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General Behavior and Motion Model for Automated Lane Change*

Hossein Tehrani, Quoc Huy Do, Masumi Egawa, Kenji Muto, Keisuke Yoneda, Seiichi Mita

Abstract— **Lane change maneuver is a cause for many severe highway accidents and automatic lane change has great potentials to reduce the impact of human error and number of accidents. Previous researches mostly tried to find an optimal trajectory and ignore the behavior model. Presented methods can be applied for simple lane change scenario and generally fail for complicated cases or in the presence of time/distance constraints. Through analysis and inspiring of human driver lane change data, we propose a multi segments lane change model to mimic the human driver for challenging scenarios. We also propose a method to convert behavior/motion selection to a time-based pattern recognition problem. We developed a simulation platform in PreScan and evaluated proposed automatic lane change method for challenging scenarios.**

I. Introduction

Statistical data of highway traffic accidents shows that the human error is a major reason for about 90% of accidents [1]. Lane change maneuver is a cause for many severe highway accidents due to wrong estimation of surrounding environment (distances, velocity or behavior of surrounding vehicles) or wrong maneuver (longitudinal and lateral motion) of human driver. Currently, ADAS (Advanced Driving Assistance Systems) or automated driving proved their great potentials to reduce the impact of human error and number of accidents through providing warning, semi or fully automated solutions.

In this paper, we address highway automated lane change problem. The lane change maneuver and behavior has been a difficult problem during the last two decades. Despite many researches and progress, we have only simple warning level applications on the market that use RADAR sensors to warn the driver during lane change. We still do not have verified semi-automated or fully automated lane change applications in the market. Previous researches in fully automated driving systems designed for lane change or overtake maneuvers, can be divided into rule-based [2], or utility-based [3], approaches. The lane changing trajectory is generated according to the vehicle states, surrounding vehicles and road information, and then the control laws are designed to use onboard sensors to track this generated trajectory. Hatipoglu et. al. [4] reported an automated lane changing controller with a two-layer hierarchical architecture. In his approach, the open loop lane change problem has been converted into an equivalent virtual reference trajectory tracking problem. CMU's Boss [5] used a model-predictive trajectory generator, the same proposed by Howard, Green, Kelly, and Ferguson [6], to produce dynamically feasible actions between the initial and the desired vehicle states by numerical linearization and inversion of the forward vehicle dynamic model. Optimal trajectory planning using non-linear cost function proposed by Stanford University [7]. The Stanford university method has been widely used by many researchers due to its effectiveness and capabilities to handle dynamic situations. The more advanced approaches include probabilistic methods to handle uncertainties in the environment [8]. And the system in study [9] uses fuzzy controllers that mimic human behavior and reactions during overtaking maneuvers. Most recently, Du et. al., [10] formulated the lane change decision making problem as an optimization problem and the vehicle dynamics and safety constraints are transformed into a mixed logical dynamical system. Ziegler and Stiller [11] proposed an effective trajectory generation for Bertha automated vehicle. They suggest a local, continuous method that is derived from a variational formulation and static and dynamic obstacle constraints are incorporated in the form of polygons. Most of discussed researches are based on the kinematic functions and try to find an optimal trajectory for lane-changing. They ignore the behavior model of lane change for complicated scenarios or in the presence of time/distance constraint for merging or exit of the highway. Later, we will show that the lane change is a complicated maneuver and it is highly depended on the behavior and dynamic of surrounding vehicles. Although, there are many researches to predict the behavior/trajectory of the surrounding vehicles [12,13] but there are few researches about how to select our own behavior in complicated scenario.

To exactly understand the human lane change model, we have conducted lane change experiments at Japan highway. The results of the human lane change motion analysis already published by the authors [14]. Through analysis of human lane change data including neighbor vehicles and motion/behavior data, we realized that the human model is not a single stage. In this paper, we propose a multi segments behavior and motion model to mimic the human driver lane change operation. At "behavior segment", the host vehicle tries to adjust the longitudinal distance, position or relative velocity to make or find suitable free space in front and the destination lane. At "motion segment", the host vehicle starts lateral motion and enters into the destination lane. We also present a method to convert the driving scene to a time based pattern and behavior selection can be considered as a multi-class classification problem. The proposed method is able to solve and generate solution for complicated lane change scenario. In section II, we present a lane change model that is inspired from human driver data in section III, IV, we discuss in detail about the behavior/motion generation and selection model, in section V we present the simulation platform and results, finally we conclude the research and discuss about future plan.

