frac{2\pi}{t_{\mathrm{LC}}}.$$

Given the lateral acceleration, the lateral speed and the lateral distance traveled during a lane change can be derived by successive integration. The assumed sinusoidal acceleration pattern seems to be appropriate for automated lane changing in order to guarantee the comfort of the passengers of the vehicle. This is obvious from the fact that the corresponding jerk function (i.e., the time-derivative of the lateral acceleration) whose value affects passenger comfort does not have any pronounced peaks.

Empirical data collected by photographing several hundreds of lane changes on multilane highways [7] have been used to determine the distribution of lateral placement, lane change distance, and total lane change time on the lateral acceleration model given by (2.1). The standard lane width on highways is 12 feet and this is the mean value of the d_I, which actually may vary from 9 feet to 15 feet. Total lane change times may vary widely. Lane changes of up to 16 s in duration are not outside the normal range, though most lane changes are significantly faster [2]. While the aerial photography method used in determining total lane change time involves a degree of under estimation due to the model used and the resolution limits available, a lower bound of about 2 s total lane change time has been determined [7]. The aggressiveness of the lane change depends primarily on the total time t_{LC} taken and also on the lane change distance.

The peak lateral acceleration A can be determined by substituting the d_I for the final distance and t_{LC} for the total time in the equation for A. Thus, a range of lateral accelerations from 0.22 ft/s^2 to 23.55 ft/s^2 (or, equivalently, from 0.068 to 0.73 g) has been found. If we assume that a nominal to slow lane change covers 12 ft in 5 s, we have a peak acceleration of 3 ft/s^2. The same distance covered in 4 s implies a peak acceleration of 4.7 ft/s^2, while 3 s produce acceleration of 8.38 ft/s^2 and 2.5 s produce acceleration of 12 ft/s^2. It becomes obvious that very fast lane changes involve very large lateral acceleration, while slow lane changes involve negligible lateral acceleration.

C. Safe and Collision-Free Lane Changing Strategies

Using the above two models for the longitudinal and lateral acceleration profiles of the merging vehicle, one can obtain different merging strategies by appropriately chosing the parameters $t_{\mathrm{long}}, t_{\mathrm{lat}}, t_{\mathrm{ch}}, a_{\mathrm{comf}}$ and t_{LC}. In other words, a particular choice for these five parameters determines the merging strategy and, moreover, affects the safety of the lane changing maneuver.

We consider a lane change to be safe and collision-free if there is sufficient spacing between the vehicles involved so that if any of the vehicles performs emergency braking at any time before, during, and after the lane changing all five vehicles could stop without colliding. In this scenario, we did not consider the case where a possible collision could be avoided by using both braking and steering. The use of only braking is considered to be a worst-case scenario and could lead to larger spacing requirements for collision-free lane change maneuvers. Moreover, this scenario is simpler since the use of both braking and steering for collision avoidance and the resulting spacing requirements for collision-free maneuvers are far more complex.

Since emergency braking can take place any time during a lane change maneuver, we could have a situation where the merging vehicle is decelerating for an emergency stop while it has both lateral and longitudinal motion. In this case, its braking capabilities are limited, primarily due to the properties and limitations of the tires. The explanation about braking dynamics, and the limitation imposed by the so-called friction-cycle during a lane change is presented in the following section.

III. Braking During Lane Changing

When a tire is operated under conditions of simultaneous longitudinal and lateral slip, the respective longitudinal and lateral forces depart markedly from those values derived under independent conditions (i.e., the values derived under only longitudinal or only lateral slip). The application of longitudinal slip generally tends to reduce the lateral force at a given slip angle[1] condition. Application of a slip angle reduces the longitudinal force developed under a given braking condition. This behavior can be seen in Fig. 4 (taken from [4]).

[1]*Slip Angle* is defined as the angle between the tire's direction of heading and its direction of travel.

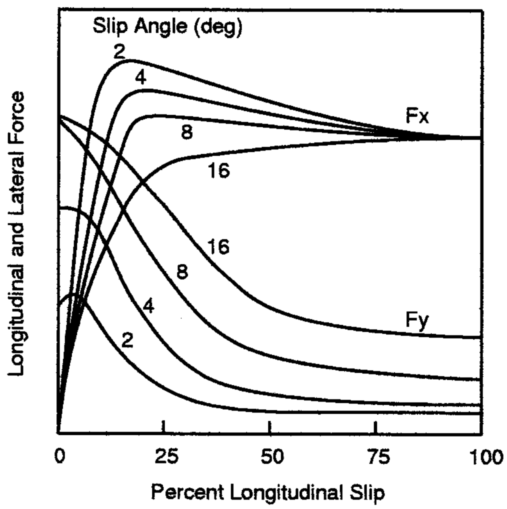

Fig. 4. Longitudinal and lateral forces (F_x, F_y) for different slip angles, as a function of longitudinal slip.

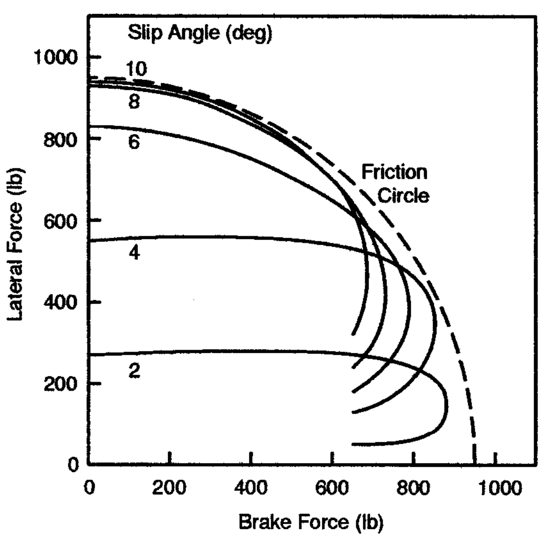

Fig. 5. Lateral force versus longitudinal force at constant lateral slip angles.

The general effect on lateral force when braking is applied is illustrated in the traction field of Fig. 5 (taken from [4]). The individual curves represent the lateral force at a given slip angle. As the brake force is applied, the lateral force gradually diminishes due to the additional slip induced in the contact area from the braking demand. This type of diagram that displays the tire's traction field is the basis for the *friction circle* concept [4]. The "circle" in most cases is actually an ellipse. Recognizing that the friction limit for a tire, regardless of direction, will be determined by the coefficient of friction multiplied by the load, it is clear that the friction can be used for lateral force or brake (longitudinal) force or a combination of the two. The direction can be positive or negative and this make little difference. *But in no case can the vector total of the two exceed the friction limit*. The limit is, therefore, a circle (ellipse) in the plane of the lateral and longitudinal forces. The position of the circle in Fig. 5 is the friction circle for the positive quadrant of the traction field. The limit is characterized as a friction circle for tires which have effectively the same limits for longitudinal and lateral forces. Certain specialized tires, however, may be optimized for lateral traction or braking traction in which case the limit is not a circle but an ellipse. We are making the simplified assumption that the longitudinal and lateral forces F_x and F_y, respectively, during simultaneous longitudinal and lateral tire slip conditions, depend linearly on the longitudinal and lateral accelerations a_x and a_y, respectively. According to the above analysis regarding the "limited friction circle", the longitudinal and lateral accelerations must satisfy

$$a_x^2 + a_y^2 \leq F_c \tag{3.1}$$

where F_c is a positive constant. It is worth noticing, that in the case of pure longitudinal motion, the maximum longitudinal acceleration of the vehicle $a_{\max}$ is larger than $\sqrt{F_c}$. In other words, formula (3.1) applies only for the case of combined longitudinal/lateral braking and not for the case of pure longitudinal braking (in which case $a_y = 0$). The above inequality simply states that braking during combined longitudinal and lateral motion significantly degrades the braking capabilities of the vehicle and, moreover, the stopping time of the vehicle depends on the time-history of the lateral accelerations; the larger the lateral acceleration is, the more distance it takes for the vehicle to stop in the longitudinal direction.

