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Scenario Model Predictive Control for Lane Change Assistance on Highways

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Abstract— **This paper presents a new algorithm for detecting the safety of lane changes on highways and for computing safe lane change trajectories. This task is considered as a building block for driver assistance systems and autonomous cars. The presented algorithm is based on recent results in Scenario Model Predictive Control (SCMPC). It accounts for the uncertainty in the traffic environment via a small number of future scenarios, which can be generated by any model-based or data-based approach. The paper describes the SCMPC design as well as the integration with scenario-based traffic predictions. The design procedure is simple and can be generalized to other control situations. An extensive case study demonstrates the effectiveness of the proposed SCMPC algorithm and its performance in lane change situations.**

I. INTRODUCTION

The past decade has witnessed a rising interest in Advanced Driver Assistance Systems (ADAS), backed by new sensor technology and the increasing capacity of computational hardware. ADAS can support drivers in many situations, e.g., collision avoidance, lane change assistance, or adaptive cruise control. Future developments will further increase the level of automation, eventually paving the way to fully autonomous cars.

This paper presents a new concept of scenario-based control design that can be integrated for ADAS and/or autonomous cars. The approach is applied to lane change assistance on highways.

A. Traffic Prediction Models

A key factor for further advances in ADAS is the estimation and prediction of *traffic scenes*. The estimation is based on measurements of the vehicle's environment via laser scanners, radars, cameras, ultrasonic sensors, vehicle-to-vehicle or vehicle-to-infrastructure communication. Most ADAS also require predictions about the future development of the current traffic scene [1], [2]. These *traffic predictions* comprise the joint trajectory prediction for all relevant vehicles, called *target vehicles* (TVs). Difficulties arise from complex vehicle dynamics, intricate tire-road interactions, and uncertainty in a human's driving actions.

The simplest approaches use *physics-based models*, which extrapolate the vehicle's movement based on the fundamental laws of motion and friction limits [3]. These models do not account for the driver's control actions. Hence they are accurate only up to a human's reaction time, i.e., about 1 second. Longer prediction times require the use of *maneuver-based models*, which are based on motion primitives for the types of maneuvers the driver may perform [4]. The classification and identification of these motion primitives works well in cases where the level of interaction with the surrounding traffic is small. In other cases, *interaction-aware models* are needed for a higher level of realism. These models base their predictions on the modeling of a joint feedback mechanism of all vehicles in the traffic scene. However, they need to cope with a high dimensional space of possible scenarios.

B. Model Predictive Control

Model Predictive Control (MPC) is a powerful approach for optimal control of multi-variable systems with constraints on the inputs and states [5], [6]. MPC has been used successfully in a variety of industrial applications [7].

Recent studies have applied MPC also for vehicle control. One benefit of MPC is the easy integration of forward information and constraints, resulting from traffic predictions or road geometry. The most basic approach, Deterministic MPC (DMPC), is based on deterministic traffic predictions. DMPC has successfully been implemented for path following under constraints [8], in absence of other traffic. A more advanced approach, Stochastic MPC (SMPC), uses a probabilistic traffic predictions. SMPC has been tested for highway driving in a simulation study [9], by considering chance constraints to keep the vehicle within safe regions.

This paper introduces a new Scenario MPC (SCMPC) approach [10], [11]. SCMPC is based on traffic predictions via scenarios, rather than a probabilistic model. The advantages are that scenario-based predictions are simple, intuitive, and amenable to numerical processing. Moreover, the scenarios can be sampled from any kind of model or data set.

Recent studies have revealed a direct connection between the number of scenarios and chance constraints on the system states [11], [12], using principles from *scenario optimization* [13], [14]. The number of scenarios required by SCMPC is low, typically less than a hundred for chance constraint probabilities of a few percent.

C. New SCMPC Algorithm

The SCMPC algorithm presented in this paper consists of two components: the generation of traffic scenarios (Section II) and the MPC formulation (Section III). It will be applied for detecting and performing safe lane change trajectories on highways. A *safe trajectory* is considered to be one that observes all actuator, comfort, and safety constraints. The algorithm requires that the lane change is initiated by a human driver, e.g., via the turn signal (using it for ADAS), or a higher-level path planner (using it for autonomous driving).

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Assumption 1 (Lane Change Initiation) *(a) The vehicle stays in its current lane until a lane change is initiated. (b) The initiation of a lane change is given as an external signal to the SCMPC controller.*

II. SCENARIO GENERATION

Given the present *traffic scene*, SCMPC requires an uncertainty model via future *traffic scenarios*. Both the scenario generation and the controller assume discretized time steps $t = 0, 1, 2, \dots$, using a common sample time t_s.

A. Background

Different model types have been used for uncertainty description in *traffic predictions* [2]. Deterministic models simply provide a single prediction trajectory for all objects in the traffic scene [15]. As such, they do not reflect the significant uncertainty associated with the possibility for different actions taken by the human drivers. Stochastic models use standard distribution functions to model the driver's behavior [1]. This approach, however, is restricted to a set of standard distributions, typically Gaussian mixtures [16]. Furthermore, it is difficult to adjust to a specific road geometry, the traffic rules, and the interaction with other vehicles.

