MCC518 Computational Fluid Dynamics Practical

Lab Manual

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Primarily

This lab manual gives programming problems in C or C++ and provides instructions to implement them. The problems in this manual are carefully selected from the accompanying theory course. The lecture plan/syllabus of the theory and this lab should be available from respective instructors. Basic knowledge of continuum mechanics is desirable. Students must also know about the simple partial differential equations (PDEs), classification of PDEs of second order into elliptic, parabolic and hyperbolic types, and PDEs governing flow and heat transfer problems.

In order to prepare for conducting lab experiments, please learn C or C++ and git by completing the corresponding (free) course from https://www.codecademy.com/

You are highly encouraged to take a look at this web-site! Our labs are equipped with Linux machines. However, you are encouraged and welcome to bring your laptops to the lab. Please install a CPP Integrated Development Environments (IDE) and compiler on your laptops before coming to the first lab.

Electronic Lab Report

The lab report in electronic format should be maintained and is due at the end of the semester. Please create a git directory and create a readme.txt file inside it with following entries:

1. Name and Admission number of the student:
2. A table of experiments and the corresponding cpp filenames, e.g.,

Date	Experiment No.	Main filename	Commit id
11.08.2020	1HE (a)	heatequn.cpp	wxyz3275f

3. You should regularly commit your assignment to the git repo.
4. Place all program files inside the same directory.

Grade

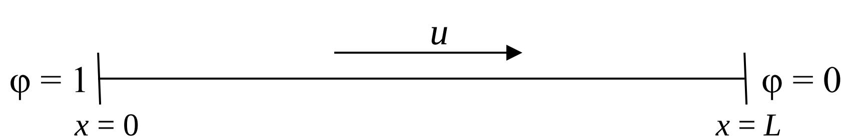
L -T - P: 0-0-3

LIST OF EXPERIMENTS

Sl. No.	Name of Experiments
Experiment 1 (Solution of Heat Equation by Explicit FDM)	
Aim	To get acquainted with explicit scheme of finite difference method for solution of parabolic partial differential equation with special reference to one dimensional heat conduction equation.
Q1	Write the explicit form of finite difference equation corresponding to the partial differential equation (PDE) $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$ or $u_t = u_{xx}$ and write a C or C++ program to solve the above PDE with prescribed initial condition and zero boundary values by explicit method.
Q2	Write a C or C++ program to solve the partial differential equation $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$ with boundary conditions $u(x,0) = \frac{1}{2}x(8-x)$ and $u(0,t) = 0, u(8,t) = 0$ by explicit method.
Experiment 2 (Solution of Heat Equation by Implicit FDM)	
Aim	To be familiar with implicit scheme of finite difference method for solution of parabolic partial differential equation with special reference to one dimensional heat conduction equation.
Q1	Write the implicit form of the difference equation corresponding to the partial differential equation (PDE) $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$ or $u_t = u_{xx}$ and write a C or C++ program to solve the above PDE with prescribed initial condition and zero boundary values by implicit method. Discuss convergence of the implicit form.
Q2	Write a C or C++ program to solve the equation $\frac{\partial u}{\partial t} = 4\frac{\partial^2 u}{\partial x^2}$ or $u_t = 4u_{xx}$ with boundary conditions $u(x,0) = \frac{1}{2}x(8-x)$ and $u(0,t) = 0, u(8,t) = 0$ by implicit method.
Experiment 3 (Solution of Heat Equation by Semi-Implicit FDM)	
Aim	To be conversant with semi-implicit scheme of finite difference method for solution of parabolic partial differential equation with special reference to one dimensional heat conduction equation.

