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Distributed model predictive control approach for cooperative car-following with guaranteed local and string stability

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ABSTRACT

In this paper, a serial distributed model predictive control (MPC) approach for connected automated vehicles (CAVS) is developed with local stability (disturbance dissipation over time) and multi-criteria string stability (disturbance attenuation through a vehicular string). Two string stability criteria are considered within the proposed MPC: (i) the l_∞-norm string stability criterion for attenuation of the maximum disturbance magnitude and (ii) l_2-norm string stability criterion for attenuation of disturbance energy. The l_∞-norm string stability is achieved by formulating constraints within the MPC based on the future states of the leading CAV, and the l_2-norm string stability is achieved by proper weight matrix tuning over a robust positive invariant set. For rigor, mathematical proofs for asymptotical local stability and multi-criteria string stability are provided. Simulation experiments verify that the distributed serial MPC proposed in this study is effective for disturbance attenuation and performs better than traditional MPC without stability guarantee.

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1. Introduction

The Connected Automated Vehicle (CAV) technologies, enabled by advanced sensing and communication, promise to fundamentally change vehicle behavior and consequently macroscopic traffic flow characteristics (Talebpour and Mahmassani, 2016). Specifically, the Cooperative Adaptive Cruise Control (CACC) system has the potential to drastically improve traffic throughput and stability through real-time control of vehicles' longitudinal movement (e.g., Shladover et al., 2012, 2015). CACC has drawn wide attention, resulting in CACC variants aiming at improving traffic flow stability, control efficiency, driving comfort, and/or fuel economy over the last decades (e.g., Lu et al., 2002; Milanes et al., 2014; Pérez et al., 2013).

Two different approaches of CACC algorithms exist: (i) closed-form linear or nonlinear state-feedback control algorithms (e.g., Morbidi et al., 2013; Stipanović et al., 2004; Zhang and Orosz, 2016, 2017) and (ii) constrained optimization control models (e.g., Hoogendoorn et al., 2012). The latter approach has evolved to model predictive control (MPC) approach implemented with a rolling horizon (e.g., Wang et al., 2014a; 2014b; Zhou et al., 2017). They each have notable advantages and drawbacks in terms of handling constraints, optimal performance, trajectory prediction, and stability analysis. For the linear control, local and string stability of CACC systems can be explicitly guaranteed and mathematically proved (e.g., Öncü et al.,

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2014; Petrillo et al., 2018; Qin and Orosz, 2017; Shladover et al., 2015). However, a collision-free constraint within reasonable acceleration/deceleration limits cannot be explicitly formulated with the linear control. Furthermore, it is difficult to design an optimization framework with multiple objectives for control performance and incorporate a prediction scheme for the leading vehicle trajectory that could significantly improve the control performance. For nonlinear state-feedback algorithms, string stability can be analyzed by the same methods as those for linear systems by linearizing the closed-loop system around equilibrium points (Talebpour and Mahmassani, 2016; Wang, 2018). However, the string stability conditions are only valid for the regions close to the equilibrium points.

In contrast, the MPC approach is capable of optimizing the acceleration portfolio considering multiple performance criteria of control efficiency, comfort, fuel consumption, etc. while respecting hard constraints on vehicles' physical limits and safety. Based on the level of coordination and cooperation among vehicles, the MPC framework can be further divided into distributed control (Wang et al., 2016) and centralized control (Zhou et al., 2017; Chen et al., 2018), where the former typically involves a pair of vehicles in a platoon while the latter involves more vehicles (e.g., all vehicles in a platoon). The centralized control can achieve the best performance, though at the computational expense that can inhibit real-time control. The distributed MPC is less computationally demanding and thus more practical, albeit at the expense of lower control performance. A major drawback of the MPC approach is that (local and string) stability analysis and proofs are largely missing in the literature despite its important implications for safety and system robustness. The present paper aims to bring together the advantages of both approaches, linear feedback control and MPC by developing an MPC framework with explicit control objectives and constraints while providing stability analysis and proofs.

Local stability and string stability are important properties of longitudinal controllers for CAV platoons. Local stability means that a vehicle is able to remain in a neighborhood of an equilibrium state (e.g., equilibrium speed and target spacing) under disturbance. String stability means that the magnitude of a disturbance (e.g., deviation from equilibrium spacing) decreases or remains constant as it propagates through a platoon of vehicles. Depending on the norm function used to characterize the disturbance magnitude, different string stability criteria exist in the literature for linear CACC controllers, such as l_2-norm, l_p-norm (Ploeg et al., 2014), and H_2-norm (Naus et al., 2010). The l_2-norm and l_∞-norm respectively represent the energy and peak of a disturbance in the time domain, and the H_2-norm represents the energy in the frequency domain.

As mentioned above, MPC-based CACC controllers effectively deal with state and control constraints but ensuring local and string stability through mathematical proofs remains a major challenge. Local stability of MPC-based CACC controllers has rarely been proved, though some notable studies provide a proof when the constraints are inactive (Gong et al., 2016; Gong and Du, 2018) or by restricting the terminal states (e.g., deviation from equilibrium spacing, speed difference, and/or acceleration at the end of a prediction horizon) to approach zero (Dunbar and Caveney, 2012). Similarly, string stability has rarely been proved for the MPC framework, though some notable attempts exist. In Dunbar and Caveney (2012), the l_∞-norm string stability was proved without considering acceleration/deceleration limits and a collision-free constraint In Kianfar et al. (2015), the l_2-norm string stability was achieved using a control matching algorithm if the state and input constraints were inactive. Other than these works, string stability was usually demonstrated by simulations (Gong et al., 2016; Wang et al., 2016), lacking mathematical guarantee. Furthermore, there is a lack of CACC controllers, including linear and nonlinear controllers, that can satisfy multiple string stability criteria (e.g., l_2 and l_∞) although such controllers would be effective in dampening disturbances in different aspects (e.g., diminish disturbance energy through l_2 and peak magnitude through l_∞). This is not surprising as designing such controller, particularly in the MPC framework, is very difficult due to the complex form of the MPC with its constraints.

This paper aims to fill the gap identified above and develop a 'serial' distributed MPC framework with guaranteed local stability as well as l_∞ and l_2 norm string stability. In the proposed framework, a vehicle transmits future predicted information, such as acceleration, to its following vehicle who then implements local optimal control based on that information. String stability is formulated as an additional constraint in the local optimization problem for each vehicle. Particularly, the proposed approach guarantees l_∞ string stability over the entire feasibility region. In addition, by tuning the weighting coefficients of the objective function, the proposed MPC also guarantees stringent l_2 string stability over the robust positive invariant set when constraints are inactive. Specifically, if the initial condition is in the robust positive invariant set, the proposed controller behaves as a linear CACC controller, wherein the system constraints are inactive, and can guarantee more rigorous string stability. Simulation experiments verify the effectiveness of the proposed controller and related theorems, as opposed to a state-of-the-art distributed CAV controller without the string stability constraint.

The paper is organized as follows. Section 2 presents the platoon dynamics model and provides the mathematical preliminaries of definitions and theorems. Section 3 provides the formulation of the proposed distributed MPC problem for longitudinal platoon control and the mathematical proofs of the asymptotical local stability and l_∞ norm string stability. Section 4 describes the tuning method to achieve the l_2 norm string stability over the constraints inactive area. Section 5 presents the results of the simulation experiments. Conclusion and future research are discussed in the final section.

2. Platoon system dynamics and stability definitions

In this study, longitudinal control of CAV platoons pertains to the control of vehicle car-following (e.g., spacing). The main objectives of the proposed controller are to regulate the inter-vehicle spacing according to a certain (equilibrium) spacing policy and to maintain zero speed difference with the preceding vehicle. According to Society of Automotive Engineers (SAE) standard for CACC, the constant time gap policy is adopted, which defines the equilibrium spacing as:

$$d_i^*(t) = v_i(t)\tau_i^* + l_i \tag{1}$$

where $d_i^*(\mathrm{t})$ denotes the equilibrium spacing or the target control spacing at time t; $v_i(t)$ is the speed of vehicle i at time t; τ_i^* is the pre-defined constant time gap; and l_i is the standstill spacing of vehicle i. Then, the deviation from $d_i^*(t)$, $\Delta d_i(t)$, and the speed difference with the leading CAV, $\Delta v_i(t)$, are respectively defined as:

$$\Delta d_i(t) = d_i(t) - d_i^*(t) \tag{2}$$

$$\Delta v_i(t) = v_{i-1}(t) - v_i(t) \tag{3}$$

where $d_i(t)$ represents the spacing at time t.

For more realistic control, an actuation time-lag is considered in vehicle dynamics, rather than assuming instantaneous implementation of acceleration, as it takes some time for a vehicle system to achieve the desired acceleration commanded by the controller (Li et al., 2011):

$$\dot{a}_i(t) = -\frac{1}{\varphi_i}a_i(\mathrm{t}) + \frac{1}{\varphi_i}u_i(t) \tag{4}$$

where $a_i(\mathrm{t})$ is the vehicle acceleration; $u_i(t)$ is the control input, the desired acceleration for vehicle i; and ϕ_i is the actuation time-lag.

By defining the system state as $x_i(t) = [\Delta d_i(\mathrm{t}), \Delta v_i(\mathrm{t}), a_i(\mathrm{t})]^T$, the individual vehicle dynamics within a CAV platoon described by Eqs. (1)–(4) can be reformulated in a state-space form as follows:

$$\dot{x}_i(t) = A_i x_i(t) + B_i u_i(t) + \mathrm{D}a_{i-1}(t) \tag{5}$$

where $A_i = \begin{bmatrix} 0 & 1 & -\tau_i^* \\ 0 & 0 & -1 \\ 0 & 0 & -1/\varphi_i \end{bmatrix}$, $B_i = \begin{bmatrix} 0 \\ 0 \\ 1/\varphi_i \end{bmatrix}$, $D = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$

