

Trajectory Prediction of a Lane Changing Vehicle Based on Driver Behavior Estimation and Classification

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Abstract—Accurate trajectory prediction of a lane changing vehicle is a key issue for risk assessment and early danger warning in advanced driver assistance systems(ADAS). This paper proposes a trajectory prediction approach for a lane changing vehicle considering high-level driver status. A driving behavior estimation and classification model is developed based on Hidden Markov Models(HMMs). The lane change behavior is estimated by observing the vehicle state emissions in the beginning stage of a lane change procedure, and then classified by the classifier before the vehicle crosses the lane mark. Furthermore, the future trajectory of the lane changing vehicle is predicted in a statistical way combining the driver status estimated by the classifier. The classifier is trained and tested using naturalistic driving data, which shows satisfactory performance in classifying driver status. The trajectory prediction method generates different trajectories based on the classification results, which is important for the design of both autonomous driving controller and early danger warning systems.

I. INTRODUCTION

Preventing transportation-related fatalities and improving traffic safety is one of the most essential issues in current intelligent transportation systems. Researches on advanced driver assistance systems(ADAS), road safety infrastructure, and autonomous vehicles play an important role in preventing collision related traffic fatalities and achieving driving safety. Currently, most driver assistance systems can only make a decision based on the current traffic situation, which makes it hard for the systems to detect a danger at an early stage[1]. Trajectory prediction of the surrounding vehicles is an effective way achieving this early-stage danger response goal. Specifically, for a vehicle running on highway, it is desirable to predict the future motion of a lane changing vehicle ahead when conducting safe behavior decisions. A driver may conduct a lane change behavior due to different reasons, e.g., keeping a relatively high traveling speed, overtaking a truck convoy, emergency obstacle avoidance in the original lane, and so forth. Different high-level driver states have great impacts on the correlated trajectory emission, which makes the prediction result relying solely on normal driving behavior inaccurate for a dangerous case.

This paper takes the different high-level driver states, i.e. states under normal or dangerous driving behavior, into consideration when predicting the future trajectory of a

lane changing vehicle. In addition, by checking whether the driver of the lane changing car is driving dangerously, the interactions between the surrounding vehicles are also considered implicitly.

II. RELATED WORK

Predicting accurate future trajectory of a lane changing vehicle is very challenging. Some approaches have been proposed using model based methods[2][3]. Another way of solving the prediction problem is to predict the future vehicle motions based on the observation of past vehicle movements[5][6]. In addition to simply using the system dynamics, approaches considering driver's intention[7], behavior difference under different traffic situation [8][11] are also proposed to improve prediction accuracy. Sorstedt, et al[4] proposed a long term motion prediction approach with respect to driver intentions by including the expected driver input in their vehicle motion model. To estimate and recognize driver intentions, approaches based on Hidden Markov Models(HMMs) are proposed to identify the underlying hidden driver states[9][10]. Gadepally, et al[12] uses HMM to estimate and predict the high-level driver states at an intersection. They also discussed the combination of HMM with a hybrid state system architecture. However, these approaches pay more attention to the driver intentions under different traffic situations rather than the driver behavior difference within one certain traffic scenario. Instead of predicting the future driving intention of a lane changing vehicle, this paper focuses on whether the driver will conduct a dangerous lane change behavior and predict different trajectories based on the classification results. The novel points that set this work apart from the former HMM approaches is that i) the long term trajectory is predicted only using observations from the beginning stage of a lane change process(the detailed stage partition is given in section III); ii) the motion prediction method takes the high-level driver states into consideration, which makes the prediction results more reasonable. For the first point, instead of observing a long term vehicle state emission and estimating the corresponding driver states, e.g. go through, turn left, turn right, or stop, similar to [12], this paper focuses on estimating the sub-state transitions during the first stage of a lane changing vehicle.

The rest of the paper is outlined as follows: Section III describes the behavior modeling and stage partition of a lane changing vehicle. Based on the stage partition, the trajectory prediction framework is also given in this section. In section IV, the driving behavior estimation and classification approach is proposed, as well as the data extraction method.

