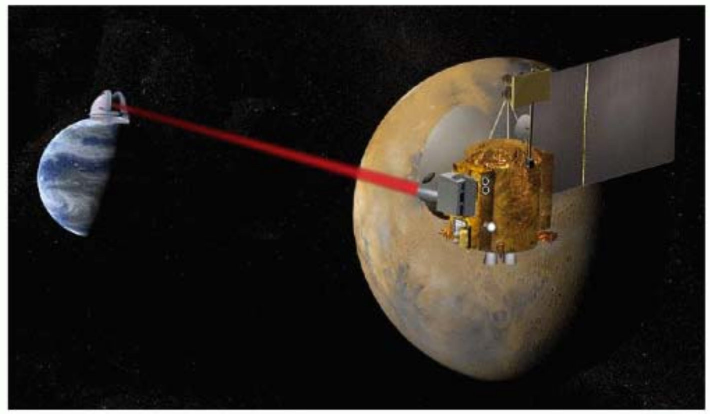
Fundamentals of Free-Space Optical Communication



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Quantum Communication, Sensing and Measurement in Space
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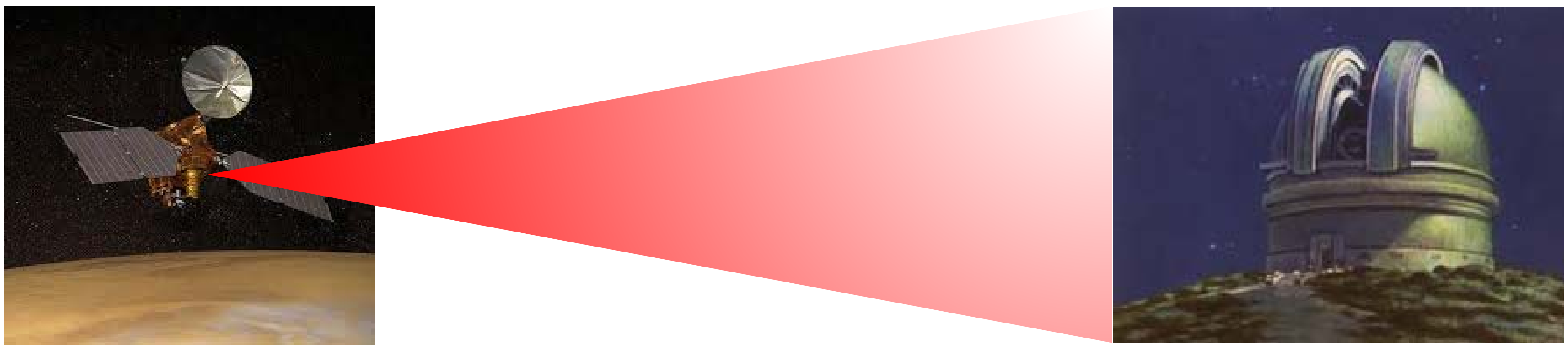
Outline of the tutorial

- System diagram and link budgets
- System elements and the deep-space communication channel
- Fundamental capacity limits
- Coding to approach capacity
- Poisson-modeled noises
- + Other losses at the detector
- Atmospheric effects on optical communication
- Conclusions

- **This talk will deal primarily with optical communication system design and analysis for *JPL's deep-space applications*. Free-space optical communication also has extensive application to near-Earth links, to space-space or space-Earth networks, and to terrestrial links and networks, but these will not be covered in this talk.**

System diagram & link budgets



In this section, we discuss:

- Basic comparison of link budgets for optical vs RF systems
- Block diagram of an optical communication system
- Detailed link budget including losses affecting optical links
- Example of a Mars-Earth optical link

Coherent Microwave (RF) vs. Non-Coherent Infrared (Optical)

Received power

$$P_r = P_w \left(\frac{\pi D_t}{\lambda}\right)^2 \left(\frac{\pi D_r}{\lambda}\right)^2 \left(\frac{\lambda}{4\pi R}\right)^2 \eta$$

Ka-band Link

f	carrier frequency	32.0 GHz
D_t	transmit diameter	3.0 m
D_r	receiver diameter	34.0 m
η	system efficiency	-10.88 dB
N_o	noise spectral density	-178.45 dB-mW/Hz
W	bandwidth	500 MHz
P_t	transmit power	35 W

Near-Infrared Link

λ	wavelength	1.55 μm
D_t	transmit diameter	22.0 cm
D_r	receiver diameter	11.8 m
η	system efficiency	-16.74 dB
α_b	noise spatial density	1.0 pW/m^2
T_s	slot width	0.5 ns
P_t	transmit power	4 W

Capacity: supportable data rate (P_r >> P_n, average power limited)

$$C_{OPT} \approx \frac{P_r \log_2 M}{(hc/\lambda)} \text{ b/s} \qquad C_{RF} \approx \frac{P_r}{\ln(2) N_0} \text{ b/s}$$

$$\frac{C_{OPT}}{C_{RF}} \approx \left(\frac{D_t^o D_r^o}{D_t^r D_r^r}\right)^2 \left(\frac{\eta^o}{\eta^r}\right) \left(\frac{N_0}{(hc/\lambda)/\log_2(M)}\right) \left(\frac{\lambda^o}{\lambda^r}\right)^2$$

aperture -33 dB

efficiency -16 dB

'noise' -12 dB

beam divergence +76 dB

But gains are less in background noise, with pointing losses, etc.

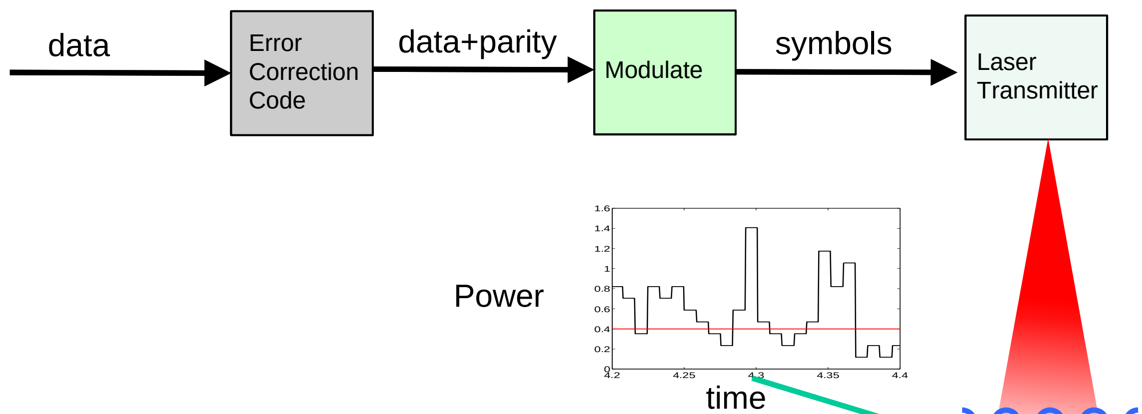
= net 15 dB gain!

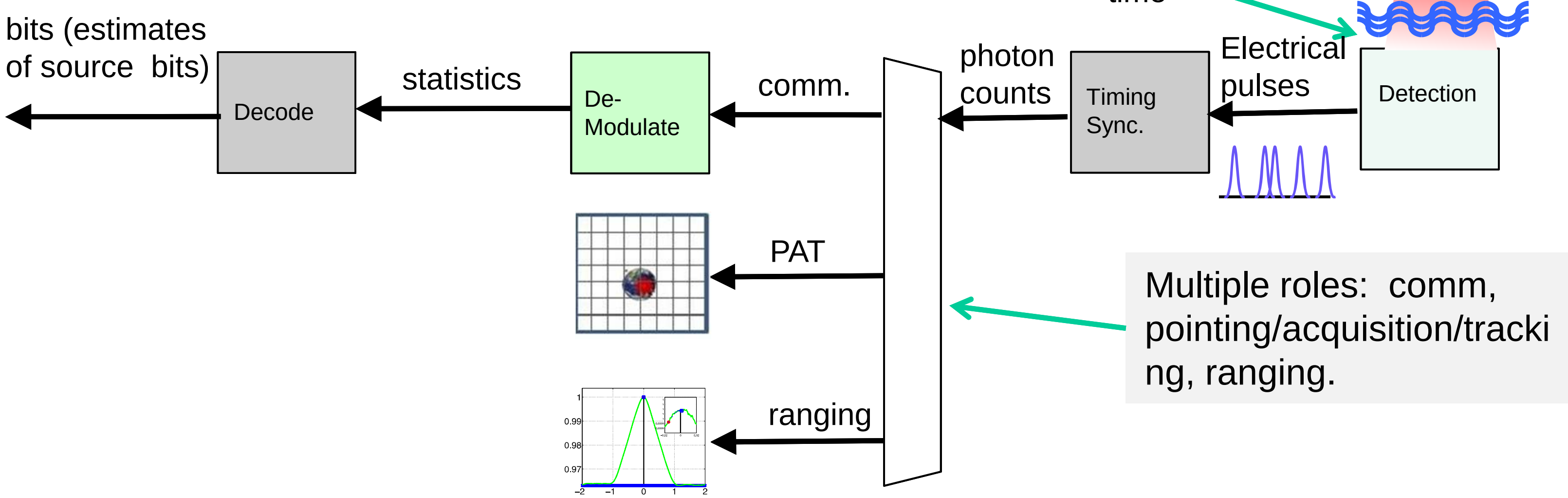
Capacity comparisons to answer the question: why optical?

Block diagram of an optical communication system

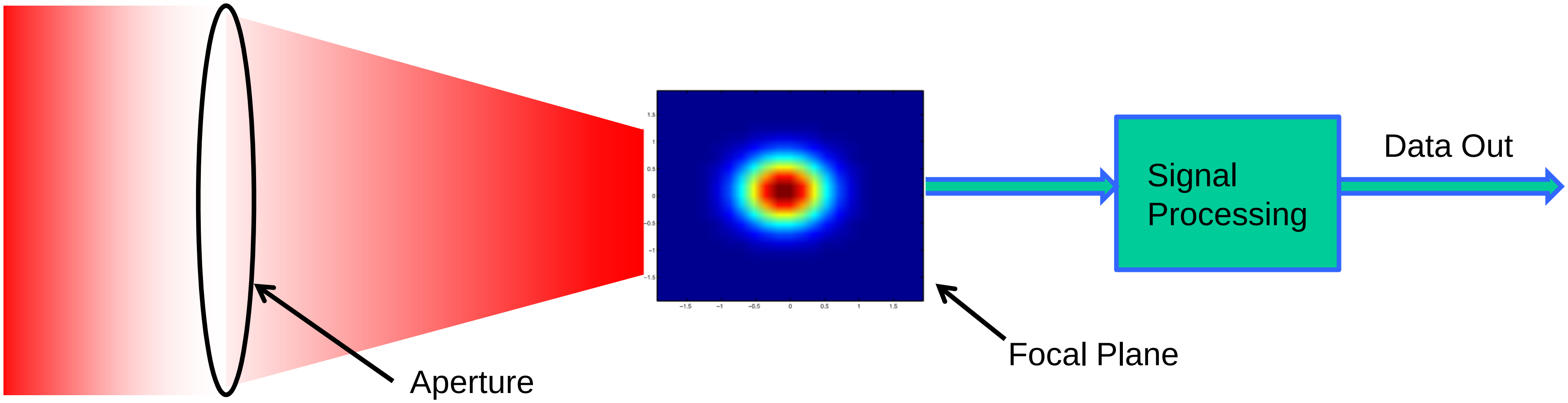
To accurately assess system performance, we must consider the context (free-space communication link) and also specify various elements of the system and the channel:

- modulation
- detection
- channel model
- channel capacity
- error correction coding





Optical system link analysis accounting for losses



- Losses due to non-ideal system components (labeled efficiencies)
- Loss due to receiving the signal power in the presence of noise
- Losses due to spatial and temporal distortion of the of the received power

Received Power: average signal power received (in focal plane)

$$P_{rx} = P_t G_t G_r L_s L_a \eta_{pt} \eta_t \eta_r$$

- Transmitted power
- Transmit & Receive aperture gains
- Space loss
- Atmospheric loss
- Pointing loss
- Transmit & Receive efficiencies

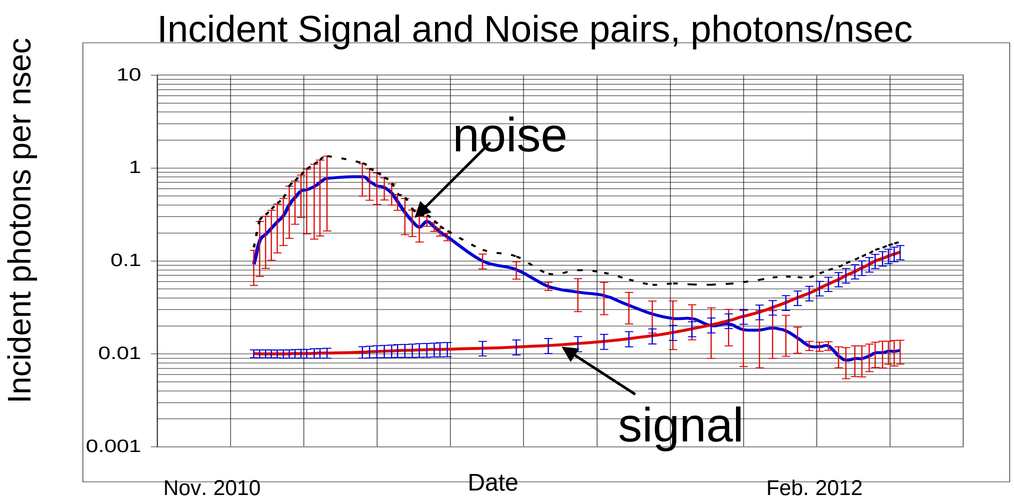
Required Power: required signal power in focal plane to support specified data rate

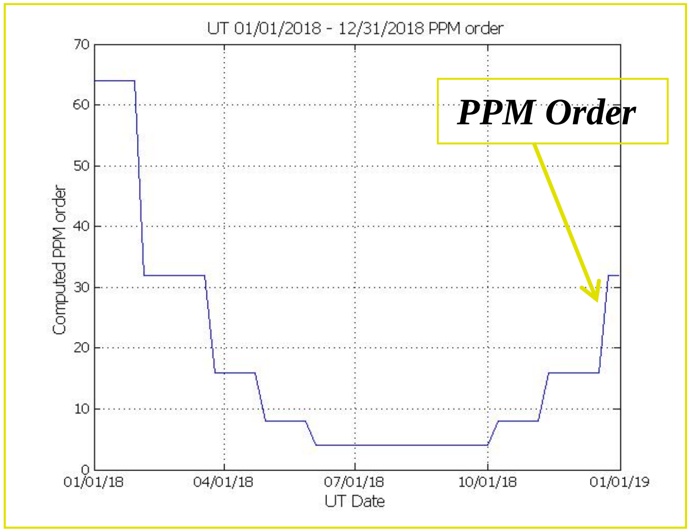
$$P_{rqd} = P_i / L_b L_j L_f L_t \eta_{det} \eta_{imp} \eta_{code} \eta_{int}$$

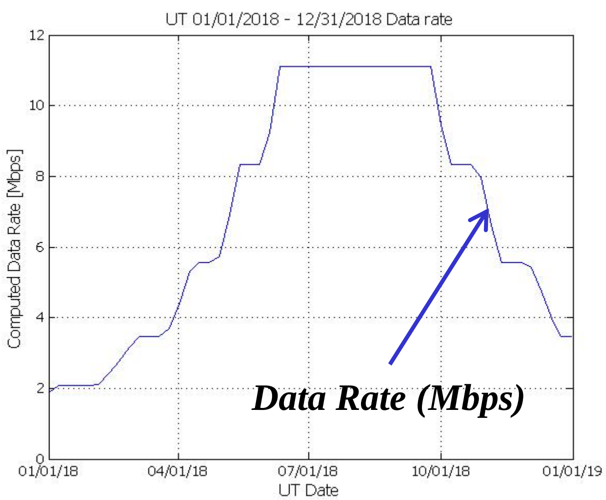
- Minimum (ideal receiver) required power
- Detector Blocking, Jitter & Efficiency losses
- Scintillation loss
- Truncation loss
- Implementation efficiency
- Code & Interleaver efficiencies

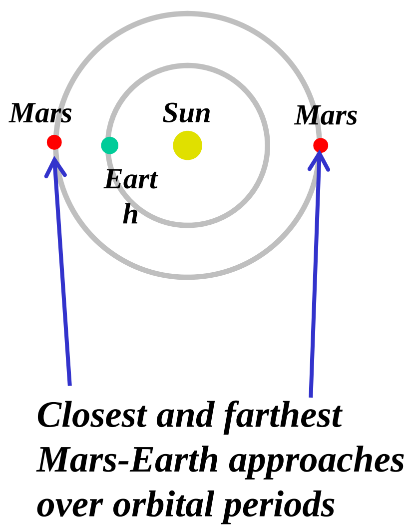
Example of Mars-Earth Link Received Signal and Noise Powers

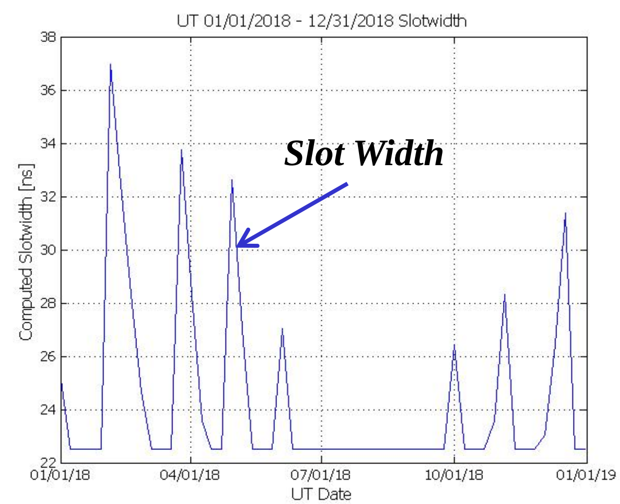
- Wide range of operating points: 20 dB range of noise power, 12 dB range of signal power, due to changes in geometry (range, sun-earth-planet angle, zenith angle) and atmosphere.











System elements and the deep-space communication channel

In this section, we discuss:

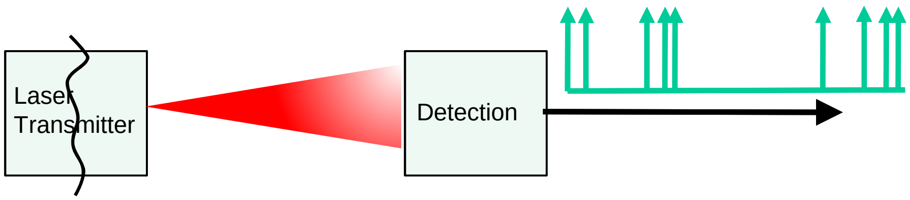
- The detection method (coherent or non-coherent)
- Intensity modulations for non-coherent detection
- Photon-counting channel model for intensity modulations
- Processing the observed photon counts to recover the data

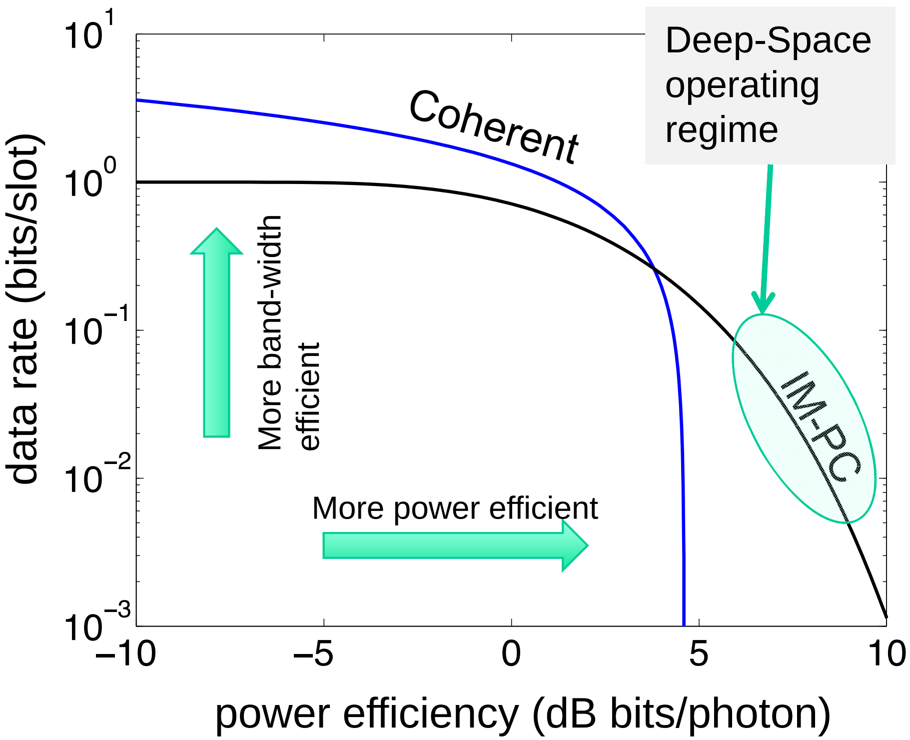
Optical signal detection methods

- Coherent-detection
  - Enables, e.g., phase-modulations (BPSK).
  - Requires correction of the phase front when transmitted through the turbulent atmospheric channel.

- Non-coherent detection
  - Enables, e.g., intensity-modulation (IM)
  - More power efficient at deep-space operating points (with low background noise).
  - Photon-counting (PC) is practical.

IM-PC is near-optimal in our region of interest (low background noise, high power efficiency).
In the remainder, we assume an IM-PC channel.





Coherent detection systems

- **Heterodyne** and **homodyne** receivers can be used with **arbitrary coherent-state modulations**.