Although the kinematic motion of a vehicle is a continuous system, the proposed control is implemented in a discrete fashion. The discrete version of the state-space system can be derived by assuming the zeroth-order hold (ZOH) for the control input, which means that the control input is constant during each update interval Δt_s: $u_i(t) = u_{i,k}$, $k\Delta t_s \le t \le (k+1)\Delta t_s$. When Δt_s is sufficiently small, the discretization process is close enough to the continuous system (Chen, 1995). Then the discrete version of Eq. (5) is shown as below:

$$x_{i,k+1} = A_i' x_{i,k} + B_i' u_{i,k} + D_i' a_{i-1,k} \tag{6}$$

where

$$A_i' = e^{A_i \Delta t_s} = \begin{bmatrix} 1 & \Delta t_s & \varphi_i\left(\tau_i^* - \varphi_i\right)\left(e^{-\frac{\Delta t_s}{\varphi_i}} - 1\right) - \Delta t_s \varphi_i \\ 0 & 1 & \varphi_i\left(e^{-\frac{\Delta t_s}{\varphi_i}} - 1\right) \\ 0 & 0 & e^{-\frac{\Delta t_s}{\varphi_i}} \end{bmatrix} \tag{7}$$

$$B_i' = \int_0^{\Delta t_s} e^{A_i t} dt B_i = \begin{bmatrix} -\varphi_i\left(\tau_i^* - \varphi_i\right)\left(e^{-\frac{\Delta t_s}{\varphi_i}} + \frac{\Delta t_s}{\varphi_i} - 1\right) - \frac{\Delta t_s{}^2}{2} \\ \varphi_i\left(1 - e^{-\frac{\Delta t_s}{\varphi_i}}\right) - \Delta t_s \\ 1 - e^{-\frac{\Delta t_s}{\varphi_i}} \end{bmatrix} \tag{8}$$

$$D_i' = \int_0^{\Delta t_s} e^{A_i t} dt D = \begin{bmatrix} \frac{\Delta t_s{}^2}{2} \\ \Delta t_s \\ 0 \end{bmatrix} \tag{9}$$

$x_{i,k} = [\Delta d_{i,k},\ \Delta v_{i,k},\ a_{i,k}]$, and $u_{i,k}$ is the state given and control input in time step k. $e^{A_i \Delta t_s}$ is a matrix exponential, obtained via the Taylor series expansion: $e^{A_i \Delta t_s} = \sum_{k=0}^{\infty} \frac{1}{k} (A_i \Delta t_s)^k$. To obtain its closed form, Jordan-Chevalley Decomposition Stewart, 1976) (or the 'c2d' function in MATLAB) can be applied. Then, Eqs. (8) and ((9) can be derived by taking the integral of the closed form.

According to Eq. (6), the above discrete-time state space system reaches the equilibrium point when $x_{i,k+1} = x_{i,k}$, given no control input $u_{i,k}$ and external disturbance $a_{i-1,\,k}$. Based on this condition, we find that the equilibrium point for vehicle i is $x_{i,e} = [0,\,0,\,0]^T$. This implies that vehicle i reaches the equilibrium spacing with zero speed difference with the preceding vehicle and maintains constant speed (Chen, 1995). Therefore, (asymptotical) local stability can be defined around $x_{i,e}$ for vehicle i, as follows.

Definition 1. ***Lyapunov Local Stability*** (adopted directly from Willems and Polderman, 2013)

For the system of $x_{i,k+1} = f(x_{i,k})$, *the equilibrium point,* $x_{i,e}$, *is said to be Lyapunov local stable if for every* ε, *there exists a* $\delta > 0$ *such that if* $\|x_{i,0} - x_{i,e}\| < \delta$, *then for every* $k \geq 0$, *we have* $\|x_{i,k} - x_{i,e}\| < \varepsilon$.

This definition states that if an initial disturbance (e.g., a deviation from equilibrium spacing, speed difference, and/or acceleration) is close enough to the equilibrium point (within δ), the disturbance will always be close enough to the equilibrium point (within ε). Note that Lyapunov local stability does not guarantee convergence to the equilibrium point. If convergence is required, a stronger local stability definition of asymptotical local stability is needed, as follows.

Definition 2. ***Asymptotical Local Stability*** (adopted directly from Willems and Polderman, 2013)

For a system of $x_{i,t+1} = f(x_{i,t})$, *the equilibrium point,* $x_{i,e}$, *is said to be asymptotically local stable if it is Lyapunov local stable, and there exists* $\delta > 0$ *such that* $\|x_{i,t} - x_{i,e}\| \to 0$ *as* $t \to \infty$ *if* $\|x_{i,0} - x_{i,e}\| < \delta$.

The asymptotical local stability definition states that when a disturbance is close enough to the equilibrium point (within δ), the disturbance will be completely resolved over time, and the CAV is able to restore the original state.

Local stability deals with the behavior of individual CAVs over time while string stability is concerned with disturbance attenuation through a vehicular string in a platoon. Using the peak magnitude of spacing error $\Delta d_{i,t}$, we define the l_∞ string stability as below:

Definition 3. l_∞ ***String Stability***

For a system of a CAV platoon with length N, it is l_∞ *string stable if and only if*

$$\|\Delta d_{i+1}\|_{l_\infty} \leq \|\Delta d_i\|_{l_\infty} \text{ for } \forall i \in [1, 2, \ldots N-1] \tag{10}$$

where $\|\Delta d_i\|_{l_\infty}$ denotes the l_∞ norm of $\Delta d_{i,k}$, given as $\|\Delta d_i\|_{l_\infty} = \sup_k(|\Delta d_{i,k}|)$ for $k \in (0, \infty)$, $k \in Z$. Physically, Eq. (10) suggests that the peak magnitude of the deviation from equilibrium spacing is not amplified through the vehicular string.

In the ensuing section, a new serial distributed MPC algorithm will be developed for CAV car-following control to satisfy the local and l_∞ string stability by Definition 1-3.

3. Serial distributed model predictive control algorithm scheme

MPC is able to handle multiple-input-multiple-output (MIMO) systems. However, when controlling the large-scale system formed by many CACC systems, a centralized scheme is time-consuming, and the computational complexity increases drastically (Zhou et al., 2017). Therefore, distributing the computational load is critical to enhance the computational speed, and we develop a distributed scheme to satisfy the on-line computing requirement.

A distributed MPC can be further divided into serial distributed MPC and parallel distributed MPC depending on the implementation and communication scheme (Negenborn et al., 2007). For the parallel MPC, local optimizers (i.e., each CAVs) solve the problem simultaneously based on the past measured and communicated information (e.g., Wang et al., 2016). This approach essentially assumes that the acceleration of the leading vehicle (last adopted) will remain the same into the future. In contrast, for serial MPC, local optimizers solve the problem sequentially in a CAV string: the future control portfolio of each CAV is solved based on that of its leading vehicle communicated via V2V communication. This means that the most recent control output and predicted future states of the leading vehicle can be considered in determining optimal control. This helps each local MPC to converge more quickly and achieve better performance (Negenborn et al., 2008). Therefore, serial MPC for CACC is considered in this study.

In the proposed framework, an optimal control problem is solved at each time step k over prediction horizon k_p based on the current state of the system and a model for system dynamics to predict the future states. The controller only implements the first control input (i.e., optimal solution for time step k) and re-computes optimal control starting from the next time step, $k+1$, with the receding prediction horizon; see Fig. 1 for the illustration of the overall control concept.

Here we carefully distinguish optimal control sequence (i.e., acceleration prescribed by the controller), predicted future states on the optimal control, and actual realized states with unmodelled and unknown disturbances: ${}^{p}_{i,k} = [u^{p,k}_{i,k}, u^{p,k}_{i,k+1}, \ldots, u^{p,k}_{i,k+m}, u^{p,k}_{i,k+m+1}, \ldots, u^{p,k}_{i,k-1+k_p}]$ denotes the optimal control sequence for vehicle i obtained at time k for the prediction horizon, k to $k+k_p$; ${}^{p}_{i,k} = [x^{p,k}_{i,k}, x^{p,k}_{i,k+1}, \ldots, x^{p,k}_{i,k+m}, x^{p,k}_{i,k+m+1}, \ldots, x^{p.k}_{i,k+k_p}]$ denotes the predicted future states for vehicle i at k for the prediction horizon, k to $k+k_p$; ${}^{r}_{i,k} = [x^{r}_{i,0}, x^{r}_{i,1}, x^{r}_{i,2}, x^{r}_{i,3}, \ldots, x^{r}_{i,k}]$ is defined as the realized states for vehicle i by time k, where $x^{r}_{i,0}$ is the initial state when the control initiates.

Based on the notations above, we formulate an MPC problem and the solution algorithm in the remainder of this section.

3.1. General optimal control formulation

Here we formulate an optimal control problem, considering the car following control efficiency and driving comfort, subject to collision-free constraints and acceleration/deceleration limits. Specifically, based on the state space in Eq. (6), the optimization framework is given as:

$$\left({}^{p}_{i,k}, {}^{p}_{i,k}\right) = \arg\min \sum_{m=1}^{k_p} L_i\left(x^{p,k}_{i,k+m}, u^{p,k}_{i,k+m-1}\right) + F_i\left(x^{p,k}_{i,k+k_p}\right) \tag{11-a}$$

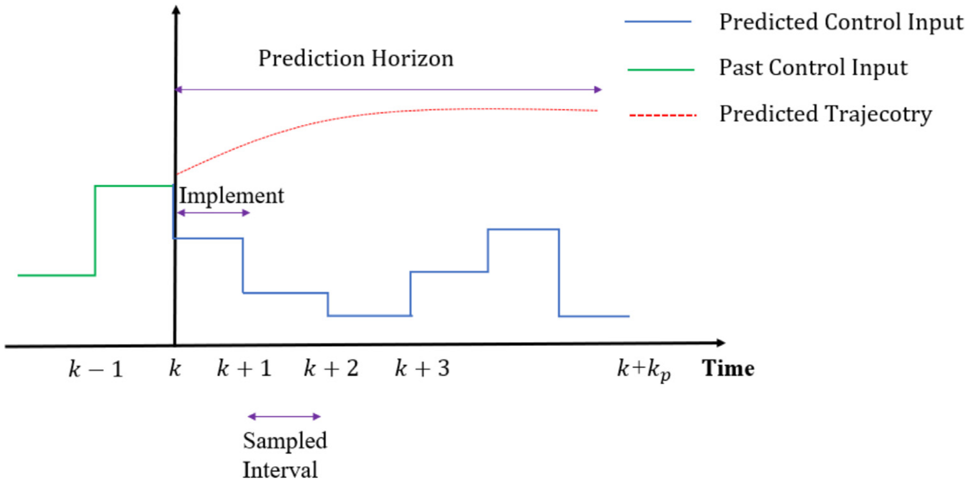


Fig. 1. Illustration of model predictive control.

s.t.

$$x^{p,k}_{i,k+m+1} = A'_i x^{p,k}_{i,k+m} + B'_i u^{p,k}_{i,k+m} + D'_i a^{p,k}_{i-1,k+m} \ \forall m \in \{1, 2, 3, \ldots, k_p - 1\} \tag{11-b}$$

$$x^{p,k}_{i,k} = x^r_{i,k} \tag{11-c}$$

$$x^{p,k}_{i,k+m} \in \chi_{i,k} \ \forall m \in \{1, 2, 3, \ldots, k_p\} \tag{11-d}$$

$$u_{i,t+k} \in U_i \ \forall m \in \{1, 2, 3, \ldots, k_p\} \tag{11-e}$$

$$x^{p,k}_{i,k+k_p} \in \chi_{i,f} \tag{11-f}$$

where k_p is the prediction horizon; $L_i(x^{p,k}_{i,k+m}, u^{p,k}_{i,k+m-1})$ is the running cost associated with the transition process of CAV before reaching an equilibrium point, considering the system state at each time step; and $F_i(x^{p,k}_{i,k+k_p})$ is the terminal cost that penalizes a deviation from equilibrium point $x_{i,e}$ at the end of the prediction horizon. Eq. (11-c) is the initial condition constraint equal to the measured state at time step k; and Eq. (11-d) is the state constraint to regulate the transition vehicle states such as regulate the maximum deviation from equilibrium spacing or acceleration constraints at each time point. The constraint in Eq. (11-e) is used to guarantee that the commanded acceleration is within reasonable range of $U_i = [u_{i,min}, u_{i,max}]$, where $u_{i,max}$ and $u_{i,min}$ represent the upper and lower bounds of acceleration, respectively. Eq. (11-f) is the terminal state constraint to regulate the terminal state to be close enough to $x_{i,e}$ for CAV i. A more detailed description of the design will be given in Section 3.3 based on the online vehicle control scheme provided in Section 3.2.