Scenario-based models do not require an explicit probability distribution for all possibilities in a traffic scene [3], [17]. This may facilitate the uncertainty description significantly, as illustrated in Section II-B.

Remark 2 (Scenario Generation Model) *The SCMPC algorithm (presented in Section III) can be based on any generic model-based or data-based method for scenario generation, if fulfills some minimal assumptions (as stated below).*

Let N be the time horizon for the traffic predictions. Denote all uncertainty involved in the traffic predictions over the horizon by an abstract variable $\delta_t \in \Delta_t$, where the support Δ_t has an arbitrary nature and dimensions. The following technical assumption on the scenario generation is needed to derive the theoretical properties in Section IV.

Assumption 3 (Traffic Scenarios) *(a) At every time step t, an arbitrary number K of samples of the uncertainty $\delta_t \in \Delta_t$ over the horizon N ('scenarios') can be drawn from the traffic prediction model. (b) These sampled scenarios $\delta_t^{(1)}, ..., \delta_t^{(K)}$ are independent and identically distributed (i.i.d.) with the real uncertainty outcome δ_t.*

Assumption 3 requires only that sufficient scenarios are available and that they are representative of the actual uncertainty in the traffic scene. Typically, the number of scenarios K turns out to be relatively small, as it will be seen later in Section IV. The availability of representative data is a minimal requirement for any uncertainty prediction.

Note that the scenario-based uncertainty model can be time-varying and hence be conditioned on the latest information available at each time step, unlike many other MPC approaches [18]. Neither the current joint probability distribution of δ_t nor the support Δ_t need to be known explicitly. These are further advantages over similar approaches using Stochastic MPC [9] or Robust MPC [19].

B. Scenario Generation Model

This paper employs an interaction-aware model for predicting the possible trajectories of each target vehicle (TV). The scenarios for individual TVs may be drawn directly from collected real-world driving data. Hence it does not require the fitting of an uncertainty model or any probabilistic assumptions. If a sampled trajectory causes a collision, it is rejected and replaced with another sample (*rejection sampling*) [3], [17]. Hence the resulting combined scenarios of all TVs in the traffic scene follows the implicit distribution of collision-free vehicle trajectories.

To obtain the *speed profile* of a TV j, first a target speed $v_{\text{ref},j}$ is sampled. This target speed equals to current speed of the TV with some high probability, but it may also differ with a non-zero probability. The acceleration or braking $a_{j,t}$ at time t is then obtained by a proportional controller on the current velocity $v_{j,t}$:

$$a_{j,t} = K_{\text{lon},j}\big(v_{\text{ref},j} - v_{j,t}\big) \ ,$$

which is saturated at the acceleration / braking limits (5). The gain of this controller, $K_{\text{lon},j}$, is also sampled and capture the aggressiveness of the acceleration or braking.

The *steering profile* of a TV j is obtained in two steps. First, a random variable decides whether the TV intends to stay in its current lane or attempts a lane change to the left or the right. The centerline of the target lane is the reference for the lateral position, $y_{\text{ref},j}$. Second, the reference is tracked by a steering angle $\delta_{j,t}$ at time t computed via proportional control on the current lateral position $y_{j,t}$:

$$\delta_{j,t} = K_{\text{lat},j}\big(y_{\text{ref},j} - y_{j,t}\big) \ ,$$

which is saturated according to steering constraints (3), (4) and lateral acceleration constraints (7). The gain $K_{\text{lat},j}$ is again sampled to capture the aggressiveness of lane change maneuver.

III. SCENARIO MODEL PREDICTIVE CONTROL

A. Vehicle Model

The SCMPC uses a *kinematic bicycle model* [20, Sec. 2.2] for the controlled vehicle (CV). The model has been shown to yield a good trade-off between model complexity and prediction accuracy for controller design [8], [21].

The kinematic bicycle model is illustrated in Fig. 1. The *center of gravity* (CoG) follows a set of nonlinear ordinary differential equations:

$$\dot{\xi} = v\cos(\psi + \beta) \ , \tag{1a}$$

$$\dot{\eta} = v\sin(\psi + \beta) \ , \tag{1b}$$

$$\dot{\psi} = \frac{v\cos(\beta)}{l_f + l_r}\tan(\gamma) \ , \tag{1c}$$

$$\dot{v} = a \ , \tag{1d}$$

where the dot indicates a derivative with respect to time and

$$\beta = \tan^{-1}\Big(\frac{l_r}{l_f + l_r}\tan(\gamma)\Big) .$$

Here (ξ, η), ψ, and v represent the vehicle's coordinates, orientation, and speed, respectively. The parameters l_f and l_r denote the lengths from the vehicle's center of gravity (CoG) to the front and rear axes, respectively. The steering angle of the front wheel γ and the acceleration a are the control inputs to the system.