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H. Tehrani, M. Egawa & K. Muto are with Corporate R&D Div.3, DENSO CORPORATION (e-mail: {hossein_tehrani, masumi_egawa, kenji_Muto}@denso.co.jp).

Q. Huy Do, K. Yoneda & S. Mita are with Research Center for Smart Vehicles of Toyota Technological Institute (e-mail: {huydq,yoneda,smita}@toyota-ti.ac.jp)

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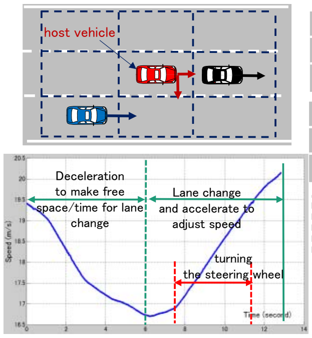


Figure 1. human lane change data.

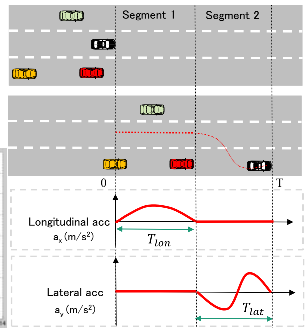


Figure 2. two segments model

II. Lane Change Model

A. What We Learned from Human Driver

We conducted lane change experiments and recorded sensing and motion data of human driver to understand the human motion/behavior model. Through analyzing the experiments data, we realized that the behavior/motion model of human driver is a complicated multi stages or segments model. Fig.1 depicts one example of the recorded lane change data for human driver. In Fig.1, the surrounding environment is shown by a grid model and the host vehicle (red vehicle) is going to do the right lane change (Japan is left hand side driving). We learned that the human driver model is not just the turning of steering wheel and it starts seconds before the steering. As shown in Fig. 1, turning the steering wheel starts at time 7.6 thought the human driver has already started to decelerate and reduce the vehicle speed from 19.5 m/s to 16.7 m/s before turning to the right lane. We see the driver has to decelerate to keep safe space from the front vehicle (black vehicle) and wait to let the blue vehicle to pass it.

By analyzing different scenarios from human driver data, we propose a two segments behavior/motion lane change model as shown in Fig. 2. In segment 1, the driver adjusts the longitudinal speed and makes safe space based on the positions and relative velocities of surrounding vehicles. The driver behavior in segment 1 is highly dependent on the number of surrounding vehicles, distances and their relative velocities. There are other important parameters such as road curvature, visibility condition or behavior of the surrounding vehicle. Through the experiments, we found that time/distance constraint is also a critical factor to select the behavior at segment 1. It occurs when we try to merge into highway traffic or exit from the highway. By analyzing different cases of human lane change data, we extracted the following behaviors for the segment 1.

$$\mathrm{B} = \{do\ lane\ change, wait, accelerate, decelerate\} \quad (1)$$

In this paper, we do not discuss about the emergency cases that may happen in the case of sudden change in the behavior of surrounding vehicles.

We did experiments by drivers with different levels of driving skills. It was interesting that even for similar driving scene the behavior of the drivers were different based on their skills and driving manner. The behavior of the human driver in lane change is dependent on the driving manner and skills. Although, it is difficult to develop a general behavior model but we can propose a standard model that guarantee the safety and smoothness of the lane change operation. For example, if there is enough space at the destination lane, we can turn the steering wheel and change the driving lane. If the relative speed of the approaching vehicles in the destination lane is relatively high, we may prefer to wait until find enough free space. In this case, we may even do deceleration to reduce the time or traveled distance. The deceleration behavior may be useful in the case of time/distance constraint when we have to change the lane to exit from the highway. In the other case, when the relative speed of the neighbor vehicles in the destination lane is relatively low, we may accelerate to pass the neighbor vehicles for doing the lane change. Later, we will discuss about these scenarios in the simulation section. In the other hand, the road curvature, speed limit and kinematic constraints of the vehicles such as maximum speed or acceleration have effect on the driver behavior.

