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Design of Emergency Maneuvers for Automated Highway System: Obstacle Avoidance Problem

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Abstract

In this paper, we analyze the problem of obstacle avoidance in an Automated Highway System (AHS). For a given scenario (traffic state, obstacle location, etc.), we synthesize the best possible avoidance maneuver for each vehicle. Our aim is to obtain a distributed strategy so that the obstacle avoidance maneuvers can be executed by vehicle based controllers (with some inter-vehicle communication) as opposed to a roadside controller making decisions and communicating it to the individual vehicles.

1 Introduction

We are interested in the design of emergency maneuvers for fully automated vehicles in an Automated Highway System (AHS). Emergencies arise in an AHS primarily due to faults and system intrusions. The management of AHS malfunctions is extensively discussed in [1] and [2]. The management of system intrusions has received less attention. An important class of system intrusions is represented by obstacles on the AHS roadway and it is desirable that the automated vehicles have obstacle avoidance capabilties. This paper is concerned with the design of obstacle avoidance maneuvers.

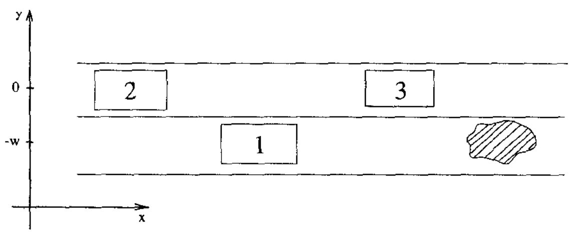

Figure 1: The three vehicles problem for gap selection

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The general obstacle avoidance scenario is as shown in figure 1. Vehicle 1 must avoid the obstacle. We refer to it as the principal vehicle. The principal vehicle can try to stop or change lanes. The selected maneuver must be executed in a safe and comfortable manner. If the obstacle is sensed at a large range the vehicle should execute either a *normal stop* or a *normal lane change.* If the obstacle is sensed at short range then safety considerations predominate over comfort and the vehicle should execute either an *emergency stop* or an *emergency lane change.* Thus we assume that there are four maneuvers to choose from. The emergency maneuvers involve braking or steering as hard as possible.

The vehicle should select the appropriate maneuver by estimating the distance required for each maneuver. If a comfortable lane change is possible then it is preferred since it will permit the vehicle to continue its journey. If the distance required for this is too great but comfortable stopping is an option then this is to be preferred over either of the emergency maneuvers. If the comfortable maneuvers are not feasible then the principal vehicle should estimate its minimum stopping distance and the minimum emergency lane change distance and pick the maneuver requiring the smaller longitudinal distance.

If a vehicle can estimate its deceleration capability then the estimation of stopping distance is a simple kinematic problem. We will not concern ourselves here any further with the estimation of stopping distance. For deceleration capability estimation refer [3]. In the rest of this paper we will address the problem of estimating the distance or time required for normal and emergency lane changes. We shall do this by designing an optimal trajectory for each maneuver as a function of the capabilities of the principal vehicle and the prevailing traffic conditions. We shall also require that the optimal trajectory be safe in the sense that the principal vehicle should not strike neighbouring vehicles during the maneuver or depart the roadway.

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It should be noted that if the lane of the principal vehicle has a shoulder adjacent to it, the principal vehicle should change lanes onto the shoulder. This is simpler since one does not have the difficulties of avoiding traffic in the target lane. However, in our approach, changing lanes onto the shoulder is a special case of changing lanes in traffic. Therefore the general scenario represented by figure 1 is addressed first. We refer to the lane of vehicle 1 as the *source lane* and that of vehicles 2 and 3 as the *target lane.* The gap defined by vehicles 2 and 3 is assumed to be the *target gap.* Vehicle 3 is the *front vehicle of the gap* and vehicle 2 is the *rear vehicle of the gap.* The lane change maneuver begins in the source lane and ends with vehicle 1 being inbetween vehicles 2 and 3, and aligned with the centerline of the target lane.

In general, the principal vehicle has more than one target gap to choose from. We refer to this as the *gap selection* problem. The gap selection problem is to be solved by solving a series of trajectory design problems for each gap and then picking the gap associated with the lowest cost trajectory. The rest of the paper discusses the trajectory design problem for a given gap.