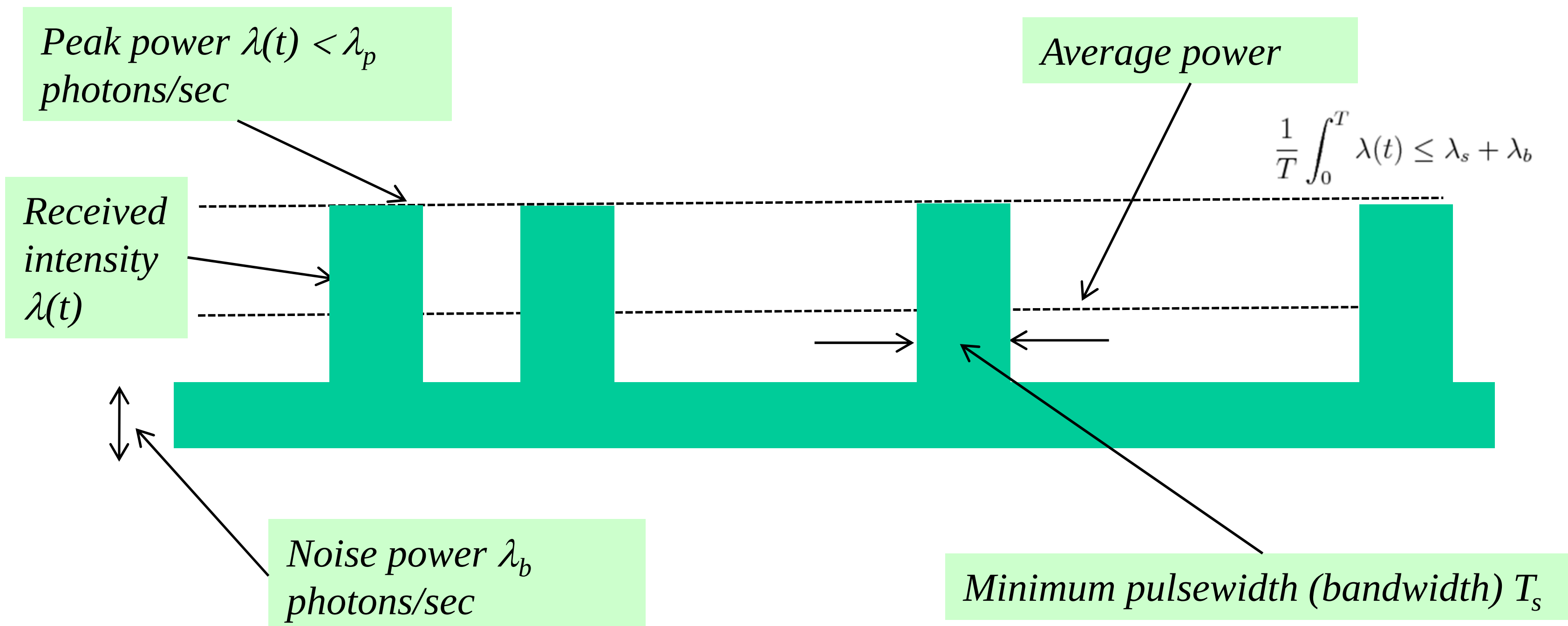
- Such receivers, teamed with high-order modulations, achieve **much higher spectral efficiency** than PPM or OOK with photon counting.
  - Coherent detection systems are generally more practical than non-coherent systems for applications requiring **extremely high data rates**.

- Coherent systems also are practical for:
  - Systems that operate through the atmosphere
  - Systems limited by background noise or interference
  - Multiple-access applications

- However, coherent receivers encounter **brick-wall limits** on their maximum achievable **photon efficiencies**:
  - Maximum of 1 nat/photon (**1.44 bits/photon**) for heterodyning
  - Maximum of 2 nats/photon (**2.89 bits/photon**) for homodyning

Optical modulations for non-coherent detection

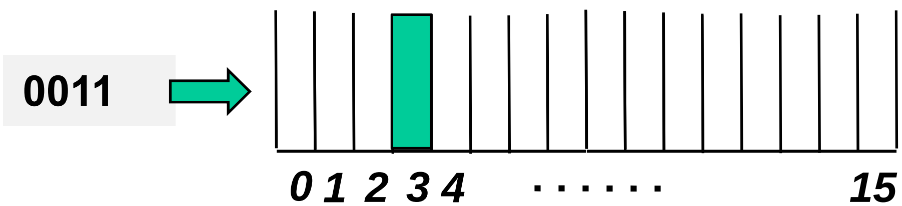
- ***Negligible loss in restricting waveforms to be slotted (change only at discrete intervals), and binary (take only two values)***
  1. ***with no bandwidth (slotwidth) constraint [Wyner]***
  2. ***in certain regions under a bandwidth constraint [Shamai]***



Optical modulations for non-coherent detection

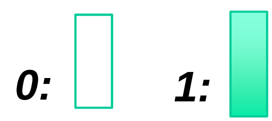
Modulation: ***Collection of waveforms used to represent information***

Pulse-Position-Modulation (PPM): ***$\log_2 M$ bits are represented by a single pulse out of M slots (here M=16).***



- ***Pulse-position-modulation (PPM) is an efficient way to implement a low duty cycle.***

On-Off-Keying (OOK): ***1 bit is represented by a slot, which may either be occupied by a pulse or not.***

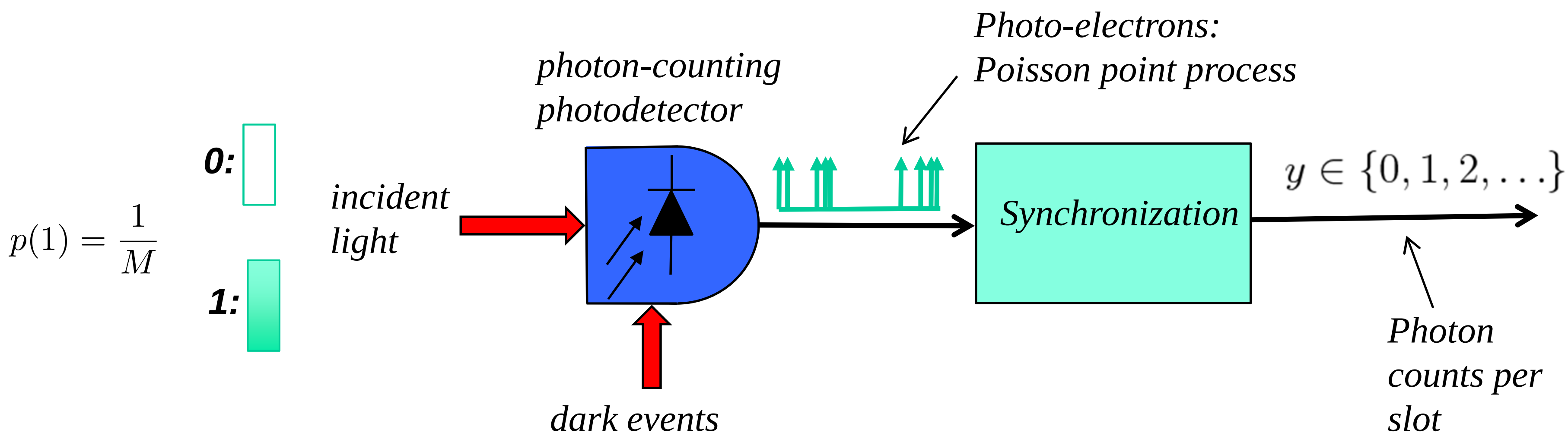


- ***But for highest energy efficiency, unpulsed OOK slots are much more probable than pulsed slots (low duty cycle).***

Given an optimum duty cycle 1/M, how do we efficiently map an unconstrained binary sequence to a duty cycle 1/M sequence?

PPM is near-optimal over all possible modulations one could use on the intensity-modulated (IM) photon-counting (PC) channel in our region of interest (low duty-cycles).

Intensity-modulated photon-counting channel model



Equivalent Channel model: Binary input, Poisson-distributed integer output

$$a \in \{0, 1\} \qquad p(a = 1) = \frac{1}{M}$$

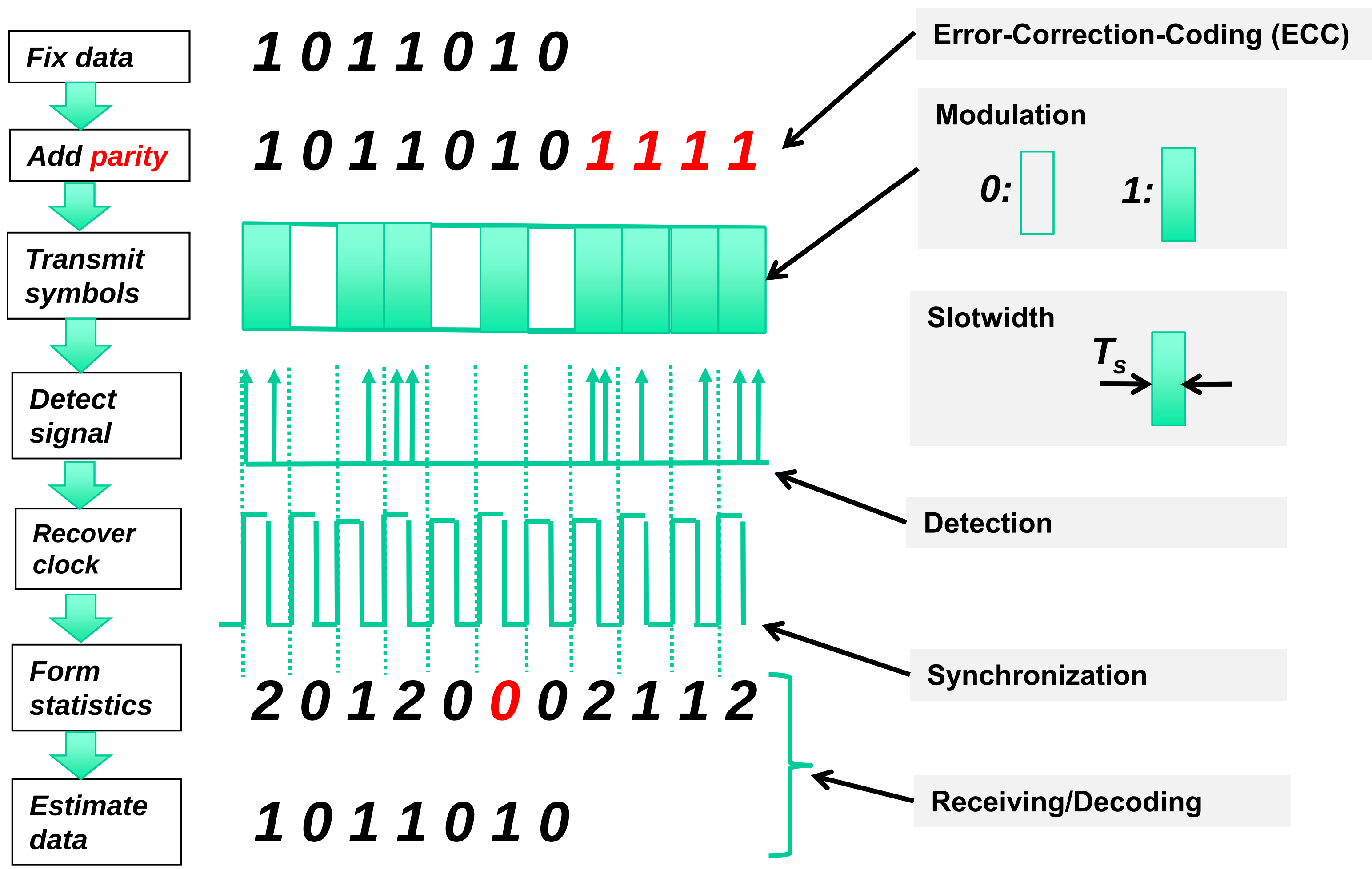
$$p(y|a = 0) = \frac{e^{-n_b}(n_b)^y}{y!}$$

$$p(y|a = 1) = \frac{e^{-(n_s+n_b)}(n_s + n_b)^y}{y!}$$

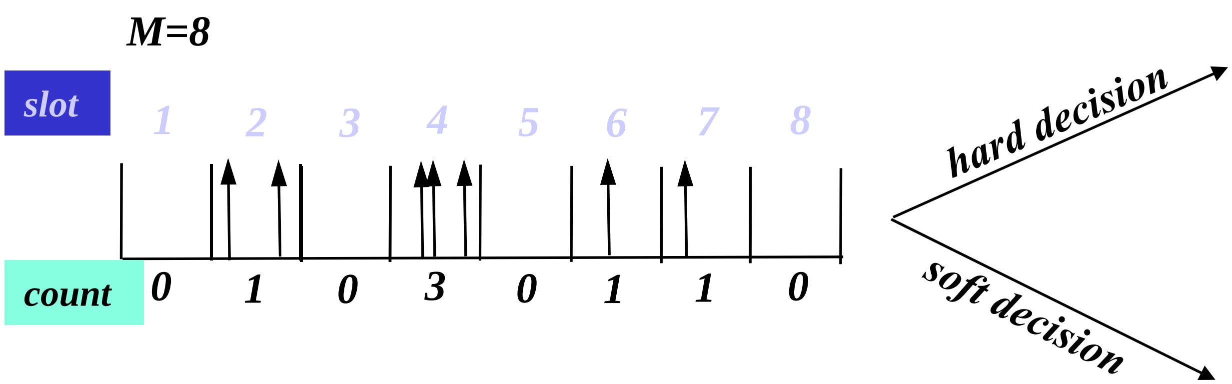
$$y \in \{0, 1, 2, \ldots\}$$

n_s = mean signal photons per pulsed slot
n_b = mean background photons per slot

Signal processing steps to communicate the data

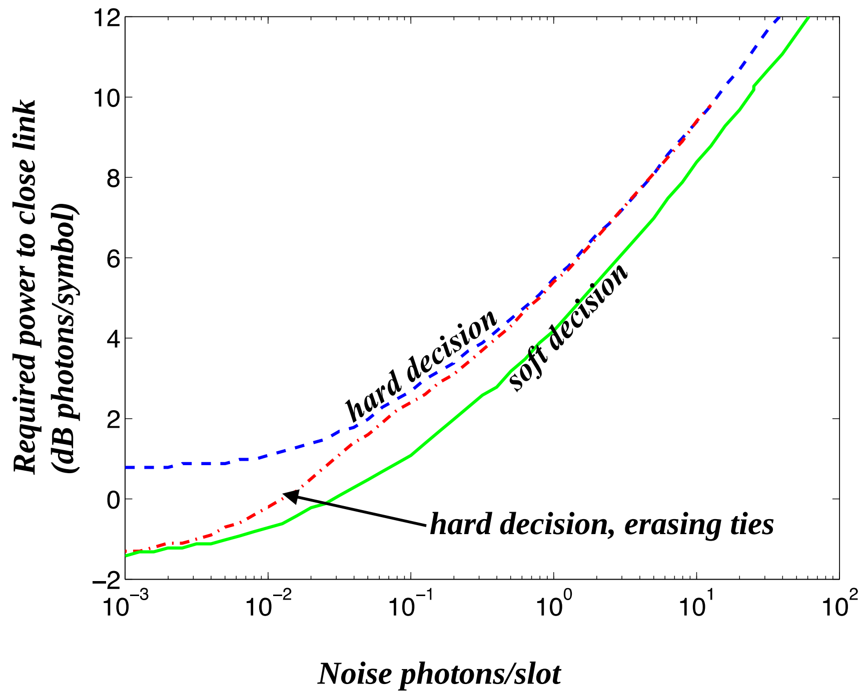


Processing the photon counts: soft vs hard decisions



$\hat{x} = 011$

p(x=000|y) = 0.05
p(x=001|y) = 0.2
p(x=010|y) = 0.05
p(x=011|y) = 0.3
…



Capacity, hard and soft decisions M = 16, Average power to achieve C = 1/8 bits/slot

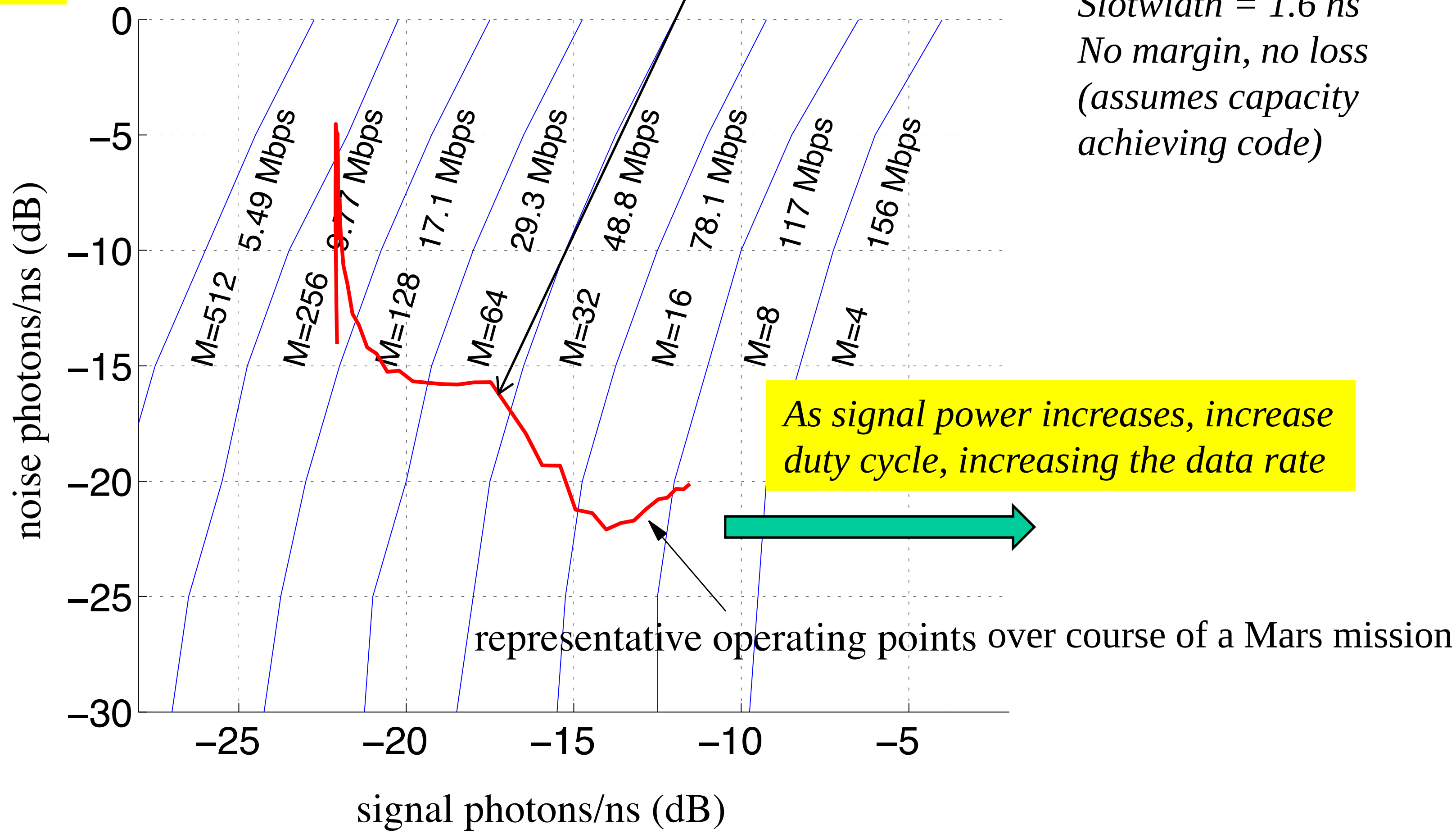
Typical region of operation is between 0.01 and 1.0 photons/slot

Near-optimal signaling for a deep-space link

Mapping of the received signal power, noise power plane to optimum PPM orders

Typical Earth-Mars link would ideally use many PPM orders

Slotwidth = 1.6 ns
No margin, no loss
(assumes capacity achieving code)



As signal power increases, increase duty cycle, increasing the data rate

Fundamental capacity limits

In this section, we discuss:

- **Fundamental capacity limits for ideal noiseless quantum channel (i.e., only "quantum noise").**
  - **Limits for given combinations of modulation and receiver.**
  - **The ultimate limit (Holevo capacity) for any quantum-consistent measurement.**
- **Capacity tradeoffs in terms of *dimensional information efficiency* (*DIE*) vs *photon information efficiency* (*PIE*).**
  - **PIE is measured in bits/photon.**
  - **DIE is measured in bits/dimension, bits/sec/Hz per spatial mode.**
- **Alternative modulations/receivers to better approach the Holevo limit.**
- **Poisson model for noisy PPM or OOK channel capacity.**

Asymptotic Holevo capacity limit

- Asymptotically, for large photon efficiency, the ultimate (Holevo) capacity efficiencies are related by:

c_d = dimensional information efficiency (DIE) [bits/dimension] or [bits/s/Hz per spatial mode]

$$c_d \rightarrow \tilde{c}_d^{(2)} \equiv e\, c_p\, 2^{-c_p}$$

c_p = photon information efficiency (PIE) [bits/photon]

- Thus, even at the ultimate limit, the dimensional efficiency (c_d) **must fall off exponentially** with increasing photon efficiency (c_p), except for a multiplicative factor proportional to c_p.

Asymptotic capacity of PPM and photon counting

- Asymptotically, for large photon efficiency, we have:

$$\log_2 M^* - c_p \to 0$$

$$c_d \to \tilde{c}_d^{(1)} \equiv \left(\frac{2}{e\,\ln 2}\right) 2^{-c_p} \approx 1.061 \times 2^{-c_p}$$

- Thus, with PPM and photon counting, and *M* optimized to achieve the best tradeoff, the dimensional information efficiency (c_d) must fall off exponentially with increasing photon efficiency (c_p).

- Comparing PPM + photon counting to the ultimate capacity, we obtain:

$$\frac{c_d(\text{ultimateHolevo})}{c_d(\text{PPM} + \text{counting})} \to \frac{\tilde{c}_d^{(2)}}{\tilde{c}_d^{(1)}} = \left(\frac{e^2 \ln 2}{2}\right) c_p \approx 2.561\, c_p$$

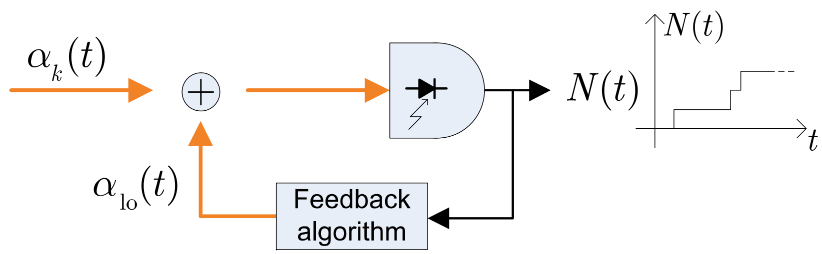
Can we approach Holevo capacity more closely than PPM/OOK + photon counting?

- Thus, the best possible factor by which the dimensional efficiency (c_d) can be improved by replacing a conventional system with PPM and photon counting with one that reaches the ultimate Holevo limit is only **linear in the photon efficiency** (c_p).