In the literature, the state machine model [15] or HMM [12] have widely applied to generate the suitable behavior for automated driving. Through discussion with test drivers, we realized that the behavior selection is a discrete decision making problem and drivers use some predefined patterns to select suitable behavior. The human driver converts the surrounding driving environment to a time based pattern and makes decision based on the predefined patterns of driving scene. In driving scene pattern, the historical information of surrounding vehicle trajectories and the future estimated trajectories (behavior and motion) are included. In this paper, we propose a method to extract the critical features from the driving scene and make a time based pattern from the surrounding environment. To define the driving scene patterns, we analysis all lane change scenarios using a grid state model. We categorize the lane change states and define alternative behaviors for each category. Later, we will discuss how to select the suitable behavior based on the driving scene pattern.

B. Lane Change Flowchart

Automatic lane change is integration of sensing/perception, planning (behavior & motion) and control. The behavior/motion flowchart for doing the lane change is shown in Fig. 3. It starts with estimation of motion parameters/ position of neighbor vehicles to estimate their trajectory. Based on the driving lane information and the behavior/motion parameters of the neighbor vehicles, we are able to estimate a trajectory for a certain period [0~T]. By using the trajectory data points $(t, x(t), y(t))$, we estimate the grid map of current and future state of the surrounding environment. In the following, we briefly explain the main components of the lane change flowchart.

C. Adaptive Grid Map

We use a grid, which is fixed to a vehicle as shown in Fig. 4. The grid cell size is dependent on the relative speed of host vehicle and corresponding neighbor vehicle as the following;

$$\mathrm{G} = d_{min} + \alpha.\Delta v \qquad (2)$$

d_{min} is the minimum safe distance between two vehicles and α is a time dependent constant to adjust the size of the grid. If the relative speed is high, more space will be required to perform the lane change task and more time for other neighbor to react. In the literature, the grid cells are generally considered to be same size [13]. Empirically through simulations, we realized that the center cells in the grid is just depends on the

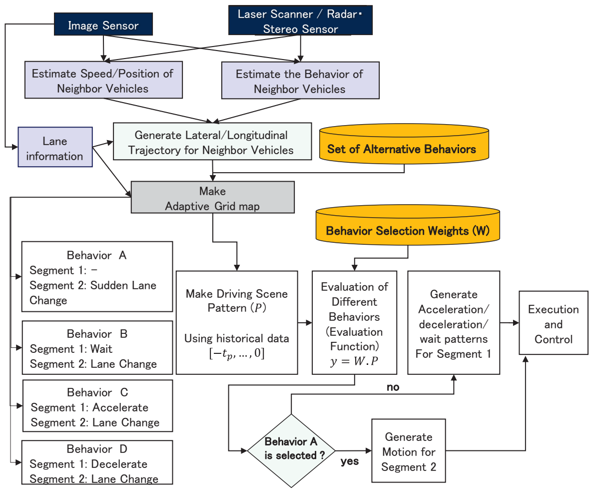


Figure 3. Automatic lane change flowchart

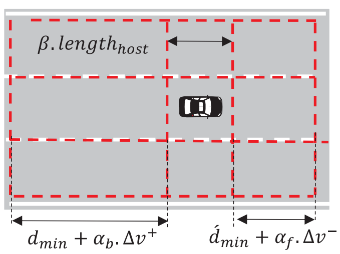


Figure 4. Discretization of the enviroment to a 9 cells grid

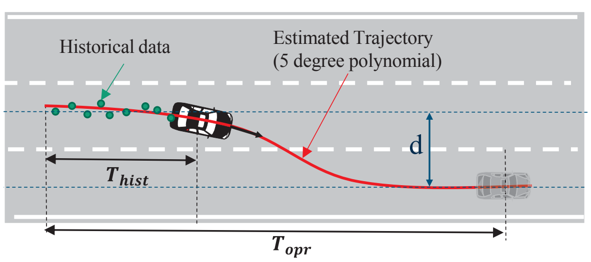


Figure 5. Neighbor vehicle trajectory estimation

vehicle size as $\beta.length_{host}$, $\beta > 1$ and it is generally smaller than the front and back cells in the grid. Consider large size for center cell causes the host vehicle does not do the lane change due to occupation of the center cell in the left or right side. We also realized that the back cells have larger size compare to front cells to mimic the human behavior.

D. Estimate Behavior of Neighbor Vehicles

It is out of scope of our research and there are many published researches to estimate the behavior of the neighbor vehicles [13]. In simulation software, we have this information by V2V communication module.