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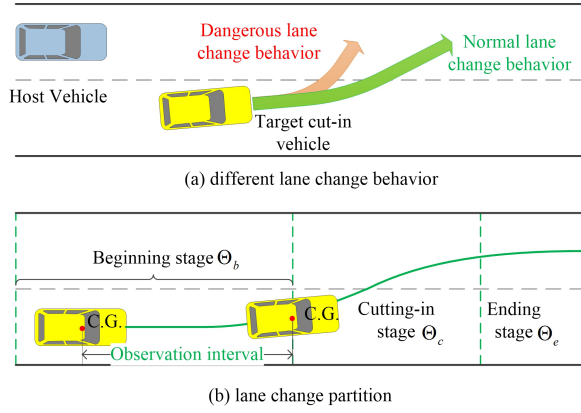


Fig. 1. Lane change scenario description and process partition

Section V shows the classification results of the proposed classifier and the trajectory prediction results. A conclusion of the work is given in section VI.

III. BEHAVIOR MODELING AND PREDICTION FRAMEWORK

A. Lane Change Behavior Modeling

This paper focuses on the trajectory prediction of a lane changing vehicle before it crossing the lane mark. The approach assumes that, for the same traffic scenario, normal driving behavior and dangerous driving behavior have different high level driver states, hence, have different emission patterns. As shown in Figure 1(a), different driving behavior will generate different lane change trajectories, which have different impacts on the control strategy of the host vehicle. In order to estimate the driver/vehicle state and to check whether the lane change behavior is dangerous at an early time, a three-stage partition of the lane change process is introduced, as shown in Figure 1(b). The first stage Θ_b involves the driver/vehicle states when the vehicle does not cross the lane mark. Θ_c , which is also the trajectory prediction interval, includes the driver/vehicle states from the end point of Θ_b to the point when the vehicle reaches the center of the other lane. The rest of the lane change behavior is allocated to Θ_e . Part of the vehicle state emission sequence in Θ_b is observed, which is defined as the *observation interval*. The observations from *observation interval* is processed to estimate the current driving status when the cut in vehicle reaches the end of Θ_b . The driving status is given as the follows:

$$\mathcal{S}_{\Theta_b} \in \{dangerous, normal\} \quad (1)$$

here *dangerous* driving is defined as driving behavior with unexpected manners involved during lane change, e.g. fatigue or aggressive driving, emergency obstacle avoidance, etc.. The driving status of the *cutting-in* stage is computed using a transition probability $p_{i,j}$ once the status of Θ_b is estimated.

$$p_{i,j} = P(\mathcal{S}_{\Theta_c} = i | \mathcal{S}_{\Theta_b} = j) \quad (2)$$

where $i, j \in \{dangerous, normal\}$.

B. Motion Prediction Framework

In order to analyze the behavior of the driver/vehicle system, and further estimate and classify different driving behavior, a hybrid control system layout is used to describe the lane changing vehicle[13]. A hybrid state system(HSS) consists of a high level discrete state system(DSS) and a low level continuous state system(CSS), which appropriately expresses the driver decision making process and the vehicle dynamic evolution of the driver/vehicle system. In this paper, the HSS model is simplified as a discrete hybrid automaton, which can be defined as follows

$$\begin{cases} \delta_e(t) = h(x_c(t), u_c(t), t) \\ i(t) = f_M(x_l(t), u_l(t), \delta_e(t)) \\ y_c(t) = C^{i(t)}x_c(t) + D^{i(t)}u_c(t) + g^{i(t)} \\ y_l(t) = g_l(x_l(t), u_l(t), \delta_e(t)) \\ x_c(t+1) = A^{i(t)}x_c(t) + B^{i(t)}u_c(t) + f^{i(t)} \\ x_l(t+1) = f_l(x_l(t), u_l(t), \delta_e(t)) \end{cases} \quad (3)$$

with the initial condition $[x_c(0), x_l(0)]^T \in \mathbb{R}^{n_c} \times \{0, 1\}^{n_l}$, and inputs $[u_c(t), u_l(t)]^T \in \mathbb{R}^{m_c} \times \{0, 1\}^{m_l}$. Here $\delta_e(t)$ and $i(t)$ are indicators from the event generator Φ and mode selector Ψ , respectively, as shown in Figure 2. $x_c(t), x_l(t), y_c(t), y_l(t)$ are the continuous and logic state and output, respectively. More details on HSS of a driver/vehicle system can be found in previous work [14].