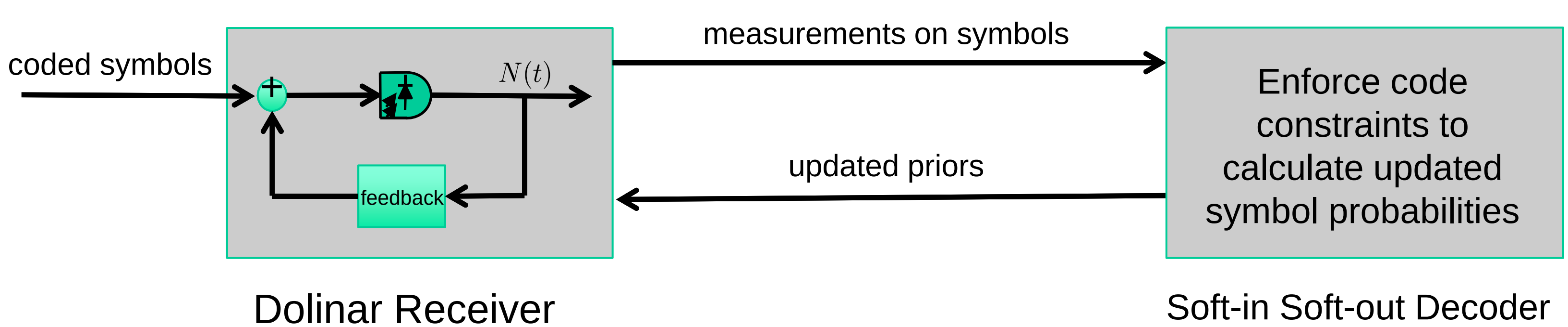
Dolinar receiver structure for BPSK or OOK

- The **Dolinar receiver** is known to be the optimal hard-decision measurement on an arbitrary binary coherent-state alphabet.
- It is also an optimal soft-decision measurement (at least for BPSK) for maximizing the mutual information.
- Unfortunately, **capacity improvements for OOK are minuscule** relative to photon counting, and there's still a **brick-wall upper limit of 2 nats/photon for BPSK**.

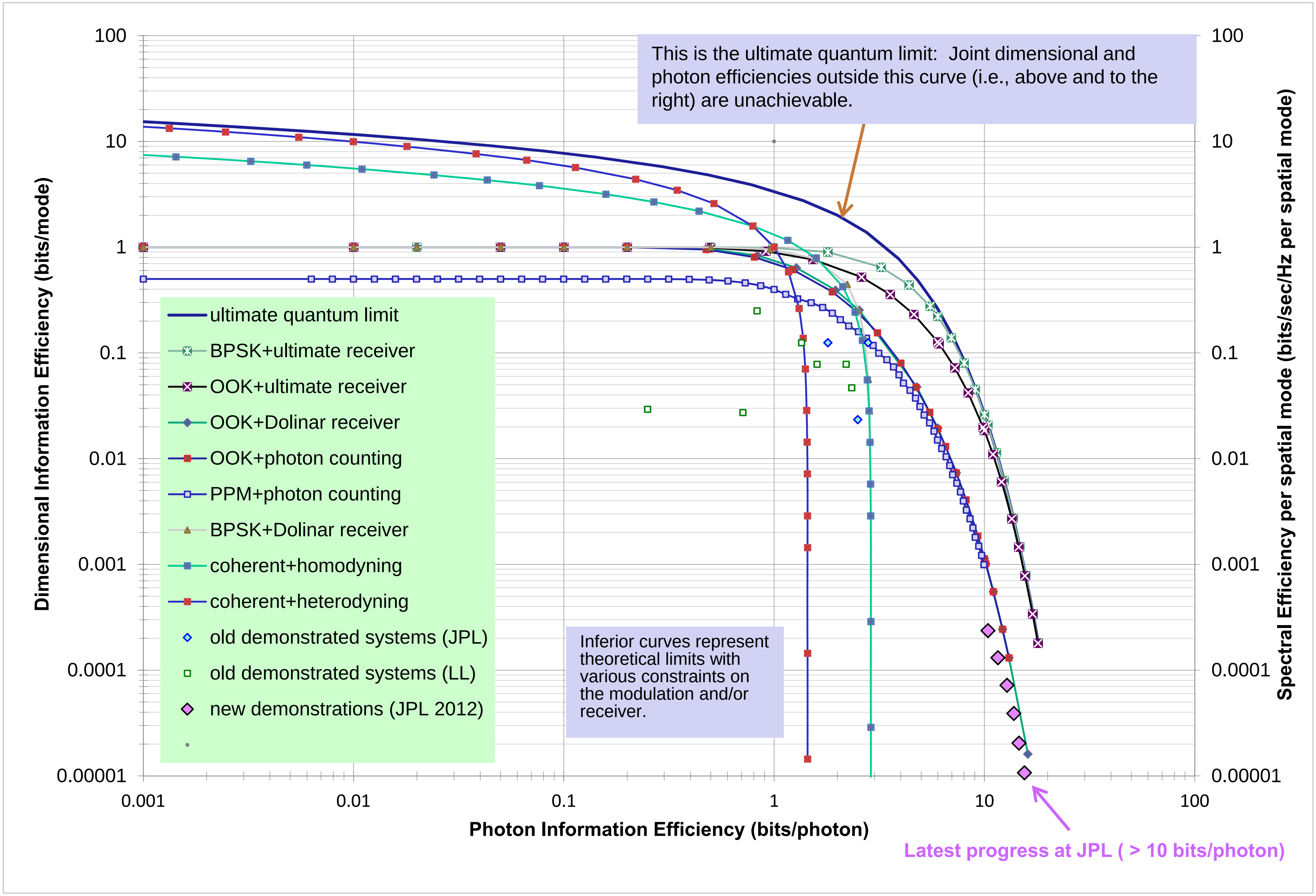
Dolinar receiver structure



- The Dolinar receiver was extended to perform ***adaptive measurements*** on a ***coded sequence*** of binary coherent state symbols.
- There was **no capacity improvement** for the Dolinar receiver with adaptive priors.



Fundamental free-space capacity limits vs state-of-the-art optical systems



Communicating with single-photon number states

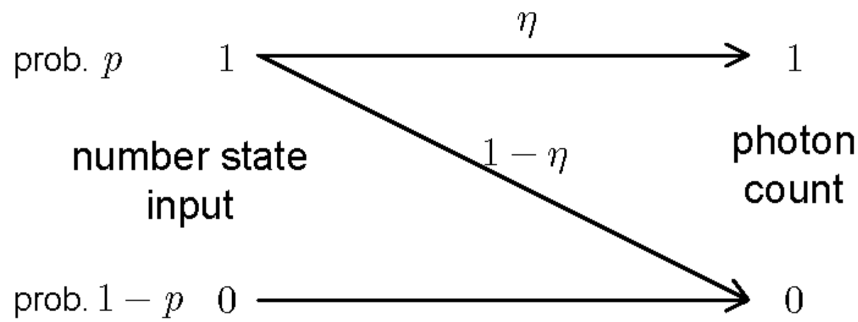
Can we do (significantly) better than than PPM/OOK + photon counting?

Yes, using quantum number states instead of coherent states.

Quantum-ideal number states:

- EM-wave with **deterministically observable energy**.
- Propagation is **degraded by channel transmissivity** η (i.e., the probability transmitted number-state photon is not received at detector).
- With ideal transmissivity, **number-states achieve Holevo limit** (with Bose-Einstein priors).
- **Binary number states are near-optimal** at large bits/photon (with ideal transmissivity).

Channel given by Z-channel of coherent-state OOK with $\epsilon \Leftrightarrow \eta$



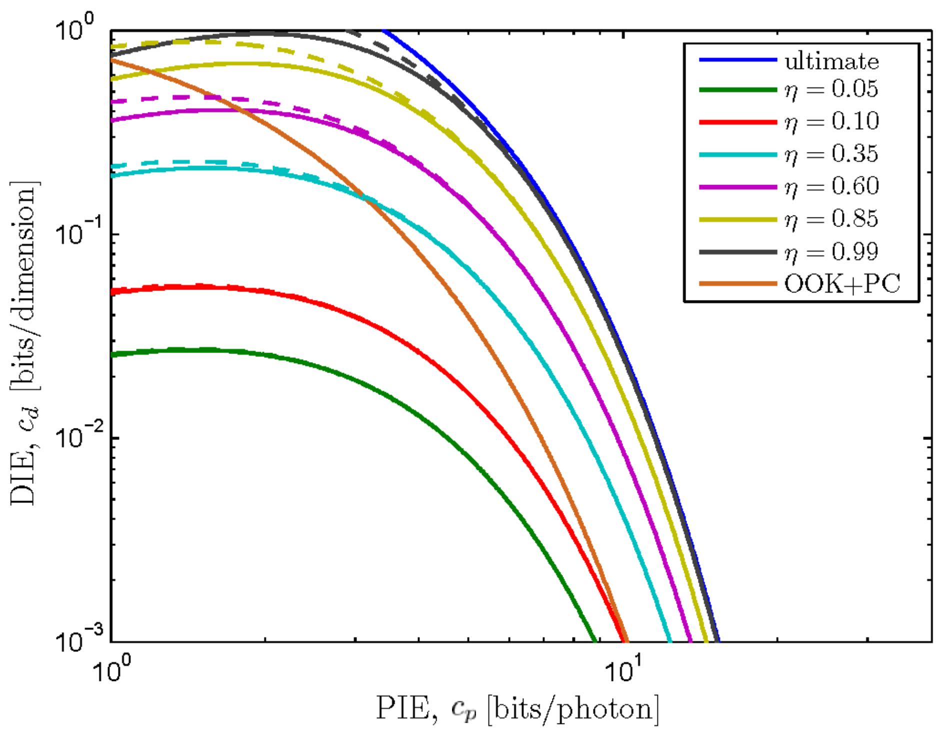
$$\Pr\left(\text{count} = 1 \middle| |1\rangle\text{sent}\right) = \eta$$

$$\Pr\left(\text{count} = 0 \middle| |1\rangle\text{sent}\right) = 1-\eta$$

Asymptotic capacity of single-photon number states

- Asymptotically at high PIE, OOK with single-photon number states achieves:

$$c_d^{\mathrm{1NS}}(c_p;\eta) \approx \underbrace{(e c_p 2^{-c_p})}_{c_d^{\mathrm{Hol}}(c_p)} \underbrace{2^{-h_2(\eta)}}_{\begin{cases}(1-\eta)^{(1-\eta)}/e^{1-\eta} \text{ for } \eta \approx 1 \\ \eta/e \text{ for } \eta \ll 1\end{cases}}$$

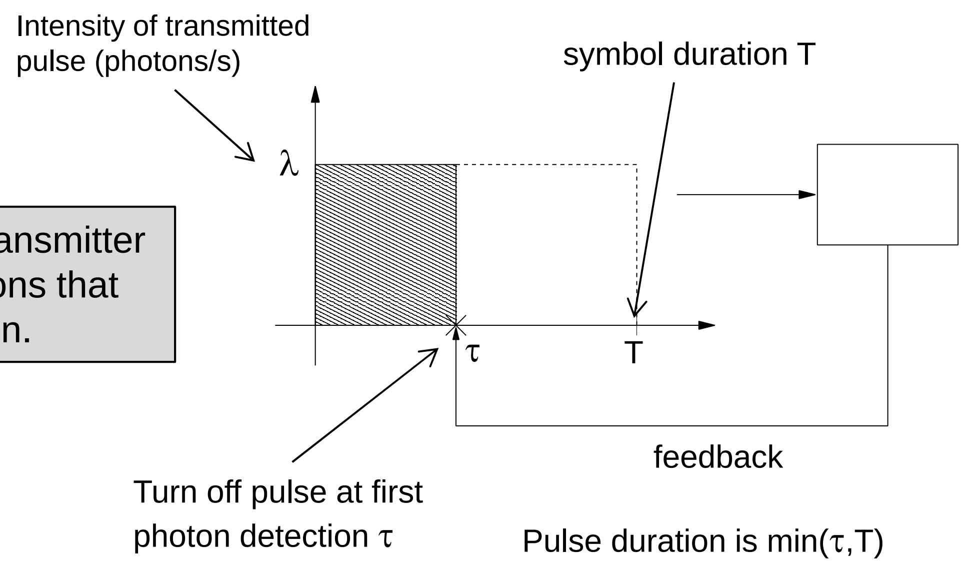


***For single-photon nunber states, the deviation from Holevo capacity is by a constant* factor for a given channel, i.e., for a given transmissivity η.**

Approximating number-state communication using coherent states with single-photon shutoff

- We can *mimic* the ideal photodetection statistics of the single-photon number state using *receiver-to-transmitter feedback*:
  - The transmitter uses standard OOK or PPM modulation, and starts sending a coherent-state pulse every time the modulator calls for an "ON" signal.
  - A standard photon-counting receiver is used.
  - Utilizing (ideal, instantaneous, costfree — i.e., **very impractical**) feedback from the receiver, the transmitter turns off its pulse as soon as the first photon is detected.
  - If the transmitted pulse has very high intensity ("***photon blasting***"), this will ensure that at least one (and therefore exactly one) photon will be detected, with very high probability.

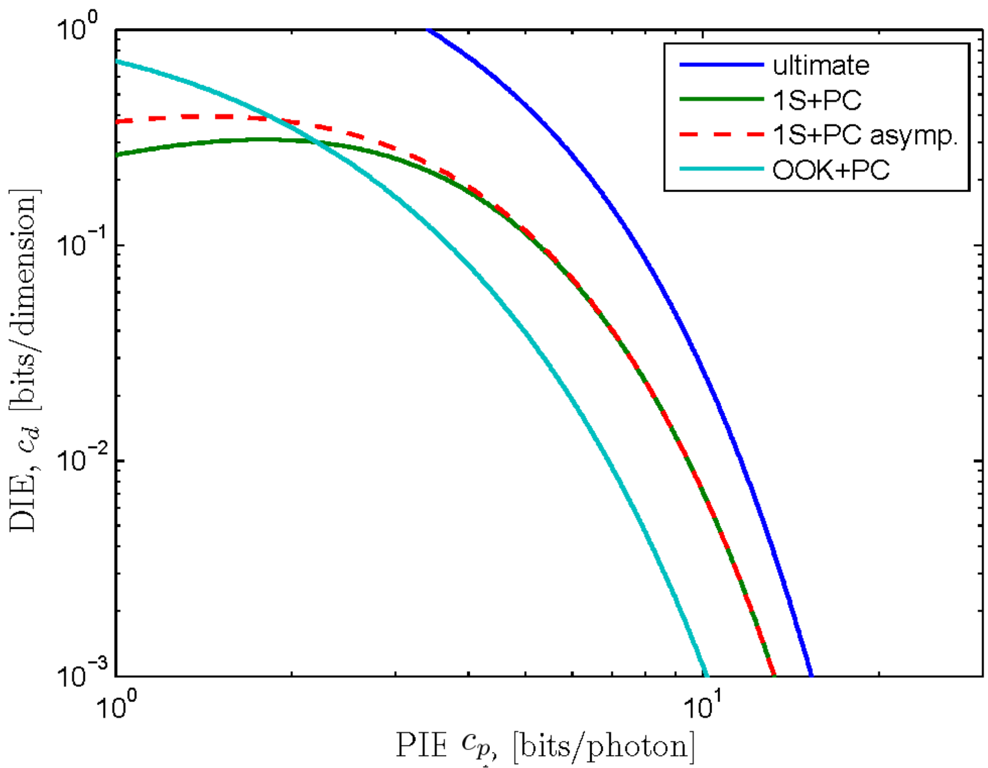
- The feedback instructs the transmitter to stop sending wasted photons that carry no additional information.



Asymptotic capacity of coherent states with single-photon shutoff

- Coherent-state OOK with single-photon shutoff economizes on photons by a factor *d*(ε), but expands bandwidth usage by the same factor *d*(ε), where ε is the pulse detection probability.
- This tradeoff is favorable at high PIE (and disadvantageous at high DIE).
- The asymptotic capacity efficiency tradeoff is:

$$c_d^{1\mathrm{S}}(c_p) \approx \left(e\, c_p\, 2^{-c_p}\right) \max_{\epsilon\in[0,1]} \left[\frac{2^{-h_2(\epsilon)}}{d(\epsilon)}\right]$$



achieved at optimal value:

$$\epsilon^* \approx 0.88$$

$$d(\epsilon^*) \approx 2.38$$

$$\lambda T \approx 2.1\ \text{photons}$$

$$\bar{n} \approx 0.876p$$

Summary of some Holevo capacity-approaching schemes

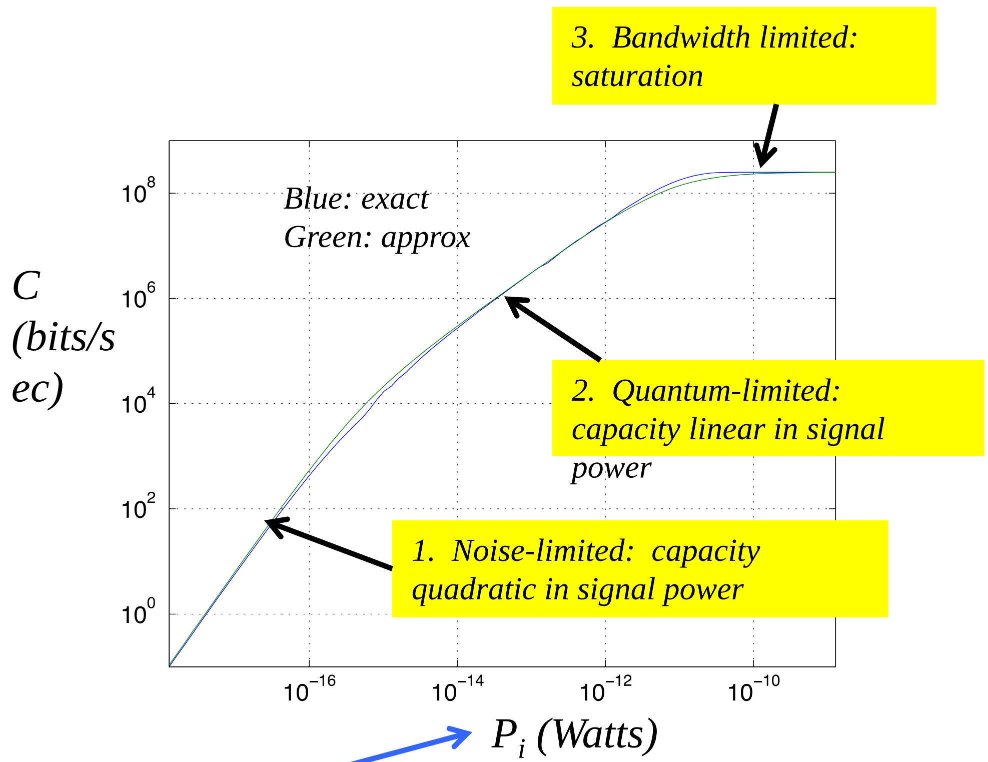
Table below shows the ***asymptotic ratio***, at high PIE, of DIE for the specified scheme to the optimal Holevo DIE at the same PIE.

- ε is the non-erasure probability (detection probability) for the coherent state cases.
- η is the end-to-end efficiency for the number state cases.

	Coherent states with single-photon shutoff		Single-photon number states		
	general ε	@ opt. ε^*	general η	@ equiv. $\eta_{eq}(\varepsilon^*)$	@ opt. η^*
OOK	$\frac{2^{-h_2(\epsilon)}}{d(\epsilon)}$	0.274 @ $\varepsilon^* = 0.876$	$2^{-h_2(\eta)}$	0.274 @ $\eta_{eq} = 0.534$	1.000 @ $\eta^* = 1$
PPM	$\frac{\epsilon/e}{d(\epsilon)}$	0.150 @ $\varepsilon^* = 0.715$	η/e	0.150 @ $\eta_{eq} = 0.407$	0.368 @ $\eta^* = 1$

Poisson model for PPM channel capacity with noise

- A Poisson channel model is used for detection of signal in background noise.
- The Poisson PPM channel capacity does not, in general, have a closed form solution.
- Approximations exist that provide insight into its behavior.
- The IM-PC channel has three regions as a function of the signal power:
    1. Noise-limited: capacity is quadratic in signal power.
    2. Quantum-limited: capacity is linear in signal power.
    3. Band-width limited: capacity saturates.
- This differs from the coherent channel which is linear or bandwidth limited.



P_i = minimum required power to close the link

- Determined by inverting the capacity function at the target data rate.
- All other system components (receiver, decoder, detector, etc.) are assumed to be ideal (no losses).

$$C \approx \frac{1}{\ln(2)E_\lambda}\left(\frac{P_i^2}{P_i\frac{1}{\ln(M)}+P_n\frac{2}{M-1}+P_i^2\frac{MT_s}{\ln(M)E_\lambda}}\right) \text{ bits/sec}$$

2. 1. 3.