3.2. Online vehicle control scheme

The vehicular optimal control scheme formulated in Eq. (11) is designed as a serial distributed rolling horizon control. Algorithm 1 below provides the details of the vehicle information communication scheme and the rolling horizon implementation procedure.

Unlike the parallel distributed control as in Wang et al. (2016), the proposed algorithm uses the most recent information from the leading vehicle and therefore, control of CAVs in a platoon is performed sequentially. This setting enhances the performance of controller by utilizing the most recent information (Negenborn et al., 2008), though the system may suffer longer communication delay accumulated over vehicles. However, the field test by Ploeg et al. (2015) suggests that the

Algorithm 1

1. Initialization: Without loss of generality, the algorithm starts from vehicle $i = 1$ at time interval $k = 0$. Set $\chi^p_{1,0} = \vec{0}$ and $x^r_{i,0}$ for the initial condition.
2. At time k, vehicle i (i) calculates the predicted optimal control sequence, $U^p_{i,k}$, by solving the local optimal control problem in Eq. (11); (ii) derives the predicted trajectory, $\chi^p_{i,k}$, according to Eq. (6); (iii) implements the control $u^{p,k}_{i,k}$; and (iv) transmits $\chi^p_{i,k}$ to vehicle $i+1$ via V2V communication.
3. Check for $i < N$, the platoon size. If yes, update the vehicle number $i = i+1$ and go back to step 2. If not, go to step 4.
4. Shift the prediction horizon, update the time, $k = k+1$, and go back to step 2.

communication delay is typically very small (≈ 0.01 s) and therefore, the communication delay is assumed insignificant for a small platoon.

3.3. Controller design to guarantee local stability and l_∞ string stability

To guarantee the local stability and l_∞ string stability (as defined in Section 2) of the proposed framework, the running and terminal cost functions, state constraints, and terminal constraints should be carefully designed. The details follow.

3.3.1. Cost function design

For the optimal control formulated in Eq. (11), we specify the running cost and terminal cost to derive the local and string stability conditions. The running cost function is specified as:

$$L_i\left(x^{p,k}_{i,k+m}, u^{p,k}_{i,k+m-1}\right) = \underbrace{\left(x^{p,k}_{i,k+m}\right)^T Q_i x^{p,k}_{i,k+m}}_{\text{control efficiency}} + \underbrace{R_i\left(u^{p,k}_{i,k+m-1}\right)^2}_{\text{comfort}} \tag{12-a}$$

where $R_i > 0$ is the comfort and fuel consumption coefficient as designed by Wang et al. (2014a), and Q_i is a positive definite diagonal coefficients matrix, which is usually designed as the diagonal matrix below:

$$Q_i = \begin{bmatrix} \alpha_{1,i} & & \\ & \alpha_{2,i} & \\ & & \alpha_{3,i} \end{bmatrix}, \alpha_{1,i},\ \alpha_{2,i},\ \alpha_{3,i} > 0 \tag{12-b}$$

$F_i(x^{p,k}_{i,k+k_p}) = (x^{p,k}_{i,k+k_p})^T P_{di} x^{p,k}_{i,k+k_p}$ is the terminal cost, where P_{di} is given as the solution of Discrete Algebraic Recatti Equation (DARE) to guarantee the local stability as suggested by Rakovic et al. (2005). A detailed proof will be given later in Propositions 1 and 2. The P_{di} is given as below:

$$P_{di} = Q_i + A_i'\left[P_{di} - P_{di}B_i'\left(R_i + B_i'^T P_{di} B_i'\right)^{-1} B_i'^T P_{di}\right]A_i' \tag{13}$$

3.3.2. State constraint design

In Eq. (11-d), $\chi_{i,k}$ is vehicle i's state constraint determined at time k. It is formulated in detail as follows:

$$a_{i,min} \le H x^{p,k}_{i,k+m} \le a_{i,max}\ \forall m \in \{1, 2, 3, \ldots, k_p - 1\} \tag{14-a}$$

$$\Delta d^-_{i,k+1} \le G x^{p,k}_{i,k+m} \le \Delta d^+_{i,k+1}\ \forall m \in \{1, 2, 3, \ldots, k_p - 1\} \tag{14-b}$$

$$\Delta d^-_{i,k+1} = \begin{cases} -s_{fi} \text{ for } i = 1 \\ -max\left(\left|\Delta d^r_{i-1,\mu}\right|\right) \text{ for } i > 1 \text{ for } \forall\mu \in \{0,\ 1, 2, k+1\} \end{cases} \tag{14-c}$$

$$\Delta d^+_{i,k} = max\left(\left|\Delta d^r_{i-1,\mu}\right|\right) \text{ for } i > 1 \text{ for } \forall\mu \in \{0,\ 1, 2, \mathrm{k}+1\} \tag{14-d}$$

Eq. (14-a) is the constraint that the realized acceleration/deceleration for vehicle i is within the physical limits of $[a_{i,min}, a_{i,max}]$, where $H = [0,\ 0,\ 1]^T$, and $a_{i,min}$ and $a_{i,max}$ are respectively vehicle i's deceleration and acceleration limits. Eq. (14-b) is the constraint that the deviation from equilibrium spacing is not too big. It also guarantees l_∞ string stability as will be proved formally later in this section. s_{fi} is the predefined safe distance, and $G = [1,\ 0,\ 0]^T$;

3.3.3. Terminal constraint design

To guarantee local stability, the terminal constraint, $\chi_{i,f}$, in Eq. (11-f) should be carefully designed. $\chi_{i,f}$ can be defined as any subset of robust positive invariant set $_{i,RPI}$ to guarantee that the CAV state will remain in $_{i,RPI}$ once it enters. We require that $\chi_{i,f} \subseteq _{i,RPI} \subseteq \chi_{i,0}$ and $x_{i,e} \in _{i,RPI}$ as suggested by Rakovic et al. (2005) and Blanchini (1999), which is given by Eq. (15):

$$_{i,RPI} = \left\{x_i = [\Delta d_i,\ \Delta v_i, a_i] \subseteq _{i,RPI} | A_i' x_{i,t} + B_i' u_{i,t} + D_i'\xi \in _{i,RPI}, u_{i,t} = h_i(x_{i,t}, \xi) \in\ U_i \text{ and } \xi \in [a_{i-1,min}, a_{i-1,max}]\right\} \tag{15}$$

In Eq. (15), the demanded acceleration function, $h(x_{i,t},\ \xi)$, is an implicit control law for the MPC when it reaches the robust positive invariant set. In this case, the implicit control law can be approximated by an unconstrained optimization problem as an extended-linear quadratic regulator (ELQR) problem (Singh and Pal, 2017) as below:

$$\min J_i\left(x^{p,k}_{i,k+m}, u^{p,k}_{i,k+m-1}\right) = \sum_{k=0}^{\infty} L_i\left(x^{p,k}_{i,k+m}, u^{p,k}_{i,k+m-1}\right) \tag{16-a}$$

s.t

$$x^{p,k}_{i,k+m+1} = A'_i x^{p,k}_{i,k+m} + B'_i u^{p,k}_{i,k+m} + D'_i a^{p,k}_{i-1,k+m} \tag{16-b}$$

By solving the ELQR, we have the implicit control law given by the discrete linear feedback and feedforward controller as in Eq. (17-a):

$$u^{p,k}_{i,k+m} = k_{i,d} x^{p,k}_{i,k+m} + k_{fi,d} a^{p,k}_{i-1,k+m} \tag{17-a}$$

where $k_{i,d}$ is the discrete linear feedback gain to account for the current state and $k_{fi,d}$ is the feedforward gain to account for the acceleration of the leading vehicle. Details are given below.

$$k_{i,d} = -\left(R_i + B_i'^T P_{di} B_i'\right)^{-1} B_i'^T P_{di} A_i' \tag{17-b}$$

$$k_{fi,d} = -\left(R_i + B_i'^T P_{di} B_i'\right)^{-1} B_i'^T P_{di} A_i' \left(A_i' - P_{di}^{-1}(P_{di} - Q_i)\right)^{-1} B_i' D_i' \tag{17-c}$$

As a special case, when $i = 1$ (the first leading vehicle in a platoon), $a^{p(k)}_{i-1,k+m} = 0$ for $\forall m \in \{1, 2, 3, \ldots, k_p - 1\}$ and $\forall t \geq 0$. In this case, there is no l_∞ norm string stability constraint, so Eq. (14-b) becomes $Gx^t_{1,t+k} \leq s_{f1}$, and $\chi_{1,t}$ is time invariant and can be represented by χ_1. Furthermore, the implicit control law for $u_{1,t} = h_1(x_{1,t})$ becomes $u_{1,t} = k^T_{1,d} x_{1,t}$, where k_1 follows the DARE as in Eq. (17-a) to (17-c). Accordingly, the terminal constraint set can be chosen as $\chi_{1,f} \subseteq_{1,RPI} \subseteq \chi_1$, which is given below:

$$\chi_{1,f} \subseteq {}_{1,PI} = \left\{x_1 = [\Delta d_1,\ \Delta v_1, a_1] \subseteq \chi_1 | A'_1 x_{i,t} + B'_1 u_{i,t} \in {}_{1,PI}, u_{1,t} = h_1(x_{1,t}) \in U_1\right\} \tag{18}$$

For the details of estimating and approximating ${}_{i,RPI}$ and ${}_{1,PI}$, the reader is referred to Marruedo et al. (2002) and Tahir and Jaimokha (2012).

Based on the constraints, running cost, and terminal cost designed above, the following propositions related to robust recursive feasibility, local stability, and l_∞ string stability can be obtained. The details follow.