E. Estimate Trajectory of Neighbor Vehicles

We use the historical motion data, road information and behavior (lane change/lane keeping) to estimate the trajectory of the neighbor vehicles. We use polynomial function to estimate the trajectory of the neighbor vehicles [16]. As shown in Fig. 5, the trajectory of a neighbor vehicles is estimated by recorded position data and lane center point (we assume that this vehicle will follow the center of the lane) and we fit the

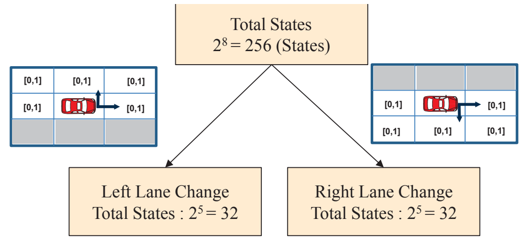


Figure 6. Total states simplification for left and right lane change

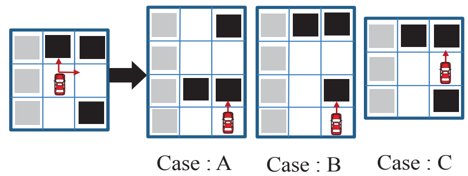


Figure 7. Different behavior and motion for doing the lane change

polynomial curve to these data. Here, we apply "Frenet Frame" method in order to be able to combine different lateral and longitudinal motions in one equation [16]. In this case, lateral and longitudinal motion of each vehicle can be presented by an equation based on the distance traveled along the center line.

F. Collision Checking

There are effective collisions checking methods in the literature to check the collision between two trajectories [17]. To check the collision in the simulation platform, we sample from the trajectory of neighbor vehicle for a certain period[0~T]. We use Inevitable Collision States (ICS) method [18] to check the collision possibility between two trajectories.

III. Behavior and Motion Model

We already proposed a two segments model for doing lane change. At segment one, "behavior segment", the host vehicle tries to adjust the longitudinal relative distance, velocity to make or find suitable free space in front or the destination lane. In this segment, the host vehicle may accelerate, decelerate or just wait to make free space/time interval at the destination lane. At segment 2, "motion segment", the host vehicle starts lateral/longitudinal motion and enter into the destination lane. Authors already presented lateral/longitudinal motion generation method for "motion segment" in [14]. In this paper, we discuss in detail about "behavior segment" and briefly review the "motion segment".

A. Behavior Segment

By discretization the surrounding environment to a nine cells grid, we are able to present different states to do the lane change. We have eight surrounding cells in the occupancy grid that can be free (cell value = 0) or occupied (cell value = 1). Then we have totally 256 (2^8) states for surrounding environment that is computational expensive for the real time application. We divide the state vector based on the left or right lane change to reduce the state vector size from 8 to 5 cells as shown in Fig. 6. In this case, the total state, is reduced from 256 to 32 based on the left or right lane change. (though in severe cases, we should consider the state of all cells). For every state, we consider different alternative behaviors as shown in Fig. 7. For the occupancy grid state in Fig. 7, different following behaviors are available to do the lane change;

- Case A: The host vehicle just waits until the right lane approaching vehicles passes it and the right lane becomes free to do right lane change.
- Case B: In this case, the host vehicle decelerates and enters to the right lane. It is preferable behavior when we have to do the lane change at limited time/distance (for example exit point in the highway).

To have exact understanding of different behaviors, we categorized 32 (2^5) occupancy gird states (left or right lane change) to the following main four categories. We limit the behavior alternatives based on the categories to reduce the calculation time. Different categories for occupancy grid states are shown in Fig. 8, and we explain the characteristics of each categories and available alternative behaviors for each state. The categories and alternative behaviors are defined based on our experiments from analyzing the human driver data for different lane change scenarios.

- Category A: We have two alternative behaviors for occupancy grid states. We either wait or do the lane change based on the relative speed and distance to the neighbor vehicles.
- Category B: We have more alternative behaviors for states in this category. We may do the lane change but sometimes acceleration/deceleration or wait is preferable to do safer/smoother lane change. In the case of time/distance constraint, acceleration/deceleration may necessary to satisfy the lane change limitations.
- Category C: It is related to complicated state during lane change. In this category, the right cell of the host vehicle is occupied and suitable behavior should be selected to provide free space at the destination lane. We may accelerate, decelerate or wait for states in this category.
- Category D: We have to wait for doing the lane change and any other behavior may cause danger.

The behavior for lane change is a complicated model and it is depend on many parameters. In most cases, simple behavior model fails and it is compatible with our observations from lane change experiments at the highway.