Using the discrete hybrid automaton expression, the high level driving states could be modeled as a finite state machine, and the low level vehicle states as a switched affine system. The high level state system interacts with the low level state system via a mode selector and an event generator set at the interface. This makes the high level decision logic and low level vehicle control system states more explicit when tracking the hidden driving states for the estimation model in the motion prediction framework. In order to obtain precise trajectory prediction of the lane changing vehicle, a trajectory prediction framework is proposed, as shown in Figure 2. The prediction framework consists of three parts: the hybrid system representing the driver/vehicle dynamic of the lane changing vehicle(target vehicle); the driver behavior classifier estimating the current driver status of the target vehicle; and the trajectory prediction module generating the future vehicle trajectories of the cutting-in stage. The driving behavior classifier takes the vehicle state emission from the target vehicle and calculate the probabilities of two HMM models representing normal and dangerous driving behavior, respectively. The trajectory prediction module generates the statistic trajectory based on the real lane-change data set when considering the driver behavior classification result. More detailed descriptions of the classifier and the trajectory prediction modules will be given in section IV.

IV. CLASSIFICATION METHOD

In order to know whether a driver is under dangerous driving, it is essential to get the driver's current decision state. However, it's difficult to directly estimate the current high-level driving states using a hybrid state system. To solve this problem, the high-level driver state transition is formulated

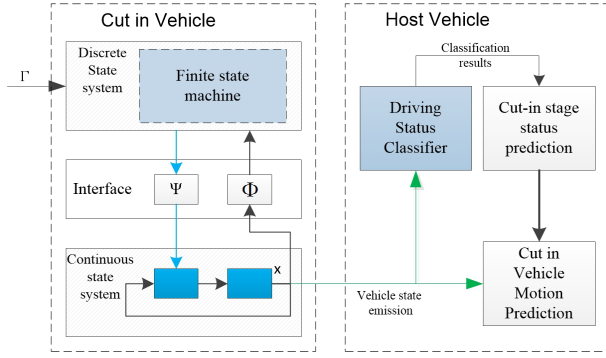


Fig. 2. Lane change trajectory prediction framework, this framework assumes that the state emissions of the cut-in vehicle could be observed via V2V network or sensors mounted on the host vehicle.

as a stochastic process and is estimated using HMM by considering the historical states and emissions.

A. HMM and Behavior Classifier

A Hidden Markov Model (HMM) can be expressed as a tuple $\lambda = \{N, M, \pi, T, e\}$, which consists of a series of N discrete hidden states and the corresponding observations for each state[15]. The observations are dependent on the hidden states, and each state has a probability distribution over each possible output. An HMM expresses the driver/vehicle dynamic system in a stochastic way properly while eliminating the low-level vehicle dynamic evolution. In addition, the modeled system will be in one of the states $q_k = s_i$ at any given time k , and the hidden states are the same as that of the finite state machine in HSS. The probability of transiting from the current state to another state is given by the transition probability matrix T .