IV. Emergency Braking During Lane Changing

In this section, we analyze the problem of emergency braking during lane changing. More precisely, we consider the problem of analyzing the behavior of the vehicles involved in a lane change maneuver in the case where one of these vehicles enters an emergency braking situation. A braking scenario, which describes exactly how the vehicles brake, is usually specified by the deceleration profiles of the vehicles as a function of time. The deceleration profile depends, in general, not only on the road conditions and the braking abilities of the vehicle but also on the particular AHS operational concept and the sensors and communication devices that the vehicle is equipped with together with the associated capabilities [3], [6].

Contrary to the longitudinal case where only two vehicles are involved, in the lane changing case we have three different emergency braking scenarios:

1) the case where the vehicle ℓ_1 enters an emergency braking situation;
2) the case where the vehicle ℓ_2 enters an emergency braking situation;
3) the case where the merging vehicle m enters an emergency braking situation.

Since the merging vehicle m is performing a lane changing when the emergency situation takes place, we may have the case where both the following vehicles f_1 and f_2 must enter an emergency braking situation as well. This is because the merging vehicle is moving in both longitudinal and lateral directions and, therefore, we might have the situation where the merging vehicle is in the originating lane when the emergency braking starts and ends up in the destination lane (or somewhere in-between) due to the lateral motion.

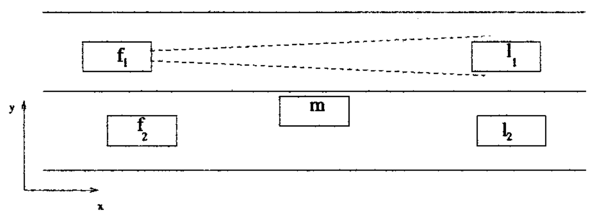

Fig. 6. The case where the vehicle ℓ_1 is "visible" by the vehicle f_1.

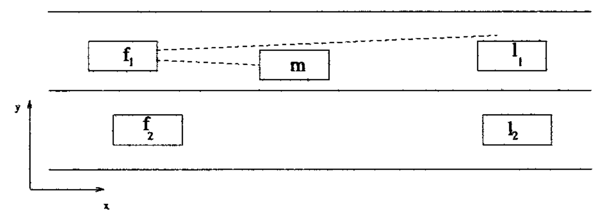

Fig. 7. The case where the vehicle ℓ_1 is not "visible" by the vehicle f_1.

In order to simplify our analysis, we will describe the case where only vehicle ℓ_1 enters an emergency braking situation. The analysis for the other cases (vehicle ℓ_2 or m enter an emergency braking situation) are similar. Similarly, we will examine the deceleration profiles only for the case of the following vehicle f_1; the deceleration profiles for the following vehicle f_2 are similar.

Similar to the longitudinal case [3], the deceleration profiles for the vehicles involved in a lane changing maneuver depend on the particular AHS operational concept the vehicles operate under. However, the problem for the case of the lane change maneuver becomes more complicated because of the degraded braking capabilities of the merging vehicle as explained in the previous section. Let us examine the deceleration profiles for each AHS operational concept.

- *Autonomous Vehicles*. A possible AHS concept is one where the vehicles operate independently, i.e., autonomously, using their own sensors without any communication between the vehicles. Each vehicle senses its environment, including lane position, adjacent vehicles, and obstacles. The infrastructure may provide basic traveler information services, i.e., road conditions and routing information.

  Since, for this particular AHS operational concept, each vehicle relies on its own sensors to determine the motion intentions of the vehicle ahead, we have to consider two different cases depending on whether the merging vehicle's position prevents the sensors of the vehicle f_1 from sensing the position of the vehicle ℓ_1. In the first case, the vehicle f_1 can sense the position and relative velocity of the vehicle ℓ_1 since the merging vehicle is either still in the originating lane or even if part of the vehicle m is already in the destination lane, its body is not in the operational range of the sensors of the vehicle f_1; in other words, in this case, the vehicle ℓ_1 is "visible" by the vehicle f_1. The second case, is the case where the vehicle ℓ_1 is not "visible" by the f_1 one, because the merging vehicle prevents the sensors of the vehicle f_1 from sensing the motion of the vehicle ℓ_1. The situation where the vehicle ℓ_1 is "visible" and not "visible" by the vehicle f_1 is illustrated in Figs. 6 and 7.

  Let us first analyze the case where the vehicle ℓ_1 is "visible" by the vehicle f_1. In this case, both the merging and the f_1 vehicles are assumed to behave similarly in the sense that they both detect the emergency braking at the same time. That is, after the vehicle ℓ_1 starts performing emergency braking, the merging and the vehicle f_1, which might have been accelerating initially, start decelerating after a detection and brake actuation delay in an effort to maintain the desired spacing. Since the two vehicles are not aware that the leader is performing emergency braking, they limit their jerks and decelerations in an effort to meet the vehicle control objective and at the same time maintain passenger comfort. The two vehicles detect and initiate emergency braking at possibly different time instants (since the spacing between the merging vehicle and the vehicle ℓ_1 is less than the spacing between the vehicle f_1 and the vehicle ℓ_1, and, thus, it is natural to assume that the vehicle f_1 detects the emergency braking after the merging vehicle does). When emergency braking is detected, passenger comfort is no longer an issue; the vehicles apply the maximum available deceleration in order to minimize the spacing needed for the vehicle to stop. The vehicles apply the maximum available deceleration at the minimum amount of time, thus resulting in maximum jerk. Due to the fact that the merging vehicle performs combined longitudinal/lateral braking it is expected (as explained in the previous section) that the maximum available decelerations are less than the ones used for pure longitudinal braking. In our analysis, we used a simplified model in order to incorporate the effect of combined longitudinal/lateral braking in the braking capabilities of the merging vehicle. More precisely, we assumed that if the merging vehicle detects and initiates emergency braking at the time-instant $t_{m_{\rm emerg}}$ and its longitudinal and lateral accelerations at this time instant are $a_1^{(m)}(t_{m_{\rm emerg}})$ and $a_2^{(m)}(t_{m_{\rm emerg}})$, respectively, then both the longitudinal and lateral acceleration will decrease linearly with respect to time, in such a way that at each time instant $t \geq t_{m_{\rm emerg}}$ they satisfy constraint (3.1). In other words, we have assumed that the longitudinal and lateral accelerations after $t_{m_{\rm emerg}}$ satisfy (4.1) and (4.2) at the bottom of the next page, where the Jerk parameters J_1 and J_2 (i.e., the derivatives of the acceleration) are such that the longitudinal and lateral accelerations $a_1^{(m)}(t)$ and $a_2^{(m)}(t)$ satisfy constraint (3.1) at each $t > t_{m_{\rm emerg}}$. Fig. 8 shows the deceleration profiles for the case where the vehicle ℓ_1 is "visible" by the vehicle f_1, where $a_{m\ \max} = \sqrt{F_c}$.

  Let us now analyze the case where the vehicle ℓ_1 is not "visible" by the vehicle f_1. In this case, the deceleration profile for the vehicle f_1 becomes more complicated, while the deceleration profiles of the other two vehicles remain the same. The fact, that the sensors of the vehicle f_1 sense only the merging vehicle has the effect the follower to detect the emergency braking situation t_d seconds after

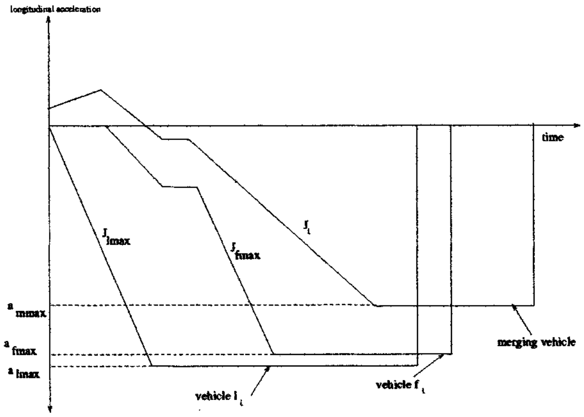

Fig. 8. Autonomous vehicles: The case where the leader is "visible" by the follower.