For the integration of the kinematic bicycle model into SCMPC, the differential equations in (1) are discretized using a forward Euler method. The state at time step $t = 0, 1, 2, \ldots$ is denoted $x_t := [\xi_t \; \eta_t \; \psi_t \; v_t]^{\mathrm{T}}$ and the input $u_t := [\gamma_t \; a_t]^{\mathrm{T}}$. The following vector-based notation is used for the resulting discrete-time nonlinear dynamics:

$$x_{t+1} = f\big(x_t, u_t\big) . \tag{2}$$

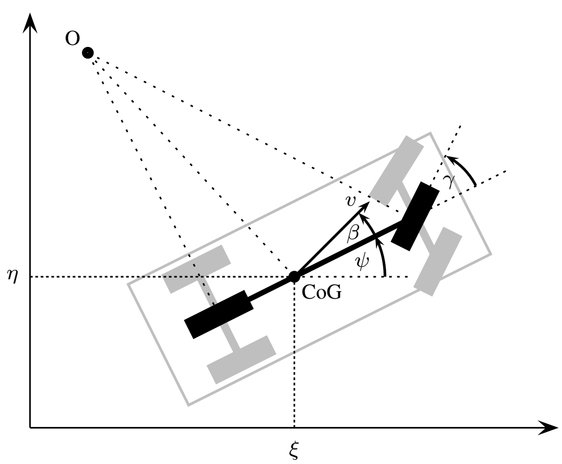


Fig. 1: Illustration of kinematic bicycle model in black, for a four-wheel car in gray (CoG: center of gravity, O: instantaneous center of rotation).

B. Input Constraints

Input constraints capture actuator limits regarding *steering angle*, *steering rate*, and *braking / acceleration*:

$$-37\,\mathrm{deg} = \gamma_{\min} \leq \gamma_t \leq \gamma_{\max} = +37\,\mathrm{deg} , \tag{3}$$

$$-6\,\tfrac{\mathrm{deg}}{\mathrm{s}} = \dot\gamma_{\min} \leq \dot\gamma \; \leq \dot\gamma_{\max} = +6\,\tfrac{\mathrm{deg}}{\mathrm{s}} , \tag{4}$$

$$-3\,\tfrac{\mathrm{m}}{\mathrm{s}^2} = a_{\min} \leq a_t \leq a_{\max} = +1.5\,\tfrac{\mathrm{m}}{\mathrm{s}^2} . \tag{5}$$

Here $\dot\gamma \approx (\gamma_{t+1} - \gamma_t)/t_{\mathrm{s}}$ is approximated by finite differences. The above parameters correspond to experimental values for the Hyundai Azera 2011 test vehicle at our laboratory; see [8] for more details.

Driving on highways at relatively high speeds typically requires small steering angles and rates. Therefore the constraints (3), (4) will not saturate almost all the time. However, steering should also be limited by the *lateral acceleration* of the vehicle, to guarantee the comfort of the passengers and to avoid skidding. The lateral acceleration $a_{\mathrm{lat},t}$ is associated with the centripetal acceleration that corresponds to the instantaneous rotation of the CoG,

$$a_{\mathrm{lat},t} = \frac{v_t^2}{l_r}\sin(\beta_t) . \tag{6}$$

The bounds are chosen as

$$-2\,\tfrac{\mathrm{m}}{\mathrm{s}^2} = a_{\mathrm{lat,min}} \leq \; a_{\mathrm{lat,t}} \; \leq a_{\mathrm{lat,max}} = +2\,\tfrac{\mathrm{m}}{\mathrm{s}^2} . \tag{7}$$

Using the same vector notation as for the vehicle model (2), the input constraints (3), (4), (5), (7) can be written as

$$u_{\min}(x_t) \leq u_t \leq u_{\max}(x_t) , \tag{8a}$$

$$\dot u_{\min} \cdot t_{\mathrm{s}} \leq u_{t+1} - u_t \leq \dot u_{\max} \cdot t_{\mathrm{s}} , \tag{8b}$$

where $u_{\min}(x_t)$, $u_{\max}(x_t)$, $\dot u_{\min}$, $\dot u_{\max}$ follow from (3)-(7).

C. Safety Distances

In highway traffic, all drivers are required to maintain a *safety distance* to the vehicle driving in their front [22, pp. 56 ff.] [23, § 4(1)]. This safety distance must be large enough to allow drivers ample reaction time in case of braking maneuvers or unexpected lane changes.

Assumption 4 (Safety Distances) *(a) If CV stays on its current lane, it is only responsible to maintain a safety distance d_{CV} to its front vehicle. (b) In an overtaking situation, the CV must respect also the safety distance of the target vehicle d_{TV} that drives behind it on the target lane. (c) These safety distances may occasionally be violated (e.g., if the TVs perform emergency braking maneuvers).*

In a particular scenario $\delta_t^{(k)}$, the safety distances translate directly into constraints on the longitudinal and lateral position of the CV. As illustrated in Fig. 2, the current lane (cl) and target lane (tl) are considered separately under each scenario. The safety distances on lane $l \in \{\mathrm{cl}, \mathrm{tl}\}$ under scenario $k \in \{1, \ldots, K\}$ are based on the trajectories of all TVs, as given by $\delta_t^{(k)}$.