B. Motion Generation

In segment 1, we try to make safe space/time interval by either acceleration/deceleration or waiting. The acceleration/deceleration patterns can be formulated as;

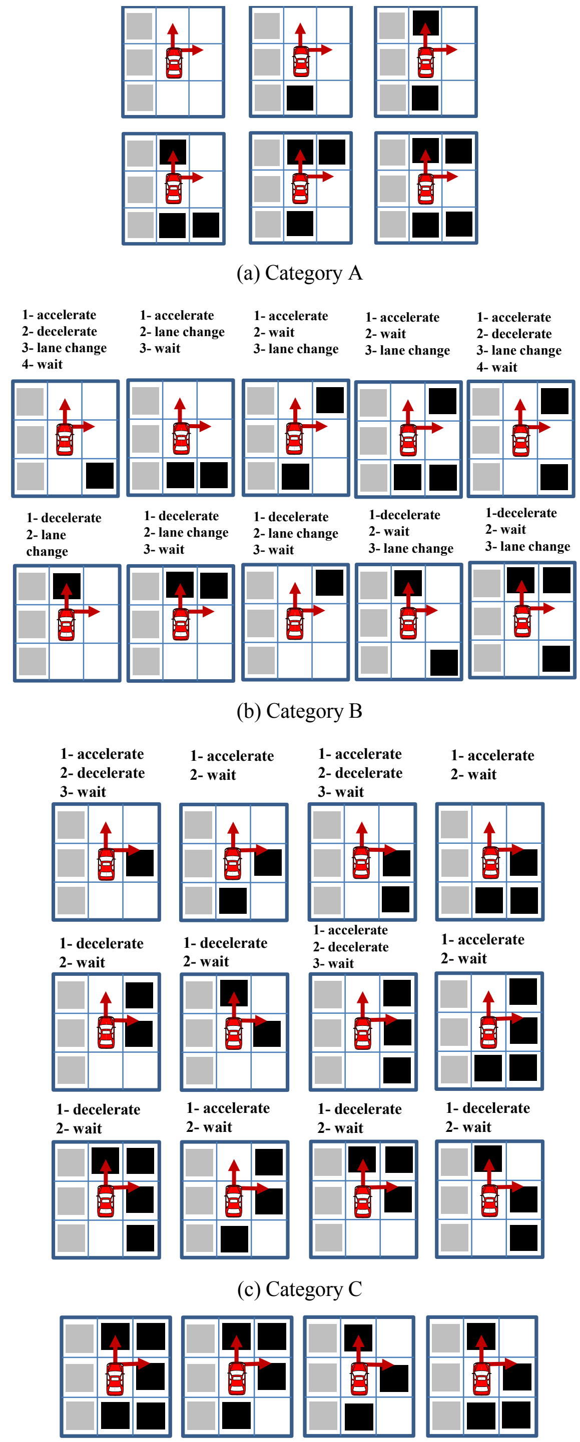
$$\ddot{x} = f(\dot{x}(t_0), x_{lead}(t_0), \dot{x}_{lead}(t_0), T, r) \quad (3)$$

Where, r is a safety reserve distance and calculated through

$$r = d_{min} + \alpha . \dot{x}_{lead}(T) \quad (4)$$

$x_{lead}(t_0)$ and $\dot{x}_{lead}(t_0)$ are the position and velocity of the leading vehicle at time t_0 respectively as shown in Fig. 9. To generate smooth and comfort acceleration/deceleration motion, we minimize the following cost function;

$$J = \int_0^T (\omega_{dist}\,[\Delta d(t)]^2 + \omega_{acc}[\ddot{x}(t)]^2)dt \quad (5)$$



(a) Category A

(b) Category B

(c) Category C

(d) Category D

Figure. 8 All 32 States Models for Right Lane Change and Available Alternative Behaviors for each of them

This cost function is similar to the method presented in [19] though, the operation time T is fixed. In our approach, the operation time T is not fixed to have more degree of freedom for behavior in segment 1. The error in the safety distance ($\Delta d(t)$) is calculated by the following;

$$\Delta d(t) = x_{lead}(t) - r + t * \dot{x}_{lead}(t) - x(t) \quad (6)$$

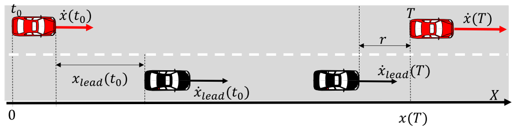


Figure 9. Acceleration pattern for segment 1.