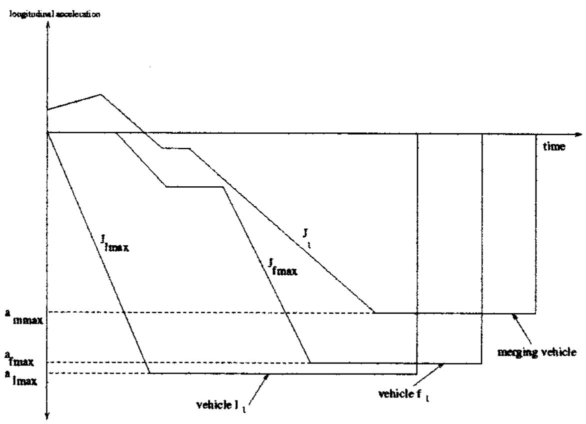

Fig. 9. Autonomous vehicles: The case where the leader is not "visible" by the follower.

the merging vehicle has detected it. The deceleration profiles for the case where the vehicle ℓ_1 is not "visible" by the follower are shown in Fig. 9.

- *Free Agents—Infrastructure Supported*. A vehicle is considered a "free agent" if it has the capability to operate autonomously but it is also able to receive communications from other vehicles and from the infrastructure. This implies that the infrastructure may get involved in a supporting role, by issuing warnings and recommendations for desired speed and headways, but the infrastructure will not have the authority to issue direct control commands. Since there exists vehicle to vehicle communication, the leader communicates with the merging vehicle and the merging vehicle, in turn, communicates with the follower and, therefore, in the case where the leader enters an emergency braking situation, the merging vehicle and the follower are informed about the emergency braking situation and verify that using their own sensors. When the merging and the vehicle f_1 detect that the vehicle ℓ_1 is braking and at the same time receive the information that this is an emergency braking, they bypass the limited jerk/limited braking stage of the autonomous vehicles case. However, as it is shown in Fig. 10, there will be a time-delay before the merging vehicle and the follower apply emergency braking. Such a delay is due to the communication delays between the three vehicles and the time needed for the sensors to verify the emergency braking situation. It must be expected that the time-delay for the follower to detect and verify emergency braking is larger than the one for the merging vehicle since similar to the autonomous vehicles case, the vehicle ℓ_1 may be "invisible" by the follower, the follower and the merging vehicle are not at the same lateral position, etc. Finally, the braking capabilities of the merging vehicle will be degraded due to the combined longitudinal/lateral braking. Fig. 10 shows the deceleration profiles for the case of infrastructure supported free-agent vehicles. Similar to the autonomous case, the braking capabilities of the merging vehicle will be degraded [see (4.1) and (4.2)].

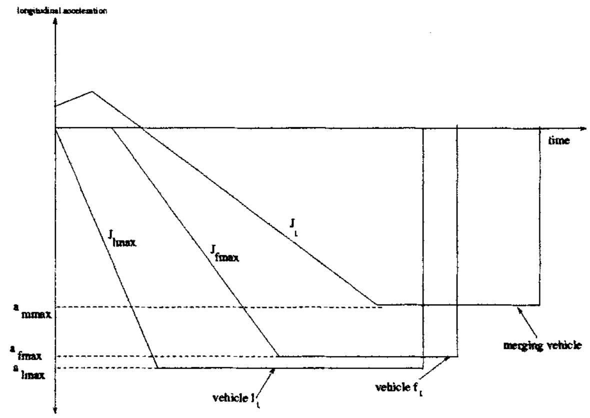

Fig. 10. Infrastructure supported free-agent vehicles.

- *Free Agents—Infrastructure Managed*. The concept of free agents with infrastructure management is based on the assumption that traffic is composed of vehicles acting as free agents while the infrastructure assumes a more active and more complex role in the coordination of the traffic flow and control of vehicles. Each vehicle is able to operate autonomously and uses its sensors to sense its position and environment, including lane position, adjacent vehicles and obstacles. The difference in this centrally managed architecture is that the infrastructure

$$a_1^{(m)}(t) = \begin{cases} a_1^{(m)}\left(t_{m_{\text{emerg}}}\right) - J_1\left[t - t_{m_{\text{emerg}}}\right], & \text{if } \left|a_1^{(m)}(t)\right|^2 < F_c \\ \sqrt{F_c}, & \text{otherwise} \end{cases} \tag{4.1}$$

$$a_2^{(m)}(t) = \begin{cases} a_2^{(m)}\left(t_{m_{\text{emerg}}}\right) - J_2\left[t - t_{m_{\text{emerg}}}\right], & \text{if } v_2^{(m)}(t) > 0 \\ 0, & \text{otherwise} \end{cases} \tag{4.2}$$

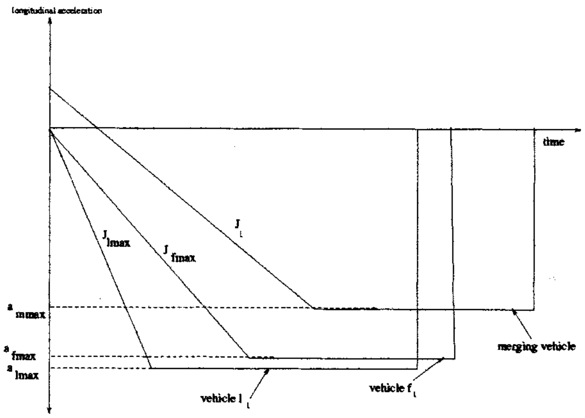

Fig. 11. Infrastructure managed free-agent vehicles.

has the ability to send commands to individual vehicles. This is envisioned to be a "request-response" type architecture, in which individual vehicles ask permission from the infrastructure to perform certain activities and the infrastructure responds by sending commands back to the requesting vehicle and to other vehicles in the neighborhood.

It is expected and assumed that the infrastructure is able to detect emergency situations and whenever it detects such an emergency, the infrastructure will have the responsibility to send an emergency braking command to all the vehicles affected. This concept minimizes the delay in performing emergency braking. The infrastructure may simply issue the command "begin emergency braking now" and all vehicles receiving this command will have to apply maximum braking without further delay. This, not only simplifies the task of determining when the vehicle ℓ_1 is performing emergency braking but also minimizes the relative delay in propagating the onset of emergency braking from each vehicle to the vehicle behind, effectively down to zero. In Fig. 11, we have plotted the deceleration profiles for the case of free agents with infrastructure management. Notice that the deceleration profiles for the three vehicles will be similar (and moreover the vehicles ℓ_1 and f_1 will stop at the same time instant in the case where the two vehicles have the same braking capabilities, i.e., in the case where $J_{\ell \max} = J_{f \max}$ and $a_{\ell \max} = a_{f \max}$). Similar to the autonomous case, the braking capabilities of the merging vehicle will be degraded [see (4.1) and (4.2)].

- *Vehicles Platoons Without Coordinated Braking*. This concept represents the possibility that the safest and possibly most cost-effective way of achieving maximum capacity is by making platoons of vehicles the basic controlling unit. Platoons are clusters of vehicles with short spacing between individual vehicles in each group and longer spacing between platoons. The characterizing differentiation is that the platoon is to be treated by the infrastructure as an "entity" thereby minimizing some of the need for communicating with and coordinating individual vehicles. The infrastructure does not attempt to control any individual vehicle under normal circumstances, keeping the cost and necessary bandwidth low. The infrastructure is expected to be an intelligent agent which monitors and coordinates the operation of the platoons. Tight coordination is required within the platoon in order to maintain a close spacing and this requires that the vehicles must be communicating with each other constantly. The significantly longer interplatoon spacing is required to guarantee no interplatoon collisions.