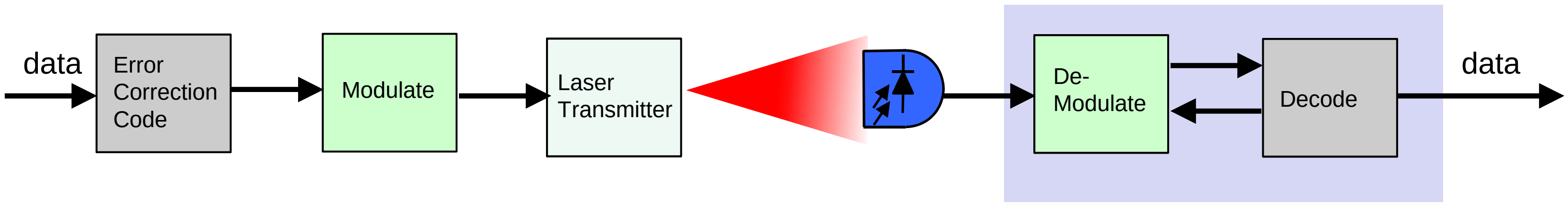
P_n = noise power
E_λ = energy per photon
M = PPM order
T_s = slot width

Coding to approach capacity

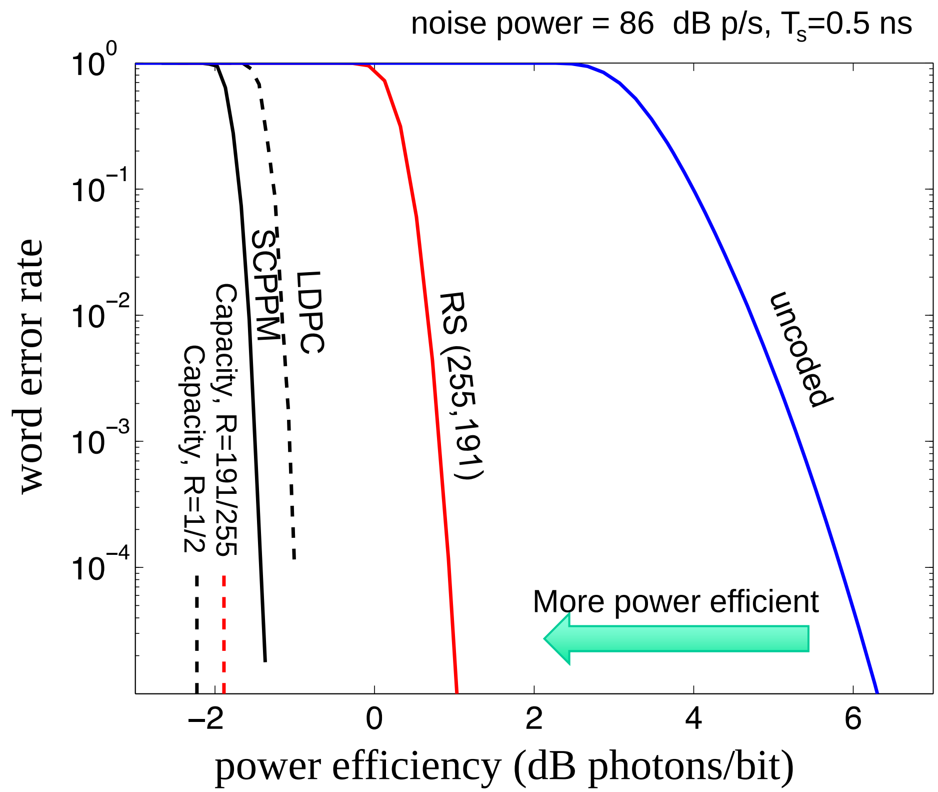
In this section, we discuss:

- **Choice of error correction code**
- **Code inefficiency relative to capacity limit**

Approaching capacity with an error correction code

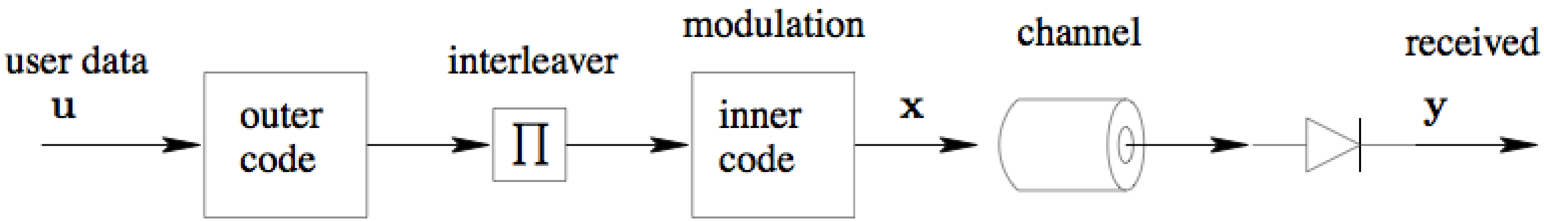


- We signal utilizing a very power efficient error-correction code (ECC) that performs close to the capacity limit.
- With high probability, a codeword error will result if the signal power drops below the channel capacity.
- Pulse-Position-Modulation (PPM) contains memory, and may be considered part of the ECC.
- Iterative demodulation and decoding (of properly designed codes) provides gains of ~1.5 dB over non-iterative decoding.
- Codes designed explicitly for use with PPM provide gains over more general-purpose codes.



Some possible choices of code

Goal: ***Choose a code type that has near-capacity performance over all operating points, and low encoding/decoding complexity.***

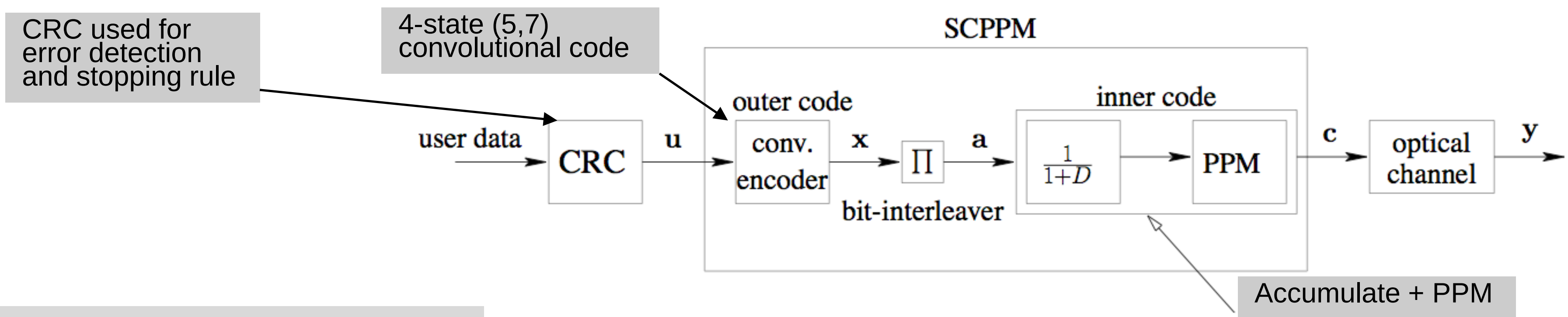


hard decisions

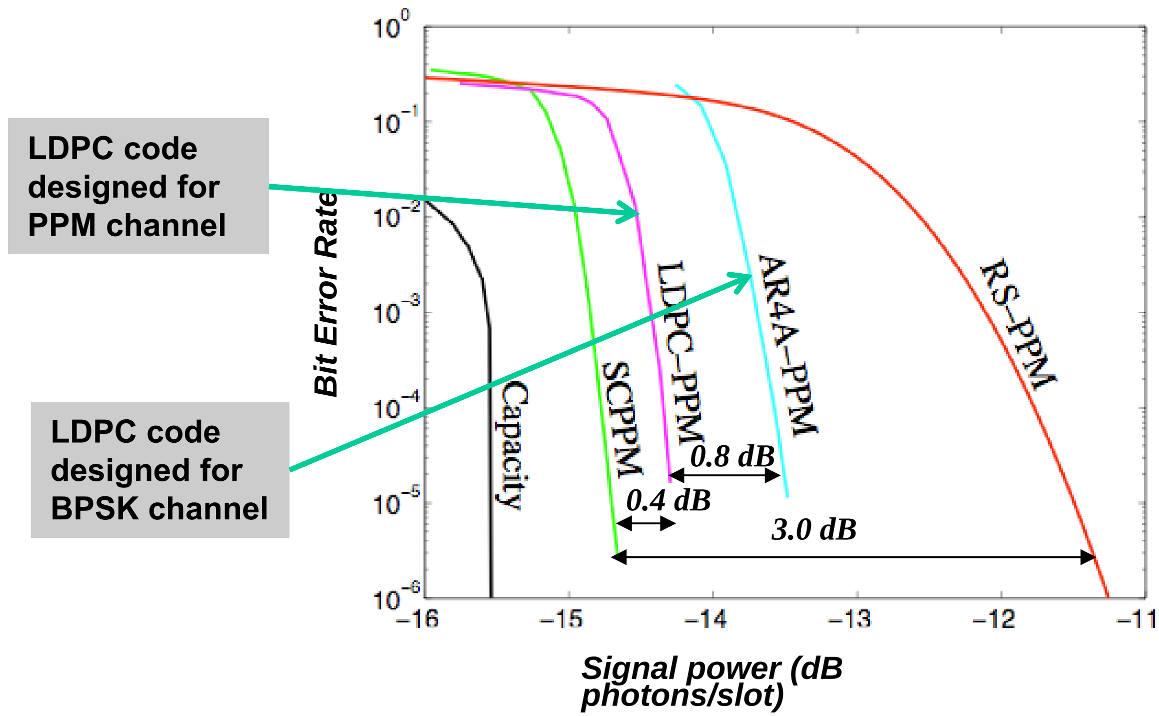
soft decisions

	outer code(s)	inner code
RSPPM	Reed-Solomon $(n,k)=(M^a-1)$, $a=1$ [McEliece, 81], $a>1$ [Hamkins, Moision, 03]	PPM
PCPPM	parallel concatenated convolutional [Kiasaleh, 98], [Hamkins, 99] (DTMRF, iterate with PPM [Peleg, Shamai, 00])	PPM
SCPPM	convolutional [Massey, 81] (iterate with APPM) [Hamkins, Moision, 02]	(accumulate) PPM
LDPC-PPM	low density parity check [Barsoum, 05]	PPM

Example of SCPPM code architecture



- The most power efficient class of codes known for the noisy Poisson PPM channel are the serial concatenation of convolutional code with an accumulator and PPM (SCPPM)
- 3.3 -- 1.8 dB gain over baseline Reed-Solomon coded PPM (R=1/2)
- 0.4 dB gain over best known LDPC-coded PPM
- Complexity, performance favors SCPPM over LDPC-coded PPM



Operating point:(n_b=0.2 photons/slot, M=64, T_s=32 nsec)

Loss due to code inefficiency with respect to capacity

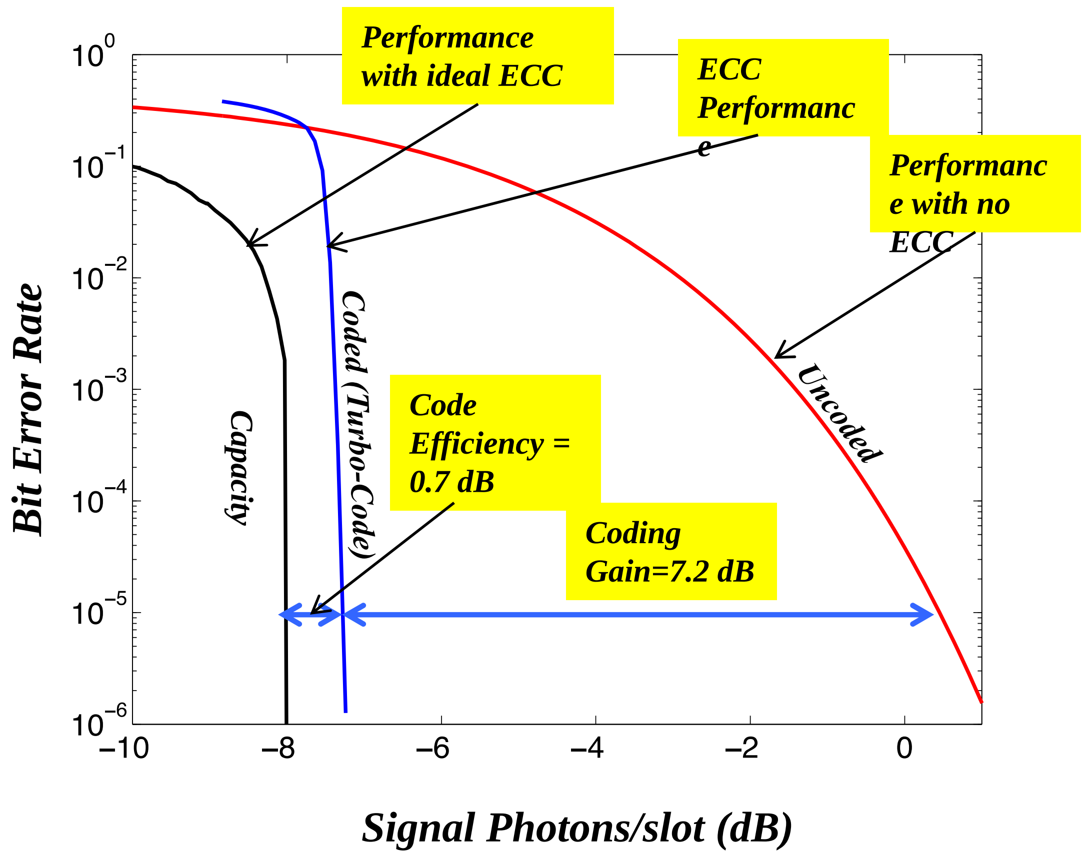
$$P_{rqd} = P_i / L_b L_j L_f L_t \eta_{det} \eta_{imp} \eta_{code} \eta_{int}$$

- Measures of the error-control-code (ECC) performance:

1. Coding Gain = (code threshold) – (uncoded threshold)

2. Code Efficiency = (capacity threshold) – (code threshold)

- We use code efficiency η_{code} to measure ECC performance:
  - Provides an immediate measure of additional gain that is possible by changing the code.
  - For modern codes (LDPC, turbo), code efficiency is well characterized as constant over varying conditions, while error-rates do not have closed-form solutions.



Poisson-modeled noises

In this section, we discuss capacity limits with:

- Thermal noise
- Finite laser transmitter extinction ratio
- Dark noise at the detector

Fundamental limit on capacity efficiency in noise

Classical (Shannon Capacity)

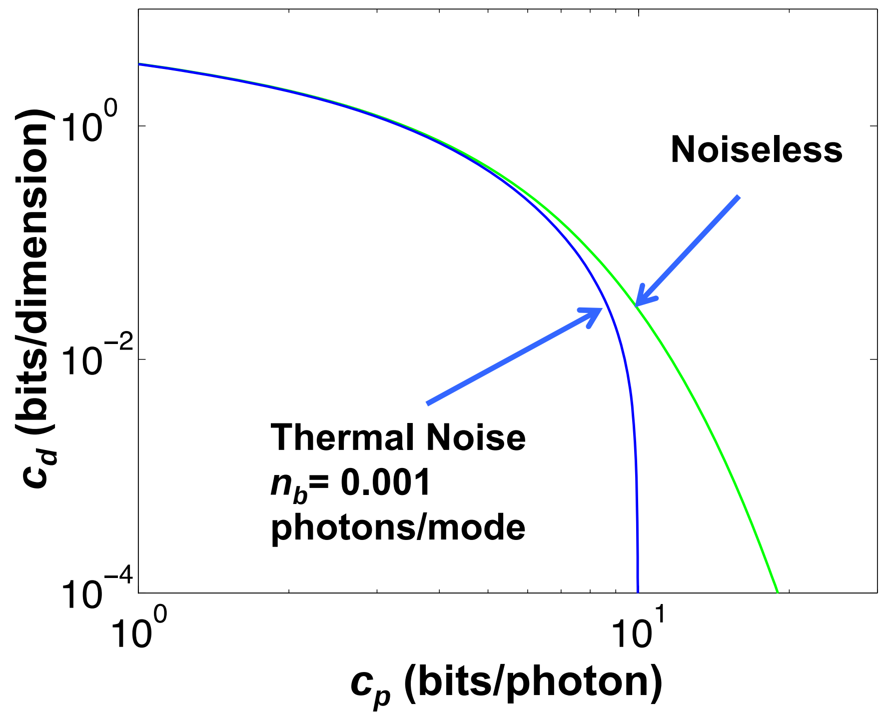
Channel described by input/output alphabets and probability map from input to output

Quantum (Holevo Capacity)

Optimize Shannon capacity over all possible measurements (select probability map)

Characterize Efficiency:

c_p = bits/photon (e.g., (bits/s)/Watt)
c_d = bits/dimension (e.g., (bits/s)/Hz)



$$g(x) = (1+x)\log_2(1+x) - x\log_2(x)$$

Noiseless

$$c_d^{\mathrm{Hol}}(n_b) = g(\bar{n}_s) \quad (\mathrm{bits/dim})$$

$$c_p^{\mathrm{Hol}} = c_d^{\mathrm{Hol}}\bar{n}_s \quad (\mathrm{bits/photon})$$

Thermal Noise (conjectured)

$$c_d^{\mathrm{Hol}}(n_b) = g(\bar{n}_s + \bar{n}_b) - g(\bar{n}_b) \quad (\mathrm{bits/dim})$$

$$c_p^{\mathrm{Hol}}(n_b) = c_d^{\mathrm{Hol}}(n_b)\bar{n}_s \quad (\mathrm{bits/photon})$$

Noisy Poisson OOK channel for thermal noise

- ***K* background noise modes (white, Gaussian), *N* counts/mode**

$$p_1(k;K) = \frac{N^k}{(1+N)^{k+K}} L_k^{(K-1)}\left(\frac{-n_s}{N(1+N)}\right) e^{-n_s/(1+N)}$$ *Negative binomial*

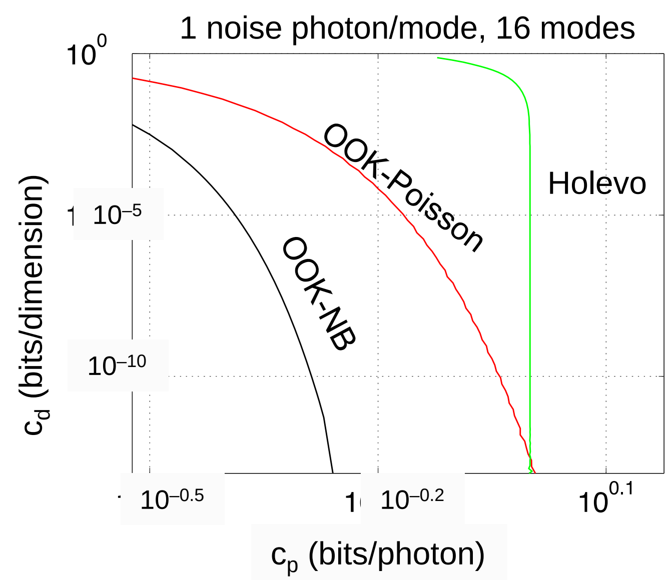
$$p_1(n;K) \xrightarrow[K\to\infty]{} \frac{(n_b+n_s)^n e^{-(n_b+n_s)}}{n!} \quad (n_b = KN)$$ *Poisson*

- Photon information efficiency of Poisson OOK channel is unbounded

$$c_p^{\mathrm{OOK}} \geq \left(\left(1+\frac{n_b}{n_s}\right)\log_2\left(1+\frac{n_s}{n_b}\right) - \frac{1}{\ln 2} - \frac{1}{n_s}\right) \xrightarrow[n_s\to\infty]{} \infty$$

- Holevo limit (conjectured) is bounded

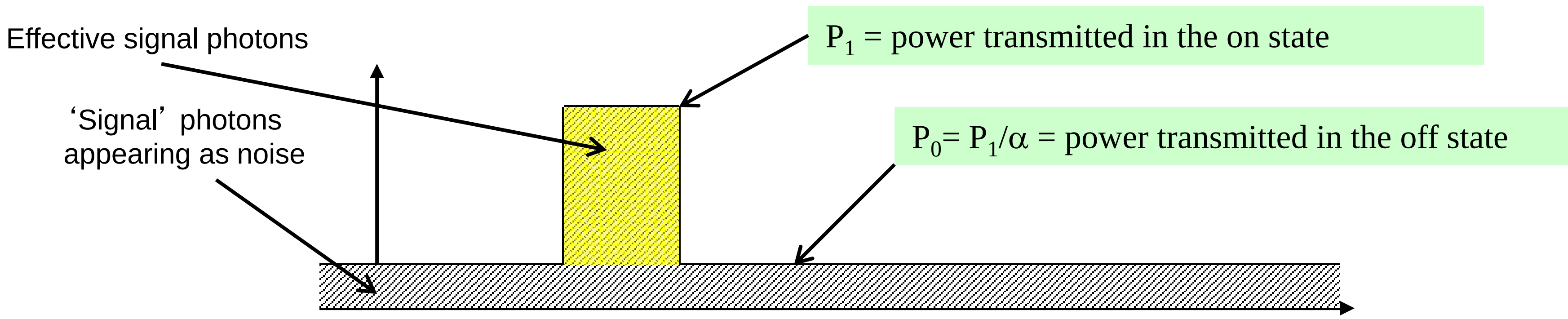
$$c_p^{\mathrm{Hol}}(n_b) \leq \log_2(1+1/n_b)$$



Poisson approximation to multimode thermal noise must become inaccurate at large c_p for any number of noise modes.

Noisy Poisson OOK channel for finite laser extinction ratio

- Non-ideal transmitters transmit some power in the "OFF" state:
  - *Power transmitted in the "OFF" state is **proportional** to power in the "ON" state; the proportionality constant is the extinction ratio α.*



- Finite transmitter extinction ratio generates a Poisson-distributed **background noise proportional to the signal**, $n_b = n_s/\alpha$.

- With finite extinction ratio α, the photon efficiency of OOK + photon counting is strictly bounded:

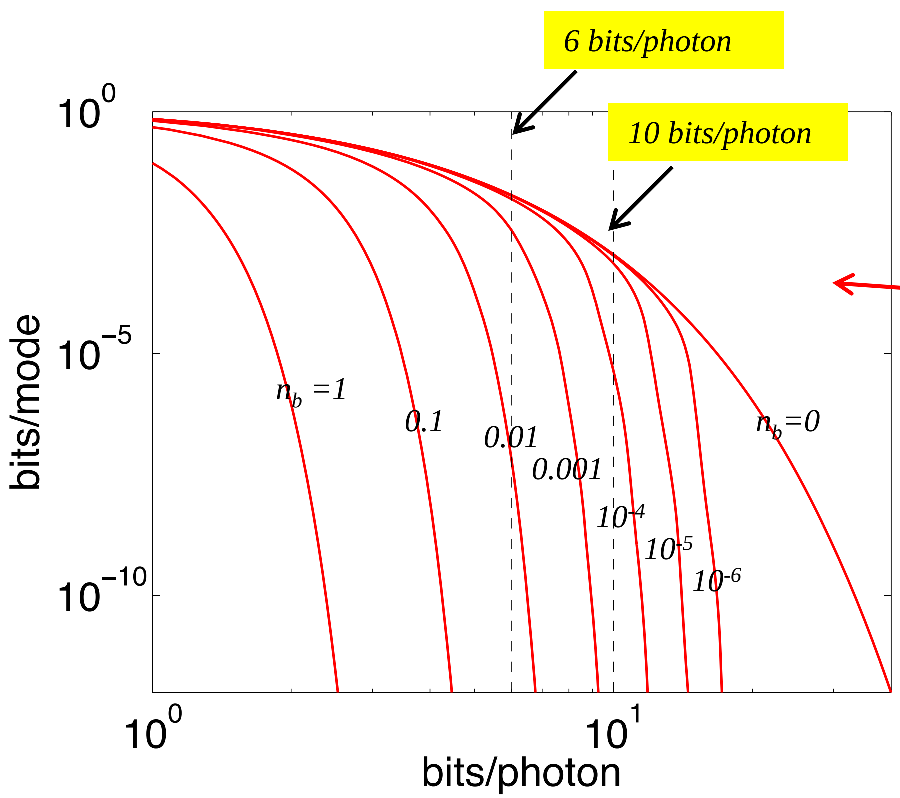
$$c_p \lessapprox \log_2(\alpha) - 1/\ln(2) \ \text{(bits/effective signal photon)}$$

Dark noise at the photodetector

- Photodetectors produce **dark noise**, which are spurious photo-electrons that are present even with no incident light.

$$n_b = l_d\, A\, T_s \quad \text{dark e/slot}$$

Dark rate — *Active area* — *Slot-width*



- Dark current generates a Poisson-distributed **signal-independent background noise** n_b.