Sl. No.	Name of Experiments
Q1	Write a C or C++ program to solve the equation $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$ or $u_t = u_{xx}$ satisfying the conditions $u(x,0) = \frac{x}{3}(16 - x^2), u(0,t) = 0, u(4,t) = 0$ by Crank-Nicolson's method, which is a semi-implicit method.
Q2	Write a C or C++ program to solve the equation $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$ or $u_t = u_{xx}$ satisfying the conditions $u(x,0) = 10x(5 - x), 0 \le x \le 5, u(0,t) = 0\,; u(5,t) = 0, t \ge 0$ by Crank-Nicolson's method, which is semi-implicit.
Experiment 4 (Solution of Elliptic PDE by FDM)	
Aim	To appreciate the importance of second order elliptic partial differential equations *viz*. Laplace and Poisson equations and its solution by finite difference method.
Q1	Write a C or C++ program to solve the Laplace equation $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$ or $u_{xx} + u_{yy} = 0$ or $\nabla^2 u = 0$ at the interior grid points of a square mesh with prescribed boundary conditions, using method of relaxation.
Q2	Find the steady state temperature distribution in a square metallic plate, one side of which is maintained at 100°C and other three sides maintained at 0°C. Hint: The problem is that of solving Laplace equation in a rectangle.
Q3	Write a C or C++ program to solve the Poisson equation $u_{xx} + u_{yy} = -4(x^2 + y^2)$ or $\nabla^2 u = -4\,(x^2 + y^2)$ over the square mesh with sides $x = 0$, $y = 0$, $x = 3$ and $y = 3$ with $u = 0$ on the boundary, using the method of relaxation.
Experiment 5 (Solution of Wave Equation by FDM)	
Aim	To solve hyperbolic PDEs such as wave equation by finite difference method.
Q1	Write a C or C++ program to solve the partial differential equation $u_{tt} = u_{xx}$, for $x = 0\ (0.2)\ 1$ and $t = 0\ (0.2)\ 1$, given that $u(0, t) = 0$, $u(1, t) = 0$, $u(x, 0) = x(1 - x)$ and $u_t(x, 0) = 1$.
Q2	Write a C or C++ program to solve the equation $u_{tt} = u_{xx}$, up to $t = 1$ with step size 0.2, given that $u(0, t) = 0$, $u(1, t) = 0$, $u_t(x, 0) = 0$ and $u(x, 0) = 10 + x(1 - x)$, $0 < x < 1$.
Experiment 6 (Solution of Heat Equation by FVM)	

Sl. No.	Name of Experiments
Aim	To understand the application of the finite volume method (FVM) to solve simple diffusion problems involving conductive heat transfer.
Q_1	Consider the problem of source-free heat conduction in an insulated rod whose ends are maintained at constant temperatures of 100°C and 500°C respectively. Calculate the steady state temperature distribution in the rod. Describe the numerical methodology behind the heat conduction equation solver based on the finite volume method (FVM) and write a C or C++ program to implement it.
Q_2	Let us have a problem that includes sources other than those arising from boundary conditions. Consider a large plate of thickness L = 2cm with constant thermal conductivity k = 0.5 W/m.K and uniform heat generation q = 1000 kW/m^3. The faces A and B are at temperatures of 100°C and 200°C respectively. Assuming that the dimensions in the y- and z-directions are so large that temperature gradients are significant in the x-direction only, calculate the steady state temperature distribution by finite volume method (FVM). Write a C or C++ program to implement it and compare the numerical results with the analytical solution.
Q_3	Let us consider the two-dimensional steady state diffusion equation, when the source term is represented in the linearised form. At the boundaries, the temperatures are known. Apply finite volume method to calculate conductive heat transfer in two-dimensional situations and write the corresponding C or C++ program to implement it.
Experiment 7 (Solution of Laplace Equation by FVM)	
Aim	To be familiar with finite volume method for the Laplace equation on two-dimensional grids.
Q_1	Describe finite volume method for solution of Laplace equation on a two-dimensional grid and write a C or C++ program to implement it.
Experiment 8 (Solution of Wave Equation by FVM)	
Aim	To understand finite volume method for the wave equation in two dimensions. and write a C or C++ program to implement it.
Q_1	Describe finite volume method for solution of linear wave equation on a two-dimensional grid and write a C or C++ program to implement it.
Experiment 9 (SIMPLE ALGORITHM)	

Sl. No.	Name of Experiments
Aim	To be conversant with the SIMPLE algorithm for calculating pressure and velocities.
Q_1	Consider the steady, one-dimensional flow of a constant-density fluid through a duct with constant cross-sectional area. Use the staggered grid, where the pressure p is evaluated at the main nodes, whilst the velocity u is calculated at the backward staggered nodes. Use the SIMPLE algorithm to calculate pressure corrections at main nodes and obtain the corrected velocity field at backward staggered nodes. Write a C or C++ program to implement it.
Q_2	A planar two-dimensional nozzle, wherein the flow is steady and frictionless and the density of the fluid is constant. Use the backward-staggered grid with five pressure nodes and four velocity nodes. The stagnation pressure is given at the inlet and the static pressure is specified at the exit. Using the SIMPLE algorithm, write down the discretised momentum and pressure correction equations and solve for the unknown pressures at the main nodes and velocities at the staggered nodes. Write a C or C++ program to implement it.
Experiment 10 (QUICK ALGORITHM)	
Aim	To get acquainted with the Quadratic upwind differencing scheme: the QUICK scheme.
Q_1	A property φ is transported by means of convection and diffusion through the one-dimensional domain as shown below.  The governing equation is given by $\frac{d}{dx}(\rho u\varphi) = \frac{d}{dx}\left(\Gamma \frac{d\varphi}{dx}\right)$; the boundary conditions are φ0 = 1 at x = 0 and φL = 0 at x = L. Using the QUICK scheme solve the problem considered for u = 0.2 m/s on a five-point grid. Write a C or C++ Program to implement it.
Q_2	Discuss stability problems of the QUICK scheme and its remedies.