$$T_{ij} = P(q_{k+1} = s_j | q_k = s_i) \quad (4)$$

The initial state distribution π shows the probability of each state from which the system starts when the lane change vehicle reaches the *observation interval*, where

$$\pi_i = P(q_1 = s_i), \forall i \in [1, N] \quad (5)$$

The probability of a driving state s_i emitting a certain observation x_k is given by $e_i(x_k)$. In this paper, the emission distribution is modeled as a Joint Gaussian distribution $\mathcal{N}(\mu, \Sigma)$, with

$$\mu = \begin{bmatrix} \mu_1 \\ \mu_2 \\ \vdots \\ \mu_M \end{bmatrix}, \Sigma = \begin{bmatrix} \sigma_{11}^2 & \sigma_{12}^2 & \cdots & \sigma_{1M}^2 \\ \sigma_{21}^2 & \sigma_{22}^2 & \cdots & \sigma_{2M}^2 \\ \vdots & \vdots & \ddots & \vdots \\ \sigma_{M1}^2 & \sigma_{M2}^2 & \cdots & \sigma_{MM}^2 \end{bmatrix} \quad (6)$$

Using an HMM, the maximum priori probability of observing a given series of observations of the vehicle states can be computed out using the *forward algorithm* [16]. The forward algorithm can determine how well a given HMM model λ fits a observation sequence $\mathbf{x} = x_1, x_2, \dots, x_K$ by calculating the conditional probability of observing \mathbf{x} given the model λ , i.e. $P(\mathbf{x}|\lambda)$. The algorithm is defined as follows. Let $\alpha_i(k)$ be the probability of observing the

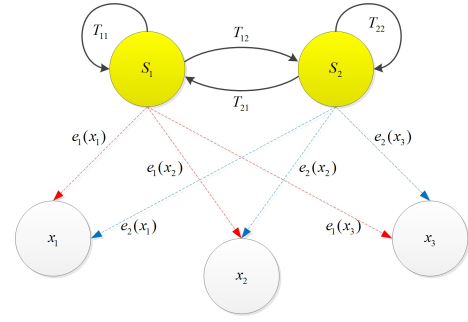


Fig. 3. A diagram of the Hidden Markov Model

beginning k observations with the k^{th} state $q_k = s_i$ given the model λ , which is denoted by

$$\alpha_i(k) = P(x_1, x_2, \dots, x_k, q_k = s_i | \lambda) \quad (7)$$

The forward algorithm is then initialized using the initial state distribution π ,

$$\alpha_i(1) = \pi_i e_i(x_1), i = 1, \dots, n. \quad (8)$$

Basing on $\alpha_i(1)$, the probability of the subsequent observation sequence $k = 2, \dots, K$ is given by

$$\alpha_j(k) = \left[\sum_{i=1}^N \alpha_i(k-1) T_{ij} \right] e_j(x_k) \quad (9)$$

After computing the probabilities at $k = K$, the desired probability is returned as

$$P(\mathbf{x}|\lambda) = \sum_{i=1}^N \alpha_i(K) \quad (10)$$

However, usually the HMM describing the driver/vehicle states is not given, other methods that could train an HMM using the observation data is needed. In order to get the HMM model before using the forward algorithm to classify the driving behavior, two HMMs representing normal and dangerous driving behavior are trained with historical observation data using the EM algorithm[17]. Given a set of K observation sequences x_1, x_2, \dots, x_K , the EM algorithm, which is also known as the Baum-Welch method, will compute the maximum-likelihood estimates of the HMM parameters, i.e.,

$$\lambda^*(\mathbf{T}, \pi, e) = \arg \max_{\lambda} P(x_1, \dots, x_K | \lambda(\mathbf{T}, \pi, e)) \quad (11)$$

To get λ^* , both forward algorithm and backward algorithm[16] are used for the training. The backward algorithm could be defined in a similar way as the forward algorithm.