Device	l_d (e/s/mm^2)
Si GM-APD	10^6
InGaAsP GM-APD	10^8
NbN SNSPD	10^2

- Noise levels with $n_b > 10^{-5}$ incur large losses at 10 bits/photon.

- Mitigation, by decreasing A and T_s, has limits
  - *A can only be decreased to the diffraction limit.*
  - *T_s can only be decreased to bandwidth limit, and we will show that decreasing T_s also exacerbates other losses.*

Noisy Poisson OOK channel for detector dark noise

- With nonzero dark rate n_b, the photon efficiency of OOK + photon counting is technically unbounded, but is ***effectively bounded***, because c_d drops off ***doubly-exponentially*** in a noisy Poisson channel.
  - c_d is approximately upper bounded by $c_d < \beta c_p 2^{-\beta c_p}$ where $\beta = \max(1, e n_b 2^{c_p})$
    - See the nearly vertical aqua curve, below its intersection with the noiseless Holevo bound (where $\beta > 1$).
  - This approximate bound crosses the noiseless OOK and Holevo curves at

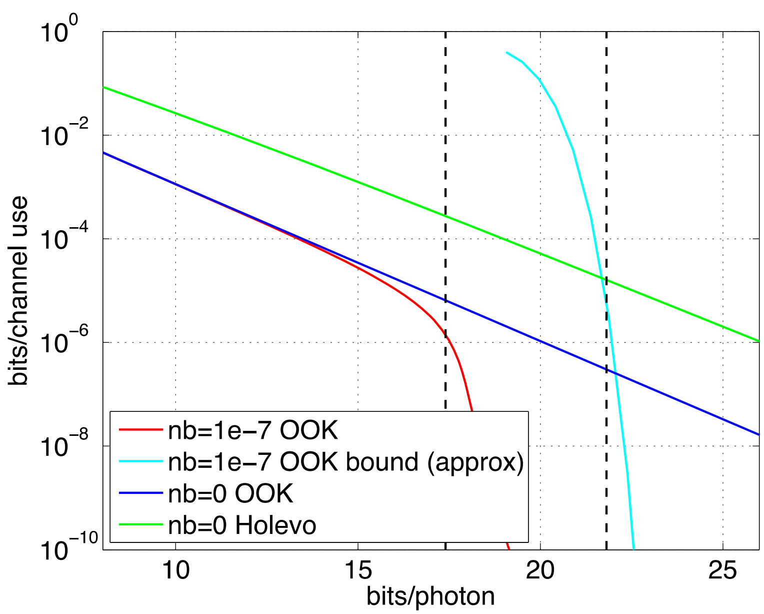
$$c_p \approx \log_2\left(\frac{1}{e n_b}\right)$$

  - The actual c_d breaks away sharply from the noiseless OOK curve starting at a lower value of c_p, estimated empirically to be:

$$c_p \approx \log_2\left(\frac{1}{e^4 n_b}\right)\Bigg|_{n_b=10^{-7}} = 17.4 \text{ bits/photon}$$

  - This breakaway point can also be interpreted as:

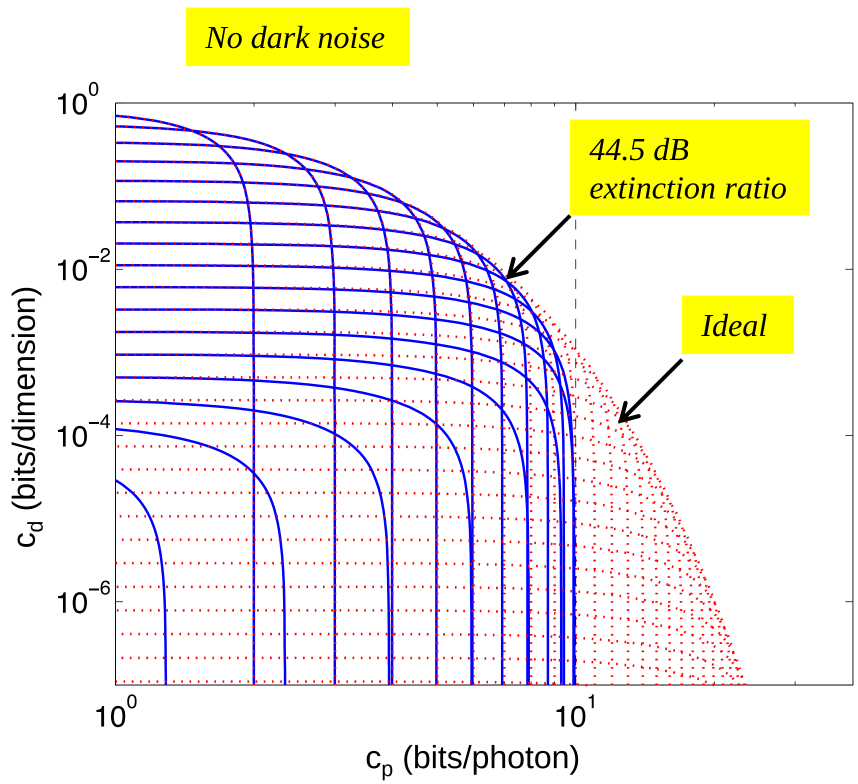
$$M n_b \approx 1/e^4 = 0.018 \text{ noise counts/PPM symbol}$$

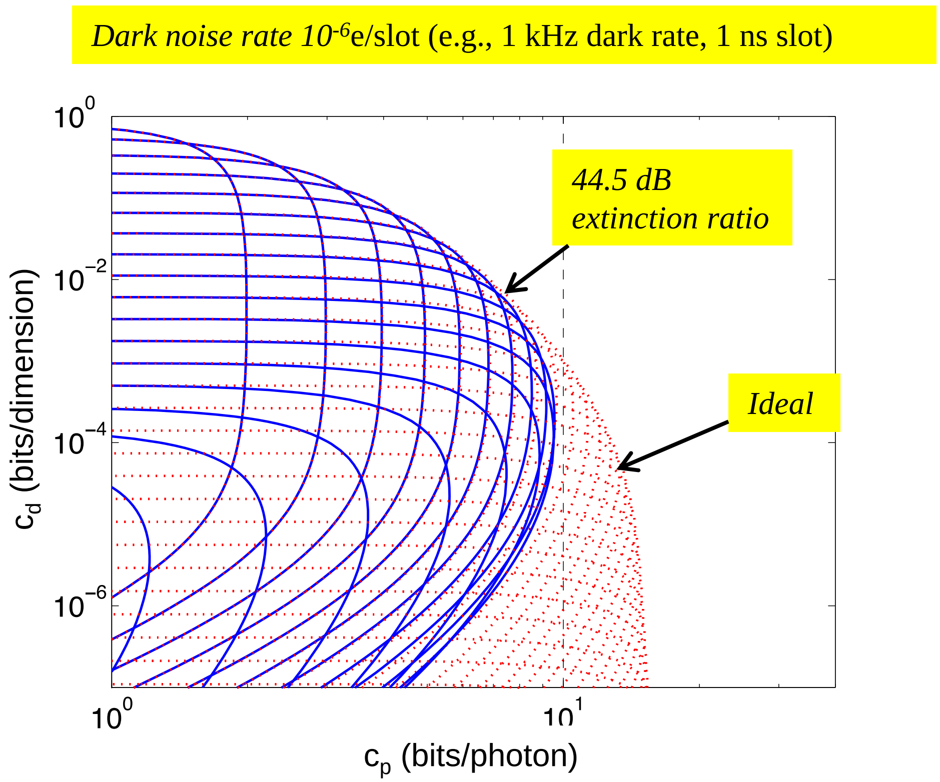


Thus, achieving arbitrarily high c_p on the noisy Poisson channel becomes impractical.

Capacity limits with dark noise & finite extinction ratio

- Each curve in these plots is the capacity efficiency tradeoff for a given PPM order *M*, and is generated by varying the average number of signal photons.



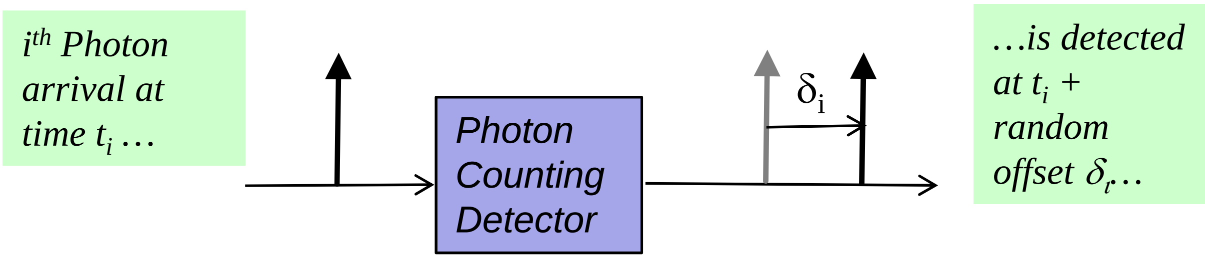


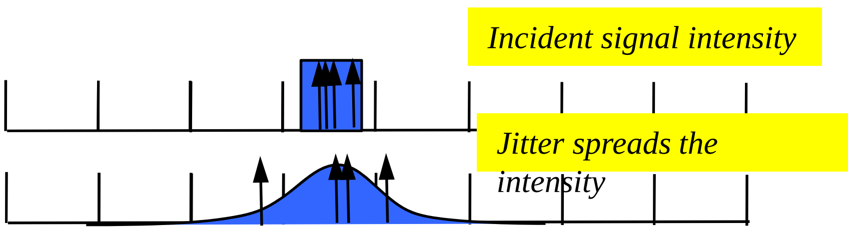
Other losses at the detector

In this section, we discuss:

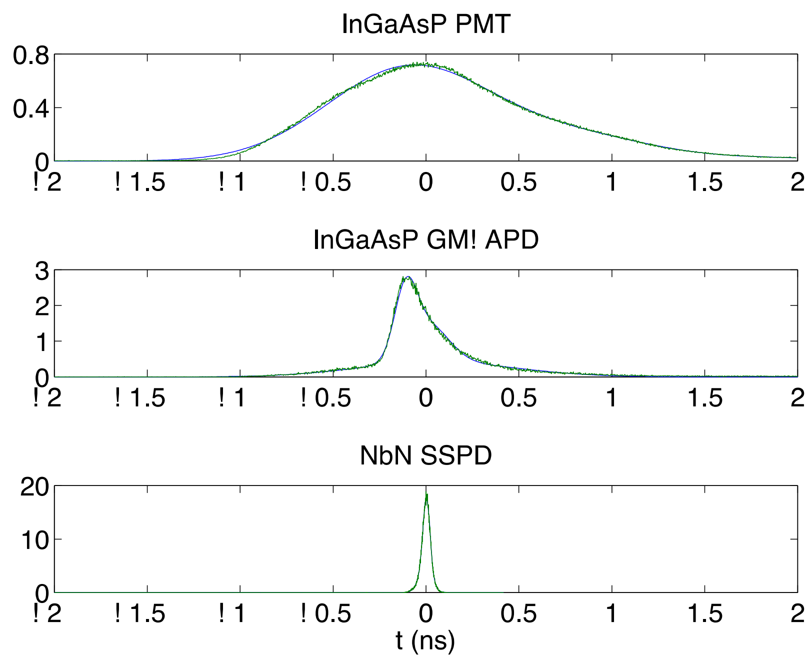
- **Detector jitter**
- **Photodetector blocking**
- **Overall system engineering**

Detector jitter





Measured hitter densities for some candidate detectors



- Jitter is the ***random delay*** from the time a photon is incident on a photo-detector to the time a photo-electron is detected.
- Jitter losses are a function of the ***normalized jitter standard deviation***:

$$\frac{\sigma_j}{T_s}$$

jitter standard deviation

Slot-width

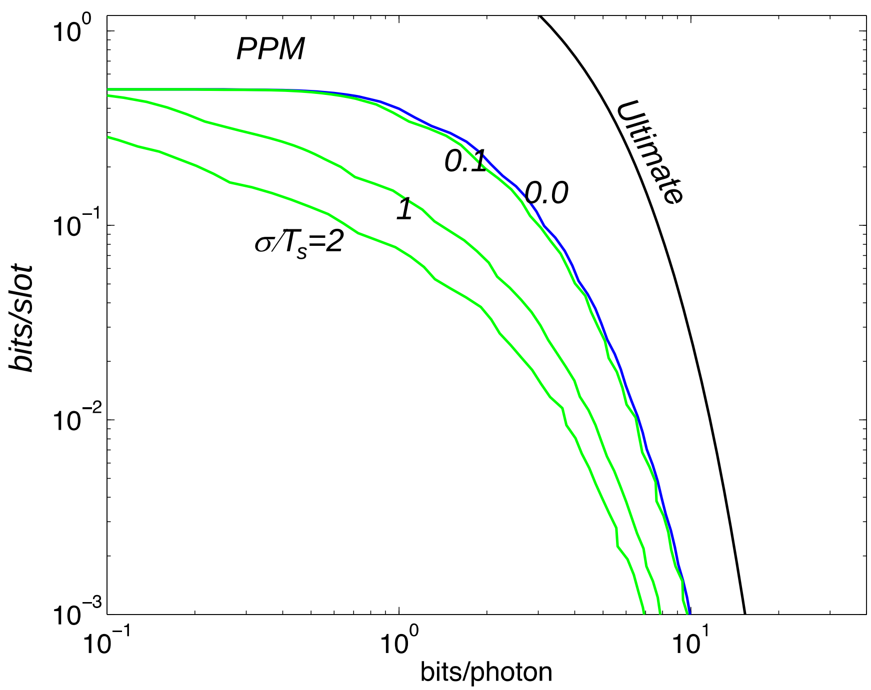
- Thus, jitter limits our ability to decrease the slot width T_s without incurring loss.

Losses due to detector jitter

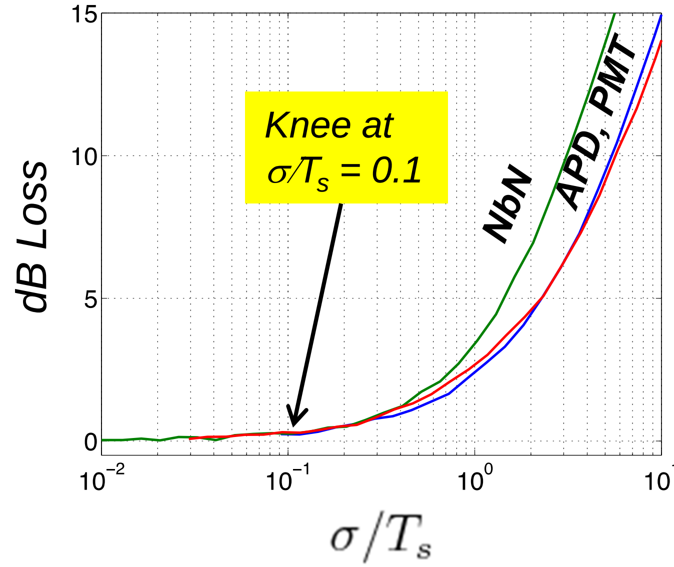
$$P_{rqd} = P_i / L_b L_j L_f L_t \eta_{det} \eta_{imp} \eta_{code} \eta_{int}$$

- *Significant losses for $\sigma/T_s > 0.1$*
- *Effectively enforces a lower bound on T_s*
  - *Limits data rate*
  - *Limits ability to mitigate dark noise*

Jitter Losses, Gaussian distributed



M=16, n_b=1



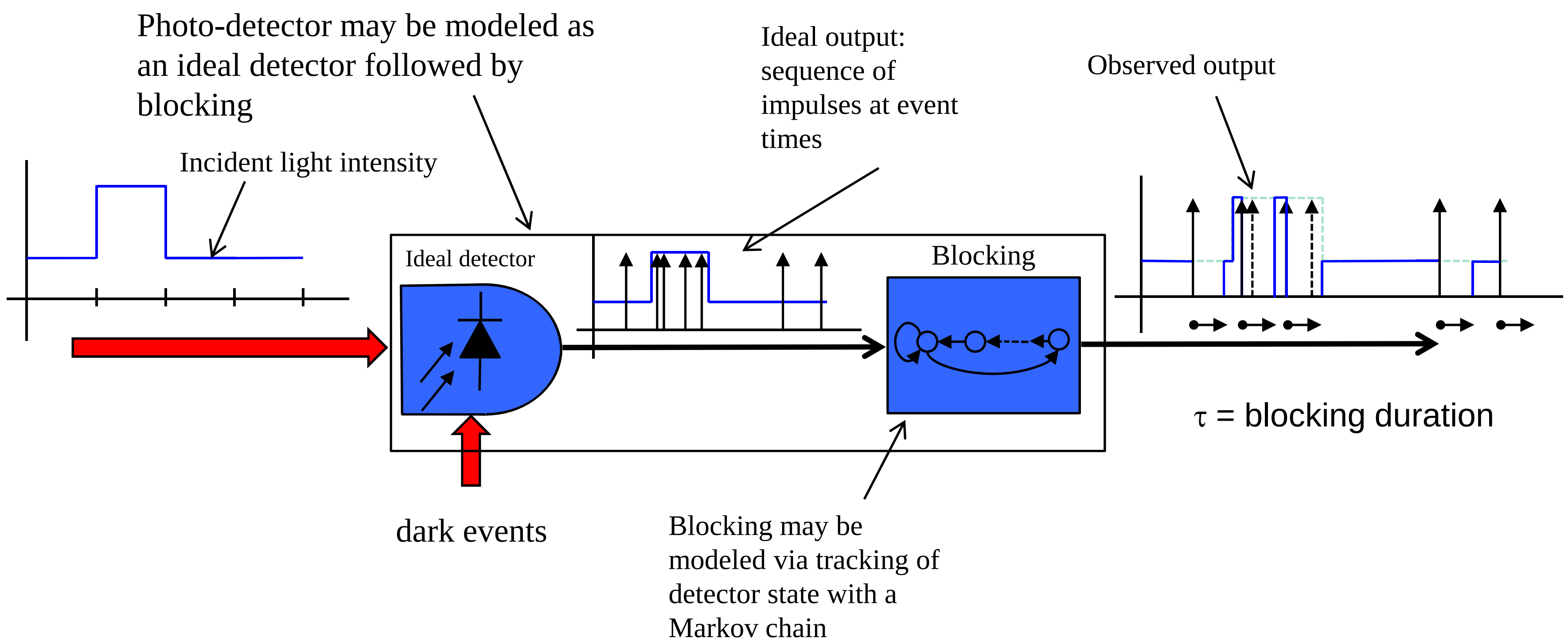
Device	σ / ns
InGaAs(P) PMT	0.9
InGaAs(P) GM-APD	0.3
Si GM-APD	0.24
NbN SNSPD	0.03

Photodetector blocking

$$P_{rqd} = P_i / {\color{red}L_b} L_j L_f L_t \eta_{det} \eta_{imp} \eta_{code} \eta_{int}$$

- *Certain photon-counting photodetectors are rendered inoperative (**blocked**) for some time τ (**dead time**) after each detection event*
  - *10—50 ns, Si GM-APD*
  - *1—10 μs, InGaAs GM-APD*
  - *3—20 ns, NbN SNSPD*

Characterize impact of blocking by
μ = probability detector is unblocked



Mitigating blocking

μ = probability detector is unblocked

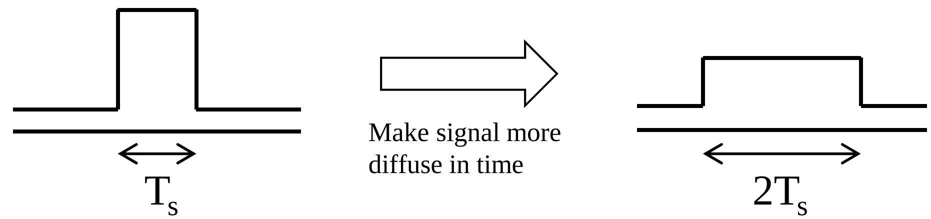
$$\mu \approx \frac{1}{1+\tau l}$$

dead-time: fixed by the device

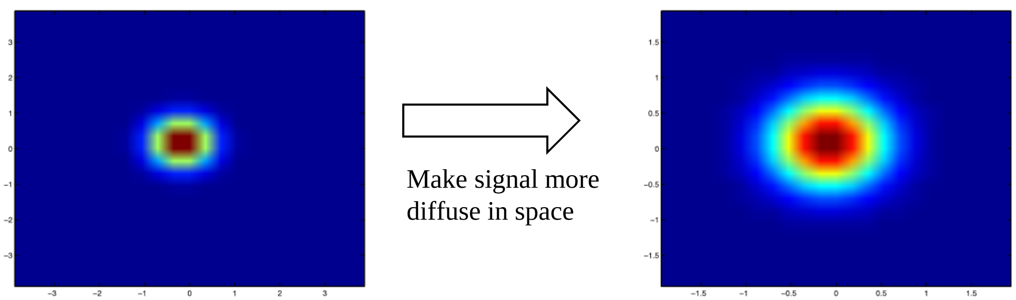
photon rate: decreased by temporal or spatial diffusion

Blocking may be mitigated by decreasing the peak incident photon rate (per detector)

- Temporally
  - *Increase the slot-width and reduce the photon rate, while preserving the photons/slot.*
  - *Reduces impact of blocking, but lowers the date rate (bits/s), and integrates more noise*



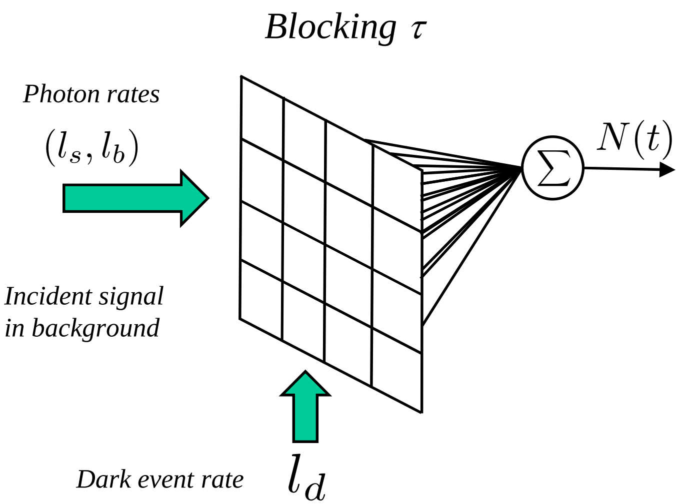
- Spatially
  - *Increase Focal length, to decrease signal intensity in the focal (detector) plane*
  - *Integrates more noise*

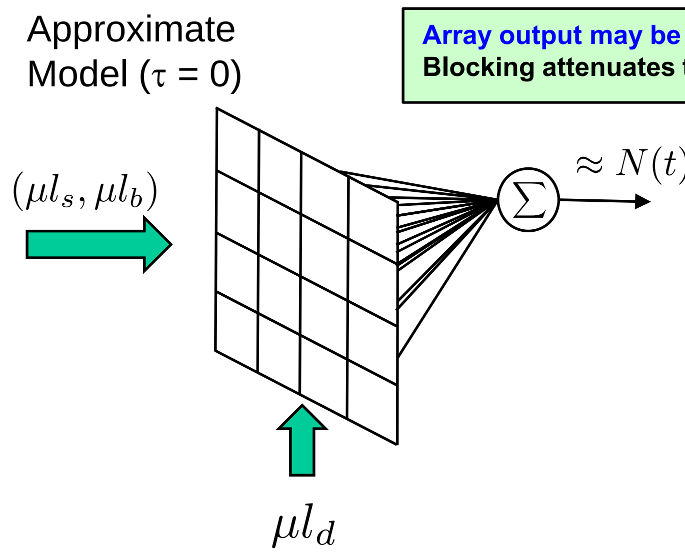


F/D=8 F/D=16

We investigated this approach, numerically determining the optimum F number for a given blocking, dark noise, and target data rate (which fixes the aggregate required signal flux)

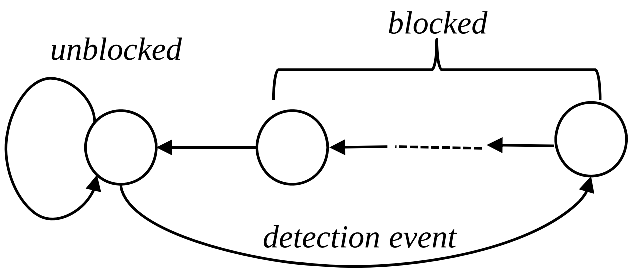
Modeling blocking loss with arrayed detectors





Array output may be approximated as Poisson. Blocking attenuates the signal and noise.