TVs in the back require the CV to maintain a safety distance d_{TV}, depending on the corresponding TV's speed. TVs in the front require the CV to maintain a safety distance of d_{CV} that varies with the CV's speed. Hence the longitudinal position of the CV (ξ coordinate) is constrained by a *safety interval* $[\xi_{\mathrm{min},t}^{l,(k)}, \xi_{\mathrm{max},t}^{l,(k)}]$, as shown in Fig. 2. The lateral position of the CV (η coordinate) is constrained by the boundaries of the current lane ($l = \mathrm{cl}$) and/or the target lane ($l = \mathrm{tl}$), $[\eta_{\mathrm{min},t}^{l}, \eta_{\mathrm{max},t}^{l}]$.

D. Lane Change Timing

The SCMPC algorithm has a prediction horizon of N time steps, i.e., a total preview time of $N \cdot t_{\mathrm{s}}$. It consists of an *outer loop* and an *inner loop*. The outer loop iterates to find the *earliest lane change time* t_{lc} within the interval $[0, N \cdot t_{\mathrm{s}}]$. The inner loop attempts to compute a safe trajectory for a fixed lane change time t_{lc}; i.e., one that observes the lane change constraints under all K scenarios.

Assumption 5 (Lane Change Constraints) *(a) When integrated into the current (or target) lane, CV must observe the*

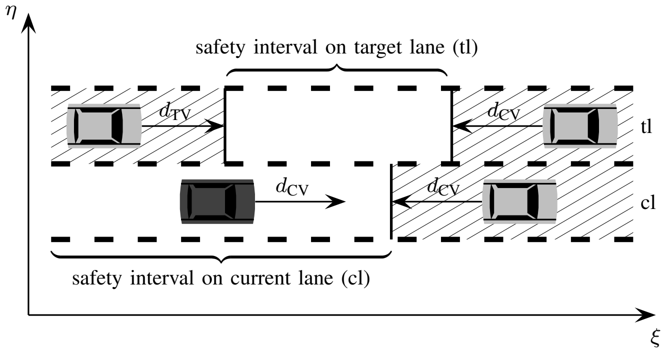


Fig. 2: Construction of safety intervals $[\xi^{l,(k)}_{\text{min},t}, \xi^{l,(k)}_{\text{max},t}]$ at time step t for a particular scenario k. The CV is shown in dark gray and three TVs in light gray. The hatched regions on the current lane $l = \text{cl}$ and the target lane $l = \text{tl}$ are unsafe.

safety interval on the current (or target) lane. (b) During a lane change maneuver, the CV must be able to safely return to the current lane, so it must observe the safety intervals on both the current and target lanes.

The CV may not able to move into the safety interval on the target lane immediately, as in Fig. 2. Then it can first adjust its position and speed on the current lane before performing the lane change. It is assumed that an earlier lane change time is always preferred. The outer loop performs a simple search over a set of candidate lane change times

$$T_{\text{lc}} \subset \{0, t_{\text{s}}, \dots, N \cdot t_{\text{s}}\} \ . \tag{9}$$

For each candidate lane change time $t_{\text{lc}} \in T_{\text{lc}}$, the inner loop computes the control inputs (steering, acceleration) and evaluates the feasibility of a lane change within t_{lc}; see Section III-F. Note that, by Assumption 5, if a lane change is feasible within t_{lc}, then it is also feasible within any candidate lane change time greater than t_{c}. Hence a simple bisection algorithm can be used for the outer loop.

E. State Constraints

Let $[\xi^{l,(k)}_{\text{min},t}, \xi^{l,(k)}_{\text{max},t}]$ denote the safety interval on lane $l \in \{\text{cl}, \text{tl}\}$ at time $t \in \{1, \dots, N\}$ under scenario $k \in \{1, \dots, K\}$:

$$\xi^{l}_{\text{min},t} := \max_{k\in\{1,\dots,K\}} \xi^{l,(k)}_{\text{min},t} \ , \tag{10a}$$

$$\xi^{l}_{\text{max},t} := \min_{k\in\{1,\dots,K\}} \xi^{l,(k)}_{\text{max},t} \ . \tag{10b}$$

The *longitudinal limits* are then defined as

$$\xi_{\text{min},t} := \begin{cases} \xi^{\text{cl}}_{\text{min},t} & \text{if} \quad t \le t_{\text{lc}} \\ \max\{\xi^{\text{cl}}_{\text{min},t}, \xi^{\text{tl}}_{\text{min},t}\} & \text{if} \quad t > t_{\text{lc}} \end{cases} , \tag{11a}$$

$$\xi_{\text{max},t} := \begin{cases} \xi^{\text{cl}}_{\text{max},t} & \text{if} \quad t \le t_{\text{lc}} \\ \min\{\xi^{\text{cl}}_{\text{max},t}, \xi^{\text{tl}}_{\text{min},t}\} & \text{if} \quad t > t_{\text{lc}} \end{cases} . \tag{11b}$$