1.1 Trajectory Design Methodology

We use an optimal control approach. For normal lane changes (NLC) we design a trajectory that is efficient in terms of the time required for the lane change. For emergency lane changes (ELC) on the other hand we design a trajectory that is efficient in terms of the longitudinal distance travelled during the lane change. We attempt to minimize the distance travelled towards the obstacle during the lane change.

The vehicles are modeled as point masses. The equations of motion are kinematic. The control inputs are the longitudinal and lateral accelerations. The longitudinal and lateral accelerations are assumed to be elliptically constrained ([4]) in a manner related to the tyre friction ellipse. Thus the optimal trajectory accounts for vehicle capability constraints thereby ensuring that the vehicle will not depart the roadway during the ELC. For NLC, we assume that the longitudinal and lateral forces are constrained by a comfortable rectangle within the ellipse. Thus the control inputs are decoupled and the trajectory design problem is simpler. It is assumed that vehicles must not travel backwards on the highway. We neglect actuator delays and lags in the dynamics though we do account for them in certain other constraints discussed later.

It is desirable that the principal vehicle not collide with the target gap vehicles during the lane change. Vehicle 3 might brake gently while the lane change is in progress in response to normal traffic disturbances or it might brake hard because the obstacle moves into its lane. If the maneuver trajectory is not suitably designed such braking disturbances will result in rear-end or sideswipe crashes. We propose to mitigate such crashes by requiring that the principal vehicle be properly aligned with the target gap before it begins to moveover. Thus for a lane change maneuver we design a *gap alignment trajectory* followed by a *moveover trajectory.* In the special case of the vehicle changing lanes onto a road shoulder, the alignment phase is not required. The moveover trajectory is designed as discussed for the general case.

The precise meaning of proper alignment is as follows. We provide a parameter in the trajectory design process that models a bound on the braking disturbances generated by the front vehicle of the gap. In other words, it is required that if the front vehicle of the gap generates any disturbance trajectory within the bound, the principal vehicle should be capable of responding with a control trajectory that will avoid collision with the front vehicle of the gap. Likewise, we require that for any control trajectory generated by the principal vehicle during the lane change, the rear vehicle of the gap should be capable of generating a control trajectory that will avoid collision with the principal vehicle. In general, these conditions can only be satisfied by deliberately driving the principal and target gap vehicles into a subset of the state space. We refer to this subset as the *safe set.* The terminal state constraint for the gap alignment trajectory is the safe set. Since moveover follows gap alignment, it always commences from an initial state in the safe set.

We provide an analytical derivation of the safe set. The derivation assumes point-mass models of the vehicles. Accelerations are assumed to be controllable subject to certain bounds and a pure time delay. This delay parameter can be used to account for vehicle sensing and actuation delays.

It should be noted that the alignment and moveover trajectories are optimized separately, i.e., first we calculate an optimal gap alignment trajectory and then use its terminal state as an intial state to calculate the moveover trajectory. The concatenation of the two pieces is the final maneuver trajectory. The concatenated trajectory, though hopefully efficient, is not necessarily optimal in the set of all possible concatenated trajectories.

1.2 Literature Review

The obstacle avoidance problem has been studied in the literature. We present a brief review in this section.

Optimal lane change maneuver trajectories are designed in [4] for lane changes onto a shoulder. A bicycle model of the car with independent constraints on lateral and longitudinal acceleration is used to calculate emergency lane change trajectories for obstacle avoiadance. In this paper we consider the more general problem of designing trajectories for obstacle avoidance in traffic, albeit with a simpler vehicle model.

In [5], a simplified point mass model is used to classify crashes arising due to unsafe, uncoordinated lane changes. The classification is then used to provide insight into the design of collision avoidance and driver warning systems for partially automated driving applications. The work of Schuster [6] analyzes obstacle avoidance on an AHS. The impact of longitudinal braking disturbances on trajectory design is not considered in [6]. For a given set of system parameters, the optimal obstacle avoiadance trajectory is found by simulating different parametrized trajectories. The approach used here is more analytical.