Markov Model of Detector State



μ = probability detector is unblocked

$$\mu \approx \frac{1}{1+\tau l}$$

l=incident photon rate

Signal Power Loss: increase in power to achieve fixed capacity

$$C_b(l'_s) = C_u(l_s)$$

$$\frac{l_s}{l'_s} \approx \begin{cases} \mu & \text{high SNR} \\ \sqrt{\mu} & \text{low SNR} \end{cases}$$

Capacity Loss: decrease in capacity at fixed signal power

blocked capacity → *unblocked capacity* →

$$\frac{C_b}{C_u} = \mu$$

Overall system engineering considerations

- *Mitigation of impairments results in conflicting demands on resources, hence requiring system engineering to optimize.*

Parameter	Blocking	Jitter	Dark Noise
F/D	green		red
T_s	green	green	red
M		green	red

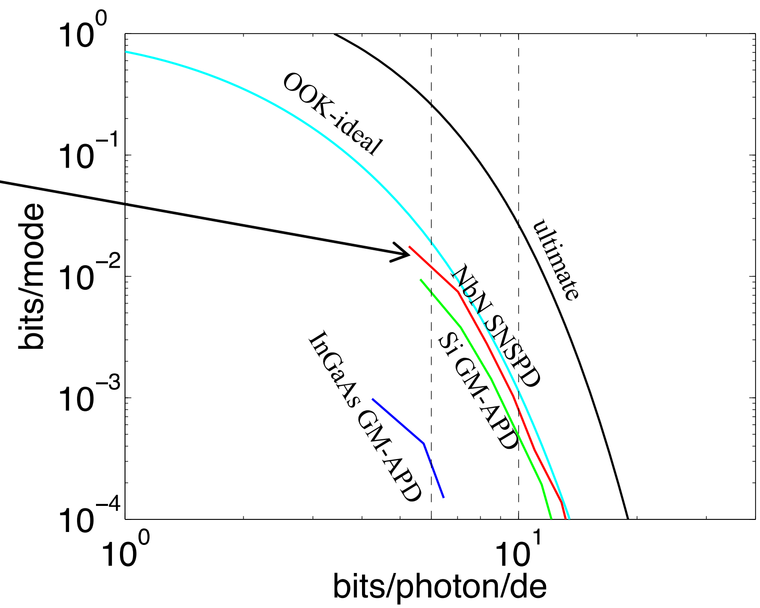
red: Mitigated by decreasing this parameter

green: Mitigated by increasing this parameter

Device requirements for high bits/photon operation

Non-ideality	Requirement	NbN SNSPD, T_s=1 ns
Extinction Ratio	α > 60 dB	(presumed infinite for graph)
Jitter	$\sigma/T_s<0.1$	0.03
Dark Noise	$l_d T_s<10^{-5}$	10^{-9}
Blocking	$l_p\ \tau << 1.0$	10^{-4}

Losses due to dark noise, blocking and jitter. Optimized over F-number and duty cycle. T_s=1 ns, no background, 1-m aperture.



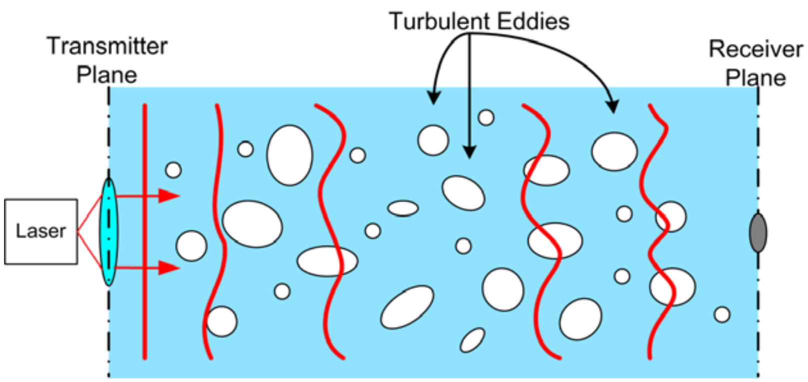
Atmospheric effects on optical communication

In this section, we discuss:

- The effects of:
  - Background radiation
  - Absorption/scattering
  - Clear sky turbulence effects
  - Pointing errors
- Fading channel models
- Mitigating the effects of fading

Atmospheric effects on optical communication

- **Background Radiation**
- **Absorption/Scattering**
- **Clear Sky Turbulence Effects**
  - **Scintillation**
  - **Angle-of-Arrival Variations**
  - **Beam Spread**
  - **Beam Wander**



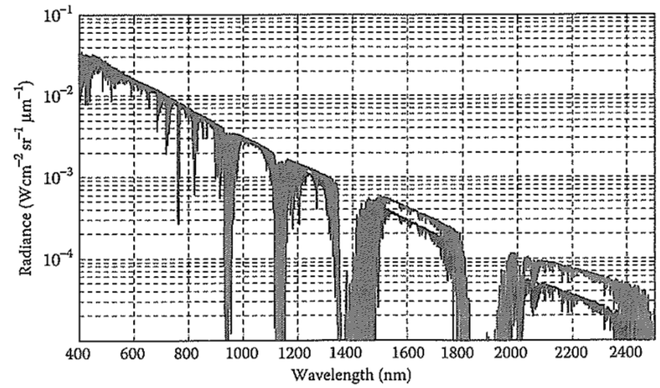


FIGURE 8.16 Daytime sky radiance at 2 km above sea level. The Sun zenith angle is 45°. Two cases (radiance curves) are shown: (1) the observer zenith angle on the ground is at 40° (higher radiance curve) and (2) the observer zenith angle on the ground is at 70° (lower radiance curve). The rural aerosol model with a visibility of 23 km at sea level was used. Data obtained after MODTRAN simulation.

[Piazzolla, '09]

264 Near-Earth Laser Communications

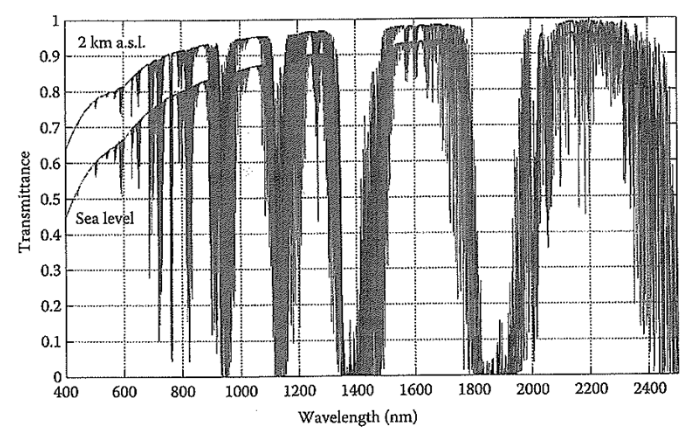


FIGURE 8.12 Atmospheric transmittance in an Earth-to-space path at zenith. A rural aerosol composition with a surface visual range of 23 km is considered. The data refers to the case of an observer located at two elevations: sea level (lower transmittance) and 2 km above sea level.

Background Scattered Light

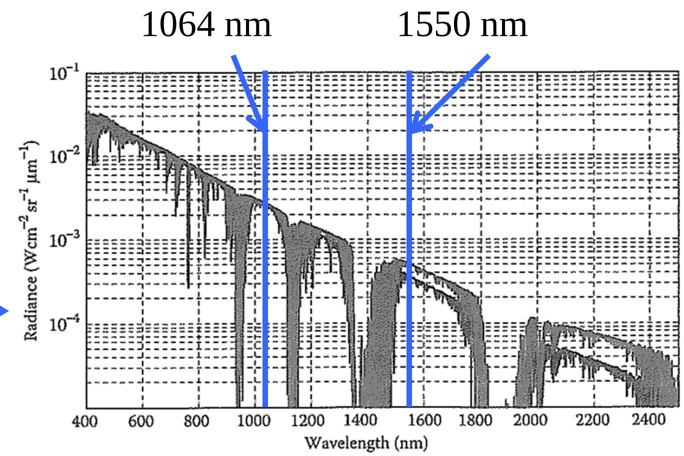
$$P_{rqd} = P_i / L_b L_j L_f L_t \eta_{det} \eta_{imp} \eta_{code} \eta_{int}$$

- **Aperture open to atmosphere also collects background light (scattered sunlight, light from point sources)**
- **Background light degrades performance**

Sky Radiance

$$P_n = B_\lambda \times A_R \times \Omega \times \Delta\lambda \times \eta_R$$

Aperture | Field-of-View | Bandwidth | Efficiency



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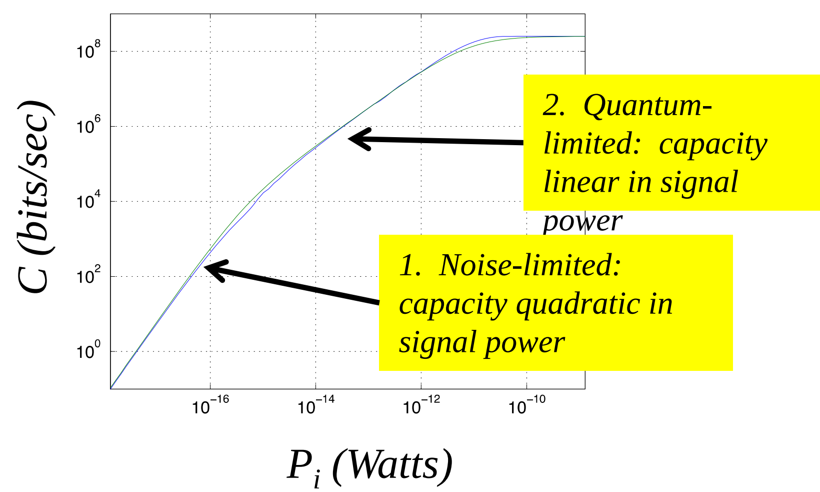
[Piazzolla, '09]

- **Impact of noise depends on the signal to noise ratio, and modulation**
- **Must be taken into account for choice of wavelength**
- **At large background noise, coherent detection becomes favorable**

$$C \approx \frac{1}{\ln(2) E_\lambda} \left(\frac{P_i^2}{P_i \frac{1}{\ln(M)} + P_n \frac{2}{M-1} + P_i^2 \frac{M T_s}{\ln(M) E_\lambda}} \right) \text{bits/sec}$$



Absorption/Scattering

$$P_{rx} = P_t G_t G_r L_s L_a \eta_{pt} \eta_t \eta_r$$

- **Absorption and Scattering from aerosols (dust, etc.) and molecules (water vapor, etc.) attenuate the signal**
- **In bad weather (rain, snow, fog), attenuation can be severe, causing dropouts**
- **In Clear Sky, must budget for attenuation**
- **Drives selection of bands with good clear sky transmissivity**
  - **Candidates for Earth-Space link: 1064, 1550 nm**
- **Typical attenuation for Space-Earth link in near-infrared at zenith 0.1—0.3 dB**
- **Outages at low elevation angles**

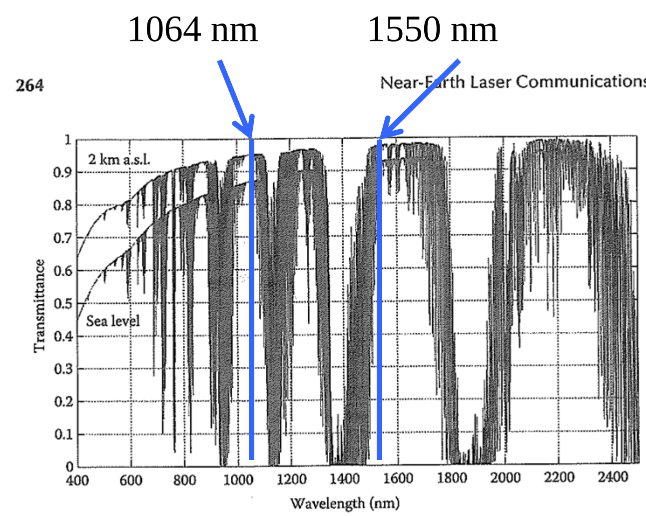
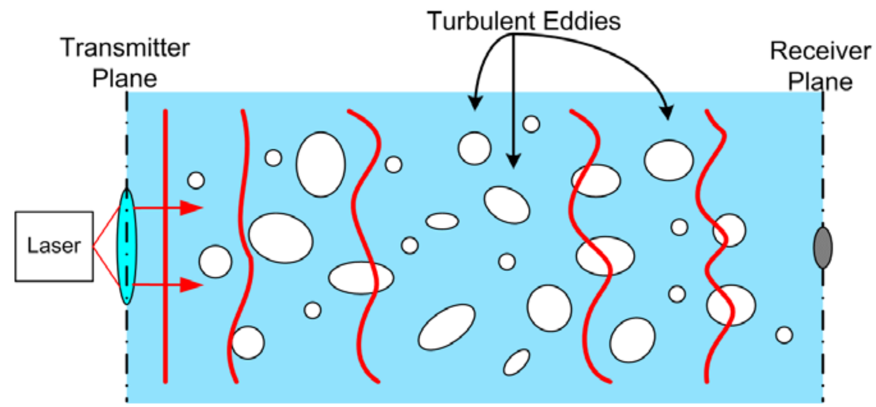


FIGURE 8.12 Atmospheric transmittance in an Earth-to-space path at zenith. A rural aerosol composition with a surface visual range of 23 km is considered. The data refers to the case of an observer located at two elevations: sea level (lower transmittance) and 2 km above sea level.

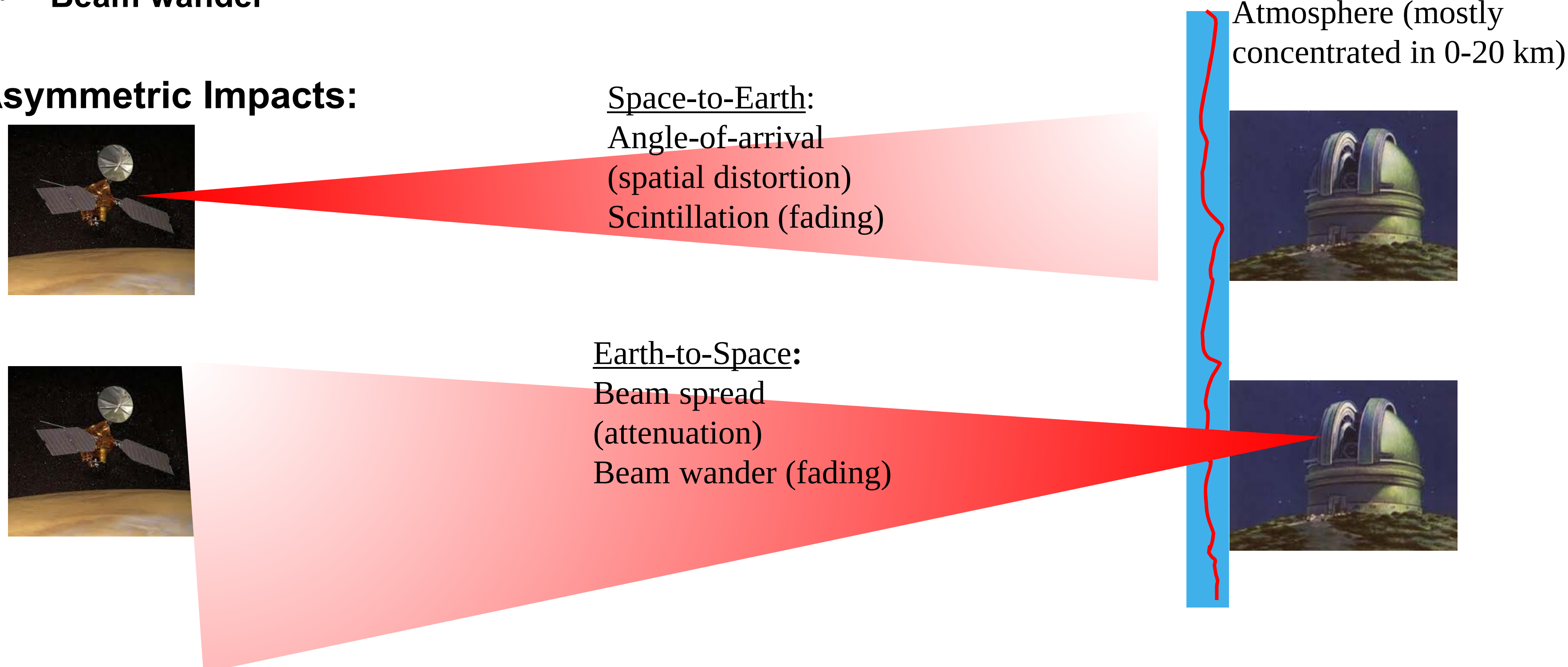
[Piazzolla, '09]

Clear Sky Turbulence

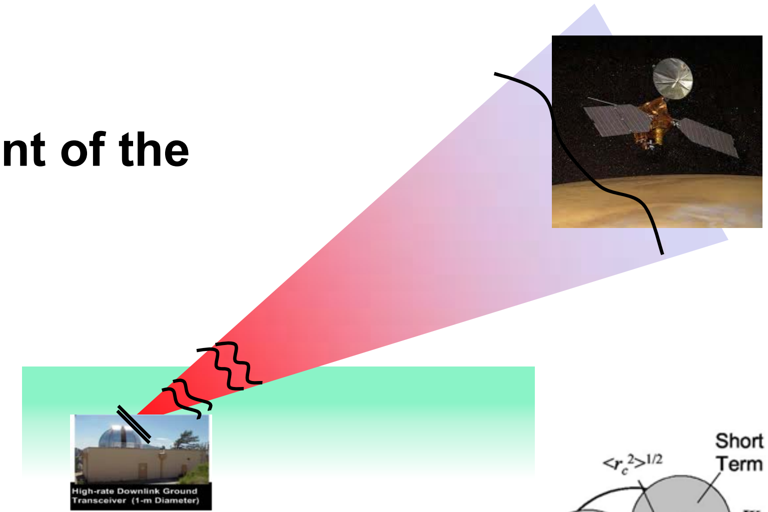
- **Random spatio-temporal mixing of air with different temperatures causes refractive-index variations**
  - **Scintillation (constructive/destructive interference)**
  - **Angle-of-arrival variations**
  - **Beam spreading**
  - **Beam wander**



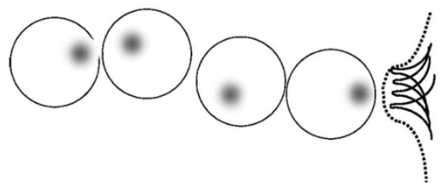
- **Asymmetric Impacts:**

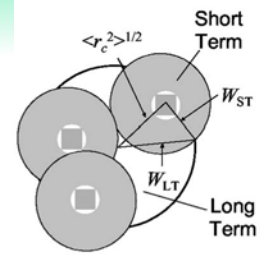


Beam Wander (Scintillation) & Beam Spread (Attenuation)



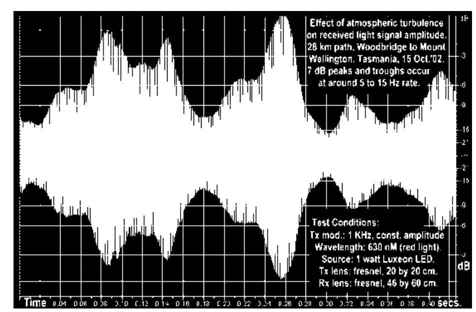
- **Turbulence is a thin phase-screen in front of the transmitter aperture**
- **Coherence length is up to meters**
  - **Receiver always sees *plane wave***
  - **Focused beam is diffraction-limited**
  - **Diffraction-limited spot moves in focal plane**





Andrews & Phillips, Opt. Eng. (2006)