The *lateral limits* are given by

$$\eta_{\text{min},t} := \begin{cases} \eta^{\text{cl}}_{\text{min},t} & \text{if} \quad t \le t_{\text{lc}} \\ \min\{\eta^{\text{cl}}_{\text{min},t}, \eta^{\text{tl}}_{\text{min},t}\} & \text{if} \quad t > t_{\text{lc}} \end{cases} , \tag{12a}$$

$$\eta_{\text{max},t} := \begin{cases} \eta^{\text{cl}}_{\text{max},t} & \text{if} \quad t \le t_{\text{lc}} \\ \max\{\eta^{\text{cl}}_{\text{max},t}, \eta^{\text{tl}}_{\text{max},t}\} & \text{if} \quad t > t_{\text{lc}} \end{cases} . \tag{12b}$$

The resulting longitudinal and lateral state constraints are

$$x_{\text{min},t} \le x_t \le x_{\text{max},t} \ , \tag{13}$$

where $x_{\text{min},t} := [\xi_{\text{min},t} \ \eta_{\text{min},t} \ -\infty \ 0]^{\text{T}}$ and $x_{\text{max},t} := [\xi_{\text{max},t} \ \eta_{\text{min},t} \ +\infty \ v_{\text{max}}]^{\text{T}}$, and v_{max} denotes the speed limit.

Given the sampled values of all scenarios $\delta^{(1)}_t, \dots, \delta^{(K)}_t$, the safety intervals for each lane and hence the state constraints (13) can be readily computed for each time step t.

F. Multi-Stage Scenario Program

For the *inner loop* of SCMPC, the lane change time t_{lc} is fixed. The control inputs $u_t, \dots, u_{t+N-1}$ over the prediction horizon are then computed by solving the following Multi-Stage Scenario Program (MSSP):

$$\min_{u_T, x_{T+1}} \sum_{\tau\in T} (x_{\tau+1} - x_{\text{ref},\tau+1})^{\text{T}} Q (x_{\tau+1} - x_{\text{ref},\tau+1}) + u_\tau^{\text{T}} R u_\tau \ , \tag{14a}$$

$$\text{s.t.} \ x_{\tau+1} = f(x_\tau, u_\tau) \qquad \forall\, \tau \in T \ , \tag{14b}$$

$$u_{\text{min}}(x_\tau) \le u_\tau \le u_{\text{max}}(x_\tau) \qquad \forall\, \tau \in T \ , \tag{14c}$$

$$\dot{u}_{\text{min}} \le u_{\tau+1} - u_\tau \le \dot{u}_{\text{max}} \qquad \forall\, \tau \in T \ , \tag{14d}$$

$$x_{\text{min},\tau+1} \le x_{\tau+1} \le x_{\text{max},\tau+1} \qquad \forall\, \tau \in T \ . \tag{14e}$$

Here $T := \{t, \dots, t+N-1\}$, and $Q \in \mathbb{R}^{4\times4}$, $R \in \mathbb{R}^{2\times2}$ are positive definite weighting matrices for tuning. The reference trajectory $x_{\text{ref},t+1}, \dots, x_{\text{ref},t+N}$ for the CV is constructed according to the reference speed and the lane change time t_{lc}. Since the SCMPC chooses the final trajectory in a safe and dynamically feasible manner, the exact shape of the reference is secondary. Here it is chosen as a simple step change in the lateral coordinate.

The MSSP is a nonlinear program, which can be solved by standard algorithms [24], [25]. The objective function (14a) is a convex quadratic function that penalizes the deviation of the state from the reference as well as the input usage. The dynamic constraints (14b) are nonlinear. The input constraints (14c,d) can be linearized by substituting the initial state $x_\tau = x_t$. The state constraints (14e) are implemented as soft constraints, to ensure feasibility of the program.

IV. BOUND ON SAFETY DISTANCE VIOLATIONS

So far, no indication has been made for the selection of the sample size K. This section uses the framework of *scenario-based optimization* [12]–[14] to establish a connection between K and the satisfaction of chance constraints, in the sense defined below [11].

Definition 6 (Chance Constraints) *The safety distance of each CV and TV may be violated in no more than a fraction $\varepsilon \in (0, 1)$ of all time steps $t = 0, 1, \dots$.*

Here $\varepsilon \in (0, 1)$ is a tuning parameter that represents the *conservatism* of the driving style. A small ε corresponds to a low-risk driving style, a large ε makes the controller perform more aggressive lane change maneuvers. To derive the theoretical results, the following technical assumption is required [11].