$$\beta_i(k) = P(x_{k+1}, \dots, x_K | q_k = s_i, \lambda) \quad (12)$$

where $\beta_i(k)$ is the probability of observing the rest of the observation sequence as x_{k+1}, \dots, x_K , given the k^{th} state $q_k = s_i$ with $\beta_i(K) = 1$, then

$$\beta_i(k) = \sum_{j=1}^N T_{ij} \beta_j(k+1) e_j(k+1) \quad (13)$$

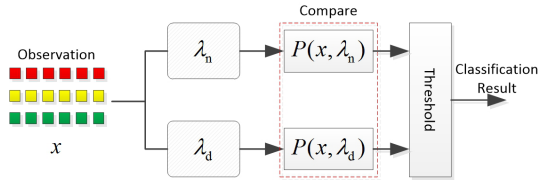


Fig. 4. Classification architecture based on HMM

The EM algorithm trains HMM models in an iterative procedure with real lane-change data. A more detailed introduction could be found in [17]. The two trained HMM models are then used for driving behavior estimation in the driving behavior classifier. The data of the lane changing vehicle obtained from the observation interval are tested in both HMM models, and the conditional probabilities $P(x|\lambda_i)$ are calculated using forward algorithm. The probabilities $P(x, \lambda_i)$ are then calculated using Bayes rule.

$$P(x, \lambda_i) = P(x|\lambda_i)P(\lambda_i), i \in \{n, d\} \quad (14)$$

where the prior probability $P(\lambda_n)$ is set equal to $P(\lambda_d)$. The classification result is computed by comparing the probabilities returned from the two HMMs, as shown in Figure 4. Further vehicle trajectory prediction is conducted in a statistical way based on the classification results.

B. Observation Sequence Extraction

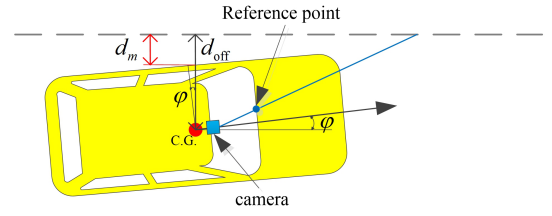
In order to train the HMM models for both normal and dangerous driving, naturalistic driving data sets containing lane change behavior are needed. In this paper, the SHRP2 sample data[18] and the 100Car near-crash data[19] are used to train the normal lane change model and dangerous lane change model, respectively. The ending position of Θ_b is calculated out when the lateral distance d_m between the middle point of the vehicle broadside and the lane mark reaches a certain threshold d_m^* , as shown in Figure 5(a). However, it's difficult to compute the lateral distance d_m directly only using the video stream from the SHRP2 sample data. Note that d_m could be calculated out using the vehicle offset from the lane mark:

$$d_m = d_{off} - \frac{W}{2 \cos \varphi} \quad (15)$$

where φ and W are the yaw angle and the width of the lane changing vehicle, respectively. Furthermore, the yaw angle is small and could be approximated as $\cos \varphi = 1$ during lane changing. d_m could be approximated as

$$d_m = d_{off} - \frac{W}{2} \quad (16)$$

For a given lateral distance threshold d_m^* , the corresponding lateral offset is calculated as $d_{off}^* = d_m^* + \frac{W}{2}$. To get d_m^* from the video stream, a reference point is calibrated as shown in figure 5(b). The reference point is a fixed point along the lower edge of the vehicle front windshield, and is defined as the intersection point with the lane mark when the vehicle lateral offset equals d_{off}^* . Here we assume the camera is mounted at the upper middle behind the front windshield.



(a) Lane offset Calculation and Measurement



(b) Reference Point Calibration in Video Stream

Fig. 5. Lane offset calculation and measurement from video stream, the vehicle is making a left lane change; two reference points are calibrated for left and right lane change, respectively.

The reference point is used to draw out time stamps when vehicle reaching the end of Θ_b . A backward data extraction is processed with a certain backward time interval t_{ob} that is shown as the *observation interval* in figure 1(b) once all the time stamps are extracted out. The extracted data are input to the behavior classifier, a classification result is then computed stating whether the vehicle is in a dangerous lane change behavior.