Proposition 1. *For the first vehicle, the serial distributed algorithm as in Algorithm 1 is recursively feasible if the MPC is initially feasible.*

Proof. If the MPC is initially feasible, there exists an optimal control sequence that ${}^{p}_{1,0} = [u^{p,0}_{1,0}, u^{p,0}_{1,1}, \ldots, u^{p,0}_{1,m}, u^{p,0}_{1,m+1}, \ldots, u^{p,0}_{1,k_p}] \in U_1$, with the predicted trajectory, ${}^{p}_{i,0} = [x^{p,0}_{1,1}, \ldots, x^{p,0}_{1,m}, x^{p,0}_{1,m+1}, \ldots, x^{p,0}_{1,k_p}] \in \chi_1$ and $x^{p,0}_{1,k_p} \in \chi_{1,f}$. According to Algorithm 1, vehicle 1 implements acceleration $u^0_{1,0}$ according to Eq. (5), which gives the realized trajectory state $x^r_{1,1} = x^0_{1,1}$. Therefore, at time interval 1, it is trivial to verify that the solution is feasible, given as $[u^{p,0}_{1,0}, u^{p,0}_{1,1}, \ldots, u^{p,0}_{1,m}, u^{p,0}_{1,m+1}, \ldots, u^{p,0}_{1,k_p}, k^T_{1,d} x^{p,0}_{1,k_p}] \in U_1$, with the predicted trajectory $[x^{p,0}_{1,2}, \ldots, x^{p,0}_{1,m}, x^{p,0}_{1,m+1}, \ldots, x^{p,0}_{1,k_p}, A'_1 x^{p,0}_{1,k_p} + B'_1 k^T_{1,d} x^{p,0}_{1,k_p}] \in \chi_1$ and $A'_1 x^{p,0}_{1,k_p} + B'_1 k^T_{1,d} x^{p,0}_{1,k_p} \in \chi_{1,f}$ by the invariance set in Eq. (18). Using the same logic, it can be proved that the feasibility can be satisfied at all time intervals using induction. □

Based on Proposition 1, the asymptotical stability for the first vehicle is given as below:

Proposition 2. *The CAV platoon leader is asymptotically locally stable if the MPC for vehicle 1 is initially feasible.*

Proof. At $t = 0$, we have ${}^{p}_{1,0} = [u^{p,0}_{1,0}, u^{p,0}_{1,1}, \ldots, u^{p,0}_{1,m}, u^{p,0}_{1,m+1}, \ldots, u^{p,0}_{1,k_p}] \in U_1$ and thus ${}^{p}_{i,0} = [x^{p(0)}_{1,1}, \ldots, x^{p(0)}_{1,m}, x^{p(0)}_{1,m+1}, \ldots, x^{p(0)}_{1,k_p}] \in \chi_1$ with the optimal cost, $J^*_{1,0}$. By Proposition 1, at time interval $k = 1$, there exists a feasible but sub-optimal control sequence $[u^{p,0}_{1,0}, u^{p,0}_{1,1}, \ldots, u^{p,0}_{1,m}, u^{p,0}_{1,m+1}, \ldots, u^{p,0}_{1,k_p}, k^T_{1,d} x^{p,0}_{1,k_p}] \in U_1$, with the predicted trajectory, $[x^{p,0}_{1,2}, \ldots, x^{p,0}_{1,m}, x^{p,0}_{1,m+1}, \ldots, x^{p,0}_{1,k_p}, A'_1 x^{p,0}_{1,k_p} + B'_1 k^T_{1,d} x^{p,0}_{1,k_p}] \in \chi_1$, which results in the cost, $J_{1,1}$ $(\geq J^*_{1,1})$. By applying Proposition 1, we only need to prove that $J^*_{1,1} - J^*_{1,0} < 0$ for the asymptotical local stability.

$$\begin{aligned} J_{1,1} - J^*_{1,0} = &\left(A'_1 x^{p,0}_{1,k_p} + B'_1 k^T_{1,d} x^{p,0}_{1,k_p}\right)^T P_{d1} \left(A'_1 x^{p,0}_{1,k_p} + B'_1 k^T_{1,d} x^{p,0}_{1,k_p}\right) + \left(k^T_{1,d} x^{p,0}_{1,k_p}\right)^T R_1 \left(k^T_{1,d} x^{p,0}_{1,k_p}\right) \\ &+ \left(x^{p,0}_{1,k_p}\right)^T Q_1 x^{p,0}_{1,k_p} - \left(x^{p,0}_{1,1}\right)^T Q_1 x^{p,0}_{1,1} - u^{p,0}_{1,0} R_1 u^{p,0}_{1,0} - \left(x^{p,0}_{1,k_p}\right)^T P_{d1} x^{p,0}_{1,k_p} \end{aligned}$$

□

We have $(A_1' x_{1,k_p}^{p,0} + B_1' k_{1,d}^T x_{1,k_p}^{p,0})^T P_{d1} (A_1' x_{1,k_p}^{p,0} + B_1' k_{1,d}^T x_{1,k_p}^{p,0}) + (k_{1,d}^T x_{1,k_p}^{p,0})^T R_1 (k_{1,d}^T x_{1,k_p}^{p,0}) + (x_{1,k_p}^{p,0})^T Q_1 x_{1,k_p}^{p,0} - (x_{1,k_p}^{p,0})^T P_{d1} x_{1,k_p}^{p,0} < 0$

Then, we have $J_{1,1}^* - J_{1,0}^* < J_{1,1} - J_{1,0}^* < -(x_{1,k_p}^{p,0})^T Q_1 x_{1,k_p}^{p,0} - u_{1,0}^{p,0} R_1 u_{1,0}^{p,0} < 0$. This means that the optimal cost is decreasing over time, satisfying the asymptotical local stability.

Definition 4. ***Robust Feasibility*** (adopted directly from Kerrigan and Maciejowski, 2001)

The MPC is controller is robustly feasible if and only if the state and control input of MPC at the next time interval remains in the feasible set for all states in the feasible set and for a predefined bounded disturbance.

Proposition 3. *The sequential distributed MPC algorithm for CACC is robust recursive feasible for $\forall i \geq 2$ if the optimization problem in Eqs. (11)–(18) is initially robustly feasible.*

Proof. Similar to the proof of Proposition 1, when the MPC for CACC is robust feasible initially, we have $[u_{i,0}^{p,0}, u_{i,1}^{p,0}, \ldots, u_{i,m}^{p,0}, u_{i,m+1}^{p,0}, \ldots, u_{i,k_p}^{p,0}] \in U_i$, and thus ${}^{p}_{i,t} = [x_{i,1}^{p,0}, \ldots, x_{i,m}^{p,0}, x_{i,m+1}^{p,0}, \ldots, x_{i,k_p}^{p,0}] \in \chi_{i,0}$ and $x_{i,k_p}^{p,0} \in \chi_{i,f}$. With the serial implementation and V2V communication, we have $x_{i,1}^r = x_{i,1}^{p,0}$. Also, since the MPC is initially robust feasible, we can verify that $[u_{i,0}^{p,0}, u_{i,1}^{p,0}, \ldots, u_{i,m}^{p,0}, u_{i,m+1}^{p,0}, \ldots, u_{i,k_p-1}^{p,0}, k_{i,d}^T x_{i,k_p}^{p,0} + k_{fi,d} a_{i-1,k_p}^{p,0}] \in U_i$; $[x_{i,2}^{p,0}, \ldots, x_{i,m}^{p,0}, x_{i,m+1}^{p,0}, \ldots, x_{i,k_p}^{p,0)}, A_i' x_{i,k_p}^{p,0} + B_i' k_{i,d}^T x_{i,k_p}^{p,0} + D_i' k_{fi,d} a_{i-1,k_p}^{p,0}] \in \chi_{i,0}$; and $A_i' x_{i,k_p}^{p,0} + B_i' k_{i,d}^T x_{i,k_p}^{p,0} + D_i' k_{fi,d} a_{i-1,k_p}^{p,0} \in \chi_{i,f}$. Furthermore, by Algorithm 1 for control implementation, the constraint in Eq. (14-c) is updated and become more relaxed over time, so we have $\chi_{i,0} \subseteq \chi_{i,1} \subseteq \chi_{i,k}$ for $\forall k \geq 2$. Thus, $[x_{i,2}^{p,0}, \ldots, x_{i,m}^{p,0}, x_{i,m+1}^{p,0}, \ldots, x_{i,k_p}^{p,0}, A_i' x_{i,k_p}^{p,0} + B_i' k_{i,d}^T x_{i,k_p}^{p,0} + D_i' k_{fi,d} a_{i-1,k_p}^{p,0}] \in \chi_{i,1}$, which satisfies the robust recursive feasibility. Like Proposition 1, by induction, it is trivial to verify that the MPC is feasible at all time steps. □

Proposition 4. *The sequential distributed MPC algorithm for CACC for $i \geq 2$ is asymptotically stable if the MPC for vehicle 1 is initially robust feasible, and the serial distributed MPC algorithm for CACC for $\forall i \geq 2$ is robust recursive feasible.*

Proof. By Proposition 3, if the sequential distributed MPC algorithm for CACC for $\forall i = 2$ is initially robust feasible, MPC for $i = 2$ is robust recursive feasible. Further, by Proposition 2, the first vehicle is asymptotically stable: i.e., it will not accelerate nor decelerate when it reaches its equilibrium. By that time, vehicle 2 will not experience any disturbance, and vehicle 2 can then be treated as vehicle 1 with no disturbance. By robust recursive feasibility, there exists a feasible solution for vehicle 2 after it reaches an equilibrium point. Vehicle 2 will then go through the process similar to vehicle 1 and becomes asymptotically stable. Again, for $\forall i > 2$, the asymptotical stability can be verified via induction. □

Proposition 5. *The sequential distributed MPC algorithm for CACC is l_∞ string stable if the optimal control problem in Eqs. (11)–(18) is feasible at all time steps.*

Proof. When the MPC is feasible at all time steps, we have $|\Delta d_{i-1,\, k+1}| \leq max(|\Delta d_{i-1,\, \mu}|)$ for $i > 1$ for $\mu \in [0, 1, 2, \ldots, k+1]$ at each time point k by the constraints in Eq. (14-c) and (14-d). Furthermore, we have $x_{i,t}^r = x_{i,k+1}^{p,k}$ by the state space in Eq. (14-b). Thus, for $\forall k > 0$, $|\Delta d_{i,\mathrm{k}+1}^r| \leq max(|\Delta d_{i-1,\mu}^r|)$ for $\mu \in [0, 1, 2, \ldots, k+1]$ when $k \to \infty$, and by the definition of l_∞ norm string stability, $\|\Delta d_i\|_{l_\infty} \leq \|\Delta d_{i-1}\|_{l_\infty}$ for $\forall i \in [2, \ldots \infty)$. □

Remark 2. The sufficient conditions for the robust feasibility and initial feasibility have been proved for the general MPC framework by Kerrigan and Maciejowski (2001) and Löfberg (2012), respectively, and therefore, the proofs are omitted here. Note that although the robust feasibility requirements appear to be strong for practical purposes, these requirements can be satisfied relatively easily if a long enough prediction horizon k_p is selected (Dunbar et al., 2012). Moreover, a long prediction horizon will also increase the chance that the proposed MPC reaches the terminal constraints. Furthermore, if no feasible solution exists, we may first relax the string stability constraints and then the terminal constraint since safety and physical acceleration/deceleration limits are more major concerns, or provide softer constraints as suggested by Richards (2015).