Assumption 7 (Feasibility) *(a) If no lane change is performed, there always exists a feasible solution to the MSSP. (b) The outer loop iterations are replaced by a fixed lane change time t_{lc}; e.g., the highest value in T_{lc}.*

Let the lane $l \in \{\text{cl}, \text{tl}\}$ and time step $t \in \{0, 1, \dots\}$ be fixed and consider the front safety distance $\tau \in \{1, \dots, N\}$ steps into the future (the back safety distance works analogously). The violation of the front safety distance can occur only if $\xi^l_{\max,t+\tau}$ has been chosen higher than

$$\bar{\xi}^l_{\max,t+\tau}\big(\delta_t\big) \ , \tag{15}$$

which is the unknown true longitudinal position for the CV under the unknown true scenario δ_t. In other words,

$$\mathbf{P}\big[V\big] \leq \mathbf{P}\big[\bar{\xi}^l_{\max,t+\tau}\big(\delta_t\big) < \xi^l_{\max,t+\tau}\big] \ , \tag{16}$$

where $\mathbf{P}\big[\,\cdot\,\big]$ denotes the probability measure and V is the event where the front safety distance is violated. Therefore, to upper bound $\mathbf{P}\big[V\big]$ by ε, it suffices to upper bound the right-hand side of (16). This can be accomplished by a basic sampling lemma [26].

Theorem 8 (Violation Bound) *Let Assumptions 3, 4, 5, 7 hold and(14b) be an exact model of the controlled vehicle. Then*

$$\mathbf{P}\big[\bar{\xi}^l_{max,t+\tau}\big(\delta_t\big) < \xi^l_{max,t+\tau}\big] \leq \frac{1}{K+1} \ , \tag{17}$$

and hence the sample size $K \geq \frac{1}{\varepsilon} - 1$ ensures satisfaction of the chance constraints of Definition 6.

Proof: All scenarios $\delta_t^{(1)}, \dots, \delta_t^{(K)}$ are independent and identically distributed with δ_t (Assumption 3), so each one is equally likely to generate the tightest bound for the safety distance [26], and hence (17). State constraint satisfaction follows by Assumption 7 along the lines of [11]. ■

The performance of the SCMPC can be enhanced by extensions to the presented basic SCMPC approach [11].

Remark 9 (Extensions) *(a) The SCMPC can include a control parameterization by any* nonlinear disturbance feedback *for performance improvements [11, Rem. 7]. (b) The SCMPC can be extended by an algorithm for* a-posteriori sample removal*, to reduce the effect of sample outliers [11, Sec. 3.3].*

V. SIMULATION RESULTS

This paper considers a highway case study as a proof of concept for the SCMPC algorithm. In the considered traffic scene, the CV is driving on the right of two lanes, with two TVs (green, violet) in the front and two TVs (gray, blue) in the back. The exact positions and speeds of the TVs vary between different configurations. The speed limit on the highway is 22.4 meters per second ($\frac{\text{m}}{\text{s}}$). The TVs actually keep their lane and retain constant speed, yet this is not known to the controller, which must consider different scenarios for the TVs.

The simulations starts at time $t = 0$, when the CV receives a lane change command. The prediction horizon is $N = 50$ steps for a sampling time of $t_{\text{s}} = 0.1\,\text{s}$, giving the SCMPC a total preview time of $N \cdot t_{\text{s}} = 5\,\text{s}$. Eight candidate lane change times $T_{\text{lc}} = \{0.0,\ 0.5,\ 1.0,\ 1.5,\ 2.0,\ 2.5,\ 3.0,\ 3.5\}$ seconds are considered. The earliest lane change time is preferred.

The reference speed v_{ref} equals $22.4\frac{\text{m}}{\text{s}}$. The lane change is commanded via a reference step in the lateral position y_{ref}. Small weights on the control inputs a and δ are used to make the control actions smooth and energy efficient. The number of scenarios has been selected as $K = 19$, corresponding to a chance constraint level of $\varepsilon = 5\%$.

The safe trajectories for three different configurations of the traffic scene are shown in Fig. 3. A sample scenario for the TVs is also depicted and serves as a reference. The depicted scenario assumes constant speed and no lane changes for each of the TVs. Fig. 4 shows the corresponding speed profile of the safe trajectory in each of the three cases, as well as each of the TVs speed.

In case 1, the CV attempts to overtake the slower TV 1 (green) in its front. In order not to violate the front safety distance of TV 4 (blue), the CV needs to accelerate before merging into the faster lane. The SCMPC finds that the earliest lane change time $t_{\text{lc}} = 1\,\text{s}$, up to which the CV needs to accelerate on its own lane; see Fig. 3(a).

In case 2, the CV intends to merge into a slower lane with TV 2 (violet) limiting its speed. In order not to violate its own front safety distance, the CV needs to slow down before merging into the target lane. The SCMPC finds that a lane change is possible immediately, $t_{\text{lc}} = 0\,\text{s}$, and it adjusts to the speed of TV 2 thereafter; see Fig. 3(b).