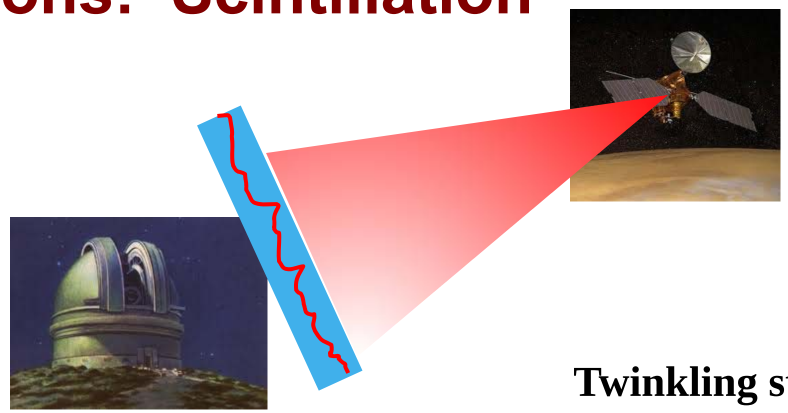
- **Beam-spread (attenuation)**
  - **Linear phase at transmitter *tilts* the beam**
  - **Higher-order phase *spreads* the beam (short-exposure < 1 msec)**
- **Beam-Wander → Scintillation**
  - **Irradiance fluctuates with log-normal distribution**
  - **Multiple transmit beams used to reduce scintillation**



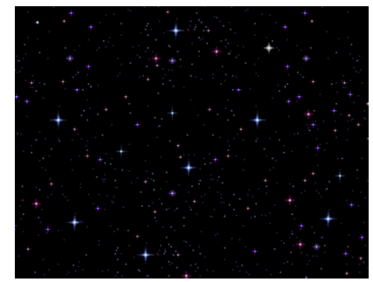
http://www.modulatedlight.org

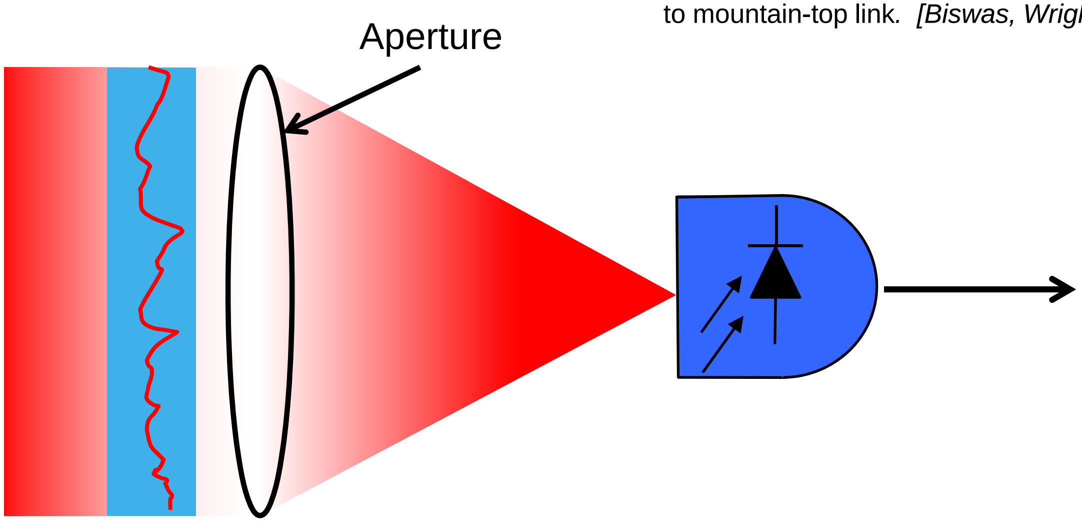
Temporal Distortions: Scintillation

- Random refractive index fluctuations also lead to phase distortions—constructive and destructive interference.
- Leads to Scintillation, random power fluctuations
- Each “coherence cell” has independent amplitude
  - Aperture averaging: averaging over multiple coherence cells reduces the fluctuation in power (law of large numbers)

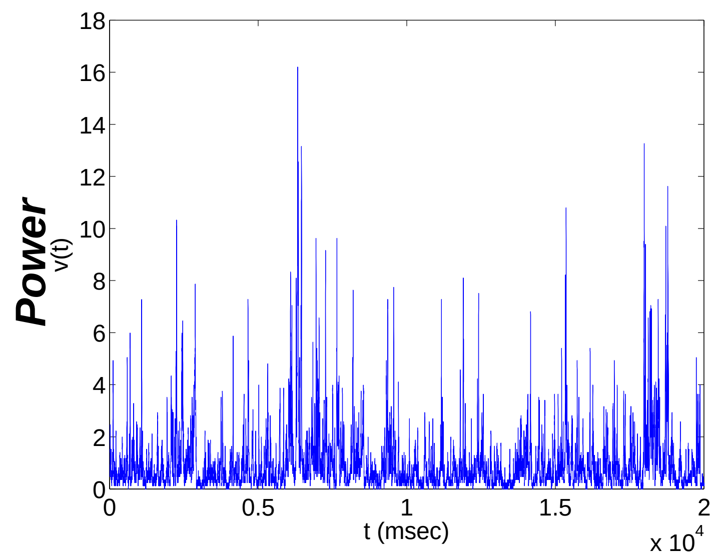


Twinkling stars

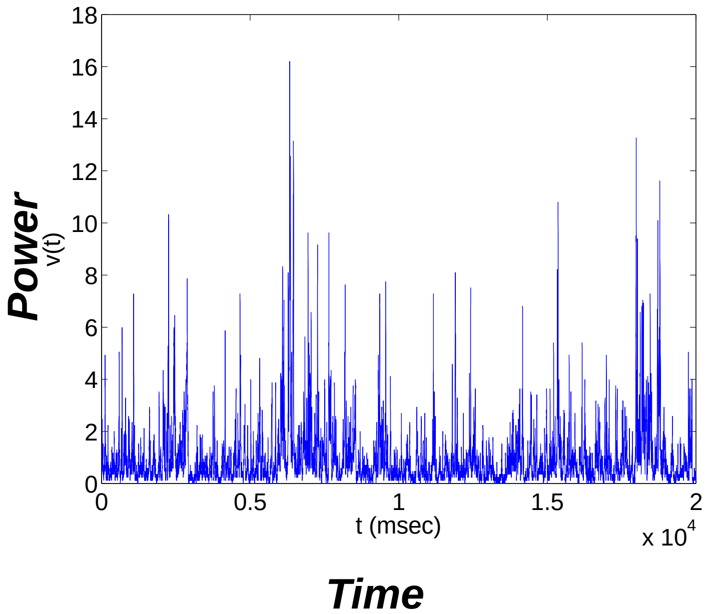


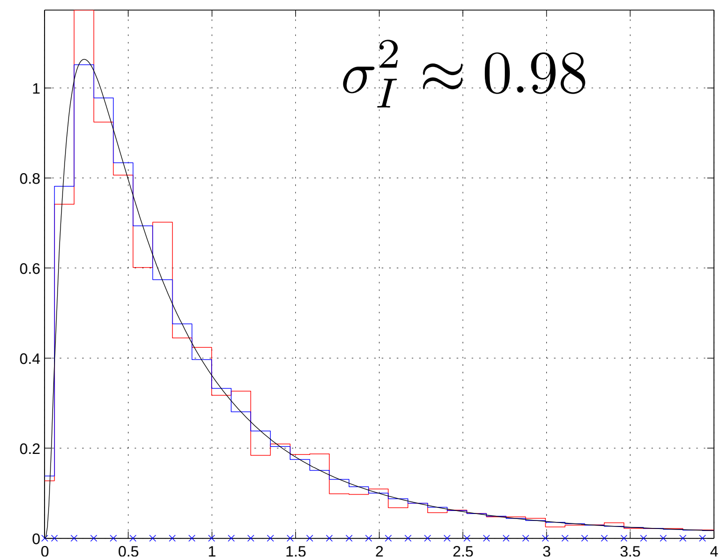


Measured fluctuations over a 45-km mountain-top to mountain-top link. *[Biswas, Wright, ‘02]*



Modeling scintillation: scintillation index



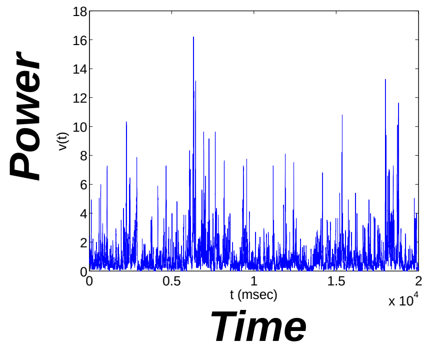


Random instantaneous power fluctuation in weak turbulence is well-modeled as log-normally distributed

σ_I^2 = scintillation index

$$f_V(v) = \frac{1}{\sqrt{2\pi\sigma_l^2}}\frac{1}{v}\exp\left(\frac{-(\log v + \sigma_l^2/2)^2}{2\sigma_l^2}\right)$$

Modeling scintillation: coherence time

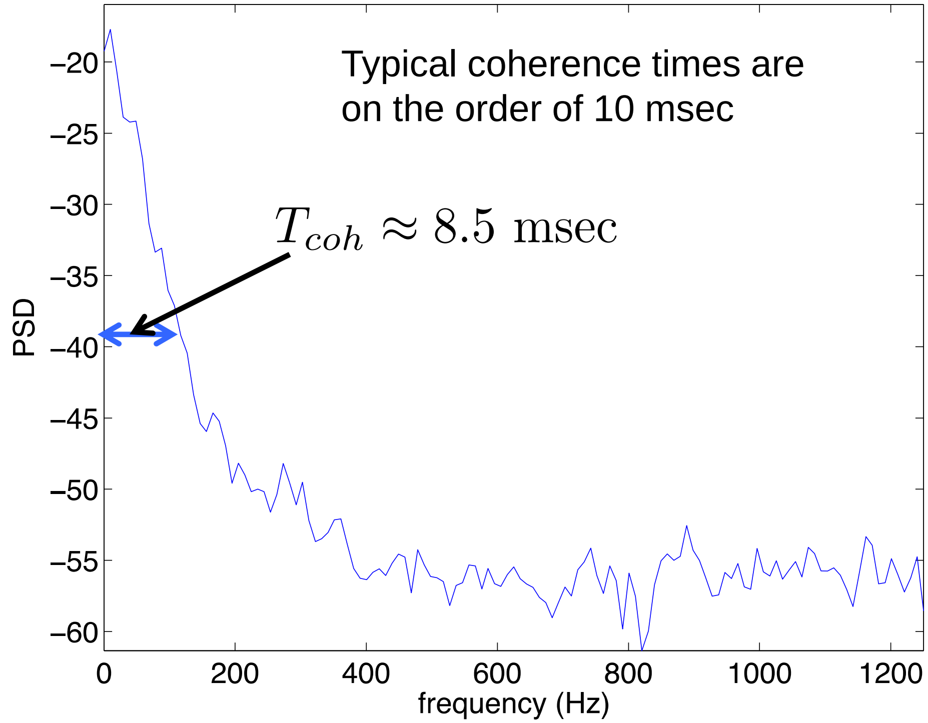


The power is highly correlated over short time intervals. The coherence time is the minimum duration over which two samples are (approximately) uncorrelated.

Coherence time goes as 1/band-width.   90% bandwidth is commonly used.

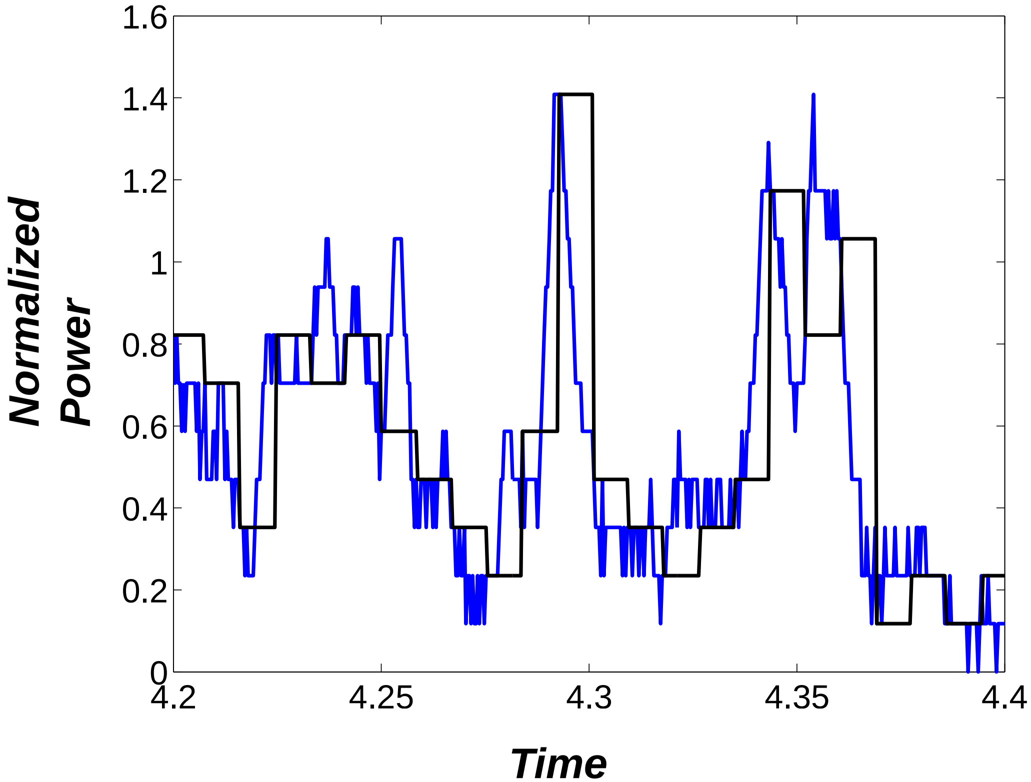
$$W(\xi) = \min\left\{2B \middle| \int_{-B}^{B} S_x(f)df = \xi \int_{-\infty}^{\infty} S_x(f)df\right\}$$

$$T_{coh} = \frac{1}{W(0.90)}$$



T_{coh} = coherence time

Block fading model



Reduce fading to a two-parameter model:

Model fades as drawn independently from a log-normal distribution every T_{coh} seconds, and constant over those intervals.

$$\{\sigma_I^2, T_{coh}\}$$

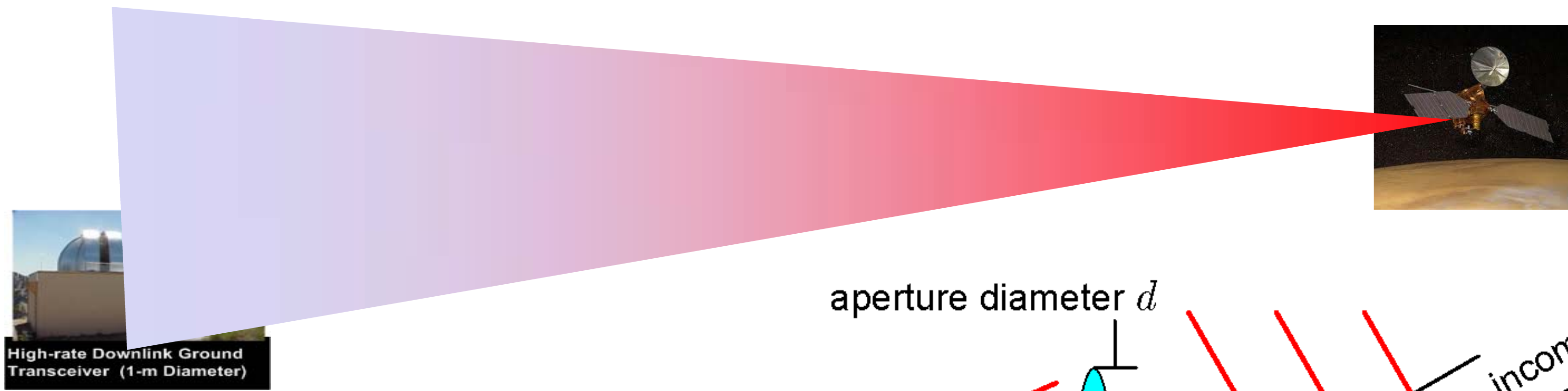
Scintillation Index

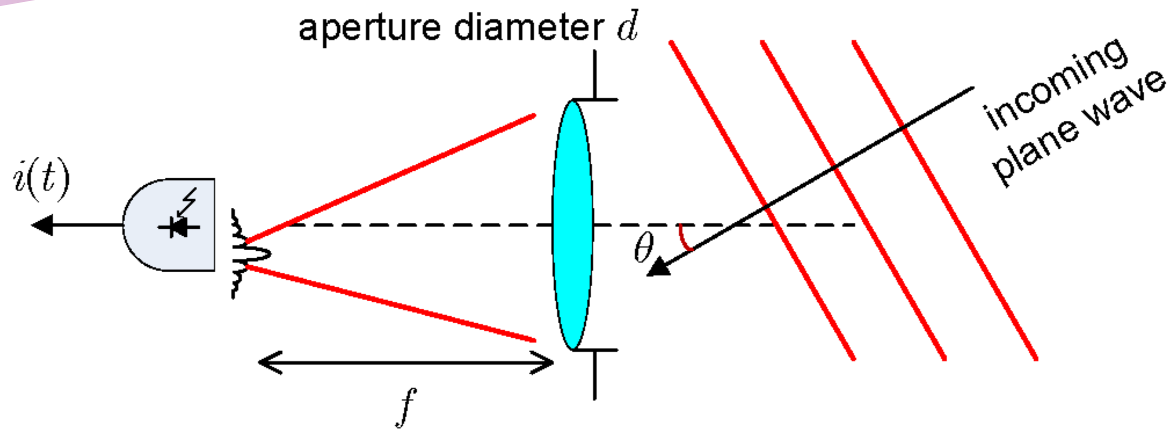
Coherence *Time*

$$T_{coh} \approx 8.5 \text{ msec}$$

$$\sigma_l^2 \approx 0.98$$

Fading due to pointing errors





[Barron, Boroson, '06]

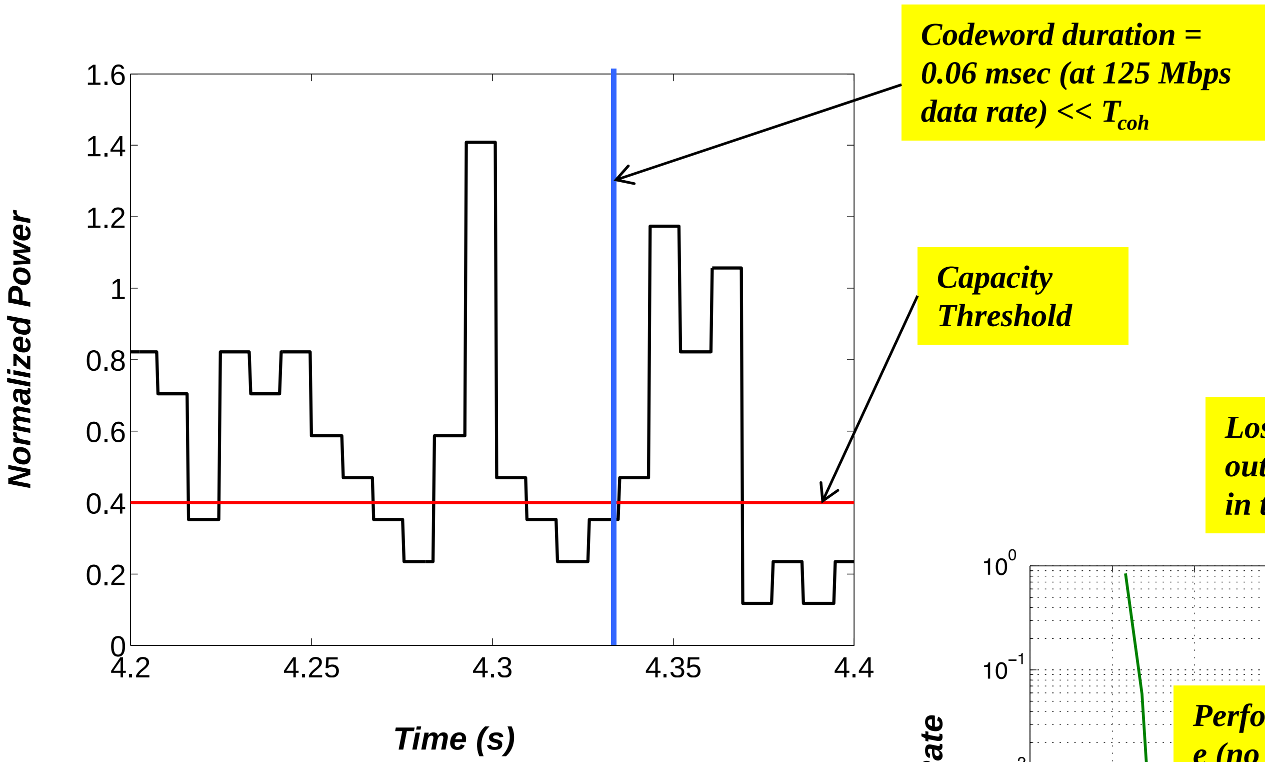
$$L_{\text{static}} \approx 10\log_{10}(1+\sigma^2) + 5\log_{10}(e)\frac{m^2}{1+\sigma^2}\ \text{dB}$$

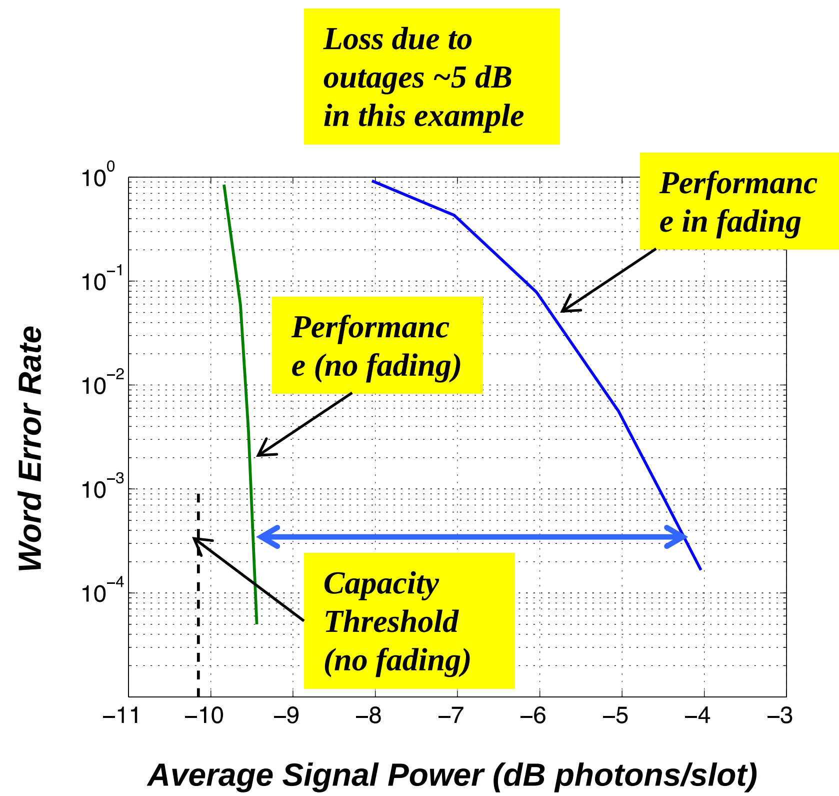
$$L_f \approx 5\log_{10}(e)(2\sigma^2 + m^2) - L_{\text{static}}\ \text{dB}$$