As Eqs. (11)–(18) suggest, the formulated problem is a quadratic programming problem that can be solved by existing methods such as interior point methods (Kruth, 2008). Furthermore, the proposed MPC may be treated as a two-stage MPC algorithm, wherein a vehicle is controlled into to the (robust) positive invariant set in the first stage to guarantee that the vehicle state is close enough to the equilibrium point, $x_{i,e}$, and when the state falls into the (robust) positive invariant set, the linear controller can be used in the second stage to control the vehicle.

4. Weight matrix tuning to guarantee l_2-norm string stability

The l_∞-norm string stability assures that the peak magnitude of a disturbance dissipates through the vehicular string. In contrast, the l_2-norm string stability guarantees that the total energy of the disturbance dissipates through the vehicular string. Both string stability criteria are useful as shown by Fig. 3. Therefore, we also propose a theorem by properly tuning the weight matrix for the objective function in Eq. (12) to achieve l_2-norm string stability when constraints are inactive.

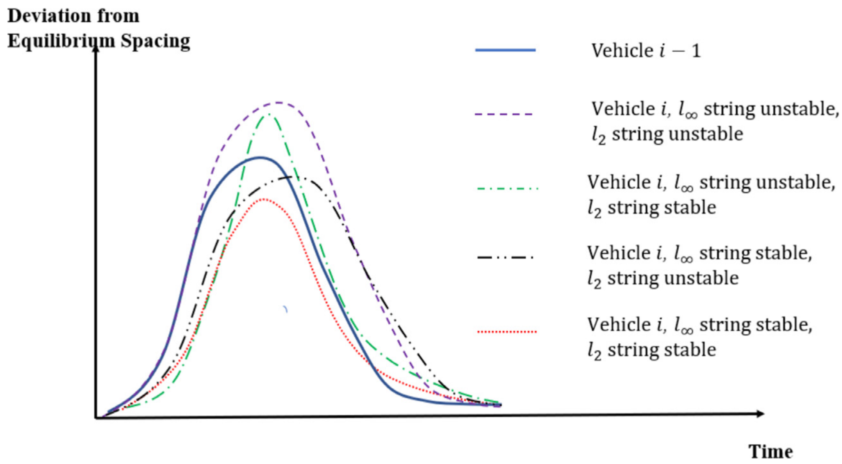


Fig. 2. Illustration of Different String Stability Definitions.

The l_2-norm string stability definition is given below:

Definition 5. ***l_2-norm string stability definition*** (Naus et al., 2010)

For a system (a CAV platoon with length N), it is l_2-norm string stable if and only if

$$\frac{\|\Delta d_i(t)\|_{l_2}}{\|\Delta d_{i-1}(t)\|_{l_2}} \le 1 \; for \, \forall i \in [1, 2, \ldots N-1] \tag{19}$$

where $\|\Delta d_i(t)\|_{l_2}$ is the l_2-norm of $\Delta d_i(t)$, equal to Riemann integral $\|\Delta d_i(t)\|_{l_2} = [\int_0^\infty (\Delta d_i(\tau))^2 d\tau]^{1/2}$ and can be approximated by Riemann sum $\|\Delta d_i(t)\|_{l_2} = \sqrt{\sum_{\tau=0}^{\infty} \Delta d_{i,\tau}^2}$ given that the discretization step t_s is sufficiently small. By the continuum approximation, the traditional l_2-norm string stability analysis method (e.g., Naus et al., 2010; Ploeg et al., 2014) can be applied with some modifications.

Then, given that t_s is sufficiently small, the implicit law, $h(x_{i,t},\ u_{i,t},\ \xi)$, can be approximated by the continuous law, $h(x_i(t),u_i(t),a_{i-1}(t))$ (Bemporad et al., 2002; Di Cairano and Bemporad, 2010), as follows:

$$\min J_i(x_i(t), u_i(t)) = \int_0^\infty x_i(\tau)^T Q_i x_i(\tau) + R_i u_i(\tau)^2 \, d\tau \tag{20-a}$$

s.t.

$$\dot{x}_i(t) = A_i x_i(t) + B_i u_i(t) + D a_{i-1}(t) \tag{20-b}$$

By solving the continuous extended linear quadratic regulator, the optimal control for the constrained problem in Eq. (13) is given as a linear feedback and feedforward controller as below:

$$u_i(\mathrm{t}) = k_i^T x_i(t) + k_{Fi} a_{i-1}(t) \tag{20-c}$$

where $k_i = [k_{si}, k_{vi}, k_{ai}]$ are vehicle i's constant continuous feedback gains for the deviation from equilibrium spacing, speed difference with the leading vehicle, and the acceleration, respectively, which can be solved by Continuous Algebraic Recatti Equation (CARE) (Anderson and Moore, 2007):

$$k_i = -R_i^{-1} B_i^T P_i \tag{21-a}$$

$$k_{fi} = -R_i^{-1} B_i^T \left[\left(A_i - R_i^{-1} B_i B_i^T P_i \right)^T \right]^{-1} P_i \mathrm{D} \tag{21-b}$$

$$P_i A_i + A_i^T P_i - P_i B_i R_i^{-1} B_i^T P_i = Q_i \tag{21-c}$$

Based on Section 3, the l_2-norm string stability can also be achieved around the equilibrium point, $x_{i,e}$, by proper tuning. Thus, the system becomes a linear system whose asymptotical l_2-norm string stability can be analyzed conveniently by a frequency domain analysis (Naus et al., 2010). Specifically, we derive sufficient conditions for l_2-norm string stability for a homogenous CAV platoon, in which Q_i, R_i, ϕ_i, and τ_i^* are same for all CAVs, as well as for a heterogeneous CAV platoon. Details follow.

By Parseval theorem (MacAulay-Owen, 1939), the energy is the same in frequency and time domains:

$$\frac{\|\Delta d_i(t)\|_{l_2}}{\|\Delta d_{i-1}(t)\|_{l_2}} = \frac{\|\Delta d_i(s)\|_{h_2}}{\|\Delta d_{i-1}(s)\|_{h_2}} \tag{22}$$

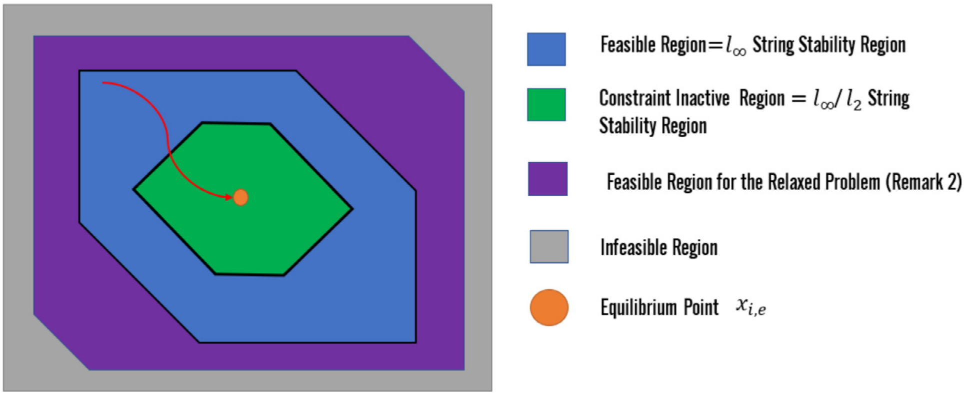


Fig. 3. Geometrical Illustration for the String Stability Region.

where $\Delta d_i(s)$ is the Laplacian transformation of $\Delta d_i(t)$, and $\|\Delta d_i(s)\|_{h_2}$ is the h_2-norm of $\Delta d_i(s)$. $s = j\omega$, where j is the imaginary unit, and ω represents the signal frequency.

For a homogenous platoon, the transfer function, $\frac{\Delta d_i(s)}{\Delta d_{i-1}(s)}$, is derived as below:

$$\frac{\Delta d_i(s)}{\Delta d_{i-1}(s)} = G_i(s) = \frac{\left(k_{s,i} + k_{v,i}s + k_{f,i}s^2\right)}{\varphi_i s^3 - (k_{a,i} - 1)s^2 + \left(\tau_i^* k_{s,i} + k_{v,i}\right)s + k_{s,i}} \tag{23}$$

Then the Cauchy inequality can be employed to make $\|G_i(s)\|_{h_\infty} \leq 1$ to satisfy Definition 5, which gives a sufficient condition for k_{si},k_{vi},k_{ai}, and k_{fi}, as shown in Proposition 6 below.

Proposition 6. *The proposed sequential distributed MPC is l_2-norm string stable over the robust positive invariant set in Eq. (15) for the homogenous platoon if the following conditions hold:*

$$(k_{a,i} - 1)^2 - 2\varphi_i\left(\tau_i^* k_{s,i} + k_{v,i}\right) - k_{f,i}^2 > 0 \tag{24-a}$$

$$2k_{s,i}k_{a,i} + \left(\tau_i^* k_{s,i} + k_{v,i}\right)^2 - k_{s,i}^2 + 2k_{s,i}k_{f,i} > 0 \tag{24-b}$$

The detailed proof is omitted here but provided in **Appendix 1**.

For the heterogeneous platoon with different Q_i, R_i, ϕ_i, and τ_i^* across CAVs, the transfer function of $\frac{\Delta d_i(s)}{\Delta d_{i-1}(s)}$ becomes

$$F_i(s) = \frac{\Delta d_i(s)}{\Delta d_{i-1}(s)} = \frac{\Delta d_i(s)}{v_i(s)} \times \left(\frac{\Delta d_{i-1}(s)}{v_{i-1}(s)}\right)^{-1} \times \frac{v_i(s)}{v_{i-1}(s)} \tag{25}$$

After simplification,

$$F_i(s) = \frac{1 - G_i(s) - s\tau_i^* G_i(s)}{1 - G_{i-1}(s) - s\tau_{i-1}^* G_{i-1}(s)} \times G_{i-1}(s) \tag{26}$$

Similar to the homogenous case, $\|F_i(s)\|_{h_\infty} \leq 1$ to satisfy Definition 4, which gives a sufficient condition for k_{si},k_{vi},k_{ai}, and k_{fi}, as presented in Proposition 7.