In case 3, the CV attempts to merge into a faster lane, blocked by TV 2 (violet) and TV 4 (blue) going at higher speeds. The CV would need to accelerate before merging into the target lane. However, the SCMPC concludes that this is not possible within the preview time of $5\,\text{s}$ and therefore remains in the original lane; see Fig. 3(c).

REFERENCES

[1] C. Laugier, I. E. Paromtchik, M. Perrollatz, M. Yong, J.-D. Yoder, C. Tay, K. Mekhnacha, and A. Négre, "Probabilistic analysis of dynamic scenes and collision risks assessment to improve driving safety," *IEEE Intelligent Transportation Systems Magazine*, vol. 2(4), pp. 4–19, 2011.

[2] S. Lefèvre, D. Vasquez, and C. Laugier, "A survey on motion prediction and risk assessment for intelligent vehicles," *Robomech Journal*, vol. 1(1), pp. 1–14, 2014.

[3] A. Eidehall and L. Petersson, "Statistical threat assessment for general road scenes using monte carlo sampling," *IEEE Transactions on Intelligent Transportation Systems*, vol. 9(1), pp. 137–147, 2008.

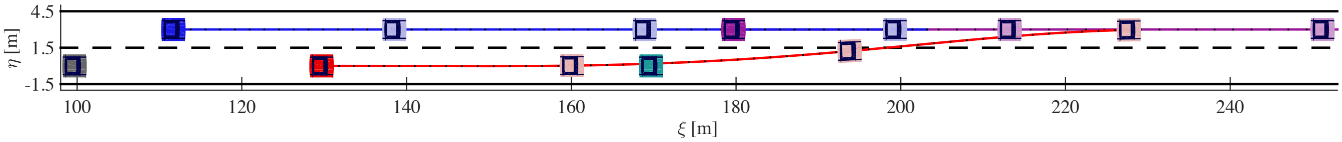
[4] G. S. Aoude, B. D. Luders, K. K. H. Lee, D. S. Levine, and J. P. How, "Threat assessment design for driver assistance system at intersections," in *13th IEEE Conference on Intelligent Transportation Systems*, Madeira Island, Portugal, 2010.

[5] J. M. Maciejowski, *Predictive Control with Constraints*. Harlow: Pearson Education, 2002.

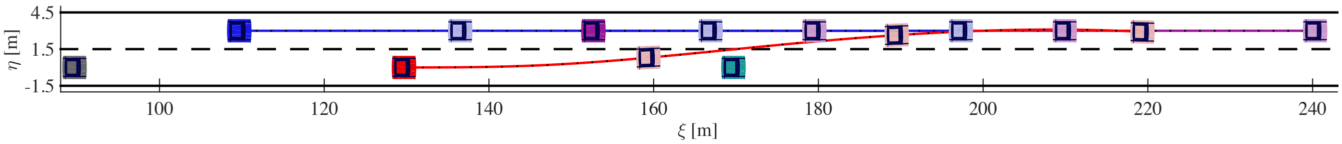
[6] J. B. Rawlings and D. Q. Mayne, *Model Predictive Control: Theory and Design*. Madison (WI): Nob Hill Publishing, 2009.

[7] S. J. Qin and T. A. Badgwell, "A survey of industrial model predictive control technology," *Control Engineering Practice*, vol. 11, pp. 733–764, 2003.

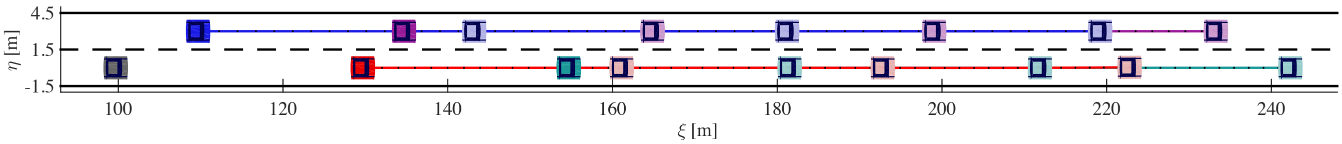
[8] J. Kong, M. Pfeiffer, G. Schildbach, and F. Borrelli, "Autonomous driving using model predictive control and a kinematic bicycle vehicle model," in *Intelligent Vehicles Symposium*, Seoul, Korea, 2015.



(a) Case 1: Merge into a faster lane.

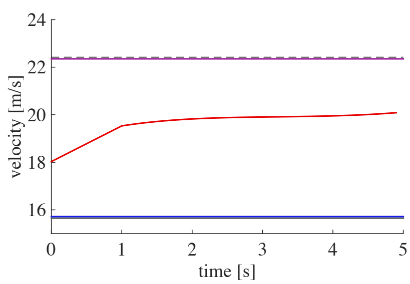


(b) Case 2: Merge into a slower lane.