2 Problem Formulation

We model the vehicles kinematically; the indices refer to the vehicle as numbered in figure 1. x and y are position coordinates. The coordinates are inertial; x is the along the lanes, and y is orthogonal to the lanes. The intial conditions are

$$\begin{array}{lll} x_i(0) = x_{i0}, & \dot{x}_i(0) = \dot{x}_{io}, & i = 1,2,3, \\ & \dot{y}_i(0) = 0, & i = 1,2,3, \\ y_{10} = -w, & y_{20} = 0, & y_{30} = 0. \end{array}$$

The parameter w is the width of a lane. The control inputs are the lateral and longitudinal accelerations represented by $u_{i,lat}$ and $u_{i,lomg}$ respectively for the i–th vehicle. Since the vehicles in the upper lane will not move laterally, we ignore their lateral dynamics. The dynamical system is linear and time invariant. In state space form it is

$$\dot{x} = \frac{d}{dt}\begin{bmatrix} \Delta x_{31} \\ \Delta \dot{x}_{31} \\ \Delta x_{12} \\ \Delta \dot{x}_{12} \\ \dot{x}_1 \\ y_1 \\ \dot{y}_1 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} x$$
$$+ \begin{bmatrix} 0 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} u_{1,long} \\ u_{2,long} \\ d \\ u_{1,lat} \end{bmatrix} \quad (1)$$

Note that the disturbance d models $u_{3,long}$. Since it is required that vehicles not travel backwards, we impose the state constraint

$$\dot{x}_i \geq 0 \;\; \forall t, i = 1,2,3.$$

The control constraints are as follows. We assume that the forces vehicle 1 is able to effect on the road in x- and y-direction are constrained by the ellipse

$$\frac{u_{1,long}^2}{\alpha_1^2} + \frac{u_{1,lat}^2}{\beta_1^2} \leq 1. \quad (2)$$

Since vehicles 2 and 3 have no lateral motion the ellipse flattens for these vehicles into the interval

$$-\alpha_i \leq u_{i,long} \leq \alpha_i; \;\; i = 2,3. \quad (3)$$

Note that the above equation also constrains the disturbance input. Since lateral motion does not start before gap alignment is complete, no lateral forces are developed by vehicle 1 during the alignment stage. Accordingly equation 2 is also replaced by its corresponding interval for design of the alignment trajectory.

In contrast to the ELC, for the NLC we assume, that the acceleration constraints are not the tyre ellipse (2). They are described by some comfortable rectangular area within the ellipse. Thus the acceleration constraints are

$$a_{i,l} \leq u_{i,long} \leq a_{i,u}, \;\; i = 1,2,3 \quad (4)$$
$$b_{i,l} \leq u_{i,lat} \leq b_{i,u}, \;\; i = 1,2,3 \quad (5)$$

We assume $b_{i,l} = -b_{i,u}, \;\; b_{i,u} \geq 0$.

In the following two subsections we discuss the cost functions and the terminal state constraints for the alignment and moveover trajectory design problems.

2.1 The Gap Alignment Problem

For nomal lane change gap alignment we design a minimum time alignment trajectory with the controls subject to rectangular constraints determined by comfortable acceleration limits. The cost function is

$$J = \int_0^{t_{f,a}} 1 \;\; d\tau. \quad (6)$$

For emergency lane change gap alignment we design a minimum distance alignment trajectory with the controls subject to vehicle capability constraints, i.e., the vehicle is permitted to decelerate or accelerate as hard as possible. The cost function is

$$J = \int_0^{t_{f,a}} \dot{x}_1 \;\; d\tau. \quad (7)$$

As stated in the introduction the alignment trajectory should terminate with the three vehicles in a state in the safe set. We next present a mathematical definition of the safe set.