  Each vehicle is expected to be equipped with sensors and intelligence to maintain its lane position, sense its immediate surroundings, and perform the functions of merging into and splitting off a platoon. It is not expected to accomplish lane changes, or merging and splitting without the infrastructure's or the platoons entity's help. The main mode of operation of the infrastructure would be of a request-response type. Each platoon's and/or vehicle's request is processed and appropriate commands are sent to the appropriate vehicles/platoons to respond to that request. The infrastructure takes a more proactive role in monitoring traffic flow, broadcasting traffic flow messages, advising lane changes to individual vehicles and platoons in addition to the usual information provider functions.

  Once a vehicle has merged into a platoon, the headway maintenance controller must take into account the braking capabilities of the vehicle ahead in order to set an appropriate separation distance that minimizes the possibility of collision. The platoon leader may also provide corrections to the individual intraplatoon headways in order to reduce the possibility of a rear-end collision between two vehicles propagating to the other members of the platoon.

  In this concept, we assume that no coordination of the braking sequence takes place within a platoon in order to distinguish it from the next one where coordinated braking is employed. Despite the fact that there is no coordinated braking, each vehicle notifies the vehicle behind about its braking capabilities and the magnitude and timing of the braking force used. When the platoon leader detects an emergency, it immediately notifies the vehicle that follows. There will be a delay while the message propagates from each vehicle to the vehicle behind, as well as an actuation delay. The deceleration profiles for the case of platoons without coordinated braking are shown in Fig. 12. Notice that the merging vehicle's profile is slightly different than the one of the vehicle f_1 due to the degraded braking capabilities of the merging vehicle.
- *Vehicles Platoons With Coordinated Braking*. The platooning concept with coordinated braking is based on the concept of maximizing capacity by carefully coordinating the timing and degree of braking among the vehicles participating in a platoon entity. This allows the minimization of spacing between vehicles without compromising safety. During a braking maneuver the platoon leader may dictate a braking sequence to be followed by each vehicle so that the maneuvers are performed without any intraplatoon collision. Such a sequence may require the last vehicle to brake first, followed by the second to last vehicle, etc. In platooning with coordinated braking,

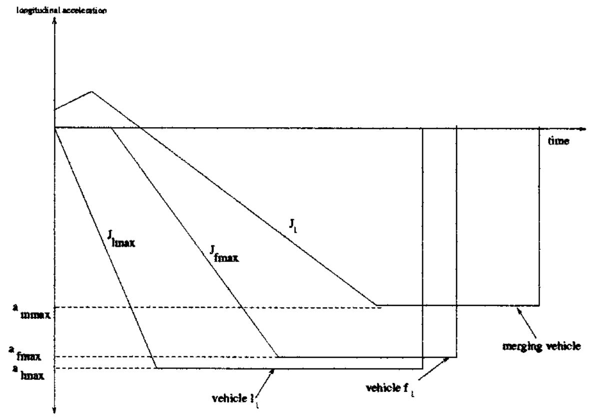

Fig. 12. Platoons without coordinated braking.

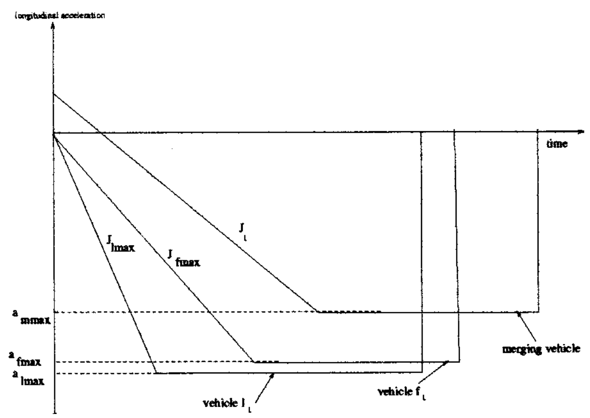

Fig. 13. Platoons with coordinated braking and no delay.

we assume that the platoon leader assumes the primary responsibility of detecting emergencies and notifying each and every vehicle in the platoon. This notification takes place through a network style vehicle to vehicle communication system that minimizes the communication delays. The platoon leader notifies all the vehicles in the platoon about the magnitude of the braking force that is to be applied and also the exact time this is to be applied. This architecture not only eliminates the need for each vehicle to detect the magnitude of the braking and if the braking should be limited or emergency braking, but also can adjust the onset of emergency braking for an effective 0 s relative delay, or even to an artificial negative relative delay. The brake actuation delay can be completely compensated for and is not affecting the scenario as long as it is approximately the same for each vehicle. Fig. 13 shows the deceleration profiles for the case of platooning with coordinated braking and no delays.

V. Safe Spacing for Lane Changing

Consider again the five vehicles involved in a lane changing maneuver as shown in Fig. 1. For simplicity, we will assume that during the lane change maneuver the leading and following vehicles ℓ_1, ℓ_2, f_1, f_2 travel with constant velocities unless an emergency braking happens. Suppose that the jth vehicle (where $j \in \{\ell_1, \ell_2, m\}$) enters an emergency braking situation at the time instant t_s. Then, we have that the longitudinal and lateral accelerations of the five vehicles are as follows:

$$a_1^{(i)}(t;t_s) = \begin{cases} 0, & \text{if } t < t_s \\ \bar{a}_1^{(i)}(t;j), & \text{otherwise} \end{cases} \quad i \in \{\ell_1, \ell_2, f_1, f_2\} \tag{5.1}$$

$$a_2^{(i)}(t) = 0, \quad \forall t, \quad i \in \{\ell_1, \ell_2, f_1, f_2\} \tag{5.2}$$

$$a_1^{(m)}(t;t_s) = \begin{cases} \tilde{a}_1^{(m)}(t), & \text{if } t < t_{m_{\rm emerg}} \\ \bar{a}_1^{(m)}(t;j), & \text{otherwise} \end{cases} \tag{5.3}$$

$$a_2^{(m)}(t;t_s) = \begin{cases} \tilde{a}_2^{(m)}(t), & \text{if } t < t_{m_{\rm emerg}} \\ \bar{a}_2^{(m)}(t;j), & \text{otherwise} \end{cases} \tag{5.4}$$

where

$\bar{a}_1^{(i)}(t;j), i \in \{\ell_1, \ell_2, f_1, f_2\}$ deceleration profile of the ith vehicle in the case where the jth vehicle performs an emergency braking as described in the previous section;

$\tilde{a}_1^{(m)}(t), \tilde{a}_2^{(m)}(t)$ longitudinal and lateral, respectively, acceleration models for the merging vehicle when it performs the lane changing maneuver;

$\bar{a}_1^{(m)}(t;j), \bar{a}_2^{(m)}(t;j)$ longitudinal and lateral, respectively, deceleration profiles of the merging vehicle in the case where the jth vehicle performs an emergency braking as described in the previous section.

More precisely, the longitudinal acceleration $\tilde{a}_1^{(m)}(t)$ is given in Figs. 2 and 3, depending on whether the destination lane moves faster than the originating one, the lateral acceleration $\tilde{a}_2^{(m)}(t)$ is the sinusoidal function given in (2.1) and, finally, the longitudinal and lateral decelerations $\bar{a}_1^{(m)}(t;j), \bar{a}_2^{(m)}(t;j)$ are given in (4.1) and (4.2), respectively. Finally, note that $t_{m_{\rm emerg}}$ is equal to $t_s + t_d$, where t_d denotes the time needed for the merging vehicle to detect and initiate emergency braking.