C. Trajectory Prediction

The prediction of vehicle trajectory in Θ_c is conducted in a statistical way by analyzing large lane change data set and computing a prediction trajectory using regression curve. For example, given a certain lane change speed range, the trajectory in Θ_c , shown in figure 1(b), can be extracted in a similar way as that in observation sequence extraction. With the real trajectory samples for both normal and dangerous driving situations, a statistical trajectory, e.g. the average trajectory, is generated and used as the prediction trajectory. In this paper, instead of using the trajectory data in the prediction interval that are difficult to get directly, the *absolute offsets* are used to analyze and compute the prediction trajectory. The absolute offset takes the absolute position offset value at each sampling time with the initial yaw angle set to 0. The advantage of using an *absolute offset* is that it could analyze left and right lane changes in the same quadrant of a Cartesian coordinate system. The predicted trajectory can then be calculated using the absolute offset values, and the corresponding initial yaw angle and

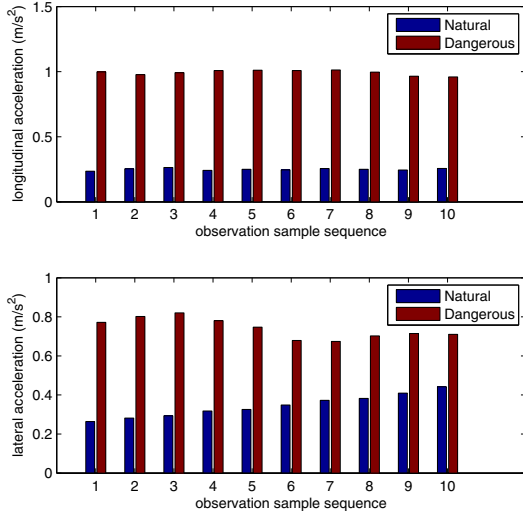


Fig. 6. Mean lateral/longitudinal acceleration values during observation interval under two different situations

speed range. In addition, interpolation methods, e.g. Bezier interpolation scheme, can be used to obtain the intermediate values.

V. RESULTS

A. Statistics of Observation Sequences

In order to see whether two different kinds of vehicle states will appear under different driving states, the statistical analysis of the observation data is conducted. The mean value of four observation parameters of the lane changing vehicle is calculated based on real driving data. The results are shown in Figure 6 and Figure 7. Figure 6 shows the mean longitudinal and lateral acceleration/deceleration value of the statistic driving data with two kinds of driving behavior. The dangerous lane change behavior generates larger longitudinal and lateral acceleration/deceleration values even with a lower mean velocity, as shown in Figure 7. Figure 7 also shows that the yaw rate of dangerous driving is larger than that of normal driving, which explains the acceleration/deceleration value difference in Figure 6.

B. Classification Results

Using the observation sequence extraction method proposed in section IV-B, 210 normal lane change instances from the SHRP2 sample data set and 140 dangerous(near crash) lane change instances from the 100Car near-crash data set are extracted and used to train the HMM models in the classifier for both normal and dangerous lane change behavior. Another 40 normal driving runs and 40 dangerous driving runs within the same speed range are used to test the classification result of the designed classifier. The classification results are shown in table I. The classifier shows a satisfactory performance dealing with lane change behavior classification, though the false negative (missing) shows a higher rate than false positive (false alarm). The higher false

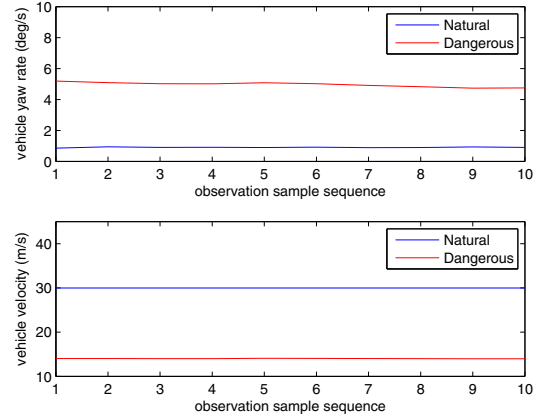


Fig. 7. Mean vehicle yaw rate and velocity during observation interval under two different situations

negative rate might be caused by the scale difference of the data sets that are used to train the HMM models.