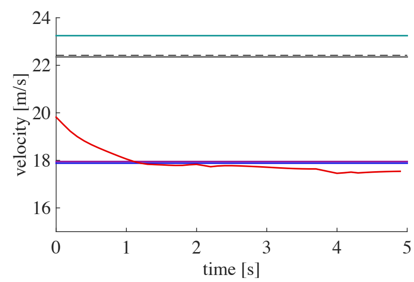


(c) Case 3: Lane change is rejected.

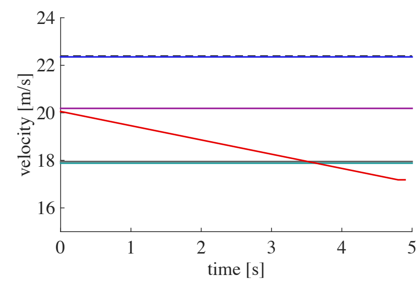
Fig. 3: Computed safe trajectories by SCMPC for three configurations of the considered traffic scene. The controlled vehicle is shown in red and the target vehicles $j = 1, 2, 3, 4$ in green, violet, gray, blue. The future trajectories are indicated for some of the vehicles, with future positions shown in pale colors.



(a) Case 1: Merge into a faster lane.



(b) Case 2: Merge into a slower lane.



(c) Case 3: Lane change is rejected.

Fig. 4: Velocity profiles corresponding to the safe trajectory of the controlled vehicle in red, and the target vehicles $j = 1, 2, 3, 4$ in green, violet, gray, blue. The dashed black line indicates the speed limit.

[9] A. Carvalho, Y. Gao, S. Lefèvre, and F. Borrelli, "Stochastic predictive control of autonomous vehicles in uncertain environments," in *12th International Symposium on Advanced Vehicle Control*, Tokyo, Japan, 2014.

[10] L. Blackmore, M. Ono, A. Bektassov, and B. C. Williams, "A probabilistic particle-control approximation of chance-constrained stochastic predictive control," *IEEE Transactions on Robotics*, vol. 26(3), pp. 502–516, 2010.

[11] G. Schildbach, L. Fagiano, C. Frei, and M. Morari, "The scenario approach for stochastic model predictive control with bounds on closed-loop constraint violations," *Automatica*, vol. 50(12), pp. 3009–3018, 2014.

[12] G. Schildbach, L. Fagiano, and M. Morari, "Randomized solutions to convex programs with multiple chance constraints," *SIAM Journal on Optimization*, vol. 23(4), pp. 2479–2501, 2013.

[13] M. C. Campi and S. Garatti, "The exact feasibility of randomized solutions of uncertain convex programs," *SIAM Journal on Optimization*, vol. 19, pp. 1211–1230, 2008.

[14] ——, "A sampling and discarding approach to chance-constrained optimization: Feasibility and optimality," *Journal of Optimization Theory and Applications*, vol. 148, pp. 257–280, 2011.

[15] J. Jansson, "Collision avoidance theory with application to automotive collision mitigation," Ph.D. Dissertation, Linköping University, Linköping, Sweden, 2005.

[16] J. Wiest, M. Höffken, U. Kreßel, and K. Dietmayer, "Probabilistic trajectory prediction using gaussian mixture models," in *Intelligent Vehicles Symposium*, Alcalá de Henares, Spain, 2012.

[17] A. Broadhurst, S. Baker, and T. Kanade, "Monte carlo road safety reasoning," in *Intelligent Vehicles Symposium*, Las Vegas (NV), United States, 2005.

[18] G. Schildbach, "Scenario-based optimization for multi-stage stochastic decision problems," Ph.D. Dissertation, Eidgenössische Technische Hochschule Zürich, Zürich, Switzerland, 2014.

[19] Y. Gao, A. Gray, H. E. Tseng, and F. Borrelli, "A tube-based robust nonlinear predictive control approach to semiautonomous ground vehicles," *Vehicle System Dynamics*, vol. 52(6), pp. 802–823, 2014.

[20] R. Rajamani, Ed., *Vehicle Dynamics and Control*, 2nd ed. New York: Springer, 2012.

[21] C. M. Kang, S.-H. Lee, and C. C. Chung, "Comparative evaluation of dynamic and kinematic vehicle models," in *53rd Conference on Decision and Control*, Los Angeles (CA), United States, 2014.

[22] California Department of Motor Vehicles, "California driver handbook," California, United States, September 2014.

[23] Bundesministerium der Justiz und für Verbraucherschutz, "Straßenverkehrs-Ordnung," Federal Republic of Germany, March 2013.

[24] D. Luenberger and Y. Ye, *Linear and Nonlinear Programming*, 3rd ed. Berlin et al.: Springer, 2008.

[25] J. Nocedal and S. Wright, *Numerical Optimization*, 2nd ed. New York: Springer, 2006.

[26] G. C. Calafiore and M. C. Campi, "Uncertain convex programs: Randomized solutions and confidence levels," *Mathematical Programming, Series A*, vol. 102-1, pp. 25–46, 2005.