Based on the above equations, we can calculate the position and velocity of the vehicle $i, i \in \{\ell_1, \ell_2, m, f_1, f_2\}$ as follows:

$$x^{(i)}(t) = x^{(i)}(0) + \int_0^t v^{(i)}(\tau)\, d\tau \tag{5.5}$$

$$v^{(i)}(t) = v^{(i)}(0) + \int_0^t a^{(i)}(\tau)\, d\tau \tag{5.6}$$

where $x^{(i)}(0), v^{(i)}(0)$ denote the initial position and velocity of the vehicle, respectively.

If the initial intervehicle spacing is large enough, then there would be no collision in the case of emergency braking during the lane change maneuver. For a given lane changing policy and a given AHS operational concept, we would like to calculate the minimum value of the initial intervehicle spacing for which there will be no collision. We refer to this value as the *Minimum Safety Spacing During Lane Changing*—(MSSLC) between those two vehicles. Note that we are interested in the following intervehicle distances $d_{\ell_1 f_1}, d_{\ell_1 m}, d_{\ell_2 f_2}, d_{\ell_2, m}, d_{m f_1}$ and $d_{m f_2}$.

Our approach in calculating the MSSLC is as follows: let us consider the intervehicle spacing d_{kh} where d_{kh} is one of the spacing of interest $d_{\ell_1 f_1}, d_{\ell_1 m}, d_{\ell_2 f_2}, d_{\ell_2, m}, d_{m f_1}$ and $d_{m f_2}$. Suppose now that each of the five vehicles travels in the freeway alone, i.e., assume that the other four vehicles are absent. Let $T_s^{(h)}(t_s; j)$ be the *stopping time* of the hth vehicle in the case where the jth vehicle $j \in \{\ell_1, \ell_2, m\}$ starts emergency braking at $t = t_s$, i.e., $T_s^{(h)}(t_s; j)$ is the time at which the hth vehicle velocity is zero. Note now that a collision occurs if the following holds: there exists a time instant $t_c \in [0, {}_s^{(h)}(T_s^{(h)}; j)]$ such that $d_{kh}(t_c)$ is negative and, moreover, the lateral positions of the vehicles k and h satisfy $|x_2^{(k)}(t_c) - x_2^{(h)}(t_c)| < L_{\rm lat}^{kh}$; here $L_{\rm lat}^{kh}$ is defined as follows: suppose that the two vehicles k and h are in two adjacent lanes and their longitudinal positions are the same. Then $L_{\rm lat}^{kh}$ denotes the minimum lateral distance of the vehicles k and h such that if the lateral distance between these two vehicles is larger than $L_{\rm lat}^{kh}$ then the two vehicles do not collide. The definition of the constant $L_{\rm lat}^{kh}$ is necessary because we may have the case where the spacing d_{hk} is negative at a given time-instant but a collision does not occur because the two vehicles are in two different lanes (or their lateral distance is large enough) at this time-instant. In order to incorporate the case where the two vehicles have large enough lateral distance we define the variable $\mathcal{I}_{hk}$ as follows: $\mathcal{I}_{hk} = 1$ if $|x_2^{(k)} - x_2^{(h)}| < L_{\rm lat}^{kh}$ and $\mathcal{I}_{hk} = 0$, otherwise. Then, the MSSLC for the spacing d_{kh} is defined as in (5.7), shown at the bottom of the page. In other words, $D_{\min}^{kh}$ is equal to the maximum distance by which the vehicle h would overtake vehicle k for all possible different emergency braking situations in the case where the two vehicles travel alone. $D_{\min}^{kh} < 0$ implies that a collision occurs, while $D_{\min}^{kh} = 0$ implies that the initial spacing between the vehicles k and h is such that there will be no collision in the case where any one of the vehicles ℓ_1, ℓ_2, m at any time instant during the lane change maneuver enters an emergency braking situation.

We employ an exhaustive search technique in order to calculate the MSSLCs $D_{\min}^{kh}$, i.e., we calculate the intervehicle spacing for all the possible cases of emergency braking, as demonstrated in the following algorithm:

Algorithm for the Calculation of MSSLCs

1. **Choose** the sampling time interval Δt.

2. **Specify** the velocities in originating lane V_o and in destination lane V_d, the initial positions of the vehicles,

and the intended lane change distance d_I. For each vehicle $j \in \{\ell_1, \ell_2, m, f_1, f_2\}$, specify the maximum available acceleration $a_{j\,\max}$ and jerk $J_{j\,\max}$; specify the friction limit constant F_c for the merging vehicle.

Finally, specify the deceleration profiles of the five vehicles based on the analysis of

Section IV, the constants $L_{\rm lat}^{kh}$ and the time constant t_d such that $t_{m_{\rm emerg}} = t_s + t_d$.

3. **Specify the Merging Strategy**: Choose the parameters $t_{\rm long}, t_{\rm lat}, t_{\rm ch}, t_{\rm LC}$ and $a_{\rm comf}$ that determine the merging strategy.

4. **FOR** all $D_{\min}^{kh} \in \{D_{\min}^{\ell_1, f_1}, D_{\min}^{\ell_1, m}, D_{\min}^{\ell_2, f_2}, D_{\min}^{\ell_2, m}, D_{\min}^{m, f_1}, D_{\min}^{m, f_2}\}$ **DO**

(a) Set $D_{\min}^{kh} = 0$.

(b) **FOR** all $j \in \{\ell_1, \ell_2, m\}$ **DO**

i. **FOR** all $t_s \in \{0, \Delta t, 2\Delta t, \ldots, t_{\rm LC}\}$ **DO**

A. Set $t = 0$.

B. Calculate $a^k(t)$ and $a^h(t)$, based on (5.1)–(5.4).

C. Update $v^{(k)}, v^{(h)}, x^{(k)}, x^{(h)}$ based on (5.5), (5.6), i.e., set

$$v^{(i)}(t + \Delta t) = v^{(i)}(t) + \int_t^{t+\Delta t} a^{(i)}(\tau)\, d\tau, \quad i = k, h$$

$$x^{(i)}(t + \Delta t) = x^{(i)}(t) + \int_t^{t+\Delta t} v^{(i)}(\tau)\, d\tau, \quad i = k, h$$

D. Calculate $d_{kh} = x^{(k)}(t + \Delta t) - x^{(h)}(t + \Delta t)$

E. **IF** $D_{\min}^{kh} > -I_{hk}(t + \Delta t) \cdot d_{kh}(t + \Delta t)$ **THEN** set

$$D_{\min}^{kh} = -I_{hk}(t + \Delta t) \cdot d_{kh}(t + \Delta t)$$

F. **IF** $v^{(h)}(t + \Delta t) > 0$ **THEN** set $t = t + \Delta t$ and **GOTO STEP B**

OTHERWISE set $T_s^{(h)}(t_s, j) = t + \Delta t$ and **ENDFOR**

(c) **ENDFOR**

5. **ENDFOR**

VI. SIMULATIONS

We used the algorithm presented in the previous section in order to calculate the *minimum safety distances* for different conditions during a lane changing maneuver. Only the case of autonomous vehicles was considered. For simplicity, all five vehicles were assumed to have the same characteristics and performance. More precisely, we considered five vehicles with length 5 m and maximum deceleration (during braking) and jerk equal to 0.5 g and 50 m/s^3, respectively. The constant F_c for the merging vehicle was set equal to 0.25. In the case where one of the leading vehicles enters in our emergency braking situation, we assumed that the merging vehicle needs 0.3 s to start decelerating and 1 s to detect the emergency braking and 0.3 s to start performing emergency braking. For the two following vehicles, we assumed the same time delays in the case where the emergency-braking leading vehicle is “visible” and in the case where it is not visible we assumed that the following vehicle needs 2 s to start decelerating, 1 s to detect

$$D_{\min}^{kh} = -\min_{t_s \in [0, t_{\rm LC}], j \in \{\ell_1, \ell_2, m\}} \left\{ \min_{t \in [0, T_s^{(h)}(t_s, j)]} \{I_{hk}(t) \cdot d_{kh}(t), 0\} \right\}. \tag{5.7}$$