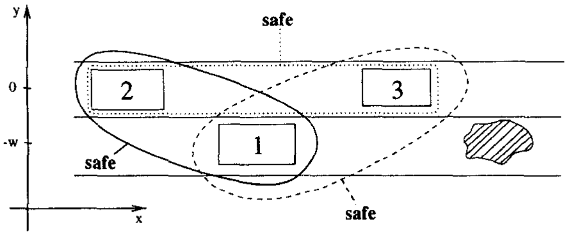

Figure 2: Pairwise safety as the end constraint of the alignment state

2.1.1 The Safe Set: The safe set is a subset of the state space. We have noted earlier that for a pair of vehicles that overlap laterally and are following each other longitudinally, safety means that for any disturbance that the front vehicle is capable of generating, the following vehicle should be capable of generating a control response that will avoid a collision. For the dynamical system described by equation 1, it is shown in [7], that the worst disturbance generated by the vehicle in front is the application of its brakes as hard as possible until it comes to rest, to which the best control response of the following vehicle is to apply its brakes as soon and as hard as possible until it too comes to rest. Therefore the safe set is the set of all states from which on application of maximum braking by the front vehicle, the rear vehicle is able to prevent a collision by also applying maximum braking with a certain reaction delay. We refer to this notion of safety as *hard braking safety.* These observations, proven in [7], provide a set of necessary and sufficient conditions that can be used to define the safe set.

Another way of defining the safe set is given in [8]. This derivation assumes the same braking and acceleration abilities for both vehicles. We believe this is unduly restrictive though considerably simpler. In this paper we make no such assumption.

Since we are dealing with a three vehicle problem we require that two sets of hard braking safety conditions be satisfied (see figure 2). Vehicle 1 must be safe against braking disturbances generated by vehicle 3 and vehicle 2 must be safe against braking disturbances generated by vehicle 1. If these two sets of conditions are satisfied then for any disturbance generated by vehicle 3, vehicle 1 may avoid collision by using any control of which it is capable, and in turn vehicle 2 will remain capable of a control trajectory that will avoid collision with vehicle 1. We describe the safe set conditions for a generic vehicle pair. The conditions are the same for the second pair.

Consider two vehicles that overlap laterally (see figure 3). The front vehicle is designated A and the rear vehicle is designated B. The state is denoted

$$z = (\Delta x, \Delta \dot{x}, \dot{x}_B)^T, \tag{8}$$

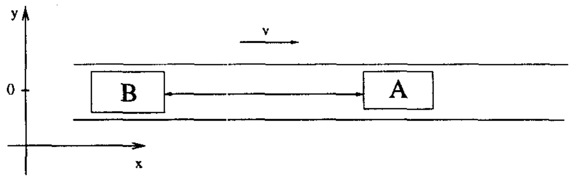

Figure 3: The two vehicles problem for pairwise longitudinal safety

where $\Delta x = x_A - x_B$ and $\Delta \dot{x} = \dot{x}_A - \dot{x}_B$. We assume that the acceleration of the vehicles is within the interval $[a_{i,l}, a_{i,u}]$, $i = A, B$. Furthermore, at time $t = 0$ vehicle A starts braking at the rate $a_{A,l}$ and vehicle B reacts to this maneuver with delay $t = \delta$, and brakes at the rate $a_{b,l}$. The stopping times of the vehicles, assuming that they do not collide, are

$$t_A = -\frac{\dot{x}_A(0)}{a_{A,l}} \tag{9}$$

$$t_B = \delta - \frac{\dot{x}_B(\delta)}{a_{B,l}} = \delta - \frac{\dot{x}_B(0) + \ddot{x}_{B0}\delta}{a_{B,l}}, \tag{10}$$

where $\ddot{x}_{B0} = const > 0$ is an allowance for acceleration by vehicle B in the time interval $[0, \delta]$.

The set of safe states is the set of all states for which the trajectory has the property

$$\{z_0 : \Delta x(t; z_0) \geq 0, \;\; t \in [0, t_B]\}. \tag{11}$$

A state is an element of the safe set if and only if the state satisfies the following inequalities. If $\delta \leq t_A \leq t_B$ or $t_A \leq \delta$ then the state must satisfy

$$\Delta x_0 - \frac{\delta^2\, \ddot{x}_{B0}}{2} + \frac{\delta^2\, \ddot{x}_{B0}^2}{2\, a_{Bl}} - \frac{\dot{x}_{A0}^2}{2\, a_{Al}} - \delta\, \dot{x}_{B0} + \frac{\delta\, \ddot{x}_{B0}\, \dot{x}_{B0}}{a_{Bl}} + \frac{\dot{x}_{B0}^2}{2\, a_{Bl}} \geq 0,.$$