TABLE I
CLASSIFICATION RESULTS OF LANE CHANGE BEHAVIOR

behavior	true	false	percentage %
normal	37	3	92.5
dangerous	32	8	80.0

C. Trajectory Prediction and Deviation Analysis

Given the driving behavior classification results of a lane changing vehicle, the following vehicle trajectory can be predicted in a statistical way using the correlated driving data set. The predicted absolute trajectory offset with 50 and 85 percentage for both normal and dangerous driving are shown in Figure 8 along with absolute offsets of the real trajectories. To use the obtained driving data reasonably, the longitudinal traveling speed range for the data samples are set from 20 m/s to 30 m/s. While a smaller speed range division can make the lane change data more consistent, too small speed range could cause excessive screening of the driving data, which causes a demand for very large data sets. The result in figure 8 shows that the dangerous lane change absolute trajectory offsets have larger variances comparing with the normal lane change instances, which corresponds to the statistical results in section V-A. In addition, the 50 percentage and 85 percentage absolute trajectory offsets have large differences for both normal and dangerous lane change cases.

Figure 9 shows the lateral/longitudinal absolute offset difference between normal and dangerous lane change with speed range from 20 m/s to 30 m/s. The absolute offset results show that there are large longitudinal position deviations between a normal driving trajectory and a dangerous one even with a small lateral offset. This result shows the importance of considering the high-level driving behavior

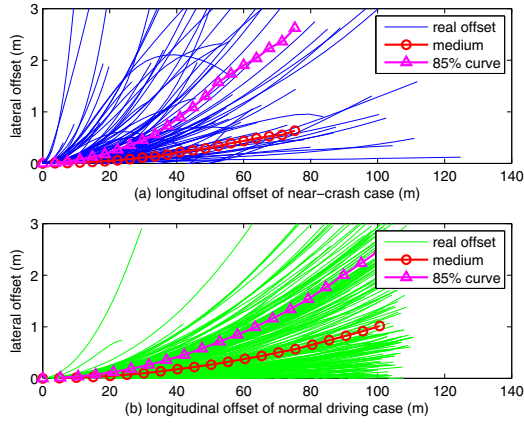


Fig. 8. Absolute trajectory offset prediction for both normal and dangerous driving

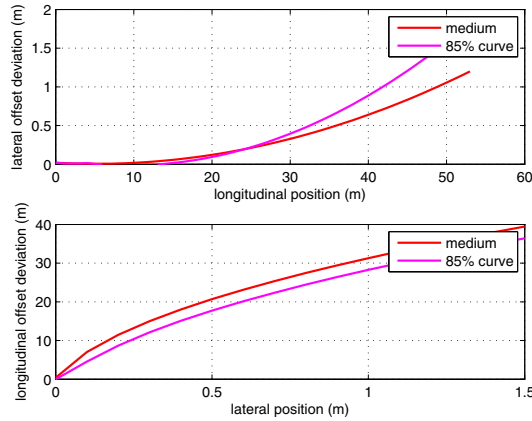


Fig. 9. Lateral/longitudinal absolute offset difference between normal and dangerous lane change

difference when predicting the future trajectory of a lane changing vehicle.

VI. CONCLUSIONS

In this paper, A trajectory prediction approach for a lane changing vehicle is proposed considering the high-level driver status. A detailed lane change partition is proposed associated with an observation interval and prediction interval. A driving behavior classifier is developed based on the HMM models and stochastic process. The classifier is trained and test with naturalistic driving data, especially the lane change data segment, and shows satisfactory performance in detecting whether the high-level driver status is normal or dangerous. Combing the classification results with the correlated naturalistic driving data sets, the prediction trajectory is generated in a statistical way. The experiments show that there are lateral offset differences for different high-level driver states with the same longitudinal speed range, which verifies the importance of estimating and classifying the high-level driving behavior before predicting future trajectories during lane change. For future work, more detailed

driving data sets that could narrow the speed ranges will be collected for more traffic scenarios besides lane change. In addition to the simple trajectory generation approach based on data percentiles of the two data sets, other approaches to measure the similarity between trajectories will be discussed in the upcoming research.

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