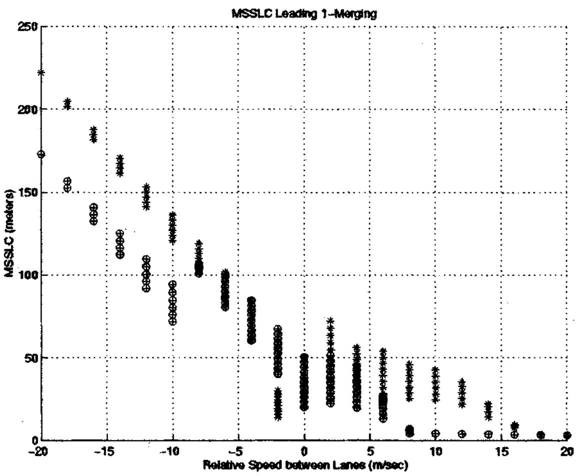

Fig. 14. MSSLC for the spacing between the vehicle ℓ_1 and the merging vehicle versus relative speed between lanes ($\circ$: $a_{\rm comf} = 0.1$ g and $t_{\rm LC} = 5$ s; $\star$: $a_{\rm comf} = 0.3$ g and $t_{\rm LC} = 5$ s; $+$: $a_{\rm comf} = 0.1$ g and $t_{\rm LC} = 10$ s).

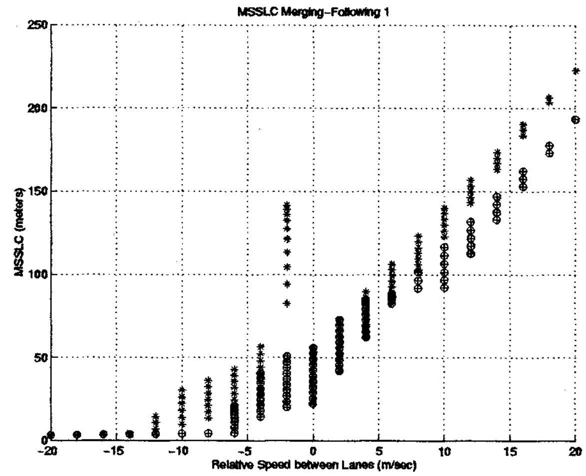

Fig. 16. MSSLC for the spacing between the merging vehicle and the vehicle f_1 versus relative speed between lanes ($\circ$: $a_{\rm comf} = 0.1$ g and $t_{\rm LC} = 5$ s; $\star$: $a_{\rm comf} = 0.3$ g and $t_{\rm LC} = 5$ s; $+$: $a_{\rm comf} = 0.1$ g and $t_{\rm LC} = 10$ s).

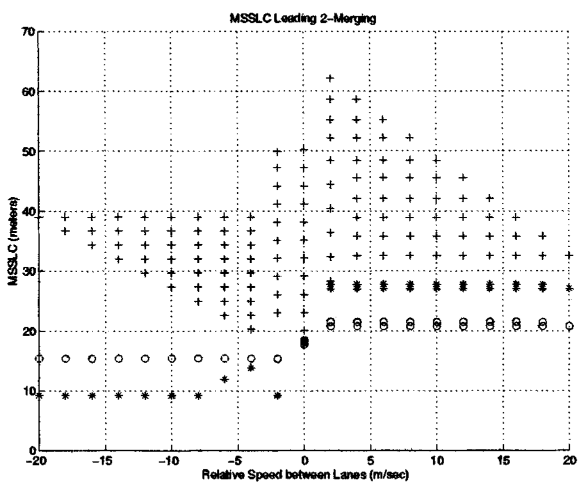

Fig. 15. MSSLC for the spacing between the vehicle ℓ_2 and the merging vehicle versus relative speed between lanes ($\circ$: $a_{\rm comf} = 0.1$ g and $t_{\rm LC} = 5$ s; $\star$: $a_{\rm comf} = 0.3$ g and $t_{\rm LC} = 5$ s; $+$: $a_{\rm comf} = 0.1$ g and $t_{\rm LC} = 10$ s).

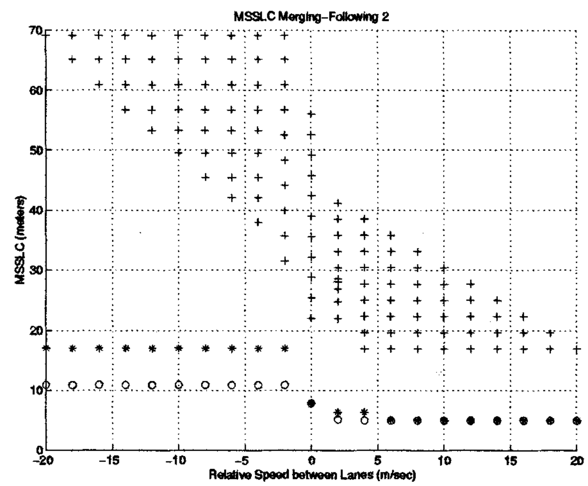

Fig. 17. MSSLC for the spacing between the merging vehicle and the vehicle f_2 versus relative speed between lanes ($\circ$: $a_{\rm comf} = 0.1$ g and $t_{\rm LC} = 5$ s; $\star$: $a_{\rm comf} = 0.3$ g and $t_{\rm LC} = 5$ s; $+$: $a_{\rm comf} = 0.1$ g and $t_{\rm LC} = 10$ s).

the emergency braking, and 0.3 s to start performing emergency braking. The constants $L_{\rm lat}^{kh}$ are set equal to 2 m.

Regarding now the particular lane changing policy of the merging vehicle we chose the various parameters as follows. In order to simplify the problem we assumed that the time (delay) needed for the merging vehicle to switch from $a_{\rm comf}$ deceleration/acceleration to 0 or $a_{\rm comf}$ acceleration/deceleration was negligible (i.e., this time was set equal to zero in the simulations). Moreover, we set $t_{\rm lat} = t_{\rm ch} = 0$ and tried different values for the $a_{\rm comf}$ in order to cover many possible cases of different lane changing maneuver strategies.

We run three different simulations. In all three simulations we calculated the MSSLC functions for the case where the speed in the destination and originating lane covers the range between 10 and 30 m/s. In the first simulation, we set $a_{\rm comf} = 0.1$ g and the time $t_{\rm LC}$ needed for the lane changing maneuver to be completed was set equal to 5 s. In the second simulation, we increased $a_{\rm comf}$ to 0.3 g and we kept $t_{\rm LC} = 5$ s. In the third simulation, we set $a_{\rm comf}$ equal to 0.1 g and we increased $t_{\rm LC}$ to 10 s.