If $\delta \leq t_B \leq t_A$ then the state must satisfy

$$\Delta x_0 + \frac{\delta^2\, (a_{Al} - \ddot{x}_{B0})}{2} + \delta\, (\dot{x}_{A0} - \dot{x}_{B0}) - \frac{(a_{Al}\, \delta - \delta\, \ddot{x}_{B0} + \dot{x}_{A0} - \dot{x}_{B0})^2}{2\, (a_{Al} - a_{Bl})} > 0.$$

Finally the state must always satisfy $\Delta x_0 \geq 0$.

For the derivation of these conditions refer [9]. The trajectory condition (equation (11)) is reduced to a condition on the initial state of the trajectory by observing that the trajectory as a whole is non-negative iff its minima are non-negative. For example if the initial state satisfies $\delta \leq t_A \leq t_B$ then the accelerations of the two vehicles are constant on the time intervals $[0, \delta]$, $[\delta, t_A]$, and $[t_A, t_B]$. At the boundaries of the intervals the acceleration is discontinuous. However, the

$\Delta x(t)$ trajectory is convex and continuous on each interval and therefore has a unique minimum on each interval. The trajectory is also twice differentiable on the interior of each interval. The equations are derived by requiring that Δx be non-negative on the boundaries of the intervals and in the interior if a minimum exists on the interior.

2.2 The Moveover Problem

For nomal lane change moveover we design a minimum time moveover trajectory with the controls subject to rectangular constraints determined by comfortable acceleration limits. The cost function is

$$J = \int_0^{t_{f,m}} 1 \; d\tau. \tag{12}$$

For emergency lane change moveover we design a minimum distance moveover trajectory with the controls subject to vehicle capability constraints, i.e., the vehicle is permitted to decelerate or accelerate as hard as possible. The cost function is

$$J = \int_0^{t_{f,a}} \dot{x}_1 \; d\tau. \tag{13}$$

The initial state of the moveover trajectory is in the safe set for both normal and emergency lane changes. The final state constraints for a moveover trajectory are

$$y_1(t_{f,m}) = 0, \;\; \dot{y}_1(t_{f,m}) = 0.$$

Therefore the moveover trajectory ends with the principal vehicle aligned with the centerline of the target lane and having zero lateral velocity.

3 Trajectory Design

In this section we describe the lane change trajectories that solve the optimal control problems formulated in the previous section. We begin with the NLC and ELC alignment problems.

3.1 Gap Alignment Design

Since we require that the three vehicles satisfy the hard braking safety constraints prior to the commencement of lateral motion, the lateral dynamics may be ignored in the gap alignment stage. Accordingly we analyze the following reduced order form of equation 1.

$$\dot{x} = \frac{d}{dt}\begin{bmatrix} \Delta x_{31} \\ \Delta \dot{x}_{31} \\ \Delta x_{12} \\ \Delta \dot{x}_{12} \\ \dot{x}_1 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} x +$$

$$\begin{bmatrix} 0 & 0 & 0 \\ -1 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & -1 & 0 \\ 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} u_{1,long} \\ u_{2,long} \\ d \end{bmatrix} \tag{14}$$

The initial state, terminal state, and control constraints are as previously described.

To solve the problem, we use the Pontryagin Principle [10]. The Hamiltonian for the time optimal problem is

$$H(x,p,u) = 1 + p^T(Ax + Bu). \tag{15}$$

From (14) $H(x,p,u)$ is

$$\begin{aligned} H(x,p,u) &= 1 + p_1\Delta\dot{x}_{31} + p_2 d + p_3\Delta\dot{x}_{12} - p_4 u_{2,long} + \\ & \quad (-p_2 + p_4 + p_5)u_{1,long}. \end{aligned} \tag{16}$$