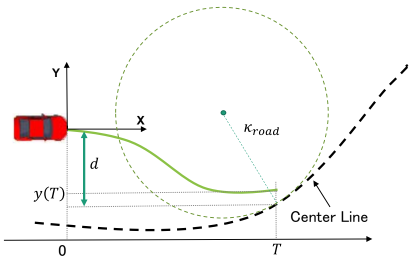


Figure 10. Steering pattern for segment 2

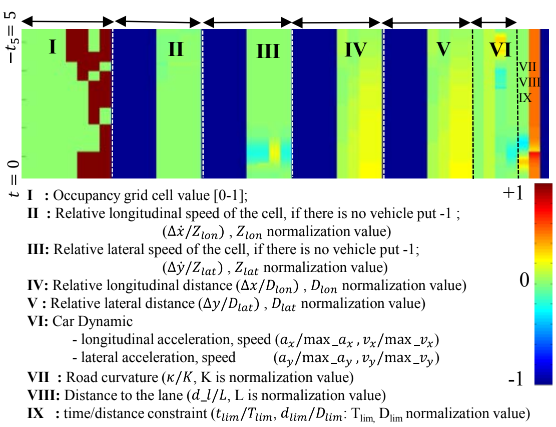


Figure 11. Pattern generation for driving scene

We use quartic polynomial to generate acceleration/deceleration motion [16]. We estimate the coefficients by considering vehicle constraints (maximum acceleration/deceleration and speed), desired velocity and operation time. We sample from valid range of operation time T and final velocity $\dot{x}(T)$ to generate alternative longitudinal trajectories while considering the boundary conditions including $\ddot{x}_{max}, \ddot{x}_{min}$.

In segment 2, we generate the lateral trajectory for steering by minimizing the following cost function;

$$J = w_{jerk}\int_0^T \dddot{y}^2(t).dt + w_{time}T + w_{error}[y(T)-d]^2 + w_{heading}[\kappa(T)-\kappa_{road}]^2 + w_{smoothness}\int_0^T \frac{\acute{\kappa}(t)^2}{\sqrt{\dot{x}(t)^2+\dot{y}(t)^2}}dt \quad (7)$$

Quintic polynomial [14] is used to generate alternative lateral trajectories for lane change as shown in Fig. 10. We sample from valid range of operation time T to generate alternative lateral trajectories considering the boundary conditions including $\ddot{y}_{max}, \ddot{y}_{min}$ and the following equation to avoid slip.

$$\kappa(t) \le \frac{\ddot{y}_{max}}{x(\dot{t})^2} \quad (8)$$

IV. Behavior Selection

To select the suitable behavior, there are many researches in the literature that are mainly based on HMM [12]. In this paper, we propose a new method and convert the behavior selection to a pattern recognition problem. For this purpose, we use information of neighbor vehicles, road, vehicle status to make a pattern by defining some features and normalizing them. It is a time based pattern that includes previous and current estimated data of surrounding. Then, the behavior selection can be converted to a multi-class classification problem and we can use machine learning techniques to train suitable behavior for each pattern. The classification function is defined as the following;

$$y \approx f\left(\left[NV,\ R, CD, CN\right]_{[-t_p,+t_f]}\right) \quad (9)$$

$$y \in \mathrm{B}\{0(LC), 1(wait), 2(acc \rightarrow LC), 3(dec \rightarrow LC)\}$$

The neighbor vehicle features (NV) come from the occupancy gird state, relative velocities and distances (lateral and longitudinal) for a period $[-t_p, +t_f]$. $-t_p$ refers to historical data and $+t_f$ is related to future estimation. We consider road curvature (R) as a feature and car dynamics (CD) features include lateral /longitudinal speed and acceleration of the host vehicle. Time/distance constraint (CN) is an important feature to show the limit of lane change operation. We normalize these data to range [-1,1] using maximum value for each of these features. Fig. 11 shows the features and a sample for feature extraction and pattern generation. $\mathrm{t} = 0$, shows the current time, $-t_p = 5\,s$ shows the historical data (5 seconds) of features.