Proposition 7. *The proposed sequential distributed model predictive control is l_2-norm string stable over constraint-inactive set in Eq. (15) for the heterogeneous platoon if the following conditions in Eq. (27) hold:*

$$\varphi_i^2\left(\varphi_{i-1} - \tau_{i-1}k_{f,i-1}\right)^2 > 0 \tag{27-a}$$

$$\varphi_i^2\left(k_{a,i-1} - 1 + k_{f,i-1} + \tau_{i-1}k_{v,i-1}\right)^2 + \left[(k_{a,i} - 1)^2 - 2\varphi_i\left(\tau_i k_{d,i} + k_{v,i}\right)\right] \times \left(\varphi_{i-1} - \tau_{i-1}k_{f,i-1}\right)^2 - k_{f,i-1}^2\left(\varphi_i - \tau_i k_{f,i}\right)^2 > 0 \tag{27-b}$$

$$\begin{aligned}&\left[(k_{a,i} - 1)^2 - 2\varphi_i\left(\tau_i k_{d,i} + k_{v,i}\right)\right] \times \left(k_{a,i-1} - 1 + k_{f,i-1} + \tau_{i-1}k_{v,i-1}\right)^2 \\ &+\left[\left(\tau_i k_{d,i} + k_{v,i}\right)^2 + 2k_{d,i}(k_{a,i} - 1)^2\right] \times \left(\varphi_{i-1} - \tau_{i-1}k_{f,i-1}\right)^2 \\ &-\left(\varphi_i - \tau_i k_{f,i}\right)^2\left(k_{v,i-1}^2 - 2k_{f,i-1}k_{d,i-1}\right) - k_{f,i-1}^2\left(k_{a,i} - 1 + k_{f,i} + \tau_i k_{v,i}\right)^2\end{aligned} \tag{27-c}$$

$$\begin{aligned}&\left[\left(\tau_i k_{d,i} + k_{v,i}\right)^2 + 2k_{d,i}(k_{a,i} - 1)\right]\left(k_{a,i-1} - 1 + k_{f,i-1} + \tau_{i-1}k_{v,i-1}\right)^2 + k_{d,i}^2\left(\varphi_{i-1} - \tau_{i-1}k_{f,i-1}\right)^2 - k_{d,i-1}^2\left(\varphi_i - \tau_i k_{f,i}\right)^2 \\ &-\left(k_{a,i} - 1 + k_{f,i} + \tau_i k_{v,i}\right)^2\left(k_{v,i-1}^2 - 2k_{f,i-1}k_{d,i-1}\right) > 0\end{aligned} \tag{27-d}$$

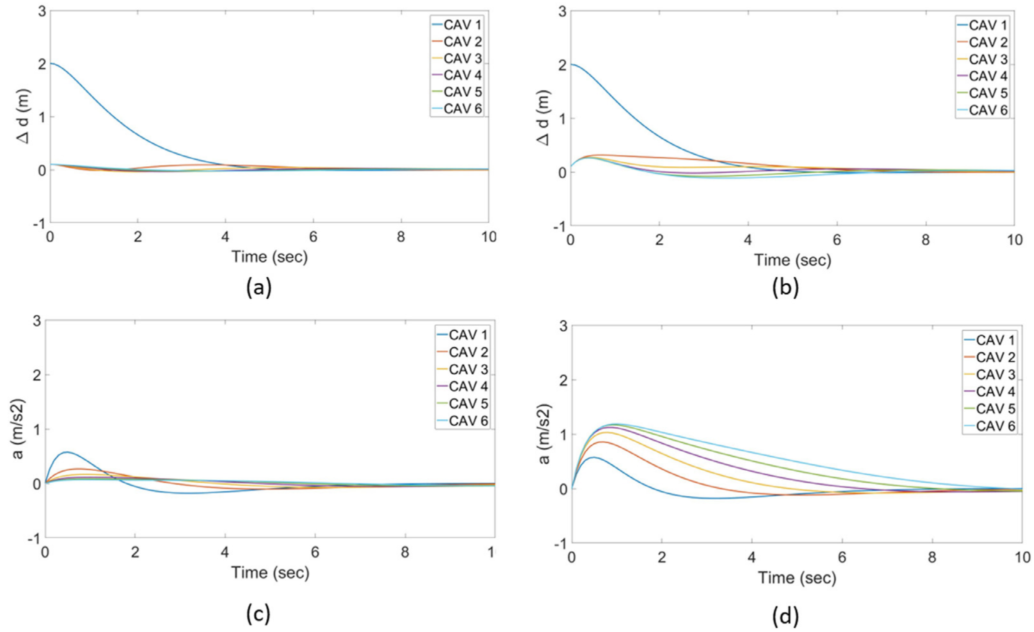


Fig. 4. $\Delta d_i(t)$ and $a_i(t)$ Comparison for Experiment 1: (a)-(b) $\Delta d_i(t)$ under the Serial Distributed MPC with and without the l_∞-norm String Stability Constraints; (c)-(d) $a_i(t)$ under the Serial Distributed CACC with and without the l_∞-norm String Stability Constraints.

Table 1
Parameter Values for Simulation.

CACC Parameters	Default Value
u_{low}	$-4\ m/s^2$
u_{up}	$4\ m/s^2$
t_s	$0.1\ s$
$a_{i,min}$	$-5\ m/s^2$
$a_{i,max}$	$3\ m/s^2$
ϕ_i	0.45
s_{fi}	$-3\ m$
τ_i^*	$1\ s$
T	$5\ s$
N	6

$$\varphi_i^2\left(\varphi_{i-1} - \tau_{i-1}k_{f,i-1}\right)^2 > 0 \tag{27-e}$$

The detailed proof is provided in Appendix 2.

Remark 4. Based on the analysis above, the l_∞-norm string stability is always satisfied by Proposition 5, and l_2-norm string stability is satisfied when the constraints are inactive. Although the l_2-norm string stability cannot always be satisfied, considering both types of string stability would still be beneficial. When a disturbance is small (and thus the constraints are not active), the system can ensure that the disturbance energy, as well as its peak magnitude, decrease through a vehicular string. When a disturbance is large and the constraints become active, the system ensures that the peak disturbance magnitude decreases through the string, which will help prevent vehicular over-reaction to the disturbance. The illustration of different regions is given in Fig. 4.

5. Simulation experiments and numerical analysis

To validate the proposed sequential distributed CACC model, we perform simulation experiments as field testing is beyond the scope of this paper. Three experiments are conducted to corroborate the mathematical proofs and evaluate the control performance. Specifically, the first experiment compares the performance of the proposed serial distributed MPC with the l_∞/l_2-norm string stability constraints against the control without the constraints, aiming to evaluate the impact of those l_∞-norm string stability constraints on the control performance. The second experiment evaluates the effectiveness of tuning the weight matrix to satisfy l_2-norm string stability when constraints are inactive. The third experiment evaluates the proposed controller with proper weighting matrix tuning under the condition that constraints are active. Parameter values are adopted from field-tested values in Ploeg et al. (2014) and Shladover et al. (2012) as presented in Table 1.

Experiment 1. Control Performance with and without $\boldsymbol{l}_\infty$-norm String Stability Constraints

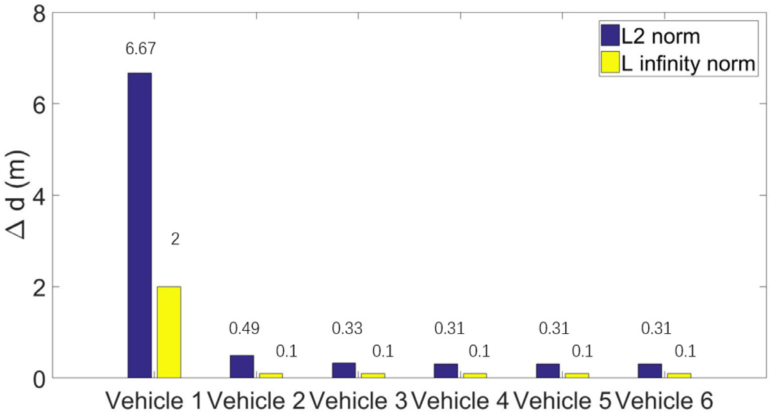
In this experiment, we compare the control performance with and without l_∞-norm string stability constraints. Firstly, we set the base control (without the l_∞-norm string stability constraints), and we set the weighting matrix $\breve{Q}_i = \begin{bmatrix} 1 & & \\ & 1 & \\ & & 1 \end{bmatrix}$ and $\breve{R}_i = 2$. Solving the DARE in Eq. (13), we obtain the terminal weight as:

$$\breve{P}_{i,d} = \begin{bmatrix} 17.07 & 8.71 & -6.38 \\ 8.71 & 27.27 & -10.56 \\ -6.38 & -10.56 & 7.64 \end{bmatrix}$$

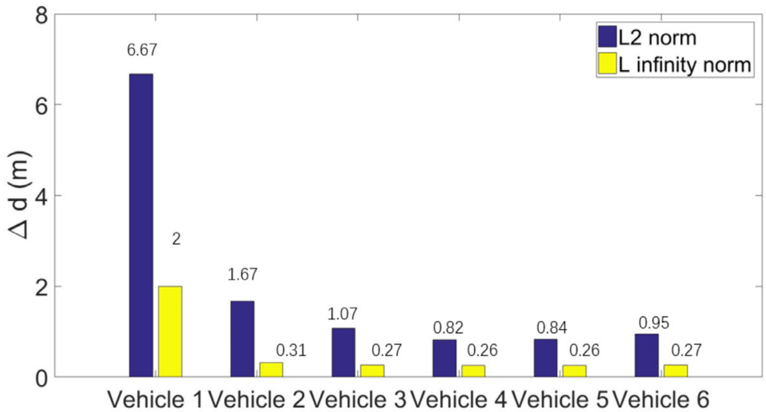
Then we set the initial condition for the deviation of spacing as $[2,\ 0.1,\ 0.1,\ 0.1,\ 0.1,\ 0.1]^T$, while the speed difference and acceleration are set as 0. As suggested by Dunbar and Caveney (2012), we choose the terminal constraint as: $\chi_{i,f} = [0,\ 0,\ 0]$, which satisfies the condition in Eq. (14), and then implement the proposed MPC according to Algorithm 1. The base case is set as a traditional MPC without the l_∞-norm constraints in Eqs. (14-c) and (14-d). The evolution of the deviation from equilibrium spacing, $\Delta d_i(t)$, and acceleration, $a_i(t)$, are shown in Fig. 5.