Since the control constraints are rectangular the minimum principle yields

$$u_{1,long}(t) = -sgn(-p_2 + p_4 + p_5)|a_{1,l\vee u}| \tag{17}$$

$$u_{2,long}(t) = -sgn(-p_4)|a_{2,l\vee u}|, \tag{18}$$

which is a bang-bang control. From (14)and (16) we get

$$\dot{p} = \begin{bmatrix} 0 \\ -p_1 \\ 0 \\ -p_3 \\ 0 \end{bmatrix} \tag{19}$$

. whence for an initial costate $p_0 = p(0)$ we get

$$\begin{aligned} p_1(t) &= p_{10}, \\ p_2(t) &= p_{20} - p_{10}t, \\ p_3(t) &= p_{30}, \\ p_4(t) &= p_{40} - p_{30}t, \\ p_5(t) &= p_{50}. \end{aligned}$$

The p-vector (costate) is never identically 0. As p_2, p_4 and p_5 are affine, the corresponding $u_{i,long}, i = 1,2$ switch their bang-bang behaviour at most once. If they switch, it is at the moment when p_4 or $-p_2 + p_4 + p_5$ change signs.

We are able to show ([9]) that the optimal control for vehicle 2 is

$$u^*_{2,long} = a_{2l}; \;\; t \geq 0, \tag{20}$$

i.e., it should brake as hard as comfortable for a NLC and as hard as possible for a ELC. We have developed a software package to compute the optimal control for vehicle 1. Since two pairwise safety conditions have to satisfied each with two equations, the package computes four trajectories and picks the lowest cost trajectory from the four as the optimal control.

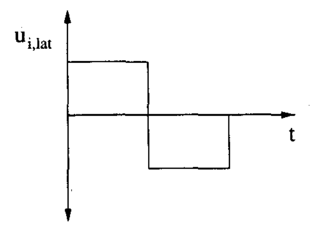

Figure 4: Moveover Lateral Acceleration

3.2 Moveover Design

In this section we describe the trajectory design process for the moveover stage. The dynamics are as described by equation (1), the terminal state constraints, initial state constraints, and control constraints are as previously described.

The Hamiltonian is formed as usual. It is a linear functional. The control constrains are compact and convex. From these two properties we can establish that the Hamiltonian is minimized on the boundary of the ellipse. Consequently, the optimal controls lie on the boundary of the ellipse. To obtain a sinusoidal lane change trajectory we assume that the lateral acceleration is as shown in figure 4.

As the lateral velocity of a vehicle is zero before and after the maneuver, the lateral acceleration a_{lat}, the lane width W and the moveover time $t_{f,m}$ are related by

$$W = \frac{a_{lat} t_{f,m}^2}{4} \tag{21}$$

If the vehicle applies longitudinal deceleration a_{long} during moveover, the longitudinal distance traveled is given by

$$x = \frac{-1}{2} a_{long} t_{f,m}^2 + v_0 t_{f,m} \tag{22}$$

where v_0 is the longitudinal speed at the beginning of the moveover. The lateral and longitudinal decelerations are related by the ellipse equation (2). The objective is to determine a_{long} and a_{lat} such that the distance traveled longitudinally in the lane of origin is minimized subject to the constraints.

The minimization problem can be solved analytically by first substituting for $t_{f,m}$ and a_{long} in equation (22) by using the other two relationships. Now, by setting the derivative of x w.r.t. a_{lat} to zero, the following cubic equation is obtained.

$$v_0^2 a_{lat}^3 - v_0^2 b^2 a_{lat} + 4W\alpha^2\beta^2 = 0 \tag{23}$$

The optimal lateral acceleration is a solution of this equation. The longitudinal deceleration is obtained by substiting this value in the ellipse equation (2).

4 Summary of results

In this paper, we have analyzed the obstacle avoiadance scenario, for automated vehicle operation on an AHS. We proposed a complete obstacle avoidance strategy. The lane change problem is divided into three steps; gap selection, gap alignment, and moveover. The trajectory synthesis for each step of the lane change is formulated as an optimal control problem. The solution to the optimal control problem provides valuable insight into the lane change design in traffic. In the future, we would like to combine the different maneuver designs into a comprehensive computational tool to evaluate the effectiveness of obstacle avoidance strategies for various scenarios. We also believe that trajectory design is an important step towards the design of feedback control laws for lane changes.

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