B. Training the Classification Function

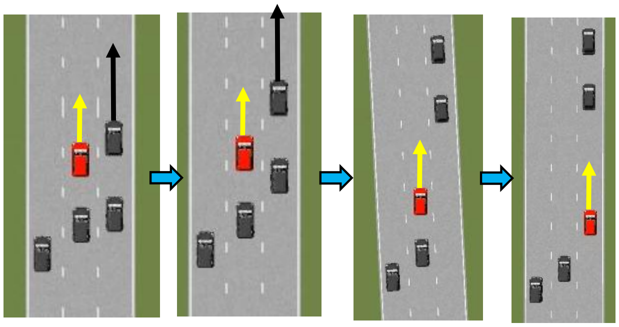
To train the classification function in (9), we have to generate many positive and negative lane change samples. We need a reference such as a human driver to evaluate the lane change motion/behavior and define the positive (safe and comfortable) and negative (unsafe or uncomfortable) samples. The evaluation is a tricky decision and there are few researches in the literature. We believe the evaluation index would consist of three main parts including safety, smoothness (comfortable) and the operation time (traveled distance). If there is time or distance constraint, evaluation can be defined as how much we can meet the limitations. In current stage of our research, we are doing the lane change experiments by human driver to define the evaluation parameters and generate positive reference samples for training the function f. To generate negative samples, we will use simulator to generate unsafe or uncomfortable samples. In this paper, we use a simple linear weighted method for classification function in (9) as following;

$$y \approx W.P; \quad P \sim \left[NV,\ R, CD, CN\right]_{[-t_p,0]} \quad (10)$$

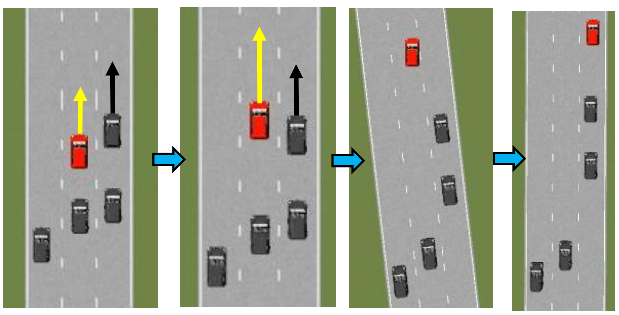
P is driving scene pattern such as shown in Fig. 11. The weights W are adjusted empirically by observing the human behavior/motion and simulation.

V. Simulation Platform.

To develop and test the proposed lane change model, we developed a simulation platform on the PreScan. It has different modules for behavior/motion planning, trajectory estimation of neighbor vehicles and control. The behavior/motion generation module is developed under C++ to increase the efficiency of the simulation platform. Fig. 12 shows simulation results for the same lane change scenario



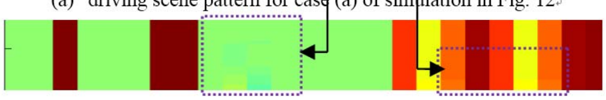
(a) wait behavior for "behavior segment



(b) acceleration behavior for "behavior segment"

Figure 12. Lane change simulation under different condition

(a) driving scene pattern for case (a) of simulation in Fig. 12



(b) driving scene pattern for case (b) of simulation in Fig. 12

Figure 13. Different driving scene pattern for case (a) & (b)

with different relative speed for neighbor vehicles (host vehicle is red one and it is going to do right lane change). Fig 12(a) shows the case when the relative speed of the two right side vehicles are relatively high and generated driving scene pattern is shown is Fig. 13 (a) (the features related to empty cells are removed for easier visualization). Fig 12 (b) shows the case when the relative speed of the two right side vehicles are relatively low and generated driving scene pattern is shown is Fig. 13 (b). The suitable behavior for each case is selected based on classification function in (10) and the driving scene patterns. Currently, we use simple linear weighted model $W.P$, and W adjusted empirically by observing the human behavior/motion and simulation. Based on driving pattern P in Fig. 13 (a), (b), the waiting $(y = 1)$ and acceleration $(y = 2)$ behavior is selected for segment 1 of simulation case (a) and (b) respectively. Through simulation, we found the proposed model is able to handle complicated lane change scenario.

VI. Conclusion

We developed a behavior/motion model for automatic lane change at highway. The proposed model is mainly inspired by human driver lane change and behavior data. We categorize the occupancy grid states and define alternative behaviors for each category. To select the suitable behavior, we generate the driving scene patterns and select the behavior based on a multi-class classification function. In current stage of research, the reference samples to train classification function is not available. We are doing lane change experiments by human drivers to extract the reference samples and train the behavior selection function. In future research, there is great potentials for deep neural network to extract the driving scene features and train the safe and smooth behavior/motion function.

References

[1] "Volvo Trucks, European Accident Research and Safety Report", http://www.volvotrucks.com/SiteCollectionDocuments/VTC/Corporate/Values/ART%20Report%202013_150dpi.pdf," 2013.