The differences in $\Delta d_i(t)$ and $a_i(t)$ between the control two algorithms, shown in Fig. 5, suggest that the string stability constraints are indeed active for the proposed control. Both algorithms can effectively regulate $\Delta d_i(t)$ and $a_i(t)$ to converge to zero, satisfying the asymptotical local stability requirement. However, the proposed control with the l_∞-norm constraints provides smoother acceleration and deceleration and regulates $\Delta d_i(t)$ better.

To gain further insight into the string-level property, we calculate the l_∞-norm and l_2-norm for each vehicle, as presented in Fig. 6. The result shows that with the l_∞-norm constraints, the vehicular string is stable (in l_∞-norm space; see Fig. 6(a)), and without them, instability is present (Fig. 6(b)). Furthermore, with the l_∞-norm constraints, the serial MPC provides lower l_2-norm of $\Delta d_i(t)$ through the string.



(a)



(b)

Fig. 5. l_∞ and l_2 Norm Comparison for Experiment 1: (a) With l_∞-norm Constraints (b) Without l_∞-norm Constraints.

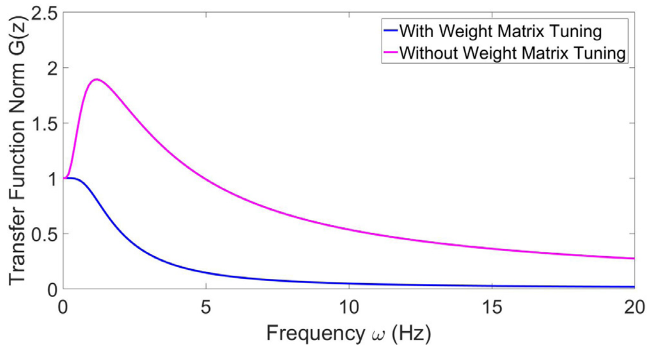


Fig. 6. Norm of Transfer Functions Over Frequency.

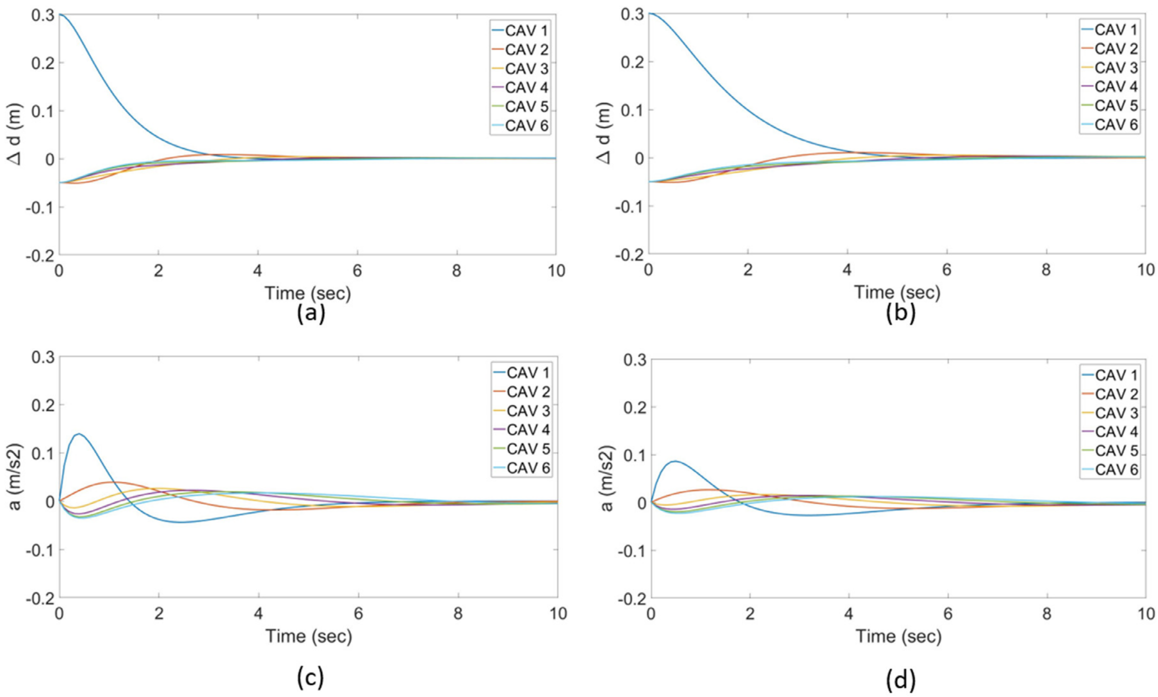


Fig. 7. $\Delta d_i(t)$ and $a_i(t)$ comparison for Experiment 2: (a)-(b) $\Delta d_i(t)$ under the Serial Distributed MPC with and without Weight Matrix Tuning; (c)-(d) $a_i(t)$ under the Serial Distributed CACC with and without Weight Matrix Tuning.

Experiment 2. **l_2**-norm String Stability with Weight Matrix Tuning over Constraint Inactive Set

In this experiment, we evaluate the effectiveness of tuning the weight matrix in achieving l_2-norm string stability. For the base case, without tuning, we set the feedback and feedforward gains at $\breve{k}_i = [0.7071,\ 1.1706, -0.7860]^T$ and $\breve{k}_{fi} = -2.4617$ by simply solving the CARE in Eq. (21). With these values, the sufficient condition for l_2-norm string stability in Eq. (24) is not satisfied. In contrast, for the case of tuning, we tune the weight matrix to $Q_i = \begin{bmatrix} 1 & & \\ & 0.5 & \\ & & 0.5 \end{bmatrix}$ and $R_i = 0.5$. This gives $k_i = [1.4142,\ 1.6100,\ -1.1730]^T$ and $k_{fi} = -0.1407$, which satisfy the sufficient condition for the l_2-norm string stability.

The l_2-norm string stability is examined via the transfer function norm, $|G_i(s)|$, as shown in Fig. 7. Note that if $\|G_i(s)\|_\infty \le 1$, it is string stable over a constraint inactive set according to Proposition 6. The result in Fig. 7 shows that l_2-norm string stability can be achieved with weight matrix tuning, but not in the base case without tuning: $\|G_i(s)\|_\infty > 1$ in some frequency range without tuning.

To provide a more detailed example, the initial deviation from equilibrium spacing is set at [0.2, −0.05, −0.05, −0.05, −0.05, −0.05] for 6 CAVs in a platoon, while the speed difference and acceleration are assumed to be zero. (By examining the Lagrangian multipliers, it can be verified that all constraints are inactive.) The evolutions of $\Delta d_i(t)$ and $a_i(t)$ for the two cases are presented in Fig. 8. It shows that the deviation from equilibrium spacing, $\Delta d_i(t)$, converges more quickly to zero

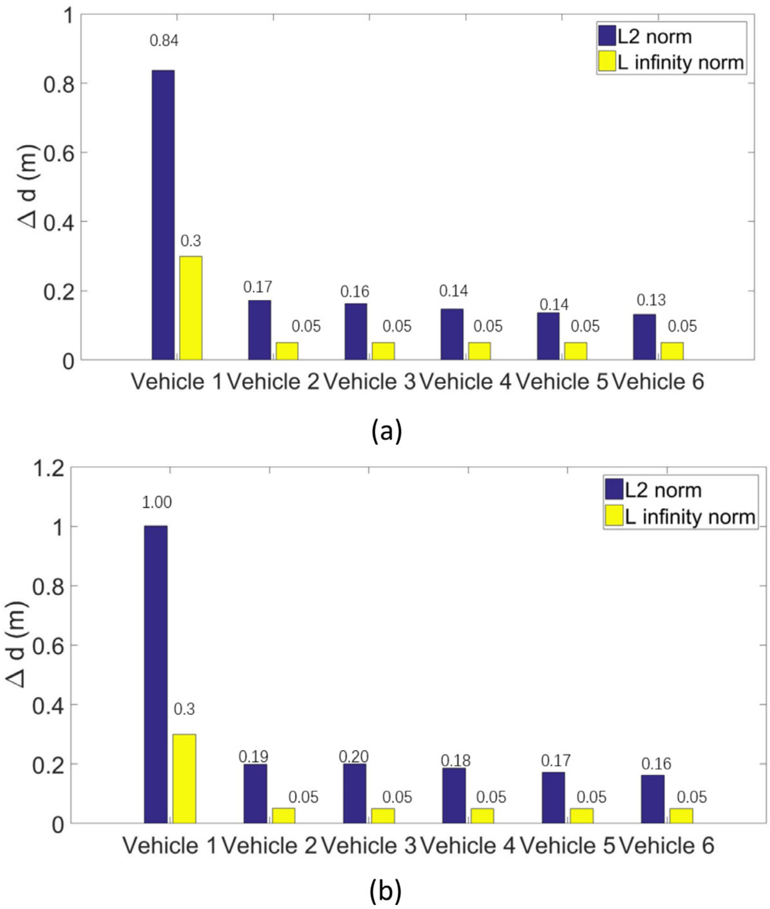


Fig. 8. l_∞ and l_2 Norms for Experiment 2 (a) with Weight Matrix Tuning (b) without Weight Matrix Tuning.

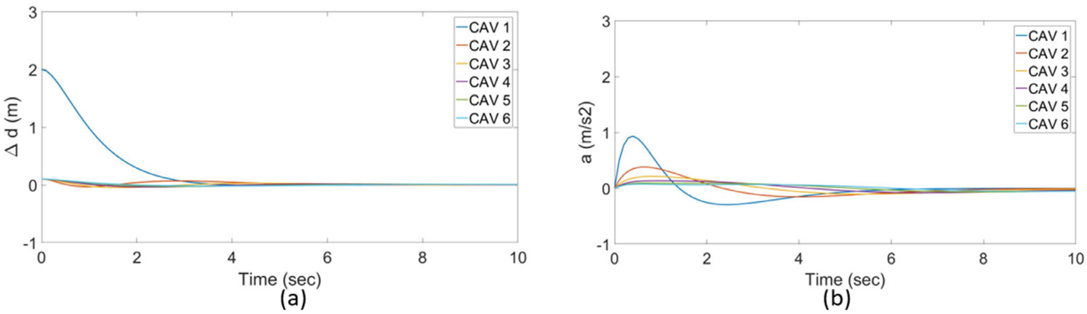


Fig. 9. $\Delta d_i(t)$ and $a_i(t)$ for Experiment 3 (a) $\Delta d_i(t)$ (b) $a_i(t)$.

with weight matrix tuning (Fig. 8(a) as compared to 8(b)), as well as the acceleration/deceleration (Fig. 8(c) as compared to 8(d)).

Fig. 9 presents the string-level performance with and without weight matrix tuning (Fig. 9(a) and (b), respectively). The result shows that the l_∞-norm string stability is satisfied for both cases as the l_∞-norm constraints are inactive. However, l_2-norm string stability is achieved only after weight matrix tuning.