- m=mean pointing error
- σ =pointing error standard deviation
- Gaussian beam, Gaussian pointing erorrs

Losses due to dynamics similarly are linear in the variance

Impact of fading on coded performance: outages

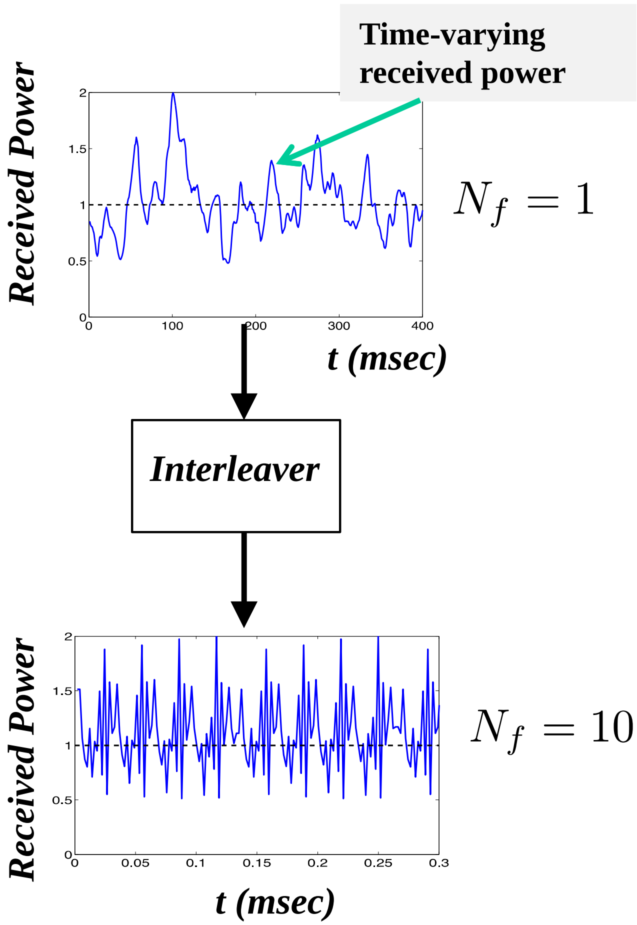


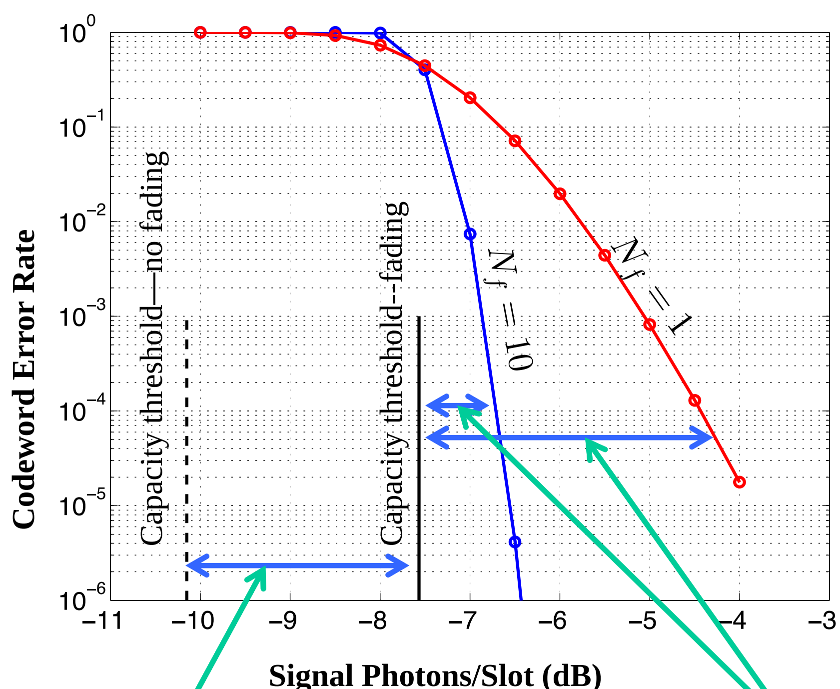


Capacity losses due to signal fading

$$P_{rqd} = P_i / L_b L_j L_f L_t \eta_{det} \eta_{imp} \eta_{code} \eta_{int}$$

- N_f = number of uncorrelated fades per codeword
- σ_I^2 = *scintillation index (variance of normal in log-normal fading)*





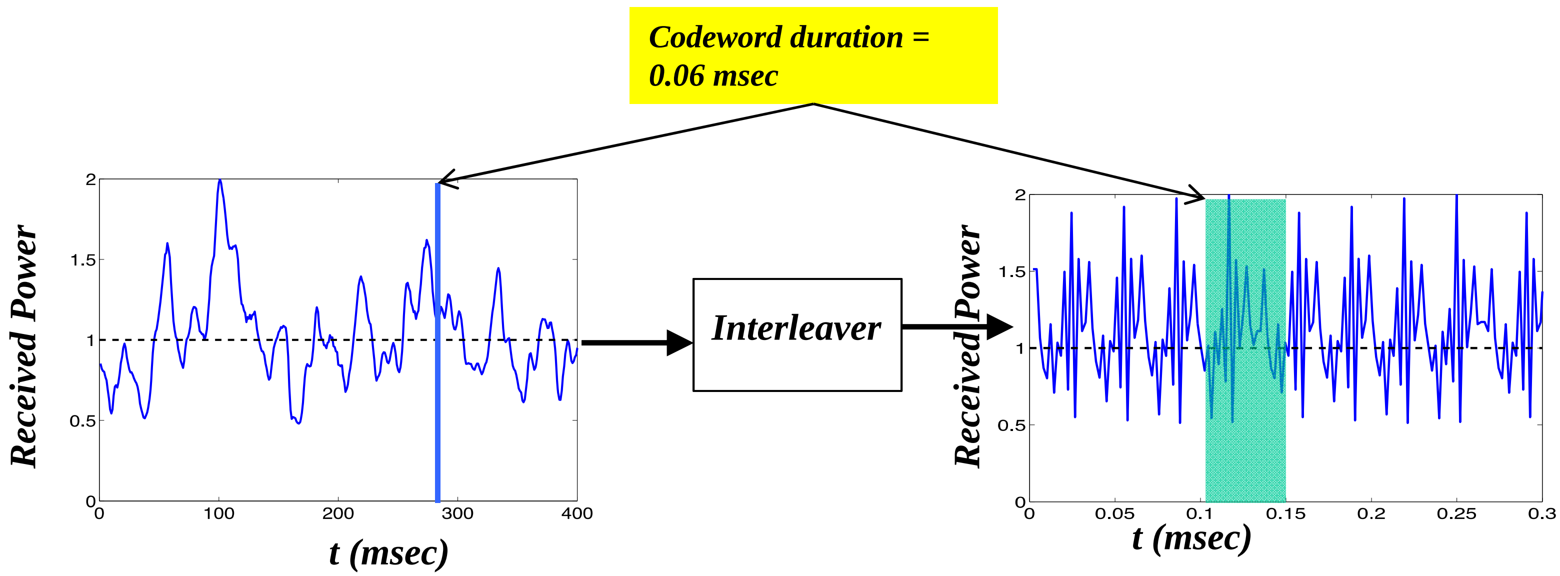
Fading capacity loss (unrecoverable)

$$L_f \approx 2.5\sigma_I^2 \text{ dB}$$

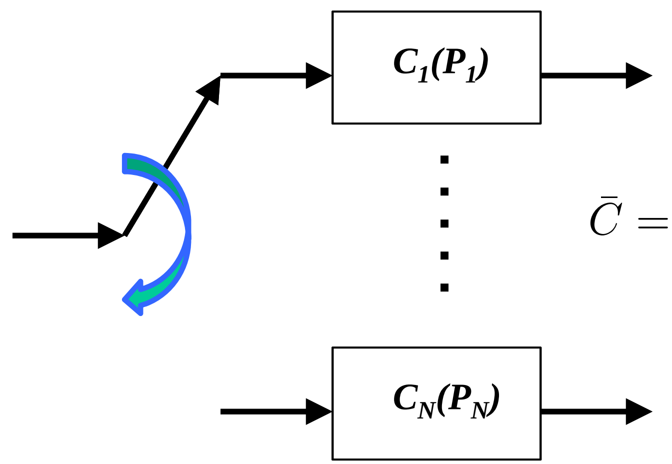
Interleaving Efficiency (mitigated with interleaving)

$$\eta_{int} \approx 16\sqrt{\frac{\sigma_I^2}{N_f}} \text{ dB}$$

Mitigating fading outages with interleaving



- Each codeword now sees N uncorrelated fades, or Powers
- Effectively transmitting over N parallel channels, each with a different power
- Relevant capacity is the *instantaneous capacity,* averaged over the N powers



$$\bar{C} = \frac{1}{N}\sum_{i=1}^{N} C(v_i P)$$

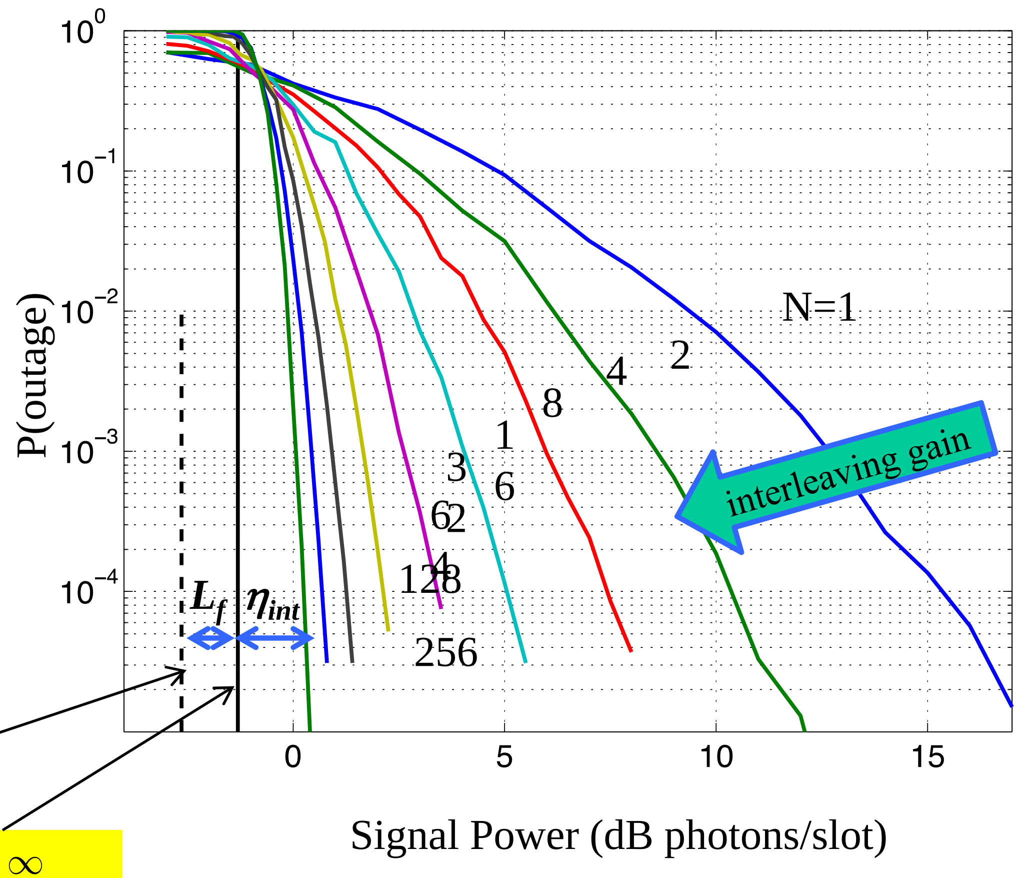
Interleaving gain and fading capacity

Instantaneous capacity is a random variable

$$P(\text{outage}) = P\left(R > \bar{C} = \frac{1}{N}\sum_{i=1}^{N} C(v_i P)\right)$$

$$\bar{C} \xrightarrow[N\to\infty]{} C_f = \int C(vP) f_V(v)\,dv$$

Fading Capacity: Fundamental limit on performance in fading.



Capacity Threshold (no fading)

$N \to \infty$ ***Fading Capacity Threshold***

Fading capacity does not approach the capacity in the absence of fading. There is a loss due to fading dynamics, even with the same average received power.

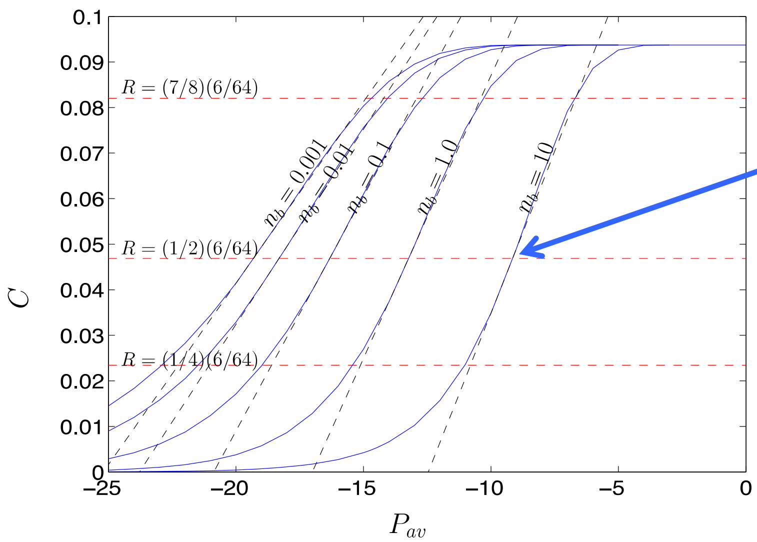
The fading loss—the gap to the fading capacity, is nonrecoverable

Divide loss into two terms:
1. η_{int} = finite interleaver loss (recoverable)
2. L_f = Fading Capacity Loss (not recoverable)

Analytic approximation of fading capacity loss

Capacity function is, in general, not known in closed form

$$C_f = \int C(vP) f_V(v) dv$$



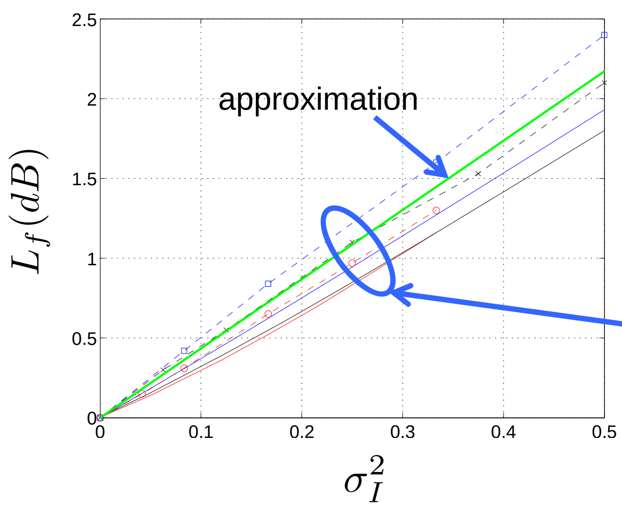
Linear approximation

$$C(P) \approx a \log(P) + \gamma$$

$$C_f(P) \approx C(P) - \frac{a}{2}\sigma_I^2$$

$$L_f \approx R_{\mathrm{ECC}} \sigma_I^2 \frac{10}{\ln(10)} \text{ dB}$$

dB Fading Capacity loss is linear in the scintillation index



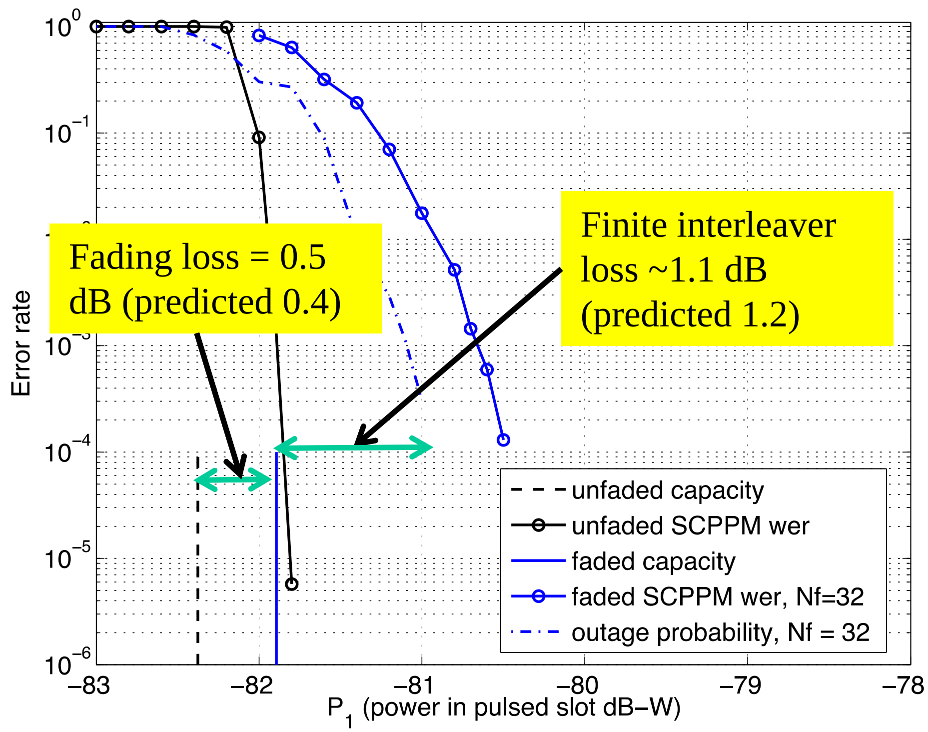
Numerically evaluated loss

Analytic approximation of finite interleaver loss

$$P(\text{outage}) = P\left(R > \bar{C} = \frac{1}{N}\sum_{i=1}^{N} C(v_i P)\right)$$

$$\underbrace{\bar{C} = \frac{1}{N}\sum_{i=1}^{N} C(v_i P)}$$

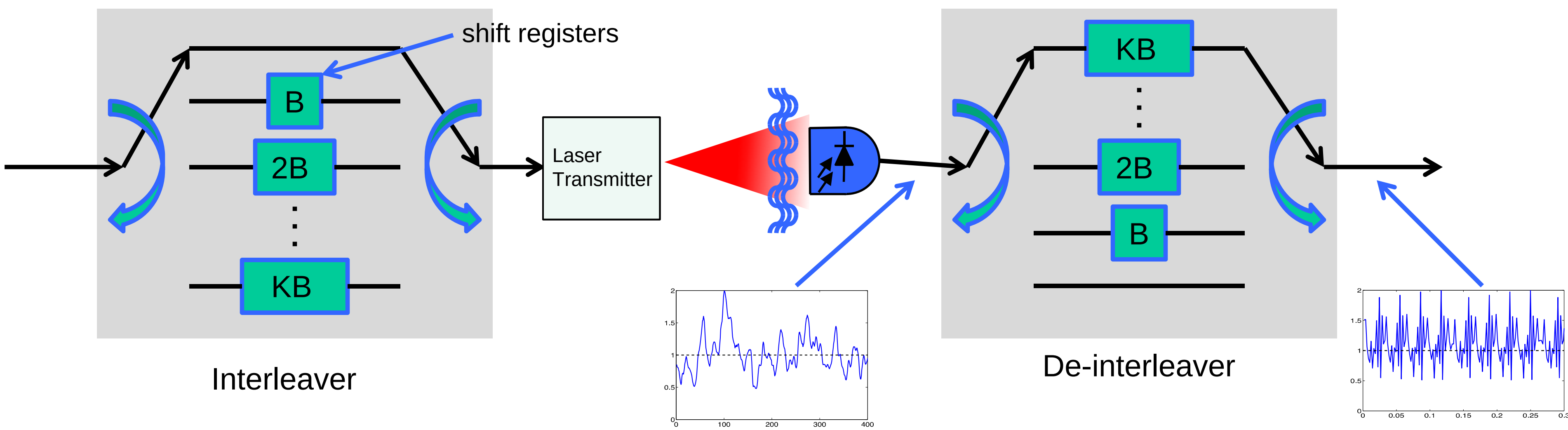
Approximate as Gaussian for large N, and apply linear approximation



$$\eta_{int,dB} \approx 16\frac{\sigma_I}{\sqrt{N_f}}$$

Interleaver Loss goes as the square root of the scintillation index

Interleaver memory requirements



- Convolutional interleaver achieves same spreading (N) as a block interleaver with half the memory

Memory (bits)

Data Rate (bits/s)

$$N_b \approx N R_b T_{coh} \text{ bits}$$

Fades/code word

Coherence Time (s)

- Example: to achieve N=100, with T_{coh}=10 msec, R_b=125 Mbps, requires a 125 Mbit interleaver.

$$\eta_{\mathrm{int}} \approx 16\sqrt{\frac{\sigma_I^2 R_b T_{coh}}{N_b}} \text{ dB}$$

Conclusions

- Free-space optical communication systems potentially gain many dBs over RF systems.
- There is no upper limit on the theoretically achievable photon efficiency when the system is quantum-noise-limited:
  – Intensity modulations plus photon counting can achieve arbitrarily high photon efficiency, but with sub-optimal spectral efficiency.
  – Quantum-ideal number states can achieve the ultimate capacity in the limit of perfect transmissivity.
- Appropriate error correction codes are needed to communicate reliably near the capacity limits.
- Poisson-modeled noises, detector losses, and atmospheric effects must all be accounted for:
  – Theoretical models are used to analyze performance degradations.
  – Mitigation strategies derived from this analysis are applied to minimize these degradations.