[2] L. Fletcher, et al., *The DARPA Urban Challenge*, volume 56 of Springer Tracts in Advanced Robotics, pages 509–548. Springer Berlin Heidelberg, 2009.

[3] J. Wei, J.M. Dolan, and B. Litkouhi. "A prediction- and cost function-based algorithm for robust autonomous freeway driving". *In Intelligent Vehicles Symposium (IV), 2010 IEEE,* pp. 512–517, June 2010.

[4] C. Hatipoglu, U. Ozguner, and K. A. Redmill, "Automated lane change controller design," *IEEE Transactions on Intelligent Transportation Systems*, vol. 4, no. 1, pp. 13–22, 2003.

[5] C. Urmson, C. Baker, J. Dolan, P. Rybski, B. Salesky, W. R. Whittaker, et al. "Autonomous driving in traffic: Boss and the urban challenge". *AI Magazine*, vol 30, pp. 17–29, 2009.

[6] T. M. Howard, C. J. Green, A. Kelly, & D. Ferguson, "State space sampling of feasible motions for high-performance mobile robot navigation in complex environments", *Journal of Field Robotics*, vol 25, pp. 325–345, 2008.

[7] J. Levinson, J. Askeland, J. Becher, J. Dolson, et al., "Toward fully autonomous driving: Systems and algorithms", *2011 IEEE Intelligent Vehicles Symposium*, pp. 163-168, 2011.

[8] R. Schubert, "Evaluating the utility of driving: Toward automated decision making under uncertainty". *Intelligent Transportation Systems, IEEE Transactions on*, vol 13, no. 1, pp. 354–364, March 2012.

[9] J.E. Naranjo, C. Gonzalez, R. Garcia, and T. de Pedro, "Lane-change fuzzy control in autonomous vehicles for the overtaking maneuver", *Intelligent Transportation Systems, IEEE Transactions on*, vol 9, no. 3, pp.438– 450, September 2008.

[10] Y. Du , Y. Wang , C. Chan, "Autonomous lane-change controller via mixed logical dynamical", *Intelligent Transportation Systems (ITSC), 2014 IEEE 17th International Conference* , pp. 1154 – 1159, 2014.

[11] Julius Ziegler, Philipp Bender, Thao Dang, Christoph Stiller. "Motion planning for Bertha - a local, continuous method". In *Intelligent Vehicles Symposium*, Dearborn, Michigan, USA, 2014. IEEE

[12] L.Peng, A. Kurt, U. Özgüner , "Trajectory prediction of lane changing vehicle based on driver behavior estimation and classification" *Intelligent Transportation Systems (ITSC), 2014* , pp. 942 – 947, 2014.

[13] D. Kasper, G. Weidl, T. Dang, et al., "Object-Oriented Bayesian Networks for Detection of Lane Change Maneuvers", *Intelligent Transportation Systems Magazine*, Vol. 4, No. 1, Jan 2014.

[14] H. Tehrani, K. Muto, K. Yoneda, S. Mita, "Evaluating human & computer for expressway lane changing", *Intelligent Vehicles Symposium Proceedings, 2014 IEEE* , pp. 382 – 387, 2014.

[15] J. Ziegler and C. Stiller, "Spatiotemporal state lattices for fast trajectory planning in dynamic on-road driving scenarios", in *IEEE/RSJ International Conference on Intelligent Robots and Systems*, pp. 1879-1884, 2009.

[16] M. Werling, J. Zeigler, S. Kammel, S. Thrun, "Optimal Trajectory Generation for Dynamic Street Scenarios in a Frenet Frame" . *2010 IEEE International Conference on Robotic and Automation*, p. 987-993.

[17] D. Althoff, M. Buss, A. Lawitzky, M. Werling, D. Wollherr "On-line Trajectory Generation for Safe and Optimal Vehicle Motion Planning", *AMS 2012*, pp. 99-107, 2012

[18] T. Fraichard and H. Asama "Inevitable Collision States. A step towards safer robots?", *Advanced Robotics*, vol. 18, pp. 1001–1024, 2004.

[19] A. Geiger, M. Lauer, F.; Moosmann, B. Ranf, et al. "Team AnnieWAY's Entry to the 2011 Grand Cooperative Driving Challenge" *Intelligent Transportation Systems, IEEE Transactions on,* vol 13, no 3, pp. 1008 – 1017, 2012.