Experiment 3. $\boldsymbol{l_2}$-norm String Stability with Weight Matrix Tuning over Constraint Active Set

In this experiment, we further examine the effectiveness of weight matrix tuning when the l_∞-norm constraints are active. The same initial condition as Experiment 1 is assumed. Fig. 10 presents the evolution of $\Delta d_i(t)$ and $a_i(t)$. It is clear that the proposed control with weight matrix tuning can achieve good performance even without theoretical guarantee l_∞-norm stability, and is more string stable than without tuning empirically.

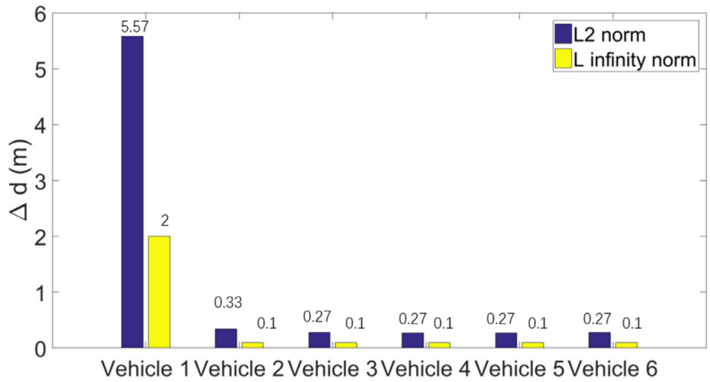


Fig. 10. l_∞ and l_2 Norm with Weight Matrix Tuning.

Fig. 11 shows the l_∞ norm and l_2 norm for each vehicle in a platoon. The result shows that even when the constraints are active, the l_2 norm surprisingly decreases with proper weight matrix tuning even without theoretical guarantee. Furthermore, compared to the result in Fig. 6(a), the l_2-norm values are smaller with tuning, underscoring the effectiveness of weighting matrix tuning in the serial MPC design.

6. Conclusions and future directions

This paper developed a serial distributed MPC approach for CACC with multi-criteria string stability. Two string stability criteria were considered within the proposed MPC: the l_∞-norm string stability criterion and l_2-norm string stability criterion. The l_∞-norm string stability was achieved by formulating constraints within the MPC based on the future states of the leading CAV, and the l_2-norm string stability criterion was achieved by proper weight matrix tuning over a (robust) positive invariant set. For rigor, this study provided mathematical proofs for asymptotical local stability and multi-criteria string stability.

To verify the proposed controller and theorems, simulation-based experiments were conducted. The experiments corroborated the correctness of the mathematical proofs for the proposed control. Furthermore, the experiments demonstrated the effectiveness of the proposed controller by comparing the performance of the serial distributed MPC with and without different string stability criteria. The results showed that the proposed controller can guarantee the multi-criteria string stability under different regions (specifically l_∞-norm string stable and l_2-norm string stable over robust positive invariant set) and provide better string stability performance than other controllers without explicit string stability constraints or weight matrix tuning.

Some future studies are desired based on the results of this study. For example, future studies should incorporate uncertainty in vehicle dynamics and potential communication or measurement delays for robust control (Zhou and Ahn, 2019). Furthermore, future development of controllers should address stabilization of a mixed vehicle platoon consisting of CAVs and human-driven vehicles. The model proposed in this paper and the insights from the mathematical proofs provide an important foundation for these future extensions.

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Supplementary materials

Supplementary material associated with this article can be found, in the online version, at doi:10.1016/j.trb.2019.07.001.

Appendix 1

To ensure that the homogenous CACC platoon is l_2-norm string stable, we apply the Cauchy inequality:

$$\|G_i(s)\|_\infty = sup\frac{\|a_{i-1}(s)\|_2}{\|a_i(s)\|_2} \le 1 \tag{A-1}$$

By applying the Laplacian transformation on Eq. (5), we obtain:

$$sX_i(s) = A_{ci}X_i(s) + Da_{i-1}(s) + Bk_{Fi}a_{i-1}(s) \tag{A-2}$$

where $A_{ci} = A + B \times k_i$.

Furthermore, we have the following relationship:

$$a_{i-1}(s) = CX_{i-1}(s) \tag{A-3}$$

Where $C = [0,\ 0,\ 1]$

Combining Eqs. (A-2) and (A-3), we obtain the relationship as below:

$$\frac{a_{i-1}(s)}{a_i(s)} = C(sI - A_{ci})^{-1}(D + Bk_{Fi}) = G_i(s) \tag{A-4}$$

$$G_i(s) = \frac{\left(k_{s,i} + k_{v,i}s + k_{f,i}s^2\right)}{\varphi_i s^3 - (k_{a,i} - 1)s^2 + \left(\tau_i^* k_{s,i} + k_{v,i}\right)s + k_{s,i}} \tag{A-5}$$

where $s = j\omega$.

We then calculate the norm of the transfer function:

$$|G_i(s)| = \sqrt{\frac{\left(-k_{f,i}^2\omega^2 + k_{s,i}\right)^2 + k_{v,i}^2\omega^2}{\left[-\omega^3\varphi_i + \left(\tau_i^* k_{s,i} + k_{v,i}\right)\omega\right]^2 + \left[(k_{a,i} - 1)\omega^2 + k_{s,i}\right]^2}} \tag{A-6}$$

By regulating that $\|G_i(s)\|_\infty < 1$, the following inequalities can be obtained.

$$\begin{aligned}&\varphi_i^2\omega^4 + \left[(k_{a,i} - 1)^2 - 2\varphi_i\left(\tau_i^* k_{s,i} + k_{v,i}\right) - k_{f,i}^2\right]\omega^2 + \\ &\left[2k_{s,i}k_{a,i} + \left(\tau_i^* k_{s,i} + k_{v,i}\right)^2 - k_{v,i}^2 + 2k_{s,i}k_{f,i}\right] > 0 \text{ for } \forall\omega > 0\end{aligned} \tag{A7}$$

The sufficient condition for the inequality in Eq. (A-7) is satisfied if all the polynomial coefficients are positive, which gives the sufficient condition for string stability as follows:

$$(k_{a,i} - 1)^2 - 2\varphi_i\left(\tau_i^* k_{s,i} + k_{v,i}\right) - k_{f,i}^2 > 0 \tag{A-8}$$

$$2k_{s,i}k_{a,i} + \left(\tau_i^* k_{s,i} + k_{v,i}\right)^2 - k_{v,i}^2 + 2k_{s,i}k_{f,i} > 0 \tag{A-9}$$

Appendix 2

For the heterogenous case, we have:

$$F_i(s) = \frac{1 - G_i(s) - s\tau_i^* G_i(s)}{1 - G_{i-1}(s) - s\tau_{i-1}^* G_{i-1}(s)} \times G_{i-1}(s) \tag{A2-1}$$

And this gives:

$$F_i(s) = \frac{\frac{(\varphi_i - \tau_i^* k_{f,i})s^3 + \left(1 - k_{a,i} - \tau_i^* k_{v,i} - k_{f,i}\right)s^2}{\varphi_i s^3 - (k_{a,i} - 1)s^2 + \left(\tau_i^* k_{d,i} + k_{v,i}\right)s + k_{d,i}} \times \left(k_{d,i-1} + k_{v,i-1}s + k_{f,i-1}s^2\right)}{(\varphi_{i-1} - \tau_{i-1}^* k_{f,i-1})s^3 + \left(1 - k_{a,i-1} - \tau_{i-1}^* k_{v,i-1} - k_{f,i-1}\right)s^2} \tag{A2-2}$$

To make $\|F_i(s)\|_\infty \le 1$, we end with the polynomial as below:

$$\begin{aligned}&\sum_{j=1}^{6} P_{f,j}\omega^{2j} > 0 \text{ for } \forall\omega > 0\\ &P_{f,5} = \varphi_i^2\left(\varphi_{i-1} - \tau_{i-1}k_{f,i-1}\right)^2\end{aligned} \tag{A2-3}$$

$$P_{f,4} = \varphi_i^2\left(k_{a,i-1} - 1 + k_{f,i-1} + \tau_{i-1}k_{v,i-1}\right)^2 + \left[(k_{a,i} - 1)^2 - 2\varphi_i\left(\tau_i k_{d,i} + k_{v,i}\right)\right] \times \left(\varphi_{i-1} - \tau_{i-1}k_{f,i-1}\right)^2 - k_{f,i-1}^2\left(\varphi_i - \tau_i k_{f,i}\right)^2 \tag{A2-4}$$

$$\begin{aligned}P_{f,3} = &\left[(k_{a,i} - 1)^2 - 2\varphi_i\left(\tau_i k_{d,i} + k_{v,i}\right)\right] \times \left(k_{a,i-1} - 1 + k_{f,i-1} + \tau_{i-1}k_{v,i-1}\right)^2 \\ &+ \left[\left(\tau_i k_{d,i} + k_{v,i}\right)^2 + 2k_{d,i}(k_{a,i} - 1)^2\right] \times \left(\varphi_{i-1} - \tau_{i-1}k_{f,i-1}\right)^2 - \left(\varphi_i - \tau_i k_{f,i}\right)^2 \\ &\left(k_{v,i-1}^2 - 2k_{f,i-1}k_{d,i-1}\right) - k_{f,i-1}^2\left(k_{a,i} - 1 + k_{f,i} + \tau_i k_{v,i}\right)^2\end{aligned} \tag{A2-5}$$

$$P_{f,2} = \left[\left(\tau_i k_{d,i} + k_{v,i}\right)^2 + 2k_{d,i}(k_{a,i} - 1)\right]\left(k_{a,i-1} - 1 + k_{f,i-1} + \tau_{i-1}k_{v,i-1}\right)^2$$
$$+k_{d,i}^2\left(\varphi_{i-1} - \tau_{i-1}k_{f,i-1}\right)^2 - k_{d,i-1}^2\left(\varphi_i - \tau_i k_{f,i}\right)^2 - \left(k_{a,i} - 1 + k_{f,i} + \tau_i k_{v,i}\right)^2\left(k_{v,i-1}^2 - 2k_{f,i-1}k_{d,i-1}\right) \quad \text{(A2-6)}$$

$$P_{f,1} = k_{d,i}^2\left(k_{a,i-1} - 1 + k_{f,i-1} + \tau_{i-1}k_{v,i-1}\right)^2 - k_{d,i-1}^2\left(k_{a,i} - 1 + k_{f,i} + \tau_i k_{v,i}\right)^2 \quad \text{(A2-7)}$$

Therefore, the sufficient condition for the heterogeneous vehicular platoon is $P_{f,j} > 0$ for $\forall j = [1, 2, \ldots, 6]$.

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