Figs. 14–17 show the MSSLC values versus relative speed between the originating and the destination lanes for the three different simulations, while Figs. 18–21 show the worst-case emergency braking time for the various intervehicle spacing versus the relative speed $V_d - V_o$ between the two lanes for the three different simulations. For a particular intervehicle spacing between two vehicles and given velocities in the two lanes, we define the time instant "worst-case emergency braking time" such that if emergency braking starts at this instant, then the required safety spacing for the particular two vehicles is the maximum. Note that in each figure, more than one point (i.e., more than one MSSLC) corresponds to each relative speed point; those MSSLC points correspond to different absolute speed values. For example, when the relative speed is -5 the various MSSLC points that correspond to this relative speed are for the cases, where $(V_o = 30, V_d = 25), (V_o = 28, V_d = 23), (V_o = 26, V_d = 21)$, etc. The points that correspond to the highest MSSLCs are the points that correspond to the highest absolute speed values. In Figs. 22–25, we plot the maximum MSSLCs versus relative speed between the two lanes, where by maximum

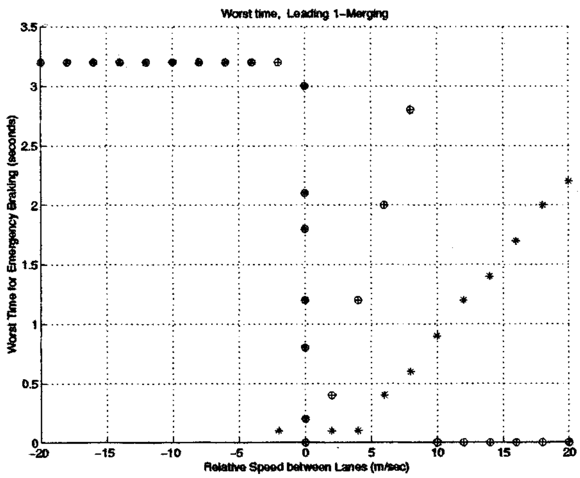

Fig. 18. Worst-case emergency braking time for the spacing between the vehicle ℓ_1 and the merging vehicle versus relative speed between lanes (∘: $a_{\rm comf} = 0.1$ g and $t_{\rm LC} = 5$ s; $\star$: $a_{\rm comf} = 0.3$ g and $t_{\rm LC} = 5$ s; $+$: $a_{\rm comf} = 0.1$ g and $t_{\rm LC} = 10$ s).

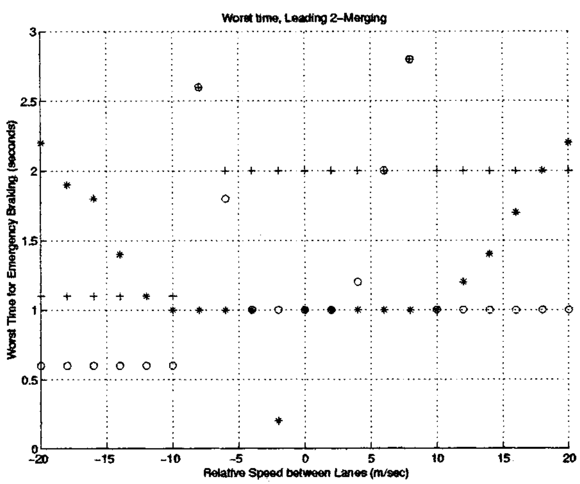

Fig. 19. Worst-case emergency braking time for the spacing between the vehicle ℓ_2 and the merging vehicle versus relative speed between lanes (∘: $a_{\rm comf} = 0.1$ g and $t_{\rm LC} = 5$ s; $\star$: $a_{\rm comf} = 0.3$ g and $t_{\rm LC} = 5$ s; $+$: $a_{\rm comf} = 0.1$ g and $t_{\rm LC} = 10$ s).

MSSLC we mean the maximum MSSLC that corresponds to a given relative speed point.

By observing Figs. 14–25, we can see that

- The MSSLC for the spacing between the leading vehicle in the destination lane and the merging vehicle decreases quickly as the relative speed between the originating and the destination lane increases. On the other hand, the MSSLC for the spacing between the merging vehicle and the following vehicle in the destination lane increases quickly as the relative speed between the originating and the destination lane decreases, while the spacing between the merging vehicle and the leading and following vehicle in the originating lane remains small. Moreover, for $a_{\rm comf} = 0.1$, the spacing between the merging vehicle and the leading and following vehicle in the originating lane remain almost constant as long as the sign of the relative speed between the two lanes remains constant.

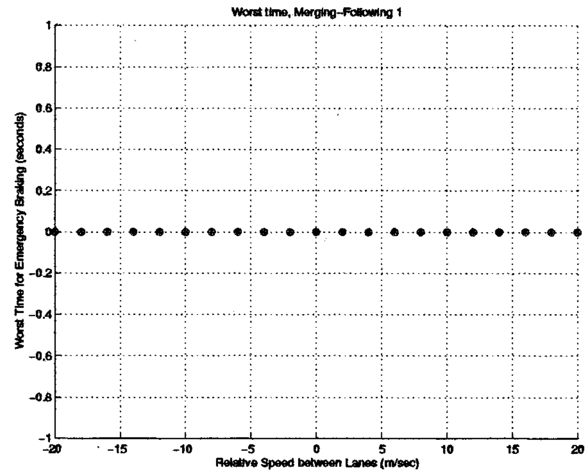

Fig. 20. Worst-case emergency braking time for the spacing between the merging vehicle and the vehicle f_1 versus relative speed between lanes (∘: $a_{\rm comf} = 0.1$ g and $t_{\rm LC} = 5$ s; $\star$: $a_{\rm comf} = 0.3$ g and $t_{\rm LC} = 5$ s; $+$: $a_{\rm comf} = 0.1$ g and $t_{\rm LC} = 10$ s).

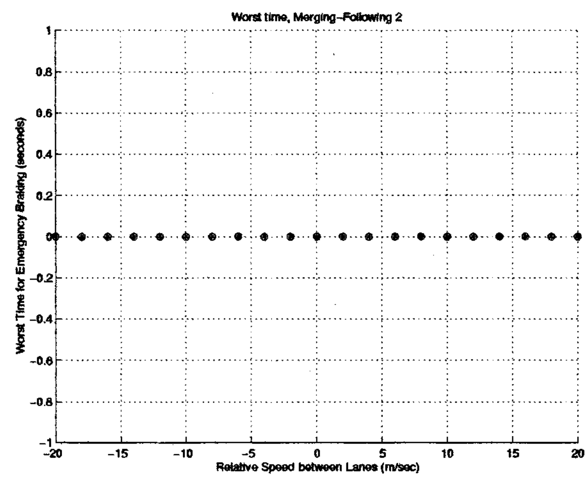

Fig. 21. Worst-case emergency braking time for the spacing between the merging vehicle and the vehicle f_2 versus relative speed between lanes (∘: $a_{\rm comf} = 0.1$ g and $t_{\rm LC} = 5$ s; $\star$: $a_{\rm comf} = 0.3$ g and $t_{\rm LC} = 5$ s; $+$: $a_{\rm comf} = 0.1$ g and $t_{\rm LC} = 10$ s).

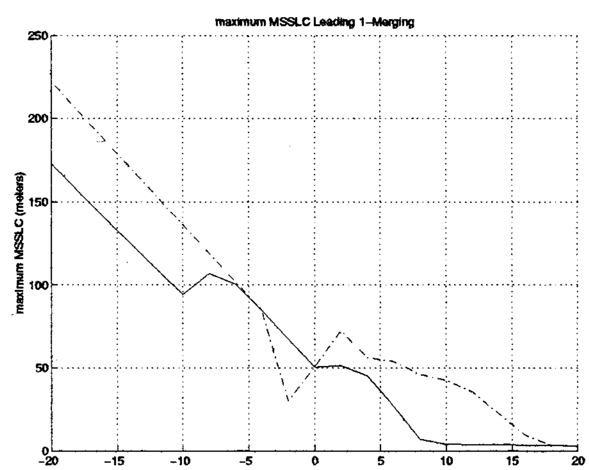

Fig. 22. Maximum MSSLC for the spacing between the vehicle ℓ_1 and the merging vehicle versus relative speed between lanes (solid curve: $a_{\rm comf} = 0.1$ g and $t_{\rm LC} = 5$ s; dash-dotted curve: $a_{\rm comf} = 0.3$ g and $t_{\rm LC} = 5$ s; dashed curve: $a_{\rm comf} = 0.1$ g and $t_{\rm LC} = 10$ s).

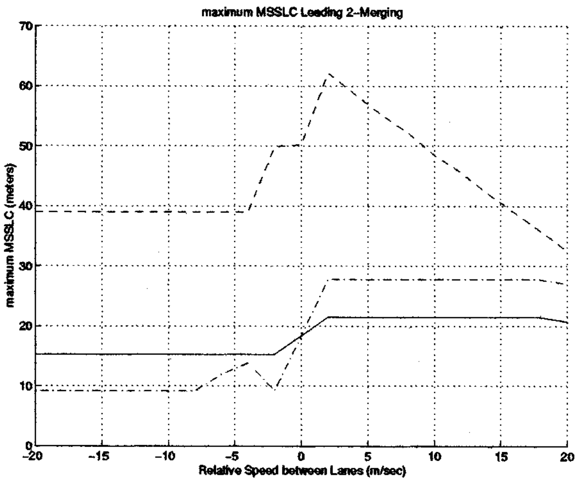

Fig. 23. Maximum MSSLC for the spacing between the vehicle ℓ_2 and the merging vehicle versus relative speed between lanes (solid curve: $a_{\rm comf} = 0.1$ g and $t_{\rm LC} = 5$ s; dash-dotted curve: $a_{\rm comf} = 0.3$ g and $t_{\rm LC} = 5$ s; dashed curve: $a_{\rm comf} = 0.1$ g and $t_{\rm LC} = 10$ s).

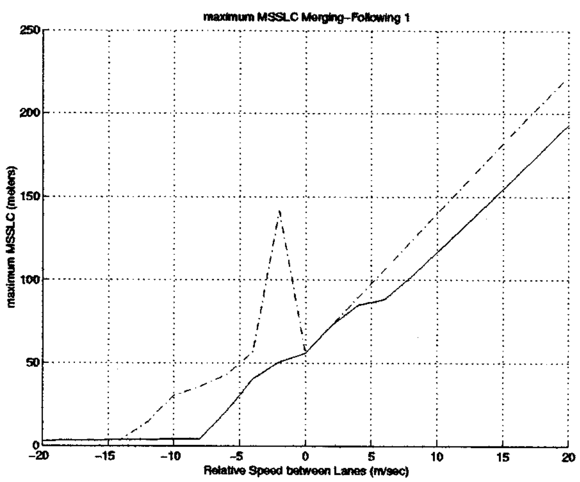

Fig. 24. Maximum MSSLC for the spacing between the merging vehicle and the vehicle f_1 versus relative speed between lanes (solid curve: $a_{\rm comf} = 0.1$ g and $t_{\rm LC} = 5$ s; dash-dotted curve: $a_{\rm comf} = 0.3$ g and $t_{\rm LC} = 5$ s; dashed curve: $a_{\rm comf} = 0.1$ g and $t_{\rm LC} = 10$ s).

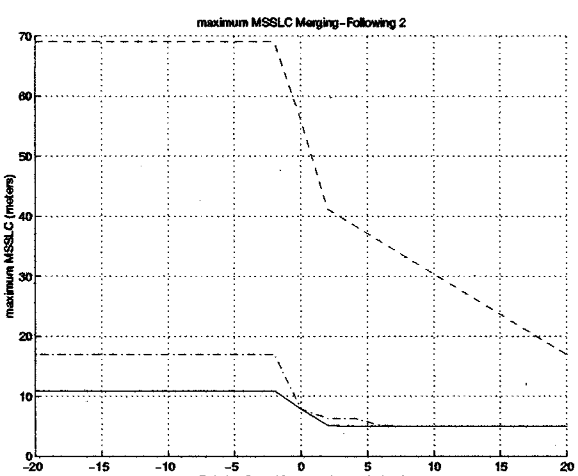

Fig. 25. Maximum MSSLC for the spacing between the merging vehicle and the vehicle f_2 versus relative speed between lanes (solid curve: $a_{\rm comf} = 0.1$ g and $t_{\rm LC} = 5$ s; dash-dotted curve: $a_{\rm comf} = 0.3$ g and $t_{\rm LC} = 5$ s; dashed curve: $a_{\rm comf} = 0.1$ g and $t_{\rm LC} = 10$ s).

- The magnitude of the "control variable" $a_{\rm comf}$ significantly affects the MSSLC. In general, as can be seen in Figs. 14–17, the more aggressive the adjustment in the longitudinal direction (i.e., the larger $a_{\rm comf}$ is), the larger the safety spacing required.
- The "aggressiveness" of the lane change maneuver specified by the magnitude of the variable $t_{\rm LC}$ does not affect the MSSLC for the spacing between the merging vehicle and the vehicles in the originating lane, but considerably affects the MSSLC for the spacing between the merging vehicle and the vehicles in the originating lane. For instance, the MSSLCs for the spacing between the merging vehicle and the vehicles in the originating lane are in the range 4–30 m in the case where $t_{\rm LC} = 5$ s and they increase considerably (in the range 16–70 m) in the case where $t_{\rm LC} = 10$ s.
- The main factor that affects the MSSLC is the relative speed between the two lanes. In general, the smaller the relative speed between the two lanes, the less the required safety spacing.
- The sensitivity of the MSSLC with respect to the absolute speed of the two lanes is large. As can be seen in the Figs. 14–17 for a given relative speed between the two lanes, the MSSLC obtained for high absolute speeds is sometimes up to 50 m larger than the MSSLC obtained for the same relative speed but for low absolute speeds. On the contrary, the worst case emergency braking time is not affected by the absolute speed, as can be seen in Figs. 18–21.
- The worst time for an emergency braking heavily depends on the relative speed between the two lanes as well. Moreover, the worst-case emergency braking time for a particular spacing between two vehicles as a function of the relative speed between the two lanes, differs a lot with the worst-case emergency braking time of the spacing between two other vehicles. Therefore, when given the lane changing policy and the relative speed between the two lanes, we cannot specify a particular time instant of the lane-changing maneuver as the most "dangerous" for emergency braking, i.e., the time instant at which emergency braking would produce the worst results.

VII. Conclusion

In this work, we analyzed the problem of collision-free merging and lane changing. We considered six different AHS operational concepts. We presented the braking profiles of the vehicles for each operational concept and we investigated the effects of the particular operational concept on the MSSLC. The braking profiles of the vehicles involved in a lane changing maneuver depend on the particular AHS operational concept, i.e., on the degree of communication between the vehicles and between the vehicles and the infrastructure. We examined various alternative scenaria for merging and lane changing and we presented an algorithm for calculating the MSSLC. The calculation of the MSSLC for the merging or lane changing maneuver is more complicated than the calculation of the minimum safety spacing for the pure longitudinal case, since in the former case we have to take into account the particular lane changing policy of the merging vehicle as well as the effect of combined lateral/longitudinal motion during the lane changing maneuver.

REFERENCES

[1] J. L. Bascunana, "Analysis of lane change crash avoidance," presented at the *Proc. Systems and Issues in ITS Conf.*, Costa Mesa, CA, Aug. 1995. SP-1106.

[2] K. Enke, "Possibilities for improving safety within the driver vehicle environment control loop," presented at the Proc. 7th Int. Technical Conf. Experimental Safety Vehicles, Washington, DC, July 1979.

[3] P. A. Ioannou, A. Kanaris, and F.-S. Ho, "Spacing and capacity evaluations for different AHS concepts," Univ. Southern California, Los Angeles, Tech. Rep., 1995.

[4] T. D. Gillespie, *Fundamentals of Vehicle Dynamics*. Warrendale, PA: SAE, 1992.

[5] S. E. Shladover, C. A. Desoer, J. K. Hedrick, M. Tomizuka, J. Walrand, W. B. Zhang, D. McMahon, and S. Sheikholeslam, "Automatic vehicle control development in the PATH program," *IEEE Trans. Veh. Technol.*, vol. 40, pp. 114–130, Feb. 1991.

[6] *A Handbook for Intervehicle Spacing in Vehicle Following*, California PATH Res. Rep., UCB-ITS-PRR-95-1, Univ. Southern California, Los Angeles, 1995.

[7] R. D. Worall and A. G. R. Bullen, "An empirical analysis of lane changing on multilane highways," *Hwy. Res. Board*, vol. 303, pp. 30–43, 1970.

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